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Liquidity Risk and Time-Varying Correlation Between Equity and Currency Returns

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October 23, 2015

Abstract

Using data on twenty major OECD countries over time, this paper documents a new evidence on real equity and real currency prices: higher real returns in the home equity market relative to foreign counterparts are generally associated with real home currency depreciation at a monthly frequency, but this negative correlation breaks down or even reverses during times of relatively higher aggregate economic uncertainty or volatility. This paper also proposes one plausible explanation for this time-varying correlation structure. The suggested model is based on a long-run risks type model, combined with time-varying liquidity risks in stock markets. With recursive preference for the early resolution of uncertainty and a negative link between the level of short-run economic growth and equity market liquidity volatility, the model demonstrates that severe short-run economic uncertainty overturns the otherwise negative link between the real currency and real relative equity returns.

Keywords: foreign exchange rates, long run risks models, liquidity risks

JEL Classification Numbers: E43, F31, G12, G15

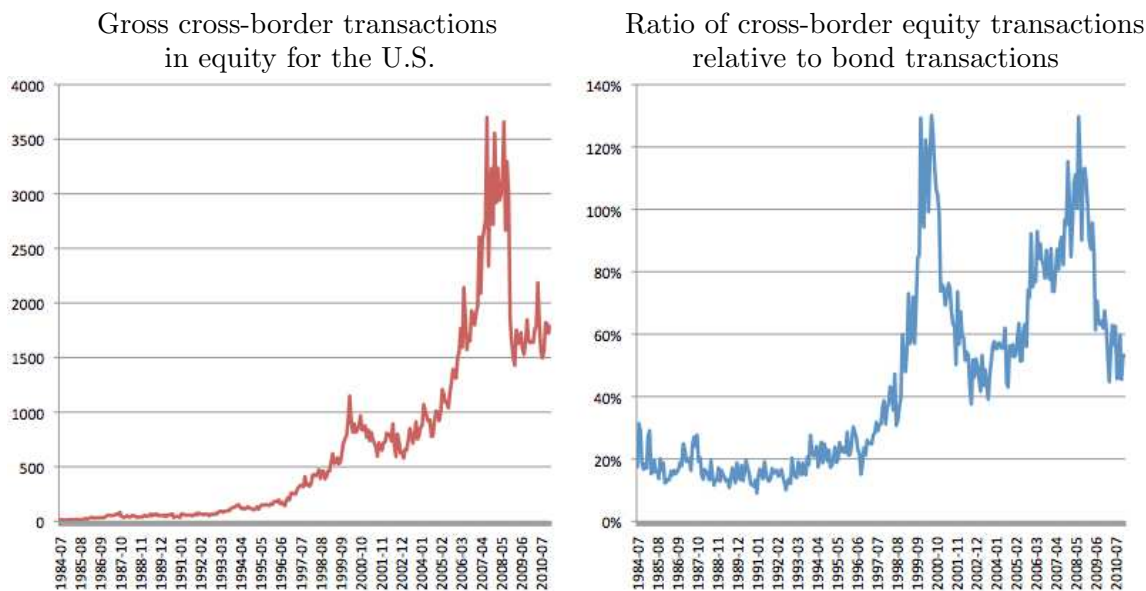
*Address: Ming Lun Street, Kaifeng, Henan, 475001, China, Phone Number: +86-187-4986-2307, e-mail: kmjung@ucdavis.edu. I am indebted to Paul Bergin for continuous advice and support for this project. I am also grateful to Katheryn Russ and Kevin Salyer for useful comments and suggestions. I would also like to thank participants at Henan University Brownbag seminar for helpful comments. I am especially indebted to William Branch and anonymous referees for help and many valuable suggestions. I also thank Niu Yongqing for excellent research assistance. All errors are mine.

1 Introduction

Since Meese and Rogoff (1983)'s study, a long-standing challenge in international economics has been the difficulty of tying floating exchange rates to macroeconomic fundamentals such as money supplies, outputs, and interest rates. While numerous studies have subsequently claimed to find success for various versions of fundamental-based exchange rate determination models (sometimes at longer horizons and over different time periods), the success of these models has not proven to be robust.

For this reason, recent exchange rate determination theory has advanced mostly outside the scope of traditional fundamental-based models. One important strand of this new literature views the link between equity and foreign exchange (FX) markets as a potential solution to the puzzle. Traditionally, international equity markets have been largely overlooked in the exchange-rate determination literature. However, a rapidly growing portion and size of the equity flows reported in Figure 1 as well as a technical development in solving DSGE models involving portfolio choice have recently started requiring for a new exchange rate theory in which exchange rates and equity-market returns are determined jointly.

Figure 1: International Equity Transaction Trend



Source: Treasury International Capital System, U.S. Treasury

As for previous literature, the empirical relationship between the exchange rates and the stock markets has been studied for a couple of decades. The results, however, are inconclusive. Most cointegration and standard granger causality tests have found no long-run association between stock prices and exchange rates.¹ On the other hand, when it comes

¹ See Granger, Huang, and Yang (2000) for details

to the relationship between *relative* equity returns and exchange rates, research has generally revealed a negative relationship at the short to medium frequency; see *2000 BIS Quarterly Review*, Brooks, Edison, Kumar, and Slock (2001), Cappiello and Santis (2007) and Hau and Rey (2006). In other words, previous studies have essentially shown that for a pair of countries, one country's currency appreciation tends to be associated with a fall in relative equity returns to the other country at the short to medium frequency.

The objective of this study is, first, to re-examine the correlation in *real* terms, i.e., the correlation between *real* currency returns and *real* relative equity returns, which is new in the literature. The sample includes a cross-section of 20 major OECD countries with flexible exchange rate regimes. Time span of data covers 1991/1 to 2014/12. Using standard panel estimators, this study generally finds similar empirical results in line with the previous studies even in real terms. However, a novel aspect of my new findings is that the generally observed negative correlation tends to disappear or even turn into a positive one for an extended period of time, especially during times of economic uncertainty. To rigorously test this hypothesis, i.e., the structural correlations are conditional on the degree of economic uncertainty, I include an interaction of (real) relative equity returns and proxy for such aggregate economic uncertainty as an additional regressor into an otherwise standard regression equation in previous studies. Through several robustness checks, the null hypothesis could not be rejected. In sum, this study empirically shows that the correlation between real relative equity and real FX returns is conditional on the degree of economic uncertainty.

To the best knowledge of the author, no previous theories could account for this newly observed evidence. The second objective of this study is, therefore, to provide one potential explanation for this newly observed evidence. To that end, the suggested model explicitly utilizes the concept of *liquidity* volatility, combined with the long run risk framework developed by Bansal and Shaliastovich (2013). Although the term liquidity is an elusive concept, the current study primarily focuses on one particular definition of liquidity, namely 'equity market liquidity', which is the ease of trading equities in stock markets. Accordingly, the liquidity volatility in the current model refers to the degree to which such trading costs in stock markets fluctuate within a given period of time. Given these concepts, the model intuition goes as follows.

First, the model assumes Epstein-Zin (EZ) preference of agents as in Bansal and Shaliastovich (2013). Introducing EZ preference with a risk aversion and intertemporal elasticity of substitution both greater than 1 would imply that agents prefer early resolution of uncertainty. Under this condition a higher unexpected or realized foreign consumption volatility tomorrow relative to the home counterpart would lead to a relatively lower *realized* returns

on foreign assets tomorrow.² In the mean time, the former would increase the *realized* foreign pricing kernel tomorrow relative to the home counterpart, i.e., foreign agents value tomorrow's consumption relatively more than their counterparts. This also indicates that the *realized* foreign currency value tomorrow would have to appreciate in order to match the *realized* higher consumption demand by foreign agents.³ This is what my model suggests as a main mechanism behind a negative correlation between currency and equity returns.

However, my model also offers a *liquidity volatility* channel through which the correlation could be overturned. First, a higher liquidity volatility increases asset prices in this framework. The intuition is similar to the one in Pastor and Veronesi (2006). Liquidity in this model *effectively* acts as a second dividend. Then, a higher volatility in this second dividend process, e.g., either a very high or a very low dividend payment, is more appreciated by consumers. This implies that a relatively higher *realized* foreign liquidity volatility tomorrow would lead to a relatively higher *realized* returns on foreign assets tomorrow. Yet, the former would affect pricing kernels differently from the consumption volatility channel. This is due to structural assumptions of the model. First, the liquidity volatility does not *directly* affect the consumption process because the short-run (SR) consumption volatility and the liquidity volatility are assumed to be independent in the model. However, the SR consumption growth level shocks are assumed to be negatively correlated with liquidity volatility shocks.⁴ In other words, a relatively higher realized foreign liquidity volatility *effectively* lowers the realized SR foreign consumption growth relative to the home counterpart. This in turn implies that the *realized* foreign pricing kernel tomorrow should increase relatively, thereby causing the realized foreign currency appreciation tomorrow. Thus, the liquidity volatility always creates a positive pressure for the correlation.

At the end of the day, the final conditional covariance depends upon which of the two volatility effects dominates. This is where the importance of time-varying SR consumption volatility or economic uncertainty comes in. Recall again that a relatively higher *realized* foreign liquidity volatility (call it A) reduces the *realized* SR foreign consumption growth (call it B) through the structural assumptions. But because the SR consumption growth is also affected by the SR consumption volatility shocks, the effect of A on B can be much amplified by a higher level of SR consumption volatility shocks. This in turn implies that the positive pressure is much more likely to dominate the negative pressure in light of a

² Standard CRRA type preferences could not simply generate consumption volatility-induced asset price changes. This is due to the fact that the pricing kernel under the CRRA preference only depends on the relative size of current and expected future consumption levels and the consumption volatility itself has nothing to do with the pricing kernel.

³ This intuition is in line with the fact that a relative currency value is positively related to the relative pricing kernel under the complete FX market assumption.

⁴ Section 4 later provides empirical support for this structural assumption.

relatively higher level of SR consumption volatility or, in a more general term, a higher SR aggregate economic uncertainty. This summarizes why and how my model can account for the empirical facts found in this paper, i.e., the correlations tend to become positive during the crisis associated with a higher degree of SR aggregate economic uncertainty.

