Indecisiveness, Undesirability and Overload Revealed Through Rational Choice Deferral

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Abstract

Three reasons why decision makers may defer choice are *indecisiveness* between feasible options, *unattractiveness* of these options and *choice overload*. This paper provides a choice-theoretic explanation for each of these phenomena by means of three deferral-permissive models of decision making that are driven by preference incompleteness, undesirability and complexity constraints, respectively. These models feature *rational* choice deferral in the sense that whenever the individual does choose an option from a menu, this is a most preferred option in that menu, so that choices are always WARP-consistent. The models also allow for the use of observable data to recover the individual's preferences and, where applicable, the indecisiveness and undesirability components of these preferences.

Keywords: Choice deferral; incomplete preferences; indecisiveness; unattractiveness; choice overload; revealed preference.

JEL Classification: D01, D03, D11

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1 Introduction

When presented with a menu of choice options such as retirement savings plans or durable consumer goods, a decision maker may decide to choose none. Experimental evidence suggests that some of the possible reasons for such behavior are (i) *decision conflict* that is caused by the agent's *indecisiveness* between the available options (Tversky and Shafir, 1992); (ii) *unattractiveness* of these options (Dhar and Sherman, 1996); and (iii) *choice overload* that ensues from the decision problem's complexity (Iyengar and Lepper, 2000). Intuition suggests that in many cases the agent may either return to the decision problem or seek alternative ones to choose from instead. Due to this generally dynamic nature of the decision process, and in line with the relevant literature in psychology, we will refer to an individual's decision to not choose from a menu as *choice deferral*.

In this paper we propose and axiomatically characterize three models of choice deferral, one for each of the three potential sources of this phenomenon that were mentioned above, i.e. indecisiveness, unattractiveness and choice overload. The common feature in all three models is that deferral is *rational* in the sense that the agent's choices are consistent with the Weak Axiom of Revealed Preference (WARP). Thus, deferral is not the result of some kind of inconsistency in the agent's underlying preferences. Indeed, whenever the agent does choose, he always does so by selecting an optimal feasible option according to his menu-independent preferences, similar to a standard utility maximizer. Moreover, in the tradition of revealed preference theory, all three models build on simple and easily falsifiable axioms on observable behavior. Therefore, an agent's conformity with either model allows an outside observer to recover (fully or, in one case, partially) the agent's preferences as well as other relevant entities such as his unattractiveness or complexity thresholds.

In the model of indecisiveness-driven deferral the agent is portrayed as having transitive but possibly incomplete preferences and as following the decision rule whereby an alternative is chosen from a menu if and only if it is preferred to all others in that menu. For this reason it is called the *maximally dominant choice* (MDC) model. Due to the incompleteness of the agent's preferences there are menus where no most preferred alternative exists, and these are precisely the ones where the agent defers. MDC is compatible with experimental evidence on conflict-induced deferral in binary menus. It also offers a dominance-based explanation of the choice overload effect, as well as of the breakdown of this effect.

The MDC model suggests a novel and generally applicable behavioral distinction between indifference and indecisiveness which complements the ones proposed by Eliaz and Ok (2006) and Mandler (2009). Specifically, an MDC agent is revealed to be indifferent between two options if and only if whenever one of these is chosen and the other is feasible, the latter is also chosen; and he is revealed to be indecisive between them if and only if neither is ever chosen in the presence of the other.

Due to the introspectively familiar idea that indecisiveness-driven deferral may be resolved over time, e.g. after suitable information acquisition, we also study a two-period extension of the MDC model in which the agent necessarily chooses an option from each menu in the second period, and does so according to some *completion* of his first-period preferences. We then apply this extension to the domain of choice under uncertainty and offer a novel choice-theoretic explanation and connection between the Bewley (2002) model of incomplete preferences under uncertainty and the Anscombe and Aumann (1963) model of subjective expected utility maximization, as well as a choice-theoretic interpretation of the objective vs subjective rationality model of Gilboa et al (2010).

Turning to the model of unattractiveness-driven deferral, the agent here is assumed to have complete and transitive preferences as well as a *desirability threshold*, which takes the form of some menu-independent alternative. The agent here behaves as an *undesirability-constrained utility maximizer* (UCUM) in the sense that he chooses his most preferred option at a menu if and only if this is strictly preferred to his desirability threshold, and defers otherwise. A novel concept of uniqueness is introduced in the context of this model, which clarifies that the agent's preferences are recoverable only up to (and excluding) his desirability threshold.

Finally, in the model of overload-driven deferral the decision maker behaves as a standard utility maximizer as long as the menu that he is faced with is considered to be "simple" according to his complexity criteria. The latter are captured by a pair that consists of a real-valued function on the set of menus and an integer, which acts as the agent's *complexity threshold*. The agent defers at a menu if and only if its complexity exceeds that threshold and chooses as a utility maximizer otherwise. This model is therefore referred to as *overload-constrained utility maximization* (OCUM). The complexity function in the OCUM model is permissive of various interpretations and functional forms, such as the number of alternatives in a menu or the time that is necessary for the agent to choose rationally from that menu. In the case where complexity coincides with cardinality, the agent's threshold can easily be recovered from observable behavior. However, unlike the previous two models, the agent's preferences here are only partially recoverable due to the fact that choices may not be made from some binary menus.

We emphasize that our analysis of choice deferral in this paper is limited to only three potential explanations of this phenomenon. There are many other sources of deferral for which we don't account. Some of those studied by economists include strategically-driven deferral that is analyzed in the consumer-search literature, as well as deferral for procedural reasons such as choice refusal considerations (Gaertner and Xu, 2004), bounded rationality (Manzini and Mariotti, 2014) or rational regret anticipation (Buturak and Evren, 2015).

2 Preliminaries

The set of all possible choice alternatives is X and is assumed finite. The set $\mathcal{M}$ denotes the collection of all nonempty subsets of X, which will be called *menus*. A choice correspondence C on $\mathcal{M}$ is a possibly multi-valued and empty-valued mapping that satisfies $C(A) \subseteq A$ for all $A \in \mathcal{M}$. Therefore, C is assumed to map $\mathcal{M}$ into $\mathcal{M}^* := \mathcal{M} \cup \{\emptyset\}$. As in Gerasimou (2015), our adopted interpretation of the situation in which $C(A) = \emptyset$ for some menu A is that the agent *defers choice* at A. Therefore, we will generally ***not assume*** the following property, which we treat explicitly as an axiom:

Nonemptiness
If $A \in \mathcal{M}$, then $C(A) \neq \emptyset$.

A binary relation $\succsim$ on X denotes a (possibly incomplete) *preorder* on X, i.e. a reflexive and transitive binary relation. When this relation is assumed complete, it will be referred to as a *weak order*. The set of greatest/maximum elements of $\succsim$ is defined and denoted by

$$B_{\succsim}(A) = \{x \in A : x \succsim y \text{ for all } y \in A\}.$$

This set is always nonempty when the preorder $\succsim$ is complete, but generally not otherwise.

3 Indecisiveness and Maximally Dominant Choice

3.1 Baseline Model of One-Shot Decisions

First, we analyze a model in which deferral is rooted in the agent's inability to compare some alternatives. Given the nature of deferral in this model, one may reason that such a decision maker may be able to eventually complete her preferences in the future, possibly after acquiring information about the alternatives, and then choose as a utility maximizer from the menus where he had originally deferred. In this section we focus on a static model where deferral is unresolved and leave this extension for the next section. The first choice axiom within this static model is the following:

Desirability
If $x \in X$, then $C(\{x\}) = \{x\}$.

This is compatible with the interpretation that whenever the decision maker is faced with only one alternative, the latter is sufficiently good for him to choose it. Under this interpretation, the axiom rules out unattractiveness of the alternatives as a potential reason for deferral.

