Multifractal Random Walk Models: Application to the Algerian Dinar exchange rates

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February 2015

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Abstract: This paper deals with a special class of multifractal models called the Multifractal Random Walk which has been widely used in finance because of its parsimonious framework, featuring many properties of financial data not considered in traditional linear models. Using the log-normal version, results confirm the Algerian Dinar is a multifractal process and has a rich wider variation spectrum versus the US Dollar than the Euro.

Keywords: multifractal processes.

1. Introduction: Multifractal processes have been widely used in finance during recent years and their statistical properties made them pertinent to model high-frequency returns as they incorporate, using few parameters, many stylized facts of financial time series. The multifractal property stems from the variation of the absolute moments of returns with the return period, using a nonlinear exponent as a function of the moment order[1]. This key characteristic is not considered in standard volatility models (GARCH family) and neglecting the influence of the sampling interval leads to a loss of useful information.

In finance, two approaches for multifractality are used: the Markov-Switching Multifractal (MSM)[2] and the Multifractal Random Walk (MRW) introduced by Bacry and Muzy[3]. The first deals with the number of volatility components, while the latter stems from the random cascades and captures the interrelation between returns and the sampling intervals using three parameters: the intermittency coefficient, the decorrelation scale (integral correlation time) and the standard deviation. Kutergin and Filiminov[4] consider the MRW as the only continuous stochastic stationary process with exact multifractal properties and Gaussian infinitesimal increments. They reported six main stylized facts captured by MRW models: absence of linear autocorrelations, volatility clustering, long memory in volatility, heavy tails

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[1] C. Sattarhoff, Statistical Inference in Multifractal Random Walk Models for Financial Time Series, Volkswirtschaftliche Analysen, Peter Lang (2011).

[2] L. Calvet, A.Fisher, Forecasting multifractal volatility, Journal of Econometrics (2001), 105.

[3] E.Bacry, J.F. Muzy, Multifractal stationary random measures and multifractal random walks with log-infinitely divisible scaling laws, Physical review (2002) E66, 56121.

[4] A.Kutergyn, V.Filiminov, On the modeling of financial time series, Financial econometrics and empirical market microstructure, Springer (2015).

in probability distribution, aggregational Gaussianity and multifractal scaling. Thus, its most used case, the log-normal MRW, will be studied and applied to the case of the Algerian Dinar versus the US Dollar and the Euro.

2. Theoretical framework:

Definition 1: A stochastic process $X(t)$ with stationary increments $\Delta_\tau X(t) = X(t) - X(t-\tau)$is called scale invariant with multifractal exponent $\zeta(q)$ and an integral scale $T \in \mathbb{R}^+$ if for all $q \in \mathbb{Q}$ we have : $E[|\Delta_\tau X(t)|^q] = E[|\Delta_1 X(t)|^q]\tau^{\zeta(q)}$ for all $\tau \in [0, T]$.

Regarding the multifractal exponent $\zeta(q)$, we identify two different types of scale invariant processes : unifractal processes in case of a linear $\zeta(q)$, and multifractal processes when $\zeta(q)$ is nonlinear, concave with a finite T.

Hence, the intermittency coefficient $\lambda^2 = -\zeta''(0)$ is used as a measure of the degree of multifractality and $\lambda^2 = 0$ refers to the traditional Brownian motion process.

Definition 2: We consider the stationary process $\omega_l(t)$, known also as a Gaussian magnitude with $\lambda^2 < \frac{1}{2}$, having its mean and autocovariance function respectively :

$$\mathbb{E}[\omega_l(t)] = -\lambda^2 \left(ln\left(\frac{T}{l}\right) + 1 \right)$$

$$\gamma_{\omega_l}(h) = \begin{cases} \lambda^2 \left(ln\left(\frac{T}{l}\right) + 1 - \frac{h}{l} \right), & 0 \le h < l \\ \lambda^2 \left(ln\left(\frac{T}{h}\right) \right), & l \le h < T \\ 0 & h \ge T \end{cases}$$

