The Surge of Wall Street and The Rise in Income Inequality - Do they share the same root ?

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The surge of Wall Street and the rise in income inequality - Do they share the same root ? *

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Abstract

Since 1980, the U.S. economy has witnessed simultaneously two macroeconomic themes: (i) the substantial growth of the financial sector, and (ii) the significant rise in income inequality. At the same time, there was a crucial change in the financial market where a wide range of new financial assets were introduced. This paper, by presenting a simple model of the interaction between the financial and real sectors, shows that the appearance of new financial instruments can generate both above themes. It can also explain the dominance of Wall Street against Main Street (non-financial sectors) in the top income earners category.

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1 Introduction

For three decades, the income share of the top 1 percent earners in US has increased at an unprecedented pace, climbing up from 8.18 percent in 1980 to 18.88 percent in 2012 (Alvaredo et al., 2012). At the same time, the economy has witnessed the expansion of the financial sector in terms of the share in GDP, the total employment and the average wage. Income from the asset market has become the growth engine for this industry since 1980. Finance also dominates all other sectors in the top income earner category (Kaplan and Rauh, 2010). Is that really a coincidence that two themes synchronized with each other ?

This paper gives a novel insight that the introduction of new financial assets might be the root of both themes. We build a simple model to explain why new assets can deepen the income inequality trend and push up the share of the financial sector in the economy. New assets allow the better risk-sharing between entrepreneurs in the different industries, making the access to the financial market become more valuable. Entrepreneurs are ready to pay more for financiers to make transactions in the financial market. Financiers themselves, due to the increase in the number of assets, can also earn the higher income from actively trading in the asset market. As the agents are risk averse, the more complete market also implies the higher rate of return on capital, which in turn transmits and amplifies the cycle between the wealth inequality and the income inequality.

The contribution of our paper is twofold. First, it gives a new explanation for the income inequality trend and the rise of the financial sector in US by putting the asset market in the main theme. There is a literature about the relationship between the development of finance and the income inequality (see Galor and Zeira (1993) and Levine (1997)), but the theme rotates around the credit market where intermediaries (banks) play the main role. This literature does not match with the data from 1980 when nearly 50 percent of the growth of finance lies in the subsector dealing with the asset market. Most closely related to our paper is the work of Greenwood and Jovanovic (1990), in which intermediaries pool and diversify the risk, leading to the higher rate of return on capital and therefore deepening the income inequality. In our model, risk-sharing is conducted in the decentralized asset market by allowing agents to trade a range of assets. The income inequality in our model is widened when the number of assets increases while the model in Greenwood and Jovanovic (1990) predicts the income inequality eventually declines with the development of finance.

Second, our model can explain the fact that more financiers go to the top income level. To my limited knowledge, this is the first paper investigating this phenomenon theoretically. Bakija, Cole and Heim (2008), by using the US income tax return data, show that the representation of the finance industry in the top 1 percent earner has increased from 7.7 percent in 1979 to 13.2 percent in 2005. Some research, like Cagetti and Nardi (2006), Jones and Kim (2014), give the insights why the income share of top entrepreneurs increases over years. However, there is a big gap in the literature in explaining the rise of the financiers into the top income, which is itself the main feature of the US inequality trend since 1980. New assets in our model generate two inequality trends simultaneously: income is redistributed from entrepreneurs to financiers and from low-wealth holders to top-wealth holders.

Our paper also touches the literature on financial innovation and security design. We follow Simsek (2013) to model an economy with a list of financial assets. In an endowment economy, Simsek (2013) illustrates the change in the portfolio risk from the appearance of the new assets. We, on the other hand, emphasize the links between the financial and real sectors; so we can evaluate the impact of new assets on the income distribution. We only concentrate into the risk sharing motive in the financial market, like Allen and Gale (1994), Duffie and Rahi (1995).

The paper is structured as follows. Section 2 introduces three stylized facts of the US economy since 1980 which motivates our theoretical model : the surge of Wall Street, the rise of the income inequality and the appearance of many new assets in the market. Section 3 describes our model. Section 4 gives some analytical results about the effect of new assets on the income distribution. Section 5 does the quantitative exercise to illustrate the impact of new financial assets on the income distribution.

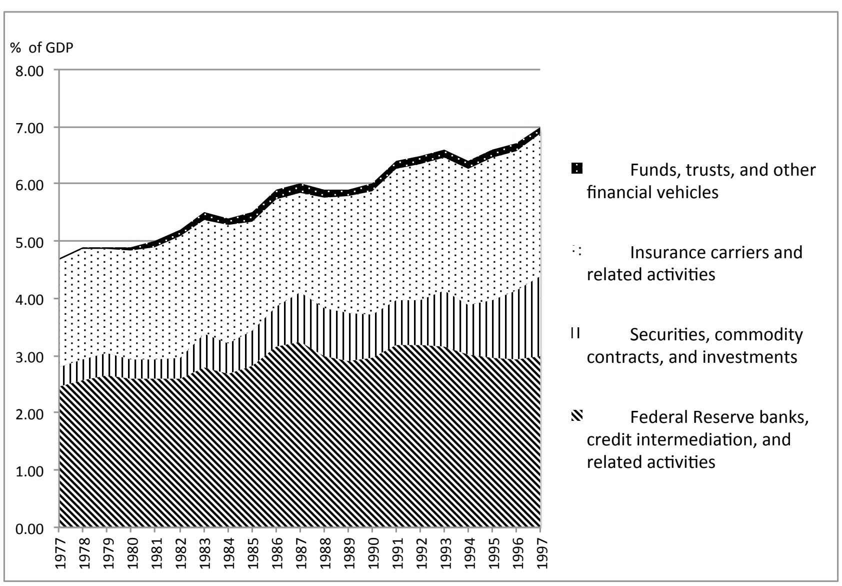
2 The three macroeconomic themes since 1980

Theme 1: The rise of the financial sector, especially the subsector trading in the asset market

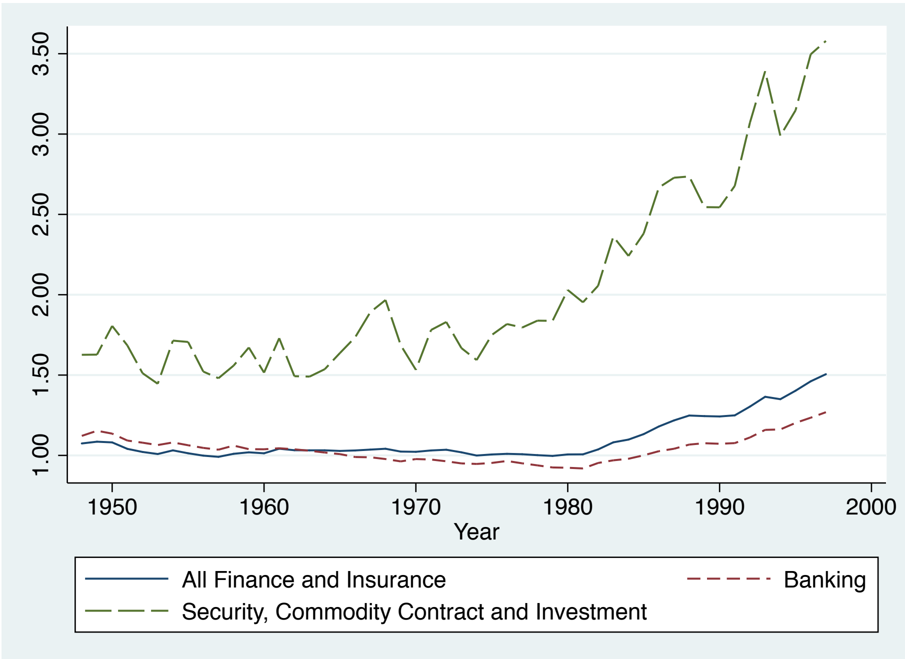
From 1977 to 1997, the share of the finance industry in GDP increases from 4.7 percent to 7 percent, in which the subsector "Securities, commodity contracts, and investments" alone contributes nearly half of this growth. This reflects a significant shift of the US financial sector since 1980s. Facilitating transactions and actively trading in the asset market becomes a new huge source of income for the financial sector beside the traditional role of providing credit to the economy.

Figure 1 depicts the growth of the finance industry during 1977-1997 with the data from BEA. If we break down the growth of the financial sector into four components, the sector dealing with the asset market "Securities, commodity contracts, and investments" contributes most to this growth (47 percent). The next one is the insurance sector (27 percent). Banking sector and "Funds, trusts, and other financial vehicles" contribute, respectively, 23 percent and 4 percent.

Figure 1: Share of subsectors of finance in GDP 1977-1997



The dominance of the finance industry against the rest of the economy is also shown in the data about the average wage. After 1980, the relative wage between the finance industry and the non-financial private sector also increases significantly. We normalize the average wage in the non-financial private sector as 1. In 1997, the wage in finance is 1.5 times as much as the one in the non-financial industry. The relative wage is very stable at 1 before 1980, indicating no difference between the average wage in finance and the other private sector. The upward trend after 1980 is again explained by the rocketing increase in the average wage of brokers in the financial market. Figure 2 documents the change in the relative wage.



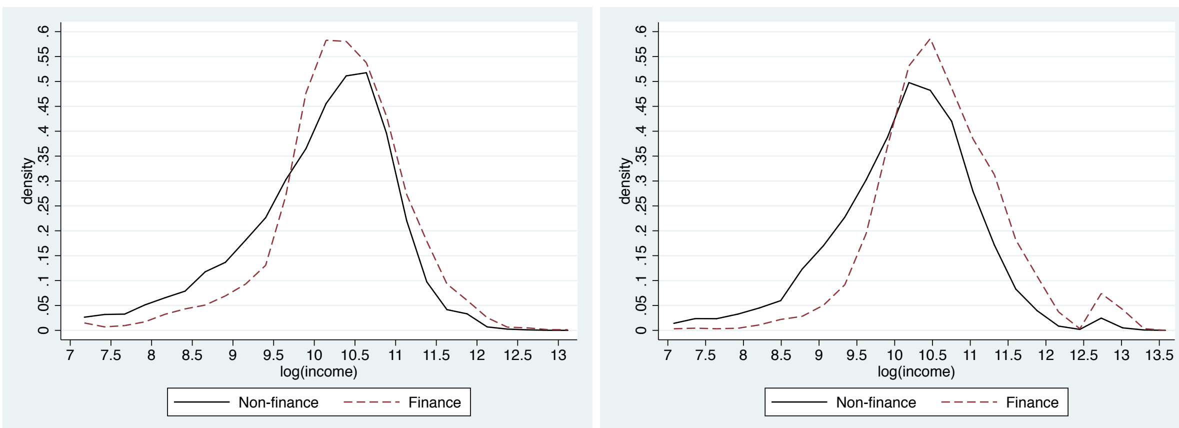
The calculation is based on Annual Industry Account data. The average wage is calculated as the ratio between total compensation of employees and the number of full-time equivalent employees. The average wage of non-finance private industry is normalized as 1. To compare data under SIC72 and SIC87, we define "credit intermediation" such that it comprises of sector coded 60 and 61 in SIC72 and sector coded 60 and 61 in SIC87.

Figure 2: The wage of the financial sector 1947-1997

Theme 2: The rise of top 1% income earners and the surge of Wall Street in top 1%

Another salient macroeconomic feature after 1980 is the rise of top 1%'s income share in US. According to Piketty and Saez (2003), the top 1%'s income share goes up from 7.9% to 18.33%

during the period 1977-2007. This surge again is contributed significantly by the rise of Wall Street financiers in top 1%, and especially in top 0.1%. According to Kaplan and Rauh (2010), Wall Street financiers comprise a higher percentage of the top income brackets than non financial executives of public companies. Using the microdata from Current Population Survey, we compare the distribution of income between people working in finance and non-financial industry in two years 1977 and 2007. (Incomes of different years are converted to the equivalent incomes in 1999)



(a) 1977

(b) 2007

Figure 3: The distribution of income

According to the Figure 3, two points are worth considering. First, both the income distributions shift right and have a larger right tail than before, referring to the concentration of income to a small amount of top earners. Second, in any cutoff income points in the right tail of the distribution, the density of top earners in finance is much higher than the general density of top earners in non-financial industry. This, in combination with the rise of finance's share in total income and total employment, give us the conclusion about the rapid surge of Wall Street in top income earners.

