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Abstract

An anchoring adjusted Capital Asset Pricing Model (ACAPM) is developed in which the payoff volatilities of well-established stocks are used as starting points that are adjusted to form volatility judgments about other stocks. Anchoring heuristic implies that such adjustments are typically insufficient. ACAPM converges to CAPM with correct adjustment, so CAPM is a special case of ACAPM. The model provides a unified explanation for the size, value, and momentum effects in the stock market. A key prediction of the model is that the equity premium is larger than what can be justified by market volatility. Hence, anchoring also provides a potential explanation for the well-known equity premium puzzle. Anchoring approach predicts that stock splits are associated with positive abnormal returns and an increase in return volatility. The approach predicts that reverse stock-splits are associated with negative abnormal returns, and a fall in return volatility. Existing empirical evidence strongly supports these predictions.

Keywords: *Size Premium, Value Premium, Behavioral Finance, Stock Splits, Equity Premium Puzzle, Anchoring Heuristic, CAPM, Asset Pricing*

JEL Classification: *G12, G11, G02*

Anchoring Adjusted Capital Asset Pricing Model

Finance theory predicts that risk adjusted returns from all stocks must be equal to each other. The starting point for thinking about the relationship between risk and return is the Capital Asset Pricing Model (CAPM) developed in Sharpe (1964), and Lintner (1965). CAPM proposes that beta is the sole measure of priced risk. If CAPM is correct then the beta-adjusted returns from all stocks must be equal to each other. A large body of empirical evidence shows that beta-adjusted stock returns are not equal but vary systematically with factors such as "size" and "value". Size premium means that small-cap stocks tend to earn higher beta-adjusted returns than large-cap stocks.[1] Value premium means that value stocks tend to outperform growth stocks.[2] Value stocks are those with high book-to-market value. They typically have stable dividend yields. Growth stocks have low book-to-market value and tend to reinvest a lot of their earnings. Value stocks are typically less volatile than growth stocks. Fama-French (FF) value and growth indices (monthly returns data from July 1963 to April 2002) show the following standard deviations: FF small value: 19.20%, FF small growth: 24.60%, FF large value: 15.39%, and FF large growth: 16.65%. That is, among both small-cap and large-cap stocks, value stocks are less volatile than growth stocks.

Intuitively, the value premium is even more surprising than the size premium as it is plausible to argue that small size means greater risk with size premium being compensation for greater risk; however, how can less volatility be more risky as the value premium seems to suggest?[3]

The existence of size and value premiums has led to a growing body of research that attempts to explain them. In particular, there is the empirical asset pricing approach of Fama and French (1993) in which these factors are taken as proxies for risks with the assumption that all risks are correctly priced.[4] The task then falls to the asset pricing branch of theory to explain the sources of these risks.

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[1] Size effect is documented in Banz (1981), Reinganum (1981), Blume and Stambaugh (1983), Brown et al (1983), and more recently in Hou, Xue, and Zhang (2014), and Fama and French (2015) among many others.
[2] Value premium is documented in Fama and French (1998) among many others.
[3] Researchers appeal to other dimensions of risk different from volatility in attempts to explain value premium. However, no consensus explanation exists as to the source of value premium.
[4] Recently Fama and French (2015) show that value premium is also captured by adding "investment" and "profitability" factors to size and beta factors.

Apart from size and value, there also exists, what is known as, the momentum effect in the stock market. Momentum effect refers to the tendency of "winning stocks" in recent past to continue to outperform "losing stocks" for an intermediate horizon in the future. Momentum effect has been found to be a robust phenomenon, and can be demonstrated with a number of related definitions of "winning" and "losing" stocks. Jegadeesh and Titman (1993) show that stock returns exhibit momentum behavior at intermediate horizons. A self-financing strategy that buys the top 10% and sells the bottom 10% of stocks ranked by returns during the past 6 months, and holds the positions for 6 months, produces profits of 1% per month. George and Hwang (2004) define "winning' stocks as having price levels close to their 52-week high, and "losing" stocks as those with price levels that are farthest from their 52-week high, and show that a self-financing strategy that shorts "losing" stocks and buys "winning" stocks earns abnormal profits over an intermediate horizon (up to 12 months) consistent with the momentum effect.

The existence of size, value, and momentum effects clearly show that CAPM falls significantly short in explaining the cross-section of market returns. It implies that at least one key assumption in CAPM is wrong. Which assumption could that be? Finance and economics literature has largely been focused on relaxing the assumption that investors consider only means and variances of payoffs while forming portfolios, and that, in the real world, there are other aspects of risk which are not captured by the simple mean-variance framework of CAPM.

In this article, I add to this literature by focusing on and relaxing another assumption of CAPM. CAPM assumes that investors are able to form correct expectations about future payoffs and their corresponding future volatilities for every stock in the market. This is a rather strong assumption especially given the fact that not all stocks are created equal. Some stocks have been around for decades and belong to well-known and well-established companies while others are relative new comers. Market participants are acutely aware of this fact, and this difference is reflected in the terminology used to classify stocks. In particular, in market parlance, there are blue chip stocks, which are stocks of well-known, well-established, financially strong companies with large cash flows. The term blue chip has its origins in poker in which the most valuable chips are known as the blue chips. As the poker analogy suggests, blue chips stocks have large market capitalizations (often in billions) and are often household names. Every sector of the economy has its own blue chip stocks, however, they commonly receive a disproportionate amount of analysts' coverage and investor attention and their business models are presumably better understood.