The Multifractal Random Walk (MRW) is defined as the weak limit process $X(t)$

$$X(t) = \lim_{l \to 0^+} \int_0^t e^{\omega_l(u)} dB(u)$$

And $dB(u)$is a Gaussian white noise with mean 0 and variance σ^2. The MRW increments of $X(t)$ are defined as :

$$\Delta_\tau X(t) = \lim_{l \to 0^+} \int_{t-\tau}^t e^{\omega_l(u)} dB(u)$$

The probability distribution of the MRW is unkown, except for statistical moments :

$$\mathbb{E}[X(t)^2] = \sigma^2 t \text{ and } \mathbb{E}[\Delta_\tau X(t)^2] = \sigma^2 \tau$$

The MRW features three parameters: the intermittency rate λ^2, the logarithmic decorrelation scale $\ln(T)$ and the logarithmic standard deviation $\ln(\sigma)$. Several approaches are used to estimate the parameters; we will use the iterated GMM estimation provided by Sattarhoff (2011)[5], which proved to be robust compared with a prior version developed by Bacry et al. (2008). Duvernet[6] proved that the decorrelation scale and the standard deviation cannot be estimated if the observation horizon is below a certain scale.

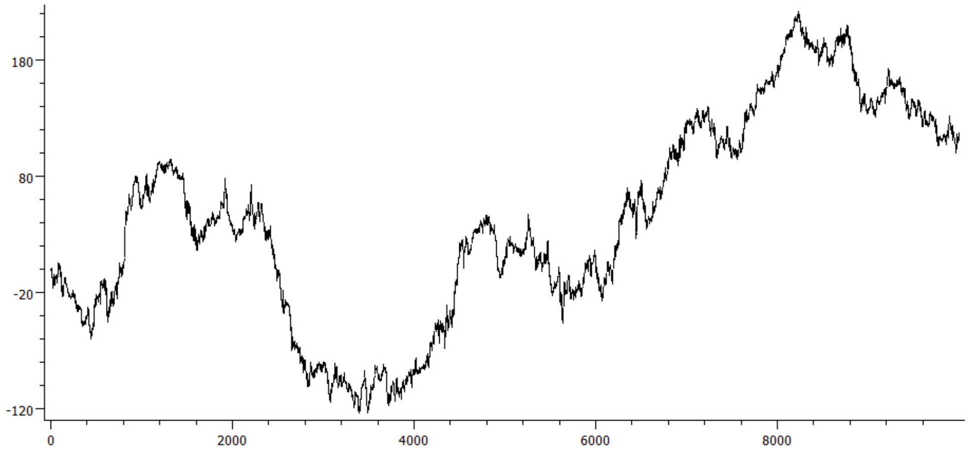

Figure 1. A generated MRW series[7] with $\lambda^2 = 0.02, \sigma^2 = 2$ *and* $T = 200$

3. Data and results: We will study the Algerian Dinar exchange rate against its two principal counterparts: the US Dollar and the Euro. Algeria, an oil exporting country, earns 98% of its revenues in US Dollar, while it imports 60% of its needs from the European Union. The series for the Algerian Dinar - US Dollar span from January 1996 to end-2014[8], while the series of the Euro has stand from January 1999 to end-2014.

[5] C. Sattarhoff, op. cit.
[6] L.Duvernet, Parametric estimation of the log-normal Multifractal Random Walk process, online document http://www.cmapx.polytechnique.fr/~duvernet/mrwstat.pdf
[7] The series was generated using *LastWave 3.1*, a software developed by Emmanuel Bacry. http://www.cmap.polytechnique.fr/~bacry/LastWave/index.html
[8] New quotation system of the Dinar entered into force in 1996, build upon 15 currencies representing the then trading partners of Algeria.

	Mean	Median	Std. Dev.	Range	Skewness	Kurtosis
Dinar – US Dollar	7.36E-05	0	0.0076	0.163	-0.163	19.541
Dinar – Euro	7.21E-05	0	0.0084	0.163	-0.023	13.195

Table 1. Descriptive statistics of the Dinar – US Dollar and the Dinar – Euro in logarithmic returns

Figures 2 to 9 show stylized facts similar to traditional financial time series as returns have zero autocorrelations (figures 4 and 5), but squared returns exhibit, over long lags, positive autocorrelations (figures 6 and 7). Evidence of long memory process, typical of fractal series, is given by the hyperbolic-decaying autocorrelation functions. Also, clusters of volatility appear in both returns' figures as a result of irregular periods of small and long variations (figures 2 and 3).