One notable advantage of this model is that it could be easily applied to bond markets, and help explaining the time-varying nature of correlations between interest rate differentials and exchange rate movements. The reason goes as follows. To begin with, bond return differentials and exchange rates in a short to medium frequency also exhibit a relationship in a way that high interest rate currencies generally tend to appreciate. This is called ‘Uncovered Interest Parity (UIP) Puzzle’ in the literature because standard two-country DSGE models would expect a negative relationship instead. What is more interesting in this so-called UIP literature is that the estimated correlations are time-varying. In particular, many evidence suggest that the estimated correlations tend to be negative, i.e., the UIP coefficient flips sign, especially when measures of market volatility soar; see [Brunnermeier, Nagel, and Pedersen \(2008\)](#) for instance. In short, there are strong parallels between the time-varying-equity-FX-correlations and the time-varying UIP conditions.

As a matter of fact, similar to existing studies on the relationship between currency and equity returns such as [Hau and Rey \(2006\)](#), [Bansal and Shaliastovich \(2013\)](#) show us a model that can only generate a *fixed* positive correlation between bond return differentials and FX rates, which resolves for the UIP puzzle. But again, they could not account for the time-varying nature of the relationship. This is where critical contribution of the current paper can possibly come in. Essentially, what this paper does is to replace bonds with equities, and to introduce exogenous aggregate liquidity process into [Bansal and Shaliastovich \(2013\)](#). If one, in fact, introduces the aggregate liquidity process into both bond and stock markets then, she could correctly account for the two time-varying correlations simultaneously. But since this paper is about how equities are related to exchange rate movements, and others have already discussed about modeling the time-varying UIP coefficient, e.g., [Brunnermeier, Nagel, and Pedersen \(2008\)](#), I chose to include only equities into the model.

2 Related Literature

Unlike a vast literature that has researched exchange rate movements through lens of the UIP condition, studies on the link between currency and equity returns are relatively scarce in the literature. Nevertheless, major strands of such studies generally found a negative link between currency and relative equity returns as already mentioned earlier. Recent studies in line with these findings include [Kim \(2011\)](#) and [Melvin and Prins \(2015\)](#). Here, we

first review how such negative correlations could come about through lens of [Hau and Rey \(2006\)](#)'s theory. They develop a theoretical model in which exchange rates, equity market returns and capital flows are jointly determined. They argue that excess equity returns over another country and currency value have a perfect negative correlation due to incomplete FX risk trading. Their key arguments are as follows. When foreign equities outperform domestic equities in terms of rate of return, the relative exposure of domestic investors to exchange rate risks increases due to incomplete FX risk hedging. To diminish the FX risks exposure, home investors should rebalance their portfolio, decreasing foreign equity holdings. This would in turn generate capital inflows into the domestic country and would therefore result in a home currency appreciation. In the end, the perfect negative correlation between equity and currency values would hold. Yet, the *perfectly* negative correlation implied by their theoretical model can not account for my new empirical results showing a time-varying correlation between equity and FX returns. Again, the main theoretical contribution of this paper is to provide one plausible explanation for this newly observed evidence.

An alternative strand of literature, though rare, also exists. In this literature, the correlation between international equity and currency returns can be non-negative; see [Griffin, Nardari, and Stultz \(2004\)](#), [Pavlova and Rigobon \(2007\)](#), [Chabot, Ghysels, and Jagannathan \(2014\)](#), and [Cenedese, Payne, Sarno, and Valente \(2015\)](#). In particular, [Pavlova and Rigobon \(2007\)](#) offer a portfolio-balance model that theoretically endogenizes the dynamics of *real* equity prices and *real* exchange rates, which is relevant to my study.⁵ Their model has implications on how equity and foreign exchange markets co-move in response to shocks, which are transmitted internationally across financial markets via the terms of trade. For example, a positive supply shock at home would have a positive effect on the *relative* domestic stock values. In line with the comparative advantages (*Ricardian*) theory, the domestic terms of trade would deteriorate, and therefore, raise the relative prices of foreign goods, i.e., domestic currency depreciation. Hence, the supply shock would generate a negative comovement between FX and equity returns.

However, the dynamics induced by demand shocks are completely different. For instance, a positive demand shock at home would improve the country's terms of trade due to a home bias assumption of domestic goods. Domestic currency would appreciate as a result, which in turn boost domestic stock values relative to foreign stock prices. Unlike supply shock, a positive correlation between FX and equity returns would be implied by demand shocks. In short, their model predicts that the relationship between FX returns and equity returns

⁵ [Heathcote and Perri \(2013\)](#) provided numerical impulse responses for excess equity returns and real exchange rates to supply and demand shocks within their theoretical model. However, the numerical results are basically the same as [Pavlova and Rigobon \(2007\)](#) and [Hau and Rey \(2006\)](#)'s analytical predictions.

critically hinges upon the dominance between demand and supply shocks. Unlike their study, this paper’s main mechanism revolves around two different *volatility* shocks; liquidity volatility and SR consumption volatility shocks.

The paper is organized as follows. Section 3 documents the newly found empirical evidence with updated data. Section 4 and 5 present the model in a rigorous manner. Section 6 discusses the model predictions on FX and equity returns. Section 7 directly tests whether the model can quantitatively replicate the empirical evidence with a calibration exercise. Section 8 concludes.

3 New Evidence

3.1 Data

This empirical study only focus on 20 OECD (including a reference or home country U.S.) countries which have a flexible exchange rate regime against the U.S. dollar and available data on variables used in this study. The sample country includes Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Korea, Spain, Sweden, Switzerland, U.K., and the U.S. Since this analysis also focuses on the relationship among variables at a monthly frequency, all collected data are based on the average monthly values.

Data on nominal FX rates in home currency per unit of foreign currency, i.e., $\$/\pounds$, were collected from *Board of Governors of the Federal Reserve System*. As for the monthly data on aggregate stock market index for each sample country, the ‘Total Share Prices for All Shares’ index constructed by *Federal Reserve Bank of St.Louis* was used. In order to convert these values into real terms, monthly data on consumer price index (CPI), collected by *Federal Reserve Bank of St.Louis*, was used as well.

For the aggregate equity market liquidity proxy, the Amihud measure of illiquidity, the most basic and widely accepted measure for aggregate liquidity in the literature; see Amihud (2002), was constructed for each sample country. It’s a ratio of absolute value of monthly stock market returns to monthly stock market trade volume. It basically tells us how much one unit of trade moves the price (indirect transaction cost measures in stock markets). Again, for a comparison purpose, this study also makes use of an alternative proxy for such stock market liquidity, namely the TED spread, i.e., a difference between LIBOR and the government bond interest rates. This particular measure is widely referred to as a proxy for ‘funding liquidity’ (the ease of trading using leverage) in the literature; see Amihud, Mendelson, and Pedersen (2013).

Data on the government bond yields (three-month bond rates) and the three-month LIBOR rates were obtained from the *Federal Reserve Bank of St. Louis*. Data for the stock market trade volume were obtained from *Yahoo Finance*. All data range starts from 1991/1 and ends 2014/12. Since this paper explicitly concerns a monthly frequency, monthly changes in stock and FX returns were chosen to analyze the correlation structure.

3.2 Variables

A *real* monthly stock market return between a month t and $t + 1$ for a country i , i.e., R_t^i , is calculated as follows.

$$R_t^i = \ln(SI_{t+1}^i) - \ln(SI_t^i) - \{\ln(CPI_{t+1}^i) - \ln(CPI_t^i)\},$$

where SI_t^i is a stock market index at a month t for a country i and CPI_t^i is the CPI for a country i at a month t . Similarly, a monthly change in *real* FX rates of the country i 's currency relative to the U.S. dollar from t to $t + 1$, i.e., Δq_t , is calculated as

$$\Delta q_t^i = \{\ln(FX_{t+1}^i) - \ln(FX_t^i)\} + \{\ln(CPI_{t+1}^i) - \ln(CPI_t^i)\} - \{\ln(CPI_{t+1}^{U.S.}) - \ln(CPI_t^{U.S.})\},$$

where FX_t^i is the nominal U.S. dollar price per unit of the country i 's currency, i.e., $\$/\mathcal{L}$, at a month t .

The main equity market liquidity proxy used in this study is the Amihud illiquidity, i.e., lq_t^i for a country i at a month t , calculated as

$$lq_t^i = \frac{|R_t^i|}{TV_t^i},$$

where TV_t^i is a measure for aggregate stock market trade volume for a country i at time t .

Finally, $\sigma_{R_t^i}^2$ and $\sigma_{lq_t^i}^2$ is defined as the volatility of R^i and lq^i at a particular month t respectively. These measures are calculated using a 2-year rolling variance of measures for R_t^i and lq_t^i respectively. Table 1 reports the summary statistics of these variables used in econometric analyses in the following section.

3.3 Econometric Analyses

Table 2 provides empirical estimates on the monthly correlations between foreign currency values relative to the U.S. \$ and excess foreign stock index returns over the U.S. counterpart. It shows two standard panel estimates on β (fixed effects (FE) estimates controlling for

Table 1: Summary Statistics

Variables	# of Observations	Mean	Std. Dev
R	5740	0.0028	0.0528
σ_R^2	5280	0.0273	0.0255
lq	1874	0.2304	0.2241
σ_{lq}^2	1576	0.0408	0.0466
Δq	5453	0.0069	0.2012

country specific fixed effects and pooled OLS estimates) for four different time periods, 1991/1 to 1998/12, 1999/01 to 2001/12, 2002/01 to 2010/12, and 2011/01 to 2014/12.

Table 2: $\Delta q_t^i = \alpha_i + \beta[R_t^i - R_t^{U.S}] + \varepsilon_t^i$

Periods	1991/01-1998/12	1999/01-2001/12	2002/01-2010/12	2011/01-2014/12
Panel with FE	-0.1710378***	1.220263**	-0.190125***	-0.0347795
Pooled OLS	-0.1757183***	1.158778**	-0.1868499***	-0.0371741
# of cross-section	19	19	19	19
# of periods	95	36	108	48
# of observations	1805	684	2052	912

Note: *, ** and *** indicates that the coefficient is significant at 10%, 5% and 1% level respectively.

First, both pooled OLS and FE estimates on β during the periods of 1991/01-1998/12 and 2002/12-2010/12 turn out to be statistically significant and negative. This regression evidence even in real terms is in line with what [Hau and Rey \(2006\)](#) and others already found in data, a negative correlation. However, this particular negative correlation certainly does not appear to hold universally over time as shown in [Table 2](#). The latter shows that the correlation overturns its sign into positive, and this positive correlation happens to be statistically significant during the period of 1999/01-2001/12. Furthermore, the beta coefficients also become statistically insignificant during the 2011/01-2014/12 period.

