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Abstract

Cement is a non-storable input in the medium- and long-term. The evidence in Chile shows that cement supply and demand are in relative equilibrium, so the demand or supply of this input can measure the activity in the structural construction or work. The aim of this paper is to backcast the series of cement production since January 2009, using as an instrument the connection of the series of cement sales, available on a monthly basis from 1991-2013. To this end, we apply the Johansen cointegration method. Then, a model of state-space is proposed to characterize the cycle of cement production, taking its connection with investment in construction into account. Indeed, cement production, technically, is a leading indicator of sectoral investment.

Key words: cement; construction investment; retropolation; cointegration; state-space; Chile

1. Introduction

From an economic perspective, cement is an intermediate input of production, and thus an important structural component of construction for social purposes or productivity \cite{Aranoff}. A structural building system is formed by a combination of materials (including cement) and labor \cite{Norman}, and it is productive when it contributes to improving the efficiency of the factors of production (labor and capital). For example, the construction of roads and bridges, among other benefits, shortens travel times; and depending on the quality (condition of roads), dampens the depreciation or deterioration of the vehicles, thus decreasing maintenance costs.

To highlight the importance of using cement, the Center for Investigations of Resources and Energy Consumption of Spain (Spanish acronym CIRCE\textsuperscript{1}), maintains that about 70\% of a building’s weight is owed to the use of cement in the construction process. In practice, the cement used during the construction of foundations is a combination of aggregates, water and additives\textsuperscript{2}. Processed cement is produced from this combination in the form of manufactured

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\textsuperscript{2}Cementos Melón, http://www.melon.cl/

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mortar concrete. Other uses include the preparation of mortar, stucco or masonry\textsuperscript{3}. Moreover, it is also extensively used in heavy construction work, like excavation, foundation building and structuring (formwork and reinforcement mats)\textsuperscript{4}.

Therefore, the use of cement processing corresponds to one of the first stages of activity in housing construction and infrastructure works (non-residential building, civil works and engineering). It is therefore reasonable to assume that the intensity of cement use could anticipate the pace of economic activity in the next stages of the construction industry’s production chain (facilities and endings), as well as predicting the need of hiring labor (Norman\textsuperscript{1979}). In this regard, [Aisen and Jones\textsuperscript{2009}], in their attempt to project the dynamics of short-term employment in Chile’s construction industry, stressed the usefulness of cement shipments as decisive determinant of employment. The Chilean Chamber of Construction (CChC\textsuperscript{5}) has used the determinant to measure indicators of sectoral activity, such as the physical index of industrial shipments, the index of Monthly Construction Activity (Spanish acronym IMACON) and the Index of Regional Construction Activity (Spanish acronym INACOR).

Moreover, cement, at least in the medium-term, is a non-storable material (non-stockable\textsuperscript{6}). Therefore, producers in the industry recommend a use-by date of not more than sixty days from the packing date indicated on the bag, and advise that after this period quality deteriorates\textsuperscript{7}. This is because the rate of deterioration is very sensitive to humidity, as well as other factors related to the warehousing environment. Indeed, the data of production (supply) of cement provided by the National Institute of Statistics (Spanish acronym INE\textsuperscript{8}) from January 2009, are relatively coincident with dispatches assumed (demand) and published by the Institute of Cement and Concrete (Spanish acronym ICH\textsuperscript{9}) from January 2009 to December 2013 - which is the last data available prior to discontinuation of the monthly publication indicator. From the above it appears that the old and discontinued series of monthly cement sales from 1991-2013 may be useful as an instrument in the backward projection of a new series of monthly manufacturing of that input. This allows us to reconstruct a wealth of historical observations of the new cement indicator (measured in tons) to stimulate future research. In this sense, Table 1 shows the close relationship between demand and supply of cement, published by ICH and INE, respectively.

In this paper the splicing methods evaluated are: (i) strictly linear and backward projection, and (ii) backward projection by cointegration. Both methods allow a new series of splicing cement manufacturing in the period hitherto unobserved (1991-2013), using as instrument for backward projection the old dispatch indicator. However, the methodological difference is in the structure of the long-term relationship to the series imposed during the period (2009-2013) they share. Particularly, strict backcasting supposes the observations of the backcasting variable and splice instruments come from the same data generating process, or probability density function, and therefore the residual structure of the relationship between variables is white noise; whereas backcasting by cointegration is less restrictive, requiring the residual stationarity series integrated relationship between variables in the same order.

\textsuperscript{3}http://www.melon.cl/
\textsuperscript{4}http://www.dictuc.cl/
\textsuperscript{5}http://www.cchc.cl/
\textsuperscript{6}http://www.imcyc.com/
\textsuperscript{7}Cemento Bío Bío, http://www.cbb.cl/
\textsuperscript{8}http://www.ine.cl
\textsuperscript{9}http://www.ich.cl
Finally, as validation method of the new spliced economic indicator of cement production, we compare the dynamics of its cycle with investment in construction, after we evidenced a significant contemporary high correlation to the annual growth rate indicator of cement with construction investment - which is one of the main macroeconomic aggregates of the National Accounts sector. The disaggregation of the cyclical component is based on the work of [Clark (1989), Kim and Nelson (1999) and Harvey (2011)], who use a state-space model to estimate unobserved variables such as cycle and the stochastic trend of a time series.

This paper therefore proceeds as follows: After this introduction, some results in Section 2 highlight the need to create an indicator of either use or production of cement. Section 3 explains the methodology of strict backcasting and of backcasting by cointegration analysis. The results of the application of these backward projection methodologies are shown in Section 4.1. The validation in economical terms of the spliced indicator of cement production will be discussed in Section 5. Conclusions are then addressed in Section 6.

2. Stylized facts

Cement, besides being an intermediate construction input, forms also part of the value of Gross National Accounts (NPP), elaborated by the Central Bank of Chile [Banco Central de Chile (2008)]. This is because engineering and construction works - partial components of gross fixed capital formation - use cement intensively as material in the construction of structural works. The percentage of intermediate use of cement in sectoral activity - defined as works of civil engineering, building and other specialized activities - is 46.1% of the total value ($433.514 million at basic prices), generated by 111 economic activities of input-output of the parent company for the base year 2008 [Banco Central de Chile (2008)]. Within building activities engineering and residential building works stand out, with percentages of intermediate use of cement around 20% and 18%, respectively (see Table 2).

Regarding the database, the number of monthly frequency of cement production is compiled by INE since January 2009. It covers the following input classifications: Portland cement, molten cement, slag cement and hydraulic analog cements, except the Clinker ones. The number of cement deliveries published by the ICH, corresponds to the monthly period from January 1991 to December 2013. This indicator is derived from the consolidated data of three large cement-producing regions in Chile: Bío Bio, Melón and Polpaico. Unfortunately, ICH discontinued the publication of this monthly indicator in January 2014. Yet, there is a need to continue such publication, given cement’s importance of being a leading indicator of construction activity and because cement is an intensively used input in the early stages of building structures. So, taking advantage of the similarity of the shipment levels with levels of cement production, consistent with the characteristics of an input or non-storable factor, we decided to backcast the new official series of cement in monthly stretches for the years 1991-2008, and so use this new series as a proxy for the intensity of cement use in Chile.

