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Abstract

Krishna and Serrano (1996) study a model of multilateral bargaining, and claim that their analysis is applicable irrespective of whether the surplus exists at the start of the game or it is created after all players agree. We show that their claim is wrong. Their analysis is not applicable when the surplus is created after all players agree. Hence, some of the important real life bargaining situations, like management-multiple unions bargaining and land assembly are not in the scope of Krishna and Serrano (1996).

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Krishna and Serrano (1996) show unique and efficient outcome in a model of multilateral bargaining with non-unanimous exit. The result is interesting, especially considering “any distribution of pie is possible” kind of result in the model with unanimous exit, studied by Shaked.[1] Shaked’s analysis is applicable irrespective of whether the surplus (under the bargaining) exists at the start of the game or it is created after all players agree. Krishna and Serrano (1996) claim that their model, and hence the analysis, is applicable to both the cases too. We, however, show that their claim is wrong. Their analysis is not applicable when the surplus is created after all players agree. This implies that some of the important

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[1]Sutton (1986) credits the work to Shaked.

and frequent economic affairs like management-multiple unions bargaining, land assembly etc, do not fall under the scope of Krishna and Serrano (1996), although claimed otherwise.

Krishna and Serrano (1996) analyse following model. There are $n(\geq 3)$ players bargaining to split a unit surplus.[2] Imagine an exogenously given sequence of the players. In the first period, the first player in the sequence proposes a distribution of the surplus, and others respond to the proposal. All those who accept, get paid as per the proposal and exit the game forever. While all those responders who reject the proposal and the proposer, bargain over the leftover surplus in the next period in the same manner. The leftover surplus is the surplus remaining after funding all the previous exits. Also, the proposer in the new period is the next player in the sequence who is still in the game. The game continues till $n-1$ players exit, leaving the proposer of the last period with the leftover surplus. All players are risk neutral and discount at a common rate $\delta \in (0,1)$ per period.
Since it is a complete information game, Krishna and Serrano (1996) employs subgame perfect nash equilibrium as the solution concept. Following is one of the key results from the paper.

Proposition 0. 1 *For $\delta \in (0,1)$, the game has an unique and efficient subgame perfect equilibrium.*[3]

Note that an equilibrium is **efficient** if the game ends in period 1, otherwise it is **inefficient**.

In the above model, the exits are funded by the surplus. Hence, the surplus must exist at the start of the game. In order to accomodate the case in which the surplus is created only after all players agree, Krishna and Serrano (1996) provide an alternate interpretation of the model, which is captured in following paragraphs extracted from Krishna and Serrano (1996, p 63).[4]

"Consider the bargaining problem of dividing a dollar among three players. Suppose player 1 proposes a division $x = (x_1, x_2, x_3)$. If both players 2 and 3 accept the proposal the game ends with that division. If both reject, in the second period player 2 proposes a division y and players 3 and 1 must respond. If player 3 accepts x and 2 rejects, 3 can "exit" the game with an amount x_3 while players

[2] Krishna and Serrano (1996) do not normalize the surplus. Either way, it does not affect any result.

[3] The proposition corresponds to Theorem 1 in Krishna and Serrano (1996).

[4] The alternate interpretation is discussed further in section 8 of Krishna and Serrano (1996).

1 and 2 are left to bargain over the division of $1 - x_3$ in period 2. This bargaining now proceeds as in the two player alternating offer game with player 2 proposing a division of $1 - x_3$.

The game outline above can also be interpreted as one where player 1 offers to purchase the right(s) to represent the accepting player(s) in any future negotiations. Thus in the example given above, player 1 purchases the right to represent player 3 in future negotiations at a price of x_3. This amount is paid to player 3 immediately by player 1, who then bargains with player 2 over whole pie. We will refer to this game, interpreted in either way, as the "exit" game."

While the first paragraph describes the model, the second one presents an alternate model, which differs in only the following way - All the exits in a period are paid by the corresponding proposer and not the surplus, which is created at the end of the game. Hence, the leftover surplus in any subgame is the entire surplus. Krishna and Serrano (1996) assumes that both the models are equivalent. If both the models are equivalent, then they must yield same result. We analyse the alternate model below and show that this is not the case.

It is straight forward to see that in the alternate model, a subgame with two players is a Rubinstein (1982) game with unit surplus. Hence, the proposer and the responder get $\frac{1}{1+\delta}$ and $\frac{\delta}{1+\delta}$ respectively in the first period of the subgame. Following proposition establishes that the alternate model suffers from inefficiency.

Proposition 1 *Consider the alternate model. For $n \geq 4$ and $\delta > \frac{1}{n-2}$, there does not exist an efficient subgame perfect equilibrium.*

Proof. Let there exists an equilibrium in which all the responders accept the proposal and the game ends in period 1. If one of the responder deviates and rejects the proposal, he gets $\frac{1}{1+\delta}$ in the ensuing Rubinstein (1982) game in the next period. Hence, the proposer must offer at least $\frac{\delta}{1+\delta}$ to each responder for all of them to accept the proposal.[5] Therefore, the maximum possible payoff to the proposer is $p_{max} = 1 - (n-1)\frac{\delta}{1+\delta}$. For $n \geq 4$ and $\delta > \frac{1}{n-2}$, p_{max} is negative, which is not possible in an equilibrium. Hence, the result. ■

Observe the contrast between proposition 1 and proposition 0.1. It is clear that the two models are not equivalent. To see the intuition behind proposition 1, con-

[5] This result is shared by some of the other multilateral bargaining models which allow for simultaneous offers and responses. See lemma 2 in Roy Chowdhury and Sengupta (2012).

sider δ close to 1. If all other responders are accepting the proposal, a responder can obtain almost half of the surplus by rejecting the proposal, in the subsequent Rubinstein type bilateral bargaining. Proposer, therefore, must offer almost half of the surplus to each responder to make them all accept, which results in loss for the proposer if there are three or more responders. Hence, the agreements must take place sequentially. This result is driven by the fact that the surplus is created at the end of the game and hence, the entire surplus is available for bargaining in every subgame. This does not happen in the model analysed in Krishna and Serrano (1996) because the surplus reduces with every exit and hence less surplus is available for bargaining to those who delay the agreements. Hence, it is less costly for the proposers to prevent responders from delaying the agreements.

Based on the difference in proposition 1 and proposition 0.1, we conclude that Krishna and Serrano (1996)'s analysis is strictly restricted to multilateral bargaining situations in which the surplus already exists and is not applicable to the case in which the surplus is created after all players agree.

References

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