QQ-plots (figures 8 and 9) of short returns exhibit fat-tailed probability distributions for both series, although they are likely to approach the normal distribution for large return periods.

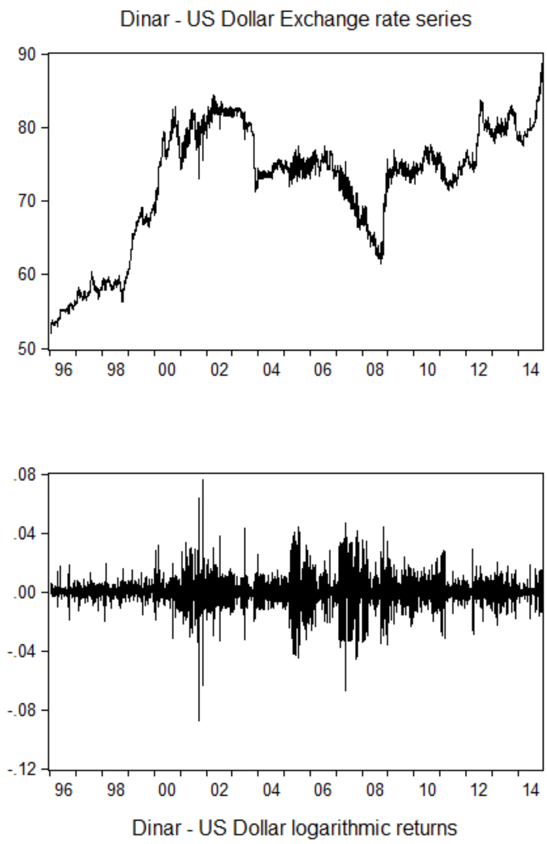

Figure 2. The Dinar-US Dollar data between January 1996 to December 2014.

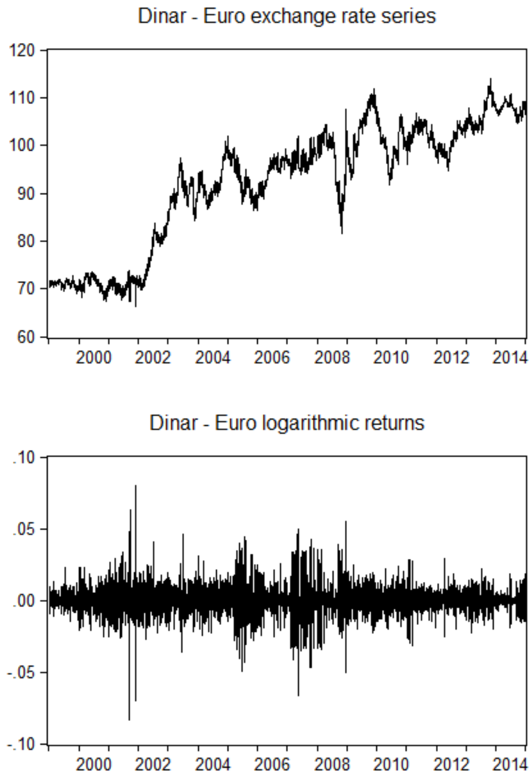

Figure 3. The Dinar-Euro data between January 1999 to December 2014.

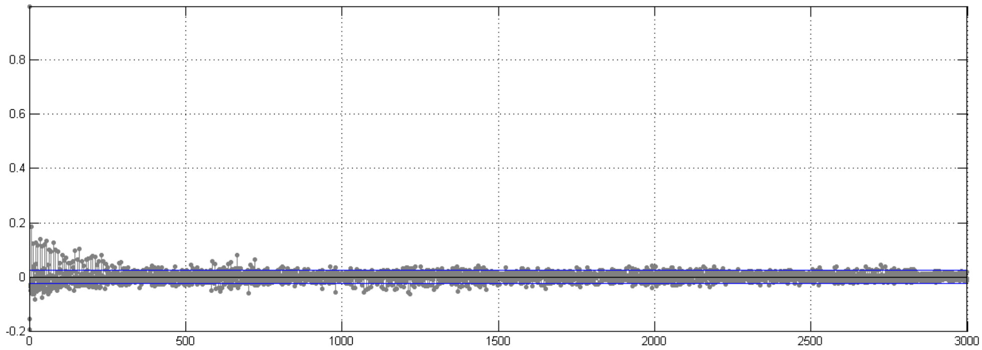

Figure 4. Empirical autocorrelation function of the Dinar – US Dollar logarithmic returns.