Considering the monthly period (2009-2013) shared by both time series, the following sections 3 and 4 show a contrast and statistically significant relationship between cement and the same inputs, based on the technique of strictly linear backward projection, the augmented unit root test of Dickey Fuller (DFA) [Dickey and Fuller (1979)] and Johansen cointegration [Johansen (1991, 1995)]. The existence of a stable and significant relationship between the two indicators, allows backcasting (splice back) the production series of cement, using the observations history of cement sales as an instrument of splice. In turn, this procedure makes it possible to reconstruct a lot of historical evidence to characterize the economic
cycle of cement production compared with the investment cycle in the construction stage of
the study that will be addressed in Section 5.

3. Linear backward projection

In this section we evaluate the possibility of backcasting (splice back) levels of cement
production (in tonnes), using as an instrument of splice the number of cement deliveries.
The first variable is provided by the INE, whose frequency is monthly and is available from
January 2009 onwards. The use or demand indicator of cement is published by the ICH, also
on a monthly basis and observed since January 1991, but truncated from January 2014.

3.1. Strict linear backward projection

The application of the strict linear backward projection method (RLE) requires both series
(cement production and sales) to be derived from the same process or function data generator
probability density. It should be noted that the application of this technique considers the
methodological assumption that cement can be treated as a non-storable input, according
to the analysis described in the introduction. To validate the relative coincidence existing
between levels of cement and tons shipped of the same input, at least during the period
shared by both series (2009-2013), we statistically test the hypothesis that the difference
between supply and demand of cement is white noise. If so, it is possible to apply the strict
linear backward projection technique. The following theorem states that two time series are
generated by the same process if their difference presents the characteristics of a process
of white noise, while matching their simple auto-correlation functions with a cross auto-
correlation function. The cross auto-correlation function (CCF) between the processes

\[ \rho(y_t, x_{t-h}) = \frac{\sum_{i=h+1}^{n} y_i x_{i-h}}{\sqrt{\sum_{i=1}^{n} y_i^2 \sum_{i=1}^{n} x_i^2}}, \]

for a lag \( h \), and where we assume that the processes \( y_t \) and \( x_t \) are independent and identically
distributed (i.i.d). The CCF measures deviation between both series with regard to their
respective means samples. Replacing \( y_t = x_t \) in (1), we have the usual simple autocorrelation
function (ACF). From there we denote \( \rho_{xy}(h) = \rho(y_t, x_{t-h}) \) the CCF between \( x_t \) and \( y_t \), and
\( \rho_x(h) = \rho(x_t, x_{t-h}) \) and \( \rho_y(h) = \rho(y_t, y_{t-h}) \) of the ACF and \( x_t \) and \( y_t \), respectively.

**Theorem 1.** Let \( \{x_t\} e \{y_t\}, t \in \mathbb{N} \), be two weak stationary processes with means \( E(x_t) = \mu_x \nolimits \)
and \( E(y_t) = \mu_y \) and auto-covariances given by:

\[
E[(x_t - \mu_x)(x_{t-j} - \mu_x)] = \gamma_x(j), \quad \forall j \in \mathbb{N},
\]

\[
E[(y_t - \mu_y)(y_{t-j} - \mu_y)] = \gamma_y(j), \quad \forall j \in \mathbb{N},
\]

respectively; under the conditions \( \sum_{j=0}^{\infty} |\gamma_y(j)| < \infty \) and \( \sum_{j=0}^{\infty} |\gamma_x(j)| < \infty \). Considering the
linear relation \( x_t = y_t + \varepsilon_t, \varepsilon_t \sim RB(0, \sigma^2) \), \( x_t \in F_{t+1}, F_{t+1} = \text{sp}\{y_s: s < t+1, s \in \mathbb{N}\} \), where
generated space or span that includes its frontier, we have to

\[ \rho_x(j) = \rho_y(j) = \rho_{xy}(j), \quad \forall j \in \mathbb{N}, \]

where \( \rho_x(j) \) and \( \rho_y(j) \) correspond to ACF between \( x_t \) and \( y_t \), respectively; and \( \rho_{xy}(j) \) is the CCF between the two processes.

**Proof.** Given the linear relationship

\[ x_t = y_t + \varepsilon_t, \]

we have \( \gamma_x(j) = E[(y_t - \mu_y)(x_{t-j} - \mu_x)] \) and \( \gamma_y(j) = E[(x_t - \mu_x)(y_{t-j} - \mu_y)] \). Furthermore, \( E(x_t) = E(y_t) \) since applying property of expected alternations \( E(E(\varepsilon_t/y_t)) = E(E(\varepsilon_t)/y_t) = 0 \), by the classical assumption that \( E(\varepsilon_t) = 0 \).

Under the assumption \( x_t \in \mathcal{F}_{t+1} \), we have \( \gamma_x(j) = \gamma_y(j) \), and thus \( \rho_x(j) = \rho_y(j) \). Then, since

\[ \frac{\gamma_x(j)}{\gamma_x(0)} = \frac{E(y_t - \mu_y)(x_{t-j} - \mu_x)}{\sqrt{\gamma_x(0)^2}}, \]

we have \( \rho(j) = \rho_{xy}(j), \forall j \in \mathbb{N} \). Since \( \rho_{xy}(j) < \infty \), the assumption is also true \( \sum_{j=0}^{\infty} \rho(j) = \sum_{j=0}^{\infty} \rho_{xy}(j) < \infty \). \[ \blacksquare \]

Theorem 1 shows that if \( x_t \) represents the levels of the new series of cement for the time \( t \), and the variable \( y_t \) refers to tons used as the same input, then the RLE method consists of simply modifying the levels of the new series, \( x_t \) such that they retain the monthly rates of change of the old series of dispatches, \( y_t \). In other words, we can backcast the new series of cement for a monthly period between 1991-2008, based on rates of implied variation of cement shipments - variables used as a splice instrument. Therefore, we must

\[ x_{t-(j+1)} = \frac{x_{t-j}y_{t-(j+1)}}{y_{t-j}}, \quad j = 0, 1, \ldots, m - 1, \]

where \( m \) is the number of months remaining of the indicator cement deliveries prior to the publication of statistics of production of this input (January 2009). In our case, \( m = 216 \) (considering the months from January 1991 to December 2008), and \( t \) is corresponding to the months when the reference variable (cement dispatches) are taken into account, used for backcasting the period of unobserved cement production (1991-2008).

However, in practice the series subject to our study proved stationary, according to the DFA test, unit root and the results are shown in Table 3. Therefore, to be consistent with what is stated in Theorem 1 the analysis of the feasibility of the application of the strict linear backward projection method will be based on the series expressed in prime differences.

Table 3 shows that the original series of production and sales of cement (\( x_t \) and \( y_t \), respectively), as well as their seasonally adjusted and into logarithms transformed versions (sa) were not stationary in levels. However, the unit root was eliminated by taking prime differences of the series in the logarithm (or monthly growth rates of the original series). So, the hypothesis to be tested in this paper is that residues of the relationship between the monthly production growth and cement demand follows a white noise process. For this purpose, the Box-Ljung test (Ljung and Box 1978) is applied to the difference or residue of the first order integrated
series, which is based on the following

\[ Q(p) = n(n + 2) \sum_{i=1}^{p} \frac{\gamma(i)^2}{n - p}, \]

where \( \gamma(i) \) is the \( i \)th autocorrelation, \( n \) is the sample size, and \( p \) are the degrees of freedom equal to the number of autocorrelations. Asymptotically, \( Q(p) \) converges in distribution \( \chi^2_p \). For details, see Harvey (1990). In each CCF and ACF plots, the peaks are compared with the 95% confidence bands computed as \( \pm z_{1-\alpha/2} \sqrt{1/n} \), where \( z_{1-\alpha/2} \) denote the standardized normal quantile related to a significance level of \( \alpha = 0.05 \). This challenges the hypothesis of absence of auto-correlation of order \( p \) in a time series. It is also used to prove that the waste follows a white noise process. Here it is utilized to test the absence of a residual auto-correlation relationship between production and demand of cement, both series expressed in prime differences or integrated first order (see Figure 1).

