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Scarcity Climate Rents in Emissions Permit Markets with Oligopoly Competition

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ABSTRACT
Prior research has shown, firstly, that firms subjected to a cap-and-trade system can enjoy scarcity rents and, secondly, that they can manipulate the price of permits as a rent seeking behaviour. We analyse these issues using a duopoly model under Cournot and Stackelberg competition. We identify the different sources of scarcity rents and determine under which conditions they can generate incentives for the firms to lobby the permit price up. We show that, under reasonable assumptions, Stackelberg competition eliminates incentives to collude if there is no grandfathering. However, such incentives appear if there is an initial free allocation of permits, which is a policy argument against grandfathering. This effect is increasing with the amount of permits allocated to the leader and those parameter changes that undermine the leader’s advantage in output production or reduce the leader’s abatement cost.

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Key words: Emissions permits, Collusion, Market power, Duopoly, Stackelberg model.

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1. Introduction

This paper examines the creation of scarcity rents for oligopolistic firms by an emission trading system. We model the mechanisms that give rise to scarcity rents and identify the circumstances under which such rents can be large enough to offset the costs to comply with the environmental policy.

The initial historical experiences of emission trading took place at regional and national levels. The US SO₂ trading system, under the framework of the Acid Rain Program, is widely recognised as the main pioneering application for its ambition and significant influence (see e.g. Stavins, 1998). This policy approach is also becoming increasingly important at an international level as a way of dealing with transboundary environmental problems, among which climate change is nowadays seen as the most salient one. It gave a reason for the creation of the United Nations Framework Convention on Climate Change (UNFCCC) in 1992 and since then its relevance within the policy agenda has steadily increased.

In 1997 the Kyoto Protocol introduced emission trading as one of the flexibility mechanisms to be used by the Annex I Parties in meeting their emission limitation commitments. The European Union Emissions Trading System (EU ETS) was launched, to a large extent, as a means to meet the Kyoto commitments. It became the first large greenhouse gas emissions trading scheme in the world, and remains the biggest. It is a major pillar of EU climate policy and it covers all 28 EU member states plus Iceland, Norway and Liechtenstein. See Newell et al. (2013) for an overview of the major carbon markets in the world.

It is well established in the literature that some environmental policies can create scarcity rents for firms, which in the case of climate policies are sometimes known as climate rents. This effect is clearly visible in most cap-and-trade (CAP) schemes,
including the US SO\textsubscript{2} market and the EU ETS. Due to the tradable nature of emission permits, some firms can take the opportunity to obtain additional revenue by selling permits, which is sometimes known as \emph{windfall profits}. Empirical evidence suggests that this phenomenon has been rather important in the first phase of the EU ETS. A less obvious effect comes from the role of permits as an input. If permits become more scarce (and hence, more expensive), this will represent a higher cost for firms, which will react by producing less output and rising its price. This induced effect can alleviate or even more than fully compensate the compliance costs, resulting in higher profits.

We examine the existence of scarcity rents for oligopolistic firms that are subject to a CAP system. Specifically, we ask if and how firms can benefit from the fact that the permits become scarcer. We set up a duopoly model in which each firm is initially endowed with a number of free permits and can purchase additional ones (or sell its surplus) in the market. In the output market we allow for Cournot or Stackelberg competition.

In the Cournot version, we show that a higher permit price increases the firms’ cost of purchasing permits, but it also has two positive effects on profits: first, it restricts output and increases the output price as well as firms' revenues (which we call "output scarcity rent") and, second, it increases the value of the firms' holding of permits (which we label as "grandfathering scarcity rent"). The share of the output scarcity rents that accrues to a specific firm depends on the elasticity of its rival's output with respect to the permit price, which implies that a monopoly can never obtain output scarcity rents as we define them.

In a Stackelberg setting with a leader and a follower, the qualitative conclusions are similar, although the calculation of the output scarcity rents are not the same for both firms due to their different market shares, which creates a gap between the effect that a higher permit price has on their profits.
We subsequently proceed to explore a particular case with a separable cost function to gain more accurate insights. As a first core finding we conclude that both firms face a profit function that is convex in the permit price, which implies that, when the price is sufficiently low, both firms will benefit from a price reduction, whereas with sufficiently high prices, they will benefit from further increases. Apart from these two regions, in the Stackelberg model there is an additional intermediate interval in which both firms' interests are decoupled because the leader's profit is decreasing while the follower's is increasing.

As a policy application of our results, we ask about the possibility that the firms could collude somehow to inflate the price of permits as a rent-seeking activity. We conclude that this type of behaviour is more likely to show up when firms compete on an equal footing, as in the Cournot setting, whereas the existence of leaders and followers tend to undermine the chances for collusion.

We pay special attention to the role of grandfathering as an allocation mechanism and conclude that it makes the firms more likely to benefit from a permit price increase and hence to engage in collusive agreements. Actually, if the number of permits distributed by grandfathering is large enough, it could even be the case that the firms are always willing to push the price up, whatever the starting point. In the Stackelberg case there is a qualitatively stronger implication that leads us to conclude that, in the absence of grandfathering, there is no room for collusion because the collusive region shrinks to such an extent that it may disappear. Therefore, in a Stackelberg setting, the mere existence of grandfathering has the qualitative implication of opening up the way for collusion. This result represents an important argument against grandfathering insofar as it can introduce incentives to foster collusive manipulation.

As a sensitivity analysis, we allow for parameter asymmetries. In the Stackelberg case, we conclude that the incentives for collusion are sensitive to the allocation of free
permits received by the leader, but not by the follower. Moreover, those parameter changes that tend to undermine the leader’s advantage in the output market (an increase in the leader’s output cost or a decrease in the follower’s) make the firms more symmetric in a certain sense and increase the likelihood of collusive behaviour. The opposite occurs with abatement costs: the likelihood of collusive behaviour tends to decrease with the leader’s abatement cost and increase with the follower’s.

The closest papers to ours in the literature are those dealing, on the one hand, with scarcity rents and, on the other hand, with permit price manipulation. For a broad discussion on scarcity rents, see e.g. Fullerton and Metcalf (2001). In a perfect competition framework, Mohr and Saha (2008) claim that, via the generation of scarcity rents, a stricter environmental regulation might have a distributional impact in the sense of increasing firm’s profits and passing the cost onto consumers. André et al. (2009) make a similar point in a strategic setting with quality competition. MacKenzie & Ohndorf (2012) claim that both revenue-raising instruments (RRIs) such as emission taxes or auctioned tradable permits, and non-revenue-raising instruments (NRRIs), e.g. freely allocated tradable permits, can create scarcity rents that may be susceptible to costly appropriation activities. Kalkuhl & Brecha (2013) analyse the impact on the overall scarcity rents of mitigation targets and find that reducing fossil resource use could actually increase scarcity rents and benefit fossil resource owners under a permit grandfathering rule.

In the specific case of emission permits trading, Newell et al. (2013) point out that the power generators extracted rents by receiving carbon allowances for free and then passing on the opportunity costs of these allowances to their customers. Moreover, some firms have taken the opportunity to sell a part of their permit allocation and get extra revenue, which is commonly referred to as windfall profits. For an analysis of this
phenomenon, see e.g. Sijm et al. (2006), Ellerman and Joskow (2008) or Ellerman et al. (2010).

Moving on to price manipulation, it can be noted that, since the seminal paper by Hahn (1984), some authors have explored the possibility that a dominant firm can exert its market power by manipulating the permit price. See, among others, Misiolek and Elder (1987), Von der Fehr (1993) and Sartzetakis (1997, 2004). Hintermann (2011, 2015) concludes that the largest European electricity producers might have found it profitable to manipulate the permit price upwards, which can partially explain the elevated allowance price level during the first 18 months of the EU ETS.

