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# The Desire for Revenge and the Dynamics of Conflicts\*

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#### Abstract

We model an infinitely-repeated conflict between two factions who both have a desire to exact revenge for past destruction suffered. The destruction suffered by a player is a stock that grows according to his opponent's destructive efforts and the rate at which past destruction is forgotten (i.e., depreciates). This gives a differential game. We find that a desire for revenge can cause a low-ability player to exert a higher effort than a high-ability player, which means that the former may have a higher probability of success in a given period. Given a desire for revenge, we find that, the conflict initially escalates and eventually reaches a steady state. When there is no desire for revenge, the conflict reaches a steady state immediately. The conflict is sufficiently less destructive if the rate at which past destruction is forgotten is sufficiently high. We briefly discuss how our results apply to the USA's invasion of Iraq, reconstruction assistance to Lebanon after the 1975-1990 war, and the Israeli-Palestinian conflict.

Keywords: conflict, differential game, open-loop equilibrium, revenge.

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*If it will feed nothing else, it will feed my revenge.* - William Shakespeare, The Merchant of Venice (Shylock act III)

Revenge is profitable. - Edward Gibbon, Decline and Fall of the Roman Empire (ch. XI)

# **1. Introduction**

The multiplicity of conflicts in Africa, Asia, Europe, and the Middle East requires the attention and efforts of academics and policy makers. Conflicts retard economic progress, lead to the destruction of human lives and property, introduce divisions among third parties (e.g., countries), and makes the world worse off. Therefore, the analysis, understanding, and possible solution of conflicts require the attention of scholars.

The desire to seek revenge in conflicts is sometimes the major cause of continuing conflict over and beyond the *original* cause of the conflict (see, for example, Kim and Smith, 1993; Chagnon, 1988; Juah, 2002). Examples are Israel/Palestine, USA response to attacks of US troops in Iraq, etc. Each faction in the conflict may want to "throw the last punch".

The desire for revenge appears to be a common human trait. As Elster (1990, p. 862) observes "[R]evenge - the attempt, at some cost or risk to oneself, to impose suffering upon those who have made one suffer, because they made one suffer - is a universal phenomenon." This desire is evidenced in the quotes at the beginning of this article.

In pre-industrial societies, revenge was seen as an integral part of justice and retribution. This still persists in certain societies. Indeed, some people justify capital punishment on the grounds that someone who has taken another human being's life deserves to have this life taken (i.e., an eye for an eye). As Nussbaum (1999, p. 157-158)

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observes "[T]he primitive sense of the just — remarkably constant from several ancient cultures to modern institutions ... — starts from the notion that a human life ... is a vulnerable thing, a thing that can be invaded, wounded, violated by another's act in many ways. For this penetration, the only remedy that seems appropriate is a counter invasion, equally deliberate, equally grave. And to right the balance truly, the retribution must be exactly, strictly proportional to the original encroachment. It differs from the original act only in the sequence of time and in the fact that it is response rather than original act — a fact frequently obscured if there is a long sequence of acts and counteracts."

To the extent that the desire for revenge is motivated by the past, it goes against economists' intuition of letting bygones be bygones. In standard economics, sunk costs should not matter. However, in reality, sunk costs matter.<sup>1</sup> And one such example is the desire to exact revenge. This desire may stem from preferences that reflect loss aversion (Kahneman and Tversky, 1979). For example, in a war, one may want to exact revenge because not doing so is tantamount to losing the war (McAfee et al, 2007).

The desire for revenge appears to be linked to a tit-for-tat strategy in repeated prisoner-dilemma-type games. Tit-for-tat strategy retaliates or punishes an opponent for not co-operating in a previous period. One would therefore think that a folk-theorem type result could sustain co-operation in an infinitely-repeated conflict even if the factions care about revenge. In such games, players co-operate because they worry about the adverse repercussions of non-cooperative behavior in the future. But given a history of conflicts, players who have a desire for revenge may only care about the past. They may want to

<sup>&</sup>lt;sup>1</sup> See the examples in McAfee et al., (2007) and the references therein.

co-operate only after they have exacted revenge. But if both factions reason in this way, then co-operation cannot be sustained.