Based on income tax return data reported by Bakija, Cole and Heim (2008), the percentage of executives, managers and supervisors (non-finance) in the top 1% in fact declined from 35.3% in 1979 to 30% in 2005, while the percentage of financiers increased by 5.5% during this time. This trend also happens with the top 0.1%, emphasizing the need of economic theory to explain this phenomenon. Table 1 documents the rise of financiers in the top income in US.

Table 1: Percentage of primary taxpayers in top 1% and top 0.1 % of the distribution of income (including capital gains) that are in each occupation

Year	**Top 1 percent**			**Top 0.1 percent**		
	Executives, managers, supervisors (non-finance)	Financial professions, including management	Others	Executives, managers, supervisors (non-finance)	Financial professions, including management	Others
1979	35.3	7.7	57.0	48.1	11.0	40.9
1993	33.3	10.8	55.9	45.7	14.1	40.2
1997	33.2	11.9	54.9	48.4	14.7	36.9
1999	32.7	12.8	54.5	47.1	16.4	36.5
2001	31.0	13.1	55.9	42.6	19.1	38.3
2002	30.8	13.0	56.2	40.6	19.0	40.4
2003	30.2	12.9	56.9	40.5	17.8	41.7
2004	30.0	13.4	56.6	40.9	18.7	40.4
2005	30.0	13.2	56.8	42.5	18.0	39.5

Theme 3: The debut of new financial instruments

The most remarkable change in the financial market during 1980s was the increase in trading volume of a wide range of new financial derivatives. Many financial products, like options or futures contract, started becoming popular and useful tools for firms in managing risk since 1980. Table 2 shows the aggregate daily trading volume of major options contracts in the world market.

Table 2: Aggregate Daily Trading Volume (Billion USD)

	1975	1980	1984	1985	1986:III
Options	0	0	8.2	24.5	34.4
Interest rate contracts	0	0	1.9	11.5	16.3
Bonds	0	0	1.9	6.5	6.7
Money market	0	0	0	5.0	9.6
Stock index contracts	0	0	6.0	12.3	15.6
Currencies	0	0	0.3	0.7	2.5

Source: Levich et al. (1988)

Beside options and futures, many types of asset backed securities (ABS) also made their debut during 1980s. The first ABS was introduced in 1985 when the Sperry Lease Finance Corporation created securities backed by its computer equipment leases. After that, the financial market introduced many types of ABS with the underlying assets ranged from automobile loans, credit card loans to student loans. Mortgage backed securities also transform the role of banks from lenders to middlemen, allowing different industries to participate in the mortgage market. In 1990s, credit default swap appeared in the market. Allen and Gale (1994) emphasizes the biggest motivation for the introduction of many new financial instruments is risk sharing. In the following section, we build a model to explain the link between new financial assets, risk sharing and the income inequality trend we observed since 1980.

3 The model

3.1 The Environment

Consider an economy with two dates, $t = 0, 1$, and one single good which can be used as either capital or consumption. Except by using this good in the production technology, it cannot be stored to the next period. There are two types of agents in the economy: entrepreneurs and financiers.

Entrepreneurs: There is a continuum measure of 1 of entrepreneurs in each sector $i \in I = \{1, 2, ..., |I|\}$. Entrepreneurs cannot move across sectors. Sectors differ from each other by the risk structure in production. The entrepreneurs in the same sector i have different initial endowments (initial wealth) e at date 0, where e follows a continuous distribution function with the density $f(e)$ on the support $[\underline{e}, \bar{e}]$. So an entrepreneur can be denoted by a pair (i, e), where i shows his sector and e shows his initial endowment level. Let $\tilde{e} = \int_{\underline{e}}^{\bar{e}} ef(e)de$ be the average initial endowment of each sector at date 0, then the total endowment of economy is $I\tilde{e}$. Entrepreneurs receive no additional endowments at date 1.

At date 0, the entrepreneur, before making his decision about production, is matched to a financier who can help him to access the financial market[1]. If the entrepreneur agrees to pay the bargained fees for the financier at date 1, he can purchase or short sell the financial assets before going into the production stage.

An entrepreneur has a CARA utility function over his net worth $n_{i,e}$ at the end of date 1:

$$U_E = E[-exp(-\theta n_{i,e})]$$

where θ is the coefficient of risk aversion, $\theta > 0$.

Financiers: There is a mass I of financiers in the economy. Financiers cannot produce; but they can make transactions in the asset market or help entrepreneurs to make transactions there. We assume that financiers do not have any endowments at date 0 and date 1. A financier also has a CARA utility function over his net worth at the end of date 1 like entrepreneurs:

$$U_F = E[-exp(-\theta n_F)]$$

Production Technology and Risk Structure: We model the risk and asset structure similar to Simsek (2013). A fundamental risk (or uncertainty) in the economy is captured by the m-dimensional random vector $\mathbf{v} = (v_1, ..., v_m)'$, which follow the multivariate normal distribution $\mathbf{v} \sim N(\boldsymbol{\mu}, \boldsymbol{\Omega})$, where $\boldsymbol{\mu} \in \mathbf{R}^m_+$. The risk when producing in the sector i is characterized by a random variable $\mathbf{W_i}'\mathbf{v}$, where $\mathbf{W_i} \in \mathbf{R}^m$. As $\mathbf{v}$ follows a multivariate normal distribution, $\mathbf{W_i'v}$ follows a normal distribution. An entrepreneur, if investing $k > 0$ amount of capital in the production at date 0, can produce the amount of output y at date 1 as:

$$y = \left(\mathbf{W_i'v} + z\right)k^{\eta} \quad , \frac{1}{2} \leq \eta < 1$$

where $z > 0$ is the parameter indicating the general productivity level of the economy, η is the coefficient showing the level of control. We assume that capital is totally depreciated.

The production function exhibits decreasing returns to scale. The normal distribution of the sector shock has a lot of advantages when we introduce the asset market. The only problem is

[1]All results of this paper still hold if we assume that entrepreneur can always access to a set of financial assets as the subset of the one that financiers can access

the output can be negative. However, if we let the the value of the general productivity level z be big enough, for example more than 10 times the standard deviation of the shock $\mathbf{W_i'v}$, the probability of $y < 0$ is almost 0. This setup of the risk structure makes the model simple and tractable.

Matching between financiers and entrepreneurs: At the beginning of date 0. financiers and entrepreneurs are matched randomly 1-to-1. Here we assume the mass of financiers is equal to the mass of entrepreneurs, so everyone is matched.[2] If an entrepreneur meets a financier, his type (i,e) is observed. To access the financial market, the entrepreneur must pay fees for the financier. They go into the Nash bargaining process with α is the bargaining power of the entrepreneur . The bargained fees $w_{i,e}$ will depend on the type of entrepreneur and only be paid at date 1. The fees here should be interpreted as the sum of consulting fees, servicing fees and transaction fees. In equilibrium, every entrepreneur will participate in the asset market.

Asset Market: There are bonds and H assets in the market, $H+1 < m$. A bond promises to pay 1 unit of goods at date 1. The inverse price of a bond is s, which is endogenous. It means that 1 unit of goods at date 0 can exchange for s units of bonds.

Unlike bonds, the payoffs of assets are uncertain. Each asset h promises to give a random payoff $a_h = \mathbf{A_h'v}$ at date 1, where $\mathbf{A_h} \in \mathbf{R}^m$. So the payoff of an asset depends on the realization of the fundamental risk $\mathbf{v}$. The vectors, $\{\mathbf{A_h}\}_{h=1,\dots,H}$, are linearly independent which ensures no assets are redundant. We can say the asset matrix $\mathbf{A} = [\mathbf{A_1} \quad \mathbf{A_2} \quad \dots \quad \mathbf{A_H}]_{m\times H}$ is full rank. The price vector of assets is $\mathbf{p} \in \mathbf{R}^H$, which is endogenous. The total net supply of assets and bonds are both 0 in equilibrium (we allow agents to short sell in the financial market).

Timing: The timing of events can be summarized as followings:

$\mathbf{t=0}$:

- Entrepreneurs and financiers are matched and they go into the bargaining process.
- Financiers and entrepreneurs make transactions in the asset market.
- Production stage: entrepreneurs make decisions about production.

$\mathbf{t=1}$:

[2] This assumption can be relaxed by letting the number of matches between financiers and entrepreneurs follows a standard matching function in search theory..

- Fundamental risk $\mathbf{v}$ is realized.
- Outputs are realized; agents receive the payoffs from the financial assets and bonds; entrepreneurs pay the committed fees to financiers.

Like Simsek (2013), we assume there is no default in the model. That assumption is justified if the fluctuation of fundamental risk is small enough. It is also standard in the literature of general equilibrium model with a set of financial assets.

3.2 The entrepreneur's problem:

When bargaining fees with the financier, the entrepreneur treats the price in the financial market as given and compares the change in his utility in two cases: with and without access to the financial market.

No access to the financial market

Let $k_{i,e}^U$ be the capital the entrepreneur of type (i,e) will put into the production if he cannot get access to the financial market. The the entrepreneur problem is:

$$V_{i,e}^U = \max_{k_{i,e}^U} \quad E[-exp(-\theta n_{i,e}^U)]$$

subject to

$$n_{i,e}^U = (\mathbf{W_i'v} + z)(k_{i,e}^U)^\eta \tag{1}$$

$$0 \leq k_{i,e}^U \leq e \tag{2}$$

As $\mathbf{W_i'v}$ follows the normal distribution, $[\exp(-\theta n_{i,e}^U)]$ follows the log-normal distribution. Under the CARA preference, we can rewrite the problem as:

$$\max_{k_{i,e}^U} \quad E(n_{i,e}^U) - \frac{\theta}{2} Var(n_{i,e}^U)$$

subject to the the constraints (1) and (2).

Let σ_i be the variance of the shock in the sector i, then $\sigma_i = \mathbf{W_i'\Omega W_i}$. The mean and variance of entrepreneur's net worth in this case are:

$$E(n^U_{i,e}) = (\mathbf{W_i'}\boldsymbol{\mu} + z)(k^U_{i,e})^\eta$$
$$Var(n^U_{i,e}) = (k^U_{i,e})^{2\eta}\mathbf{W_i'\Omega W_i} = (k^U_{i,e})^{2\eta}\sigma_i$$

As the entrepreneur is risk averse, he must consider the trade-off between the expected return from the production and his sector risk σ_i- the variance of the shock in the sector i. Without access to the financial market, he must bear the risk himself. Lemma 1 shows the unique solution for the entrepreneur.

Lemma 1. *The solution for the entrepreneur (i,e) in case of no access to the financial market is:*

$$k^U_{i,e} = \min\{\hat{k}^U_{i,e}, e\}$$

where

$$\hat{k}^U_{i,e} = \left(\frac{\mathbf{W_i'}\boldsymbol{\mu} + z}{\theta\sigma_i}\right)^{1/\eta}$$

In equilibrium, every entrepreneur will pay financiers to get access to the financial market so $V^U_{i,e}$ only plays the role as the reference point to identify the bargaining fees.