This article puts forward a modified version of CAPM which assumes that investors use the payoff volatilities of well-established stocks with large market capitalizations as starting points which are then adjusted to form volatility judgments about other stocks. Starting from Kahneman and Tversky (1974), over 40 years of research shows that people have a tendency to start from what they know and make adjustments to it to form judgments. However, adjustments typically fall short. This observation is known as the anchoring bias (see Furnham and Boo (2011) for a literature review). Adjustments are typically insufficient because people tend to stop adjusting once a plausible value is reached (see Epley and Gilovich (2006)). Hence, assessments remain biased towards the starting value known as the anchor.

I show that anchoring adjusted CAPM (ACAPM) provides a unified explanation for the size, value, and momentum effects in the stock market. Of course, it is impossible to prove conclusively that any one explanation is correct. The purpose of this article is to demonstrate that anchoring must be considered a plausible explanation for the size, value, and momentum effects. ACAPM converges to CAPM if the adjustments made to volatilities of well-established stocks to arrive at volatility judgments of other stocks are correct. If ACAPM converges to CAPM, the size, value, and momentum effects disappear. If ACAPM deviates from CAPM, the size, value, and momentum effects re-emerge. So, CAPM can be considered a special case of ACAPM corresponding to correct adjustments. Furthermore, ACAPM approach makes the following predictions: 1) stock splits generate positive abnormal returns and an increase in return volatility, 2) reverse stock splits generate negative abnormal returns and a fall in return volatility, and 3) the equity premium is larger than what market volatility suggests. Existing empirical evidence strongly confirms all three predictions.

Hirshleifer (2001) considers anchoring to be an *"important part of psychology based dynamic asset pricing theory in its infancy"* (p. 1535). Shiller (1999) argues that anchoring appears to be an important concept for financial markets. This argument has been supported quite strongly by recent empirical research on financial markets. Anchoring has been found to matter for credit spreads that banks charge to firms (Douglas et al (2015), it matters in determining the price of target firms in mergers and acquisitions (Baker et al (2012), and it also affects the earnings forecasts made by analysts in the stock markets (Cen et al (2013)). Furthermore, Siddiqi (2015) shows that anchoring provides a unified explanation for a number of key puzzles in options market.

Well-established stocks, which are typically big-cap or large-cap stocks, constitute a small fraction of the total number of firms whose stocks are traded. In the US market, less than 4% of the stocks are classified as large-cap, however, they receive a substantially greater amount of attention from full-time professional stock analysts. A study suggests that roughly 83% of analysts cover large-cap stocks, which are less than 4% of the stocks, leaving only 17% analysts for the remaining 96%.[5]

Arguably, this disproportionate interest is partly due to the fact that well-established firms have a long history behind them, That is, there is sufficiently rich dataset available to study and analyze. Smaller firms, with much shorter histories, have not been around long enough to generate a rich dataset. Whatever the reason, the substantially smaller relative attention that they get does not make them any easier to value. Imagine one is interested in Cisco system's stock in February 1990. Cisco in 2015 is a network technology giant and considered a blue-chip stock with over 30 years of history behind it. However, back in 1990, its stock was launched at a price of 6 cents (on split-adjusted basis). Not much was known about Cisco in 1990, then only 6 years old, in the relevant market segment largely dominated by IBM. How would one go about forming a judgment about Cisco's stock in 1990? Where-else would one start if not by looking at the performance of the established market leader at that time, which was IBM, and attempt to make appropriate adjustments for much smaller size, greater riskiness, and growing nature of the new firm? Of course, with time, the business model of Cisco was better understood; however, the firm also grew and now is among large-cap blue chip stocks. Other start-ups and relative new comers now occupy the same spot that Cisco had in 1990. And, arguably, just like for Cisco in 1990, for these newer small companies, one may start from Cisco's stock and attempt to make appropriate adjustments to form relevant judgments. The point is that a given firm may go through several classifications over its lifetime. A small-cap stock of yesterday, if it does not go bust, may be a large-cap stock of today, with newer small cap stocks taking its place. The identities of firms within the categories of large-cap and small-cap change, but the percentages of stocks in each category remain more or less the same. So, the impact of the anchoring bias may never disappear, as there will always be small-cap stocks that are valued by making adjustments to large-cap stocks. Learning may alleviate the bias in the stock of a particular small company if it does not go bust, but the time it takes to do that, may mean a classification change to large-cap stock, with some other small-cap taking its place.

[5] http://punchinvest.com/wp-content/uploads/2011/11/The-Blind-Spot-Impact-of-Analyst-Coverage1.pdf

Anchoring is among the most deep rooted cognitive biases, and short of getting one's hands on a crystal ball that reveals future payoff volatilities of all stocks, it is hard to see how one can escape from it. For a typical stock, forming judgments about future payoff volatility is essentially forming judgments about something which is largely unknowable. When faced with this task, the obvious thing to do is to start from what one knows and make adjustments to it. Where else can one start if not from a well-known and a well-established stock in the same sector? Plausibly one starts from there and then make adjustments. If adjustments happen to be correct, then the size, value, and momentum effects disappear and CAPM becomes the correct model. If adjustments fall short, the size, value, and momentum effects emerge. Anchoring heuristic implies that adjustments typically fall short.

To my knowledge, adjusting CAPM for anchoring is the smallest deviation from its basic framework that allows one to capture the size, value, and momentum effects. By this criterion, it offers the simplest explanation.