Indeed, a key difference between tit-for-tat strategy and exacting revenge (as modeled in this paper) is that in tit-for-tat, the player who retaliates does not derive utility from the revenge *per se*. He only derives a positive utility if his retaliatory action causes his opponent to co-operate in the future. Hence tit-for-tat is forward-looking<sup>2</sup> while exacting revenge - as modeled in this paper and in reality – need not be: revenge is backwardlooking. This distinction is akin to the legal and philosophical discussions of punishment for the purpose of deterrence and reform vis-à-vis punishment for the purpose of atonement (justice). It is the basis of the legal debate on the merits of retributive justice vis-à-vis restorative justice. This difference in perspectives explains why some South Africans were not satisfied with the mandate and job of the Truth and Reconciliation Commission<sup>3</sup> in post-apartheid South Africa.

To be sure, there is a wide literature on contests and conflicts. However, our work will be the first in the economics literature to examine revenge in conflicts. There is, of course, a literature which studies the conditions under which conflicts escalate or end. Examples are Nalebuff (1986), Carlson (1995), Fearon (1996), Bester and Konrad (2005), Molinari (2000) and Konrad and Kovenock (2005). But none has focused on the

<sup>&</sup>lt;sup>2</sup> Commenting on the attractiveness of tit-for-tat, Axelrod (1984, p. 54) observed that "[W]hat accounts for TIT-FOR-TAT's robust success is its combination of being nice, retaliatory, forgiving and clear. Its niceness prevents it from getting into unnecessary trouble. Its retaliation discourages the other side from persisting whenever defection is tried. Its forgiveness helps restore mutual co-operation. And its clarity makes it intelligible to the other player, thereby eliciting long-term co-operation." For a critique of this farreaching claim, see Martinez-Coll and Hirshleifer (1991) and Binmore (1998).

<sup>&</sup>lt;sup>3</sup> The official government webpage of South Africa's Truth and Reconciliation Commission is http://www.doj.gov.za/trc/. The report of the commission is available at http://www.info.gov.za/otherdocs/2003/trc/

role of a desire for revenge.<sup>4</sup> Amegashie and Kutsoati (2007), Carment and Rowlands (1998), Chang et al. (2007), and Siqueira (2003) examined third-party intervention in conflicts; Garfinkel and Skarpedas (2000) studied how conflict can arise in a world of complete information; and Skaperdas (1992) investigated the conditions for peace and conflict in world with no property rights. None of these works focuses on how the desire for revenge fuels conflict.

Unlike economists, the role of revenge in wars, conflicts, and social relationships has been studied by philosophers, legal scholars, political scientists, and psychologists. Examples are the works of Elster (1990), Juah (2002), Chagnon (1988), Suny (1993), Stuckless and Goranson (1992) and Kim and Smith (1993).

The paper is organized as follows: the next section present a differential game model of revenge in conflicts. Section 3 presents an open-loop equilibrium analysis of the model. Section 4 concludes the paper.

#### 2. A model of revenge in conflicts

Consider two factions, 1 and 2, in an infinitely-repeated conflict (war). The beginning of time is period 0. Effort of faction i in period t is given by  $x_i(t)$ , i = 1,2.

The original cause of the conflict is a given resource.<sup>5</sup> The factions compete for this resource in every period t. For example, in the Israel/Palestine conflict the resource may be land which is used for settlements. In every period t, both factions value the resource at v > 0. Moreover, there is a second benefit from being successful in the conflict, namely

<sup>&</sup>lt;sup>4</sup> Glaeser (2005) is an interesting analysis of hatred.