With access to the financial market

If the entrepreneur participates in the asset market, he can buy bonds and assets to hedge the risk when producing in the sector i. Let $x^h_{i,e}$ be the position of the entrepreneur (i,e) on the asset h. Let $\mathbf{x_{i,e}}$ be the vector showing his position for all H assets and $k_{i,e}$ be the capital invested in the production. The entrepreneur problem can be written as:

$$V_{i,e} = \max_{k_{i,e},\mathbf{x_{i,e}}} \quad E[-exp(-\theta n_{i,e})]$$

subject to

$$n_{i,e} = \underbrace{(e - k_{i,e} - \mathbf{x_{i,e}}'\mathbf{p})s}_{\text{Bond Payoffs}} + \underbrace{(\mathbf{W_i}'\mathbf{v} + z)(k_{i,e})^{\eta}}_{\text{Production}} + \underbrace{\mathbf{x_{i,e}}'\mathbf{A}'\mathbf{v}}_{\text{Asset Payoffs}} - \underbrace{w_{i,e}}_{\text{Fees for financier}} \tag{3}$$

Like the case of no access to the financial market, the problem can be rewritten in the mean-variance form:

$$\max_{k_{i,e},\mathbf{x_{i,e}}} \quad E(n_{i,e}) - \frac{\theta}{2} Var(n_{i,e})$$

And the mean and variance of net worth are:

$$E(n_{i,e}) = (e - k_{i,e} - \mathbf{x_{i,e}}'\mathbf{p})s + (\mathbf{W_i}'\boldsymbol{\mu} + z)(k_{i,e})^{\eta} + \mathbf{x_{i,e}}'\mathbf{A}'\boldsymbol{\mu} - w_{i,e}$$
$$Var(n_{i,e}) = \sigma_i (k_{i,e})^{2\eta} + \mathbf{x_{i,e}}'\boldsymbol{\Omega_A}\mathbf{x_{i,e}} + 2\mathbf{x_{i,e}}'\boldsymbol{\lambda_i}(k_{i,e})^{\eta}$$

where $\boldsymbol{\lambda_i} = \mathbf{A}'\boldsymbol{\Omega}\mathbf{W_i}$ is the covariance between the risk in the sector i and the asset payoffs, $\boldsymbol{\Omega_A} = \mathbf{A}'\boldsymbol{\Omega}\mathbf{A}$ is the variance-covariance of assets' payoffs matrix.

The capability of making transactions in the asset market adds two layers to the entrepreneur's decision. First, he can borrow and lend through the bond market or short-sell through the asset market, so the resource restriction is no longer binding. Second, he can hedge the production risk through the asset market. His capital investment still depends on the sector risk σ_i; but it also depends on the asset market. He wants to buy more asset h if it offers him the high expected return $(\mathbf{A}'\boldsymbol{\mu} - \mathbf{p}s)$ or its return is negative correlated to his sector risk $\boldsymbol{\lambda_i}$. His decision also depends on the variance-covariance of all asset payoffs $\boldsymbol{\Omega_A}$. Gaining access to the asset market helps the entrepreneurs to hedge the risk better and therefore leveraging and investing more into the risky production process.

Assumption 1 ensures that market is incomplete, so that entrepreneurs could not share all the idiosyncratic risks through trading H assets.

Assumption 1. $\sigma_i \neq \boldsymbol{\lambda_i}'\boldsymbol{\Omega_A}^{-1}\boldsymbol{\lambda_i}, \quad \forall i = 1,...,I$

Lemma 2. *Under Assumption 1 and $s > 0$, the solution for the entrepreneur (i,e) if he can access to the financial market is unique. Let $\beta = 1/\eta$, his capital choice $k_{i,e}$ and portfolio* *choice* $\mathbf{x_{i,e}}$ *are the solution of the following system:*

$$(\mathbf{W_i'}\boldsymbol{\mu}+z)-s\beta(k_{i,e})^{\eta(\beta-1)}-\theta\sigma_i(k_{i,e})^{\eta}-\theta\mathbf{x_{i,e}}'\boldsymbol{\lambda_i}=0 \tag{4}$$

$$(\mathbf{A}'\boldsymbol{\mu}-\mathbf{p}s)-\theta\left(\boldsymbol{\Omega_A}\mathbf{x_{i,e}}+(k_{i,e})^{\eta}\boldsymbol{\lambda_i}\right)=0 \tag{5}$$

It is also easy to see that the value of gaining access to the financial market (less the fees paid to the financier) is always greater than (or at least equal to) the autarky case. Moreover, the decision rules for invested capital $k_{i,e}$ and portfolio $\mathbf{x_{i,e}}$ do not depend on the initial endowment level e.

3.3 The financier:

Financiers can alway access and make transactions in the financial market. The financier's problem can also be written in the mean-variance trade-off like the entrepreneurs. Financiers solve:

$$\max_{\mathbf{x_F}} E(n_F)-\frac{\theta}{2}Var(n_F)$$

subject to

$$n_F=(-\mathbf{x_F'p})s+\mathbf{x_F'A'v}+w_{i,e} \tag{6}$$

Lemma 3. *The mean and variance of financiers' net worth are:*

$$E(n_F)=(-\mathbf{x_F'p})s+\mathbf{x_F'A'}\boldsymbol{\mu}+w_{i,e}$$
$$Var(n_F)=\mathbf{x_F}'\boldsymbol{\Omega_A}\mathbf{x_F}$$

The solution for financiers is:

$$\mathbf{x_F}=\frac{\boldsymbol{\Omega_A^{-1}}(\mathbf{A}'\boldsymbol{\mu}-\mathbf{p}s)}{\theta} \tag{7}$$

Unlike the entrepreneurs, the financiers only care about the expected return of asset payoffs $(\mathbf{A}'\boldsymbol{\mu}-\mathbf{p}s)$ and the variance-covariance of asset's payoff ($\boldsymbol{\Omega_A}$). The purpose of the financiers when holding risky assets is fundamentally different from the entrepreneurs' choice. There is no hedging motivation there, the financiers hold the risky assets to earn the positive income from making transactions in the financial market.

3.4 Fee bargaining:

When an entrepreneur (i,e) meets a financier, his type is observed by the financier. They treat the bond price s and asset prices $\mathbf{p}$ in the financial market as given and go into Nash bargaining. Recall $V_{i,e}$, $V_{i,e}^{U}$ are the indirect utilities of the entrepreneur in case he can and cannot access the financial market. Let V_F, V_F^N be respectively the indirect utility of the financier in case of bargaining is successful and not. We shorten the notation as all indirect utility functions depend on $\mathbf{p}$ and s.

Let $\hat{V}_{i,e}$ be the indirect utility of the entrepreneur if he can access the asset market without paying fees to the financier.

$$\hat{V}_{i,e} = -exp(-\theta \hat{n}_{i,e})$$

$$\text{where} \quad \hat{n}_{i,e} = (e - k_{i,e} - \mathbf{x}_{\mathbf{i,e}}'\mathbf{p})s + (\mathbf{W_i}'\boldsymbol{\mu} + z)(k_{i,e})^{\eta} + \mathbf{x}_{\mathbf{i,e}}'\mathbf{A}'\boldsymbol{\mu}$$

Then $V_{i,e} = exp(\theta w_{i,e})\hat{V}_{i,e}$. We also have $V_F = exp(-\theta w_{i,e})V_F^N$. With the entrepreneur's bargaining power as α ($0 < \alpha < 1$), the bargained fees will solve this problem:

$$\max_{w_{i,e}} \left(exp(\theta w_{i,e})\hat{V}_{i,e} - V_{i,e}^{U} \right)^{\alpha} \left(exp(-\theta w_{i,e})V_F^N - V_F^N \right)^{1-\alpha}$$

Lemma 4. *The bargain fee $w_{i,e}^{*}$ is:*

$$exp(\theta w_{i,e}^{*}) = \frac{2\alpha - 1}{2\alpha} + \sqrt{\left(\frac{2\alpha-1}{2\alpha}\right)^2 + \frac{(1-\alpha)}{\alpha}\left(\frac{V_{i,e}^{U}}{\hat{V}_{i,e}}\right)}$$

In the special case $\alpha = 1/2$, then:

$$w_{i,e}^{*} = \frac{1}{2}\left[\left(E(\hat{n}_{i,e}) - \frac{\theta}{2}Var(\hat{n}_{i,e})\right) - \left(E(n_{i,e}^{U}) - \frac{\theta}{2}Var(n_{i,e}^{U})\right)\right]$$

Like all the other Nash bargaining problems, the bargained fee is set up to divide the total surplus from the successful match. As $\hat{V}_{i,e} \geq V_{i,e}^{U}$, all matches are successful in equilibrium.

4 Equilibrium:

Definition: Market equilibrium consists of asset price vector $\mathbf{p}$ and bond price $s > 0$, an allocation $\{k_{i,e}\}$, a vector of asset holdings $\{\{\mathbf{x_{i,e}}\}, \mathbf{x_F}\}$, and the fees $\{w_{i,e}\}$ such that:

(i) Given $\mathbf{p}$ and s, $\{k_{i,e}, \mathbf{x_{i,e}}\}$ solves the problem of the entrepreneur of type (i,e); $\{\mathbf{x_F}\}$ solves the problem of financiers; $\{w_{i,e}\}$ solves the problem of fee bargaining between entrepreneurs and financiers.

(iii) All markets clear:

$$\text{Asset Market:} \quad \sum_{i=1}^{I}\left(\int_{\underline{e}}^{\bar{e}} \mathbf{x_{i,e}} f(e)de\right) + I\mathbf{x_F} = \mathbf{0} \tag{8}$$

$$\text{Goods Market:} \quad \sum_{i=1}^{I}\left(\int_{\underline{e}}^{\bar{e}} k_{i,e} f(e)de\right) = I\tilde{e} \tag{9}$$

This paper only considers the market equilibrium when the inverse bond price is positive (bonds will exist in equilibrium). When the amount of endowment is bigger than the level economy can absorb, the risk averse entrepreneurs might want to get rid of excess resources rather than put it into the production function (see Lemma 1). In this case, $s \leq 0$. However, the Assumption 2 ensures that the case with negative s will not happen.

Assumption 2. *The total endowment of the economy satisfies:*

$$\sum_{i=1}^{I}(\mathbf{W_i'}\boldsymbol{\mu} + z) - \theta\sigma_i(\tilde{\sigma}_i I\tilde{e})^{\eta} > 0; \quad \textit{where} \quad \tilde{\sigma}_i = \frac{\sigma_i^{1/(1-\eta)}}{\sum_{j=1}^{I}\sigma_j^{1/(1-\eta)}}$$

4.1 The existence and uniqueness of the equilibrium

We prove the existence and the uniqueness of the equilibrium in two steps. First, we prove that the set of allocation of market equilibria (if exist) is identical to the set of solutions of a problem faced by a social planner. Second, we prove the solution for the social planner's problem is unique; and, therefore, the market equilibrium exists and must be unique. We can also recover all the prices in the market equilibrium from the social planner's problem.

Assume there is a social planner who wants to maximize the aggregate certainty equivalent net worth of the whole economy Y, which is defined by:

$$Y = \left[\sum_{i=1}^{I} \int_{\underline{e}}^{\bar{e}} \left(E(n_{i,e}) - \frac{\theta}{2} Var(n_{i,e})\right) f(e) de\right] + I\left(E(n_F) - \frac{\theta}{2} Var(n_F)\right) \tag{10}$$

The social planner takes the structure of assets as given and chooses the portfolios as well as the capital invested in the production for each agent. However, she faces four constraints. First, she must allocate the capital invested in the production and the portfolios for all agents in the same sector identically. It means that $k_{i,e} = k_i$ and $\mathbf{x_{i,e}} = \mathbf{x_i}$. Second, the total allocated capital must be less than or equal to the available endowment in date 0. Third, the portfolios for all agents must satisfy the net supply of assets equal to 0. Fourth, she could not interfere and reallocate the agents' income at date 1.

