The Costs of Implementing a Unilateral One-Sided Exchange Rate Target Zone

Hertrich, Markus

2015

The Costs of Implementing a Unilateral One-Sided Exchange Rate Target Zone

Markus Hertrich[a,b]

[a] *Department of Finance, University of Basel, Peter Merian-Weg 6, 4052 Basel (Switzerland). Email:* `markus.hertrich@unibas.ch`

[b] *Institute for Finance, University of Applied Sciences Northwestern Switzerland, Peter Merian-Strasse 86, 4002 Basel (Switzerland). Email:* `markus.hertrich@fhnw.ch`.

Abstract

In the aftermath of the recent financial crisis, the central banks of small open economies such as the Czech National Bank and the Swiss National Bank (SNB) both implemented a unilateral one-sided exchange rate target zone vis-à-vis the euro currency to counteract deflationary pressures. Recently, the SNB abandoned its minimum exchange rate regime of CHF 1.20 per euro, arguing that after having analyzed the costs and benefits of this non-standard exchange rate policy measure, it was no longer sustainable. This paper proposes a model that allows central banks to estimate ex-ante the costs of implementing and maintaining a unilateral one-sided target zone and to monitor these costs during the period where it is enforced. The model also offers central banks a tool to identify the right timing for the discontinuation of a minimum exchange rate regime. An empirical application to the Swiss case shows the actual size of these costs and reveals that these costs would have been substantial without the abandonment of the minimum exchange rate regime, which accords with the official statements of the SNB.

Keywords: Foreign exchange reserves, minimum exchange rate, reflected geometric Brownian motion, target zone costs, Swiss National Bank.

JEL classifications: E42, E52, E58, E63, F33, F45.

1. Introduction

In the aftermath of the recent financial crisis that erupted in 2007/08, the central banks of small open economies (e.g., measured by the open market index of the International Chamber of Commerce (ICC Research Foundation, 2013)) such as the Czech National Bank (CNB) and the Swiss National Bank (SNB) both implemented a unilateral one-sided exchange rate target zone vis-à-vis the euro currency since November 7, 2013 and from September 6, 2011 to January 15, 2015, respectively. Before this crisis, the central banks of other small open economies such as the Swedish Riksbank from 1993 to 2002 (Humpage and Ragnartz, 2006), the Croatian National Bank in 1993 (Cottarelli and Doyle, 1999) and the Central Bank of Hong Kong in 2005[1] had temporarily introduced a one-sided target zone to prevent the domestic currency from appreciating vis-à-vis a specific foreign currency beyond some announced minimum exchange rate level, herewith implementing a so-called strong-side commitment. Moreover, in 1978, the SNB already had temporarily a one-sided target zone in place, when the SNB set a minimum exchange rate vis-à-vis the Deutsche mark (DEM) to impede a strengthening of the Swiss franc (CHF) beyond the level of DEM-CHF 0.80.[2]

Recently, a strand of literature has emerged that analyzes the credibility of the Swiss target zone vis-à-vis the euro currency in the aforementioned period of interest (Hertrich and Zimmermann, 2015) and estimates the latent spot EUR-CHF exchange rate that would have prevailed without the SNB's non-standard exchange rate policy measure (Hanke et al. (2014), Hanke et al. (2015) and Jermann (2015)). Other studies explore whether the exchange rate target model developed by Krugman (1991) provides a good explanation of the Swiss currency in the period where the EUR-CHF 1.20 exchange rate floor was enforced (Studer-Suter and Janssen, 2014). This paper contributes to this strand of literature and proposes a model that allows central banks to estimate ex-ante the costs of implementing and maintaining a unilateral one-sided target zone and to monitor these costs during the period where it is enforced. The model also offers central banks a tool to identify the right timing for the discontinuation of a minimum exchange rate regime. An empirical

[1]In 2005, Hong Kong installed a two-sided exchange rate target zone, allowing to implement a strong-side commitment for the value of the Hong Kong dollar vis-à-vis the U.S. dollar (USD), see Genberg and Hui (2011) or Chen et al. (2013), among others.

[2]Furthermore, the empirical results in Chen and Giovannini (1992) indicate that under the European Monetary System in the 1990s some central banks indeed were enforcing an implicit upper or lower boundary. Similarly, the Japanese monetary authorities have actively intervened in the foreign exchange market to impede appreciations of the yen vis-à-vis the USD (Chaboud and Humpage, 2005) and as such have created a temporary lower boundary.

application to the Swiss case shows the actual size of these costs in the period where the minimum EUR-CHF exchange rate regime was in place and reveals that these costs would have been substantial without the abandonment of the minimum exchange rate regime, which accords with the official statements of the SNB.

The paper is structured as follows: Section 2 shows how the exchange rate can be modeled under a strong-side commitment when the domestic central bank enforces a minimum exchange rate on a continuous basis. In Section 3, it is shown how the costs of implementing and maintaining a unilateral one-sided target zone (or strong-side commitment) can be measured ex-ante and during the period where it is enforced using the observed spot EUR-CHF exchange rate. Section 4 considers an alternative way of modeling these costs based on the spot EUR-CHF exchange rate that would have prevailed without the minimum exchange rate regime in the period of interest, while Section 5 analyzes the actual costs of the SNB's exchange rate policy regime from September 6, 2011 to January 15, 2015 and discusses the timing of the SNB's decision to abandon this policy. Section 6 summarizes the main findings of the paper.

2. The Exchange Rate Dynamics in a One-Sided Target Zone

Assume that before the implementation of the strong-side commitment the spot exchange rate F_t (quoted as the number of units of domestic currency required to buy one unit of foreign currency as of time t, see, for instance, Reiswich and Wystup (2010)) follows a geometric Brownian motion (GBM) process (which is a commonly used assumption in finance, see Glasserman (2004), Wystup (2010a), Musiela and Rutkowski (2009) or Geman (2015), among others) in a free floating exchange rate system with drift coefficient μ and diffusion coefficient σ, respectively:

$$dF_t = \mu F_t d_t + \sigma F_t dW_t, \tag{1}$$

where dW_t denotes the increment of a standard Wiener process. By applying Ito's lemma to $\ln(F_t)$, we find that

$$d\ln(F_t) = \left(\mu - \frac{\sigma^2}{2}\right) d_t + \sigma dW_t. \tag{2}$$

The exchange rate is a martingale if the drift coefficient μ is replaced by

$$\mu^* = r - r^f, \tag{3}$$

where r and r^f are the annual, continuously compounded risk-free interest rates in the domestic and foreign currency, respectively.

Inspired by the recent non-standard exchange rate policy of the SNB that enforced a minimum EUR-CHF 1.20 exchange rate from September 6, 2011 to January 15, 2015, this paper focuses on a unilateral (in the following, this term will be omitted for the sake of simplicity) one-sided target zone for the exchange rate F_t subject to a lower boundary b, whereby a central bank intervenes to buy a specific foreign currency[3] (e.g., the euro currency in the case of the SNB) to maintain a minimum exchange rate b (which will be called "floor" in the following), whenever necessary. As a consequence, the observed spot exchange rate under the minimum exchange rate regime S_t will be equal to or larger than the latent (or now shadow) exchange rate F_t. The observed exchange rate S_t then equals:

$$S_t = F_t \cdot \max\left\{1, \max_{0 \le s \le t} b/F_s\right\}, \text{ for } 1 \le t \le T, \tag{4}$$

where T denotes the finite lifetime (in years) of the strong-side commitment.