As the ACF shows, it is possible to reject the hypothesis that residues of the relationship between demand for cement production follow a white noise process, as some values exceed (in absolute value) the Bartlett confidence bands. This finding is endorsed by \( p \) values of the Ljung-Box test, situated in the rejection region distribution \( \chi^2 \), considering criteria of up to 1% of significance. Yet, since this result is not consistent with the provisions of Theorem 1, we rule out the possibility of applying the method of strict linear backward projection in a series of cement production. This leads us to the alternative, splicing cointegration time series, since the manufacturing and sales of cement were being built by the same order. Also, the residual term follows a regular rational process with spectral density, based on the ARIMA(2, 0, 1) specification that follows from the correlogram (Contreras-Reyes and Idrovo 2011). This method will be addressed in the next section.

3.2. Linear backcasting by co-integration

In this section we evaluate an alternative series of cement production backcasting through the method of cointegration. In broad strokes, this technique assumes that - in a horizontal or relatively wide time window - cement production fluctuates in line with the number of cement sales. So their difference or residue has been stable over time, even though initially both variables (production and delivery) are not stationary or integrated of order one - denoted by \( I(1) \). From an economic perspective, the existence of cointegration sees a relationship equilibrium or long-term relation between the variables of interest (demand and supply of cement) while residues of this relationship represent the imbalances or transitionary misalignments of the cement market. Although differences between levels of production and sales of cement can be observed within the economy, cointegration supposes that there will always be latent market forces that lead to the reestablishment of equilibrium in the long run. That is, we say that the time series \( x_t \) (production) and \( y_t \) (firms) only cointegrate if a parameter set \( \{\alpha, \beta\} \) is non-zero, such that the linear combination of the type \( \alpha x_t + \beta y_t \) is stationary or integrated of order zero. In this case, it is said that the stochastic processes share common trends, so that the presence of spurious correlations is either discarded or relationships without economic function are present. For a full explanation of the above, consider the following system of two equations:

\[
\begin{align*}
   x_t &= \mu_t + \varepsilon_t, \\
   y_t &= \xi_t + \nu_t,
\end{align*}
\]
where $\mu_t$ is the stochastic trend series of sales of cement ($y_t$) and $\xi_t$ is the corresponding trend of cement ($x_t$). It is recalled that all these variables are not stationary or integrated $I(1)$. In addition, there is no a priori reason to suppose equal trend measures, so $\mu_t \neq \xi_t$. System disturbances ($\epsilon_t, \nu_t$) follow each stationary stochastic process or $I(0)$.

Consider the following linear combination of the systemic variables $x_t$ and $y_t$ with $\alpha$ and $\beta$ parameters: $\alpha x_t + \beta y_t = \alpha \mu_t + \beta \xi_t + \alpha \varepsilon_t + \beta \nu_t$. In principle, not all lineal combinations are stationary. The combination $\alpha \mu_t + \beta \xi_t$ is $I(1)$, shows the result of the stochastic trends and the combination of the error terms $\alpha \varepsilon_t + \beta \nu_t$ is stationary, since disturbances are individual processes $I(0)$. Thus, for $x_t$ and $y_t$ to cointegrate, the stationarity of the combination on the right side should be achieved of the equation, i.e. $\beta y_t + \alpha x_t \sim I(0)$. This conjecture is only alone if the following restriction $\alpha \mu_t + \beta \xi_t = 0$ is imposed, which means that both time series share common trends, because one ends up being the function of the other. So that while the trends can differ on levels in the presence of cointegration, the changing of $t$ is proportional to the other trend measure $\mu_t$. Indeed, if $\alpha \mu_t + \beta \xi_t = 0$, then $\mu_t = -(\beta/\alpha) \xi_t$.

Having verified the existence of cointegration, the backward projection of cement production - conditional on the observed dispatches of that input as splice variable - the following expression is obtained:

$$x_{t-(j+1)} = -\left(\frac{\beta}{\alpha}\right) y_{t-(j+1)},$$

where $j = 0, 1, \ldots, m - 1$ (with $m$ the number of remaining months being the indicator of cement dispatches). Finally, the radius of parameters $-\beta/\alpha$ is the result of the normalizing of the cointegrating vector $(\alpha, \beta)^\top$, this is in order to clear the variable we seek to backcast depending on the observed series, $E(x_t/y_t) \sim I(0)$.

### 3.3. Johansen cointegration test

Here we focus on the Johansen cointegration test for the specification and estimation of a Vector Autoregressive Model (VAR) of order $p$ (see e.g. Crowder and Wohar 2004). Specifically, we analyze a bivariate VAR model to inferring a long-term relationship of two time series: production and sales of cement. Without loss of generality, consider the following model $VAR(1)$:

$$
\begin{pmatrix}
x_t \\
y_t
\end{pmatrix}
= 
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix}
+ 
\begin{pmatrix}
\phi_{11} & \phi_{12} \\
\phi_{21} & \phi_{22}
\end{pmatrix}
\begin{pmatrix}
x_{t-1} \\
y_{t-1}
\end{pmatrix}
+ 
\begin{pmatrix}
\epsilon_{1t} \\
\epsilon_{2t}
\end{pmatrix},
$$

where the variables $x_t$ e $y_t$ are processes $I(1)$ and the error vector $(\epsilon_{1t}, \epsilon_{2t})^\top$ are innovations with zero mean and constant variance. Alternatively, based on the Johansen technique, if we subtract the vector $(x_{t-1}, y_{t-1})^\top$ on both ends of the system of equations of the model VAR (1), then this can be restated in a similar manner to the specification used in the contrast of the Dickey-Fuller unit root:

$$
\begin{pmatrix}
\Delta x_t \\
\Delta y_t
\end{pmatrix}
= 
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix}
+ 
\begin{pmatrix}
\phi_{11} - 1 & \phi_{12} \\
\phi_{21} & \phi_{22} - 1
\end{pmatrix}
\begin{pmatrix}
x_{t-1} \\
y_{t-1}
\end{pmatrix}
+ 
\begin{pmatrix}
\epsilon_{1t} \\
\epsilon_{2t}
\end{pmatrix}.
$$

The rank of the coefficient matrix $\Pi$ that accompanies the lagged variables in the bivariate VAR (1) model, contains information on the number of existing cointegrating vectors in the
system. Here, if $\Pi$ has rank 1, then there are two non-zero vectors of the dimension $2 \times 1$, $\delta$ and $\beta$, such that $\Pi = \delta \beta^T$, where the parameter $\beta$ is the cointegrating vector or long-term relationship between production and sales of cement, and $\delta$ is the vector parameter of speed and short-term adjustment of the error correction vector (VEC). In this case, $\beta^T(x_t, y_t)^T$ is a stationary linear combination $I(0)$.