Lemma 5. *A social planner's problem can be defined as*

$$\max_{k_i, \mathbf{x_i}} \quad Y = \left[\sum_{i=1}^{I} (\mathbf{W_i'}\boldsymbol{\mu} + z)(k_i)^{\eta} - \frac{\theta}{2}\left(\sigma_i (k_i)^{2\eta} + \mathbf{x_i}'\boldsymbol{\Omega_A}\mathbf{x_i} + 2(k_i)^{\eta}\mathbf{x_i}'\boldsymbol{\lambda_i}\right)\right] - \frac{\theta I}{2}\mathbf{x_F'}\boldsymbol{\Omega_A}\mathbf{x_F}$$

subject to

$$\sum_{i=1}^{I} k_i \leq I\tilde{e}$$

$$\sum_{i=1}^{I} \mathbf{x_i} + I\mathbf{x_F} = \mathbf{0}$$

Then under the Assumptions (1), the social planner's solution is unique.

Theorem 1. *Under the Assumptions (1)-(2), the market equilibrium exists and is unique.*

We can also recover the inverse price of bond in the market equilibrium from solving the social planner's problem. From the proof of Theorem 1, the Lagrangian multiplier for the resource constraint in the solution of social planner problem is s. In the following sections, we analyze the impact of new assets on the income distribution in the economy.

4.2 New assets and the rise of the financial sector

We characterize the change in the economy and in the financial sector if the list of assets expands. An economy can be denoted by $\xi = (\{\mathbf{W_i}\}_{i=1}^{I}, \{\mathbf{A_h}\}_{h=1}^{H})$. Assume there is a new additional asset $\mathbf{A_{H+1}}$ for trading, the new economy can be denoted by $\xi' = (\{\mathbf{W_i}\}_{i=1}^{I}, \{\mathbf{A_h}\}_{h=1}^{H+1})$. We make the following assumption for this new asset.

Assumption 3. $\mathbf{A_{H+1}} \in \mathbf{R}^m$, $\mathbf{A_{H+1}} \notin span(\{\mathbf{A_1}, \mathbf{A_2}, ..., \mathbf{A_H}\})$.

This assumption makes sure the new asset is not redundant and the asset matrix is still full rank. Now we characterize the rise of the financial sector when a new asset is added. Let Y_F be the "adjusted" certainty equivalent (CE) net worth of all financiers (Y_F contains all the consulting fees but only half of financiers' transaction income) and φ be the share of Y_F in total economy:

$$Y_F = \underbrace{\left(\sum_{i=1}^{I} \int_{\underline{e}}^{\bar{e}} w_{i,e} f(e) de \right)}_{\text{Financier's wages}} + \frac{I}{2} \left(\underbrace{(-\mathbf{x_F'}\mathbf{p})s + \mathbf{x_F'}\mathbf{A}'\boldsymbol{\mu} - \frac{\theta}{2}\mathbf{x_F'}\boldsymbol{\Omega}_A\mathbf{x_F}}_{\text{Half of CE financiers' transaction income}} \right) \tag{11}$$

$$\varphi = \frac{Y_F}{Y} \tag{12}$$

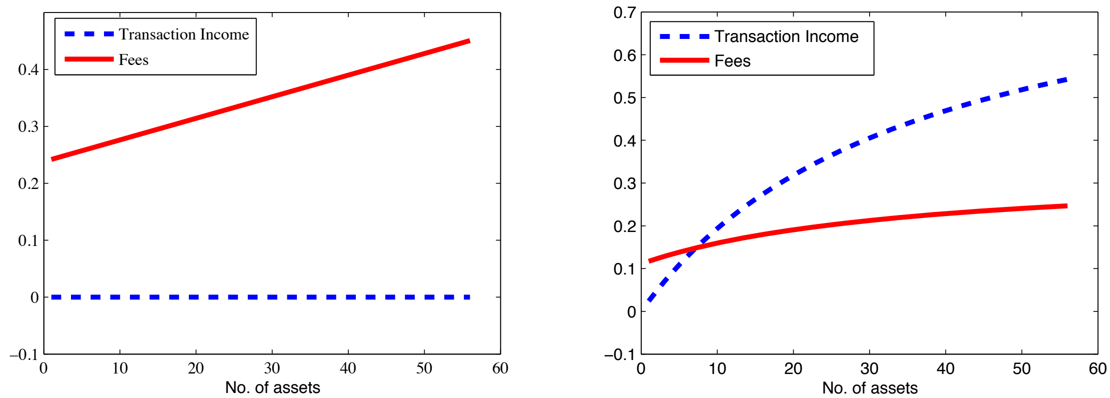
Theorem 2. *If both economies ξ and ξ' satisfy the Assumption (1)-(2), the new asset satisfies the Assumption 3 and the bargaining power of entrepreneur $\alpha = 0.5$, then:*

(i) The certainty equivalent net worth of the whole economy Y' in (ξ') is greater than or equal to Y in (ξ).

(ii) The certainty equivalent net worth of all financier Y_F' in (ξ') is greater than or equal to Y_F in (ξ).

(iii) The financial sector's share φ is (weakly) increasing when the economy transforms from (ξ) to (ξ').

Theorem 2 shows the rise of the financial sector when a new asset is added in the economy. The appearance of the new financial asset can increase the risk-sharing efficiency; therefore, it raises the value of being accessed to the financial market. Entrepreneurs are ready to pay



(a) $\mathbf{W_1} + \mathbf{W_2} = \mathbf{0}$

(b) $\mathbf{W_1} = \mathbf{W_2}$

Figure 4: The financiers' fees income and transaction income

more for the financiers to hedge their risks in production. So the fees $w_{i,e}$ increase to reflect the benefit from accessing the financial market to entrepreneurs. Intuitively, if the CE net worth of the whole economy Y increases due to the innovation in the financial sector, financiers will capture half of this gain in the Nash bargaining process with entrepreneurs.

What do we know about the financiers' expected transaction income $\mathbf{x}'_{\mathbf{F}}(\mathbf{A}'\boldsymbol{\mu} - s\mathbf{p})$ when new assets appear? First, it is surely not negative, otherwise they would be better by not making transaction $\mathbf{x_F} = 0$. While entrepreneurs join the financial market to hedge the production risk, financiers join the market to earn the transaction income.

Second, the change in the financiers' transaction income depends on the risk correlation between the production sectors. Let's assume that the economy has two production sector $I = 2$. If the risk of two production sectors are perfectly negative correlated, transaction income is always zero; and fees income are increasing with the appearance of new asset (Figure 4a). In the opposite case, when risk of two sectors are extremely positive correlated, the financier's transaction income is generally increasing with the appearance of new assets as there are more financial tools for financiers to share risks with the production sectors (Figure 4b). We formalize two above ideas for some specific economies.

Lemma 6. *If the risks of two production sectors are perfectly negative correlated $I = 2$, $\mathbf{W_1} + \mathbf{W_2} = 0$ and $\mathbf{W}'_1\boldsymbol{\mu} = \mathbf{W}'_2\boldsymbol{\mu} = \mathbf{0}$, then financiers' expected transaction income $\mathbf{x}'_{\mathbf{F}}(\mathbf{A}'\boldsymbol{\mu} - s\mathbf{p}) = 0$.*

To illustrate for the change in the financiers' transaction income when the risks of two production sectors are perfectly positive correlated, we consider the case when two sectors are identical $\mathbf{W_1} = \mathbf{W_2}$, or we can consider I=1.

Lemma 7. *If both economies ξ and ξ' satisfy the Assumption (1)-(2), the new asset satisfies the Assumption 3 and $I = 1$, then financiers' expected transaction income $\mathbf{x'_F}(\mathbf{A}'\boldsymbol{\mu} - s\mathbf{p})$ is weakly increasing when the economy transforms from (ξ) to (ξ').*

4.3 New assets and the rise of income inequality

New assets also deepen the income inequality in the economy. From the result of Theorem 2, new assets can transfer the income from entrepreneurs to financiers. This creates the income inequality between the finance industry and non-finance industries. However, new assets also widen the income gap within entrepreneurs. New assets allow the better chance to hedge the risk in the production, pushing up the capital demand and therefore the rate of return on capital s and the income share of top wealth holders.

To see the mechanism clearly, we consider the economy with two production sectors with perfectly negative risk correlation $I = 2$, $\mathbf{W_1} + \mathbf{W_2} = \mathbf{0}$ and $\mathbf{W'_1}\boldsymbol{\mu} = \mathbf{W'_2}\boldsymbol{\mu} = \mathbf{0}$, then $\boldsymbol{\lambda_1} = -\boldsymbol{\lambda_2}$ and $k_1^* = k_2^*$ (result in the proof of Lemma 6). Risk sharing in this economy is conducted between two sectors. Solving the social planner problem with bond price s as the Lagrangian multiplier with the resource constraint, we get: (equation 28 in the Appendix)[3]

$$(\mathbf{W'_i}\boldsymbol{\mu} + z) - s\beta(k_i^*)^{1-\eta} = \theta\sigma_i(k_i^*)^{\eta} - \frac{\theta}{2}(k_i^*)^{\eta}\boldsymbol{\lambda'_i}\boldsymbol{\Omega_A^{-1}}\boldsymbol{\lambda_i}, \qquad i = 1,2$$

Recall that $\beta = 1/\eta$, multiply both sides by $\eta(k_i^*)^{\eta-1}$:

$$\underbrace{(\mathbf{W'_i}\boldsymbol{\mu} + z)\eta(k_i^*)^{\eta-1}}_{\text{Marginal Benefit}} = \underbrace{\overbrace{s}^{\text{Give up bond payoffs}} \overbrace{+\theta\sigma_i\eta(k_i^*)^{2\eta-1}}^{\text{Production Risk}} \overbrace{-\frac{\theta}{2}\eta(k_i^*)^{2\eta-1}\boldsymbol{\lambda'_i}\boldsymbol{\Omega_A^{-1}}\boldsymbol{\lambda_i}}^{\text{Risk Sharing}}}_{\text{Marginal Cost}} \tag{13}$$

[3]The detail derivation is conducted in the proof of the Theorem 1.

The above equation shows the capital demand in each sector in the economy. The left hand side of (13) is the marginal benefit for entrepreneurs from investing one more unit of capital in the production. The right hand side could be considered as the marginal cost of putting capital into the production. The first component of this cost is the opportunity cost from giving up the chance to buy bonds. This second component is associated with the risk-averse attitude of entrepreneurs, as more investment means the higher risk. However, this cost can be reduced if the list of financial assets allows the entrepreneurs to hedge the production risk. The effect of new asset on the capital demand comes from the increase of the risk-sharing effect $(\theta/2)\eta(k_i^*)^{2\eta-1}\boldsymbol{\lambda}_i'\boldsymbol{\Omega}_A^{-1}\boldsymbol{\lambda}_i$, when entrepreneurs have more financial instrument tools to hedge their production risk.

First, we prove formally the important result about the impact of new asset on the right hand side of (13). From now, we use the notation $\hat{a}$ for variable (or parameter) a in the economy (ξ) and $\tilde{a}$ for variable (or parameter) a in (ξ').

As the risk structures in two sectors are symmetric, we have $\sigma_1 = \sigma_2$ and $\boldsymbol{\lambda}_1'\boldsymbol{\Omega}_A^{-1}\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_2'\boldsymbol{\Omega}_A^{-1}\boldsymbol{\lambda}_2$. We drop the subscript i to shorten the notation.