<sup>&</sup>lt;sup>5</sup> Based on our model, the reader can easily verify that no conflict (i.e., zero effort levels) is not a Nash equilibrium in period 0.

a value of revenge. Efforts by the factions result in the destruction of human life and property. This destruction fuels the conflict and creates the desire for revenge. In period t, the value of revenge to faction i is

$$R_{i}(t) = R_{i}[s_{i}(t), s_{j}(t)]$$
 (1)

where i = 1, 2, j  $\neq$  i and s<sub>i</sub>(t) is the accumulated stock of destruction inflicted by faction i on faction j.

The revenge function in (1) has the following properties:

(i) 
$$R_i[s_i(t), s_j(t)] > 0$$
, if  $s_j(t) > 0$ ,  $j \neq i$ .

(ii) 
$$R_i[s_i(t), s_j(t)] = 0$$
, if  $s_j(t) = 0$ ,  $j \neq i$ 

(iii) 
$$\partial R_i / \partial s_i(t) > 0$$
,  $j \neq i$ .

$$(iv)\partial R_i/\partial s_i(t) \leq 0$$
.

Property 1 means that a faction has a positive valuation for revenge, if the cumulative destruction of its property and human life is positive. Property 2 means this valuation is zero, if there has not been any loss of human lives or property. Property 3 means that the higher is the cumulative destruction suffered the higher is the value of revenge. Property 4 means that the value of revenge is non-increasing in the cumulative destruction that is inflicted on the other faction.<sup>6</sup>

The stock of destruction evolves through time as follows

$$\dot{s}_{i}(t) = \gamma x_{i}(t) - \delta s_{i}(t)$$
<sup>(2)</sup>

<sup>&</sup>lt;sup>6</sup>It is important to note that revenge is fueled by past destruction not current destruction. However, the continuous-time formulation of the problem may not make this fact clear enough. If we were using a discrete-time formulation, we would have written the value of revenge in period t as a function of accumulated destruction up to period t-1.

where  $\gamma$ ,  $\delta \ge 0$ . The stock of destruction decays at rate,  $\delta$ . This decay is due to the fact that, for example, destroyed buildings are rebuilt resulting in people forgetting *some* of the past destruction of buildings (i.e.,  $0 < \delta < 1$ ) and so may no longer receive a value of revenge from it. People may also forget past destruction or their desire for revenge may wane even if there is no such reconstruction of buildings. This decay may be a reason why the conflict will perhaps end in some period or, at least, why the conflict may attain a steady state in which the intensity of the conflict is low.

In each period, the probability that faction i can successfully exact revenge is

$$P_{i}(x_{i}, x_{j}) = \frac{1}{2} + \eta(x_{i} - x_{j}), \qquad (3)$$

where i = 1,2,  $j \neq i$  and  $\eta$  is a positive parameter.<sup>7</sup> This probability function is increasing in a faction's own effort but decreasing in the effort of his opponent. The latter effect reflects the fact that his opponent will counter his attempt to exact revenge while also trying to exact his (i.e., the opponent) own revenge. We could make a distinction between offensive effort and defensive effort but this will only complicate the model without affecting our main results. Indeed, this distinction is not necessary insofar as offensive effort could also be a form of defense.

For simplicity, we assume that the equation in (3) is also the probability that faction i will successfully capture or defend the resource (e.g., land) in a given period.

To elaborate, the conflict erupts in period 0 over the resource of value v > 0. Suppose, for a moment, that the conflict over the initial resource is resolved after the period 0 battle, where each faction captures a proportion of the resource according to the success

<sup>&</sup>lt;sup>7</sup> See Che and Gale (2000) for analysis of this type of difference-form contest success function. For our purposes, this function is easier to work with than the Tullock ratio-form function.

function above. Therefore, further conflicts are fueled by the need to seek revenge due to the destruction of human lives on both sides. The assumption is that period 0 efforts resulted in the loss of human lives and property. Then in an attempt to seek revenge in period 1, there will also be efforts which will lead to further loss of human lives, giving cause for further revenge and so on.<sup>8</sup> But in reality, the conflict over the initial resource (e.g., land) need not be resolved in period 0. There is indeed a fine line between the desire for revenge fuelling the conflict and the quest to appropriate the resource fuelling the conflict. We shall therefore assume that in their desire to seek revenge, each faction takes into account that winning the battle over the resource is also a possibility. Thus in each period, the resource (e.g., land) is also up for grabs. This assumption also allows us to compare our results to the benchmark case where there is no revenge and there is a contest over the resource in every period.