What is interesting is that these two particular periods are closely linked to times of global economic uncertainty. For instance, the 1999/01-2001/12 period coincided with a series of various world-wide economic crises such as Asian financial crisis, Russian default crisis, Long-Term-Capital-Management (LTCM) crisis, and dot-com bubble crisis. The 2011/01-2014/12 period also relates to current European debt crisis, which mainly causes financial market turmoils in major Euro countries. In fact, [Table 3](#) also shows that the individual OLS estimates for major Euro countries during that time. 8 out of 11 Euro countries' beta coefficient turn out to be positive, though not statistically significant.

In sum, all these interesting new findings lead one to set up a null hypothesis if the correlation between currency and relative equity returns are in fact conditional on the degree of economic uncertainty or volatility such that the correlation tends to show a strong tendency

Table 3: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \varepsilon_t$

Countries	$\hat{\beta}$
Austria	0.1660517
Belgium	-0.3707395**
Finland	0.0539352
France	0.0575972
Germany	0.1842392
Greece	0.0681877
Ireland	-0.5453559***
Italy	0.2453416**
Netherlands	-0.4549546**
Portugal	0.1160395
Spain	0.1219099

Note: The sample periods is for European Debt Crisis (2011/01-2012/12).
The same significance level applies to *, ** and *** as before.

to become positive during uncertainty crises. In order to test this hypothesis, I include an interaction term between $[R_t - R_{US,t}]$ and $[\sigma_{R_t^i}^2 + \sigma_{R_t^{U.S.}}^2] \equiv X_t^i$, a proxy for the sum of SR economic uncertainty measures for the two countries in a pair, into the baseline regression equation in Table 2 as an additional regressor. Table 4 reports the estimation results.

Table 4: $\Delta q_t^i = \alpha_i + \beta[R_t^i - R_t^{U.S.}] + \gamma[R_t^i - R_t^{U.S.}]X_t^i + \varepsilon_t^i$

Methods	Panel with FE		Pooled OLS	
	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
	-0.1680585	9.937949*	-0.175161	9.976492*
# of cross-section	19		19	
# of periods	264		264	
# of observation	5016		5016	

Note: The same significance level applies to *, ** and *** as before.

The beta coefficients are all insignificant for both FE panel and pooled OLS estimation cases. On the contrary, the gamma coefficients for both FE panel and pooled OLS cases turn out to be positive and statistically significant. Thus, the null hypothesis could not be rejected at a 10% significance level.⁶ Intuitively, this result indicates that the more the economy gets uncertain the more likely the correlation between currency and relative equity returns becomes positive. For a comparison purpose, Table 5 also reports such estimates using OLS estimators on individual cross-country cases. The results are basically in line with Table 4's results. 13 out of 19 country pair cases show positive gamma estimates, though most of them are statistically insignificant.

Lastly, to illustrate the time-varying correlations between Δq_t^i and $[R_t^i - R_t^{U.S.}]$, Figure 2 plots the two-year rolling correlations between Δq_t^i and $[R_t^i - R_t^{U.S.}]$ together with $[\sigma_{R_t^i}^2 + \sigma_{R_t^{U.S.}}^2]$,

⁶ In fact, the *p-value* for both estimation cases were close to 5%, i.e., that the null hypothesis could have been not rejected even at a 5% significance level.

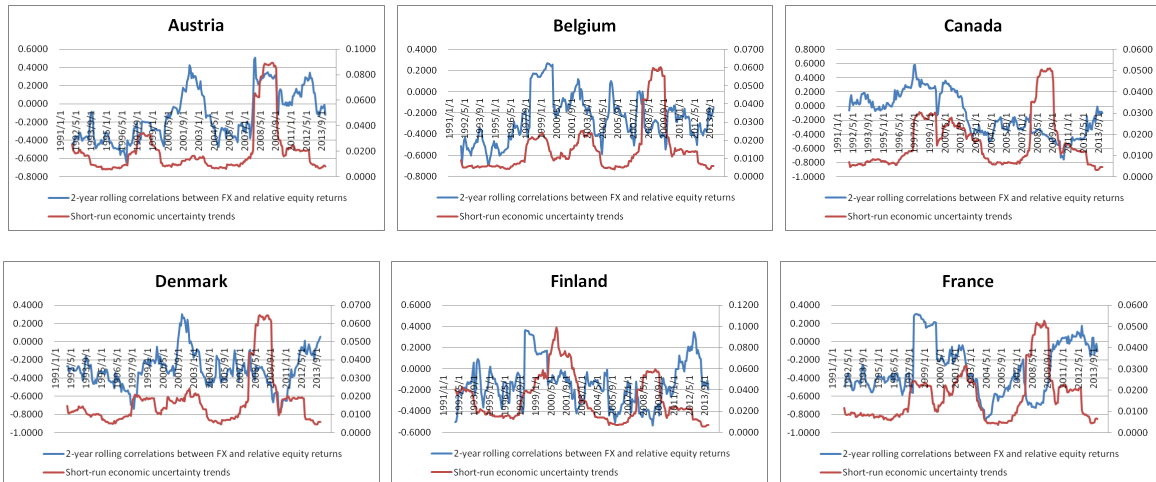
Table 5: $\Delta q_t = \alpha + \beta[R_t - R_{US,t}] + \gamma[R_t - R_{US,t}]X_t + \varepsilon_t$

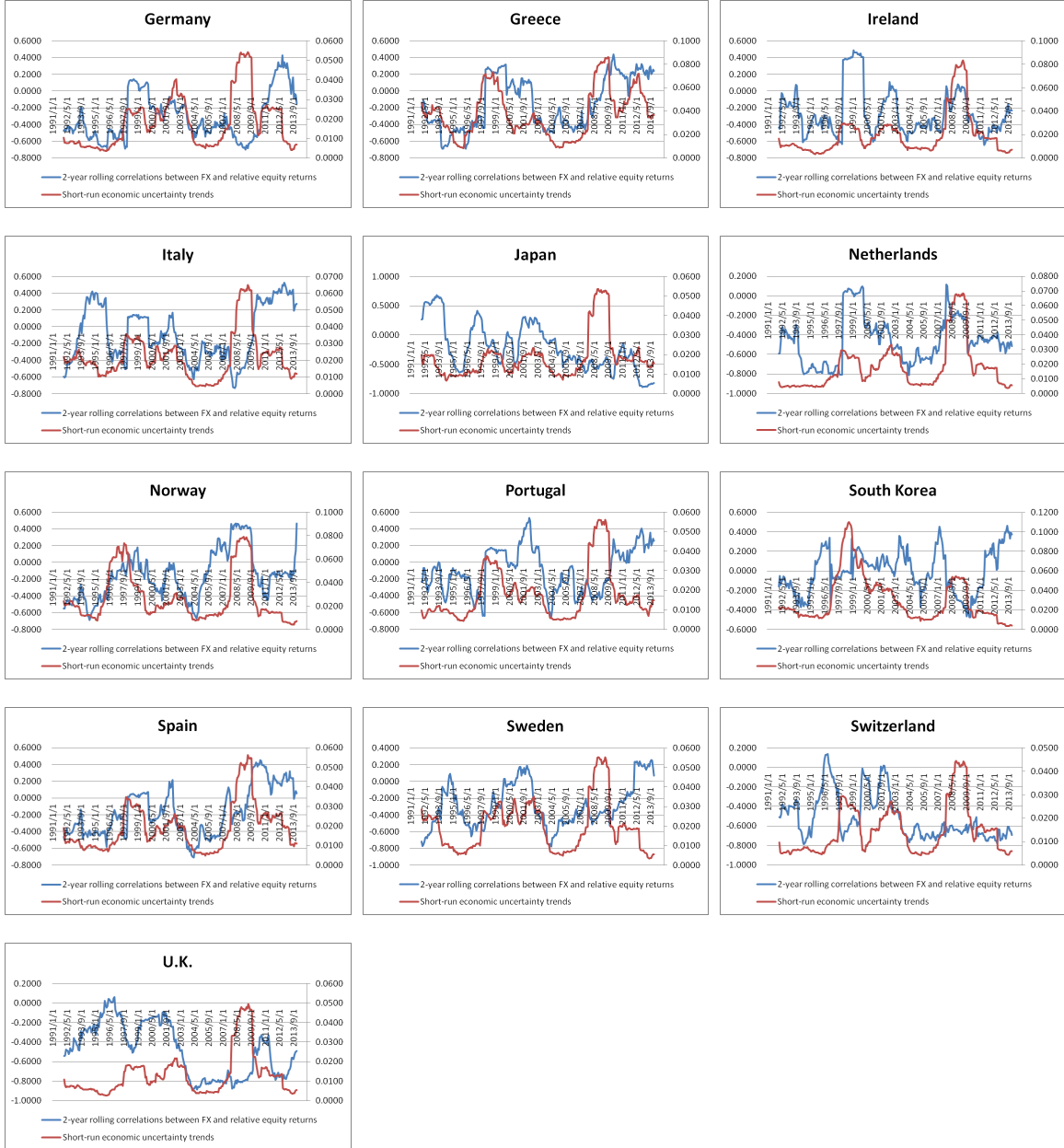
Countries	$\hat{\beta}$	$\hat{\gamma}$
Austria	-0.6285739*	6.551034
Belgium	0.0730233	15.85387
Canada	0.0176795	-3.437809**
Denmark	-0.2253558***	-0.0485715
Finland	-0.0218074	3.115712
France	-0.2578065	30.51101
Germany	-0.250619*	4.238589
Greece	-1.46096**	46.47461***
Ireland	0.9100167	16.04203
Italy	-0.8357776	66.95506
Japan	0.0148532	-7.140679*
Netherlands	-0.5183074***	8.739108
Norway	-0.268392***	5.075721***
Portugal	-0.4271297	44.80914
South Korea	-0.1388388	3.423984
Spain	-0.4306854	27.5968
Sweden	-0.1592045*	-3.11975
Switzerland	-0.5088915***	-0.8674161
U.K.	-0.4982304**	-11.54311

Note: The same significance level applies to *, ** and *** as before.

i.e, SR economic uncertainty index, for 19 country pairs. This figure clearly illustrates that the correlations are neither perfectly positive or negative for most country pairs. Further, they are certainly time-varying and tend to show a strong tendency to be negative during relatively *tranquil* times, consistent with previous findings in the literature. However, these trends also tend to overturn during time of economic stress for many pairs. To sum up, all these evidence call for a new model that could account for this sign-switching correlation structure, which is pursued in the following sections.