Lemma 8. *Let $\tilde{\boldsymbol{\Omega}}_A$, $\hat{\boldsymbol{\Omega}}_A$ be respectively the variance-covariance payoffs of assets in the economy (ξ') and (ξ). Then, under the Assumption (3) and $H+1<m$:*

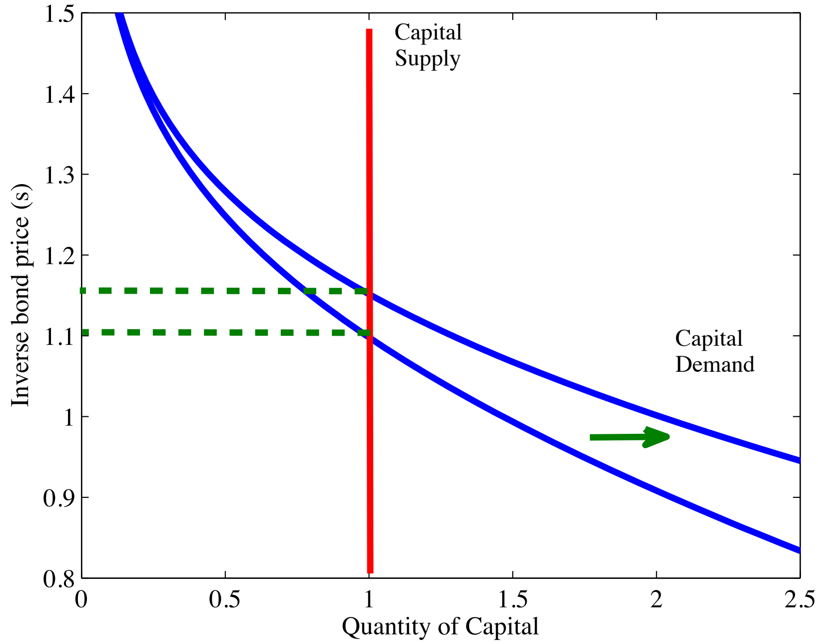
$$\left(\tilde{\boldsymbol{\Omega}}_A^{-1} - \begin{bmatrix} \hat{\boldsymbol{\Omega}}_A^{-1} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix}\right) \quad \textit{is positive semidefinite}$$

One direct result we get from Lemma 8 is:

$$\hat{\boldsymbol{\lambda}}'\hat{\boldsymbol{\Omega}}_A^{-1}\hat{\boldsymbol{\lambda}} = \begin{bmatrix} \hat{\boldsymbol{\lambda}} \\ \mathbf{A}_{\mathbf{H+1}}'\boldsymbol{\Omega}\mathbf{W} \end{bmatrix}' \begin{bmatrix} \hat{\boldsymbol{\Omega}}_A^{-1} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\lambda}} \\ \mathbf{A}_{\mathbf{H+1}}'\boldsymbol{\Omega}\mathbf{W} \end{bmatrix} \leq \tilde{\boldsymbol{\lambda}}\tilde{\boldsymbol{\Omega}}_A^{-1}\tilde{\boldsymbol{\lambda}}$$

For the case two symmetric sectors, under the Assumption (2), the total capital demand will equate the total capital supply $I\tilde{e}$ in equilibrium, or $k_1^* = k_2^* = \tilde{e}$. With the appearance of new

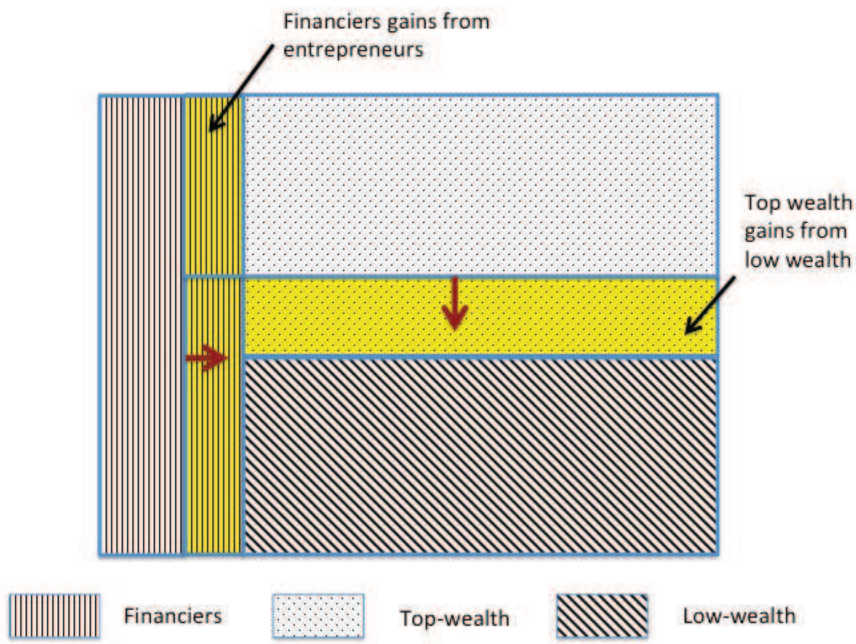
Figure 5: The effect of new assets on capital return



asset, the only way to balance the equation (13) is the inverse bond price s must go up. Intuitively, because new financial securities allow the better risk sharing between two sectors, the capital demand increases. While the total supply of capital in inelastic at $I\tilde{e}$, the rate of return on capital s will go up.

Figure 5 shows the effect of the new assets on the inverse bond price. The introduction of new financial assets shift the capital demand to the right, raising up the inverse of bond price. This implies that the people in the top of the wealth distribution enjoy the bigger chunk in GDP as their capital income increases. Recall that the difference between the net worth at date 1 of two entrepreneurs in the same sector with initial wealth e_1 and e_N will be $(e_N - e_1)s$ (as they will chose the same portfolio and capital invested); therefore, the new financial assets, by increasing s, also deepen the income inequality from the initial wealth inequality. The entrepreneurs with the low initial wealth must borrow the capital with the higher interest rate while the one with high initial wealth enjoys the higher income from lending capital. The higher rate of return on capital creates the inequality cycle when transmitting the wealth inequality to income inequality and reverse.

Figure 6: The effect of new assets on the income distribution



To see it clearly, let $y(e)$ be the expected income of all entrepreneurs with the initial endowment e and the financiers who are matched to them. We know that transaction income and the fees from the financial market only transfer from members in this group, so it does not affect the expected income of the whole group. The capital invested in equilibrium will be $k_{i,e} = \tilde{e}$. We have:

$$y(e) = (e - \tilde{e})s + \sum_{i=1}^{2} (\mathbf{W}_{\mathbf{i}}' \boldsymbol{\mu} + z)\tilde{e}^{\eta} = (e - \tilde{e})s + 2z\tilde{e}$$

Let y be the expected GDP of the economy, then:

$$y = 2z\tilde{e}$$

For the group in the bottom of wealth distribution, $\underline{e} < \tilde{e}$. The increase in s reduces their income while the group at the top of wealth distribution $\bar{e} > \tilde{e}$ enjoys the bigger share in GDP. The effect of new assets on the distribution of income can be summarized by the Figure 6.

The introduction of new financial assets increases the income share of financiers in GDP. It also pushes up the share of top wealth people in the country. Although the model is very simple, it matches two income inequality trends we observe from the 1980: the rapid rise of top wealth people as well as many top financial executives in the top 0.1% income in the economy.

5 Numerical Example

5.1 Calibration

In our numerical example, we set up the number of sector $I = 2$. Each sector in the model consists of 28 industries in the data from BEA. The sector 1 contains all the industries in the manufacturing sector and the agriculture sector. The sector 2 contains all the industries in the service sector. The total number of industries is 56. This list of 56 industries is ordered in a way such that all the industries belonging to sector 1 are in the odd positions.[4]

We calibrate the fundamental risk based on the industry risk in the data. The dimension of fundamental risk $m = 56$, where each v_i represents the risk of one industry. The vector $\mathbf{W_1}$=[1 0 1 0 ... 1 0]' showing the risk for the sector 1 is set up to have zero in the even positions and 1 in the odd position. It means that sector 1's risk ($\mathbf{W_1 v} = v_1 + v_3 + ... + v_{55}$) only contains the risk of 28 industries in the odd positions. Similarly, the vector $\mathbf{W_2}$=[0 1 ... 0 1]' showing the risk for sector 2 means that the sector 2 only contains the risk of 28 industries in the even positions in the list.

Naturally, the entry (i, j) of matrix $\mathbf{\Omega}$ shows the covariance between the risk of industry i and the risk of industry j in the data. However, this calibration must introduce a matrix data of size 56×56, which means that model depends on more than 1,000 parameters . We choose the alternative way to calibrate $\mathbf{\Omega}$, so it only depends on two parameters. We estimate the covariance matrix of output growth between 56 industries in US between 1970-1982, the period before new financial instruments appear. Then we calculate the average covariance between the growth of two industries, which is $1.05e-04$. We set up all entries $\mathbf{\Omega}(i, j) = 1.05e-04$, where $i \neq j$. For all the diagonal entries, we set $\mathbf{\Omega}(i,i) = 39e-04$, which is the average variance of

[4] The particular order does not matter to the trend in our result. We choose this order so the set of assets insure the risks for both sectors.

growth of 56 industries. We set $\boldsymbol{\mu} = \mathbf{0}$. [5]

The asset matrix $\mathbf{A}$ in this experiment is set up so that asset h can fully insure the risk in the industry h. In this way $\mathbf{A_h}$ is the unit vector with the entry in the position h equal to 1.

For the production function, the level of span of control η is set to equal 0.9 and the general productivity z is set at 1.38 to match with the target that return on capital in the economy with only bond is around 7 percent in 1980.

The wealth distribution of entrepreneurs is assumed to follow a Pareto distribution with density:

$$f(e) = \frac{\varsigma \underline{e}^{\varsigma}}{e^{\varsigma+1}}$$

We calibrate ς and $\underline{e}$ to satisfy two conditions. First, the aggregate wealth $\tilde{e}$ of each sector is 1. Second, the initial wealth share of the top 1% entrepreneurs is 24% to match the data in 1980 from Saez and Zucman (2014). The parameters are summarized in the Table 3.

Table 3: Parameter values

Param.	Value	Note
m	56	Dimension of fundamental risk
I	2	Number of sectors
θ	1	Risk aversion coefficient
z	1.38	General technology level
η	0.9	Span of control level
α	0.5	Bargaining power of entrepreneurs
$\underline{e}$	0.29	Scale of initial wealth distribution
ς	1.42	Shape of initial wealth distribution

5.2 Results

We start from the economy with only bonds (zero assets) and keep adding the new asset into the economy and calculate the new equilibrium. As the dimension of fundamental risk is $m = 56$,

[5] We experimented with the model when $\boldsymbol{\Omega}(i, j)$ matches the real variance-covariance matrix of 56 industries in data and the asset matrix is random, all the trends of results are identical to our simplified version.

the market is complete when we have 56 assets. All results are calculated in the expected value as the actual income of agents in the economy at date 1 depends on the realization of shock $\mathbf{v}$. Table 4 shows the main statistics of the economy when the financial innovations happen.

Table 4: The change in economy with new financial assets

No of assets (H)	Rate of return on capital (s)	GDP	Share of finance in GDP	Share of top 0.1% entrepreneurs	Share of top 0.1% financiers
0	1.072	2.76	11.06	11.88	8.24
1	1.075	2.76	11.21	11.91	8.27
2	1.078	2.76	11.35	11.93	8.30
3	1.081	2.76	11.49	11.96	8.33
4	1.084	2.76	11.63	11.98	8.35
5	1.087	2.76	11.75	12.00	8.38
6	1.089	2.76	11.88	12.02	8.40
7	1.091	2.76	12.00	12.04	8.42
8	1.094	2.76	12.11	12.06	8.45
9	1.096	2.76	12.22	12.08	8.47
10	1.098	2.76	12.32	12.10	8.49
...	...	...	...	...	...
54	1.155	2.76	15.01	12.58	9.02
55	1.156	2.76	15.05	12.59	9.03
56	1.157	2.76	15.09	12.60	9.04

There are three important observations. First, the expected level of GDP does not change with the introduction of new assets. In our model, the technology level remains constant; therefore, the introduction of new assets mainly affects the welfare and the distribution of income rather than the level of output. Second, new assets increase the income share of the financial sector in the economy. In our simulation, ten new assets can increase the share of the finance industry by 4%. Third, the top-wealth entrepreneur and the top financiers enjoy the bigger chunk of GDP when the financial innovation happens. The income share of entrepreneurs at bottom in fact shrinks. That replicates the two important income inequality trends we observe since 1980: the rise of financiers and top 0.1% in wealth.