In every period t, faction i incurs the cost of effort, given by  $C_i(x_i(t))$  and suffers a cost of destruction given by  $\beta x_j(t)$ ,  $\beta \ge 0$ , and  $j \ne i$ , i = 1, 2. We assume that  $C_i(x_i(t))$  is increasing and strictly convex.

The present value of faction i's net benefit could be written as

$$W_{i} = \int_{0}^{\infty} \left( P_{i}(t) [v + R_{i}(\cdot)] - \beta x_{j}(t) - C_{i}(x_{i}(t)) \right) e^{-rt} dt$$
(4)

where r > 0 is the common discount rate and  $i \neq j$ ,  $i = 1,2, j = 1,2.^9$ 

<sup>&</sup>lt;sup>8</sup>For the sake of exposition, we sometimes couch the discussion in parts of this article within a discrete-time framework.

<sup>&</sup>lt;sup>9</sup> As noted above, revenge is a function of past destruction which enters the objective function as a positive value.

#### 2.1 Some remarks on modeling revenge

Before we proceed to the solution of the model, we think a few remarks about how we have modeled revenge are in order.

It is important to note that the positive value of revenge does not reflect masochistic preferences. A faction does not derive satisfaction from suffering destruction. Destruction is costly to the victim. However, given that destruction has been suffered *in the past*, the victim derives satisfaction from exacting revenge.

For the sake of exposition, consider faction 1. It is obvious that the ability to exact revenge must be positively related to the effort of the faction seeking revenge. We have captured this effect by making  $x_1$  to positively influence faction 1's benefit function through the P<sub>1</sub> function but to negatively affect faction 2 through the –  $\beta x_1$  term in faction 2's payoff function.

To be sure, the true cost of faction 1's destructive effort on faction 2 could be  $\beta x_1$ but faction 1 might possibly think it is  $\alpha x_1$ , where  $\alpha > \beta$ . That is, faction 1 could overestimate the effect of his destructive effort on faction 2. Faction 1 may genuinely believe that this is the case or it may be part of an inflammatory political rhetoric. However, this issue is irrelevant in our open-loop equilibrium because the equilibrium efforts are independent of  $\beta$ . Also, a successful revenge need not imply that  $\beta x_1$  must be equal to  $R_1$ , since the cost of past destruction could be different for the avenger (i.e., the victim) and the perpetrator. Furthermore, the value of revenge to the avenger may be different from the cost of destruction to his opponent. For example, a faction that wants to avenge the loss of a hundred lives may see his opponent's loss of more than a hundred lives or less than a hundred lives as the desired or successful revenge. This may stem

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from different sensitivities to the loss of life which may be driven, for example, by different religious and secular beliefs. We find, though, that in equilibrium  $x_1$  is positively related to  $R_1$ .

One may argue that past destruction should be completely forgotten or taken out of future revenge-seeking calculus after it has been successfully avenged. This means that  $\delta = 1$  after past destruction has been successfully avenged.<sup>10</sup> We do not make this assumption for the following reasons: First, the destruction suffered by a faction is narrated by older generations to younger generations, regardless of whether they were successfully avenged. Long-standing foes continually remind themselves of the destruction suffered from the other group. The idea is to maintain group solidarity, keep the struggle ongoing, and to be wary of their foes. Such constant reminders could bring back memories of past destruction that was successfully avenged and fuel current revenge.<sup>11</sup> Second, since the avenger will invariably suffer some losses in the current period even if the revenge was successful, there is usually nothing like a costless revenge in the current period. And even if there is, the enemy will also attempt to avenge previous revenge. This is what Nussbaum (1999) referred to as "... a long sequence of acts and counteracts". Hence current destruction suffered in the process of exacting revenge or an enemy's destructive response to one's previous revenge could bring back memories of past destruction that was successfully avenged. Revenge creates an unending cycle of animosity which may draw its momentum from past destruction suffered several periods

 $<sup>^{10}</sup>$  It is important to note that our results will not change if faction i derived some value of revenge in the event of an unsuccessful attempt at exacting revenge, so long as this value is smaller than  $R_i$ .