Since the variables $x_t$ and $y_t$ are not stationary or integrated of first order, then the equation of lag operators $(I_2 - \Phi L = 0)$, $L = (\lambda_1, \lambda_2)^T$, contains at least one root unit. So, from the canonical decomposition of $\Phi$, the roots $\lambda_1$ and $\lambda_2$ satisfying $|\Phi - \lambda I_2| = 0$ envision the existence of a cointegrating relationship to be one of them equal to the unit. This is equivalent to say that two variables $(x_t$ and $y_t$) share common trends. For example, if we choose $\lambda_1 = 1$ and $|\lambda_2| < 1$, then $\phi_{11} = 1 - \phi_{12}\phi_{21}/(1 - \phi_{22})$. Using the latter and returning to the definitions of $x_t$ and $y_t$ raised in (2) and (3), the VAR(1) bivariate model can be expressed by

$$
\begin{pmatrix}
\Delta \mu_t \\
\Delta \xi_t
\end{pmatrix} =
\begin{pmatrix}
a_1 \\
a_2
\end{pmatrix} +
\begin{pmatrix}
\phi_{11} - 1 & \phi_{12} \\
\phi_{21} & \phi_{22} - 1
\end{pmatrix}
\begin{pmatrix}
\mu_{t-1} \\
\xi_{t-1}
\end{pmatrix} + \Omega
$$

where

$$
\Omega =
\begin{pmatrix}
\phi_{11}\xi_{t-1} + \phi_{12}\nu_{t-1} + \epsilon_{1t} - \xi_t \\
\phi_{21}\xi_{t-1} + \phi_{22}\nu_{t-1} + \epsilon_{2t} - \xi_t
\end{pmatrix}
$$

Then, replacing $\phi_{11}$ in the coefficient matrix $\Phi$, cointegration satisfies the following system according to the stochastic trends $x_t$ and $y_t$:

$$
\begin{pmatrix}
-\phi_{12}\phi_{21} \\
\phi_{21}
\end{pmatrix}
\begin{pmatrix}
\phi_{12} \\
\phi_{22} - 1
\end{pmatrix}
\begin{pmatrix}
\mu_{t-1} \\
\xi_{t-1}
\end{pmatrix} =
\begin{pmatrix}
0 \\
0
\end{pmatrix}
$$

with

$$
|\Phi - I_2| =
\begin{vmatrix}
-\frac{\phi_{12}\phi_{21}}{1 - \phi_{22}} & \phi_{12} \\
\phi_{21} & \phi_{22} - 1
\end{vmatrix} = 0
$$

(here $\lambda_1 = 1$ prevailed). Therefore, variables $x_t$ and $y_t$ share common trends:

$$
\mu_{t-1} = \frac{1 - \phi_{22}}{\phi_{21}}\xi_{t-1}.
$$

To perform the statistical test of the presence of one or more cointegrating relationships between variables $I(1)$, Johansen proposes the following test of two statistics, based on the range or values of the characteristics of matrix $\Pi$:

(i) $J = -n \sum_{i=r+1}^{p} \ln(1 - \lambda_i)$ (trace test),

(ii) $J = -n \ln(1 - \lambda_{r+1})$ (test of the maximum value of the characteristic root),

where $\lambda$ is the largest eigenvalue of $\Pi$, $p$ is the number of variables VAR model, and $n$ is the sample size. The trace test contrasts the null hypothesis of $r$ vectors cointegration versus the alternative hypothesis $p$ cointegrating relationships. The test of the maximum eigenvalue evaluates the hypothesis $r$ cointegrating vectors versus the alternative $r + 1$ cointegrating relationships. These tests asymptotically converge to $\chi^2$ distribution with critical values developed by [Johansen and Juselius (1990)].
4. Results

In this section, we use the Johansen test to backcast the new series of cement production through the cointegrating relationship between supply and demand. If there is a stable long-term relationship between production and sales of cement, it is feasible to backcast the first series (of cement production) according to historical demand. However, prior to the application of the Johansen cointegration test, it is necessary to define the optimal order of lags for the unrestricted bivariate model estimation VAR.

The selection is based on the minimum value reached by the following statisticians: likelihood ratio (LR), the Akaike Information Criterion (AIC) \cite{Akaike1974}, Schwarz’s Bayesian Information Criterion (SC) \cite{Schwarz1978}, and the Hannan-Quinn criterion (HQ) \cite{Hannan1979}. Table 4 shows that for a maximum of five potential setbacks, we conclude that the optimal number of lags for the VAR model is equal to 3. This analysis is complemented by the contrast of the hypothesis of the absence of VAR residual autocorrelation, based on the $\chi^2$ statistic multiplier Lagrange test (LM-test). Table 5 shows that there is no evidence that would warrant rejection of the hypothesis of serial autocorrelation absence in the residuals of the unrestricted VAR(3) model, because the statistics of the $\chi^2$ tests for each of the 10 delays considered in the residual structure exceed largely their respective critical residual values, as noted in the p values at 5% significance. Therefore, the data suggest that the specification of a VAR with three lags is appropriate to obtain wastes of white noise and continue to the next step of our analysis, the application of the Johansen cointegration test.

Considering the estimation of the unrestricted VAR(3) model, we apply the trace test and maximum characteristic value, both corresponding to the Johansen cointegration test. The trace test confirms with 5% the significance of the existence of a stable long-term relationship between cement production and sales during the monthly period from 2009-2013 (Table 6). Specifically, the hypothesis of absence of a cointegration connection between the variables of interest is rejected, since the value $p = 0.01$ indicates that the trace calculated from test data (19.96) is greater than the critical value (15.5), with a level of significance $\alpha = 0.05$. Then, there is evidence to reject the hypothesis that there is at most one cointegrating vector because the trace statistic (2.2) was less than the critical value (3.84), endorsed by the $p$ value that was obtained to contrast this hypothesis (0.14). This finding is confirmed by the test of the maximum eigenvalue in Table 6.

Table 7 shows the cointegration vector normalized to the parameter series accompanying tons of cement, so as to project back (backcast) the series of cement production in the period not observed (January 1991 to December 2008). The estimate is conditional on the historical behavior of the number of sales of that input used in this paper as a tool for splicing. Moreover, it appears that 67% of the variability in cement production is explained by changes in the demand for the input, which is a good fit between the two series. Noting the value of the coefficient of determination $R^2$ and the Durbin Watson (DW) statistic, we rule out the presence of spurious correlation between the variables considered in our analysis. Indeed, Figure 2 compares the evolution of cement supply and its estimate, based on a cointegrating relationship with a confidence interval of 95%. Figure 3 shows the backward projection made to the series of cement for the unobserved monthly period 1991-2008, with a confidence interval of 95%.