To see the income inequality trend more clearly, we take the economy with only bonds as the benchmark and calculate the income growth of top 0.1% entrepreneurs by wealth, top 0.1%

financiers (who are matched with top 0.1% entrepreneurs) and the entrepreneurs at the bottom 10% when the number of assets increases. Figure 7 displays the income growth of three groups of agents with the financial innovations. The introduction of new assets benefits the top financiers most. Compared to the economy with only bonds, the income of top financiers grows by 28 percent if the market becomes complete. This explains the dominance of Wall Street against Main Street observed in the empirical research by Kaplan and Rauh (2010) since 1980. The top wealth entrepreneurs also enjoys the higher growth of income due to the higher rate of return on capital; however, their income growth is less than the top financiers. The agents at the bottom of wealth ladder suffer the decline in income share.

We break down the financiers' income into two parts to understand more clearly the force behind the rise of the financial sector. In our model, financiers have two sources of income: the consulting fees is paid from entrepreneurs and the income they earn from making transaction in the financial market. Both income sources go up with the the financial innovations, but the transaction income grows with the faster rate. Figure 8 shows the financiers' fees and transaction income when the number of assets increases.

The transaction income grows due to the fact that new asset open more opportunities for financiers earns money from the asset market. There is no uncertainty in the fees financiers are paid, so they only hold assets if the expected income outweighs the variance of assets' payoffs. Financiers always earn the positive expected transaction income. This fundamentally differs from the purpose of hedging the production risks when entrepreneurs hold assets.

The divergent trend of income growth between the different types of agents is the most crucial insight in our simple model. This numerical result confirms the two income inequality trends we characterized in the theoretical results. Financial innovations push up the income of financiers and the people in the top distribution of wealth.

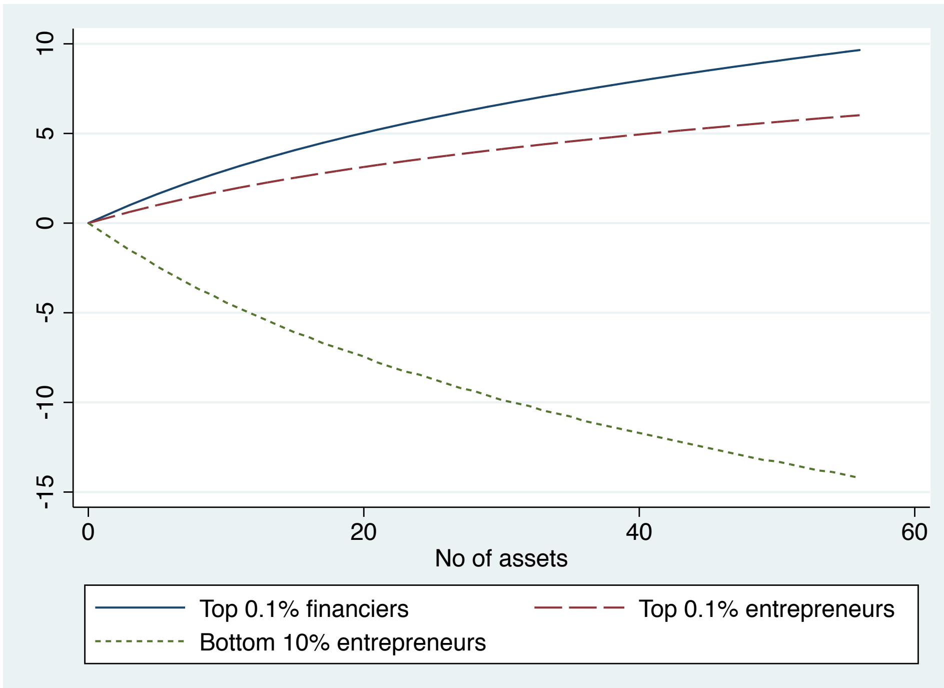


Figure 7: Income growth of agents in the economy with financial innovations

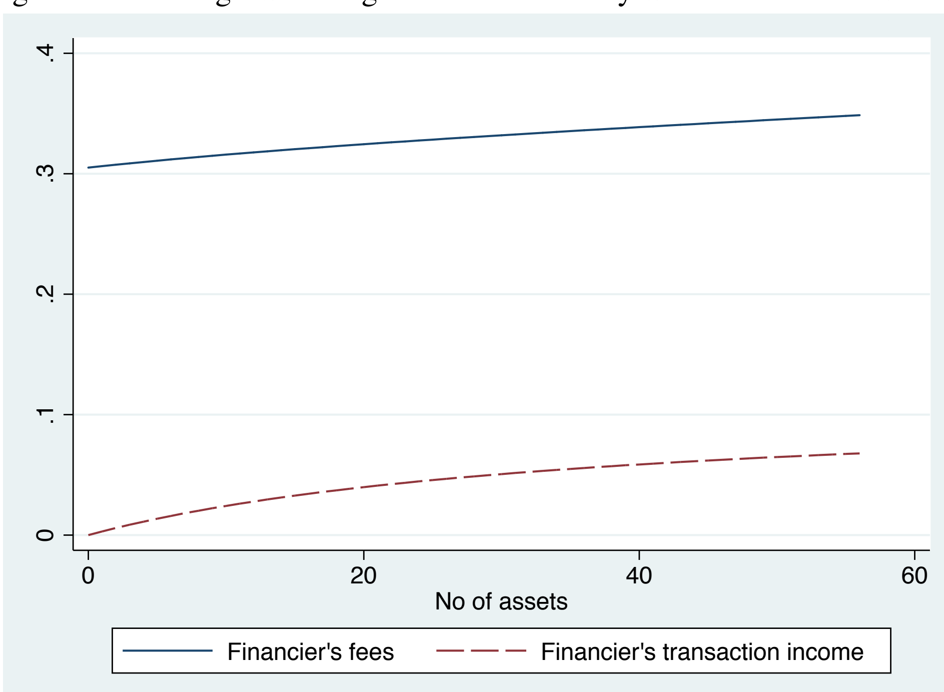


Figure 8: Fees and transaction income of finance

6 Conclusion

This paper builds the simple model to understand the link between the appearance of new financial assets and the income inequality trend we observed in US since 1980. The model predicts that new assets lead to two trends in the income distribution. First, the income share of the financial sector will go up, due to both the consulting fees and the profits they earn from the transactions in asset market. Second, the cycle between wealth inequality to income inequality will become more severe, as new assets allow the better risk-sharing and therefore raise the rate of return on capital. Both predictions of the model are very consistent with the data trend from 1980.

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A Mathematical Appendix

Lemma 1:

In case of no access to the financial market, the entrepreneur problem can be rewritten as:

$$\max_{k^U_{i,e}} \quad (\mathbf{W_i'}\boldsymbol{\mu}+z)(k^U_{i,e})^{\eta}-\frac{\theta}{2}\sigma_i(k^U_{i,e})^{2\eta} \tag{14}$$

subject to

$$k^U_{i,e}\leq e$$

With $1/2\leq\eta<1$, the objective function is strictly concave. Let γ be the Lagrangian multiplier with the resource constraint, take the first order condition of (14) with respect to $k^U_{i,e}$:

$$\begin{aligned}&\eta(\mathbf{W_i'}\boldsymbol{\mu}+z)(k^U_{i,e})^{\eta-1}-\theta\eta\sigma_i(k^U_{i,e})^{2\eta-1}+\gamma=0\\&\gamma\geq 0,\quad e-k^U_{i,e}\geq 0,\quad \gamma(e-k^U_{i,e})=0\end{aligned}$$

Assume $\gamma=0$, then let $\hat{k}^U_{i,e}$ be the unique positive solution of the first order condition:

$$\hat{k}^U_{i,e}=\left(\frac{\mathbf{W_i'}\boldsymbol{\mu}+z}{\theta\sigma_i}\right)^{1/\eta}$$

The optimal choice of capital for the entrepreneur in the autarky case will be:

$$k^U_{i,e}=\min\{\hat{k}^U_{i,e}\,,\,e\}$$

Lemma 2:

First, let $t=(k_{i,e})^{\eta}$ and $\beta=1/\eta$ $(1<\beta\leq 2)$, so we rewrite the problem as:

$$\max_{t\geq 0} \quad (e-\mathbf{x_{i,e}}'\mathbf{p})s+(\mathbf{W_i'}\boldsymbol{\mu}+z)t-st^{\beta}+\mathbf{x_{i,e}}'\mathbf{A}'\boldsymbol{\mu}-w_{i,e}-\frac{\theta}{2}\left(\sigma_i t^2+\mathbf{x_{i,e}}'\boldsymbol{\Omega_A}\mathbf{x_{i,e}}+2t\mathbf{x_{i,e}}'\boldsymbol{\lambda_i}\right) \tag{15}$$

We prove the Lemma 2 in two steps. First, under the Assumption (1) and $s>0$, we prove the objective function (15 is strictly concave. Then we only need to examine the first order condition.
Let $\mathbf{y_{i,e}}=[\mathbf{x_{i,e}}' \quad t]'\in\mathbf{R}^{H+1}$, so

$$Var(n_{i,e})=\mathbf{y_{i,e}'}\boldsymbol{\Lambda_i}\mathbf{y_{i,e}}\geq 0 \quad \forall\mathbf{y_{i,e}}\in\mathbf{R}^{H+1}$$

$$where \quad \boldsymbol{\Lambda_i}=\begin{bmatrix}\boldsymbol{\Omega_A} & \boldsymbol{\lambda_i}\\ \boldsymbol{\lambda_i'} & \sigma_i\end{bmatrix}$$

So, we have $\boldsymbol{\Lambda_i}$ is positive-semidefinite. Moreover $\boldsymbol{\Omega_A}$ is positive definite, then the Schur complement of $\boldsymbol{\Omega_A}$ in $\boldsymbol{\Lambda_i}$ is $S=\sigma_i-\boldsymbol{\lambda_i'}\boldsymbol{\Omega_A}^{-1}\boldsymbol{\lambda_i}\geq 0$. Under the Assumption 1, we have $(\sigma_i-\boldsymbol{\lambda_i'}\boldsymbol{\Omega_A}^{-1}\boldsymbol{\lambda_i})>0$.
Now we are ready to prove the objective function is strictly concave. Let $\mathbf{H_i}$ be the Hessian matrix of the

objective function (15). Then:

$$-\mathbf{H_i} = \theta \begin{bmatrix} \mathbf{\Omega_A} & \boldsymbol{\lambda_i} \\ \boldsymbol{\lambda_i}' & \sigma_i + \frac{s}{\theta}\beta(\beta-1)t^{\beta-2} \end{bmatrix}$$

With $1 < \beta \leq 2$, $s > 0$ and $(\sigma_i - \boldsymbol{\lambda_i}'\mathbf{\Omega_A}^{-1}\boldsymbol{\lambda_i}) > 0$ we have the Schur complement of $\mathbf{\Omega_A}$ in the matrix $(-1/\theta)\mathbf{H_i}$ is positive:

$$\sigma_i + \frac{s}{\theta}\beta(\beta-1)t^{\beta-2} - \boldsymbol{\lambda_i}'\mathbf{\Omega_A}\boldsymbol{\lambda_i} > 0$$

So $(-\mathbf{H_i})$ is positive definite or $\mathbf{H_i}$ is negative definite. We finish the first part of the proof such that the objective function is strictly concave when $t > 0$.
Take the first order condition with respect to t and $\mathbf{x_{i,e}}$ (then replace $t = (k_{i,e})^\eta$):

$$\begin{aligned}
&(\mathbf{W_i}'\boldsymbol{\mu} + z) - s\beta(k_{i,e})^{\eta(\beta-1)} - \theta\sigma_i(k_{i,e})^\eta - \theta\mathbf{x_{i,e}}'\boldsymbol{\lambda_i} = 0 \\
&(\mathbf{A}'\boldsymbol{\mu} - \mathbf{p}s) - \theta(\mathbf{\Omega_A}\mathbf{x_{i,e}} + (k_{i,e})^\eta\boldsymbol{\lambda_i}) = 0
\end{aligned}$$