This article is organized as follows. Section 2 develops ACAPM. Section 3 shows that effects akin to size, value, and momentum arise with ACAPM. Section 4 provides a numerical illustration of ACAM vs. CAPM. The implications of ACAPM for the equity premium puzzle are discussed in section 5. Section 6 discusses the predictions of the anchoring approach for stock-splits and reverse stock-splits. Section 7 concludes.

2. Anchoring Adjusted CAPM

Consider an overlapping-generations (OLG) economy in which agents are born each period and live for two periods. For simplicity, in the beginning, I assume that they trade in the stocks of two firms and invest in a risk-free asset. One firm is well-established with large payoffs (the leader firm), and the second firm is a relative new-comer with much smaller payoffs (normal firm). The next period payoff per share of the leader firm is denoted by $\delta_{Lt+1} = P_{Lt+1} + d_{Lt+1}$ where P_{Lt+1} is the next period share price and d_{Lt+1} is the per share dividend of the large firm. Similarly, the next period payoff per share of the normal firm is defined by $\delta_{St+1} = P_{St+1} + d_{St+1}$. The risk-free rate of return is r_F and $\delta_{Lt+1} \gg \delta_{St+1}$. At time t, each agent chooses a portfolio of stocks and the risk-free asset to maximize his utility of wealth at $t+1$. There are no transaction costs, taxes, or borrowing constraints.

The market dynamics are described by a representative agent who maximizes utility:

$$n_L\{E_t(\delta_{Lt+1}) - (1+r_F)P_{Lt}\} + n_S\{E_t(\delta_{St+1}) - (1+r_F)P_{St}\} - \frac{\gamma}{2}\{n_L^2\sigma_L^2 + n_S^2\sigma_S^2 + 2n_L n_S\sigma_{LS}\}$$

where $n_L, n_S, and\ \gamma$ denote the number of shares of the leader firm, the number of shares of the normal firm, and the risk aversion parameter respectively. Next period variances of the leader firm and the normal firm payoffs per share are $\sigma_L^2 = Var(\delta_{Lt+1})$ and $\sigma_S^2 = Var(\delta_{St+1})$ respectively with $\sigma_L^2 > \sigma_S^2$, and σ_{LS} denotes their covariance. Note, that payoff variances are different from return variances. The payoff variance of the normal firm's stock, σ_S^2, is smaller than the payoff variance of the leader firm's stock, σ_L^2, because of the much smaller size of its payoffs. In contrast, the return variance of the normal firm is much larger than the return variance of the large firm's stock because of the smaller share price of the normal firm. That is why, when considering variances, it is important to be clear whether one is considering the payoff variance or the return variance. To see this clearly, consider an example. Suppose the possible payoffs of the leader firm stock, in the next period, are 300, 350, and 400 with equal chance of each. The variance of these payoffs can be calculated easily and is equal to 1666.667. In a risk-neutral world, with zero risk-free interest rate, the price must be 350, so corresponding (gross) returns are: 0.857, 1, 1.143. The return variance is 0.010. Assume that the next period payoffs of the normal firm are 0, 35, and 70. The variance of these payoffs is 816.667. The risk neural price (with zero risk-free rate) is 35 leading to possible returns of 0, 1, and 2. The corresponding return variance is 0.66. As can be seen in this example, the payoff variance of the normal firm stock is smaller than the payoff variance of the leader firm stock, whereas the return variance of the normal firm is much larger.

The first order conditions of the maximization problem are:

$$E_t(\delta_{Lt+1}) - (1+r_F)P_{Lt} - \gamma n_L\sigma_L^2 - \gamma n_S\sigma_{LS} = 0 \quad (1)$$

$$E_t(\delta_{St+1}) - (1+r_F)P_{St} - \gamma n_S\sigma_S^2 - \gamma n_L\sigma_{LS} = 0 \quad (2)$$

Solving (1) and (2) for prices yields:

$$P_{Lt} = \frac{E_t(\delta_{Lt+1}) - \gamma n_L \sigma_L^2 - \gamma n_S \sigma_{LS}}{1 + r_F}$$

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma n_S \sigma_L^2 - \gamma n_L \sigma_{LS}}{1 + r_F}$$

If the number of shares of the leader firm outstanding is n_L', and the number of shares of the normal firm outstanding is n_S', then the equilibrium prices are:

$$P_{Lt} = \frac{E_t(\delta_{Lt+1}) - \gamma n_L' \sigma_L^2 - \gamma n_S' \sigma_{LS}}{1 + r_F} \quad (3)$$

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma n_S' \sigma_S^2 - \gamma n_L' \sigma_{LS}}{1 + r_F} \quad (4)$$

Next, I show how anchoring alters the above equilibrium.

Suppose, the representative agent is unsure about the variance of the normal firm's payoffs, and to form his judgment, he starts from the variance of the leader firm and subtracts from it. I continue to assume that his judgments about the covariance and expected payoff are correct. Note, that covariance and expected payoff vary linearly with size, whereas variance varies non-linearly, so one is more likely to make errors in estimating variance when size varies. Alternatively, one can assume that erroneous judgments are formed for covariance and expected payoff as well as the variance with a relatively larger error in variance judgment, without any change in the results that follow. So, for simplicity and clarity of exposition, I choose to assume that there is no error in covariance and expected payoff judgments.