Scaling the stochastic process S_t by the floor b and taking logs, the resulting stochastic process $\{\ln(S_t/b)\}$ results from the (scaled) exchange rate process $\{\ln(F_t/b)\}$ in a free-floating exchange rate regime by introducing a reflecting barrier at zero (see, e.g., Gerber and Pafumi (2000), Ko et al. (2010), Veestraeten (2013) and Hertrich and Zimmermann (2015)), a so-called reflected (or regulated) GBM.

Assume that the drift rate of S_t is identical to the drift rate μ^* of F_t, whenever $S_t > b$. Let S_1 denote the exchange rate that is observed in the market just after announcing the introduction of a lower floor level for the exchange rate F_t. Assuming that the domestic currency is (potentially) overvalued vis-à-vis the involved foreign currency prior to the announcement of the floor,[4] which induces the domestic central bank to introduce a non-standard exchange rate policy measure in the form of a minimum exchange rate that either equals or is below the "fair" (or fundamental) equilibrium exchange rate at inception, it is assumed that the spot exchange rate "jumps" to $S_1 = F_1 > F_0$ just after officially announcing the introduction of the floor.[5] Hence, assuming that the latent exchange rate

[3]Given that before the SNB discontinued the minimum exchange rate, other alternatives to a removal of the minimum EUR-CHF 1.20 exchange rate were discussed as well, most notably a pegging of the Swiss franc to a currency basket (see, e.g., Baltensperger (2015) and Bernholz (2015)), the domestic central bank may alternatively peg its currency to a currency basket to enforce a minimum exchange rate.

[4]Which accords with the official statements of the SNB.

[5]Assuming that the announcement of the floor is interpreted as the "fully credible" willingness of the domestic central bank to not tolerate an overvaluation of the domestic currency anymore and to actively intervene in the FX market with unsterilized transactions (affecting the current or future fundamentals),

F_t is the equilibrium exchange rate for $t \geq 1$ that would prevail without the enforcement of the floor (which accords with the assumption in Hanke et al. (2014), Hanke et al. (2015) and Jermann (2015)), under the strong-side commitment $S_t \geq F_t$ for $t \geq 1$, whereby the domestic currency will be either "fairly" priced or undervalued (due to the interventions of the domestic central bank) with respect to the foreign currency compared to the situation in a free-floating exchange rate regime.

Let $p(x; k, T)$, $x > 0$, denote the transition probability density of the stochastic process $\{\ln(S_t/b)\}$ with the log-moneyness $k = \ln(S_1/b)$. As already mentioned, $p(x; k, T)$ is the probability density function of a reflected (or regulated) GBM and under the risk-neutral measure (Cox and Miller, 1965) equals:[6]

$$\begin{aligned} p(x; k, T) &= n\left(x; k + \mu^* \cdot T, \sigma^2 \cdot T\right) + \left(\frac{b}{S_1}\right)^{\theta - 1} \cdot n\left(x; -k + \mu^* \cdot T, \sigma^2 \cdot T\right) \\ &- (\theta - 1) \cdot \exp^{(\theta-1)x} \cdot \left[1 - \Phi\left(\frac{x + k + \mu^* \cdot T}{\sigma \cdot \sqrt{T}}\right)\right], \end{aligned} \tag{5}$$

with $\theta = \frac{2 \cdot \mu^*}{\sigma^2}$, where $n(x; \mu, \sigma^2)$ denotes the probability density function of the normal distribution with mean μ and standard deviation σ and $\Phi(x)$ denotes the cumulative distribution function of the standard normal distribution.

3. The Costs of Implementing a One-Sided Target Zone

Assume that the central bank of the domestic country has a finite lifetime T for the strong-side commitment in mind that is, however, not publicly announced when introducing a one-sided target zone (or an unknown maturity that is constant, as in Hanke et al. (2015)). Let $V(S_1, T)$ denote the costs (as of time $t = 1$) per unit of foreign currency of implementing a strong-side commitment for a lifetime T, in the sense of the extra amount of foreign currency (measured in units of the domestic currency) that a central bank has to buy due to its commitment to a publicly announced intervention level b for the previously free-floating exchange rate F_t. Hence, the unit cost $V(S_1, T)$ can be modeled as the difference between the discounted expected value of holding one unit of foreign currency both under the

whereby initially the spot exchange rate S_1 equals the "new" fundamental exchange rate. Notice that the SNB in actual fact stopped sterilizing interventions in the indicated period of interest. Alternatively, F_1 can be set equal to F_0, which would give a proxy for the maximum costs of implementing a one-sided target zone in the described setting. For more details on this statement, see the last paragraph in Section 3.

[6]The transition probability density $p(x; k, T)$ corresponds to Equation 2.4 in Gerber and Pafumi (2000), Equation 2.5 in Ko et al. (2010) and Equation 5 in Veestraeten (2008).

strong-side commitment and in a free-floating exchange rate system $(S_T - F_T)$, measured in units of the domestic currency, as the "upgraded" spot exchange rate S_t is the result of the (expected) interventions by the domestic central bank in order to maintain a strong-side commitment for T years, buying just enough foreign currency at the minimum exchange rate, whenever necessary, so that S_t does not fall below b:

$$\begin{aligned} V(S_1, T) &= V(F_1, T) = \exp^{\left(r^f - r\right)T} \left[\mathrm{E}^{Q^{RGBM}}\left(S_T\right) - \mathrm{E}^{Q^{GBM}}\left(F_T\right)\right], \\ &= \exp^{\left(r^f - r\right)T} \mathrm{E}^{Q^{RGBM}}\left(S_T\right) - F_1, \end{aligned} \tag{6}$$

where the discounted expected values $\left\{\exp^{\left(r^f - r\right)T}\left[\mathrm{E}^Q\left(\cdot\right)\right]\right\}$ are calculated under the corresponding risk-neutral measure Q with respect to the parameters μ^* and σ.[7] In the following, this model will be denoted by "model 1".

Notice that the approach in this paper can be compared to the pricing of a dynamic investment fund protection (Gerber and Pafumi (2000) and Ko et al. (2010)), assuming that the investment fund consists of one unit of foreign currency. Hence, initially the investment fund has a price of F_1. Offering a dynamic investment fund protection then corresponds to the action of a central bank that buys just enough foreign currency at the enforced floor level to maintain a minimum exchange rate in the spot market, whenever necessary. The value or price of this investment fund with dynamic protection minus the initial value of the investment fund can then be interpreted as the costs of the central bank's minimum exchange rate policy in domestic currency units per unit of foreign currency.

Plugging the transition probability density $p(x; k, T)$ into Equation 6 and following the steps in Gerber and Pafumi (2000) or Ko et al. (2010), using the interest parity condition for S_T and the integrals in Appendix A, the following costs (as of time $t = 1$) of maintaining a minimum exchange rate are obtained:

$$\begin{aligned} V(S_1, T) &= b\exp^{\left(r^f - r\right)T}\left(1 - \frac{1}{\theta}\right)\Phi\left(\frac{\ln\left(b/S_1\right) - \left[\mu^* - \sigma^2/2\right]T}{\sigma\sqrt{T}}\right) \\ &\quad + \frac{b}{\theta}\left(\frac{b}{S_1}\right)^{\theta}\Phi\left(\frac{\ln\left(b/S_1\right) + \left[\mu^* + \sigma^2/2\right]T}{\sigma\sqrt{T}}\right) \\ &\quad + S_1\Phi\left(\frac{\ln\left(S_1/b\right) + \left[\mu^* + \sigma^2/2\right]T}{\sigma\sqrt{T}}\right) - F_1. \end{aligned} \tag{7}$$

[7] Notice that uncovered interest parity has been applied in Equation 6, which is a standard assumption in the target zone literature (see, e.g., the credibility tests of target zones that rely on this parity (e.g., Svensson (1991) or Bertola and Svensson (1993)) or the theoretical work on the foreign exchange risk premium in target zone models (Svensson, 1992)).