Unlike most papers in the related literature, we consider an oligopoly market structure instead of a dominant firm and a competitive fringe. Thus, our setting is more consistent with collusive agreements between firms rather than dominant position abuse by a single firm. Moreover, instead of studying the distortion of competition on (product or permit) markets caused by price-setting firms on the allowance market, we focus on the output market and do not explicitly model the permit market, but simply study how the permit price affects the firms’ results. In this sense, our paper is closer to Ehrhart et al. (2008), who show that under some conditions firms can benefit from a higher price of permits. They also claim that in the EU ETS there are loopholes in the trading law that allow for collusive behaviour among firms to manipulate the price of permits.\(^3\)

In contrast to Ehrhart et al. (2008), we consider grandfathering, which allows us to make an explicit characterisation of the different sources of scarcity rents. Moreover, we compare Cournot vs. Stackelberg settings, while their analysis is restricted to situations in which the firms play exactly the same role in the market. We are not aware

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\(^3\) These include, first, the possibility to influence the initial allocation of permits (to make it more stringent), second, the ‘opt-in’ rule, which enables outside firms to participate in the system, third, the implementation of project-based mechanisms paying more for these credits than they would in the market and fourth, paying additional emissions duties.
of any paper in the related literature that compares Cournot and Stackelberg settings. Another difference is our detailed study of a particular case, which renders some insights that cannot be derived in a general model, such as the fact that incentives for collusion could totally disappear under Stackelberg competition.

Section 2 presents the basic model. A particular abatement cost function is considered in Section 3. Section 4 focuses on the interpretation of our results in terms of incentives for price manipulation. Section 5 concludes.

2. The general model

Let us consider a simple duopoly model of a polluting industry that is subject to a tradable permit system. We assume that the firms compete in the output market and, once their output levels, \( x_1 \) and \( x_2 \), are determined, they simultaneously choose their cost-minimising emission levels, \( e_1 \) and \( e_2 \).

The cost function of firm \( i \in \{1, 2\}, \ C_i(x_i, e_i), \) depends on output \( (x_i) \) and emissions \( (e_i) \) and is continuous and twice differentiable in both arguments with the following properties:

\[
\frac{\partial C_i}{\partial x_i} > 0, \quad \frac{\partial C_i}{\partial e_i} < 0, \quad \frac{\partial^2 C_i}{\partial e_i^2} > 0, \quad \frac{\partial^2 C_i}{\partial x_i \partial e_i} < 0.
\]  

(1)

This function integrates production and abatement costs and reflects the fact that producing clean (with low emissions) is more costly than producing dirty. Each unit of emissions must be covered by an emission permit. Initially, each firm \( i \) is endowed with a given amount of free permits \( S_i \), and additionally required permits, \( e_i - S_i \), can be obtained on the market at an exogenously given price, \( p \). Considering the cost of permit purchasing, the total cost of firm \( i \) is given by
\[
TC_i(x_i, e_i) := C_i(x_i, e_i) + p(e_i - S_i).
\] (2)

To find a subgame-perfect Nash equilibrium, the model is solved by backward induction. In the final stage, both firms decide on their emission levels to minimise their total cost, \(TC_i(x_i, e_i)\), while taking their output levels and the price of permits as given. If the solution is interior, we obtain the standard first-order condition (FOC),

\[
\frac{\partial C_i}{\partial e_i} + p = 0,
\] (3)

which implicitly defines each firm’s demand for permits, \(e^*_i(x_i, p)\). Using this demand function in (2) we obtain the minimised total cost function of each firm in terms of its output and the price of permits:

\[
TC^*_i(x_i, p) := TC(x_i, e^*_i(x_i, p)) = C(x_i, e^*_i(x_i, p)) + p[e^*_i(x_i, p) - S_i]
\] (4)

and using the envelope theorem we know that

\[
\frac{\partial TC^*_i}{\partial p} = e^*_i - S_i,
\] (5)

which simply states that the marginal impact of an increase of the permit price on total cost equals the amount of purchased permits.

Now we move on to the first stage of the game, the output market, in which both firms face the inverse demand function \(P(X)\), where \(X := x_1 + x_2\), \(\frac{dp}{dX} < 0\) and they compete between them with the aim of maximising its profit defined as

\[
\Pi_i = P \cdot x_i - TC^*_i(x_i, p).
\]

We consider two alternative market structures, with simultaneous (Cournot) and sequential (Stackelberg) decisions respectively. Both structures are analysed separately in the following subsections.

\(^4\) The second order condition is always fulfilled due to the convexity of \(C_i\) in emissions. Throughout the paper, we restrict to interior solutions. Asterisks denote equilibrium values.
2.1 Cournot Model

Consider first a Cournot framework, in which each firm decides its output level to maximise its profit taking the rival’s output as given. Firm $i$’s first order condition is

$$ p(x_i + x_{-i}) + \frac{\partial p}{\partial x} x_i - \frac{\partial TC_i}{\partial x_i} = 0, $$

where $"-i"$ refers to the firm other than $i$. This equation implicitly defines the reaction function of firm $i$:

$$ x_i = x_i^b(x_{-i}, p). $$

Using the implicit function theorem, we conclude that one firm’s output is decreasing in its rival’s output and the price of permits:

$$ \frac{\partial x_i^b}{\partial x_{-i}} = \frac{-dP/dX}{2dP/dX - \frac{\partial^2 TC_i}{\partial x_i^2}} < 0 \quad \frac{\partial x_i^b}{\partial p} = \frac{\frac{\partial^2 TC_i}{\partial x_i^2}}{2dP/dX - \frac{\partial^2 TC_i}{\partial x_i^2}} < 0. $$

The system of equations formed by both reaction functions determine the final equilibrium as a function of the permit price and using the equilibrium value of output we can also write both agents’ profits as a function of the permit price:

$$ \Pi_i^*(p) := p(x_i^* + x_{-i}^*) x_i^* - TC(x_i^*, e_i^*). $$

Our main research question is to what extent, and by which channels, the scarcity of permits can benefit firms. Taking the price of permits as an indicator of scarcity, our question can be stated in an operational way by measuring what is the impact of a permit price increase on profits. Differentiating (9) with respect to $p$ and taking into account equations (4) and (6) we obtain

$$ \frac{d\Pi_i^*}{dp} = \frac{dP}{dx} \frac{\partial x_i^*}{\partial p} x_i^* + s \frac{\partial e_i^*}{\partial p}. $$
from which we conclude that the marginal effect of the permit price on the profit can be split in three components. The two first effects ($SR^i$ and $SR^G$) are positive and account for the scarcity rents for firm $i$. The third component is negative and determines what part of the scarcity rents each firm can capture.

The first component of scarcity rents is endogenous and represents the additional revenue that each firm will receive thanks to the reduction in output supply and the resulting increase in the output price. We call it “output scarcity rent”. The second component is purely exogenous and simply equals the amount of permits that each firm receives for free. We label this term as “grandfathering scarcity rent” and corresponds to what is commonly known as windfall profit.

The output scarcity rent is given by

$$SR^i = \frac{dP}{\partial X} \frac{\partial x^i}{\partial p} x^i. \tag{10'}$$

It is interesting to note that a higher value of $p$ causes the output of both firms, and hence total output, to decrease, which pushes the price up, but each firm can only benefit from the part of this effect that is due to its rival’s output reduction ($\partial x^r_i/\partial p$ in equation (10')). The reason is that decreasing the own output has a positive effect (increasing the price and decreasing the cost) and a negative effect (decreasing the number of sold units) and in equilibrium both effects cancel out each other because of the first order maximum profit conditions. The reduction in the rival’s production, on the contrary, causes output price to rise without having any negative side-effect for firm $i$.