The agent knows that as the normal firm has smaller payoffs, its payoff variance must also be smaller. So, he starts from the variance of the leader firm and subtracts from it to form his judgment about the normal firm's variance: $\hat{\sigma}_S^2 = \sigma_L^2 - A$. If he makes the correct adjustment, then $A = \sigma_L^2 - \sigma_S^2$. Anchoring bias implies that the adjustment falls short. That is, $A = m(\sigma_L^2 - \sigma_S^2)$ with $0 < m < 1$. Hence, $\hat{\sigma}_S^2 = (1 - m)\sigma_L^2 + m\sigma_S^2$. Note, if the adjustment is correct then $m = 1$.

With such anchoring, the equilibrium price of the normal firm falls, however, the equilibrium price of the leader firm remains unchanged.

The equilibrium price of the normal firm with anchoring is:

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma n_S' m\sigma_S^2 - \gamma n_S'(1-m)\sigma_L^2 - \gamma n_L'\sigma_{LS}}{1+r_F} \tag{5}$$

Adding and subtracting $\gamma n_S'\sigma_S^2$ to the numerator of the above equation and using $cov(\delta_{St+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1}) = n_L'\sigma_{LS} + n_S'\sigma_S^2$ leads to:

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma[cov(\delta_{St+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1}) + n_S'(1-m)(\sigma_L^2 - \sigma_S^2)]}{1+r_F} \tag{6}$$

Not only the price in (6) is smaller than in (4), but there is also another interesting aspect to it. The impact of anchoring is larger, smaller the actual payoff volatility of the normal firm's stock. That is, keeping all else the same, higher actual payoff volatility lowers the impact of anchoring. We will see shortly that this provides a potential explanation for the value premium. Of course, with correct adjustment, that is, with $m = 1$, there is no anchoring and (6) reduces to (4).

Expressing (6) in terms of the expected stock return leads to:

$$E_t(r_S) = r_F + \frac{\gamma}{P_{St}}[cov(\delta_{St+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1}) + n_S'(1-m)(\sigma_L^2 - \sigma_S^2)] \tag{7}$$

Anchoring does not change the share price of the leader firm. By re-arranging (3), the expected return expression for the stock price of the leader firm is obtained:

$$E_t(r_L) = r_F + \frac{\gamma}{P_{Lt}}[cov(\delta_{Lt+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1})] \tag{8}$$

Expected return on the total market portfolio is obtained by multiplying (7) by $\frac{n_S'P_{St}}{n_L'P_{Lt}+n_S'P_{ST}}$

and (8) by $\frac{n_L'P_{St}}{n_L'P_{Lt}+n_S'P_{ST}}$ and adding them:

$$E_t[r_M] = r_F + \frac{\gamma}{n_L' P_{Lt} + n_S' P_{St}} [Var(n_L' \delta_{Lt+1} + n_S' \delta_{St+1}) + n_S'^2 (1-m)(\sigma_L^2 - \sigma_S^2)] \qquad (9)$$

Proposition 1 ***The expected return on the market portfolio with anchoring is larger than the expected return on the market portfolio without anchoring.***

Proof.

Follows directly from (9) by realizing that with anchoring P_{St} is smaller than what it would be without anchoring, and the second term, $n_S'^2(1-m)(\sigma_L^2 - \sigma_S^2)$, which is positive with anchoring is equal to zero without anchoring. ■

Equation (9) has implications for the equity premium puzzle put forward in Mehra and Prescott (1985). We will see in section 4 that anchoring offers at least a partial explanation for the puzzle.

From (9), one can obtain an expression for the risk aversion coefficient, γ, as follows:

$$\gamma = \frac{(E_t[r_M] - r_F) \cdot (n_L' P_{Lt} + n_S' P_{St})}{Var(n_L' \delta_{Lt+1} + n_S' \delta_{St+1}) + n_S'^2 (1-m)(\sigma_L^2 - \sigma_S^2)} \qquad (10)$$

Substituting (10) in (7) and (8) and using $P_{Mt} = n_L' P_{Lt} + n_S' P_{St}$ leads to:

$$E_t(r_S) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_S, r_M) + \frac{n_S'(1-m)(\sigma_L^2 - \sigma_S^2)}{P_{St} P_{Mt}}}{Var(r_M) + \frac{n_S'^2(1-m)(\sigma_L^2 - \sigma_S^2)}{P_{Mt}^2}} \qquad (11)$$

$$E_t(r_L) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_L, r_M)}{Var(r_M) + \frac{n_S'^2(1-m)(\sigma_L^2 - \sigma_S^2)}{P_{Mt}^2}} \qquad (12)$$

Equations (11) and (12) are the expected return expressions for the normal stock and the leader stock respectively with anchoring. They give the expected return under the anchoring adjusted CAPM (ACAPM). It is straightforward to see that substituting $m = 1$ in (11) and (12) leads to the classic CAPM expressions. That is, without anchoring ACAPM converges to CAPM, with beta being the only price risk factor, $\beta_S = \frac{Cov(r_S,r_M)}{Var(r_M)}$, and $\beta_L = \frac{Cov(r_L,r_M)}{Var(r_M)}$.

Proposition 2 shows that anchoring implies a small-size premium.

Proposition 2 ***There is a small-size premium with anchoring, as the beta-adjusted excess return on the normal stock is larger than the beta-adjusted excess return on the leader stock.***

Proof.

From (12):

$$\frac{E_t[r_L - r_F]}{\frac{Cov(r_L, r_M)}{Var(r_M)}} < E_t[r_M - r_F]$$

and from (11)

$$\frac{E_t[r_S - r_F]}{\frac{Cov(r_s, r_M)}{Var(r_M)}} > E_t[r_M - r_F]$$

Hence, the beta-adjusted excess return on the normal stock must be larger than the beta-adjusted excess return on the leader stock.