<sup>&</sup>lt;sup>11</sup> Indeed, the revenge function (i.e., satisfaction from revenge) could change over time depending how radical the current generation is relative to previous generations. For simplicity, we keep the revenge function constant and only allow the state variables to change over time.

ago. For these reasons, we do not assume that  $\delta = 1$ . Instead, we assume that  $0 < \delta < 1$  even if past destruction is successfully avenged.

# 3. Open-loop Equilibrium

We look for an open-loop equilibrium. An open-loop equilibrium seems more realistic within the context of revenge in conflicts. As noted by Hume (1898), revenge is an emotionally charged reaction leading people who seek revenge to knowingly ignore certain consequences of their actions.<sup>12</sup> When people decide to take revenge, they may not take into account the possibility that their actions could lead to retaliation by their opponent. They just do it (i.e., seek revenge). This is consistent with the rationale behind the open-loop strategy where the state variable (i.e., the level of accumulated destruction) is ignored by a faction. In a closed-loop case, the factions will take into account the fact that their destructive efforts in the current period could lead to retaliation by their opponent in future periods and therefore they will suffer more destruction.<sup>13</sup> This may have a desirable restraining effect. However, we doubt that conflicts like civil wars and wars in the Middle East and other parts of the world, once they had begun and were somewhat fueled by a desire for revenge, necessarily involved these kinds of calculations. Indeed, our open-loop analysis leads to very intuitive results that are consistent with casual empiricism.

<sup>&</sup>lt;sup>12</sup> Hume (1898, appendix II) notes that "[W]ho see not that vengeance, from the force alone of passion, may be so eagerly pursued as to make us knowingly neglect every consideration of ease, interest, and safety and, like some vindictive animals, infuse our very souls into the wounds we give an enemy." This, however, does not mean that the desire for revenge is not driven by some modicum of rational decision making. It is only less so. However, the fact remains, as echoed by Hume, that people could go to great lengths to exact revenge.

<sup>&</sup>lt;sup>13</sup> As noted earlier, we sometimes couch certain arguments within a discrete-time framework to ease exposition.

To obtain the open-loop equilibrium, we assume that the factions condition their effort choices on calendar time t only and take as given the strategy of the other faction. Faction i maximizes the present value of its net benefit in (4) subject to

$$s_i(0) \equiv s_i^0 > 0, \ \dot{s}_i(t) = \gamma x_i(t) - \delta s_i(t) \text{ and } \dot{s}_j(t) = \gamma x_j(t) - \delta s_j(t), \ j \neq i, \ i = 1, 2.$$

The current value Hamiltonian for faction 1 is

$$H_{1} = P_{1}(x_{1}, x_{2}(t))[v + R_{1}(s_{1}, s_{2})] - \beta x_{2}(t) - C_{1}(x_{1}) + \lambda_{1}[\gamma x_{1} - \delta s_{1}] + \mu_{1}[\gamma x_{2}(t) - \delta s_{2}]$$
(5)

where the  $\lambda$ s and the  $\mu$ s represent co-state variables. Denoting the first derivative by a prime and by indicating the variable with respect to which a derivative is taken on the left-hand-side below in square brackets beside the relevant equation, we obtain the following optimality conditions from (5):

$$[x_1]: P'_1(x_1, x_2(t))[v + R_1(s_1, s_2)] - C'_1(x_1) + \lambda_1 \gamma = 0$$
(6)

$$[s_1]: \ \lambda_1 = (r+\delta)\lambda_1 - H_1' = (r+\delta)\lambda_1 - P_1(x_1, x_2(t))R_1'(s_1, s_2)$$
(7)