From (10’) we immediately obtain $SR^i \geq SR^r_2 \Leftrightarrow e_{x^i_{p, r}} \geq e_{x^r_{p, i}}$, where $e_{A,B}$ denotes the elasticity of $A$ with respect to $B$, i.e., a firm can enjoy more output scarcity rent than its rival if its rival’s output is more sensitive to the permit price than its own output.
A consequence of this result is that a monopoly would never obtain positive output scarcity rents because of an increase in the permit price. The only channel by which a monopoly could benefit from a higher value of $p$ is the grandfathering scarcity rent and equation (10) implies that the monopoly profit could increase because of such an effect only if $S_i > e_i^*$, i.e., if it is a net seller of permits.

The last term in (10) is the equilibrium amount of emissions, i.e., the required amount of permits for firm $i$, which determines the part of the scarcity rents that firm $i$ is not able to capture. Who gets that part of the rents depends on how the firm obtains the permits. If they are auctioned, it is the auctioneer (typically the environmental authority) who gets the rents. If they are bought in the secondary market, the rents are transferred to the seller.

Altogether, the second and third summands in (10) represent the net purchase (if $S_i < e_i^*$) or sell ($S_i > e_i^*$) of permits by firm $i$. If $S_i > e_i^*$, firm $i$ initially receives more permits than needed in equilibrium and then it would get an extra profit by selling some permits in the market, which can be most naturally interpreted as an scarcity rent since the firm is getting some revenue by selling a scarce asset. If $S_i < e_i^*$ the firm is a net buyer of permits and the combination of the second term and third terms in (10) determines the cost increase due to a higher permit price. Even in this case we continue to interpret $S_i$ as an scarcity rent in the sense that it allows the firm to finance for free part of the external cost caused by its own emissions. In the limiting case, $S_i = e_i^*$, all the equilibrium emissions are exactly covered with free permits. Then, an increase in the price of permits would not have any effect on the firm’s cost and the final effect is simply the output scarcity rent.
If $S_i \geq e^*_i$, we know for sure that firm $e_i - S_i$ would benefit from an increase in $p$, i.e., the scarcity rents would be large enough to overcompensate the additional cost. On the other hand, if $S_i < e^*_i$, in general we cannot tell which effect prevails at this level of generality. Anyway, it's interesting to note that, since the final effect is strictly positive when $e_i^* = S_i$, by continuity we can assert that there is some interval of $S_i$ such that firm $i$'s profit increases with $p$ even if the firm is a net buyer of permits. Recall that this event is discarded in monopoly as the output scarcity rent is absent.

2.2 Stackelberg model

We now consider a setting in which there are a leader (firm 1) and a follower (firm 2) in the output market. The interest of this variation is to find out how different positions in the market, as a leader or a follower, determine the ability of a firm to capture scarcity rents. The FOC of the follower’s problem is

$$P + \frac{dP}{dX} x_2 - \frac{\partial TC^*_2}{\partial x_2} = 0,$$

which, solving for $x_2$, gives the reaction function $x_2^*(x_1, p)$. Differentiating (11) and operating, we conclude that the optimal follower’s output is decreasing in the leader’s output and the permit price:

$$\frac{\partial x_2^*}{\partial x_1} = -\frac{\frac{dP}{dX}}{2\frac{dP}{dX} - \frac{\partial^2 TC^*_2}{\partial x_2^2}} < 0,$$

$$\frac{\partial x_2^*}{\partial p} = \frac{\frac{dP}{dX} - \frac{\partial^2 TC^*_2}{\partial x_2^2}}{2} < 0.$$

(12)

The leader’s FOC is

$$P(x_1 + x_2) + \frac{dP}{dX} x_1 \left(1 + \frac{\partial x_2^*}{\partial x_1}\right) - \frac{\partial TC_1}{\partial x_1} = 0,$$

(13)
from which we obtain the leader’s optimal output as a function of the permit price, \( x_i^*(p) \). By differentiating (13), we conclude that the leader’s output supply is also decreasing in \( p \):

\[
\frac{dx_i^*}{dp} = \frac{\partial^2 TC_i^*}{\partial x_i \partial p} < 0. \tag{14}
\]

Equations (12) and (14) show how the leader and the follower react to a permit price increase. While the follower only takes into account the effect of its own output variation on the output price, the leader incorporates, not only its own, but also the follower’s. This tends to make the denominator of (14) smaller in absolute value and, hence, the whole expression greater in absolute value, i.e., the leader’s output tend to be more sensitive to the permit price than the follower’s.

Using the equilibrium output values we can express the profit of both firms solely as a function of the permit price. By direct differentiation with respect to \( p \), and dropping the terms that cancel out due to the FOCs, we obtain

\[
\frac{d\Pi_i^*}{dp} = \frac{dP}{dX} \frac{\partial x_i^*}{\partial p} \frac{\partial x_i^*}{\partial x_i} + \frac{S_i}{\text{SR}_i} - e_i^*, \tag{15}
\]

\[
\frac{d\Pi_f^*}{dp} = \frac{dP}{dX} \frac{\partial x_f^*}{\partial p} \frac{\partial x_f^*}{\partial x_i} + \frac{S_f}{\text{SR}_f} - e_f^*. \tag{16}
\]

where we find again the same qualitative effects as in the Cournot model: two components of the scarcity rents, \( S_{R_i} \) and \( S_{R_f} \), together with the marginal cost effect due to the higher cost of permit purchase and the resulting sign is undetermined. Regarding the output scarcity rent, note that the effect of a price increase on the follower’s output has two components: a direct one and an indirect one through the
leader’s output. Formally, \\
$$\frac{dx_i^*}{dp} = \frac{\partial x_i^R}{\partial p} + \frac{\partial x_i^k}{\partial x_i} \frac{dx_i^*}{dp}.$$ \\
Nevertheless, the latter effect is already accounted for in the leader’s optimising process and hence only the former matters to determine the leader’s scarcity rent.

Direct comparison of (15) and (16) shows that \\
$$SR_i^* \geq SR_2^* \iff \varepsilon_{x_i^*},p \geq \varepsilon_{x_i^*},p,$$ \\
i.e., the relative size of the output scarcity rent depends again on the relative elasticities, with the difference that what matters for the leader is the elasticity of its equilibrium output whereas for the follower it is the elasticity of its reaction function. The rest of the discussion presented in the Cournot case largely applies to the Stackelberg model.

The main conclusion that we can draw from (15) and (16) is that, unlike the Cournot case, the conditions under which a price increase is profit-enhancing are different for the leader and the follower and this is due, not only to the fact that they may have different cost structures, but also to the fact that their reactions to a price increase, given in (12) and (14), are different and, therefore, it may be the case that an increase in the permit price causes the profit of one firm to increase and the profit of the other firm to decrease. To gain more accurate insights, we explore a specific case in the next section.

3. A Separable Function

3.1. Basic elements

Assume that production and abatement costs are separable in the following way. The production cost of firm $i$ is given by $cx_i$, so there is a constant marginal production cost equal to $c$. The inverse demand function for output is linear: $P(X) = a - bX$. Each unit of output generates $r$ units of pollution, where $r > 0$ is a constant coefficient of
pollution intensity (the gross emissions of firm \( i \) are hence given by \( rx_i \)). By undertaking abatement activities, firms can reduce their flow of pollution. Let us denote as \( q_i \geq 0 \) the amount of emissions abated by firm \( i \). Thus, net emissions are given by \( e_i = rx_i - q_i \). Following Sartzetakis (1997), we assume the following quadratic abatement cost function \( (AC) \), which is common to both firms:

\[
AC(q_i) = q_i (d + tq_i),
\]

where \( d \) and \( t \) are positive parameters. Assume that both firms initially receive an equal allocation of free permits, \( S_1 = S_2 = S \), and denote as \( y_i \) the amount of permits that firm \( i \) buys (if \( y_i > 0 \)) or sells (if \( y_i < 0 \)) in the market, which can be calculated as the difference between net emissions and the initial allocation:

\[
y_i = e_i - S = rx_i - q_i - S.
\]