WARP
If $x \in C(A)$, $y \in A \setminus C(A)$ and $y \in C(B)$, then $x \notin B$.

When Nonemptiness is also assumed, WARP can be written in a number of equivalent ways. Without it, however, such equivalences are no longer valid. The above statement of WARP is in the spirit of Samuelson's (1938) original version of the axiom: *If an alternative x is chosen over some other alternative y in some menu, then there is no menu where y is choosable and x is feasible.* The axiom's status as a core principle of choice consistency remains intact (if not strengthened) in an environment where deferral is permissible.

Intuitively, when the decision maker chooses x over y from a menu A even though she had the opportunity to defer, this may be a stronger indication that x is preferred to y relative to the case where choice from A was forced. To the extent that this is so, it becomes even more plausible to expect that a rational decision maker will not choose y from any menu where x is also feasible.

Strong Expansion
If $x \in C(A)$, $y \in A$ and $y \in C(B)$, then $x \in C(A \cup B)$.

To our knowledge, this axiom is novel. It is implied by WARP when Nonemptiness is assumed and is distinct from WARP otherwise. It strengthens the well-known "Expansion" or "Property γ" (Sen, 1971) axiom, which states that an alternative x that is choosable in both A and B is also choosable in $A \cup B$. Indeed, the proposed axiom requires that if x is chosen in the presence of y in some menu A and y is chosen in B, then x is chosen in $A \cup B$. This obviously reduces to Expansion in the special case where $x = y$. With regard to intuition, if a rational agent chooses x in the presence of y at menu A when deferral is possible, this suggests that he finds x at least as good as y and everything else in A. Likewise, the fact that y is chosen in B suggests that y is at least as good as everything else in B. Such an agent should therefore also consider x to be at least as good as everything else in $A \cup B$ and hence would choose x from this expanded menu, as required by the axiom. More compactly, Strong Expansion ensures that the individual's revealed preference relation is transitive.

Contraction Consistency
If $x \in C(A)$ and $x \in B \subset A$, then $x \in C(B)$.

This standard axiom is also weaker than WARP when Nonemptiness is assumed, but logically distinct from it otherwise. For example, $x \in C(A)$, $x \in B \subset A$ and $C(B) = \varnothing$ is consistent with WARP but violates Contraction Consistency. However, ruling out this kind of behavior is normatively appealing. Indeed, when choice reveals preference, x being choosable at A suggests that x is at least as good as every other alternative in A, hence in $B \subset A$ too. Therefore, to allow for the possibility that nothing is chosen from B amounts to saying that the most preferred option in the menu is not chosen, which is irrational.

Proposition 1
The following are equivalent for a choice correspondence $C : \mathcal{M} \to \mathcal{M}^*$:
1. C satisfies Desirability, WARP, Contraction Consistency and Strong Expansion.
2. There exists a unique preorder $\succsim$ on X such that

$$C(A) = \begin{cases} \varnothing, & \text{iff } B_{\succsim}(A) = \varnothing \\ B_{\succsim}(A), & \text{otherwise} \end{cases} \tag{1}$$

We will refer to (1) as the model of *maximally dominant choice (MDC)*. This model portrays a cautious decision maker who chooses if and only if he can find a most preferred option in a menu and defers otherwise. In so doing, such an agent makes fully consistent choices across menus and therefore is fully hedged against manipulations of a money-pump kind (Mandler, 2009; Danan, 2010).

The model's predictions are compatible in a straightforward way with experimental findings on choice from conflict-inducing multi-attribute alternatives such as those reported by Tversky and Shafir (1992). These findings suggest that, given two alternatives x and y that dominate each other in some important attribute, many people are willing to choose x when x is the only feasible option and y when y is the only feasible option, but nothing when both x and y (and only them) are feasible. If preferences are let to coincide with the usual partial ordering on attribute space, then one would have $x \succsim x$, $y \succsim y$ and $x \not\succsim y$, $y \not\succsim x$, in which case (1) would indeed predict $C(\{x\}) = \{x\}$, $C(\{y\}) = \{y\}$ and $C(\{x, y\}) = \varnothing$, consistent with the findings.

Conceptually closest to the MDC model are the *Conflict Decision Avoidance* (CDA) procedure (Dean, 2008), and the *Extended Partial Dominance* (EPD) procedure (Gerasimou, 2015). Following the notation introduced in Masatlioglu and Ok (2005), in both these papers the domain of choice includes decision problems of the form (A, s) and $(A, \diamond)$. A decision problem (A, s) consists of a menu A and an alternative $s \in A$, which is interpreted as the problem's status quo and takes the form of an alternative like every other in that menu. By contrast, a decision problem $(A, \diamond)$ is one without a status quo (Masatlioglu and

Ok, 2005) or with a "non-explicit" status quo whose nature is such that it cannot be thought of as being an element of A (Gerasimou, 2015).

In the CDA model the agent has an incomplete preference relation $\succsim$ and chooses a maximizer of this relation as in (1) if one exists. If not, and he is faced with a problem (A, s), then he chooses the status quo s if no alternative "better" than s is available in A (where "better" here is captured by a correspondence that is distinct from the agent's preferences), and he chooses an option that maximizes a *completion* of $\succsim$ otherwise (the latter is also the case when he is faced with a problem $(A, \diamond)$). The main differences between the MDC and CDA model is that the former restricts attention to decision problems where inaction is associated with *choice deferral* and **not** with status quo maintenance as defined above, and that MDC obeys WARP whereas CDA does not. As a result of the first difference, the MDC model cannot explain the *status quo bias* phenomenon (Samuelson and Zeckhauser, 1988), whereas CDA can. On the other hand, choice deferral is not allowed in the CDA model, as the agent chooses an option from A when he is faced with $(A, \diamond)$. Finally, the EPD model features WARP-inconsistent choices that are based on the selection criteria of total undomination *and* partial dominance, which again are defined in terms of an incomplete preference relation. That model, however, allows for a behavioral distinction between inaction that results to choice deferral vs status quo bias, and is compatible with both phenomena.

3.1.1 Revealed Preference, Indifference and Indecisiveness

The MDC model suggests a purely behavioral criterion to disentangle indifference and indecisiveness from a given set of decision observations that are consistent with it. Specifically, for an agent who has generated such data the psychological state concerning two alternatives x and y is

indifference *iff for every menu $A \in \mathcal{M}$ such that $x, y \in A$, it holds that $y \in C(A)$ whenever $x \in C(A)$;*

indecisiveness *iff for every menu $A \in \mathcal{M}$ such that $x, y \in A$ it holds that $x, y \notin C(A)$;*

preference (for x over y) *iff there exists a menu $A \in \mathcal{M}$ such that $x \in C(A)$ and $y \in A \setminus C(A)$.*

As in Eliaz and Ok (2006), and consistent with any model of rational choice, the agent here is revealed to be indifferent between x and y if and only if both options are always choosable whenever both are feasible and one is choosable. However, unlike the Eliaz-Ok criterion for revealed indecisiveness which necessitates one alternative to be chosen over the other in some menu and is therefore associated with a violation of WARP, the agent in the MDC model *is revealed to be indecisive between x and y if and only if neither of these options is ever chosen in the presence of the other*. This novel criterion is the first to allow for a distinction between revealed indifference and indecisiveness that applies to WARP-consistent agents with incomplete preferences. Moreover, unlike Eliaz and Ok (2006), no a priori restrictions on the agent's preferences are necessary for this distinction to be made (a condition called "regularity" must be satisfied by the agent's preferences in the Eliaz-Ok model).