Figure 2: Correlation between FX and equity returns with a rolling window of 2-year periods





4 The Model

4.1 Epstein-Zin Recursive Utility

The representative investor preference over the uncertain aggregate consumption stream C_t are assumed to have a functional form of the EZ utility function.

$$U_t = [(1 - \beta)C_t^{\frac{1-\gamma}{\theta}} + \beta(E_t U_{t+1}^{1-\gamma})^{\frac{1}{\theta}}]^{1-\gamma}, \quad (1)$$

where β , ψ and γ are the time discount factor, the intertemporal elasticity of substitution (IES) and the risk aversion parameter respectively. Parameter θ is defined as $\theta = (1 - \gamma)/(1 - 1/\psi)$. As pointed out in [Bansal and Shaliastovich \(2013\)](#), the logarithm of the intertemporal marginal rate of substitution (IMRS) for these preferences is given by

$$m_{t+1} = \theta \log \beta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1}, \quad (2)$$

where $\Delta c_{t+1} = \log(C_{t+1}/C_t)$ is the growth rate of aggregate consumption and $r_{c,t+1}$ is the log of the return on an *imaginary* asset which delivers aggregate consumption as its dividend each time period. This return is not observed in data.

4.2 Aggregate Consumption Process

I adopt the exact same consumption process in [Bansal and Shaliastovich \(2013\)](#) where home and foreign countries differ only in consumption volatility, i.e., $\sigma_{g,t}$, $\sigma_{g,t}^*$, and consumption growth innovations, i.e., η_{t+1} , η_{t+1}^* . From now on, foreign country variables are indexed by a superscript *. The consumption dynamics for home are the following;

$$\begin{aligned} \Delta c_{t+1} &= \mu_g + x_t + \sigma_{g,t} \eta_{t+1} \\ x_{t+1} &= \rho x_t + \sigma_{x,t} e_{t+1} \\ \sigma_{g,t+1}^2 &= v_g \sigma_{g,t}^2 + \omega_{g,t+1} \\ \sigma_{x,t+1}^2 &= v_x \sigma_{x,t}^2 + \omega_{x,t+1}, \end{aligned} \quad (3)$$

where x_t is a persistent long-run expected growth component. The fact that home and foreign countries share the same long-run component reflects upon the historical fact that long-run growth prospects across the countries are similar. Notice here that this long run consumption shock (e_t) is persistently transmitted into the future consumption process whereas the short run consumption shock (η_t) is not. The consumption growth differences between the two countries in this model are captured by the differences in short-run consumption shocks and volatilities. For tractability, η_{t+1} and e_{t+1} are assumed to follow the standard normal distribution. The innovations in volatility processes $\omega_{g,t+1}$ and $\omega_{x,t+1}$ are assumed to follow a gamma distribution with a mean of $\bar{\omega}_g$ and $\bar{\omega}_x$ respectively, and a variance of σ_{gw}^2 and σ_{xw}^2 respectively. No contemporaneous correlation between the $\omega_{g,t+1}$ and $\omega_{x,t+1}$ is assumed for simplicity. Finally, empirical justifications for the time-varying volatilities of the consumption and long-run components are presented in [Bansal and Shaliastovich \(2013\)](#).

4.3 Aggregate Dividend Process

Following [Bansal and Shaliastovich \(2013\)](#), the aggregate dividend in this economy follows the following process.

$$\Delta d_{t+1} = \mu_g + \phi x_t + \varphi_d \sigma_{g,t} \eta_{d,t+1}, \quad (4)$$

where the average dividend growth rate is equal to the rate for aggregate consumption, i.e., μ_g , and the volatility of dividend growth is simply φ_d times greater than the consumption counterpart. For tractability, independence between consumption and dividend growth shocks is assumed.

4.4 Aggregate Equity Market Liquidity Process

Since this is (to the best of my knowledge) the first long-run risks model that explicitly incorporates the liquidity process in asset markets, some of the structural assumptions in this section might appear to be non-standard, and, hence, deserve some detailed explanation.

First, this model only focuses on the ‘equity market liquidity’. As mentioned before, (equity market) liquidity in this model is defined as the ability to buy or sell large quantities of assets quickly and at low cost in stock markets. Specifically, we follow [Acharya and Pedersen \(2005\)](#)’s precise definition, the per-share cost of selling aggregate security e.g., f_t .⁷ Moreover, following [Acharya and Pedersen \(2005\)](#), I take the process of $\{f_t\}_{t=0}^{\infty}$ exogenously given by,

$$\Delta f_{t+1} = -ax_t - \sigma_{l,t} \zeta_{t+1}, \quad (5)$$

where ζ_{t+1} is the liquidity (level) shock and $\sigma_{l,t}$ is the time-varying volatility of the ζ_{t+1} .

The assumption that Δf_{t+1} has a persistent long-run growth component as in Δc_{t+1} is empirically supported. [Acharya and Pedersen \(2005\)](#) report highly persistent U.S. equity market liquidity with an autocorrelation of around 0.9 at a monthly frequency. [Brunnermeier and Pedersen \(2008\)](#) demonstrate the pro-cyclical nature of the asset market liquidity provision and offer a theoretical explanation based on the funding-liquidity constrained in-

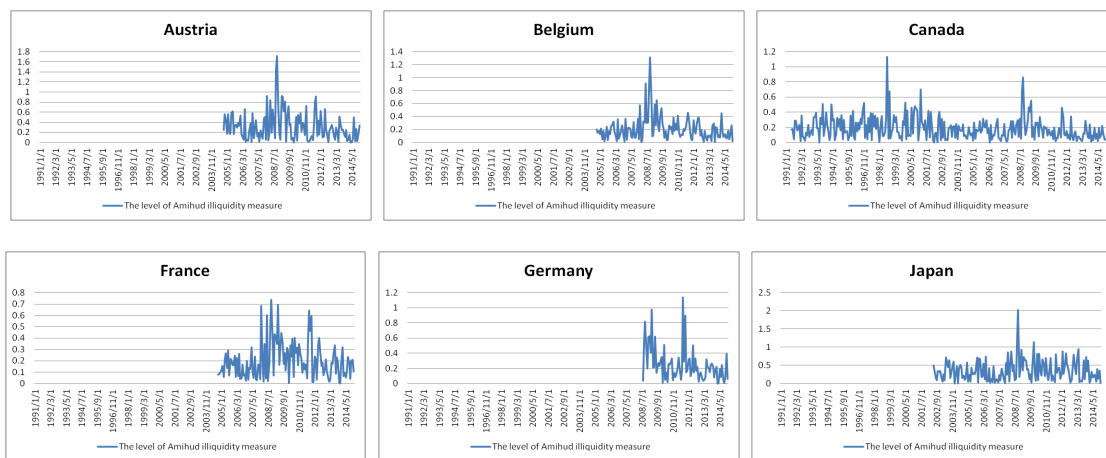
⁷ Potentially, one could also introduce FX market liquidity into this model. Focusing on the last concept above, aggregate FX market liquidity can be defined as the per currency unit cost of selling currencies. However, I *implicitly* assume that such costs are zero in my model. This simplification has been chosen for two reasons. First, it would greatly reduce the complexity of theoretical analysis in the paper. If one decided to introduce the liquidity of currencies as well as equities in this framework, she would have to be very precise about how to model the correlation between currency and equity liquidities. One could not simply assume i.i.d. process for each since the liquidity exhibits the ‘commonality effect’ across different asset markets; please see [Amihud, Mendelson, and Pedersen \(2013\)](#). Second and most importantly, the FX market has been perceived as arguably the most ‘liquid’ market in the world by both academics and practitioners. As a matter of fact, bid-ask spreads in FX markets are razor thin, and trading volume as well as market depth are by far the greatest among different kinds of asset markets; please see [Evans and Lyons \(2002\)](#).

vestors' decisions. Hence, this evidence justifies the liquidity process in eq.(5) containing a persistent long-run component, i.e., pro-cyclicality.

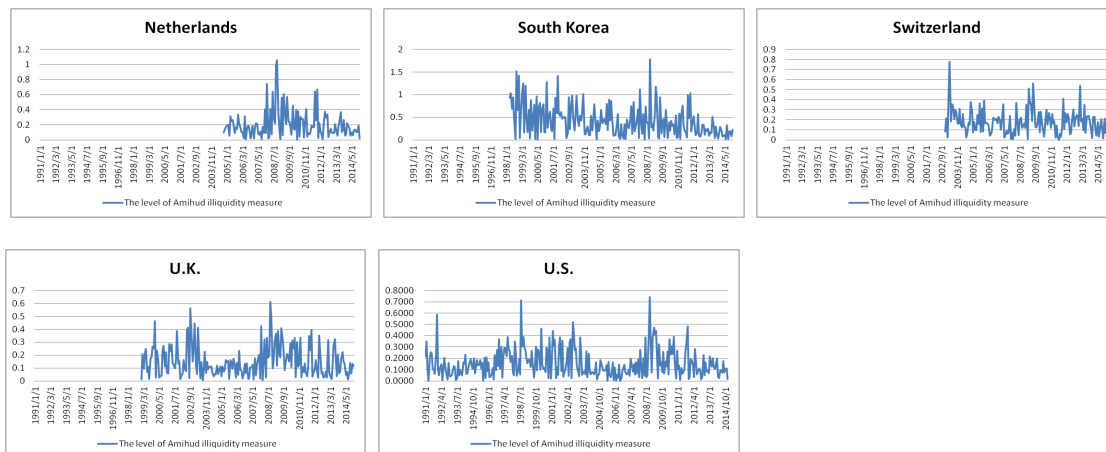
ζ_{t+1} could be viewed as shocks to aggregate transaction costs, e.g., broker fees and bid-ask spreads, in stock markets. In reality, these aggregate liquidity shocks can be easily linked to macroeconomic events in which equity market liquidity suddenly dries up in a sense that dealers dramatically widen bid-ask spreads, take the phone off the hook, or close down operations as their trading houses run out of cash and take their money off the table. Clearly, much evidence on these kinds of events have been documented in the literature. Furthermore, these events are found to be recurring. Amihud, Mendelson, and Wood (1990) show that the stock market crash of October 19, 1987 can be partly explained by a decline in investors' perceptions of the market's liquidity. Amihud, Mendelson, and Pedersen (2013) also show that equity market liquidity dried up during the collapse of the hedge fund Long Term Capital Management (LTCM) and the Russian default. In recent times, stock markets around the world also experienced the drying up of liquidity during the collapse of Lehman Brothers and Bear Sterns in 2008. Amihud, Mendelson, and Pedersen (2013) also argue that the 'flash crash' of 2010 in the U.S. stock market is another recent example of a liquidity event.