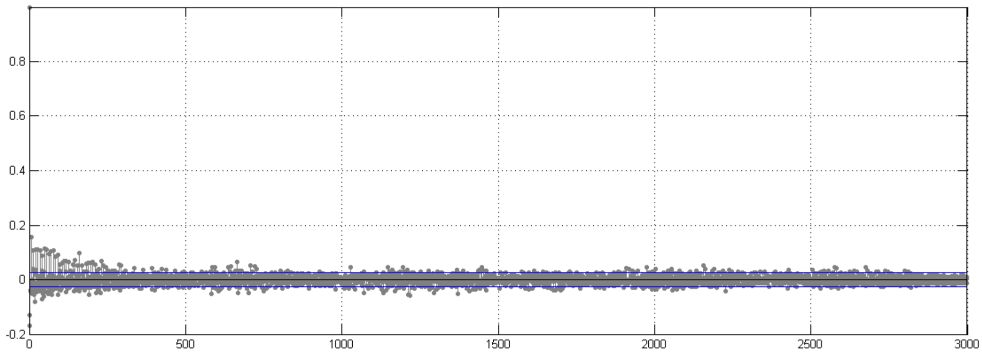

Figure 5. Empirical autocorrelation function of the Dinar – Euro logarithmic returns.

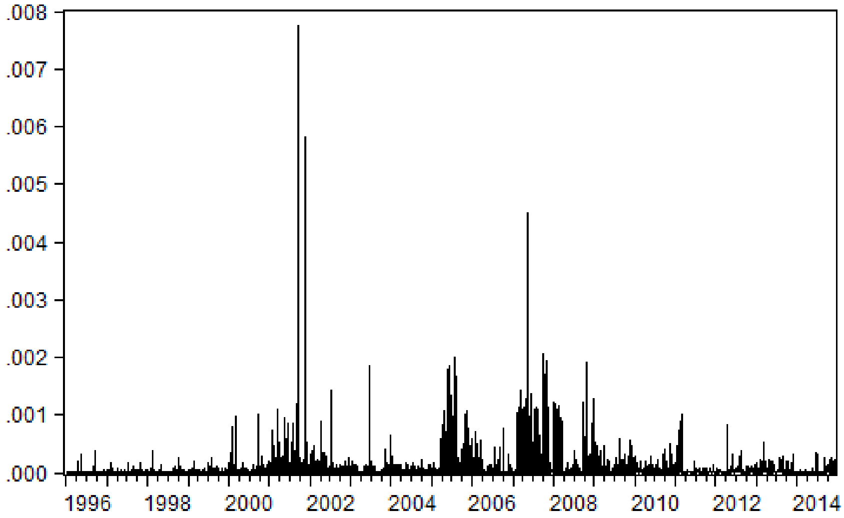

Figure 6. Squared logarithmic returns of the Dinar – US Dollar.

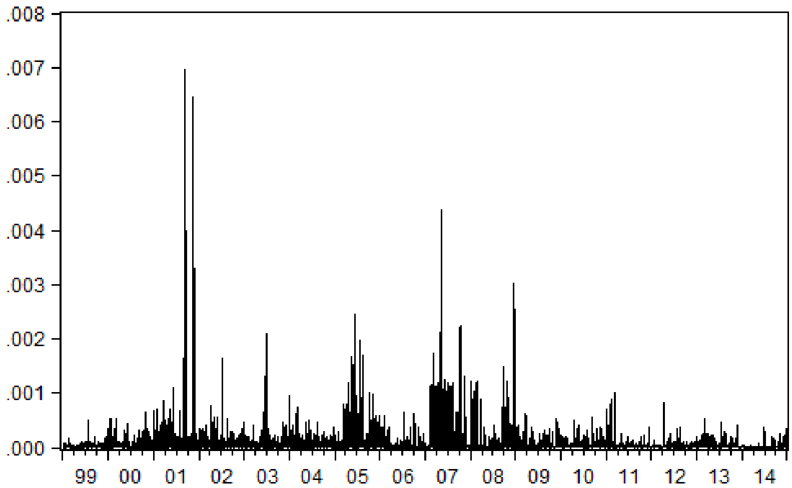

Figure 7. Squared logarithmic returns of the Dinar – Euro.