Mandler (2009) proposed a distinction between the two concepts that is based on data from sequential pairwise trades of options that are not ranked by strict preference. Specifically, if such sequential trades eventually result in the decision maker owning an alternative that is either strictly better or strictly worse to the one he started off with, then Mandler's criterion suggests that he must have been indecisive at some point along the sequence of pairs that were involved in the trades. On the other hand, in the absence of such a strict preference ranking it cannot be ruled out that the agent was indifferent throughout. Clearly, the MDC distinction between indifference and indecisiveness is compatible with Mandler's distinction because the revealed indifference relation is transitive whereas the incomparability relation is generally not.

An interesting relationship exists between the properties of the MDC model and the revealed-preference analysis in Bernheim and Rangel (2009). The authors proposed that, in a model-free world, an alternative x may be thought of as being strictly preferred to another alternative y if there is no menu in which y is chosen and x is feasible. In the MDC model, x is revealed preferred to y if, in addition to this, there is a menu in which x is chosen *over* y. Thus, although the Bernheim-Rangel condition is necessary for x to be revealed preferred to y under the MDC model, it is not sufficient. In fact, as is evident from the preceding discussion, this condition is also necessary for x to be revealed incomparable to y in the MDC model. Interestingly, however, the *absence* of a revealed preference relation à la Bernheim and Rangel between x and y exactly coincides with the revealed incomparability relation of the MDC model, as can be easily verified.

3.1.2 A Dominance-Based Explanation of the "Choice-Overload" Phenomenon

The MDC model is also compatible with evidence suggesting the occurrence and disappearance of the so called *"choice overload"* effect. This refers to the phenomenon whereby decision makers defer significantly more often when faced with large/complex menus that contain many alternatives than when they are faced with smaller ones. The first evidence for this effect came from the field and was reported in Iyengar and Lepper (2000). One of the explanations that have been proposed is that deferral in large menus is caused by the lack of familiarity and the absence of a most preferred option (Iyengar and Lepper, 2000; Scheibenhenne, Greifeneder, and Todd, 2010). Intuitively, the higher the degree of incompleteness in the agent's preferences, the more likely it is that, as the size of the menu increases, the number of incomparable pairs will become so large that no alternative is preferred to all others. This explanation is compatible with (1). The model is also compatible with the explanation of how the effect breaks down when a dominant option is added to a large menu at which choice would have otherwise been deferred (Scheibenhenne *et al*, 2010). Indeed, as explained above, this happens when $C(A) = \varnothing$, regardless of the size of A, and when an alternative x is added to A such that $x \succsim y$ for all $y \in A$, in which case $C(A \cup \{x\}) = \{x\}$. Thus, Proposition 1 offers incompleteness-driven theoretical predictions for both the occurrence and the disappearance of the choice overload effect.

3.2 Sequential Choice and the Resolution of Indecisiveness

Intuition suggests that indecisiveness between alternatives can be resolved over time, after reflection or suitable information acquisition. As a result, choice deferral that is driven solely by indecisiveness should vanish too. To model this, our primitive in this subsection is a *pair* of choice correspondences $C_1, C_2 : \mathcal{M} \to \mathcal{M}^*$. We interpret C_i as capturing the agent's behavior in period $i = 1, 2$. Given that the domain of both C_1 and C_2 is the same collection of menus, and each menu in this collection represents a decision problem, we are implicitly assuming that first-period problems neither disappear nor change in the second period.

The axioms that follow impose some structure on the pair C_1, C_2 and are mainly normative.

Eventual Nonemptiness
If $A \in \mathcal{M}$, then $C_2(A) \neq \varnothing$.

This axiom precludes indefinite deferral by requiring the decision maker to choose an alternative from every menu in the second period, regardless of whether he initially deferred at this menu or not. This restriction is relevant in decision problems where the individual must ultimately make a choice, e.g. when deciding among job offers. When the agent defers at some menu A initially but chooses from it eventually, we can think of this choice as being the result of the agent receiving additional information about the alternatives in A during the interim stage through some unmodelled process.

Sequential Choice Consistency
If $x \in C_1(A)$, then $x \in C_2(A)$.

This axiom requires the agent's second-period choice at every menu A to be consistent with the choice (if any) that would have been made by him had he faced that menu in the first period.

Proposition 2
The following are equivalent for two choice correspondences $C_1, C_2 : \mathcal{M} \to \mathcal{M}^*$:
1. C_1 and C_2 satisfy Desirability, WARP, Contraction Consistency, Strong Expansion, Eventual Nonemptiness and Sequential Choice Consistency.
2. There exists a unique preorder $\succsim_1$ and a completion $\succsim_2$ of this preorder such that, for all $A \in \mathcal{M}$

$$C_i(A) = \{x \in A : x \succsim_i y \text{ for all } y \in A\}, \qquad i = 1, 2. \tag{2}$$

This can be interpreted as a two-period model in which deferral is permissible in the first period but a choice is necessarily made in the second. Moreover, the choice that is made in the second period is by maximization of the agent's first-period preferences which have meanwhile been completed through some unmodelled process (e.g. via information acquisition).

3.2.1 Application: Indecisiveness and deferral under uncertainty

A specific domain in which the sequential MDC model of Proposition 2 is potentially illuminating is that of choice under uncertainty. In this domain, the finite set of outcomes X is coupled by a pair (S, Σ), where S is a set of possible states of the world and Σ a σ-algebra of events derived from S. Moreover, the objects of choice are Anscombe-Aumann (1963) acts that belong to the set $\mathcal{F}$ of all Σ-measurable functions mapping S into $\Delta(X)$, the latter being the set of probability distributions on X. Let $\mathcal{F}$ be endowed with a suitable topology and let $\mathcal{D}$ denote the set of all nonempty convex, compact subsets of $\mathcal{F}$, with $\mathcal{D}^* := \mathcal{D} \cup \{\emptyset\}$. Finally, assume that the choice correspondences C_i, $i = 1, 2$, map $\mathcal{D}$ into $\mathcal{D}^*$.

The benchmark model of incomplete preferences under uncertainty is Bewley (2002). In the weak-preference analogue of this model that was axiomatized in Ghirardato et al (2003) and Gilboa et al (2010), the decision maker is portrayed as having preferences over $\mathcal{F}$ that are captured by a preorder $\succsim$ (which is necessarily complete only in the subdomain of constant acts), and also as having beliefs over S that are captured by a non-singleton closed, convex set Π of probability measures on (S, Σ). In the Bewley model the agent compares two acts f and g by the following, partially applicable rule:

$$f \succsim g \iff \int_S \mathbb{E}_{f(s)} u d\pi(s) \geq \int_S \mathbb{E}_{g(s)} u d\pi(s) \qquad \text{for all } \pi \in \Pi. \tag{3}$$

Here, $u : X \to \mathbb{R}$ is a von Neumann-Morgenstern utility function, which exists due to the completeness of $\succsim$ in the subdomain of constant acts, $\Delta(X)$, while $\mathbb{E}_{f(s)} u = \sum_{x \in X} f(s)(x)u(x)$.

Being derived from a model of *preference* and not *choice* under uncertainty, the Bewley rule does not directly specify what the agent's decision is in those cases where no feasible act is preferred to all others in a given menu. However, Bewley (2002), followed by Mandler (2004), Rigotti and Shannon (2005) and Masatlioglu and Ok (2005), among others, argued that the agent *preserves the status quo* unless he can find an act that is preferred to it. As we also argued in Gerasimou (2015), there are cases in which the status quo does indeed take the form of a feasible option just like all others, and hence it is potentially comparable with these other options so that this rule can be applied. Choosing from a set of insurance policies when already endowed with one of them is an example of such a problem. Yet, there are also cases where the status quo does not take this form. An example may be the problem of choosing an insurance policy when not already endowed with one. Assuming that every option is acceptable in principle, the status quo here does not actually enter the decision problem. One way forward for the decision maker here is to choose among the preference-undominated acts, as studied in Stoye (2015). Another possibility that we analyzed in Gerasimou (2015) is that the lack of a partially or totally dominant option in such cases can lead to *choice deferral*.