■

In the next two sub-sections, the above results are generalized. In section 2.1, the results are generalized to include a large number of normal firms while keeping the number of leader firm at one. In section 2.2, the results are generalized to include a large number of leader firms as well. We will see that effects similar to size, value, and momentum arise naturally with anchoring.

2.1 Anchoring adjusted CAPM with many normal firms

It is straightforward to extend the anchoring approach to a situation in which there are a large number of normal firms. I make the further assumption that all stocks have positive CAPM-betas. This simplifies the following discussion considerably. In general, stocks almost always move with the market, and it is rare to find a stock that has a negative beta. Equation (6) remains unchanged. However, equation (9) changes slightly to the following:

$$E_t[r_M] = r_F + \frac{\gamma}{P_{Mt}}\left[Var(\delta_{Mt+1}) + \sum_{i=1}^{k} n'^2_{Si}(1-m)(\sigma_L^2 - \sigma_{Si}^2)\right] \tag{13}$$

where δ_{Mt+1} is the payoff associated with the aggregate market portfolio in the next period, and k is the number of normal firms in the market.

From (13), it follows that:

$$\gamma = \frac{(E_t[r_M] - r_F)\cdot(P_{Mt})}{Var(\delta_{Mt+1}) + \sum_{i=1}^{k} n'^2_{Si}(1-m)(\sigma_L^2 - \sigma_{Si}^2)} \tag{14}$$

The corresponding expression for a normal firm j's expected return can be obtained by substituting (14) in (7):

$$E_t(r_{Sj}) = r_F + E_t[r_M - r_F]\cdot\frac{Cov(r_{Sj}, r_M) + \dfrac{n'_{Sj}(1-m)(\sigma_L^2 - \sigma_{Sj}^2)}{P_{Sjt}P_{Mt}}}{Var(r_M) + \sum_{i=1}^{k}\dfrac{n'^2_{Si}(1-m)(\sigma_L^2 - \sigma_{Si}^2)}{P_{Mt}^2}} \tag{15}$$

The corresponding expression for the leader firm is obtained by substituting (14) in (8):

$$E_t(r_L) = r_F + E_t[r_M - r_F]\cdot\frac{Cov(r_L, r_M)}{Var(r_M) + \sum_{i=1}^{k}\dfrac{n'^2_{Si}(1-m)(\sigma_L^2 - \sigma_{Si}^2)}{P_{Mt}^2}} \tag{16}$$

(15) and (16) provide the expected return expressions corresponding to a situation in which there are a large number of normal firms and one leader firm. It is straightforward to check that in the absence of the anchoring bias, that is, when $m = 1$, the anchoring model converges to the classic

CAPM expressions of expected returns. In the next section, I generalize the model to include a large number of leader firms as well.

2.2 ACAPM with Q leader firms and $Q \times k$ normal firms

It is natural to expect that every sector has its own leader firm whose stock is used as a starting point to form judgments about other firms in the same sector. I assume that there are Q sectors and every sector has one leader firm. I assume that the number of normal firms in every sector is k. That is, the total number of normal firms in the market is $Q \times k$. As the total number of leader firms is Q. The total number of all firms (both leader and normal) in the market is $Q + (Q \times k)$.

Following a similar set of steps as in the previous two sections, the expected return expression for a normal firm j in sector $q \in Q$ is given by:

$$E_t(r_{qSj}) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_{qSj}, r_M) + \frac{n'_{qSj}(1-m)(\sigma_{qL}^2 - \sigma_{qSj}^2)}{P_{qSjt}P_{Mt}}}{Var(r_M) + \sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{n'^2_{qSi}(1-m)(\sigma_{qL}^2 - \sigma_{qSi}^2)}{P_{Mt}^2}} \tag{17}$$

The corresponding expression for the leader firm in sector q is given by:

$$E_t(r_{qL}) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_{qL}, r_M)}{Var(r_M) + \sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{n'^2_{qSi}(1-m)(\sigma_{qL}^2 - \sigma_{qSi}^2)}{P_{Mt}^2}} \tag{18}$$

(17) and (18) are the relevant expected return expressions under ACAPM. As before, it is easy to see that, in the absence of the anchoring bias, that is, if $m = 1$, ACAPM expressions converge to the classic CAPM expressions.

3. The Size, Value, and Momentum Effects under ACAPM

In equations (17) and (15), effects that correspond to the well documented size, value, and momentum premiums can be seen.

Size premium means that beta-adjusted excess returns on small-cap stocks are larger than beta-adjusted excess returns on large-cap stocks. To demonstrate the existence of a size effect in ACAPM, we need to see whether beta-adjusted excess returns on smaller-size normal firms are bigger than the beta-adjusted excess returns on relatively larger-size normal firms. In a given cross-section of firms, one expects that large-cap stocks have larger payoff covariance with aggregate market payoff when compared with small-cap stocks. This is due to the relatively larger size of their payoffs. Note that payoff covariance is different from return covariance. One expects large-cap stocks to have lower return covariance with the market return when compared with small-cap stocks. This is due to their larger prices. Similarly, one expects that large-cap stocks have larger payoff variance due to their larger payoff size, while having smaller variance of returns due to their larger prices.

Proposition 3 shows that larger payoff covariance and payoff volatility correspond to lower beta-adjusted returns under ACAPM.

Proposition 3 (The Size Premium):

Beta-adjusted excess returns with anchoring fall as payoff covariance with the market and payoff volatility increase in a given cross-section of stocks. In the absence of the anchoring bias, beta-adjusted excess returns do not change as payoff covariance and payoff volatility change, and are always equal to the market risk premium.