Similarly, the corresponding costs as of time $1 \leq t \leq T$ equal:[8]

$$
\begin{aligned}
V(S_t, \widetilde{T}) &= b \exp^{\left(r^f - r\right)\widetilde{T}} \left(1 - \frac{1}{\theta}\right) \Phi \left(\frac{\ln\left(b/S_t\right) - \left[\mu^* - \sigma^2/2\right] \widetilde{T}}{\sigma \sqrt{\widetilde{T}}} \right) \\
&\quad + \frac{b}{\theta} \left(\frac{b}{S_t} \right)^{\theta} \Phi \left(\frac{\ln\left(b/S_t\right) + \left[\mu^* + \sigma^2/2\right] \widetilde{T}}{\sigma \sqrt{\widetilde{T}}} \right) \\
&\quad + S_t \Phi \left(\frac{\ln\left(S_t/b\right) + \left[\mu^* + \sigma^2/2\right] \widetilde{T}}{\sigma \sqrt{\widetilde{T}}} \right) - S_t. \qquad (8)
\end{aligned}
$$

with $\widetilde{T} \equiv T - t$.

In the following, the impact of the parameters b and T (i.e., the two parameters that the domestic central bank can choose freely) on the unit costs $V(S_1, T)$ is analyzed. Tables 1 and 2 and Figures 1 and 2 illustrate how increasing the lifetime T of a one-sided target zone increases the costs of implementing a minimum exchange rate regime, irrespective of the sign of the drift rate μ^*. Similarly, the larger the implemented floor level b is, the more

Table 1: Unit Costs of a One-Sided Target Zone ($\mu^* < 0$; Model 1)

	b					
T	1.00	1.05	1.10	1.15	1.20	1.25
1m	0.000	0.000	0.000	0.000	0.001	0.025
2m	0.000	0.000	0.000	0.000	0.005	0.036
3m	0.000	0.000	0.000	0.001	0.010	0.045
6m	0.000	0.000	0.001	0.007	0.026	0.068
1y	0.000	0.002	0.009	0.026	0.057	0.104
2y	0.008	0.018	0.038	0.070	0.112	0.165
3y	0.024	0.045	0.075	0.115	0.164	0.220
4y	0.047	0.077	0.115	0.161	0.215	0.273
5y	0.076	0.112	0.157	0.208	0.265	0.327
10y	0.260	0.322	0.388	0.457	0.529	0.602

Notes: The table displays the unit costs $V(S_1, T)$ of implementing a unilateral one-sided target zone for a lifetime T of 1 month (1m), 2 months (2m), 3 months (3m), 6 months (6m), 1 year (1y), 2 years (2y), 3 years (3y), 4 years (4y), 5 years (5y) or 10 years (10y) and different floor levels b. The domestic and foreign risk-free interest rates equal $r = 1\%$ and $r^f = 4\%$, respectively. The spot and latent exchange rate at time $t = 1$ (i.e., S_1 and F_1) both equal 1.25 with a volatility level of $\sigma = 8\%$.

[8]See Equations 2.11 and 5.2 in Gerber and Pafumi (2000).

it costs to maintain a one-sided target zone, as the domestic central bank has to (potentially) intervene more often to implement the minimum exchange rate b.

Table 2: Unit Costs of a One-Sided Target Zone ($\mu^* > 0$; Model 1)

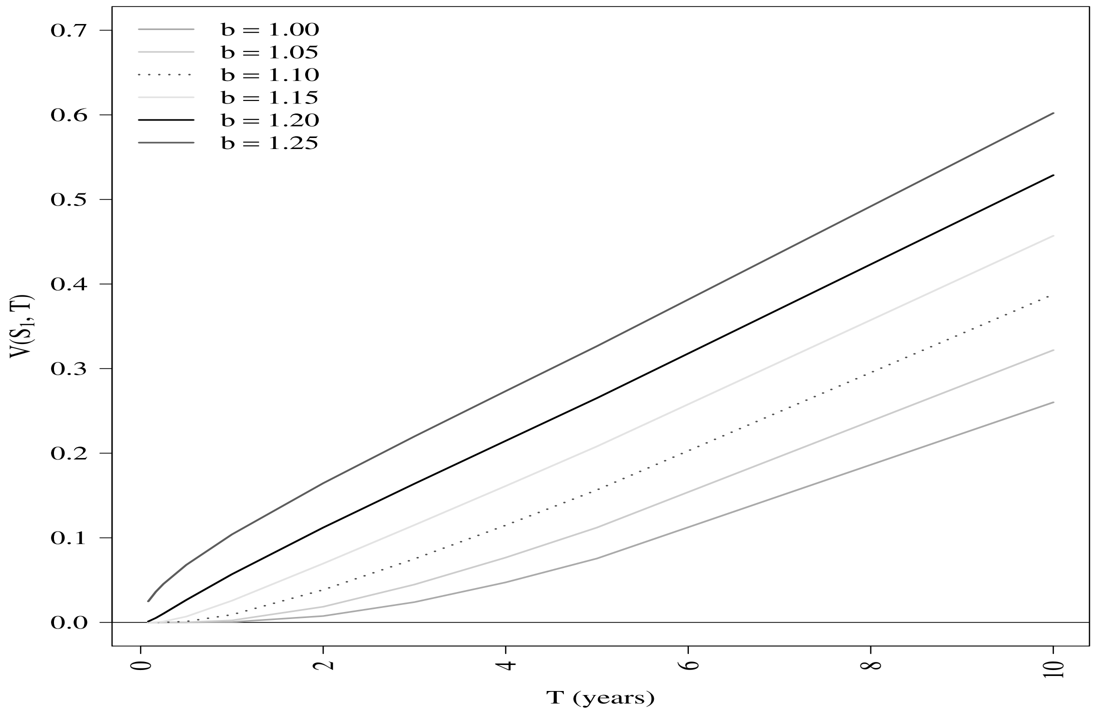
	b					
T	1.00	1.05	1.10	1.15	1.20	1.25
1m	0.000	0.000	0.000	0.000	0.001	0.022
2m	0.000	0.000	0.000	0.000	0.003	0.030
3m	0.000	0.000	0.000	0.000	0.006	0.036
6m	0.000	0.000	0.000	0.003	0.014	0.048
1y	0.000	0.000	0.002	0.008	0.026	0.064
2y	0.001	0.002	0.007	0.018	0.041	0.082
3y	0.002	0.005	0.012	0.025	0.051	0.093
4y	0.003	0.007	0.015	0.031	0.058	0.101
5y	0.004	0.009	0.018	0.035	0.063	0.107
10y	0.008	0.016	0.028	0.047	0.077	0.122

Notes: The table displays the unit costs $V(S_1, T)$ of implementing a unilateral one-sided target zone for a lifetime T of 1 month (1m), 2 months (2m), 3 months (3m), 6 months (6m), 1 year (1y), 2 years (2y), 3 years (3y), 4 years (4y), 5 years (5y) or 10 years (10y) and different floor levels b. The domestic and foreign risk-free interest rates equal $r = 4\%$ and $r^f = 1\%$, respectively. The spot and latent exchange rate at time $t = 1$ (i.e., S_1 and F_1) both equal 1.25 with a volatility level of $\sigma = 8\%$.