In light of the above, we can use Proposition 2 to provide an alternative choice-theoretic interpretation of the Bewley model for the case of decision problems in which the original Bewley decision rule may be inapplicable due to the different nature of the status quo and where the agent is unwilling to choose an option that is merely undominated. Specifically, given the above primitives one can model an agent faced with a menu A in $\mathcal{D}$ as possibly deferring à la Bewley (2002) in the first period and choosing as a subjective expected utility maximizer à la Anscombe and Aumann (1963) in the second period, as follows:

$$C_1(A) = \bigcap_{\pi \in \Pi} \arg\max_{f \in A} \int_S \mathbb{E}_{f(s)} u d\pi(s) \tag{4a}$$

$$C_2(A) = \arg\max_{f \in A} \int_S \mathbb{E}_{f(s)} u dp(s). \tag{4b}$$

Under (4) the individual chooses a most preferred act in the first period if and only if one exists; when such an act does not exist, choice is deferred to the second period, by which time he has narrowed down his beliefs to a *single* prior p from the original set Π, and chooses as a subjective expected utility maximizer according to the same, first-period utility function u that dictates his unchanged tastes.

In the same vein, one can use Proposition 2 to provide a choice-theoretic foundation and reinterpretation of the objective- subjective rationality model of preferences under uncertainty that was axiomatized in Gilboa et al (2010) by attaching a temporal dimension to the associated objective and subjective preference relations. Specifically, there is a pair $\succsim_1$, $\succsim_2$ of preference relations on $\mathcal{F}$, the former of which is incomplete à la Bewley and the second is complete and ambiguity-averse à la Gilboa and Schmeidler (1989). The authors provided necessary and sufficient conditions on this pair of relations so that there exist a common set of priors Π and utility function u such that the former is represented by this pair as in (3) and the latter according to the maxmin rule

$$f \succsim_2 g \iff \min_{\pi \in \Pi} \int_S \mathbb{E}_{f(s)} u d\pi(s) \geq \min_{\pi \in \Pi} \int_S \mathbb{E}_{g(s)} u d\pi(s). \tag{5}$$

Moreover, in this model $\succsim_2$ is a completion of $\succsim_1$, as in our Proposition 2. Similar to the case above, when faced with menu A in $\mathcal{D}$ the agent decides as follows:

$$C_1(A) = \bigcap_{\pi \in \Pi} \arg\max_{f \in A} \int_S \mathbb{E}_{f(s)} u d\pi(s) \tag{6a}$$

$$C_2(A) = \arg\max_{f \in A} \min_{\pi \in \Pi} \int_S \mathbb{E}_{f(s)} u d\pi(s) \tag{6b}$$

The difference between (4) and (6) lies in the second-period choice criterion, where subjective expected utility maximization in (4b) has been replaced by *maxmin* subjective expected utility maximization in (6b). We note that a similar choice-theoretic interpretation can also be given to the dual-preference model of Kopylov (2009).

3.3 Rational Choice as a Special Case

When preferences are complete in the MDC model of Proposition 1, the latter clearly reduces to utility maximization. From the axiomatic point of view this is equivalent to Nonemptiness being satisfied on top of the other axioms. In this case, Desirability as well as Contraction Consistency and Strong Expansion follow directly from Nonemptiness and from Nonemptiness together with WARP, respectively.

The result below provides a novel decomposition of Nonemptiness + WARP that characterize utility maximization in this context. This decomposition is in terms of the other consistency axioms that are involved in Proposition 1. Before stating the result we recall that a choice correspondence C satisfies *Expansion Consistency* or *Sen's* β if $x, y \in C(A)$, $B \supset A$ *and* $x \in C(B)$ *implies* $y \in C(B)$.

Proposition 3
The following are equivalent for a choice correspondence $C : \mathcal{M} \to \mathcal{M}^*$*:*
1. C satisfies Nonemptiness and WARP.
2. C satisfies Nonemptiness, Contraction Consistency and Expansion Consistency.
3. C satisfies Nonemptiness, Contraction Consistency and Strong Expansion.
4. There exists a unique weak order $\succsim$ *on X such that, for all* $A \in \mathcal{M}$*,*

$$C(A) = B_{\succsim}(A). \tag{7}$$

The novel statement in Proposition 3 is the third one and clarifies that Expansion Consistency and Strong Expansion are equivalent under Nonemptiness and Contraction Consistency, and that either of these axiomatic combinations characterizes rational choice. The equivalence between the first and fourth statement is due to Arrow (1959), while that between the second and fourth is due to Sen (1971). Proposition 3 is clearly a corollary to Proposition 1.

4 Desirability-Constrained Utility Maximization

A second potential source of choice deferral is inferiority of all feasible alternatives relative to a desirability threshold that is specified by the agent. We will refer to this kind of deferral as *undesirability-* or *unattractiveness-driven*.

One condition that must clearly be satisfied for the observed deferral to be of this kind is the following:

Undesirability
There exists $x \in X$ *such that* $C(\{x\}) = \varnothing$.

Indeed, if an option x is considered not to be "good enough", then it is intuitive that the decision maker will not choose it whenever this is the only feasible option. In contract theory the Undesirability axiom manifests itself through the agent's participation constraint: unless a contract's expected utility exceeds the agent's reservation utility, the agent will not choose it even if it's the only feasible one. For experimental evidence related to this axiom the reader is referred to Zakay (1984) and Mochon (2013).

Contractive Undesirability
If $C(A) = \varnothing$ and $B \subset A$, then $C(B) = \varnothing$.

This condition too is clearly necessary to model unattractiveness-driven deferral. Intuitively, if an individual considers nothing to be sufficiently good in some menu A, then this must also be true in every submenu B of A, as required by Contractive Undesirability. This is in sharp contrast to the MDC model of indecisiveness-driven deferral where removing an option from a menu at which no choice was made may result in a dominant feasible option to arise and therefore in a choice to be made.

We will write that a preference relation $\succsim$ on X ***quasi-rationalizes a choice correspondence*** $C : \mathcal{M} \to \mathcal{M}^*$ if $C(A) = B_{\succsim}(A)$ whenever $C(A) \neq \varnothing$. We will also write that a weak order $\succsim$ quasi-rationalizes C ***uniquely up to an element*** $x^* \in X$ if for any other weak order $\succsim'$ on X where $x \succsim' y$ holds whenever $x \succsim y$ and $x, y \succsim x^*$, the weak order $\succsim'$ also quasi-rationalizes C.

Proposition 4
The following are equivalent for a choice correspondence $C : \mathcal{M} \to \mathcal{M}^*$:
1. C satisfies WARP, Contraction Consistency, Undesirability and Contractive Undesirability.
2. There exists a weak order $\succsim$ on X and an element $x^* \in X$ such that, for all $A \in \mathcal{M}$,

$$C(A) = \begin{cases} \varnothing, & \text{iff } z \in B_{\succsim}(A) \text{ implies } x^* \succ z \\ & \\ B_{\succsim}(A), & \text{otherwise} \end{cases} \tag{8}$$

Moreover, $\succsim$ quasi-rationalizes C uniquely up to x^*.