In other words, \( e_i = y_i + S \); i.e., the net emissions of a firm must be covered by permits that either come from its free allocation or are bought at the market. Firm \( i \)'s total cost function is now given by

\[
TC_i(x_i, y_i) = cx_i + (rx_i - y_i - S)(d + t(rx_i - y_i - S)) + py_i,
\]

which, following the notation used above, can be written in terms of output and net emissions as

\[
TC_i(x_i, e_i) = cx_i + (rx_i - e_i)(d + t(rx_i - e_i)) + p(e_i - S).
\]

Solving the third stage of the game, we can compute the optimal amount of emissions of firm \( i \) as a function of output:

\[
e^*_i(x_i, p) = rx_i - \frac{p - d}{2t}, \quad i = 1, 2,
\]

and hence firm \( i \)'s optimal abatement is
which, due to separability, is independent of output and, due to cost symmetry, is common for both firms. Using (21) in (18) and (20) we obtain the optimal traded permits and the corresponding minimised cost function:

\[ y_i^* (x_i, p) = \frac{d - p}{2t} + r x_i - S , \]

\[ TC_i^* (x_i, p) = x_i (c + pr) - \frac{(p - d)^2}{4t} - p S , \]

and the latter expression reveals that the marginal production cost is constant in output and increasing in the permit price. The market value of the grandfathered permits plays the role of a lump-sum cost reduction. Now we move on to the output stage considering separately the Cournot and the Stackelberg cases.

3.2 Cournot

First, we consider the Cournot model. To ensure interior solution (with output, emissions and abatement being non-negative), we introduce the following assumption.

**Assumption 1.** The price of permits is bounded in the following way: \( d \leq p \leq \overline{p}_c \) where

\[ \overline{p}_c := \frac{2tr(a-c)+3bd}{2tr^2+3b}. \]

The lower bound for \( p \) introduced in Assumption 1 prevents abatement from being negative (see (22)). Note that \( d \) is the marginal cost of abatement at \( q = 0 \). If the price of permits is even lower than the cost of the first unit of abatement, it will never be profitable to abate, since buying permits is always a cheaper option. The upper bound for \( p \) prevents emissions from being negative, which implies that output is also positive.\(^5\)

\(^5\) If net emissions and abatement are nonnegative, gross emissions must also be nonnegative, i.e., \( rx_i \geq 0 \), which implies \( x_i \geq 0 \).
From the FOCs of the firms we obtain their reaction functions and the equilibrium output level, which is common for both firms:

$$x_i^* = x_{-i}^* = \frac{a - c - rp}{3b}. \quad (25)$$

As in the classical Cournot model with linear demand and constant marginal cost, the output of both firms depend positively on the demand intercept, $a$, and negatively on the demand slope, $b$, and all the cost parameters $c$, $r$ and $p$. The equilibrium profits are also constant across firms and can be computed as $\Pi_i^* = (a - 2bx_i^*)x_i^* - TC_i^*(x_i^*, p)$.

To determine the effect of a permit price increase, we differentiate to get

$$\frac{\partial \Pi_i^*(p)}{\partial p} = \frac{r x_i^*}{S_{iG}} + \frac{S_{iG}}{S_{iG}} - \left(\frac{rx_i^* - p - d}{\frac{2t}{q_i}}\right), \quad (26)$$

which is the particular version of (10) in this example. The first term in (26) is the output scarcity rent, the second is the grandfathering scarcity rent and the third (the whole parenthesis) is the cost effect (determined by net emissions). We can get some useful intuitions by simple inspection of expression (26).

First, by assigning a large enough amount of free permits, $S$, it's always possible to make a firm willing that the permit price increases thanks to the grandfathering scarcity rent. In simple words, any firm would benefit if it could hold a large amount of an asset (permits) that is becoming scarcer (and thus more expensive) in the market.

Assume now that the grandfathering effect is eliminated by setting $S = 0$ in (26) (consider, as a particularly relevant example, that the permits are sold in an auction instead of grandfathering). There are still two positive effects, the combination of which may cause the firm to benefit from an increase in the permit price. The first is the output scarcity rent and the second is due to the adaptation ability of the firm by means of
abatement (the second term in the parenthesis). If there was not an available abatement
technology or it was prohibitively expensive,\textsuperscript{6} then the right-hand side of (26) would
collapse to \(-(2r/3)x_i\), whose sign is unambiguously negative. The economic
consequence of this result is that the output scarcity rent by itself would never be enough
to fully compensate the cost effect of a higher permit price.

To get some additional insight, we can use the equilibrium values of output and
emissions to write the profit function in terms of the permit price:

\[
\Pi^i(p) = \frac{(a-c-pr)^2}{9b} + \frac{(p-d)^2}{4t} + pS.
\]  

(27)

The main features of this function and their economic consequences are
summarised in Proposition 1:

**Proposition 1.** Under assumption 1, \( \Pi^i(p) \) is a strictly convex function of \( p \) with a
global minimum at \( \hat{p}^c \), with \( d < \hat{p}^c < \bar{p}^c \). Therefore, \( \Pi^i(p) \) is decreasing for \( p < \hat{p}^c \)
and increasing for \( p > \hat{p}^c \) \hfill \blacksquare

According to Proposition 1, the profit function of both firms is U-shaped and,
therefore, there is a positive critical value of the permit price, \( \hat{p}^c \),\textsuperscript{7} below which the
negative effect dominates, i.e., a marginal increase of the permit price will reduce the
firms’ profit, whereas above it further increments of the price will generate more than
enough scarcity rents to offset the negative effect and make the firms better off.

To understand the shape of this functions it’s instructive to investigate the
behaviour of each component of (26) with respect to \( p \). According to (25), output is
decreasing in \( p \) and thus the size of the output scarcity rent is decreasing in \( p \) while the

\textsuperscript{6} This can be seen by making \( t \) arbitrarily large in (26).

\textsuperscript{7} Specifically, \( \hat{p}^c = \frac{4tr(a-c) + 9b(d-2S)}{4tr^2 + 9b} \).
grandfathering scarcity rent is constant in $p$. Combining (21) and (25) it can be shown that the cost effect (determined by $e_i^*$) is a decreasing function of $p$. This is due to the ability of the firm to adapt by a reducing output (and emissions) and increasing abatement. It turns out that the latter effect dominates the former, which gives the profit function a convex shape as stated in Proposition 1.

### 3.3 Stackelberg

Assume that firms 1 and 2 are a leader and a follower respectively. In this case, we need to impose Assumption 2 to ensure interior solution.

**Assumption 2.** The price of permits is bounded in the following way: $d \leq p \leq \bar{p}^s$ where

$$\bar{p}^s := \frac{2bd + rt(a - c)}{2b + tr^2}. \quad (28)$$

As above, this assumption prevents abatement, emissions (and hence output) from being negative.\(^8\) By standard methods,\(^9\) we obtain the equilibrium outputs:

$$x_1^* = \frac{a - c - rp}{2b}, \quad (29)$$

$$x_2^* = \frac{a - c - rp}{4b}. \quad (30)$$

From (29) and (30), we conclude that the leader’s output is twice that of the follower. Using these expressions we obtain the equilibrium profits as a function of $p$ and, by differentiation, the effect of the permit price on equilibrium profits can be measured as

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8. In this case the upper bound is set at the point at which the emissions of the follower become zero, which implies that the rest of relevant variables are non-negative.

9. The follower chooses $x_2$ to maximise its profit while taking $x_1$ as given. The leader chooses $x_1$ to maximise its own profit taking into account the follower’s reaction function.
\[
\frac{\partial \Pi_i}{\partial p} = \frac{r}{2} x_i^* + \frac{S}{sk_i} - \left( \frac{rx_i^* - p - d}{2t} \right),
\]  

(31)

which looks very similar to (26) and qualitatively the same effects are present. The grandfathering scarcity rent and the abatement effect are identical as in the Cournot model. Once again, the output scarcity rent by itself cannot compensate for the cost effect due to a higher permit price. Also, the positive abatement effect is increasing in \( p \), and so the higher the permit price the more the firms can benefit by doing abatement to adapt themselves to the market conditions.