Proof.

$$BetaAdjusted\ excess\ return = \frac{E(r_{qSj}) - r_F}{\frac{Cov(r_{qSj}, r_M)}{Var(r_M)}}$$

Substituting from (17) and re-arranging leads to:

$BetaAdjusted\ excess\ return$

$$= \left[\frac{Var(r_M) \cdot E_t[r_M - r_F]}{Var(r_M) + \sum_{q=1}^{Q} \sum_{i=1}^{k} \frac{n_{qSi}'^2 (1-m)\left(\sigma_{qL}^2 - \sigma_{qSi}^2\right)}{P_{Mt}^2}} \right] \cdot \left[1 + \frac{n_{qSj}'(1-m)\left(\sigma_{qL}^2 - \sigma_{qSj}^2\right)}{P_{qSjt} P_{Mt} \cdot Cov(r_{qSj}, r_M)} \right]$$

That is, beta-adjusted excess return can be written in the form:

$$BetaAdjusted\ excess\ return = [h] \cdot [1 + g]$$

where $g = \frac{n_{qSj}'(1-m)\left(\sigma_{qL}^2 - \sigma_{qSj}^2\right)}{P_{qSjt} P_{Mt} \cdot Cov(r_{qSj}, r_M)} = \frac{n_{qSj}'(1-m)\left(\sigma_{qL}^2 - \sigma_{qSj}^2\right)}{Cov(stock's\ payoff, market\ payoff)}$

and $h = \frac{Var(r_M) \cdot E_t[r_M - r_F]}{Var(r_M) + \sum_{q=1}^{Q} \sum_{i=1}^{k} \frac{n_{qSi}'^2 (1-m)\left(\sigma_{qL}^2 - \sigma_{qSi}^2\right)}{P_{Mt}^2}} = constant$

Clearly, as payoff covariance and payoff variance of a normal firm stock increase in the given cross-section of stocks, g falls. It follows that beta-adjusted excess returns must fall as payoff covariance and payoff variance increase when there is anchoring bias. In the absence of anchoring bias, that is, with $m = 1$, it follows that $g = 0$ and $h = E[r_M - r_F]$. Hence, in the absence of anchoring, beta-adjusted excess return does not change with payoff covariance and payoff variance, and remain equal to the market risk premium.

■

Proposition 3 demonstrates that an effect equivalent to the size effect exists with anchoring adjusted CAPM as larger size firms typically have larger payoff covariance with market payoff and larger payoff volatilities, when compared with smaller size firms.

As mentioned in the introduction value premium means that value firms earn higher beta-adjusted excess returns than growth firms. Value firms have higher book-to-market ratios when compared with growth firms. Among firms of similar size, that is, firms having similar prices and the number of shares outstanding, a growth firm would have a lower book of value of equity due to its smaller asset base. As the name suggests, a growth firm is attempting to increase its asset base at a rapid pace. Consequently, it has higher payoff volatility. Keeping other things the same, higher payoff volatility reduces the impact of anchoring. Hence, an effect akin to the value premium naturally arises with anchoring as proposition 4 shows.

Proposition 4: (The Value Premium):

Controlling for size, beta-adjusted excess return on stocks with smaller payoff volatility is larger than the beta-adjusted excess returns on stocks with higher payoff volatility.

Proof.

Proof follows by following nearly the same steps as in the proof for proposition 3. Note, that $g = \frac{n'_{qSj}(1-m)\left(\sigma_{qL}^2 - \sigma_{qSj}^2\right)}{Cov(stock's\ payoff, market\ payoff)}$. As payoff volatility rises, g falls. It follows that beta-adjusted excess return falls as payoff volatility rises.

■

Corollary 4.1: (The Value Premium Falls with Size):

At higher level of payoff covariance with the market, the impact of an increase in payoff volatility on beta-adjusted excess returns is smaller.

Proposition 4 shows that value premium arises due to higher payoff volatility of growth firms. However, the relative impact of higher volatility is smaller at larger sizes. This is because the denominator in g gets bigger with size. So, if the anchoring approach is correct, one expects value premium to decline with size. Intriguingly, this is exactly what the empirical evidence suggests. Fama and French (2004) among others show that the value premium declines with size.

As proposition 2, proposition 3, and proposition 4 show, the size and value premiums arise naturally with anchoring. Proposition 5 shows that an effect similar to the momentum effect can also be seen in (17).

Proposition 5: (The Momentum Effect):

As "m" falls, that is, as anchoring gets stronger, beta-adjusted returns rise.

Proof.

Can be seen directly from (17) that if "m" falls, holding other parameters constant, beta-adjusted excess returns rise. ■

The effect described in proposition 5 can be described as the momentum effect. Stocks that have received unusually good news recently are "winning stocks", and stocks that have received unusually bad news recently are "losing stocks". Winning stocks are likely to get more strongly anchored to the leader stock as their recent success makes them more like the leader. For losing stocks, their recent bad spell makes them less like the leader. That is, "m" falls for winning stocks and rises for losing stocks. So, winning stocks continue to outperform losing stocks till the effect of differential news on "m" dissipates, and "m" returns to its normal level.

Without anchoring, ACAPM converges to CAPM, and the size, value, and momentum effects disappear. This can be seen directly from (17) by substituting $m = 1$ in (17). Hence, CAPM is a special case of ACAPM.