Lemma 3:

The mean and variance of financers' net worth:

$$\begin{aligned}
&E(n_F) = (-\mathbf{x_F}'\mathbf{p})s + \mathbf{x_F}'\mathbf{A}'\boldsymbol{\mu} + w_{i,e} \\
&Var(n_F) = \mathbf{x_F}'\mathbf{\Omega_A}\mathbf{x_F}
\end{aligned}$$

The financier's problem can be rewritten as:

$$\max_{\mathbf{x_F}} \quad \big((-\mathbf{x_F}'\mathbf{p})s + \mathbf{x_F}'\mathbf{A}'\boldsymbol{\mu} + w_{i,e}\big) - \frac{\theta}{2}\mathbf{x_F}'\mathbf{\Omega_A}\mathbf{x_F} \tag{16}$$

As $\mathbf{\Omega_A}$ is positive definite, the objective function in (16) is strictly concave. Take the first order condition of (16) with respect to $\mathbf{x_F}$, we have:

$$\begin{aligned}
&-\mathbf{p}s + \mathbf{A}'\boldsymbol{\mu} - \theta\mathbf{\Omega_A}\mathbf{x_F} = 0 \\
\Longleftrightarrow\ &\mathbf{x_F} = \frac{\mathbf{\Omega_A}^{-1}(\mathbf{A}'\boldsymbol{\mu} - \mathbf{p}s)}{\theta}
\end{aligned}$$

Lemma 4:

Let $\chi = exp(\theta w_{i,e})$. As the values $\hat{V}_{i,e}$, $V^U_{i,e}$ and V^N_F are independent of $w_{i,e}$ and negative, we can rewrite the problem as:

$$\min_\chi \big(\chi\hat{V}_{i,e} - V^U_{i,e}\big)^\alpha \big(\frac{1}{\chi} - 1\big)^{1-\alpha}$$

Take the first order condition of the above function with respect to χ:

$$\begin{aligned}
&\alpha\hat{V}_{i,e}\big(\chi\hat{V}_{i,e} - V^U_{i,e}\big)^{\alpha-1}\big(\frac{1}{\chi} - 1\big)^{1-\alpha} + (1-\alpha)\left(-\frac{1}{\chi^2}\right)\big(\chi\hat{V}_{i,e} - V^U_{i,e}\big)^\alpha\big(\frac{1}{\chi} - 1\big)^{-\alpha} = 0 \\
\Longleftrightarrow\ &\alpha\hat{V}_{i,e}\big(\frac{1}{\chi} - 1\big) + (1-\alpha)\left(-\frac{1}{\chi^2}\right)\big(\chi\hat{V}_{i,e} - V^U_{i,e}\big) = 0
\end{aligned}$$

$$\iff -\alpha\hat{V}_{i,e}\chi^2 + (2\alpha-1)\hat{V}_{i,e}\chi + (1-\alpha)V_{i,e}^U = 0 \tag{17}$$

We have $-\alpha(1-\alpha)\hat{V}_{i,e}V_{i,e}^U < 0$, so the equation (17) always have an unique positive solution χ^*, which is:

$$\chi^* = \frac{2\alpha-1}{2\alpha} + \sqrt{\left(\frac{2\alpha-1}{2\alpha}\right)^2 + \frac{(1-\alpha)}{\alpha}\left(\frac{V_{i,e}^U}{\hat{V}_{i,e}}\right)}$$

In the case $\alpha = 0.5$, we have

$$\chi^2 = \frac{V_{i,e}^U}{\hat{V}_{i,e}}$$

In all case, the wage is increasing with the difference between certainty equivalent net worth of matched entrepreneur and unmatched entrepreneur. To see it clearly, let $\hat{n}_{i,e}$ be the net worth of a matched entrepreneur if he can access to the financial market and pay nothing to financiers:

$$\frac{V_{i,e}^U}{\hat{V}_{i,e}} = exp\left\{\theta\left[\left(E(\hat{n}_{i,e}) - \frac{\theta}{2}Var(\hat{n}_{i,e})\right) - \left(E(n_{i,e}^U) - \frac{\theta}{2}Var(n_{i,e}^U)\right)\right]\right\}$$

Lemma 5:

Let $t_i = (k_i)^\eta \geq 0$ and $1 < \beta = 1/\eta \leq 2$. We can rewrite the the social planner's problem as:

$$\min_{t_i\geq 0,\mathbf{x_i}} Y = -\left[\sum_{i=1}^{I}(\mathbf{W_i'}\boldsymbol{\mu} + z)t_i - \frac{\theta}{2}\left(\sigma_i(t_i)^2 + \mathbf{x_i}'\boldsymbol{\Omega_A}\mathbf{x_i} + 2t_i\mathbf{x_i}'\boldsymbol{\lambda}_i\right)\right] + \frac{\theta I}{2}\mathbf{x_F'}\boldsymbol{\Omega_A}\mathbf{x_F} \tag{18}$$

subject to

$$\sum_{i=1}^{I}(t_i)^\beta \leq I\tilde{e} \tag{19}$$

$$\sum_{i=1}^{I}\mathbf{x_i} + I\mathbf{x_F} = \mathbf{0} \tag{20}$$

First we prove the objective function is strictly convex. Recall the matrix $\boldsymbol{\Lambda_i}$ we define from the Lemma 2. Let $\mathbf{H}$ be the Hessian matrix of the objective function (18). Then:

$$\mathbf{H} = \begin{pmatrix} \theta\boldsymbol{\Lambda}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \theta\boldsymbol{\Lambda}_2 & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \theta\boldsymbol{\Lambda}_I & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \theta I\boldsymbol{\Omega_A} \end{pmatrix}$$

From the Lemma 2, all $\boldsymbol{\Lambda_i}$ are positive definite under the Assumption (1). We also have $\boldsymbol{\Omega_A}$ is positive definite. So $\mathbf{H}$ is positive definite. The objective function is strictly convex. We also have the feasible set C is convex and closed (set is created by the intersection of two constraints (19) and (20)). Now, we

prove the objective function is coercive on C.
From the constraint (19), $0 \leq t_i \leq (I\tilde{e})^\eta$, $\forall i$. So :

$$\left(\sum_{i=1}^{I}(\mathbf{W_i'}\boldsymbol{\mu}+z)t_i\right) \quad \text{is bounded}$$

Let $\mathbf{y_i} = [\mathbf{x_i}' \quad t_i]'$ and recall positive definite matrix $\boldsymbol{\Lambda_i}$ from the Lemma 2. Let f be the objective function in (18)

$$f(\mathbf{y_1},...,\mathbf{y_I},\mathbf{x_F}) = \underbrace{\frac{\theta}{2}\left(\sum_{i=1}^{I}\mathbf{y_i}'\boldsymbol{\Lambda_i}\mathbf{y_i} + I\mathbf{x_F'}\boldsymbol{\Omega_A}\mathbf{x_F}\right)}_{\to+\infty \text{ when } \|(\mathbf{y_1},...,\mathbf{y_I},\mathbf{x_F})\|\to\infty} - \underbrace{\left(\sum_{i=1}^{I}(\mathbf{W_i'}\boldsymbol{\mu}+z)t_i\right)}_{\text{bounded on } C}$$

So we have the objective function is coercive on C as $\forall(\mathbf{y_1},...,\mathbf{y_I},\mathbf{x_F}) \in C$, $f(\mathbf{y_1},...,\mathbf{y_I},\mathbf{x_F}) \to +\infty$ whenever $\|(\mathbf{y_1},...,\mathbf{y_I},\mathbf{x_F})\| \to +\infty$.

To shorten the notation we denote $\mathbf{z} = (\mathbf{y_1},...,\mathbf{y_I},\mathbf{x_F})$. As f is coercive on C, we prove the set $S = \{\mathbf{z} \in C | f(\mathbf{z}) \leq 0\}$ is non-empty and compact. First, if we set $\mathbf{z} = \mathbf{0}$, then $f(\mathbf{z}) = 0$, so the set $\{\mathbf{z} \in C | f(\mathbf{z}) \leq 0\}$ is non empty. Function f is continuous on C, implying the set S is closed. So to prove the compactness, we only need to prove $\{\mathbf{z} \in C | f(\mathbf{z}) \leq 0\}$ is bounded. Assume it is not bounded, then there must exist a sequence $\{\mathbf{z}^\nu\} \subset C$ with $\|\mathbf{z}^\nu\| \to \infty$. By the coercivity of f, we must also have $f(\mathbf{z}^\nu) \to +\infty$. This contradicts the fact that $f(\mathbf{z}) \leq 0$. So the set S must be bounded. Then S must be compact.

Now apply the Weierstrass's Theorem, as f is continuous in the compact set S, there must exist $\mathbf{z}^*$ such that f attains the minimum value in S. So f also attains the minimum value in C at $\mathbf{z}^*$. Moreover, f is strictly convex in C, so $\mathbf{z}^*$ is unique.

Theorem 1:

We prove the existence and uniqueness of market equilibrium in three steps. First, we prove every market equilibrium's allocation (if exists) is also the solution of social planner's problem in Lemma 5. Second, we prove the unique solution of social planner's problem in Lemma 5 is one of market equilibria. From the first two steps and Lemma 5, we can conclude the market equilibrium exists and be unique.

Step 1: Under the Assumption (1)-(2), every market equilibrium's allocation (if exists) is also the solution of social planner's problem in Lemma 5.

Consider again the social planner problem in Lemma 5. Let $\nu \geq 0$ be the Lagrangian multiplier with the resource constraint (19) and a $\boldsymbol{\gamma} = (\gamma_1,...,\gamma_H)$ be the Lagrangian multipliers with the portfolio constraints (20).

Take the first order condition (for t_i after we take the FOC w.r.t t_i then we replace $t_i = (k_i)^\eta$) , we have:

$$-(\mathbf{W_i'}\boldsymbol{\mu}+z) + \nu\beta(k_i)^{\eta(\beta-1)} + \theta\sigma_i(k_i)^\eta + \theta\mathbf{x_i}'\boldsymbol{\lambda_i} = 0, \qquad i = 1,...,I \tag{21}$$

$$\theta\boldsymbol{\Omega_A}\mathbf{x_i} + \theta(k_i)^\eta\boldsymbol{\lambda_i} - \boldsymbol{\gamma} = \mathbf{0}, \qquad i = 1,...,I \tag{22}$$

$$\theta I\boldsymbol{\Omega_A}\mathbf{x_F} - I\boldsymbol{\gamma} = \mathbf{0} \tag{23}$$

$$\nu \geq 0, \quad \sum_{i=1}^{I}k_i \leq I\tilde{e}, \quad \nu\left(\sum_{i=1}^{I}k_i - I\tilde{e}\right) = 0 \tag{24}$$

$$\sum_{i=1}^{I} \mathbf{x_i} + I\mathbf{x_F} = \mathbf{0} \tag{25}$$

From the Lemma 5, every allocation satisfies the system of equations from (21)-(25) will be the solution of social planner's problem.

If we set $\nu = s \geq 0$, $\boldsymbol{\gamma} = (\mathbf{A}'\boldsymbol{\mu} - \mathbf{p}s)$, then from Lemma 2, every market equilibrium allocation will be the solution of the system (21)-(25), so they are solutions of the social planner's problem.

Step 2: Under the Assumption (1)-(2), the planner's solution is one of the market equilibrium.

Let $\{k_i^*, \mathbf{x_i}^*, \mathbf{x_F}^*, \nu^*, \boldsymbol{\gamma}^*\}$ be the solution of the social planner problem. We will prove $\nu^* > 0$ under the Assumption (1)-(2).