Since the time series used in our analysis are expressed in logarithms, cement levels (tonnes) and standard error were obtained (based on the properties of the log-normal dis-
\[ \tilde{x}_{t-(j+1)} = \exp \left( \hat{\varphi} + \hat{\tau} \ln y_{t-(j+1)} + \frac{\hat{\sigma}^2_x}{2} \right), \]
\[ \hat{\sigma}^2_x = (\exp(\hat{\sigma}^2_x) - 1)\tilde{x}_{t-(j+1)}, \]
respectively; where \( \tilde{x}_t \) is the backward projection of tons of cement production, \( \ln y_t \) is the natural logarithm of dispatches, \( j = 0, 1, \ldots, m-1 \) is the backcast period with \( m \) the number of remaining months as indicator of cement deliveries (January 2009 to December 2008), and the parameter \( \tau \) is obtained from the vector normalization cointegrating \([\alpha, \beta]^T\) (this is in order to clear the variable we seek to backcast in the function of the observed series). The variance \( \hat{\sigma}^2_x \) is interpreted as the short-term imbalance or mismatch (transient) between supply and demand of cement. The variance of cement supply, \( \hat{\sigma}^2_x \), is the parameter used for the design of the confidence interval \[ [\tilde{x}_{t-(j+1)} - z_{0.025}\hat{\sigma}_x, \tilde{x}_{t-(j+1)} + z_{0.025}\hat{\sigma}_x]. \]

To validate the backward projection indicator of cement from an economic perspective, we calculate the cross-correlation between the annual growth of the quarterly series of cement production (spliced) and the annual change in the key macro-economic National Accounts (NPP), prepared on a quarterly basis by the Banco Central de Chile [2008]. Considering Eq. [1] we are interested in obtaining the cross-correlation between cement production \( x \) and each one of the macroeconomic variables \( y \): Gross domestic product (GDP) of the economy, construction GDP, investment in the economy and sectoral investment. Figure [4] shows that the annual growth rate of cement production is simultaneously and significantly correlated with major NPP indicators, both at the national and sectoral levels. However, it can be appreciated that the annual growth of cement production scores correlates somehow with greater investment in construction (0.78) on the other macroeconomic indicators. So, in the specification of the structural time series model (see the following section), we present this relationship when estimating the cyclical component of cement production.

5. Characterization of the economic cycle of cement production

In Chile exist a lot of studies that measure aggregated economic cycles, but there is little documented research on cycles in the construction area [Piguillem 2004]. In this section, we analyze the characteristics of recessive states of the cement production cycle in terms of duration, depth and relationship with the cycle of investment in construction. The idea of linking the cement cycle indicator with the investment cycle stems from the high correlation (0.78) found between their respective rates of inter-annum change.

For this purpose, we use state-space models in order to disaggregate both cement production and investment into two components not observed: cyclical/transitory and the stochastic trend. The cyclical component accounts for short-term imbalances in the series with regard to its evolution trend. Such detours could be due to transitory productivity shocks or disturbances from other sources, but be of short duration, as during the recession in Chile between 2008 and 2009, generated by the systemic effect of the international financial crisis [Cerda and Vergara 2008]. For its part, the stochastic trend path includes, among other factors, permanent productivity shocks and structural or economic policy changes, such as fixing an inflation target and the structural surplus rule, as stabilizing mechanisms of the economic cycle [Banco Central de Chile 2007, Marcel et al. 2001], and the subsequent boom of public works concessions which generated a level change in the evolution of investment [Idrovo 2012].
5.1. Univariate model

The series of cement used in this section is normalized to the quarterly frequency in order to be consistent with the sectoral investment of the Banco Central de Chile (2008). In addition, we incorporate two lags in the dynamics of the cycle, in line with their level of significance and the original structure put forward in the Kim–Nelson model (Kim and Nelson 1999). The specification of a state-space model to estimate unobserved univariate components in the construction sector is based on the work of Clark (1989), Kim and Nelson (1999) and Harvey (2011). First, we conducted an exploratory exercise in which we ignored the ratio of cement to construction investment, i.e.:

\[
x_t = e_t + c_t,
\]

\[
e_t = f_{t-1} + e_{t-1} + \nu_t,
\]

\[
c_t = \phi_1 c_{t-1} + \phi_2 c_{t-2} + \varepsilon_t,
\]

\[
f_t = f_{t-1} + \omega_t,
\]

where \(x_t\) represents the production of cement, \(e_t\) is the stochastic trend component, and \(c_t\) is the cycle indicator. The \(e_t\) component follows random walk processes with time-varying drift, \(f_t\), and noise \(\nu_t \sim i.i.d. N(0, \sigma_\nu)\). Also, since the process \(f_t\) has a noise \(\omega_t \sim i.i.d. N(0, \sigma_\omega)\), the dynamic drift trend collected persists over structural changes or structural shocks in the economy, so the trend series can experience level changes over time. The non-observed component \(c_t\) is the cycle of cement supply, which follows an autoregressive process and noise \(\varepsilon_t \sim i.i.d. N(0, \sigma_\varepsilon)\) (Kim and Nelson 1999). Finally, it is assumed that the system errors are distributed as a normal with zero mean and constant variance. This assumption guarantees the success of the maximum likelihood estimation of the parameters of the Kalman filter, because it allows to decompose the prediction error of the state-space system and represents the model as a dynamic recursive process (Harvey 1990).

Table 8 (top) first presents the results of the estimation by maximum plausibility of the parameters of the univariate state-space model, i.e., the one that considers only the series of cement production, ignoring their relation to the investment cycle in construction. In this case, all coefficients were significant, according to the magnitudes of their standard errors. The dispersion of the stochastic trend of producing cement is similar to its cycle. For its part, the evolution of the drift of the trend was relatively stable. The following process cycle indicator is stationary but with signs of long memory \((\phi_1 + \phi_2 = 0.8)\) (Contreras-Reyes and Palma 2013). Figure 5a shows the evolution of the cyclical component of the cement production, spliced by the cointegration method. We first present some cyclical behavior irregularities. Particularly, a certain asymmetry in its trajectory can be observed, i.e. the episodes of recession tend to be more profound and lasting versus expansive states of the indicator. In principle, this finding is inconsistent with the observations in the aggregate cycle of the economy (Idrovo 2010), given the significant correlation between the production of cement and macroeconomic variables (see Section 4). This is because the recessionary periods have not been so deep and lasting with respect to expansive episodes of historical average growth rates with a negative sign.

5.2. Modelo bivariado

To add value to the above analysis we considered to leverage the significant relationship discovered between the annual growth rates of cement production and investment in construction. Therefore, we incorporate this relation in the cyclic structure of the state-space model
for the decomposition of cement in the stochastic trend and transitional component. In this case, the model of unobserved components is of bivariate kind, and additionally to cement we include construction investment:

\[ y'_t = u_t + v_t, \]  
\[ u_t = g_{t-1} + u_{t-1} + \eta_t, \]  
\[ v_t = \phi_1 v_{t-1} + \phi_2 v_{t-2} + \epsilon_t, \]  
\[ g_t = g_{t-1} + \lambda_t, \]

where \( y'_t \) is the logarithm of investment in construction \( (y'_t = \ln y_t) \), \( u_t \) is the component of unobservable stochastic trend with noise \( \eta_t \sim i.i.d. N(0, \sigma_\eta) \), and \( v_t \) is the cyclical component of investment with stationary noise \( \epsilon_t \sim i.i.d. N(0, \sigma_\epsilon) \). On the other hand, changes in the country’s productivity occurred in the wake of the Asian crisis in 1999 (Durán-Palma et al. 2005), the significant development of infrastructure works concessions (Cámara Chilena de la Construcción 2014), the international financial crisis of 2009 (Humphrey et al. 2009) and cycle effects of mining investment (Cámara Chilena de la Construcción 2015), are evident in the evolution of sectoral investment. Therefore, these productivity shocks are considered in the dynamics of the term drift, \( g_t \), as the stochastic trend whose process of investment in construction follows a random walk with noise \( \lambda_t \sim i.i.d. N(0, \sigma_\lambda) \).