Comparing Table 1 with Table 2 and Figure 1 with Figure 2, it becomes evident that the unit costs are larger when the drift rate μ^* is negative (Table 1 and Figure 1), as in this case the foreign currency is expected to depreciate vis-à-vis the domestic currency over time according to uncovered interest parity. Hence, implementing a strong-side commitment becomes ceteris paribus more "expensive", as the undervaluation (the value of S_t vs. the value of F_t, for $1 \leq t \leq T$) induced by the interventions of the domestic central bank increases faster over time than in the case where the drift rate rate μ^* is positive. Consequently, in this case the domestic central bank could increase the domestic interest rate (whereby μ^* would increase and become either positive or less negative) to lower the costs $V(S_1, T)$.

Before concluding this section, notice that when a central bank is considering whether to implement a one-sided target zone or not, the maximum costs of this exchange rate policy can be estimated ex-ante by setting $S_1 = b$ and $F_1 = F_0$ in $V(S_1, T)$, assuming that the spot exchange rate is close to the announced floor level just after announcing a minimum exchange rate policy and that the domestic currency is either fairly priced or overvalued prior to the announcement of the floor.[9]

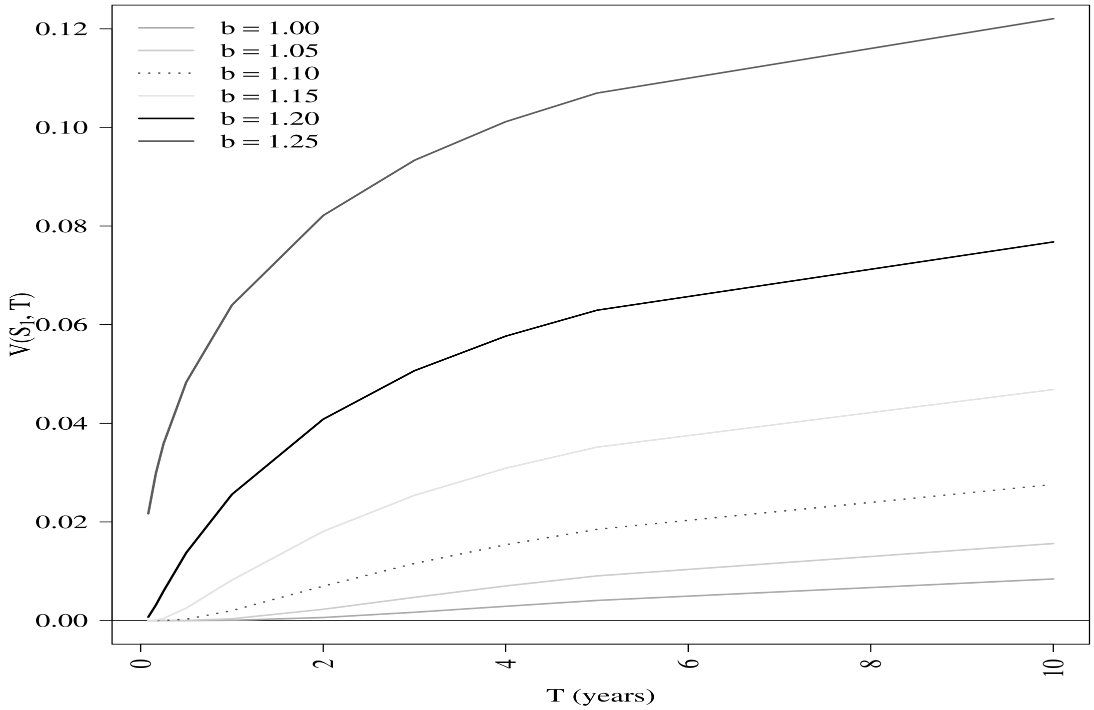
Figure 1: Unit Costs of a One-Sided Target Zone ($\mu^* < 0$; Model 1)



Notes: The figure shows the unit costs $V(S_1, T)$ of implementing a unilateral one-sided target zone for a lifetime T of 1 month (1m), 2 months (2m), 3 months (3m), 6 months (6m), 1 year (1y), 2 years (2y), 3 years (3y), 4 years (4y), 5 years (5y) or 10 years (10y) and different floor levels b. The domestic and foreign risk-free interest rates equal $r = 1\%$ and $r^f = 4\%$, respectively. The spot and latent exchange rate at time $t = 1$ (i.e., S_1 and F_1) both equal 1.25 with a volatility level of $\sigma = 8\%$.

[9]It can be easily shown that $V(S_1, T)$ is decreasing in S_1 and F_1 (see Appendix C). Hence, since $S_1 \geq b$, $S_t \geq F_t$ and assuming that the domestic currency was previously either fairly priced or overvalued (hence, the following condition holds: $F_1 \geq F_0$), it must be the case that these costs are at its maximum when $S_1 = b$ and $F_1 = F_0$.

Figure 2: Unit Costs of a One-Sided Target Zone ($\mu^* > 0$; Model 1)



Notes: The figure shows the unit costs $V(S_1, T)$ of implementing a unilateral one-sided target zone for a lifetime T of 1 month (1m), 2 months (2m), 3 months (3m), 6 months (6m), 1 year (1y), 2 years (2y), 3 years (3y), 4 years (4y), 5 years (5y) or 10 years (10y) and different floor levels b. The domestic and foreign risk-free interest rates equal $r = 4\%$ and $r^f = 1\%$, respectively. The spot and latent exchange rate at time $t = 1$ (i.e., S_1 and F_1) both equal 1.25 with a volatility level of $\sigma = 8\%$.

4. The Costs under an Alternative Modeling Approach

Of course, there is room for using alternative models: The observed spot exchange rate (S_t in Sections 2 and 3) can as well be modeled as the sum of the latent exchange rate (F_t in Sections 2 and 3) and the price of an American put option on the latent exchange rate $P^A(F_t, T)$ with an uncertain, but constant lifetime T, whereby the domestic central bank implicitly writes (cancels) put options, when it sells (buys) the foreign currency (Hanke et al., 2015):

$$S_t = F_t + P^A(F_t, T). \tag{9}$$

Since there are no closed-form formulas for non-perpetual American options when it is assumed that the underlying follows a geometric Brownian motion, Hanke et al. (2015) argue that the American put option will never be exercised before maturity, given the fact that both the interest rates in the domestic country they analyze (Switzerland) are close to zero and given that the interest rates in the foreign country (the euro zone) are larger. Consequently, the American put option on the latent exchange rate in Hanke et al. (2015) can be replaced by a European put option $P(F_t, T)$:[10]

$$S_t = F_t + P(F_t, T). \tag{10}$$

Moreover, from the point of view of the domestic central bank, the uncertain lifetime T in their model can be replaced by a known maturity date T. The European put option in Hanke et al. (2015) can then be priced using the Garman and Kohlhagen (1983) model.