$$[s_2]: \dot{\mu}_1 = (r+\delta)\mu_1 - H'_1 = (r+\delta)\mu_1 - P_1(x_1, x_2(t))R'_2(s_1, s_2)$$
(8)

and the transversality conditions

$$\lim_{t \to \infty} e^{-rt} \lambda_1(t) \widetilde{s}_1(t) = 0 \qquad \qquad \lim_{t \to \infty} e^{-rt} \mu_1(t) \widetilde{s}_2(t) = 0 \qquad (9)$$

where the tilde over the state variable means that (9) has to be satisfied for each feasible value of the state variables. By the same arguments, we obtain for faction 2

$$[x_2]: -P'_1(x_1, x_2(t))[v + R_2(s_1, s_2)] - C'_2(x_2) + \lambda_2 \gamma = 0$$
(10)

$$[s_2]: \dot{\lambda}_2 = (r+\delta)\lambda_2 - H_1' = (r+\delta)\lambda_2 - [1 - P_1(x_1, x_2(t))]R_2'(s_1, s_2)$$
(11)

$$[s_1]: \dot{\mu}_2 = (r+\delta)\mu_2 - H'_1 = (r+\delta)\mu_2 - [1 - P_1(x_1, x_2(t))]R'_2(s_1, s_2)$$
(12)

and the transversality conditions

To simplify the analysis, we assume that  $R_1(s_1,s_2) = as_2$ ,  $R_2(s_2,s_1) = as_1$ , where a > 0. Hence,  $\partial R_i / \partial s_i = 0$  for all i. This is a reasonable assumption. A player's desire for revenge may depend on the destruction suffered but not on the destruction caused to his opponent.<sup>14</sup> This is akin to a psychological phenomenon, formalized by Kahneman and Tversky (1979), where gains (in this case, destruction inflicted on others) are weighted less than losses (i.e., destruction suffered). Here we assume that the destruction inflicted on others is completely disregarded in a faction's revenge-seeking calculus. We also

assume that 
$$C_1(x_1) = \frac{c_1}{2}x_1^2$$
,  $C_2(x_2) = \frac{c_2}{2}x_2^2$ , where  $c_1, c_2 > 0$ .

Given  $\partial R_i / \partial s_i = 0$ , it can be shown using (7) and (9) that  $\lambda_1(t) = 0$  and using (11) and (13) that  $\lambda_2(t) = 0$  for all t and i.<sup>15</sup> Then (6) and (10) yield the optimal effort levels, which are functions of accumulated destruction, as:

$$\mathbf{x}_{1}^{*} = \frac{\eta \mathbf{v}}{\mathbf{c}_{1}} + \frac{\eta \mathbf{a}}{\mathbf{c}_{1}} \mathbf{s}_{2}, \qquad \mathbf{x}_{2}^{*} = \frac{\eta \mathbf{v}}{\mathbf{c}_{2}} + \frac{\eta \mathbf{a}}{\mathbf{c}_{2}} \mathbf{s}_{1}$$
 (14)

To see that the effort levels in (14) are indeed equilibrium efforts, note that the effort levels can be rewritten as  $x_i^* = (\eta/c_i)(v + R_i)$ , i = 1,2. In an open-loop equilibrium, a player chooses the time path of his actions assuming that the time path of his opponent's actions is fixed. In any period t, a player's does not care about his own state variable but cares about his opponent's state variable in the previous period via its effect on his desire

<sup>&</sup>lt;sup>14</sup> A simple revenge function which satisfies our assumptions including  $\partial R_i/\partial s_i \neq 0$  is  $R_i = as_j(s_j - s_i)^2$ ,  $i \neq j$ , i = 1, 2, j = 1, 2. However, the dynamics of conflict is very difficult to analyze in this case because it requires the solution to a two-dimensional system of complicated non-linear differential equations. <sup>15</sup>Intuitively, if player i's own state variable does not enter his payoff function and he does not condition his strategies on this state variable, then the shadow price of this state variable must be zero.