In this model the agent behaves as if he had complete and transitive preferences over the set X and as making decisions like a utility maximizer as long as his most preferred feasible option in a given menu is strictly preferred to some pre-specified alternative x^*. The latter sets his *undesirability threshold* in the sense that the agent defers choice from a menu if and only if the best alternative in the menu is inferior to x^*. Since the behavior captured in (8) deviates from rational choice only in that some alternatives are never chosen because they are undesirable, we will refer to it as the model of *undesirability-constrained utility maximization (UCUM)*. Given the close link between this model and rational choice it is not surprising that WARP and Contraction Consistency are satisfied here too, as Proposition 4 clarifies.

With regard to preference revelation in the UCUM model, quasi-rationalizability of the choice correspondence C by the weak order $\succsim$ uniquely up to x^* suggests that such revelation is possible for the part of the agent's preference order that ends on the (possibly degenerate) indifference set to which alternative x^* belongs. This part of the agent's preferences is completely and uniquely recoverable from his choices. The "usefulness" of his deferring behavior in this regard is limited to the identification of those alternatives in X that are undesirable in the sense that they will never be chosen. Clearly, the agent's preference ranking of these alternatives cannot be recovered using decision data. This is the reason for the potential multitude of weak orderings that are compatible with C in the quasi-rationalizability sense that is alleged in Proposition 4. Finally, given the lack of choice data that might indicate a preference between such options, there is no way for an outside observer to know that the agent can in fact rank these options in the first place. Proposition 4 merely states that such a ranking is possible.

5 Overload-Constrained Utility Maximization

The last potential reason for choice deferral that we study in this paper is rooted in the decision maker's limited cognitive or time resources, which may urge him to defer choice in decision problems that he does not consider "easy". We will refer to this third source of deferral as *choice overload*. In Section 3.1.2 we argued that the MDC model of indecisiveness-driven deferral is compatible with one of the suggested explanations for this phenomenon. Here we propose a different explanation that builds on a model of utility maximization constrained by *complexity thresholds*.

In addition to some of the axioms that have already been presented, this model necessitates explicit statement of the following standard condition:

Binary Choice Consistency
If $x \in C(\{x, y\})$ and $y \in C(\{y, z\})$, then $x \in C(\{x, z\})$.

The central property in the model that we introduce in this section is the following:

Overload Monotonicity
If $C(A) = \emptyset$ and $B \supset A$, then $C(B) = \emptyset$.

This axiom suggests that if the individual finds a menu A to be complex, he also finds every menu $B \supset A$ to be complex, so that overload is monotonic with respect to set inclusion. While the intuitive appeal of this axiom is obvious at one level, we note that the axiom can fail descriptively when insertion of a clearly superior alternative in a complex menu may override the complexity of the decision problem and nudge the individual towards that alternative. On the other hand, one may argue that the axiom is "normatively appealing" for a cognitive- or resource-constrained decision maker on the grounds that the complexity criterion that such an individual may employ incorporates a break-even cost-benefit analysis, so that once a menu is complex according to this criterion, then any menu that includes it must be at least as complex even if it contains stand-out options, possibly because their discovery may be too costly.

Proposition 5
The following are equivalent for a choice correspondence $C : \mathcal{M} \to \mathcal{M}^*$:
1. C satisfies Desirability, WARP, Overload Monotonicity and Binary Choice Consistency.
2. There exist a unique preorder $\trianglerighteq$ on X, a completion $\succsim$ of this preorder, a function $\psi : \mathcal{M} \to \mathbb{R}$ and an integer n such that, for all $A, B \in \mathcal{M}$ and all $x, y, z \in X$

$$C(A) = \begin{cases} \emptyset, & \textit{iff } \psi(A) > n \\ B_{\succsim}(A), & \textit{otherwise} \end{cases} \tag{9a}$$

$$\psi(\{x,y\}) \leq n \;\;\&\;\; \psi(\{y,z\}) \leq n \quad \Longrightarrow \quad \psi(\{x,z\}) \leq n \tag{9b}$$

$$\psi(\{x\}) \leq n \tag{9c}$$

$$B \supset A \quad \Longrightarrow \quad \psi(B) \geq \psi(A). \tag{9d}$$

This model portrays the decision maker as a utility maximizer who is non-standard in that he employs a complexity/overload criterion that determines whether he will engage in utility maximization at some menu or whether he will *defer* at that menu instead. It will therefore be referred to as the *Overload-Constrained Utility Maximization (OCUM)* model. Specifically, the individual's complexity criterion here is captured by the *complexity function* ψ and *complexity threshold* n. These may correspond to the number of elements in a menu and a cut-off menu size, respectively, or to the time it takes the decision maker to make a choice from a menu and the total time he has available. Whichever the case, (9a) suggests that the agent defers when and only when the decision problem is sufficiently complex according to the pair (ψ, n).

It follows directly from (9b) that the complexity function is increasing with respect to menu inclusion, which is a consequence of the Overload Monotonicity axiom. It also follows from (9a) and (9c) that a decision problem consisting of a singleton menu is never complex. Finally, (9d) suggests that if the binary menus $\{x,y\}$ and $\{y,z\}$ are not complex, then the same will be true for menu $\{x,z\}$. When ψ is the cardinality function, for example, then non-complexity of one binary menu implies that all such menus must also be non-complex, consistent with (9d).

Unlike the models of indecisiveness- and undesirability-driven deferral that were presented above and where the agent's preferences are fully and almost fully recoverable from decision data, respectively, in the OCUM model such recovery is only partially possible. Indeed, behavior compatible with the four axioms of Proposition 5 uniquely pin down a generally incomplete preference relation $\trianglerighteq$ which is defined by the rule $x \trianglerighteq y$ iff $x \in C(A)$ and $y \in A$ for some menu A. Given the complexity function ψ and its associated threshold n there may be alternatives x and y for which $\psi(\{x,y\}) > n$ and $x, y \notin C(A)$ for every menu A with $\psi(A) \leq n$. For such alternatives it holds that $x \ntrianglerighteq y$ and $y \ntrianglerighteq x$. While the model is well-defined in terms of a completion of this preorder, since completions are generally not unique this fact restricts the extent to which preferences can be recovered in the OCUM model.

In the special case where the complexity function ψ captures the number of alternatives in a menu, i.e. where $\psi(A) = |A|$ for all A, the model suggests a second explanation of the choice overload phenomenon that was first discussed in Section 3.1.2. Specifically, in this case the model's interpretation is that the decision maker always counts the number of alternatives in a menu and if this exceeds his pre-specified menu-size threshold n, he defers. This prediction is compatible with the choice-overload findings in Iyengar and Lepper (2000). Unlike the explanation of this effect that is suggested by the MDC model, however, the OCUM model cannot account for the observed breakdown of choice overload when a clearly dominant option is added to a complex menu.

6 Between the Three Models

Table 1 illustrates the axiomatic similarities and differences between the three proposed models of rational choice deferral by checking conformity of each model with each of the axioms that were presented above (excluding those of Section 3.2).

Table 1: Axiom Compatibility of the Three Models of Rational Choice Deferral

	Indecisiveness (MDC)	**Unattractiveness (UCUM)**	**Overload (OCUM)**
WARP	YES	YES	YES
Contraction Consistency	YES	YES	YES
Strong Expansion	YES	YES	NO
Desirability	YES	NO	YES
Undesirability	NO	YES	NO
Contractive Undesirability	NO	YES	NO
Binary Choice Consistency	YES	YES	YES
Overload Monotonicity	NO	NO	YES

In all three models the decision maker's choices satisfy WARP, Contraction Consistency and Binary Choice Consistency. In particular, since the OCUM model predicts $C(B) \neq \emptyset$ whenever $C(A) \neq \emptyset$ and $B \subset A$, it is trivially compatible with Contraction Consistency. Compatibility of the MDC model with Binary Choice Consistency is also obvious in view of the fact that preferences in this model are transitive. To see that this axiom is also compatible with the UCUM model observe that $x \in C(\{x, y\})$ and $y \in C(\{y, z\})$ here suggest that both x and y are above the desirability threshold. Moreover, since preferences are transitive and the "desirable" option x is preferred to z, we also have $x \in C(\{x, z\})$, as required.