Under this modeling approach, EUR-CHF option contracts on the spot exchange rate S_t can be interpreted as compound options in the period of interest. Hence, the costs associated with the monetary policy regime described in Hanke et al. (2015) can easily be estimated by applying their approach to estimate F_t using EUR-CHF option prices and plugging the estimated values into the European put option price formula. Nevertheless, since the approach in Hanke et al. (2015) is not completely free from errors (Hertrich, 2015), since Equation 10 does not hold in general[11] and since the model in Section 3 is based on FX spot market data, this paper prefers an alternative estimation procedure due to Hanke et al. (2014) that relies on spot currency data instead. Their approach is based on the structural credit risk model on page 42ff. in Lando (2004). Moreover, the dynamics of the estimated latent EUR-CHF FX rate in Hanke et al. (2014) are closer to the results in Jermann (2015) than the results in Hanke et al. (2015), which makes us feel more comfortable with the approach in Hanke et al. (2014).

Hanke et al. (2014) use the transition probability density of the stochastic process $\{\ln(F_t)\}$ which is assumed to follow a geometric Brownian motion and condition this density on the exchange rate just before the minimum exchange rate was announced (i.e., $\ln(S_0)$). Using the definition of the observed spot exchange rate in Equation 10 and the Garman-Kohlhagen put option price formula, applying both the transformation and the inverse function theorem, taking the natural logarithm and maximizing the resulting

[10]However, as the domestic central bank has the power to cancel the European put option at every point of time, from an investor's point of view, they should model the European put option as an Israeli option in the spirit of Kifer (2000) and Kühn and Kyprianou (2007), an option which puts an upper bound on the time value conceded to the option buyer by introducing the callable feature (Kühn and Kyprianou, 2007). Modeling the put option as an Israeli option would then make the put option less valuable.

[11]As already mentioned, the approach in Hanke et al. (2014) and Hanke et al. (2015) both require the domestic interest rate to remain close to zero and that the foreign interest rate is larger than the domestic interest rate throughout the period of interest and is therefore less general then the presented model.

log-likelihood, they obtain the time series of the latent exchange rate F_t in the period of interest by "inverting" the put option price formula.[12] The costs of implementing a one-sided target zone can then be estimated by plugging in the parameter estimates into Equation 10. The results of this approach will be denoted by "model 2".

Alternatively, the costs of enforcing a minimum exchange rate regime can be proxied by the modeling approach in Imai and Boyle (2001), adjusting Equation 14 in their paper for the case of currencies. Their model is based on a lookback option on the latent exchange rate F_t and is closely related to the model in Section 3. Hence, using the estimated latent exchange rate $\hat{F}_t$ in Hanke et al. (2014), setting $b = 1.20$ and $T = 1\text{m}$, 6m and 1y, respectively, the costs approximately equal:[13]

$$\begin{aligned}
V(F_t, \widetilde{T}) \quad = \quad & b \exp^{(r^f - r)\widetilde{T}} \left(1 - \frac{1}{\theta}\right) \Phi \left(\frac{\ln(b_t^{'}/F_t) - [\mu^* - \sigma^2/2]\,\widetilde{T}}{\sigma\sqrt{\widetilde{T}}} \right) \\
& + \frac{b}{\theta} \left(\frac{b_t^{'}}{F_t} \right)^{\theta} \Phi \left(\frac{\ln(b_t^{'}/F_t) + [\mu^* + \sigma^2/2]\,\widetilde{T}}{\sigma\sqrt{\widetilde{T}}} \right) \\
& + F_t \left\{ M_t \Phi \left(\frac{\ln(F_t/b_t^{'}) + [\mu^* + \sigma^2/2]\,\widetilde{T}}{\sigma\sqrt{\widetilde{T}}} \right) - 1 \right\} \qquad (11)
\end{aligned}$$

with $M_t = \max\left\{1, b/\max_{0 \le s \le t} F_s\right\}$, $b_t^{'} = b/M_t$ and $\widetilde{T} \equiv T - t$. This model is called "model 3" in the following.

5. Empirical Results

In this section, the costs of implementing a one-sided target zone are estimated for the case of Switzerland from September 6, 2011 to January 14, 2015. The present study focuses on the Swiss case, since for the Czech case the lower boundary is not fixed to EUR-CZK 27, but only to be close to that floor level and because there the step of proxying the American put option by a European put option (arguing that the foreign interest rates are consistently larger than the domestic interest rates and that the latter are close to zero; see Section 3 for more details) is less convenient, since for this currency pair the drift rate μ^* has changed the sign several times since November 7, 2013,[14] which is another argument

[12] Which is obtained numerically. To date there is no closed-form formula to express the underlying asset price for a given put price and a set of parameters as a function of the price of the underlying.

[13] Since both the transition probability density and F_1 are given, alternatively, the expectation maximization algorithm can be applied to obtain the expected value of F_t.

[14] Measured by the corresponding CZK PRIBOR and EUR LIBOR interest rates.

why the approach in Section 3 is preferred to the alternative models in Section 4.

5.1. Data

To calculate the costs of implementing a one-sided target zone in Equations 8, 10 and 11, the domestic and foreign risk-free interest rates r and r^f are proxied by the corresponding CHF LIBOR and EUR LIBOR interest rates for contract maturities of 1 month (1m), 6 months (6m) and 12 months (1y). For specifying the volatility level σ, option implied volatilities for call and put options on the EUR-CHF spot FX rate from Bloomberg with an option delta of $\Delta \pm 25\%$ and contract maturities of 1m, 6m and 1y are used, covering the period from September 6, 2011 to January 14, 2015. Specifically, it is assumed that the domestic central bank uses the previous day's implied volatility as an estimate for today's implied volatility, following Whaley (1993) and Bakshi et al. (1997), among others, as compared to alternative measures, this procedure has good forecasting power (see, e.g., Satchell (2007) and Wang and Daigler (2011)). The volatility smile effect is captured by applying the Vanna-Volga approximation as discussed in Castagna and Mercurio (2005), thereby getting implied volatilities that are consistent with the previous day's smile curve.[15] The minimum exchange rate b equals EUR-CHF 1.20. The exchange rate after announcing the implementation of a strong-side commitment S_1 is set equal to the recorded EUR-CHF exchange rate on September 6, 2011, which is (approximately) EUR-CHF 1.21. The lifetime of the one-sided target zone T is set equal to the aforementioned FX option contract maturities.