As a difference from the Cournot case, the output scarcity rent that accrue to each firm represents a larger proportion of its own output \((r/2\) instead of \(r/3\)). Another difference is due to the fact that the leader produces twice as much as the follower, and thus, it enjoys more output scarcity rents, but its gross emissions are also bigger than the follower's. Simple manipulation of (31), together with (29) (30), gives

\[
\frac{\partial \Pi_1^r}{\partial p} - \frac{\partial \Pi_2^r}{\partial p} = -\frac{r}{2} (x_1^* - x_2^*) = -\frac{r(a - c - rp)}{8b} < 0,
\]

(32)

where the inequality always holds under interior solution. Therefore, a rise in the permit price will always benefit more (or harm less) the follower than the leader. The reason lies in the output difference: since the leader produces more than the follower, it also pollutes more and its cost is more sensitive to the permit price. The impact of \( p \) on both firms’ profit is summarised in the following proposition:

**Proposition 2.** \( \Pi_1^r(p) \) and \( \Pi_2^r(p) \) are strictly convex functions of \( p \) with a global minimum at \( \hat{p}_1^* \) and \( \hat{p}_2^* \) respectively, with \( d < \hat{p}_2^* < \hat{p}_1^* \). 

According to Proposition 2, the minima of the profit functions are ordered such that \( \hat{p}_2 < \hat{p}_1 \); i.e., the follower reaches a minimum for a lower price than the leader.
Hence, we have that, if \( p < \hat{p}_2 \), both firms are situated in the decreasing part of their profit functions (and so would prefer that the price decreases). If, instead, \( \hat{p}_2 < p < \hat{p}_1 \), the follower is situated in the increasing part (and so will benefit from a price increase), whereas the leader is still in the decreasing part (and therefore will still prefer the price to decrease).

So, from the point of view of our research, one important novelty of the Stackelberg model with respect to Cournot is the fact that both firms can have different interests regarding the evolution of \( p \). As we discussed in the next section, this has important implications for the chances of collusive behaviour.

4. Scarcity rents and price manipulation

In this section we explore the question of how the existence of scarcity rents can generate incentives for the firms to lobby or collude in order to manipulate the price of permits upwards. As it was discussed in the introduction, some authors have claimed that this is a plausible possibility and there is some empirical evidence to suggest that it has happened in the past. As in Ehrhart et al. (2008), we do not explicitly model the permit market. Consequently, we restrict ourselves to testing the existence of incentives for collusion to manipulate the price of permits rather than modelling price manipulation itself or determining if such manipulation has taken place in practice.

In our framework, it is natural to consider that both firms have incentives to collude in order to manipulate the price upwards when the profit of both is increasing in \( p \) simultaneously. To get sharper results about this question, we focus on the separable case introduced in Section 3, although the qualitative insights we obtain can be extended to a more general setting. We have shown that, both in the Cournot and the Stackelberg cases, for each specific firm there is a threshold below which the firm prefers the permit
price to decrease and above which it prefers an increase. The comparison of these thresholds is crucial to determine the generation of incentives for collusive agreements.

4.1 Cournot vs. Stackelberg

In the Cournot model, according to Proposition 1, the threshold value is \( \hat{p}^C \), which splits the range of possible values for \( p \) into two non-empty regions, that we call C-I:=\([d, \hat{p}^C]\) and C-II:=\((\hat{p}^C, \overline{p}^C)\). As a function of \( p \), the profit of both firms is decreasing in the first region and increasing in the second. Therefore, the incentives for firms to collude in order to push the price up exist in region C-II and not in C-I.

Things are a somewhat different in the Stackelberg case. From Proposition 2, we know \( \hat{p}_2^S < \hat{p}_1^S \), which means that the follower reaches the threshold value sooner (i.e., for a lower value of \( p \)) than the leader. As it is shown in Figure 1, this can give rise to three different regions. In region S-I:=\([d, \hat{p}_2^S]\), the profit of both firms is decreasing in \( p \). In region S-II:=\((\hat{p}_2^S, \hat{p}_1^S)\), the profit of the leader is still decreasing while the follower’s is increasing. Finally, in region S-III:=\((\hat{p}_1^S, \overline{p}_1^S]\) the profit of both firms is increasing in \( p \). It is only in region S-III that both firms have incentives to collude in order to push the price upwards. Proposition 3 compares the relevant thresholds for both models.

Proposition 3. The critical values of the permit price in the Cournot and the Stackelberg model are ordered in the following way: \( \hat{p}_2^S < \hat{p}^C < \hat{p}_1^S \leq \overline{p}^F < \overline{p}^C \)

According to Proposition 3, \( \hat{p}_2^S < \hat{p}^C < \hat{p}_1^S \), which means that a Cournot firm reaches the threshold value later than a Stackelberg follower but sooner than a
Stackelberg leader. Moreover, the last three inequalities in Proposition 3 imply that region S-III is strictly contained in region C-II, which means that collusion for price inflation is more likely under Cournot than under Stackelberg competition, in the sense that there is a wider range compatible with such an event. So, one key conclusion of our study is that the existence of a leader-follower relationship between the firms reduces the chances of collusive behaviour with respect to a symmetric setting such as the Cournot framework.

![Equilibrium profits as a function of p in the Stackelberg model](image)

**FIGURE 1:** Equilibrium profits as a function of $p$ in the Stackelberg model

The focus of this paper is on region S-III, as this is the only one within which firms can find it profitable to collude in order to push the price up. One natural question is how large this region is, or, in other words, how likely it is to fall within this region. Region S-III is delimited by two threshold values for $p$. First, $\hat{p}_1^S$, which is the price above which it is profitable, not only for the follower, but also for the leader to push the price up. The second threshold is the upper bound, $\bar{p}^S$, which is the highest value of the price compatible with an interior solution. The size of region III is thus given by the difference between these two thresholds, which can be computed as

$$\bar{p}^S - \hat{p}_1^S = \frac{4btS}{2b + tr^2},$$

(33)
and so the size of region S-III depends positively on the number of free permits received by the firms, as well as the slope of the demand curve, \( b \), and the abatement cost parameter \( t \), whereas it depends negatively on the emissions intensity parameter, \( r \). In the next subsection we focus on the role of grandfathering.

### 4.2 The role of grandfathering

In this section we ask about the effect of modifying the initial allocation of permits on the generation of incentives for collusion. For convenience, define the following threshold:

\[
\tilde{S}^C := \frac{2r(a - c - dr)}{9b}.
\]

**Proposition 4.** In the Cournot model, under Assumption 1, the following results hold:

a) As \( S \) increases, \( \tilde{p}^C \) decreases, which implies that the size of region C-I decreases and that of region C-II increases.

b) For any value \( S \geq \tilde{S}^C \), region C-I disappears.

c) The size of region C-II is strictly positive for \( S = 0 \)

Proposition 4 shows that increasing the number of free permits, \( S \), shifts the lower bound of region C-II to the left while the upper bound price remains the same. Therefore, region C-I shrinks and region C-II gets bigger, which implies that the chances for collusive behaviour increase. Moreover, Proposition 4 shows that region C-I disappears if \( S \) is large enough. The main consequence is that the more free permits the firms own, the more likely they are to be willing that the permit price increases. The reason is that the existence of free permits makes permit purchasing less costly. Moreover, if \( S \) is large enough it opens the way for obtaining positive revenues by selling some permits. Nevertheless, result c) in the proposition implies that, even without
grandfathering, there is a positive range such that both firms prefer the permit price to increase and thus they have incentives to lobby it up.