The Desirability axiom is clearly satisfied in the MDC and OCUM models (in the case of the latter this is due to (9c)), while its logical complement, Undesirability, is satisfied by the UCUM model only. It is intuitive that Contractive Undesirability too is satisfied only by the latter model. Indeed, moving from A where $C(A) = \emptyset$ to $B \subset A$ in the MDC model can render some option in B a most preferred one in that menu, and may therefore result in $C(B) \neq \emptyset$. In the OCUM model, one may have $\psi(A) > n$ but $\psi(B) \leq n$, in which case too one obtains $C(B) \neq \emptyset$.

Like MDC, the UCUM model is compatible with Strong Expansion. The logic behind this is analogous to the logic that shows conformity of this model with Binary Choice Consistency. On the other hand, OCUM may lead to violations of Strong Expansion. For example, if $\psi(\cdot) = |\cdot|$ and $n = 2$ we would have $x \in C(\{x, y\})$, $y \in C(\{x, z\})$ and $C(\{x, y, z\}) = \emptyset$.

Finally, neither MDC nor UCUM obey Overload Monotonicity in general. In the former case this can happen when $C(A) = \emptyset$ and the menu $B \supset A$ includes an option $x \in B \setminus A$ that is weakly preferred to all other options in B. In the latter case we have $C(A) = \emptyset$ when A consists of "undesirable" options, and $C(B) \neq \emptyset$ for every $B \supset A$ that contains at least one "desirable" alternative.

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Appendix 1: Proofs

Proof of Proposition 1:

This is included in the proof of Proposition 2 below, hence omitted. ∎

Proof of Proposition 2:

We only prove that 1 ⇒ 2 (the proof of the converse implication is straightforward). Let C_1, C_2 satisfy the axioms and define the relation $\succsim_1$ on X by $x \succsim_1 y$ if there exists $A \in \mathcal{M}$ such that $x \in C_1(A)$ and $y \in A$. Since all singletons are included in $\mathcal{M}$, it follows from Desirability that $\succsim_1$ is reflexive. Suppose now that $x \succsim_1 y$ and $y \succsim_1 z$ for some $x, y, z \in X$. There exist $A, B \in \mathcal{M}$ such that $x \in C_1(A)$, $y \in A$ and $y \in C_1(B)$, $z \in B$. From Strong Expansion, $x \in C_1(A \cup B)$. Since $\{x, y, z\} \in \mathcal{M}$ by assumption and $\{x, y, z\} \subseteq A \cup B$, it follows from Contraction Consistency that $x \in C_1(\{x, y, z\})$. Hence, $x \succsim_1 z$. This shows that $\succsim_1$ is transitive, hence a preorder.

We must first prove that

$$C_1(A) = \{x \in A : x \succsim_1 y \text{ for all } y \in A\}. \tag{10}$$

Suppose that (10) is not satisfied. Assume that there is $A \in \mathcal{M}$ such that $x \in A$, $x \succsim_1 y$ for all $y \in A$ and $x \notin C_1(A)$. If $y \in C_1(A)$ for some $y \neq x$, then this fact and $x \succsim_1 y$ together contradict A1. Suppose $C_1(A) = \emptyset$ instead. The postulate that $x \succsim_1 y$ for all $y \in A$ is equivalent to the postulate that for each $y \in A$ there exists $B_y \in \mathcal{M}$ such that $y \in B_y$ and $x \in C_1(B_y)$. Let $S := \bigcup\{B_y : y \in A\}$. Clearly, $A \subseteq S$. Moreover, the full domain assumption ensures that $S \in \mathcal{M}$. Repeated application of Strong Expansion gives $x \in C_1(S)$. Since $A \subseteq S$, $x \in A$ and $A \in \mathcal{M}$, it follows from A3 that $x \in C_1(A)$. Conversely, if $x \in C_1(A)$ and $y \in A$, then, by definition of $\succsim_1$, $x \succsim_1 y$. Thus, (10) holds. Uniqueness of $\succsim_1$ follows from the fact that all binary menus are included in $\mathcal{M}$.

Now define $\succsim_2$ on X by $x \succsim_2 y$ if there is $A \in \mathcal{M}$ such that $x \in C_2(A)$ and $y \in A$. We know from Arrow (1959) that, since Eventual Nonemptiness and WARP hold, $\succsim_2$ is a weak order that satisfies

$$C_2(A) = \{x \in A : x \succsim_2 y \text{ for all } y \in A\}. \tag{11}$$

Finally, from Sequential Choice Consistency we have $x \succsim_2 y$ whenever $x \succsim_1 y$. Hence, $\succsim_2$ is indeed a completion of $\succsim_1$. ∎

Proof of Proposition 3

We will show that 2 ⇔ 3. For the ⇒ direction, suppose $x \in C(A)$, $y \in A$ and $y \in C(B)$. Assume to the contrary that $x \notin C(A \cup B)$. Let $z \in C(A \cup B)$. From CC we have $z \in C(\{x, z\})$ and $z \in C(\{y, z\})$. Consider first the case where $y \in C(A \cup B)$. CC implies $C(\{y, z\}) = \{y, z\}$ and $C\{x, y\} = \{x, y\}$. Since $A \cup B \supset \{x, y\}$, the latter fact and EC together with $y \in C(A \cup B)$ imply $x \in C(A \cup B)$, a contradiction. Now consider the case where $y \notin C(A \cup B)$. If $z \in A$, then $x \in C(A)$ and CC imply $C(\{x, z\}) = \{x, z\}$. EC now again implies $x \in C(A \cup B)$, a contradiction. If $z \in B$, then $y \in C(B)$ and CC imply $C(\{y, z\}) = \{y, z\}$. EC again implies $y \in C(A \cup B)$, a contradiction.

For the converse implication, suppose $x, y \in C(A)$, $B \supset A$ and $x \in C(B)$. We must show $y \in C(B)$. Since $x, y \in C(A)$, Contraction Consistency implies $C(\{x, y\}) = \{x, y\}$. Since $y \in C(\{x, y\})$ and $x \in C(B)$, it follows from Strong Expansion and $y \in B$ that $y \in C(B)$. ∎

Proof of Proposition 4:

Necessity of the axioms is straightforward to check. We prove sufficiency. First, we show that C satisfies Strong Expansion. Suppose not. We have $x \in C(A)$, $y \in A$, $y \in C(B)$ and $C(A \cup B) = \emptyset$ (WARP is violated if $x \notin C(A \cup B) \neq \emptyset$). It follows from Contractive Undesirability and $C(A \cup B) = \emptyset$ that $C(A) = C(B) = \emptyset$, a contradiction.