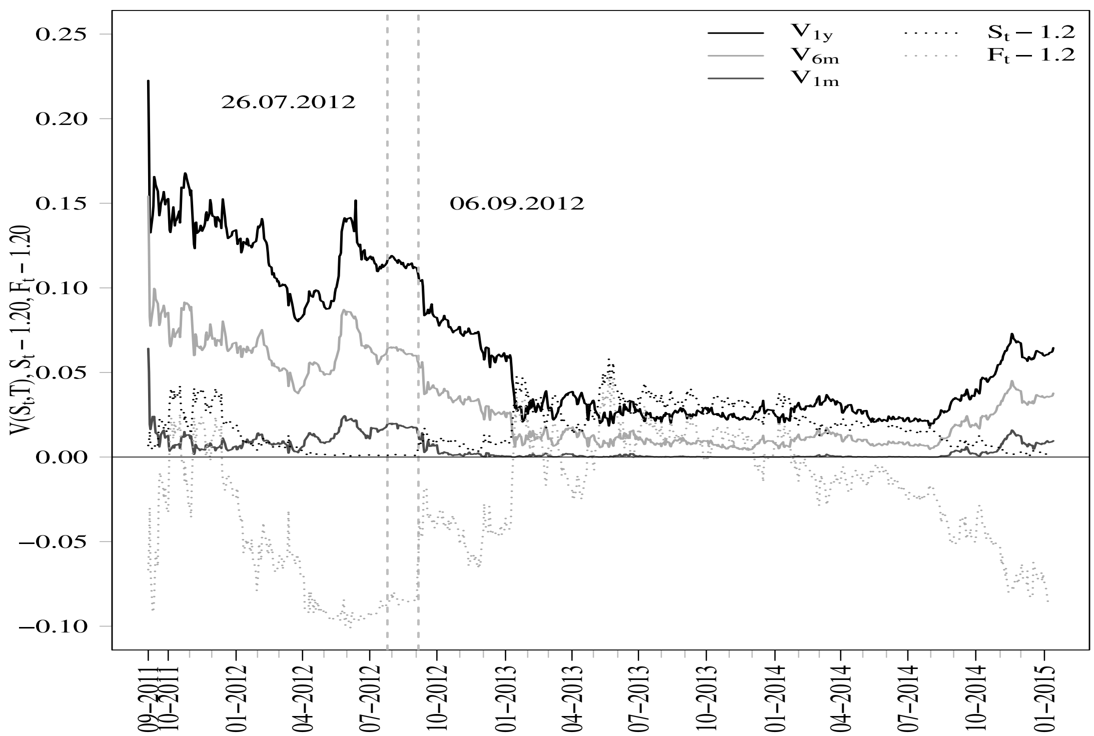
5.2. The Costs of Implementing a Unilateral One-Sided Exchange Rate Target Zone

In this section and to exemplify the theoretical models (models 1, 2 and 3), the unit costs of the SNB's exchange rate policy vis-à-vis the euro from September 6, 2011 to January 14, 2015 are analyzed. Figure 3 shows the estimated target zone costs for several maturities, ranging from 1 month to up to 1 year, assuming that the EUR-CHF exchange rate follows a RGBM after September 6, 2011 (called model 1, see Section 3). The figure indicates that initially, i.e., in the weeks and months after announcing the minimum exchange rate of EUR-CHF 1.20, the costs of maintaining the floor were relatively large. Evidently, the larger the lifetime T of the one-sided target zone (compare, for instance, V_{1m} with V_{1y}), the larger are the costs of enforcing a minimum exchange rate regime, since it becomes more

[15] More details about the Vanna-Volga method can be found in, e.g., Castagna and Mercurio (2005), Castagna and Mercurio (2007), Wystup (2010b) and Bossens et al. (2010).

likely that the domestic central bank will have to intervene in the future.[16] Interestingly, Mario Draghi's "Whatever it takes"-statement on July 26, 2012 had a minor effect on the target zone costs. It was not until the ECB launched the Outright Monetary Transaction (OMT) program on September 6, 2012 that the costs decreased significantly. This date initiated a period where the latent EUR-CHF FX rate F_t continually increased and the target zone costs continually fell, until these costs reached a global minimum level for V_{1y} in summer 2014.[17] This low cost level may explain why the SNB continued implementing

Figure 3: Target Zone Costs under RGBM (Model 1)



Notes: The figure shows the costs (in CHF) of implementing a unilateral one-sided target zone for a lifetime of 1 month (V_{1m}), 3 months (V_{3m}) or 1 year (V_{1y}), respectively, under the assumption that the EUR-CHF exchange rate follows a reflected geometric Brownian motion (RGBM) from September 6, 2011 to January 14, 2015. The first marked date (26.07.2012) refers to the announcement of the "Draghi put" ("Whatever it takes""), the second marked date (06.09.2012) to the date when the European Central Bank launched the Outright Monetary Transaction (OMT) program. The dotted lines display the deviation of the spot and the latent FX rate S_t (in black) and F_t (in grey) from the EUR-CHF 1.20 floor, respectively. Data source: Bloomberg.

[16]Given the prominent role of the CHF as a safe haven currency in periods of global financial instability, this argumentation seems especially plausible in the period of interest.

[17]V_{6m} and V_{1y} reached their global minima on November 27, 2013 and July 31, 2014, respectively.

the minimum exchange rate until the SNB finally decided to abandon the target zone regime on January 15, 2015. Figure 3 also indicates that presumably in spring 2013, when both the difference between the spot and latent EUR-CHF FX rate had reached an all-time low on May 22, 2013,[18] whereby the Swiss currency had reached an all-time low since the introduction of the EUR-CHF 1.20 floor vis-à-vis the euro and was close to being priced as in a free-floating exchange rate regime, and when the costs of maintaining a one-sided target zone were relatively high (in the period from March 2013 until the end of August 2014, the highest cost levels for V_{6m} and V_{1y} were reached on April 3, 2013), it would have been a good period to let the EUR-CHF exchange rate float freely again, as removing the lower boundary may not have caused the EUR-CHF exchange rate to fall as extremely as it happened on January 15, 2015 and as the costs of further enforcing the minimum exchange rate policy for a further year (V_{1y}) were high.

Comparing the situation in spring 2013 to the period when the SNB decided to abandon the EUR-CHF 1.20 floor on January 15, 2015, Figure 3 indicates that both especially the short-term costs (V_{1m} and V_{6m}) and the gap between the spot and latent EUR-CHF FX rate were considerably larger in the latter period. Consequently, without the abandonment, the SNB presumably would have had to intervene in the currency market to enforce the EUR-CHF 1.20 floor (as it was already the case in 2012) in winter and spring 2015. Moreover, the results in Figure 3 show that the SNB could have anticipated the EUR-CHF FX rate volatility that was observed after abandoning the floor, since the difference between S_t and F_t was rather large in January 2015. Alternatively, instead of completely abandoning the EUR-CHF 1.20 floor, the SNB could have lowered the floor to a lower level, in the spirit of Hertrich and Zimmermann (2015), an alternative also supported by Bernholz (2015). All in all, however, it seems that the timing of the SNB in deciding to discontinue the floor was rather good.