Considering the system (4)–(7), we studied the following system for production cycles \( y'_t \) based on the cement trend system (8)–(11):

\[ x'_t = e_t + c_t, \]  
\[ e_t = e_{t-1} + \nu_t, \]  
\[ c_t = \theta_0 v_t + \theta_1 v_{t-1} + \theta_2 v_{t-2} + \epsilon_t, \]

where \( x'_t \) is the logarithm of cement production \( (x'_t = \ln x_t) \), and \( e_t \) and \( c_t \) are the components of the trend and unobservable cycle noise \( \nu_t \sim i.i.d. N(0, \sigma_\nu) \) and \( \epsilon_t \sim i.i.d. N(0, \sigma_\epsilon) \), respectively. Given the significant correlation of cement production with investment in construction (section 4), the model considers a structure where the cycle of cement, \( c_t \), is related to the cycle of investment in construction, \( v_t \), as shown in Eq. (14) of the unobserved model components.

Table 8 shows the estimated parameters by maximum likelihood of the bivariate model of unobserved components. It demonstrates that the volatility of investment in construction is mostly explained by the dispersion of the stochastic trend, regarding the volatility of the cyclical component \( (\sigma_\eta \gg \sigma_\lambda) \). For cement production, both terms (trend and cycle) have similar variances \( (\sigma_\nu \approx \sigma_\epsilon) \) as observed in the univariate model. However, the difference lies in the magnitudes of their dispersions, i.e., the trend and the cycle indicator of cement throws in the bivariate model a variance lower than when compared with that observed in the univariate model, which is indicative of an improvement in fit. This is corroborated in part by a higher value of the function log-likelihood, reported in the bivariate state-space model of Eqs. (8)–(14). Moreover, no evidence of asymmetries are evident in the cyclical component of investment, partly because it records a low persistence in its dynamics \( (\phi_1 + \phi_2 \ll 1) \). This result is analogous to the case of cement production, especially because their cyclical behavior is strongly correlated with investment in the sector. Therefore, the fluctuations in the cement indicator (monthly rate) can be a natural measure and advanced investment in construction (whose publication is quarterly with a and two months lag).
Figures 5(b)–(c) compare the evolution of the series of the trend and cycle of cement production. Data were calculated with the uni- and bivariate state-space models. While the univariate model assumes absence of a relationship between the cement cycle and investment in construction, the bivariate model accounts for such a relationship. Figure 5(d) shows the evolution of the stochastic trend in construction investment in the state-space bivariate model. We can identify lower levels of sectoral investment in the crisis 1998-99, during which the level of investment fluctuates below its stochastic trend. Furthermore, we can see that the recessive state was preceded by an investment boom. Then, from 2006 on, a new growth cycle in investment decisions, consistent with the boom of concessions in public works and the higher economic growth during early 2008, can be observed (Idrovo 2012). In 2009 a negative gap or low sector investment regime emerges in line with the effects of the international financial crisis (Humphrey et al. 2009). After that, a marked acceleration of investment is observed to settle in on its trend or long-term level. This is due to the high productivity of capital as consequence of its scarcity in the aftermath of the 2010 earthquake (Siembieda et al. 2012). Recently, construction investment fluctuates below its trend, which is a signal of low sectoral activity. This result is explained by the process of normalization of the mining investment cycle and construction which are derived therefrom (Cámara Chilena de la Construcción 2014, 2015).

Based on the structure of the bivariate state-space model, we need to extract the component (not shown) of the stochastic trend series in investment in the construction sector. It is possible to retrieve its cyclical evolution as a non-observed variable. Figure 6(a) demonstrates that the investment cycle is highly correlated to cement production, so the lowest correlation relative to their annual growth rates (Figura 6(b)) is due to the greater dispersion of the trend’s component of sectoral investment versus the stochastic volatility trend of cement production ($\sigma_\eta > \sigma_\nu$). Therefore, by correcting the dispersion of the tendency of both variables, they exhibit practically coincident trajectories.

Table 9 summarizes the main characteristics of the production cycle of cement and investment in construction in the respective recessions of 1998-99 and 2008-09. It can be noted that the recessionary conditions of the economic cycle for both indicators were selected on the basis of recessive periods of aggregate activity. According to economic literature, recession occurs when the quarterly GDP adjusted for seasonal effects, experiences negative changes for two or more consecutive quarters (Bravo et al. 2002). Indeed, as a result of the Asian crisis, seasonally adjusted GDP, decreased by 0.8% and 3.2% per quarter over the last two quarters in 1998 (http://www.bcch.cl). Then in the years 2008-09, four consecutive seasonally adjusted declines can be observed, whose magnitudes were -0.1% in the second quarter of 2008, -0.3% during the third quarter of the same year, -1.8% by the end of 2008, and -1.2% in the first three months of 2009.

Comparing these recessions in the depth or magnitude of falls and cycle time, we see in Table 9 that the deadline is the period where the cycle peaked before declining, or experiences a turning point into a recessive state. The minimum date is when the recession hit the bottom, i.e., when it exhibited a positive turning point to head for a neutral state of the cycle. The return refers to the date on which the cycle is normalized, i.e. arriving at its neutral level. Duration is the time (in quarters) from deadline to minimum date. The length of recovery is the time (in quarters) from when the recession bottomed (minimum date) until the cycle (return) normalized. The gap max-min is the difference in percent between the maximum
and minimum measurement stochastic trend, which reflects the depth of the cycle. The cycle is before the points of gap percentage of the indicator on its stochastic trend, which indicates the state prior to recession. In order to explore if the amplitude of the previous recession cycle is able to anticipate the severity of it, we see that strong recessions can be preceded by strong expansions and vice versa. The cycle corresponds to the gap after the percentage points under its stochastic trend indicator.

6. Conclusions

In order to backcast a series of unobserved cement production on a monthly basis 1991-2008 two backward projection methodologies were evaluated: strictly linear backward projection and backward projection for cointegration. However, we discarded backcasting based on the strict method, after verifying statistically that the series subject to our analysis are not embodiments of the same data generating process. This considered the monthly period that share supply and demand for cement (2009-13).

Given the above, we proceeded to test the presence of a long-term relationship or cointegration between the series of cement production supplied by INE and the series of cement sales supplied by ICH, both in the monthly period 2009-2013. In this case, the requirement for residual behavior between the two series is a weak stationarity and not the restrictive properties of a white noise process. Indeed, the statistical evidence leaned toward the spicing method of co-integration and the instrument of backcasting of the historical monthly series provided by ICH (1991-2008).

Note that in this paper no deeper analysis of the origin of the disturbances in the sector, in the sense that an external crisis or structural reforms, was undertaken and thus some parameters may sensitize construction workers’ and enterprises’ policy preferences. To do so, a general stochastic model of dynamic balance (DSGE) is required (Gupta et al. 2015). So, our analysis focuses only on the behavior of the production cycle of cement and its link with the cycle of sectoral investment. This is because our aim is to economically validate the utility of a new monthly cement indicator as a leading indicator of the construction sector, which should be based on its relationship with the quarterly series of investment in construction, published by the Banco Central de Chile (2008).