Now let $X^* := \{x \in X : x \in C(A) \text{ for some } A \in \mathcal{M}\}$. Let $Y = X \setminus X^*$. From Undesirability, $Y \neq \emptyset$. Define the relation $\triangleright$ on X by $x \triangleright y$ if $x \in C(A)$, $y \in A$ for some $A \in \mathcal{M}$, and the relation $\succsim^*$ by

$$\succsim^* = \triangleright \cup \{(x,x) : x \in Y\}.$$

In view of Contraction Consistency and the definition of $\succsim^*$, this relation is reflexive. We will also show that the restriction of $\succsim^*$ on X^* is complete and transitive, hence a weak order. Consider $x, y, z \in X^*$ and suppose $x \succsim^* y \succsim^* z$. Applying the same argument with Strong Expansion that was used in the proof of Proposition 2 shows that $x \succsim^* z$, which establishes transitivity. Now suppose $x \not\succsim^* y$ and $y \not\succsim^* x$ for some $x, y \in X^*$. It follows that $C(\{x,y\}) = \emptyset$. From Contractive Undesirability we then obtain $C(\{x\}) = C(\{y\}) = \emptyset$. Since $x \succsim^* y$ and $y \succsim^* z$, this contradicts Contraction Consistency and the definition of $\succsim^*$. Therefore, $\succsim^*$ is a reflexive relation on X and a weak order on $X^* \subset X$. Define x^* as a minimum element of $\succsim^*$ on X^*, i.e. such that $x \succsim x^*$ for all $x \in X^*$.

Now order the $\sim$-equivalence classes of X^* by $[x_i] \gg [x_j]$ if $x \succ^* x'$ for $x \in [x_i]$ and $x' \in [x_j]$. Since X^* is finite, there is some integer k such that $[x_1] \gg [x_2] \gg \ldots \gg [x_k]$. By definition, $x^* \in [x_k]$. Next, let R be an arbitrary weak order on Y with its symmetric and asymmetric parts denoted by I and P, respectively, and order the I-equivalence classes of Y by $[y_i]P[y_j]$ if yPy' for $y \in [y_i]$ and $y' \in [y_j]$. Again, there exists an integer n such that $[y_1]P[y_2]P \ldots P[y_n]$. Finally, define the relation $\succsim$ by

$$\succsim = \succsim^* \bigcup R \bigcup_{i \leq k,\, j \leq n} ([x_i] \times [y_j]) \bigcup_{i \leq k} ([x_i] \times [x_i]) \bigcup_{j \leq n} ([y_j] \times [y_j]).$$

where

$$[x_i] \times [y_j] = \bigcup \{(x,y) : x \in [x_i],\ y \in [y_j]\}.$$

It is easy to check that $\succsim$ is a weak order that extends $\succsim^*$ from X^* to X.

To establish the first part in (8), let $C(A) = \emptyset$ for some $A \in \mathcal{M}$. Suppose $z \succsim x^*$ for some $z \in B_{\succsim}(A)$. Then, $z \in X^*$ holds, which implies (in view of Contraction Consistency) $C(\{z\}) = \{z\}$. Since $\{z\} \subset A$, this and $C(A) = \emptyset$ together contradict Contractive Undesirability. Thus, $C(A) = \emptyset$ implies $x^* \succ z$ for $z \in B_{\succsim}(A)$. Conversely, suppose $x^* \succ z$ for $z \in B_{\succsim}(A)$. This implies $x^* \succ z$ for all $z \in A$. In view of the definition of x^* and Y, this in turn implies $A \subset Y$. It follows then that $z \notin C(A)$ for all $z \in A$ and therefore $C(A) = \emptyset$. To establish the second part, let $z \succsim x^*$ for $z \in B_{\succsim}(A)$. From above, $C(A) \neq \emptyset$. Suppose $z \notin C(A)$ and $y \in C(A)$. It holds that $y \succ z$. But since $z \in B_{\succsim}(A)$ implies $z \succsim x$ for all $x \in A$, we arrive at a contradiction. Therefore, $C(A) \neq \emptyset$ implies $C(A) = B_{\succsim}(A)$.

Finally, consider a weak order $\succsim'$ on X such that $x \succsim y \Leftrightarrow x \succsim' y$ for all $x, y \in X^*$ such that $x, y \succsim x^*$. Clearly, $B_{\succsim}(A) = B_{\succsim'}(A)$ for all $A \in \mathcal{M}$ such that $A \cap X^* \neq \emptyset$. Hence, $C(A) \neq \emptyset$ implies $C(A) = B_{\succsim'}(A)$. Therefore, $\succsim$ quasi-rationalizes C uniquely up to x^*. ■

Proof of Proposition 5.

We establish sufficiency of the axioms first. Start by defining the collection

$$\mathcal{N} = \{A \in \mathcal{M} : C(A) = \emptyset \ \& \ A \supset D \Longrightarrow C(D) \neq \emptyset\}$$

Next, let

$$\begin{aligned}
G_1 &= \arg\min\{|A| : A \in \mathcal{N}\} \\
G_2 &= \arg\min\{|A| : A \in \mathcal{N} \setminus G_1\} \\
G_3 &= \arg\min\{|A| : A \in \mathcal{N} \setminus (G_1 \cup G_2)\} \\
&\vdots \\
G_k &= \arg\min\{|A| : A \in \mathcal{N} \setminus (G_1 \cup G_2 \cup \ldots \cup G_{k-1})\}
\end{aligned}$$

where G_k is determined by the condition

$$\mathcal{N} \setminus \bigcup_{i=1}^{k} G_i = \emptyset \quad \& \quad \mathcal{N} \setminus \bigcup_{i=1}^{k-1} G_i \neq \emptyset$$

Now, for a given $n \in \mathbb{N}$, define the function $\psi : \mathcal{M} \to \mathbb{R}$ as follows:

$$\begin{aligned}
\psi(A) = n &\iff C(A) \neq \emptyset \\
\psi(A) = n+1 &\iff A \in G_1 \\
&\vdots \\
\psi(A) = n+k &\iff A \in G_k
\end{aligned}$$

By construction, $C(A) = \emptyset$ iff $\psi(A) > n$. Moreover, Binary Choice Consistency and Desirability ensure that ψ also satisfies (9b) and (9c), respectively. We will now show that (9d) holds, i.e. for all $A \in \mathcal{M}$, $B \supset A \Rightarrow \psi(B) \geq \psi(A)$. Suppose $A \in \mathcal{M}$ is such that $C(A) \neq \emptyset$. Then $\psi(A) = n$. If $B \supset A$ is also such that $C(B) \neq \emptyset$, then $\psi(B) = n$ too. If $C(B) = \emptyset$, then, from the definition of ψ, $\psi(B) > n$. Now suppose $A \in \mathcal{M}$ is such that $C(A) = \emptyset$. Thus, $A \in G_i$ for some $i \geq 1$. If $B \supset A$, then, in view of Overload Monotonicity, $B \in G_{i+l}$ for some $l > 0$. It now follows from the definition of ψ that $B \supset A \Rightarrow \psi(B) > \psi(A)$. Thus, (9d) holds.

From the above we have $C(A) \neq \emptyset$ iff $\psi(A) \leq n$. It remains to be shown that there exists a unique preorder $\trianglerighteq$ and a completion $\succsim$ of $\trianglerighteq$ such that $C(A) \neq \emptyset$ implies $C(A) = B_{\succsim}(A)$ for all $A \in \mathcal{M}$. The proof of this relies on a suitable adaptation of the argument in Richter (1966, p. 640). Define the relation $\trianglerighteq$ on X by $x \trianglerighteq y$ if there is $A \in \mathcal{M}$ such that $x \in C(A)$ and $y \in A$. From Desirability, $\trianglerighteq$ is reflexive. Now suppose $x \in C(A)$, $y \in A$ and $y \in C(B)$, $z \in B$. Since all binary menus are in $\mathcal{M}$ by assumption, in view of WARP and (the contrapositive of) Overload Monotonicity, these assumptions imply $x \in C\{x, y\})$ and $y \in C(\{y, z\})$, respectively. It now follows from Binary Choice Consistency that $x \in C(\{x, z\})$, which implies $x \succsim z$. Therefore, $\trianglerighteq$ is transitive, hence a preorder.