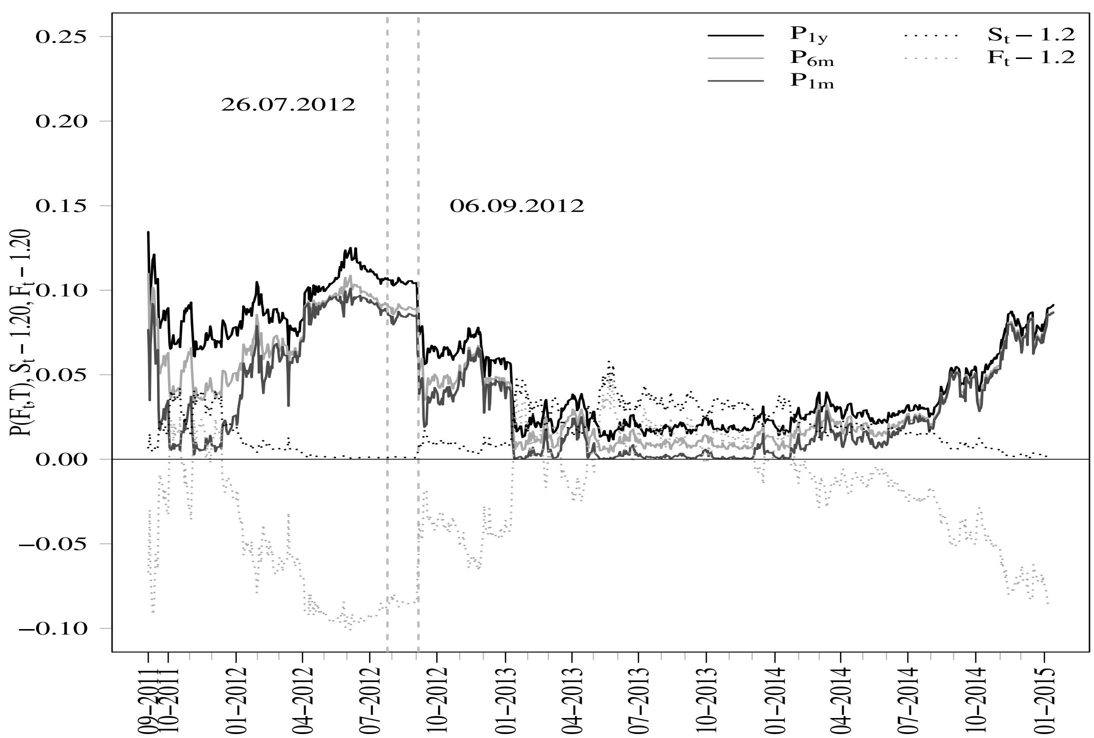
Interestingly, the dynamics of the target zone costs accords with the results in López and Mendizábal (2003), who develop a target zone model and compare the costs of implementing a target zone in terms of both the interest rate variability and the exchange rate variability to alternative regimes, whereby the more credible a target zone is, the smaller the costs of a target zone are, since Figure 2 in Hertrich and Zimmermann (2015) indicates that the credibility of the SNB's minimum exchange rate policy was relatively large in the period where the target zone costs were rather low (in the period from March 2013 until the end of August 2014 in Figure 3). Similarly and in general, a lack of or imperfect

[18]Moreover, on this day, both the spot and latent EUR-CHF FX rate had reached an all-time high.

credibility of a target zone regime makes the enforcement of a floor more expensive, since the domestic central bank has to buy relatively larger amounts of the foreign currency. If these interventions are unsterilized, the domestic interest rate will fall accordingly, increasing the target zone costs in, for instance, the framework of López and Mendizábal (2003).

A closer look at Figures 4 and 5, where the target zone costs according to models 2 and 3 (see Section 4) are plotted, confirms the conjectures in the previous paragraphs, since the evolution of the target zone costs over time accords with the dynamics of the data in Figure 3.

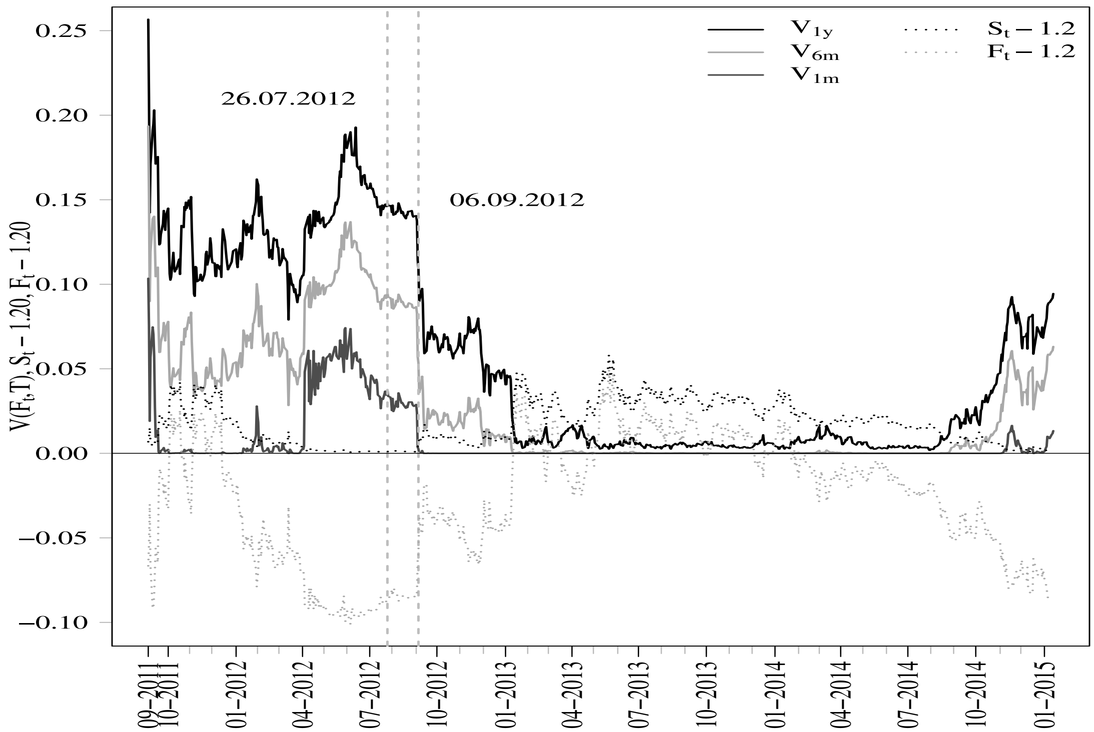
Figure 4: Target Zone Costs Writing Put Options (Model 2)



Notes: The figure shows the costs (in CHF) of implementing a unilateral one-sided target zone for a lifetime of 1 month (P_{1m}), 6 months (P_{6m}) or 1 year (P_{1y}), respectively, under the assumptions of model 2 from September 6, 2011 to January 14, 2015. The first marked date (26.07.2012) refers to the announcement of the "Draghi put" ("Whatever it takes""), the second marked date (06.09.2012) to the date when the european Central Bank launched the Outright Monetary Transaction (OMT) program. The dotted line displays the deviation of the FX rate $\tilde{S}_t$ from the EUR-CHF 1.20 floor. Data source: Bloomberg.

The results in Figures 3, 4 and 5 can also be related to the SNB's exchange market interventions since September 6, 2011. Taking the average daily volume of EUR-CHF spot exchange trades (on a "net-net" basis) of EUR 31.9 billion (BIS, 2014), calculating with 20 trading days per month, assuming that the SNB had to buy on average 10.8% of the daily turnover concerning EUR-CHF spot deals[19] and multiplying this number by the average

Figure 5: Target Zone Costs under RGBM based on the Latent EUR-CHF FX Rate (Model 3)



Notes: The figure shows the costs (in CHF) of implementing a unilateral one-sided target zone for a lifetime of 1 month (V_{1m}), 6 months (V_{6m}) or 1 year (V_{1y}), respectively, under the assumptions of model 3 from September 6, 2011 to January 14, 2015. The first marked date (26.07.2012) refers to the announcement of the "Draghi put" ("Whatever it takes""), the second marked date (06.09.2012) to the date when the european Central Bank launched the Outright Monetary Transaction (OMT) program. The dotted line displays the deviation of the FX rate $\tilde{S}_t$ from the EUR-CHF 1.20 floor. Data source: Bloomberg.

target zone costs of EUR-CHF 0.0631 (Figure 3), EUR-CHF 0.0362 (Figure 4) and EUR-CHF 0.0562 (Figure 5) for a lifetime of 1 year, respectively, the costs of the interventions of the SNB approximately equal CHF 39.6, CHF 22.6 or CHF 35.3 billions per year. If these numbers are multiplied with the duration of the EUR-CHF target zone of approximately

[19] Using the information that before the end of 2014 the SNB only had to intervene during three months in 2012 (Bernholz, 2015), the information that in that year the SNB bought CHF 188 billion in foreign currency (Swiss National Bank, 2012) and using the EUR-CHF 1.20 floor as the intervention level, the SNB's monthly EUR-CHF activity equals approximately $(188/3)/(1.2 * 0.7595 * 31.9 * 20) \approx 10.8\%$.