In the next section, ACAPM and CAPM are illustrated with a numerical example.

4. Anchoring Adjusted CAPM: A Numerical Example

Suppose there are four types of stocks with next period payoffs as shown in Panel A of Table 1. Type "Large" belongs to a large well-established firm with large cash flows. Types S1, S2, and S3 are smaller firms with equal expected payoffs, however, their payoff volatilities are 416.667, 216.667, and 66.667 respectively. That is, among the small firms, S1 is the most volatile, followed by S2, and then S3. Panel B of Table 1 shows the associated covariance matrix. The risk aversion parameter is assumed to be 0.001, and the one period risk-free rate is 0.01. Every type is assumed to have exactly one share outstanding.

Prices implied by CAPM can be calculated for each stock from (3) and (4) and are shown in Panel C of Table 1. Panel C also shows expected returns, the value of the aggregate market portfolio, the variance of the market portfolio's return, and the covariance of each stock's return with the market portfolio's return. Panel D shows each stocks beta and the corresponding beta-adjusted excess return. It can be seen that all stocks have the same beta-adjusted excess return, which is equal to the excess return on the market portfolio.

(**Insert Table 1 here)**

The key prediction of CAPM can be seen in the last line of Table 1. That is, beta-adjusted excess returns of all assets must be equal. In other words, beta is the only measure of priced risk in CAPM. And, investors are rewarded based on their exposure to beta-risk. Once beta-risk has been accounted for, there is no additional return.

Next, we will see what happens with anchoring. Table 2 shows the results from ACAPM. Everything is kept the same except that now anchoring is allowed in variance judgments. The anchoring prone marginal investor starts from the variance of the large firm and subtracts from it to form variance judgments about the small stocks. For the purpose of this illustration, I assume that he goes 90% of the way. That is, $m = 0.90$. As can be seen, price of the large firm does not change, however, the prices of small firms change, and can be calculated from (6). As expected, expected return on the large firm's stock does not change. However, as the market portfolio changes, all betas change. Expected returns on small firms can be calculated from (15).

Both the size and value premiums can be seen in Table 2. As can be seen from the table, beta-adjusted excess returns on small stocks are larger than the beta-adjusted excess return on the large stock. This is the size premium. Furthermore, among small firms, highest volatility S1 has the smallest excess return of 0.03425, whereas the lowest volatility S3 has the highest excess return of 0.039274. As value stocks typically have lower volatility than growth stocks, in this example, S3 is like a value stock, and S1 is like a growth stock. Hence, the value premium is also seen in this example. Next, the implications of ACAMP for the equity premium puzzle and two predictions of the model are discussed in the light of empirical evidence.

(**Insert Table 2 Here)**

5. The Equity Premium Puzzle

Since its identification in Mehra and Prescott (1985), a large body of research has explored what is known in the literature as the equity premium puzzle. It refers to the fact that historical average return on equities (around 7%) is so large when compared with the historical average risk-free rate (around 1%) that it implies an implausibly large value of the risk aversion parameter. Mehra and Prescott (1985) estimate that a risk-aversion parameter of more than 30 is required, whereas a much smaller value of only about 1 seems reasonable.

If one is unaware of the phenomenon of anchoring, and he uses CAPM to estimate risk-aversion then he would use the following equation:

$$E_t[r_M] = r_F + \frac{\gamma}{P_{Mt}}[Var(\delta_{Mt+1})] \tag{16}$$

By substituting for average return on the market portfolio, volatility of aggregate market payoff, the risk-free rate, and the market level in (16), one may recover the corresponding value of γ, the risk-aversion parameter. The equity premium puzzle, in the CAPM context, is that the recovered value of risk-aversion parameter is implausibly large.

A key prediction of ACAPM is that the average return on the market portfolio is larger than what can be justified by market volatility. That is, with CAPM adjusted for anchoring, the expected return on the market portfolio is given by:

$$E_t[r_M] = r_F + \frac{\gamma}{P_{Mt}}\left[Var(\delta_{Mt+1}) + \sum_{i=1}^{k} n_{Si}'^2(1-m)(\sigma_L^2 - \sigma_{Si}^2)\right] \tag{17}$$

A comparison of (17) and (16) shows that, with anchoring accounted for, a much smaller value of the risk-aversion parameter is required to justify the observed equity premium. Hence, ACAPM offers at least a partial explanation for the equity premium puzzle.

6. ACAPM, Stock-Splits, and Reverse Stock-Splits

A stock-split increases the number of shares proportionally. In a 2-for-1 split, a person holding one share now holds two shares. In a 3-for-1 split, a person holding one share ends up with three shares and so on. A reverse stock-split is the exact opposite of a stock-split. Stock-splits and reverse stock-splits appear to be merely changes in denomination, that is, they seem to be accounting changes only with no real impact on returns. With CAPM, stock-splits and reverse stock-splits do not change expected returns. To see this clearly, consider equation (4), which is reproduced below:

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma n'_S \sigma_S^2 - \gamma n'_L \sigma_{LS}}{1 + r_F} \tag{18}$$

A 2-for-1 split in the small firm's stock divides the expected payoff by 2, divides the variance by 4 and covariance by 2, while multiplying the number of shares outstanding by 2. That is, a 2-for-1 split leads to:

$${P_{St}}^{Split} = \frac{\frac{E_t(\delta_{St+1})}{2} - \gamma \frac{2n'_S \sigma_S^2}{4} - \frac{\gamma n'_L \sigma_{LS}}{2}}{1 + r_F} \tag{18}$$

It follows that ${P_{St}}^{Split} = \frac{P_{St}}{2}$.