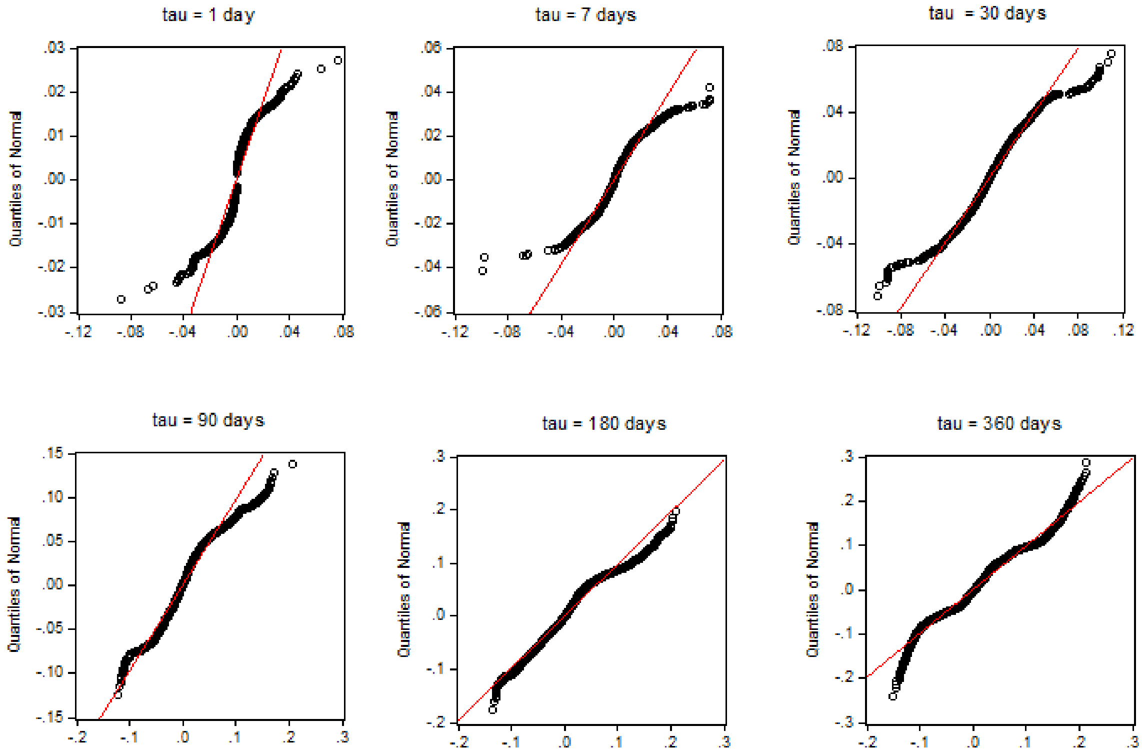

Figure 8. QQ plots of the Dinar – US Dollar returns versus normal distribution for various return periods τ.