Further, the model allows for time-varying liquidity volatility as shown in $\sigma_{l,t}$. This specification is well supported by existing studies; see Amihud (2002) and Acharya and Pedersen (2005). As shown in section 3, this study directly calculates two proxies for such aggregate equity market liquidity, i.e., Amihud measure of illiquidity and TED spreads, as well. Figure 3 and 4 clearly illustrate time-varying trends (both in levels and volatility) of Amihud illiquidity measures for 11 selected OCED countries.⁸

Figure 3: Amihud illiquidity measures (levels)



⁸ The other 9 countries in the sample do not have reasonably long enough span of time series data. For this reason, they are not reported here.



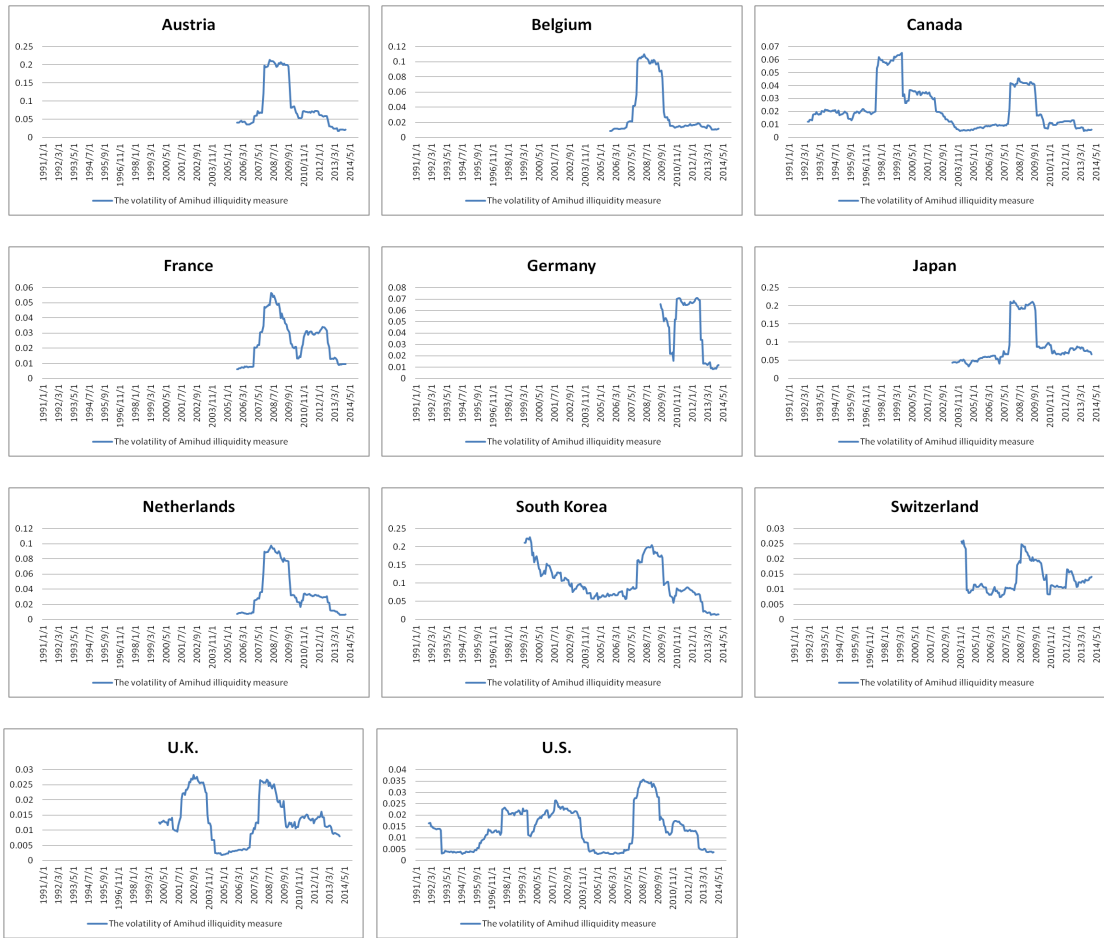
It is important to emphasize that my model is simply based on exogenous liquidity shocks. In other words, there is no endogenous mechanism whereby the liquidity shocks become more volatile during any recession period for the economy. However, this exogenous shock is enough to show that time-varying correlations between equity and FX returns are triggered by the magnitude of economic uncertainty in my model. I leave the task of endogenizing the liquidity shock for future research.

The main advantage of introducing this exogenous liquidity process into the equity market in this economy is that it would allow one to compute *liquidity-adjusted* asset prices in equilibrium easily as suggested by [Acharya and Pedersen \(2005\)](#). Basically, agents in this economy can buy the aggregate security at, say for instance, P_t , but must sell at $P_t - f_t$. [Acharya and Pedersen \(2005\)](#) show that the equilibrium asset price process $\{P_t\}_{t=0}^{\infty}$ under exogenously given Δd_{t+1} and $\Delta f_{t+1}, \forall t$ is equivalent to the one under a imaginary dividend process of $\Delta d_{t+1} - \Delta f_{t+1}, \forall t$.

This equivalence result also holds true in the current framework for the following reasons. First, the assumption of Epstein-Zin preference in my model does not affect the equivalence result since [Acharya and Pedersen \(2005\)](#) chose to work with CARA preferences for a tractability reason. They show that the equivalence result holds for an arbitrary increasing and concave utility defined on $(-\infty, \infty)$ as long as conditional expected net returns are normal, which is exactly the characteristics of [Bansal and Shaliastovich \(2013\)](#) upon which the current model builds.

However, the critical assumption needed to establish [Acharya and Pedersen \(2005\)](#)'s result in this framework is that agents ought to sell the security every period. For this reason, they work on a simple overlapping generation model where the old always sells securities after one period (when they die). With an infinitely living representative agent as in this framework, one could only apply the equivalence result to the case where the representative agent buys and sells the aggregate security every period.

Figure 4: Amihud illiquidity measures (volatility)



As Acharya and Pedersen (2005) point out, deriving a general equilibrium equity price level in a more general setting with endogenous holding periods would be an onerous task. This study avoids such task in this model, and instead take a reduced form approach. Thus, equilibrium asset prices in the current model are restricted to the case where agents are exogenously assumed to trade assets every period.

Based on this equivalence result, the current study also works on a (liquidity-adjusted) asset pricing model where the aggregate equity's dividend follows an *imaginary* process of $\Delta D_{t+1} \equiv \Delta d_{t+1} - \Delta f_{t+1}, \forall t$. Once again, the liquidity itself does not affect the dividend process at all. However, I incorporate the exogenous liquidity process into the dividend process to bring about the liquidity factor-adjusted equilibrium equity prices as in Acharya and Pedersen (2005).

To sum up, the *imaginary* home and foreign aggregate dividend processes take the fol-

lowing forms respectively.

$$\begin{aligned}\Delta D_{t+1} &= \mu_g + (\phi + a)x_t + \varphi_d \sigma_{g,t} \eta_{d,t+1} + \sigma_{l,t} \zeta_{t+1}, \\ \Delta D_{t+1}^* &= \mu_g + (\phi + a)x_t + \varphi_d \sigma_{g,t}^* \eta_{d,t+1}^* + \sigma_{l,t}^* \zeta_{t+1}^*.\end{aligned}\tag{6}$$

For simplicity, the dividend growth shocks, i.e., $\eta_{d,t+1}$ and $\eta_{d,t+1}^*$ and liquidity shocks, i.e., ζ_{t+1} and ζ_{t+1}^* , are assumed to be an independent, identically distributed normal process with no covariance. The volatility process of liquidity shocks for each country are given by

$$\begin{aligned}\sigma_{l,t+1}^2 &= v_l \sigma_{l,t}^2 + \omega_{l,t+1}, \\ \sigma_{l,t+1}^{*2} &= v_l \sigma_{l,t}^{*2} + \omega_{l,t+1}^*.\end{aligned}\tag{7}$$

The innovations in volatility processes $\omega_{l,t+1}$ and $\omega_{l,t+1}^*$ are assumed to follow a gamma distribution with no contemporaneous correlations. $\bar{\omega}_l$ and σ_{lw}^2 are the mean and variance of $\omega_{l,t+1}$ respectively (the same applies to the foreign case).

Last, I impose one crucial structural assumption on shock processes in this economy. As explained so far, all the shock processes are assumed to be idiosyncratic. Yet, one exception is introduced as follows.

Assumption 1 *A variance-covariance matrix for $(\omega_{l,t}, \eta_t)$, Σ is given by*

$$\Sigma = \begin{bmatrix} \sigma_{lw}^2 & \tau \\ \tau & 1 \end{bmatrix}$$

where $\tau < 0$. The same applies to the foreign counterpart.

In words, the SR consumption growth level shock and the liquidity volatility shock for each country follows a joint distribution with a negative contemporaneous correlation. This particular assumption plays a pivotal role in generating a time-varying correlation between currency and equity return differentials as intuitively explained in section 1. Section 6 will rigorously show the underlying mechanism much in detail. Empirical support for this assumption is also provided in Table 6.

The third column shows correlations between a two-year rolling variance of Amihud measure of equity market illiquidity (proxy for $\omega_{l,t}$) and stock market returns (proxy for SR economic growth, i.e., η_t) for 11 major OECD countries.⁹ They all turn out to be negative.

⁹ σ_{TED}^2 ranges from 1992/11 to 2014/01 and 1992/01 to 2012/06 respectively for Belgium and Canada. σ_{lq}^2 ranges from 1992/06 to 2014/01, 1999/04 to 2014/01, 2000/01 to 2014/01, 2003/06 to 2014/01, 2003/12 to 2014/01, 2005/10 to 2014/01, and 2009/08 to 2014/01 respectively for Canada, South Korea, U.K, Japan, Switzerland, (Austria, Belgium, France and Netherlands) and Germany.