We now perform a similar exercise for the Stackelberg case. Define the following threshold value for $S$:

$$\bar{S} := \frac{r(a-c-dr)}{8b}. \quad (34)$$

**Lemma 1.** In the Stackelberg model, under Assumption 2, the following results hold:

a) If $0 < S < \bar{S}$, then $d < \hat{p}_2 < \hat{p}_1 < \bar{p}_s$.

b) If $\bar{S} < S < 2\bar{S}$, then $\hat{p}_2 < d < \hat{p}_1 < \bar{p}_s$.

c) If $S > 2\bar{S}$, then $\hat{p}_2 < \hat{p}_1 < d < \bar{p}_s$.

d) If $S = 0$, then $d < \hat{p}_2 < \hat{p}_1 = \bar{p}_s$. \hfill \blacksquare

**Proposition 5.** In the Stackelberg model, under Assumption 2, the following results hold:

a) If $0 < S < \bar{S}$, regions S-I, S-II and S-III are non-empty.

b) If $\bar{S} < S < 2\bar{S}$, region I disappears and region II is delimited by $d < \bar{p} < \hat{p}_s$.

c) If $S > 2\bar{S}$, regions I and II disappear and region III is defined by the entire feasible range, $[d, \bar{p}_s]$.

d) If $S = 0$, region S-III disappears \hfill \blacksquare

The consequences of Lemma 1 and Proposition 5 are the following. The threshold values $\hat{p}_1^s$ and $\hat{p}_2^s$ decrease with $S$, which implies that, for each firm, there is a wider range such that its profit is increasing in $p$. This renders a similar conclusion as in the Cournot model: the changes of observing collusive behaviour increase with grandfathering. If the initial allocation is large enough, region I disappears, which implies that the follower is always interested in increasing $p$ and, if it is even larger, both
regions I and II disappear, which implies that both the leader and the follower are always interested in manipulating the price upward. This is the most favourable case for collusion.

But there is a qualitative difference with respect to the Cournot model: if no permits are allocated for free, the chances of collusive behaviour shrink to the extent that they totally disappear because a price increase is never profitable for the leader although it can be for the follower. This result has an important policy implication. If the output market is characterised by Stackelberg competition and there is no grandfathering, the firms have no incentives to collude in order to manipulate and the inclusion of grandfathering opens up the possibility of collusion.

### 4.3. Parameter asymmetries in the Stackelberg model

In the previous subsections we have assumed both firms to be fully symmetric in terms of their cost functions and the initial allocation they receive. In the Cournot case, moreover, they are also symmetric regarding their role in the market. In the Stackelberg model, although they are symmetric in terms of the parameters there is an asymmetry in their role as one acts as a leader and the other as a follower. One central conclusion is that this asymmetry makes collusive agreements less likely as their interests are decoupled.

As a sensitivity analysis, in this subsection we consider the possibility that firms are asymmetric in terms of cost and/or initial permit endowment. In the Cournot case, since we start from a fully symmetric situation, it is rather straightforward to conclude that, in general, introducing any asymmetry between the firms will make their interests to diverge and the chances for collusive agreements will decrease. So, we focus on the Stackelberg case, in which the results are less obvious as we start from a situation that is already asymmetric in nature.
We denote the production cost of firm $i$ as $c_i x_i$, where $c_i$ is a firm-specific unit cost parameter. Analogously, firm $i$’s abatement cost function is given by:

$$AC_i(q_i) = q_i \left(d_i + t_i q_i\right), \quad i = 1, 2.$$  \hspace{1cm} (35)

Each firm receives an initial free endowment of permits, $S_i$, which is not necessarily constant across firms. Proceeding as in the basic case, we obtain the optimal amounts of emissions, abatement and purchase of permits for each firm in the emissions stage:\textsuperscript{10}

\begin{align*}
e^*_i(x_i, p) &= \frac{d_i - p}{2t_i} + r x_i, \quad (36) \\
n^*_i(p) &= \frac{p - d_i}{2t_i}, \quad (37) \\
y^*_i(x_i, p) &= r x_i - \frac{p - d_i}{2t} - S_i, \quad (38)
\end{align*}

and, moving on to the output stage, we can compute the equilibrium levels of output:

\begin{align*}
x^*_1 &= \frac{a + c_2 - 2c_1 - rp}{2b}, \quad (39) \\
x^*_2 &= \frac{a + 2c_1 - 3c_2 - rp}{4b}. \quad (40)
\end{align*}

To investigate the likelihood of observing collusive behaviour, we proceed by analysing the effect of different parameters on the size of region S-III. Due to the larger number of parameters, by choosing the right combination of them we could generate almost any imaginable case. Hence, we need to bound the range of possibilities so as to avoid meaningless results. For this reason, we introduce the following assumptions:

\textsuperscript{10} Unlike the other parameters, we assume that the emissions intensity parameter, $r$, is common to both firms; i.e., $r_i = r$. There are two reasons for this simplification. First, the sensitivity analysis results related to these parameters are unclear and so we do not gain any valuable insight by exploring them. Second, the sign of some equilibrium values for some of the key variables are affected by the terms $2r_i - r$ and/or $3r_i - 2r_i$ and this fact forces us to keep the asymmetry between these parameters bounded so as to avoid meaningless results.
Assumption 3: \( d_1, d_2 < p < \bar{p}^{S-2} \), where \( \bar{p}^{S-2} \) is the value of \( p \) such that \( \hat{e}_2^* = 0 \).

Assumption 4: \( e_1^* > e_2^* \).

Assumption 5: \( \hat{p}_2^S < \hat{p}_1^S \).

The two first assumptions ensure nonnegative values for all the relevant variables. The idea is that the leader will still be the one who produces a larger amount of output and emissions. Hence, the follower will still be the one who finds it profitable to pollute zero for a lower value of \( p \) and such a value determines the upper bound for the interval that is compatible with an interior solution, \( \bar{p}^{S-2} \) (where “\( S-2 \)” stands for “Stackelberg, case 2”). If this is the case, it is natural to accept that Assumption 5 also holds; i.e., it is easier for the follower than it is for the leader to benefit from a price increase.

Under these assumptions, region S-III is still delimited by the leader’s critical price, call it \( \hat{p}_1^{S-2} \), and the upper bound \( \bar{p}^{S-2} \) and hence its size increases if \( \bar{p}^{S-2} \) increases and/or \( \hat{p}_1^{S-2} \) decreases. Proposition 3 summarises how the size of this region depends on the parameters of the model and Table 1 presents a taxonomy of all the relevant effects.

**Proposition 6.** In the Stackelberg case, under assumptions 1, 2 and 3, the size of region S-III increases in the following cases:

a) If the leader’s marginal production cost, \( c_1 \), increases or the follower’s marginal production cost, \( c_2 \), decreases.

b) If the parameter of the linear term in the abatement cost function decreases for the leader (\( d_1 \)) or increases for the follower (\( d_2 \)).
c) If the parameter of the quadratic term in the leader’s abatement cost function, \( t_1 \), decreases (provided the number of free permits is moderate) or the equivalent follower’s parameter, \( t_2 \), increases.

d) If the number of free permits received by the leader, \( S_1 \), is increasing regardless of the free permits received by the follower.

<table>
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<th>Effects on thresholds</th>
<th>( \Delta \hat{p}^{S-2} )</th>
<th>( \Delta \hat{p}_1^{S-2} )</th>
<th>( \Delta \left( \hat{p}^{S-2} - \hat{p}_1^{S-2} \right) )</th>
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<td>( c_1 )</td>
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<td>( c_2 )</td>
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<td>( S_2 )</td>
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Table 1. Summary of sensitivity analysis results.

(*) For a moderate value of \( S_1 \).