for revenge. <sup>16</sup> Since, for example, player 1 does not care about  $s_1$ , has no control over  $s_2$  and treats the time path of  $x_2$  and, for that matter  $s_2$ , as fixed, it follows that player 1 in each period sees himself as being in a stationary (static) contest with prize (v + R<sub>1</sub>). A similar argument holds for player 2. The efforts levels in (14) are the unique Nash equilibrium values of a static contest with prize (v + R<sub>i</sub>) for player i. Hence, the effort levels in (14) are the unique open-loop equilibrium effort levels.<sup>17</sup> If there were no revenge, the problem in each stage will obviously be stationary. The solution is

 $x_i^* = (\eta/c_i)v$ , which is the solution when  $R_i = 0$  in (14) for all t and i.

Inserting (14) into the state equations in (2) yields a two-dimensional system of linear differential equations in  $s_1$  and  $s_2$ :

$$\dot{s}_1 = \frac{\eta v}{c_1} - \delta s_1 + \frac{\eta a}{c_1} s_2, \qquad \dot{s}_2 = \frac{\eta v}{c_2} + \frac{\eta a}{c_2} s_1 - \delta s_2$$
 (15)

By determining the eigenvalues and the eigenvectors of this system, it can be shown that the solution is

$$s_{1}(t) = s_{1}^{\infty} + \frac{k_{1}\sqrt{c_{1}} - k_{2}\sqrt{c_{2}}}{2\sqrt{c_{1}}}e^{\theta_{1}t} + \frac{k_{1}\sqrt{c_{1}} + k_{2}\sqrt{c_{2}}}{2\sqrt{c_{1}}}e^{\theta_{2}t}$$
(16)

$$s_{2}(t) = s_{2}^{\infty} + \frac{k_{2}\sqrt{c_{2}} - k_{1}\sqrt{c_{1}}}{2\sqrt{c_{2}}}e^{\theta_{1}t} + \frac{k_{1}\sqrt{c_{1}} + k_{2}\sqrt{c_{2}}}{2\sqrt{c_{2}}}e^{\theta_{2}t}$$
(17)

<sup>17</sup> Given the above reasoning, the equilibrium effort levels with a Tullock contest success function and a linear cost function,  $c_i x_i$ , is  $x_i^* = c_j (v + R_i)^2 (v + R_j) / [c_j (v + R_i) + c_i (v + R_j)]^2$ , i = 1,2. However, the dynamics of conflict is very difficult to analyze in this case because it requires the solution to a two-dimensional system of non-linear differential equations. Note that the equilibrium effort levels in (14) are independent of  $\beta$ . This is a consequence of the open-loop equilibrium.

<sup>&</sup>lt;sup>16</sup>In a closed-loop equilibrium, a player would take into account the fact that the future value of his state variable affects his opponent's future value of revenge and hence his opponent's future effort. The absence of this effect makes the open-loop equilibrium very easy to find. As argued earlier, the absence of this effect is consistent with revenge-taking calculus.

where

$$\theta_1 = -\delta - \frac{a\eta}{\sqrt{c_1 c_2}}, \qquad \theta_2 = -\delta + \frac{a\eta}{\sqrt{c_1 c_2}}$$
(18)

are the two eigenvalues of (15),

$$s_{1}^{\infty} = \frac{\eta v(\delta c_{2} + \eta a)}{\delta^{2} c_{1} c_{2} - \eta^{2} a^{2}}, \quad s_{2}^{\infty} = \frac{\eta v(\delta c_{1} + \eta a)}{\delta^{2} c_{1} c_{2} - \eta^{2} a^{2}}$$
(19)

are the steady state values of accumulated destruction and

$$k_1 = s_1^0 - s_1^\infty, \ k_2 = s_2^0 - s_2^\infty$$
 (20)

are given constants.