Table 6: Correlations between SR economic growth and liquidity volatility

Countries	$corr(R, \sigma_{TED}^2)$	$corr(R, \sigma_{lq}^2)$
U.S.	-0.1592	-0.1905
Germany	-0.0432	-0.0648
U.K.	-0.0341	-0.2105
Switzerland	-0.1535	-0.0702
South Korea	0.0250	-0.0198
France	-0.0268	-0.3090
Austria	0.0007	-0.2040
Netherlands	-0.0832	-0.2694
Belgium	-0.0925	-0.2937
Japan	-0.0460	-0.2398
Canada	-0.1434	-0.1019

Note: σ_{TED}^2 refers to the 2-year rolling variance of TED spreads.

For robustness check, I use different proxy for $\omega_{l,t}$, i.e., a two-year rolling variance of TED spreads. The second column also shows the same results qualitatively.

5 Asset Markets

5.1 Stochastic Discount Factor

First, [Bansal and Shaliastovich \(2013\)](#) show that the log-linearized return on the *imaginary* asset that pays out the aggregate consumption every period is given by the following processes, which are linear in state variables.¹⁰

$$\begin{aligned} r_{c,t+1} &= \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}, \\ pc_t &= A_0 + A_x x_t + A_{gs} \sigma_{g,t}^2 + A_{xs} \sigma_{x,t}^2, \end{aligned} \tag{8}$$

where pc_t is the log wealth or price to consumption ratio. The solutions coefficients for $\kappa_0, \kappa_1, A_0, A_{xs}$ are shown in the appendix of [Bansal and Shaliastovich \(2013\)](#). These coefficients are not important for the later analysis. For this reason, they are not reported here. The solution coefficients A_x and A_{gs} are given by

$$A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \quad A_{gs} = \frac{(1 - \gamma)(1 - \frac{1}{\psi})}{2(1 - \kappa_1 v_g)}.$$

One thing to note is that A_x and A_{gs} would have been negative and positive values,

¹⁰ [Bansal and Shaliastovich \(2013\)](#) show that the log-linearized solution to the model is very close to the solution of the model based on numerical methods.

respectively, under the CRRA preference with risk aversion greater than 1. Under such conditions, a higher consumption volatility, $\sigma_{g,t}^2$, today would raise asset prices today which is certainly counterintuitive. In contrast, if the intertemporal elasticity of substitution and risk aversion are larger than one, A_{gs} becomes negative, resulting in a negative relationship between contemporaneous consumption volatility and asset prices. This is precisely what [Bansal and Yaron \(2004\)](#) argue as a theoretical explanation for the relationship between consumption volatility and asset prices.

Combining the Euler condition in eq.(2) and the equilibrium price-consumption ratio in e.q.(8), an analytical expression for the intertemporal marginal rate of substitution (IMRS) can be obtained as follows.¹¹

$$m_{t+1} = m_0 + m_x x_t + m_{gs} \sigma_{g,t}^2 + m_{xs} \sigma_{x,t}^2 - \lambda_\eta \sigma_{g,t} \eta_{t+1} - \lambda_e \sigma_{x,t} e_{t+1} - \lambda_{gw} \omega_{g,t+1} - \lambda_{xw} \omega_{x,t+1}, \quad (9)$$

where λ_η and λ_e are the market prices of short-run and long-run risks.

λ_{gw} and λ_{xw} are the market prices of short-run and long-run volatility risks. They are given by

$$\lambda_{gw} = - \left(\gamma - \frac{1}{\psi} \right) (\gamma - 1) \frac{\kappa_1}{2(1 - \kappa_1 v_g)},$$

$$\lambda_{xw} = - \left(\gamma - \frac{1}{\psi} \right) (\gamma - 1) \frac{\kappa_1}{2(1 - \kappa_1 v_g)} \left(\frac{\kappa_1}{1 - \kappa_1 \rho} \right)^2.$$

Note that these risk compensation parameters, λ_{gw} and λ_{xw} , are zero in the CRRA utility case, while they all become negative under the situation where agents prefer early resolution of uncertainty, i.e., $\gamma > 1$ and $\psi > 1$.

m_{gs} and λ_η are two most important solution coefficients since the expected difference between home and foreign stochastic discount factor (SDF) only comes from different consumption volatility levels, i.e., $\sigma_{g,t}^2$ and $\sigma_{g,t}^{*2}$, which will turn out to be critical in FX movements.

$$m_{gs} = -\frac{1}{2} \left(\gamma - \frac{1}{\psi} \right) (\gamma - 1), \quad \lambda_\eta = \gamma.$$

With IES and risk aversion both being large than one, IMRS's sensitivity to current consumption volatility, i.e., m_{gs} , becomes negative. In other words, the pricing kernel's negative sensitivity to consumption volatility is entirely consistent with eq.(8) where asset

¹¹ The computation of IMRS is greatly facilitated through the fact that the expectations of the exponential of the state variables is exponentially linear in the current states. See [Bansal and Shaliastovich \(2013\)](#) for details

prices fall when consumption volatility increases. This is the unique feature of the EZ preference since the typical CRRA utility would have implied no impact of consumption volatility on the pricing kernel, i.e., $m_{gs} = 0$.

With this analytical expression for the pricing kernel, equilibrium currency and equity prices as well as expected returns can be obtained, which is discussed in the following two sections.

5.2 Equilibrium Real Foreign Exchange Rate

Backus, Allan, and Chris (2001) show that in the case of complete markets, investing in foreign currency amounts to shorting a claim that pays off home SDF and going long in a claim that pays off the foreign SDF. In other words, the following condition holds

$$s_{t+1} - s_t = m_{t+1}^* - m_{t+1}, \quad (10)$$

where s_t is the real FX rate in home currency per unit of foreign currency and m_{t+1}^* and m_{t+1} are the pricing kernel for foreign and home countries respectively. Intuitively, a higher foreign SDF is consistent with foreign consumers valuing tomorrow's consumption goods more than home consumers. This would in turn mean higher relative real price for foreign goods tomorrow, i.e., foreign currency appreciation.

Under the complete market assumption, the eq.(9) and (10) give the equilibrium real FX process in this economy as below.

$$s_{t+1} = s_t + m_{gs} \{ \sigma_{g,t}^{*2} - \sigma_{g,t}^2 \} - \lambda_\eta \{ \sigma_{g,t}^* \eta_{t+1}^* - \sigma_{g,t} \eta_{t+1} \} - \lambda_{gw} \{ \omega_{g,t+1}^* - \omega_{g,t+1} \}. \quad (11)$$

Similar to Bansal and Shaliastovich (2013), the expected FX changes depend upon the current consumption volatility difference, i.e., $\sigma_{g,t}^{*2} - \sigma_{g,t}^2$.

$$E_t[s_{t+1} - s_t] = \frac{1}{2} \left(\gamma - \frac{1}{\psi} \right) (\gamma - 1) (\sigma_{g,t}^2 - \sigma_{g,t}^{*2}). \quad (12)$$

The intuition behind the eq.(12) is straightforward. A higher domestic consumption volatility today would lower the domestic pricing kernel under the EZ preference with γ and ψ both being larger than one. In consequence, the relative price of home goods is expected to fall tomorrow, indicating the expected home currency depreciation.

5.3 Equilibrium Real Equity Returns

The appendix shows that the log-linearized real return on home equity is a linear process in state variables.

$$\begin{aligned} r_{d,t+1} &= \ell_0 + \ell_1 pd_{t+1} - pd_t + \Delta D_{t+1}, \\ pd_t &= B_0 + B_x x_t + B_{gs} \sigma_{g,t}^2 + B_{xs} \sigma_{x,t}^2 + B_{ls} \sigma_{l,t}^2, \end{aligned} \quad (13)$$

where pd_t is the log price to (imaginary) dividend ratio and the solution coefficients for B_{gs} and B_{ls} are the following:

$$B_{gs} = \frac{0.5(\varphi_d - \gamma)^2 - (\gamma - 1/\psi)(\gamma - 1)}{1 - \ell_1 v_g} < 0, \quad (14)$$

$$B_{ls} = \frac{1}{2(1 - \ell_1 v_l)} > 0. \quad (15)$$

Again, other equilibrium solution coefficients are not reported here since they do not affect the following analyses.

It is worth interpreting the sign of the two coefficients, B_{gs} and B_{ls} . First, the sign of B_{gs} depends on model and preference parameters. Nevertheless, its sign is most likely to be negative under the typical parameter values widely used in the long-run risks literature. This will become clear in the calibration section later. As already discussed, the assumption of IES and risk aversion both being larger than one is critical in bringing about the negative B_{gs} .

Second, B_{ls} is always positive, meaning that the higher liquidity volatility, i.e., $\sigma_{l,t}^2$, *ceteris paribus* boosts equity prices, hence reducing expected equity returns in the future. This is one crucial point of the current model. In contrast to consumption volatilities, liquidity volatility has a positive effect on asset prices. As explained earlier, the mechanism behind this is similar to [Pastor and Veronesi \(2006\)](#). Since the liquidity in this model effectively acts as an extra and exogenously given i.i.d. dividend, more variations in the latter cause agents to value equities more.

Finally, following [Bansal and Yaron \(2004\)](#), the risk premium on the aggregate equity security in this model can be described as;

$$RP_t \equiv E_t [r_{d,t+1} - r_{f,t+1}] + \frac{1}{2} var_t [r_{d,t+1}] = -cov_t [m_{t+1}, r_{d,t+1}]$$

where $r_{f,t+1}$ is the risk-free rate in this economy. The following lemma shows a closed form solution for the risk premium in this economy.

Lemma 1 *The risk premium has the following form.*

$$RP_t = \lambda_l \ell_1 B_x \sigma_{x,t}^2 + \lambda_{gw} \ell_1 B_{gs} \sigma_{gw}^2 + \lambda_{xw} \ell_1 B_{xs} \sigma_{xw}^2 + \lambda_\eta \ell_1 B_{ls} \tau$$

With IES and risk aversion both being larger than one, $\lambda_l \ell_1 B_x > 0$, $\lambda_{gw} \ell_1 B_{gs} > 0$, $\lambda_{xw} \ell_1 B_{xs} > 0$, and $\lambda_\eta \ell_1 B_{ls} \tau < 0$.

Proof. See the appendix. ■

First, the risk premium on the aggregate security is time-varying as the long-run growth trend's volatility, $\sigma_{x,t}^2$, fluctuates. Second, loadings in front of $\sigma_{x,t}^2$, σ_{gw}^2 , and σ_{xw}^2 become positive under the assumption of both IES and the risk aversion parameters being greater than 1. This in turn intuitively implies that during periods of high economic uncertainty, risk premia will rise. All these characteristics are standard in long run risks models.