Define the relation P on X by xPy if $x \trianglerighteq y$ and $y \not\trianglerighteq x$. Since P is the asymmetric part of the preorder $\trianglerighteq$, P is a strict partial order. Define the relation J on X by xJy if $x \trianglerighteq y$ and $y \trianglerighteq x$. J is the symmetric part of the preorder $\trianglerighteq$, hence an equivalence relation on X. Let the J-equivalence class of $x \in X$ be denoted by $[x]$. Let $\mathcal{X}$ be the quotient set derived from X by J. Let $\mathcal{P}$ be a relation on $\mathcal{X}$ defined by $[x]\mathcal{P}[y]$ if xPy. The relation $\mathcal{P}$ is a strict partial order on $\mathcal{X}$. From Szpilrajn's (1930) theorem, it admits an extension into a strict linear order $\mathcal{R}$ on $\mathcal{X}$. Now define the relation $\succsim$ on X by $x \succsim y$ iff xJy or $[x]\mathcal{R}[y]$. It is easy to show that $\succsim$ is a complete preorder on X.

Suppose $C(A) \neq \emptyset$. Let $x \in C(A)$. For all $y \in A$, it holds that xPy or xJy. Hence, $x \in C(A)$ implies $x \succsim y$ for all $y \in A$. Conversely, suppose $C(A) \neq \emptyset$, $x \in A$ and $x \succsim y$ for all $y \in A$. Let $z \in C(A)$. If $x = z$, there is nothing to prove. Suppose $z \neq x$. We have zJx or $[z]\mathcal{R}[x]$. At the same time, $x \succsim z$ implies xJz or $[x]\mathcal{R}[z]$. Suppose $x\mathcal{R}y$ and $y\mathcal{R}x$. Since $x \neq z$, this violates the asymmetry of $\mathcal{R}$. Hence, xJz. This implies $x \in C(B)$ and $z \in B$ for some $B \in \mathcal{M}$. Suppose $x \notin C(A)$. Since $z \in C(A)$, this contradicts WARP. Therefore, $x \in C(A)$. This completes the proof that (9a) holds.

With regards to necessity of the axioms, it follows from (9a) that WARP is satisfied. It also follows from (9a) and (9b) that Overload Monotonicity is satisfied, and from (9a) and (9c) that Desirability is satisfied. Finally, it follows from (9a) and (9d) that Binary Choice Consistency is satisfied too. ■

Appendix 2: Axiom Independence

Let $X = \{w, x, y, z\}$ and $\mathcal{M} = \{A : A \subseteq X, A \neq \emptyset\}$. Each of the examples below presents a set of choices from elements of $\mathcal{M}$ that satisfy all but one of the axioms involved in Proposition 1.

Not WARP

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = \{w,x\},\ C(\{w,y\}) = \{w\},\ C(\{w,z\}) = \emptyset,\ C(\{x,y\}) = C(\{x,z\}) = \{x\},\ C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w,x\},\ C(\{w,x,z\}) = \{x\},\ C(\{w,y,z\}) = \emptyset,\ C(\{x,y,z\}) = \{x\}$$
$$C(\{w,x,y,z\}) = \{x\}$$

Not Desirability

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \emptyset$$

$$C(\{w,x\}) = \{w,x\},\ C(\{w,y\}) = \{w\},\ C(\{w,z\}) = \emptyset,\ C(\{x,y\}) = \{x\},\ C(\{x,z\}) = C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w,x\},\ C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \emptyset$$

Not Contraction Consistency

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = \emptyset,\ C(\{w,y\}) = C(\{w,z\}) = \{w\},\ C(\{x,y\}) = C(\{x,z\}) = C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w,x\},\ C(\{w,x,z\}) = \emptyset,\ C(\{w,y,z\}) = \{w\},\ C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \{w,x\}$$

Not Strong Expansion

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = C(\{w,y\}) = C(\{w,z\}) = \{w\},\ C(\{x,y\}) = C(\{x,z\}) = C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w\},\ C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \emptyset$$

Next, each of the examples below presents a set of choices from elements of $\mathcal{M}$ that satisfy all but one of the axioms involved in Proposition 4.

Not WARP

$$C(\{w\}) = \emptyset,\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \{\emptyset\}$$
$$C(\{w,x\}) = \{w,x\},\ C(\{w,y\}) = \{w,y\},\ C(\{w,z\}) = \{w\},\ C(\{x,y\}) = C(\{x,z\}) = \{x\},\ C(\{y,z\}) = \{y\}$$
$$C(\{w,x,y\}) = \{w,x\},\ C(\{w,x,z\}) = \{x\},\ C(\{w,y,z\}) = \{y\},\ C(\{x,y,z\}) = \{x\}$$
$$C(\{w,x,y,z\}) = \{x\}$$

Not Contraction Consistency

$$C(\{w\}) = \{w\},\ C(\{x\}) = C(\{y\}) = C(\{z\}) = \emptyset$$
$$C(\{w,x\}) = \{w,x\},\ C(\{w,y\}) = C(\{w,z\}) = \{w\},\ C(\{x,y\}) = C(\{x,z\}) = C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w,x\} = C(\{w,x,z\}),\ C(\{w,y,z\}) = \{w\},\ C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \{w,x\}$$

Not Undesirability

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = \{w,x\},\ C(\{w,y\}) = C(\{w,z\}) = \{w\},\ C(\{x,y\}) = C(\{x,z\}) = \{x\},\ C(\{y,z\}) = \{y,z\}$$
$$C(\{w,x,y\}) = C(\{w,x,z\}) = \{w,x\},\ C(\{w,y,z\}) = \{w\},\ C(\{x,y,z\}) = \{x\}$$
$$C(\{w,x,y,z\}) = \{w,x\}$$

Not Contractive Undesirability

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \emptyset$$
$$C(\{w,x\}) = \{w,x\},\ C(\{w,y\}) = \{w\},\ C(\{w,z\}) = \emptyset,\ C(\{x,y\}) = \{x\},\ C(\{x,z\}) = C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w,x\},\ C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \emptyset$$

Finally, each of the examples below presents a set of choices from elements of $\mathcal{M}$ that satisfy all but one of the axioms involved in Proposition 5.

Not WARP

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = \{w,x\},\ C(\{w,y\}) = \{w\},\ C(\{w,z\}) = \emptyset,\ C(\{x,y\}) = C(\{x,z\}) = \{x\},\ C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w,x\},\ C(\{w,x,z\}) = C(\{w,z\}) = \emptyset,\ C(\{x,y,z\}) = \{x\}$$
$$C(\{w,x,y,z\}) = \emptyset$$

Not Desirability

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \emptyset$$
$$C(\{w,x\}) = \{w,x\},\ C(\{w,y\}) = \{w\},\ C(\{w,z\}) = \emptyset,\ C(\{x,y\}) = \{x\},\ C(\{x,z\}) = C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w,x\},\ C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \emptyset$$

Not Binary Choice Consistency

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = \emptyset,\ C(\{w,y\}) =,\ C(\{w,z\}) = C(\{x,y\}) = C(\{x,z\}) = \emptyset,\ C(\{y,z\}) = \{y\}$$
$$C(\{w,x,y\}) = C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \emptyset$$

Not Overload Monotonicity

$$C(\{w\}) = \{w\},\ C(\{x\}) = \{x\},\ C(\{y\}) = \{y\},\ C(\{z\}) = \{z\}$$
$$C(\{w,x\}) = C(\{w,y\}) = C(\{w,z\}) = \{w\},\ C(\{x,y\}) = C(\{x,z\}) = C(\{y,z\}) = \emptyset$$
$$C(\{w,x,y\}) = \{w\},\ C(\{w,x,z\}) = C(\{w,y,z\}) = C(\{x,y,z\}) = \emptyset$$
$$C(\{w,x,y,z\}) = \emptyset$$