Increasing the leader’s production cost or reducing the follower’s cost tends to erode the leader’s advantage with respect to the follower in the output market, which has the effect of making the firms more symmetric in a certain sense. The more symmetric the firms are, the more aligned their interests will be and hence it is more likely for them to find it profitable to collude. Increasing \( c_1 \) has a twofold effect. On the one hand, \( \hat{p}^s \) grows because the output of the follower increases, which makes it less likely for firm 2 to decide not to emit at all (in other words, the range of prices under which there is an interior solution widens). On the other hand, \( \hat{p}_1^s \) decreases, as, due to the higher cost, firm 1 tends to produce less and to emit less and hence its total cost will be less sensitive to an increase in the price of permits. Both of these effects tend to enlarge the collusive
region. Just the opposite occurs when $c_2$ increases. Firm 1 tends to produce more and pollute more and hence its cost becomes more sensitive to an increase in the price of permits (which increases the value of $\hat{p}_1^s$), whereas the follower tends to produce less and to reach the point where it finds it profitable to stop polluting sooner ($\bar{p}^s$ decreases), which reduces the size of the collusion region.

The abatement cost parameters ($d_i$ and $t_i$) are only relevant for the own firm, but not for its rival. Both $d_2$ and $t_2$ are irrelevant in determining the value of $\hat{p}_1^s$. However, increasing either of them enlarges the relevant feasible range because the follower’s abatement cost increase, which makes it less likely to reach the point where it pollutes zero. The corresponding parameters for firm 1 are immaterial in determining the value of $\bar{p}^s$, their only relevant effect being on $\hat{p}_1$. Assuming a moderate value of the leader’s initial endowment of permits, any increase in $d_1$ and $t_1$ makes the leader’s abatement cost higher, which makes firm 1 become more sensitive to increases in the price of permits.

Finally, the initial allocation of permits is irrelevant for the upper bound $\bar{p}^s$, as it represents simply a fixed term in the cost (and the profit) function and so the optimal decisions are not affected. The value of a firm’s profits is affected by its own endowment (not the rival’s) and hence only $S_1$ is relevant in determining the size of region S-III. When the leader’s free endowment increases, its cost becomes less sensitive to an increase in the price of permits and it will hence be more receptive to the idea of pushing the price up, thereby increasing the likelihood of observing collusive behaviour.
5. Conclusions and policy implications

We have studied the creation of scarcity rents for oligopolistic firms due to the requirement to buy emission permits in a cap-and-trade system. Along with a higher purchase cost, a permit price increase can generate scarcity rents in two ways: firstly, by restricting output and pushing up its price, and secondly, when the firms are endowed with an initial permit allocation, a higher permit price makes such an allocation more valuable. We claim that these scarcity rents can generate incentives for the firms to collude in order to inflate the permit price.

We have demonstrated that the extent to which so-called output scarcity rents can be obtained by a specific firm increases with the elasticity of its rival's output to the permit price as the effect of own output variation cannot be profitable in equilibrium.

A leadership position tends, ceteris paribus, to make a firm produce more, so its cost becomes more sensitive to a permit price increase. As a consequence, a leader is less prone to be willing to face a higher price and thus to accept taking part in a price-inflation collusive agreement. In short, a leader-follower relationship in the output market reduces the scope for collusion to manipulate the price of permits upward. In a specific example we have shown that the region within which there are incentives to collude can shrink to such an extent that it can disappear. A policy implication of this finding is that, when there is a leader in the product market, this can result in a lack of incentives for collusion in the permit market.

Another policy implication is that distributing some permits for free by means of grandfathering allows firms to obtain larger scarcity rents and strengthens the incentives to collude in order to manipulate permit prices up. In the case of Stackelberg competition this effect may be particularly important as the existence of grandfathering is actually a necessary condition for any profitable collusive agreement to take place. The greater the
number of permits distributed by a non-market scheme (particularly to firms that enjoy market power) the more incentives there are for collusion. The European Union is reducing the use of grandfathering and increasing the use of auctioning to distribute emission permits. The 2008 revised European Emission Trading Directive established the mandate that auctioning is to be the default method for allocating allowances as a fundamental change for the third trading period, starting in 2013. The arguments put forward by the European Commission (EC) to support the introduction of auctions are that auctioning “best ensures the efficiency, transparency and simplicity of the system, creates the greatest incentives for investment in a low-carbon economy and eliminates windfall profits”.

Our results provide an additional argument to reduce the use of grandfathering (and arguably to increase the use of auctioning), as it might introduce incentives for price manipulation.

Our final insight is that the likelihood of firms finding collusion profitable is very sensitive to the cost asymmetries between them. In general terms, the more asymmetric the firms are, the more difficult collusion becomes. Moreover, if a grandfathering scheme exists, the more permits are allocated to firms enjoying market power, the more likely it is that collusion takes place.

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References


APPENDIX

Proof of Proposition 1

The equilibrium value for emissions can be obtained from equations (21) and (25). Equating this value to zero we conclude that emissions are positive under Assumption 1:

\[ e^*_i = \frac{2tr(a-c-rp) - 3b(p-d)}{6bt} \geq 0 \iff p \leq \hat{p}^c := \frac{2tr(a-c) + 3bd}{3b + 2tr^2} \]  \hspace{1cm} (A.1)

Using equations (24) and (25) together with the inverse demand function yields the following expression for the equilibrium profit:

\[ \Pi^*_i(p) = \frac{4t(a-c-rp)^2 + 9b(p-d)^2 + 36btps}{36bt}, \]

which is strictly convex in \( p \) because the second derivative is positive. From the first derivative we get the critical price

\[ \frac{\partial \Pi^*_i}{\partial p} \geq 0 \iff p \geq \hat{p}^c := \frac{4tr(a-c) + 9bd - 18btps}{9b + 4r^2t}. \]  \hspace{1cm} (A.2)

By direct comparison, we conclude

\[ \hat{p}^c - \hat{p}^c = \frac{6brt(a-c-dr) + 18btps(3b + 2r^2t)}{(3b + 2r^2t)(9b + 4r^2t)} > 0, \]  \hspace{1cm} (A.3)

which is positive under Assumption 1 because all the parenthesis are positive. To prove that \( a-c-dr \) is positive, using (22) and the definition of abatement \( q_i = rx_i - e_i \), we conclude that, within the relevant range, \( x_i^* > e^*_i / r > 0 \). Using the expression for \( x_i^* \) given in (25), we conclude that \( x_i > 0 \) implies \( a-c > rp \), and that this inequality, together with \( d < p \) (Assumption 1), implies \( a-c > dr \).

QED.
Proof of Proposition 2

Using (29) and (30) in (21), we obtain the equilibrium values for emissions:

\[ e^*_1 = \frac{db + rt(a-c) - p(b+tr^2)}{2bt}, \]  
\[ (A4) \]

\[ e^*_2 = \frac{2bd + tr(a-c) - p(2b+tr^2)}{4bt}, \]  
\[ (A5) \]

By direct comparison we conclude that under Assumption 2, equilibrium emissions are nonnegative,

\[ e^*_1 > e^*_2 \geq 0 \iff p \leq \overline{p}^s := \frac{2bd + rt(a-c)}{2b + tr^2}. \]  
\[ (A6) \]

and following the same reasoning as in the proof of Proposition 2, output must also be nonnegative. Using the equilibrium values of output and emissions, together with the expression of the inverse demand and the cost function (20) we obtain the equilibrium profits of both firms:

\[ \Pi_1^s(p) = P(x^*_1 + x^*_2)x^*_1 - TC_1^s(x^*_1, p) = \frac{t(a-c-pr)^2 + 2b(p-d)^2 + 8btpS}{8bt}, \]

\[ \Pi_2^s(p) = P(x^*_1 + x^*_2)x^*_2 - TC_2^s(x^*_2, p) = \frac{t(a-c-pr)^2 + 4b(p-d)^2 + 16btpS}{16bt}. \]

The second derivative reveals that these functions are strictly convex. Differentiating them with respect to \( p \), we conclude that they have respective minima at

\[ \text{Arg min}_p \Pi_1^s(p) \equiv \hat{p}_1^s = \frac{rt(a-c) + 2bd - 4bts}{2b + tr^2}, \]

\[ \text{Arg min}_p \Pi_2^s(p) \equiv \hat{p}_2^s = \frac{rt(a-c) + 4bd - 8bts}{4b + tr^2}, \]

and it follows straightforwardly that both \( \hat{p}_1^s \) and \( \hat{p}_2^s \) depend negatively on \( S \).