Last thing to note here is that a fixed negative covariance, τ , between the liquidity volatility shock, ω_l , and the short-run consumption level shock, η , is assumed in this model. This implies that changes in τ do have a level effect on the risk premium although they could not command the time-varying risk premium. The intuition is that a higher $|\tau|$ makes the aggregate liquidity volatility and the aggregate consumption move in opposite way to a greater extent. Since the aggregate liquidity volatility *effectively* increases asset returns through a similar mechanism in [Pastor and Veronesi \(2006\)](#), a higher $|\tau|$ lowers the risk premium, i.e., it lowers the level of constant in the risk premium.

6 Correlations on FX and Equity Returns

This section focuses on the time-varying correlations between FX and relative equity returns implied by the model. What is critical in triggering the sign-switching behavior of the correlation turns out to be the magnitude of SR consumption volatility or SR economic uncertainty in this model. The following proposition summarizes the model prediction on the time-varying correlations.

Proposition 1 *The conditional covariance of unexpected (or realized) FX movements and unexpected (or realized) relative equity returns have the following closed form solution in this model economy.*

$$cov_t [FX_{t+1}, RD_{t+1}^*] = -\lambda_{gw} B_{gs} 2 [\sigma_{gw}^2 + (\bar{\omega}_g)^2] - \tau \lambda_\eta B_{ls} [\sigma_{g,t}^* + \sigma_{g,t}], \quad (16)$$

where $FX_{t+1} = s_{t+1} - s_t$ and $RD_{t+1}^* = r_{d,t+1}^* - r_{d,t+1}$.

Under the assumption that $\gamma > 1$, $\psi > 1$ and $\tau < 0$ there exists a unique positive threshold level of Q such that if $\sigma_{g,t}^* + \sigma_{g,t} > Q$ then, the conditional covariance becomes a positive value, otherwise its sign is reversed. The Q is given by

$$Q = \frac{-\lambda_{gw} B_{gs} 2 [\sigma_{gw}^2 + (\bar{\omega}_g)^2]}{\tau \lambda_\eta B_{ls}} > 0. \quad (17)$$

Proof. See the appendix. ■

In order to develop intuition behind this result, it is convenient to work with the following equation instead.

$$\text{cov}_t [FX_{t+1}, RD_{t+1}^*] = E_t [(FX_{t+1} - E_t[FX_{t+1}])(RD_{t+1}^* - E_t[RD_{t+1}^*])]. \quad (18)$$

Thus, the conditional covariance can be intuitively understood as how the realized FX movement, i.e., $FX_{t+1} - E_t[FX_{t+1}]$, and the realized equity return differentials, i.e., $RD_{t+1}^* - E_t[RD_{t+1}^*]$, comove in response to various different shocks.

The appendix shows that the realized FX movement and the realized equity return differentials can be expressed as

$$FX_{t+1} - E_t[FX_{t+1}] = -\lambda_{gw} \{\omega_{g,t+1}^* - \omega_{g,t+1}\} - \lambda_\eta \{\sigma_{g,t}^* \eta_{t+1}^* - \sigma_{g,t} \eta_{t+1}\}, \quad (19)$$

$$RD_{t+1}^* - E_t[RD_{t+1}^*] = B_{gs} \{\omega_{g,t+1}^* - \omega_{g,t+1}\} + B_{ls} \{\omega_{l,t+1}^* - \omega_{l,t+1}\} \\ \varphi_d \{\sigma_{g,t}^* \eta_{d,t+1}^* - \sigma_{g,t} \eta_{d,t+1}\} + \{\sigma_{l,t}^* \zeta_{t+1}^* - \sigma_{l,t} \zeta_{t+1}\}. \quad (20)$$

First thing to note here is that the realized SR consumption volatility differentials, i.e., $\omega_{g,t+1}^* - \omega_{g,t+1}$ make the realized FX movement and the realized equity return differentials move the opposite way. Regarding the FX movements, since the market price of SR volatility risks is negative, i.e., $\lambda_{gw} < 0$, in this framework, a higher realized SR consumption volatility makes agents value consumptions more. In other words, the realized pricing kernel rises as a response. This explains why the higher realized SR consumption volatility differentials would lead to a realized foreign currency appreciation, shown in eq.(19). On the contrary, a higher realized SR consumption volatility depresses the realized equity return differentials as standard in long-run risks models. This is shown in e.q.(20) with a negative value for B_{gs} . In sum, the $\omega_{g,t+1}^* - \omega_{g,t+1}$ always induces the conditional covariance to be negative. This effect is captured by the (negative constant) first term in eq.(16).

Other shocks' differentials do not affect the conditional covariance since they are i.i.d. except through a negative contemporaneous correlation between ω_l and η . Specifically, as can be seen in eq.(20) with $B_{ls} > 0$, the higher realized liquidity volatility differentials, i.e.,

$\omega_{l,t+1}^* - \omega_{l,t+1}$ boost the realized equity return differentials *effectively* through the second independent dividend effect, explained earlier. Importantly, the increase in $\omega_{l,t+1}^* - \omega_{l,t+1}$ is also likely to cause a fall in $\sigma_{g,t}^* \eta_{t+1}^* - \sigma_{g,t} \eta_{t+1}$, i.e., the realized SR consumption growth differentials, through a negative τ . Since the market price of SR risks, i.e., λ_η , is positive in this framework, a reduction in $\sigma_{g,t}^* \eta_{t+1}^* - \sigma_{g,t} \eta_{t+1}$ is equivalent to an increase in the realized pricing kernel differentials. This in turn would lead to the realized foreign currency appreciation. Furthermore, the magnitude of such realized foreign currency appreciation is amplified by the level of current SR consumption volatility, i.e., $\sigma_{g,t}^*$ and $\sigma_{g,t}$, as implied in eq.(19). This summarizes why $\omega_{l,t+1}^* - \omega_{l,t+1}$ creates a positive pressure for the conditional covariance, and more importantly induces the covariance to be time-varying. This effect is captured by the (positive and time-varying) second term in eq.(16).

Eventually, the SR economic uncertainty level, i.e., $\sigma_{g,t}^* + \sigma_{g,t}$ relative to the Q in Proposition 1, determines whether the correlations become positive or negative. Again, this model prediction is consistent with the empirical evidence found in section 3. The following section finally examines to what quantitative extent this model economy can replicate the empirical evidence through calibration exercises.

7 Calibration of FX and Equity Returns

7.1 Parameterization

The baseline calibration parameter values of Table 7, were adapted from [Bansal and Shaliastovich \(2013\)](#). Notice here that the model is calibrated at a monthly frequency, so these parameter values were transformed into monthly values. [Bansal and Shaliastovich \(2013\)](#) provided detailed explanation for these values. Basically, they chose these values such that consumption processes in this model economy correspond well to U.S and UK (mostly U.S) business-cycle data. As for the preference parameters, nothing is at odds with the standard values in the literature except for the magnitude of the IES (greater than 1), which is still debatable. However, the IES value of 1.5 is chosen to match the inverse relationship between asset values and consumption volatility, which is well supported in data.

The parameter values for aggregate liquidity dynamics deserve an explanation since they are unique features of this model. Most importantly, the aggregate liquidity volatility level, σ_l , and the aggregate liquidity volatility of volatility, σ_{lw} , was chosen to match the data on Amihud measure of equity market illiquidity volatility used in section 3; see summary statistics in Table 1. The aggregate liquidity volatility persistence parameter, v_l was chosen to match the mean of the estimated AR1 coefficients for the Amihud measure of illiquidity

for each country; see Table 8 for details. A fixed parameter, τ , for the contemporaneous covariance between η and ω_l was chosen to match the average of correlation coefficients between stock returns and volatility of Amihud illiquidity measures; see Table 6 for details. Finally, these parameter values confirm the negative marginal effect of consumption volatility on asset prices, i.e., $B_{gs} = -357$, and the positive marginal effect of liquidity volatility on asset prices, i.e., $B_{ls} = 2.1$.

Table 7: Model parameter values

<i>Consumption Dynamics</i>	
Mean of consumption growth	$\mu_g = 0.0016$
Expected growth persistence	$\rho = 0.991$
Short-run volatility level	$\sigma_g = 0.0042$
Short-run volatility persistence	$v_g = 0.803$
Short-run volatility of volatility	$\sigma_{gw} = 1.57 * 10^{-5}$
Long-run volatility level	$\sigma_x = 1.67 * 10^{-4}$
Long-run volatility persistence	$v_x = 0.9799$
Long-run volatility of volatility	$\sigma_{xw} = 1.96 * 10^{-6}$
<i>Aggregate Dividend and Liquidity Dynamics</i>	
Aggregate dividend sensitivity to long-run news	$\phi + a = 1.25$
Aggregate dividend growth volatility level	$\varphi_d = 10$
Aggregate liquidity volatility level	$\sigma_l = 0.2$
Aggregate liquidity volatility persistence	$v_l = 0.97$
Covariance parameter for SR growth and liquidity volatility	$\tau = -0.0235$
Aggregate liquidity volatility of volatility	$\sigma_{lw} = 2.16 * 10^{-3}$
<i>Preference Parameters</i>	
Discount factor	$\beta = 0.9978$
Intertemporal elasticity of substitution	$\psi = 1.5$
Risk aversion coefficient	$\gamma = 10$

7.2 Quantitative Results

Under the parameter values specified above, a model simulation of a 20-year period was conducted. First, the ‘uncertainty index’, the sum of σ_{gt}^2 and σ_{gt}^{*2} , for the 20-year period was simulated. Then, I estimated the coefficient on the interaction term between equity return differentials and the uncertainty index in a similar fashion as γ in Table 4. Based on 5000 simulations of a 20-year period, the average regression coefficient turns out to be

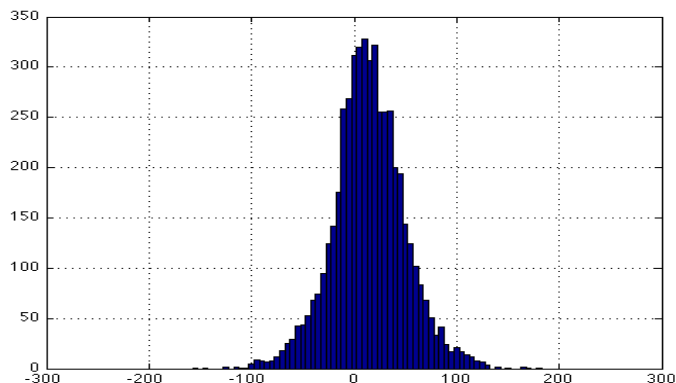
Table 8: $\sigma_{lq,t}^2 = \alpha + \beta\sigma_{lq,t-1}^2 + \varepsilon_t$