A state-space model is then proposed to characterize the production cycle of cement, considering its connection with investment in construction. The study found that cement is a natural leading indicator of a cycle of sectoral investment. From the estimation of the cyclical component of cement production and sectoral investment, both measured as the percentage gap with their tendencies, we conclude that the qualitative characteristics (in terms of timing) and the sizes of the gaps in the percentage cycle before and after their recessive stages are analogous. Moreover, both indicators recorded strong contractions regarding their trend measures during the 1998-99 and 2008-09 recessions. For now, there is no precise explanation for this close behavior between the cement indicator and sectoral investment. However, it is assumed that part of that could be the fact that cement is used highly concentrated in the first production stages of the investment, which is very sensitive to variations in the confidence of entrepreneurs and consumers about the economic outlook (Cámara Chilena de la Construcción 2015). The results of the gap before the recessions’ cycle and the gap in the cycle after them, are inconclusive as to whether the deepest recessions are usually preceded by booms. It is probable that this is more evident in the case of series that involve more recessive periods, such as the debt crisis of 1982 (Díaz-Alejandro 1985) and the 1990 recession, due to
changing government (Durán-Palma et al. 2005; Cerda and Vergara 2008). Finally, cement is a non-storable input in the medium and long-terms. Our results showed that the supply and demand of cement are relatively coincident, so indiscriminately both indicators measure activity in structural construction or structural work, which are in the first stages of the production chain in the sector (Cámara Chilena de la Construcción 2015).

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References


Siembieda, W., Johnson, L., Franco, G. (2012). Rebuild fast but rebuild better: Chile’s initial recovery following the 27 February 2010 earthquake and tsunami. Earthquake Spectra 28, S621-S641.
Table 1: Tons of Cement production and inputs (2009-2013). The test \(t\) (1) of equal average (\(p\) value = 0.991 monthly variance, \(p\) value = 0.159 annual variance), the Wilcoxon / Mann-Whitney (2) (Bergmann et al. 2000) of equal medians (\(p\) value = 0.974 monthly variance, \(p\) value = 0.170 annual variance), the \(F\) test (3) of equal variances (\(p\) value = 0.323 monthly variance, \(p\) value = 0.777 annual variance), and the normality test (4) (Jarque and Vera 1987) are calculated at 5% of significance.

<table>
<thead>
<tr>
<th>Levels</th>
<th>Monthly change</th>
<th>Annual change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Production</td>
<td>Offices</td>
</tr>
<tr>
<td>Mean(1)</td>
<td>374,361</td>
<td>359,124</td>
</tr>
<tr>
<td>Median(2)</td>
<td>377,377</td>
<td>356,298</td>
</tr>
<tr>
<td>Maximum</td>
<td>453,783</td>
<td>446,803</td>
</tr>
<tr>
<td>Minimum</td>
<td>305,420</td>
<td>275,145</td>
</tr>
<tr>
<td>Std. Dev.(3)</td>
<td>34,139</td>
<td>41,801</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.230</td>
<td>0.188</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.742</td>
<td>2.254</td>
</tr>
<tr>
<td>Jarque-Bera(4)</td>
<td>0.604</td>
<td>1.747</td>
</tr>
<tr>
<td>Probability</td>
<td>0.707</td>
<td>0.418</td>
</tr>
<tr>
<td>n</td>
<td>60</td>
<td>60</td>
</tr>
</tbody>
</table>
Table 2: Use of the product cement, lime and gypsum in the various activities of the economy (input-output matrix, reference compilation of National Accounts 2008 [Banco Central de Chile 2008]). The manufacture of concrete and other products is based on nonmetallic minerals (1). These activities are evaluated at basic prices (2).

<table>
<thead>
<tr>
<th>Activities</th>
<th>Million$^{(2)}$</th>
<th>Distribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper Mining</td>
<td>12,338</td>
<td>2.85</td>
</tr>
<tr>
<td>Other non-ferrous ores mining</td>
<td>7,657</td>
<td>1.77</td>
</tr>
<tr>
<td>Manufacturing cement, lime and plaster</td>
<td>25,147</td>
<td>5.80</td>
</tr>
<tr>
<td>Manufacturing concrete and other products$^{(1)}$</td>
<td>178,597</td>
<td>41.20</td>
</tr>
<tr>
<td>Building residential buildings</td>
<td>78,458</td>
<td>18.10</td>
</tr>
<tr>
<td>Nonresidential building construction</td>
<td>18,074</td>
<td>4.17</td>
</tr>
<tr>
<td>Construction civil engineering works</td>
<td>87,498</td>
<td>20.18</td>
</tr>
<tr>
<td>Specialized construction activities</td>
<td>15,872</td>
<td>3.66</td>
</tr>
<tr>
<td>Others</td>
<td>9,874</td>
<td>2.28</td>
</tr>
<tr>
<td>Total</td>
<td>433,514</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Source: Own calculations based on statistics from the Central Bank of Chile.
Table 3: Augmented Dickey-Fuller unit root test production ($x_t$) and offices ($y_t$) cement (01/2009-12/2013). $p$ values in different $y$-levels are calculated with intercept. The lags are based on the Hannan-Quinn test (Hannan and Quinn 1979). The series $x_t$ corresponds to cement (tonnes), $y_t$ corresponds to cement shipments (tonnes) and ($sa$) corresponds to the seasonally adjusted version via the EX-12 program the Central Bank of Chile (Bravo et al. 2002).

<table>
<thead>
<tr>
<th>Series</th>
<th>levels</th>
<th>$p$ values</th>
<th>Lag</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_t$</td>
<td>0.085</td>
<td>0.095</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>ln($x_t$)</td>
<td>0.095</td>
<td>0.095</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$y_t$</td>
<td>0.397</td>
<td>0.397</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>ln($y_t$)</td>
<td>0.554</td>
<td>0.554</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$x_{(sa)t}$</td>
<td>0.400</td>
<td>0.400</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>ln($x_{(sa)t}$)</td>
<td>0.362</td>
<td>0.362</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$y_{(sa)t}$</td>
<td>0.740</td>
<td>0.740</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>ln($y_{(sa)t}$)</td>
<td>0.746</td>
<td>0.746</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 4: Selection Criteria lags VAR model with endogenous variables: $\log(c_{new})$ and $\log(c_{old})$, and for the period 2009–13 ($n = 55$). The criteria used are: sequential modified LR test statistic (each test at 5% level), Akaike information criterion (AIC), Schwarz information criterion (SC), Hannan-Quinn information criterion (HQ); and where the lag order selected by the criterion is marked in bold.

<table>
<thead>
<tr>
<th>Lag</th>
<th>LogL</th>
<th>LR</th>
<th>AIC</th>
<th>SC</th>
<th>HQ</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>131.797</td>
<td>3.29e-05</td>
<td>-4.647</td>
<td>-4.501</td>
<td>-4.591</td>
</tr>
<tr>
<td>2</td>
<td>143.209</td>
<td>2.51e-05</td>
<td>-4.917</td>
<td>-4.625</td>
<td>-4.804</td>
</tr>
<tr>
<td>3</td>
<td>154.315</td>
<td>1.94e-05</td>
<td>-5.175</td>
<td>-4.737</td>
<td>-5.006</td>
</tr>
<tr>
<td>4</td>
<td>155.102</td>
<td>2.19e-05</td>
<td>-5.058</td>
<td>-4.474</td>
<td>-4.832</td>
</tr>
<tr>
<td>5</td>
<td>156.977</td>
<td>2.37e-05</td>
<td>-4.981</td>
<td>-4.251</td>
<td>-4.699</td>
</tr>
</tbody>
</table>
Table 5: VAR (3) residual serial correlation LM tests where the null hypothesis is not serial, the correlation at lag order 3, \( p \) values are related to chi-square distribution with 4 degrees of freedom, and \( n = 57 \).