That is, the price with split is exactly half of what the price would have been without the split. As both the expected payoff and the price are divided by two, there is no change in expected returns associated with a stock-split under CAPM.

With anchoring in volatility judgments, things change considerably. Before the split, the volatility of a well-established stock's payoffs is the starting point for forming judgments about the volatility of a small firm. However, stock-split (reverse stock-split) creates an even clearer starting point. A stock-split reduces the payoffs per share without changing anything else, and reverse stock-split increases the payoffs per share without any other impact on the firm. To an anchoring prone investor, his volatility judgment before the split (reverse stock-split) is a clear starting point for forming volatility judgments after the stock-split (reverse stock-split). An anchoring-prone investor uses his judgment about the variance of payoffs before the split (reverse-split) and reduces it (increases it) to form his volatility judgment after the stock-split (reverse-split). Anchoring heuristic leads to insufficient adjustment. This implies that stocks are underpriced after a split, and overpriced after a reverse split.

Using $\hat{\sigma}_S^2$ for volatility judgment before the split, a 2-for-1 split leads to dividing $\hat{\sigma}_S^2$ by an amount less than 4 due to the anchoring bias. Hence, the price after the split is given by:

$$P_{St}{}^{Split} = \frac{\frac{E_t(\delta_{St+1})}{2} - \gamma \frac{2n'_S\sigma_S^2}{f} - \frac{\gamma n'_L\sigma_{LS}}{2}}{1 + r_F} \tag{19}$$

with $f < 4$. Hence, $P_{St}{}^{Split} < \frac{P_{St}}{2}$. Similarly, anchoring implies the following for a reverse stock-split of 1-for-2: $P_{St}{}^{Reverse-Split} > 2P_{St}$.

In other words, if anchoring is a robust phenomenon then one expects to see positive abnormal returns after stock-splits and negative abnormal returns after reverse-splits. Furthermore, if the anchoring approach is correct, then one expects to see an increase in return volatility after the split. This is because the price falls more than the fall in expected payoffs causing an increase in volatility of returns. For reverse stock-splits, if the anchoring approach is correct, then one expects to see a fall in return volatility. This is because price rises more than payoffs causing a decline in return volatility.

Empirical evidence strongly supports the above predictions. Using data from 1975 to 1990, Ikenberry et al (1996) shows that stock-splits are associated with 8% positive abnormal returns after one year, and 16% abnormal returns over three years. Ikenberry et al (2003) uses data from 1990 to 1997 and confirms the earlier findings. Gharghori et al (2015) find that option market traders do expects an increase in return volatility after the split. Kim et al (2008) examine the long-run performance of 1600 firms with reverse stock-splits and reports negative abnormal returns. Illahi (2012) finds that return volatility decreases after a reverse stock-split. Hence, ACAPM provides a potential explanation for the empirical findings regarding stock-splits and reverse stock-splits.

7. Conclusions

In this article, an anchoring adjusted Capital Asset Pricing Model is put forward. Adjusting CAPM for anchoring provides a plausible unified explanation for the size, value, and momentum effects in the stock market. The anchoring model predicts that the expected return on the market portfolio must be larger than what can be justified by observed market volatility. This prediction is in line with the well-known equity premium puzzle. Hence, the anchoring approach provides at least a partial explanation for the puzzle. The anchoring approach also predicts that stock-splits have positive abnormal returns, and reverse stock-splits have negative abnormal returns. Existing empirical evidence strongly supports these predictions.

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Table 1 CAPM Returns and Prices $\gamma = 0.001, and\ r_F = 0.01$				
Panel A: Payoffs				
	Large	S1	S2	S3
	100	5	10	20
	150	30	30	30
	200	55	50	40
Expected Payoff	150	30	30	30
Panel B: The Covariance Matrix				
	Large	S1	S2	S3
Large	1666.667	833.333	666.667	333.333
S1	833.333	416.667	333.333	166.667
S2	666.667	333.333	216.667	133.333
S3	333.333	166.667	133.333	66.667

Panel C: CAPM Prices					
	Large	S1	S2	S3	Mkt Portfolio Value
Price	145.0495	27.9703	28.3168	29.0099	230.3465
Expected Returns	0.03413	0.0726	0.0594	0.03413	0.0419
Variance of Mkt Portfolio's Return	0.1385				
Covariance with Mkt Portfolio's Return	0.10475	0.2716	0.2146	0.10475	0.1385
Panel D: CAPM Beta and Beta-Adjusted Excess Returns					
CAPM Beta	0.75622	1.96081	1.54945	0.75622	1
Beta Adjusted Excess Returns	0.03191	0.03191	0.03191	0.03191	0.03191

Table 2 ACAPM Returns and Prices $\gamma = 0.001, r_F = 0.01, and\ m = 0.90$					
Panel A: ACAPM Prices					
	Large	S1	S2	S3	Mkt Portfolio
Price	145.0495	27.84653	28.18722	28.85149	229.9257
Expected Return	0.03413	0.0773	0.0647	0.0398	0.0438
Variance of Mkt Portfolio's Return	0.139031				
Covariance with Mkt Portfolio's Return	0.1049	0.2733	0.2161	0.1055	0.139031
Panel B: CAPM Beta and Beta Adjusted Returns under ACAPM					
CAPM Beta	0.7548	1.9659	1.5542	0.75898	1
Beta Adjusted Excess Returns	0.031967	0.03425	0.035164	0.039274	0.0338