Countries	AR(1)
Austria	0.9795409***
Belgium	0.9783027***
Canada	0.9710265***
France	0.9737169***
Germany	0.9400317***
Japan	0.9545403***
Netherlands	0.9800469***
South Korea	0.9754176***
Switzerland	0.8938113***
U.K.	0.9824603***
U.S.	0.9794583***

Note: The same significance level applies to *, ** and *** as before.

approximately 14 (about 10 in reality as reported in Table 4). About 67% of the simulated coefficients are positive numbers (13 out of 19 country pairs, 68% of the total pairs, report positive estimates on γ in Table 5). These results are well within the reach of the empirical evidence.¹²

Figure 5: Frequency distribution of regression coefficients, $\hat{\gamma}$, based on 5000 simulations



8 Concluding Remarks

This paper finds new evidence on the time-varying correlation structure between (real) equity and (real) currency returns. In particular, the negative correlation, found in many exiting studies, is found to become very weak or even overturn its sign during times of

¹² To make the comparison quantitatively appropriate, the ‘uncertainty index’ is multiplied by 10 as in the empirical exercise in section 3. Further, the average coefficient on the simulated β s turn out to be centered around zero as the $\hat{\beta}$ in Table 4. Since this empirically estimated beta is statistically insignificant. I do not report its simulated counterpart here.

economic stress or uncertainty. Given this newly found evidence, this paper also provides one plausible explanation for the time-varying correlations. A key mechanism behind the possible positive link between equity and currency returns lies in a negative correlation between the *level* of SR economic growth and equity market liquidity volatility, empirically supported findings as well. Since this positive force gets stronger whenever the *volatility* of SR economic growth soars, the correlations exhibit strong tendency to become positive during economic uncertainty crises.

This particular explanation is, of course, not without limitations. The ‘uncertain’ economic times are surely a combination of potentially interrelated economic events, e.g., flight to quality episodes, unconventional monetary policy, and etc, which creates various adverse consequences for many aspects of the economy and asset markets. Equity market liquidity could be just one of those channels through which relative prices are severely distorted during uncertain economic times. Thus, it would be interesting to endogenize the equity market liquidity process especially in accordance with various macroeconomic fundamentals such as monetary policy and endogenous portfolio choice of international investors. I leave this fruitful exercise to future research.

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Appendix

Proof for the equation (13), (14) and (15)

Define W_t as a price of equity before dividend at time t and then the formula for this price should be as below.

$$W_t = E_t \sum_{j=0}^{\infty} M_{t+j} D_{t+j},$$

where M_{t+j} is the stochastic discount factor at time $t+j$. The rate of return on this equity is then given by

$$R_{d,t+1} = \frac{W_{t+1} + D_{t+1}}{W_t} = \frac{D_{t+1}}{D_t} \frac{(1 + Z_{t+1})}{Z_t},$$

where Z_t is defined as a price to dividend ratio. The standard log linearization of $R_{d,t+1}$ gives a following equation

$$r_{d,t+1} = \ell_0 + \ell_1 pd_{t+1} - pd_t + \Delta D_{t+1}. \quad (21)$$

Now the proof for the e.q.(13) follows as below.

First, the log price to dividend ratio, pd_t is conjectured as

$$pd_t = B_0 + B_x x_t + B_{gs} \sigma_{g,t}^2 + B_{xs} \sigma_{x,t}^2 + B_{ls} \sigma_{l,t}^2. \quad (22)$$

Second, a standard Euler equation for equities is given by

$$E_t[\exp(m_{t+1} + r_{d,t+1})] = 1. \quad (23)$$

Third, substitute e.q.(22) into (21) and then into e.q.(23). This will give

$$\begin{aligned}
E_t[\exp(m_{t+1} + r_{d,t+1})] &= E_t[\exp\{m_0 + m_x x_t + m_{gs} \sigma_{g,t}^2 + m_{xs} \sigma_{x,t}^2 \\
&\quad - \lambda_\eta \sigma_{g,t} \eta_{t+1} - \lambda_e \sigma_{x,t} e_{t+1} - \lambda_{gw} \omega_{g,t+1} - \lambda_{xw} \omega_{x,t+1} \\
&\quad + \ell_0 + \ell_1 (B_0 + B_x x_{t+1} + B_{gs} \sigma_{g,t+1}^2 + B_{xs} \sigma_{x,t+1}^2 + B_{ls} \sigma_{l,t+1}^2) \\
&\quad - (B_0 + B_x x_t + B_{gs} \sigma_{g,t}^2 + B_{xs} \sigma_{x,t}^2 + B_{ls} \sigma_{l,t}^2) \\
&\quad + \mu_g + (\phi + \tau) x_t + \varphi_d \sigma_{g,t} \eta_{d,t+1} + \sigma_{l,t} \zeta_{t+1}\}] \\
&= 1.
\end{aligned} \tag{24}$$

Even though the volatility shocks are non-Gaussian, this model specification belongs to the exponentially affine class. One of the nicest features of the exponentially affine function is that the expectations of the exponential of the state variables is exponentially linear in the current states. In consequence, solving for the equilibrium solution coefficients, B_{gs} would only require us to sum up all the loadings in front of $\sigma_{g,t}^2$ and to set them equal to zero. Similar logic applies to B_{ls} as well. The loadings in front of $\sigma_{g,t}^2$ and $\sigma_{l,t}^2$ are respectively given by

$$\begin{aligned}
0 &= m_{gs} + \ell_1 v_g B_{gs} - B_{gs} + \frac{1}{2} (\varphi_d - \gamma)^2, \\
0 &= \ell_1 B_{ls} v_l - B_{ls} + \frac{1}{2}.
\end{aligned}$$

Finally, rearranging the two equations above gives e.q.(14) and (15). As mentioned already, equilibrium solutions for all the other coefficients are omitted here because they are irrelevant for the purpose of this study. The exact derivation for those coefficients are almost identical as the ones in [Bansal and Shaliastovich \(2013\)](#). *Q.E.D.*

Proof for Lemma 1

First, $-cov_t[m_{t+1}, r_{d,t+1}] = -E_t[(m_{t+1} - E_t[m_{t+1}])(r_{d,t+1} - E_t[r_{d,t+1}])]$. From eq.(9) it is easy to construct $m_{t+1} - E_t[m_{t+1}]$ as

$$m_{t+1} - E_t[m_{t+1}] = -\lambda_\eta \sigma_{g,t} \eta_{t+1} - \lambda_e \sigma_{x,t} e_{t+1} - \lambda_{gw} \omega_{g,t+1} - \lambda_{xw} \omega_{x,t+1}. \tag{25}$$

By using eq.(13) and its expected value, one could derive $r_{d,t+1} - E_t[r_{d,t+1}]$ as

$$\begin{aligned}
r_{d,t+1} - E_t[r_{d,t+1}] &= \varphi_d \sigma_{g,t} \eta_{d,t+1} + \sigma_{l,t} \zeta_{l,t+1} \\
&\quad + \ell_1 \{B_x \sigma_{x,t} e_{t+1} + B_{gs} (\omega_{g,t+1} - \bar{\omega}_g) + B_{xs} (\omega_{x,t+1} - \bar{\omega}_x) + B_{ls} (\omega_{l,t+1} - \bar{\omega}_l)\},
\end{aligned} \tag{26}$$

where $\bar{\omega}_g, \bar{\omega}_x$ and $\bar{\omega}_l$ are the unconditional mean of consumption growth volatility, long-

run growth volatility and liquidity volatility respectively. Finally, by exploiting i.i.d shock processes and combining eq.(25) and (26), one can derive the closed form solution in Lemma 1. *Q.E.D.*

Proof for Proposition 1

Equation (19) can be easily obtained through using eq.(11). $RD_{d,t+1}^*$ can be computed using eq.(13). The result is given by

$$\begin{aligned} r_{d,t+1}^* - r_{d,t+1} = & -B_{gs}(1 - \ell_1 v_g)(\sigma_{g,t}^{*2} - \sigma_{g,t}^2) - B_{ls}(1 - \ell_1 v_g)(\sigma_{l,t}^{*2} - \sigma_{l,t}^2) \\ & + B_{gs} \{ \omega_{g,t+1}^* - \omega_{g,t+1} \} + B_{ls} \{ \omega_{l,t+1}^* - \omega_{l,t+1} \} \\ & + \varphi_d \{ \sigma_{g,t}^* \eta_{d,t+1}^* - \sigma_{g,t} \eta_{d,t+1} \} + \{ \sigma_{l,t}^* \zeta_{t+1}^* - \sigma_{l,t} \zeta_{t+1} \}. \end{aligned}$$

By taking expectation into this expression one could derive eq.(20).

By replacing the equation (19) and (20) into (18) and using the i.i.d. assumptions on relevant shocks one could finally get the following.

$$\begin{aligned} cov_t [FX_{t+1}, RD_{t+1}^*] = & E_t [-\lambda_{gw} B_{gs} E_t [\omega_{g,t+1}^{*2} + \omega_{g,t+1}^2] \\ & - \lambda_\eta B_{ls} \{ E_t [\eta_{t+1} \omega_{l,t+1}] \sigma_{g,t} + E_t [\eta_{t+1}^* \omega_{l,t+1}^*] \sigma_{g,t}^* \}] \\ = & -\lambda_{gw} B_{gs} 2 [\sigma_{gw}^2 + \bar{\omega}_g^2] - \tau \lambda_\eta B_{ls} \{ \sigma_{g,t}^* + \sigma_{g,t} \}. \end{aligned}$$

Note that the second equation above uses two facts. First, $E_t [\eta_{t+1} \omega_{l,t+1}] = cov_t [\eta_{t+1} \omega_{l,t+1}] = \tau$ due to $E_t [\eta] = 0$ (the same applies to the foreign case). Second, $E_t [\omega_{g,t+1}^2] = Var_t [\omega_{g,t+1}] + (E_t [\omega_{g,t+1}])^2$ (the same applies to the foreign case). *Q.E.D*