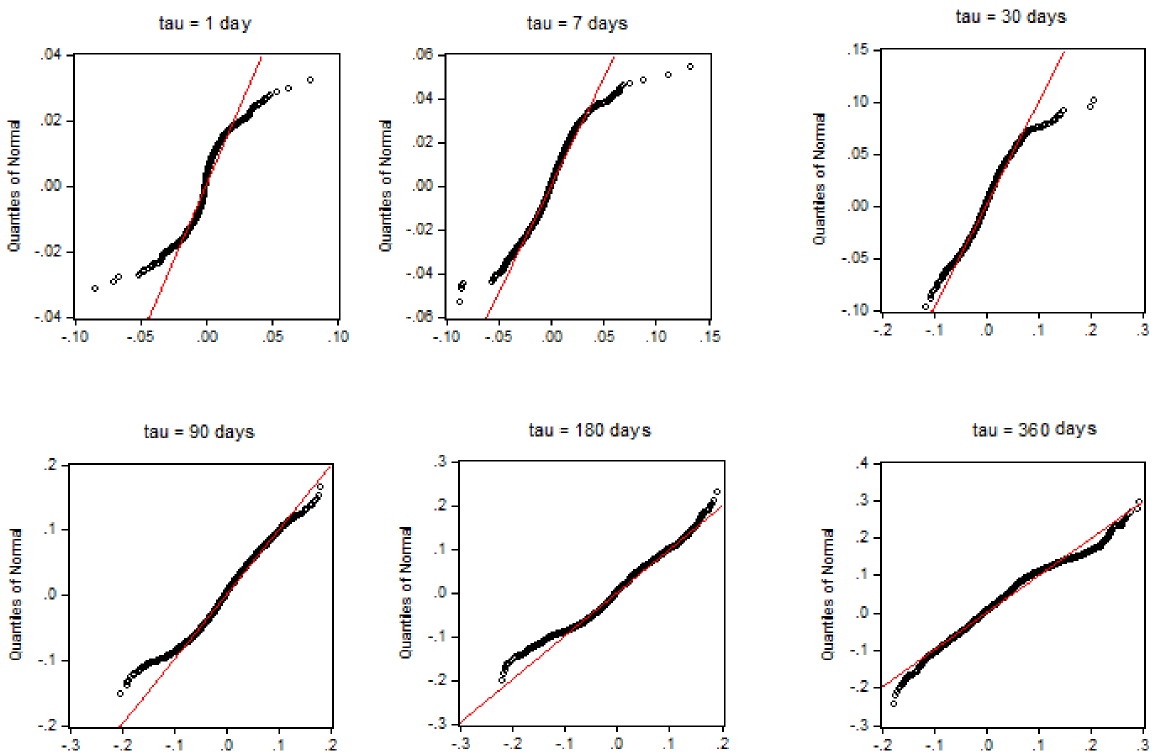

Figure 9. QQ plots of the Dinar – Euro returns versus normal distribution for various return periods τ.

	λ^2	$\ln(\sigma)$
Dinar – US Dollar	0.0530 *[0.0375 , 0.0685]*	-5.5756 *[-5.6840 , -5.4672]*
Dinar – Euro	0.0269 *[0.0175 , 0.0363]*	-5.3056 *[-5.3719 , -5.2393]*

Table 2. Parameters estimation and 95% confidence intervals using the iterated GMM procedure provided by Sattarhoff (2011)[9].

4. Discussion: Results shows the logarithmic standard deviations are very close, although significantly different from each other. The intermittency rates of both series are different from zero, proving that the Algerian Dinar is a multifractal process as mentioned previously by Diaf and Toumache[10]. However, the Dinar – US Dollar series has an intermittency rate of 0.053, twice the intermittency rate of the Dinar – Euro. This estimate (0.053) lies far from the most commonly reported value (0.02) advocated by Bacry, Kozhemyak and Muzy[11] but seems close to the suggested value of 0.04 given by Sornette[12]. Thus, the multifractal degree of the Dinar – Euro remains significant but relatively weak to Dinar – US Dollar which exhibits a strong multifractal behavior, probably due to the monetary policy interventions which aims to stabilize the Dinar.

5. Conclusion: The MRW approach sheds light on the multifractal structure of the exchange rate and tells us about its intensity, as it incorporates main stylized facts of financial time series. The findings confirm the complex behavior of the Dinar regime against its two main counterparts (the Euro and the US Dollar) which cannot not be captured by traditional linear models. Given the structural composition of the Algerian economy, less diversified and heavily relying on hydrocarbon exports, the Dinar – US Dollar parity is called to serve as a tool for the both the monetary and the fiscal policies. Hence, this prime role, combined with the oil prices' volatility, make the series subjected to intense turbulences than other exchange rate series.

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[9] Starting values set to 0.02 for λ^2 and 8 for $\ln(T)$, corresponding to a decorrelation scale of 3,000 observations.
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