Regarding the order of the thresholds, by direct comparison we conclude that

\[ \hat{p}_1^s > \hat{p}_2^s \iff 2b(2rt(a-c-dr+2rSt)) > 0; \] however, we have already proved \( a - c - dr \geq 0 \)
in Proposition 1, which ensures that \( \hat{p}_i^s > \hat{p}_i^d \). Moreover, using (A6) we also conclude that \( \hat{p}_i^s = \hat{p}_i^s + \frac{4btS}{2b + tr^2} > \hat{p}_i^s \). Hence, we have that \( \hat{p}_i^s < \hat{p}_i^d < \hat{p}_i^s \). QED.

**Proof of Proposition 3**

To prove the proposition it is enough to compare the corresponding expressions, compute the difference and check the sign.

\[
\hat{p}_i^s - \hat{p}_i^c \iff \frac{brt(a - c - dr) + 2br^2tS}{(2b + r^2t)(9b + 4r^2t)} > 0,
\]

\[
\hat{p}_i^c - \hat{p}_i^d \iff \frac{7brt(a - c - dr) + 14br^2tS}{(9b + 4r^2t)(4b + r^2t)} > 0,
\]

\[
\hat{p}_i^c - \hat{p}_i^s = \frac{rbr(a - c - dr)}{(3b + 2r^2t)(2b + r^2t)} > 0.
\]

QED

**Proof of Proposition 4**

The first part of the proposition results from deriving the expression for \( \hat{p}_i^c \) given in (A.2) with respect to \( S \). To prove the second part note that Region I disappears when \( d \geq \hat{p}_i^c \), and using the expression for \( \hat{p}_i^c \):

\[
\hat{p}_i^c \leq d \iff \frac{4rt(a - c) + 9bd - 18btS}{9b + 4r^2t} \leq d \iff S \leq \frac{2r(a - c - dr)}{9b} := \tilde{S}.
\]

The third part follows immediately from (A.3). QED

**Proof of Lemma 1**

From Proposition 2 we know \( \hat{p}_i^s < \hat{p}_i^d < \hat{p}_i^s \). To determine the relative position of \( d \), first note that Assumption 2 implies \( d < \hat{p}_i^s \) and hence we only have to check...
whether \( d \) is below \( \hat{p}_2^\delta \), in the interval \( (\hat{p}_2^\delta, \hat{p}_1^\delta) \) or in the interval \( (\hat{p}_1^\delta, \bar{p}) \). By direct comparison, we conclude the following, which prove statements b) and c):

\[
\hat{p}_1^\delta > d \iff S < \frac{r(a-c-dr)}{4b} = 2\bar{S}, \quad (A7)
\]

\[
\hat{p}_2^\delta > d \iff S < \frac{r(a-c-rd)}{8b} = \bar{S}, \quad (A8)
\]

To prove statement d) note that \( \hat{p}_1 = \bar{p}^\delta \) when \( S = 0 \). QED.

**Proof of Proposition 5**

Statement a) follows from a similar reasoning to that used in the proof of Proposition 4. Statements b), c) and d) follow straightforwardly from the corresponding statements in Lemma 1. QED.

**Proof of Proposition 6**

Using (39) and (40) in (36), we obtain the equilibrium values for emissions:

\[
e_1^*(x_1^*, p) = \frac{b(d_1 - p) + t_r[a + c - 2c_1 - rp]}{2bt_1},
\]

\[
e_2^*(x_2^*, p) = \frac{2b(d_2 - p) + r_t[a + 2c_1 - 3c_2 - rp]}{4bt_2}.
\]

By imposing the non-negativity conditions on the follower’s emissions, we obtain the upper bound value for the permit price, \( \bar{p}^{c-2} \):

\[
e_2^* \geq 0 \iff p \leq \bar{p}^{c-2} := \frac{2bd_2 + rt_2(a - 3c_2 + 2c_1)}{2b + r^2t_2}.
\]

(A9)

By substitution of the relevant variables in the profit function, we obtain the expression for the leader’s profit function in terms of the model parameters:
\[\Pi'_i(p) = \frac{[a + c_z - 2c_i - rp]}{8b} + \frac{(p - d_i)^2}{4t_i} + pS_i\]

Differentiating with respect to \(p\), we obtain

\[
\frac{\partial \Pi'_i}{\partial p} = \frac{2b(p - d_i) + 4bt_iS_i - rt_i[a + c_z - 2c_i - rp]}{4bt_i}
\]

and, by equating this derivative to zero, we get the minimum value of \(p\) such that the leader finds it profitable to push the price up, \(\hat{p}_{i}^{s-2}\):

\[
\frac{\partial \Pi'_i}{\partial p} \geq 0 \iff p \geq \frac{rt_i(a + c_z - 2c_i) + 2bd_i - 4bt_iS_i}{r^2t_i + 2b} = \hat{p}_{i}^{s-2}.
\]

(A10)

By direct differentiation of the values of \(\bar{p}^{s-2}\) and \(\hat{p}_i^{s-2}\), we obtain the results in the proposition:

\[
\frac{\partial \bar{p}^{s-2}}{\partial c_1} = \frac{2rt_2}{2b + r^2t_2} > 0; \quad \frac{\partial \hat{p}_i^{s-2}}{\partial c_1} = \frac{-2rt_1}{2b + rt_1} < 0;
\]

\[
\frac{\partial \bar{p}^{s-2}}{\partial c_2} = \frac{-3rt_2}{2b + r^2t_2} < 0; \quad \frac{\partial \hat{p}_i^{s-2}}{\partial c_2} = \frac{rt_1}{2b + rt_1} > 0;
\]

\[
\frac{\partial \bar{p}^{s-2}}{\partial d_1} = 0; \quad \frac{\partial \hat{p}_i^{s-2}}{\partial d_1} = \frac{2b}{2b + rt_1} > 0;
\]

\[
\frac{\partial \bar{p}^{s-2}}{\partial d_2} = \frac{2b}{2b + r^2t_2} > 0; \quad \frac{\partial \hat{p}_i^{s-2}}{\partial d_2} = 0;
\]

\[
\frac{\partial \bar{p}^{s-2}}{\partial S_1} = \frac{\partial \bar{p}^{s-2}}{\partial S_2} = 0;
\]

\[
\frac{\partial \hat{p}_i}{\partial S_1} = \frac{-4bt_1}{2b + rt_1} < 0; \quad \frac{\partial \hat{p}_i^{s-2}}{\partial S_2} = 0;
\]
\[ \frac{\partial \hat{p}_{1}^{S-2}}{\partial t_{1}} = \frac{2br(a + c_{2} - 2c_{1} - d_{1}r) - 8b^{2}S_{1}}{[r^{2}t_{1} + 2b]} \leq 0 \iff S_{1} \leq \frac{r(a + c_{2} - 2c_{1} - d_{1}r)}{4b}; \]

\[ \frac{\partial \hat{p}^{S-2}}{\partial t_{2}} = \frac{2br[a - 3c_{2} + 2c_{1} - rd_{2}]}{(2b + r^{2}t_{2})^{2}} > 0, \]

where, in an interior solution, the numerator of the last expression must be positive for the follower’s output to be positive. \( \text{QED.} \)