<table>
<thead>
<tr>
<th>Lags</th>
<th>LM stat.</th>
<th>( p ) value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.467</td>
<td>0.483</td>
</tr>
<tr>
<td>2</td>
<td>4.160</td>
<td>0.385</td>
</tr>
<tr>
<td>3</td>
<td>4.250</td>
<td>0.373</td>
</tr>
<tr>
<td>4</td>
<td>6.499</td>
<td>0.165</td>
</tr>
<tr>
<td>5</td>
<td>5.632</td>
<td>0.228</td>
</tr>
<tr>
<td>6</td>
<td>5.234</td>
<td>0.264</td>
</tr>
<tr>
<td>7</td>
<td>6.614</td>
<td>0.158</td>
</tr>
<tr>
<td>8</td>
<td>2.024</td>
<td>0.731</td>
</tr>
<tr>
<td>9</td>
<td>1.184</td>
<td>0.881</td>
</tr>
<tr>
<td>10</td>
<td>2.570</td>
<td>0.632</td>
</tr>
</tbody>
</table>
Table 6: Unrestricted cointegration rank test (trace and maximum eigenvalue). Trace indicates test 1 cointegrating eqn(s) at the 0.05 level. (1) Denotes rejection of the hypothesis at the 0.05 level. \( p \) values correspond to MacKinnon-Haug-Michelis test (MacKinnon et al. 1999).

<table>
<thead>
<tr>
<th>Test</th>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>( J ) Statistic</th>
<th>Critical Value</th>
<th>( p ) values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trace</td>
<td>None(^{(1)})</td>
<td>0.272</td>
<td>19.960</td>
<td>15.495</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>At most 1</td>
<td>0.039</td>
<td>2.196</td>
<td>3.841466</td>
<td>0.138</td>
</tr>
<tr>
<td>Maximum eigenvalue</td>
<td>None(^{(1)})</td>
<td>0.272</td>
<td>17.764</td>
<td>14.265</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>At most 1</td>
<td>0.039</td>
<td>2.196</td>
<td>3.842</td>
<td>0.138</td>
</tr>
</tbody>
</table>
Table 7: Normalized vector cointegration.

<table>
<thead>
<tr>
<th>ln $x_t$</th>
<th>$\hat{\varphi}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\alpha}$</th>
<th>$R^2$</th>
<th>DW</th>
<th>$\hat{\sigma}_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-4.635</td>
<td>-0.641</td>
<td>0.67</td>
<td>1.51</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>S.E.</td>
<td>0.751</td>
<td>0.059</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t$ value</td>
<td>6.175</td>
<td>10.917</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 8: Estimation of univariate and bivariate unobserved components with cement production models.

<table>
<thead>
<tr>
<th>Model</th>
<th>parameters</th>
<th>Est.</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Univariate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\nu$</td>
<td>0.020</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.025</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>0.000</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.116</td>
<td>0.112</td>
<td></td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.311</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>176.748</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td><strong>Bivariate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>0.019</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.021</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\lambda$</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.283</td>
<td>0.142</td>
<td></td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.412</td>
<td>0.091</td>
<td></td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>1.093</td>
<td>0.258</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.105</td>
<td>0.086</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
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</tr>
<tr>
<td>$\sigma_\nu$</td>
<td>0.013</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>$\sigma_\varepsilon$</td>
<td>0.012</td>
<td>0.003</td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>477.581</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
Table 9: Characteristics of past recessions. Events marked with (1) consider the 2010 earthquake (Siembieda et al. 2012).

<table>
<thead>
<tr>
<th>Years</th>
<th>event</th>
<th>cement</th>
<th>construction investment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1998</td>
<td>maximum date</td>
<td>1998-II</td>
<td>1998-II</td>
</tr>
<tr>
<td></td>
<td>minimum date</td>
<td>1999-III</td>
<td>1999-II</td>
</tr>
<tr>
<td></td>
<td>return</td>
<td>2001-I</td>
<td>2001-I</td>
</tr>
<tr>
<td></td>
<td>duration</td>
<td>5 quarters</td>
<td>4 quarters</td>
</tr>
<tr>
<td></td>
<td>duration recovery</td>
<td>6 quarters</td>
<td>7 quarters</td>
</tr>
<tr>
<td></td>
<td>gap max-min</td>
<td>23.18%</td>
<td>21.39%</td>
</tr>
<tr>
<td></td>
<td>gap cycle before</td>
<td>7.84%</td>
<td>7.33%</td>
</tr>
<tr>
<td></td>
<td>gap cycle after</td>
<td>-8.23%</td>
<td>-7.12%</td>
</tr>
<tr>
<td>2008–09</td>
<td>maximum date</td>
<td>2008-III</td>
<td>2008-II</td>
</tr>
<tr>
<td></td>
<td>minimum date</td>
<td>2009-II</td>
<td>2009-II</td>
</tr>
<tr>
<td></td>
<td>estimated return</td>
<td>2009-IV</td>
<td>2009-IV</td>
</tr>
<tr>
<td></td>
<td>return(1)</td>
<td>2010-IV</td>
<td>2010-IV</td>
</tr>
<tr>
<td></td>
<td>duration</td>
<td>3 quarters</td>
<td>4 quarters</td>
</tr>
<tr>
<td></td>
<td>duration recovery</td>
<td>2 quarters</td>
<td>2 quarters</td>
</tr>
<tr>
<td></td>
<td>duration recovery(1)</td>
<td>6 quarters</td>
<td>6 quarters</td>
</tr>
<tr>
<td></td>
<td>gap max-min</td>
<td>16.55%</td>
<td>11.20%</td>
</tr>
<tr>
<td></td>
<td>gap cycle before</td>
<td>9.04%</td>
<td>7.80%</td>
</tr>
<tr>
<td></td>
<td>gap cycle after</td>
<td>-3.51%</td>
<td>-2.43%</td>
</tr>
</tbody>
</table>
Figure 1: a) Correlogram residue or difference between inputs and cement (01/2009–12/2013). b) $p$ values of Ljung-Box test.
Figure 2: Adjustment of cement production ($x_t$, tonnes) from the cointegration regarding cement sales ($y_t$, tonnes), with a quarterly frequency for the period 2009-13.
Figure 3: Backward projection of cement production (in tonnes) on a monthly basis for the period 1991-2015.
Figure 4: Cross-correlation function (CCF) between the cycle of production of cement and lags-lead main indicators of National Accounts.
Figure 5: a) cyclical component of cement production (tons percentage points from the trend), b) cement (tonnes) and its trend component, c) cyclical component of cement production (tons, points percentage from trend), d) investment in construction (millions of pesos of 2008) and its trend component.
Figure 6: Cycles and annual growth rates (%) of investment in construction and production of cement.