For a meaningful steady state we must have  $s_i^{\infty} \ge 0$  and, hence,  $\delta^2 c_1 c_2 > \eta^2 a^2$ . The eigenvalues in (18) are then both negative which implies that the solution in (16) and (17) converges to the stable steady state given by (19). This will not be the case if  $\delta = 0$ . Therefore decay stabilizes the conflict and eventually leads to a steady state. Note that  $x_i(0) = (\eta/c_i)v < x_i^{\infty} = (\eta/c_i)(v + as_j^{\infty})$ ,  $i = 1, 2, j = 1, 2, j \neq i$ . This gives the following

proposition:

**Proposition 1:** If factions in a conflict are motivated by the desire to seek revenge, then the conflict escalates through time and eventually reaches a steady state. With no desire for revenge, the conflict reaches a steady state right from the beginning of the conflict. Also, the steady state effort level is higher with revenge than with no revenge.

Note that if  $c_1 > c_2$ , then  $s_1^{\infty} < s_2^{\infty}$ . Then using equation (14) we obtain the result that it is possible to have  $x_1^* > x_2^*$  even if  $c_1 > c_2$ . Notice that this result is possible although both factions have the same revenge functions. It is not driven by different

degrees of satisfaction from exacting revenge. This result might explain why a faction with less efficient conflict technology can squarely match one with a more efficient conflict technology. Note, however, that this result is impossible when none of the parties has a desire for revenge. We summarize this result in the following proposition:

**Proposition 2:** If factions in a conflict are motivated by the desire to seek revenge, then the militarily inferior faction might exert a greater effort than the militarily superior faction. Hence, the militarily inferior faction may have a higher probability of winning the conflict in some periods.<sup>18</sup> If they both have no desire for revenge, then the militarily superior faction will always exert a greater effort.

Recall that the higher is  $\delta$ , the higher is the rate at which past destruction is forgotten. Since  $\partial s_i^{\infty} / \partial \delta < 0$  for all i, this result and the equilibrium effort levels imply the following proposition:

**Proposition 3:** If factions in a conflict are motivated by the desire to seek revenge, then the higher is the rate at which past destruction by an opponent is forgotten, the lower is the steady-state level of effort in the conflict.

#### 4. Discussion and conclusion

Proposition 3 implies that historical narratives of the atrocities of one group against another, while useful as a way of understanding the past, may also have the undesirable effect of increasing the cost of conflict especially between groups with a

<sup>&</sup>lt;sup>18</sup> This seemingly counter-intuitive result is not surprising when one thinks of contests with endogenous valuations. In a recent contribution, Sela and Cohen (2005) find that in a contest where the winner's effort is fully re-imbursed, a low-ability contestant could have a higher success probability than a high-ability contestant. Proposition 1 is based on the fact that a high-ability contestant's effort positively influences the valuation of a low-ability contestant and vice versa.

history of conflict such as the Israelis and Palestinians, the Serbs and Croats, and the Hutus and Tutsis. In contrast, reconstruction assistance such those given to Lebanon after the 1975-1990 war may help in increasing the rate at which past destruction (physical) is forgotten, although memories of the destruction of human lives is unlikely to be affected by such reconstruction assistance.

In war, it is sometimes the case that superior military strength may not be enough against the resolve and determined spirit of an enemy. A case in point is the USA's invasion of Iraq. Proposition 2 suggests that the desire for revenge may partly account for this resolve. A faction with inferior military technology might exert a higher effort than one with superior military technology, so long as the former faction is motivated by revenge. This is because the desire for revenge increases the valuation of the faction who has suffered more destruction. If the low-ability faction's valuation is sufficiently high, then he will exert a higher effort. If this happens in a steady state, then it will continue forever. So in a steady state, the low-ability faction's higher effort does not increase the cumulative destruction suffered by the high-ability faction to the point where the high-ability faction begins to exert a higher effort because it has a higher valuation.

We also found that while revenge may cause a conflict to escalate, it eventually reaches a steady state.

In conclusion, we hope that our work has shed some light on our understanding of the effect of revenge on the dynamics of conflict and will lead to further work in this area. For example, an interesting but very challenging extension is to endogenize preferences for revenge. Why are some groups more revenge-driven than others? There may be evolutionary reasons for revenge.

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