Sum the equations (22) across i, then add with (23):

$$\theta \boldsymbol{\Omega_A}(\sum_{i=1}^{I} \mathbf{x_i}^* + I\mathbf{x_F}^*) + \theta \sum_{i=1}^{I} (k_i^*)^\eta \boldsymbol{\lambda_i} = 2I\gamma$$
$$\rightarrow \boldsymbol{\gamma} = \frac{\theta}{2I} \sum_{i=1}^{I} (k_i^*)^\eta \boldsymbol{\lambda_i} \qquad \text{(Use 25)} \tag{26}$$

Substitute (26) back into (22):

$$\mathbf{x_i}^* = -(k_i^*)^\eta \boldsymbol{\Omega_A}^{-1} \boldsymbol{\lambda_i} + \frac{\boldsymbol{\Omega_A}^{-1}}{2I} \sum_{i=1}^{I} (k_i^*)^\eta \boldsymbol{\lambda_i} \tag{27}$$

We put (27) into (21):

$$(\mathbf{W_i'}\boldsymbol{\mu} + z) - \nu\beta(k_i^*)^{1-\eta} - \theta\sigma_i(k_i^*)^\eta = -\theta(k_i^*)^\eta \boldsymbol{\lambda_i'} \boldsymbol{\Omega_A}^{-1} \boldsymbol{\lambda_i} + \frac{\theta}{2I} \sum_{j=1}^{I} (k_j^*)^\eta \boldsymbol{\lambda_i'} \boldsymbol{\Omega_A}^{-1} \boldsymbol{\lambda_i} \tag{28}$$

Sum (28) across i:

$$\sum_{i=1}^{I} (\mathbf{W_i'}\boldsymbol{\mu} + z) - \nu\beta(k_i^*)^{1-\eta} - \theta\sigma_i(k_i^*)^\eta = -\frac{\theta}{2} \sum_{i=1}^{I} (k_i^*)^\eta \boldsymbol{\lambda_i'} \boldsymbol{\Omega_A}^{-1} \boldsymbol{\lambda_i} \tag{29}$$

We prove under the Assumption (2), (29) fails to happen when $\nu = 0$. Assume $\nu = 0$, the RHS of (29) is negative in the feasible set, we prove the LHS is positive. Consider the following subproblem:

$$\min \quad g(k_1, ..., k_I) = \sum_{i=1}^{I} (\mathbf{W_i'}\boldsymbol{\mu} + z) - \theta\sigma_i(k_i)^\eta$$

subject to

$$k_i \geq 0, \quad \sum_{i=1}^{I} k_i \leq I\tilde{e}$$

Solve the above subproblem, we have:

$$\min g = \sum_{i=1}^{I} (\mathbf{W_i'}\boldsymbol{\mu} + z) - \theta\sigma_i(\tilde{\sigma}_i I\tilde{e})^\eta; \quad \text{where} \quad \tilde{\sigma}_i = \frac{\sigma_i^{1/(1-\eta)}}{\sum_{j=1}^{I} \sigma_j^{1/(1-\eta)}}$$

Under the Assumption (2), $\min g > 0$. So LHS of (29) is positive. We must have ν>0.

Then there is a allocation with $k_{i,e} = k_i^*$, $\mathbf{x_{i,e}} = \mathbf{x_i}^*$, $s = \nu^* > 0$ and $\mathbf{p} = (\mathbf{A}'\boldsymbol{\mu} - \boldsymbol{\gamma}^*)/\nu^*$ which is a market equilibrium.

As from Lemma 5, the social planner's solution exists and be unique. Then from the proof in Step 1 and 2, the market equilibrium exists and be unique.

Theorem 2:

(i) From the Theorem 2, we know that the social planner problem is identical to the market solution under two assumptions. For the economy ξ', the certainty equivalent net worth of the whole economy Y' with $(H+1)$ asset is:

$$Y' = \max_{k_i, \mathbf{x_i}} \quad \left[\sum_{i=1}^{I} (\mathbf{W_i'}\boldsymbol{\mu} + z)(k_i)^{\eta} - \frac{\theta}{2}\left(\sigma_i (k_i)^{2\eta} + \mathbf{x_i}'\boldsymbol{\Omega_A}\mathbf{x_i} + 2(k_i)^{\eta}\mathbf{x_i}'\boldsymbol{\lambda_i}\right)\right] - \frac{\theta I}{2}\mathbf{x_F'}\boldsymbol{\Omega_A}\mathbf{x_F}$$

subject to

$$\sum_{i=1}^{I} k_i \leq I\tilde{e}$$
$$\sum_{i=1}^{I} \mathbf{x_i} + I\mathbf{x_F} = \mathbf{0}$$

If we impose the another constraint to restrict the use of asset $(H+1)th$ such that $x_i^{H+1} = 0 \quad \forall i \in I$, and $x_F^{H+1} = 0$ then the problem become the one when the social planner faces with the economy with only H assets. So we must have $Y' \geq Y$ as we have less constraint.

(ii) Let Y_N be the sum of certainty equivalent net worth for all entrepreneurs if they cannot access to the financial market. We have Y_N be a constant, in detail:

$$Y_N = \sum_{i=1}^{I} \int_{\underline{e}}^{\bar{e}} \left(E(n_{i,e}^U) - \frac{\theta}{2}Var(n_{i,e}^U)\right) f(e) de$$

Use the equation wage bargaining in case $\alpha = 0.5$ and take log both sides:

$$w_{i,e} = \frac{1}{2}\left[\left(E(\hat{n}_{i,e}) - \frac{\theta}{2}Var(\hat{n}_{i,e})\right) - \left(E(n_{i,e}^U) - \frac{\theta}{2}Var(n_{i,e}^U)\right)\right]$$

So the difference between Y and Y_N could be consider as the value added of the financial sector and $Y - Y_N = 2Y_F$. As $Y' \geq Y$ and Y_N does not change when we add a new asset, we have $Y_F' \geq Y_F$.

(iii) We rewrite:

$$\varphi = \frac{Y_F}{Y} = \frac{Y_F}{2Y_F + Y_N}$$

When we add a new asset, Y_F increases, Y_N is a positive constant so φ also increases.

Lemma 6 :

If the risks of two sectors are perfectly negative correlated, $\boldsymbol{\lambda}_1 = -\boldsymbol{\lambda}_2$, $\sigma_1 = \sigma_2 = \sigma$ and $\boldsymbol{\lambda}_1'\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_2'\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda}_2 = \boldsymbol{\lambda}'\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda}$. From the equation (28) in the proof of theorem 1, we have:

$$z - \nu\beta(k_1^*)^{1-\eta} - \theta(\sigma - \boldsymbol{\lambda}'\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda})(k_1^*)^{\eta} = z - \nu\beta(k_2^*)^{1-\eta} - \theta(\sigma - \boldsymbol{\lambda}'\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda})(k_2^*)^{\eta} \tag{30}$$

Under the Assumption 1, in the proof of Lemma 2, we have $\sigma - \boldsymbol{\lambda}'\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda} > 0$. Consider the function:

$$f(k) = z - \nu\beta(k)^{1-\eta} - \theta(\sigma - \boldsymbol{\lambda}'\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda})(k)^{\eta}$$

This function is decreasing in k, therefore the equation (30) only happens when $k_1^* = k_2^*$. Now substitute this result in the equation (27), we have $\mathbf{x_1} = -\mathbf{x_2}$, therefore $\mathbf{x_F} = \mathbf{0}$. The financiers' transaction income is 0.

Lemma 7 :

Consider the economy with only one production sector, then $k_1^* = \tilde{e}$. Substitute this result into (26) and (27), we get:

$$\mathbf{x_1^*} = -\frac{1}{2}\tilde{e}^{\eta}\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda}_1$$
$$\boldsymbol{\gamma} = \frac{\theta}{2}\tilde{e}^{\eta}\boldsymbol{\lambda}_1$$

From $\mathbf{x_F} = -\mathbf{x_1}$ and $\boldsymbol{\gamma} = \mathbf{A}'\boldsymbol{\mu} - s\mathbf{p}$, the expected financiers' transaction income :

$$\mathbf{x_F}'(\mathbf{A}'\boldsymbol{\mu} - s\mathbf{p}) = \frac{\theta}{4}\tilde{e}^{2\eta}\boldsymbol{\lambda}_1'\boldsymbol{\Omega}_{\boldsymbol{A}}^{-1}\boldsymbol{\lambda}_1$$

Using the direct result from the Lemma 8 , we have the expected financiers' transaction income is weakly increasing when new asset is added into the economy.

Lemma 8 :

First, both $\tilde{\boldsymbol{\Omega}}_{\boldsymbol{A}}$ and $\hat{\boldsymbol{\Omega}}_{\boldsymbol{A}}$ are positive definite matrices. We denote $\mathbf{A}$ as the asset matrix in (ξ) and $\mathbf{A_{H+1}}$ as the new asset:

$$\mathbf{Q} = \tilde{\boldsymbol{\Omega}}_{\boldsymbol{A}}^{-1} - \begin{bmatrix} \hat{\boldsymbol{\Omega}}_{\boldsymbol{A}}^{-1} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{\kappa}\hat{\boldsymbol{\Omega}}_{\boldsymbol{A}}^{-1}\mathbf{b}\mathbf{b}'\hat{\boldsymbol{\Omega}}_{\boldsymbol{A}}^{-1} & -\frac{1}{\kappa}\hat{\boldsymbol{\Omega}}_{\boldsymbol{A}}^{-1}\mathbf{b} \\ -\frac{1}{\kappa}\mathbf{b}'\hat{\boldsymbol{\Omega}}_{\boldsymbol{A}}^{-1} & \frac{1}{\kappa} \end{bmatrix}$$

$$\text{where} \quad \mathbf{b} = \mathbf{A}'\boldsymbol{\Omega}\mathbf{A_{H+1}}; \quad \kappa = \mathbf{A_{H+1}}'\boldsymbol{\Omega}\mathbf{A_{H+1}} - \mathbf{b}'\hat{\boldsymbol{\Omega}}_{\boldsymbol{A}}^{-1}\mathbf{b}$$

Now we prove $\kappa > 0$. As $\tilde{\boldsymbol{\Omega}}_{\boldsymbol{A}}$ is invertible, $\kappa \neq 0$. We have:

$$\kappa = \mathbf{A_{H+1}}'\underbrace{\left(\boldsymbol{\Omega} - \boldsymbol{\Omega}\mathbf{A}(\mathbf{A}'\boldsymbol{\Omega}\mathbf{A})^{-1}\mathbf{A}'\boldsymbol{\Omega}\right)}_{=\mathbf{D}}\mathbf{A_{H+1}}$$

Consider a matrix $\mathbf{M}$:

$$\mathbf{M} = \begin{bmatrix} \mathbf{A}'\boldsymbol{\Omega}\mathbf{A} & \mathbf{A}'\boldsymbol{\Omega} \\ \boldsymbol{\Omega}\mathbf{A} & \boldsymbol{\Omega} \end{bmatrix}$$

$\mathbf{M}$ is positive-semidefinite as $\boldsymbol{\Omega}$ is positive definite and the Schur complement of ($\boldsymbol{\Omega}$) in $\mathbf{M}$ is $\mathbf{A}'\boldsymbol{\Omega}\mathbf{A} - \mathbf{A}'\boldsymbol{\Omega}\boldsymbol{\Omega}^{-\mathbf{1}}\boldsymbol{\Omega}\mathbf{A} = \mathbf{0}$. So we have the Schur complement of $\mathbf{A}'\boldsymbol{\Omega}\mathbf{A}$ in $\mathbf{M}$ is positive semidefinite, and it is $\mathbf{D}$. So we have $\kappa \geq 0$ and $\kappa \neq 0$ then $\kappa > 0$.
The matrix $(\kappa\mathbf{Q})$ is positive definite as $1 > 0$ and the Schur complement of 1 in $(\kappa\mathbf{Q})$ is $\mathbf{0}$. As $\kappa > 0$, $\mathbf{Q}$ is also positive semidefinite.