3.25 years, these costs amount to CHF 128.9, CHF 73.5 and CHF 114.8 billions, respectively. Comparing these numbers with the size of the SNB's current FX reserves ($\sim$ CHF 526.2 billions) with the same metric before the announcement of the floor ($\sim$ CHF 372.3 billions),[20] it is interesting to notice how close the estimates for the target zone costs according to model 1 and 3 are to the actual size of the changes in the FX reserves in the SNB's balance sheet.

Concluding this section, the empirical analysis reveals important insights about the costs (in terms of the extra amount of foreign currency) of implementing and maintaining a minimum exchange rate regime and the factors that have an impact on these costs. Furthermore, the results show how financial market information can be used by central banks to assess the costs associated with the implementation of unilateral exchange rate policies.

6. Conclusion

In the aftermath of the recent financial crisis, the central banks of small open economies, such as the Czech and the Swiss National Bank introduced a lower boundary vis-à-vis the euro currency, the currency of their most important trading partners (i.e., the countries from the euro zone). This policy has generated a strand of literature that analyzes these episodes. The present paper contributes to this target zone literature by proposing a model that allows to quantify the expected costs of implementing a strong-side commitment for an exchange rate that is (potentially) "overvalued" prior to the announcement of the exchange rate floor. The paper proceeds and analyzes the impact that the parameters that are under the control of the domestic central bank have on these target zone costs, namely the lifetime of the target zone, the domestic risk-free interest rate and the implemented floor level. The presented model suggests first, that these costs are larger when the drift rate of the exchange rate is negative, as this implies that the foreign currency is expected to depreciate vis-à-vis the domestic currency over time according to interest parity. Second, a shorter lifetime of the target zone in general implies lower target zone costs. Third, the lower the implemented floor level is, the lower are these costs.

An empirical application to the recently discontinued minimum exchange rate regime that the Swiss National Bank implemented from September 6, 2011 to January 15, 2015 complements the paper and adds insights into the presented model, emphasizing the relevance of applying models that allow central banks both to estimate these costs before

[20]The difference equals CHF 153.9 billions.

announcing a unilateral one-sided target zone and to monitor these costs during the period where it is enforced. The model also offers central banks a tool to identify ex-ante a suitable date to discontinue the exchange rate target zone regime.

The empirical evidence suggests first, that there may have been better dates to abandon the EUR-CHF target zone from an ex-post perspective (although the specific date when the SNB decided to discontinue the floor was well chosen, given the circumstances) and second, that the actual size of these costs might have otherwise been substantial. The robustness of the presented model is assessed by applying two alternative approaches, which yield qualitatively comparable results. The presented model therefore allows central banks to infer relevant foreign exchange rate information (from spot and/or from option markets), especially when considering ex-ante the announcement of an exchange rate floor or when monitoring the development of a target zone while it is in place.

References

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical Performance of Alternative Option Pricing Models. Journal of Finance 52 (5), 2003–2049.

Baltensperger, E., 2015. SNB-Doyen will neue Untergrenze. In: Interview with Neue Zürcher Zeitung (January 11, 2015).

Bernholz, P., 2015. The Swiss Experiment: From the Lower Bound to Flexible Exchange Rates. Cato Journal 35 (2), 403–410.

Bertola, G., Svensson, L. E. O., 1993. Stochastic Devaluation Risk and the Empirical Fit of Target-Zone Models. Review of Economic Studies 60 (3), 689–712.

BIS, 2014. Global Foreign Exchange Market Turnover in 2013. Triennial Central Bank Survey, Bank for International Settlement.

Bossens, F., Rayée, G., Skantzos, N., Deelstra, G., 2010. Vanna-Volga Methods Applied to FX Derivatives: From Theory to Market Practice. International Journal of Theoretical and Applied Finance 13 (8), 1293–1324.

Castagna, A., Mercurio, F., 2005. Consistent Pricing of FX Options. Internal Report, Banca IMI.

Castagna, A., Mercurio, F., 2007. The Vanna-Volga Method for Implied Volatilities. Risk Magazine, 39–44.

Chaboud, A., Humpage, O., 2005. An Assessment of the Impact of Japanese Foreign Exchange Intervention: 1991-2004. International Finance Discussion Papers 824, 1–41.

Chen, Y.-F., Funke, M., Glanemann, N., 2013. Off-the-record Target Zones: Theory with an Application to Hong Kongs Currency Board. Studies in Nonlinear Dynamics and Econometrics 17 (4), 373–393.

Chen, Z., Giovannini, A., 1992. Target Zones and the Distribution of Exchange Rates. Economics Letters 40, 83–89.

Cottarelli, C., Doyle, P., 1999. Disinflation in Transition, 1993-97. International Monetary Fund Occasional Paper 179, 1–45.

Cox, D., Miller, H., 1965. The Theory of Stochastic Processes. London: Chapman and Hall.

Garman, M. B., Kohlhagen, S. W., 1983. Foreign Currency Option Values. Journal of International Money and Finance 2 (3), 231–237.

Geman, H., 2015. Agricultural Finance: From Crops to Land, Water and Infrastructure. Chichester: John Wiley & Sons.

Genberg, H., Hui, C.-H., 2011. The Credibility of Hong Kongs Link from the Prespective of Modern Financial Theory. Journal of Money, Credit and Banking 43 (1), 185–206.

Gerber, H., Pafumi, G., 2000. Pricing Dynamic Investment Fund Protection. North American Actuarial Journal 4 (2), 28–37; Discussion 37–41.

Glasserman, P., 2004. Monte Carlo Methods in Financial Engineering. Heidelberg: Springer.

Hanke, M., Poulsen, R., Weissensteiner, A., 2014. Analyzing the Swiss National Bank's Euro Exchange Rate Policy: A Latent Likelihood Approach. Working Paper, available at SSRN 2515210.

Hanke, M., Poulsen, R., Weissensteiner, A., 2015. Where Would the EUR/CHF Exchange Rate be Without the SNB's Minimum Exchange Rate Policy? Working Paper, forthcoming in the Journal of Futures Markets.

Hertrich, M., 2015. On the EUR/CHF Exchange Rate thatWould Have Prevailed Without the SNBs Minimum Exchange Rate Policy. Working paper.

Hertrich, M., Zimmermann, H., 2015. On the Credibility of the Euro/Swiss Franc Floor: A Financial Market Perspective. Working Paper, available at SSRN 2290997.

Humpage, O., Ragnartz, J., 2006. Swedish Intervention and the Krona Float, 1993-2002. Sveriges Riksbank Working Paper Series 192, 1–40.

ICC Research Foundation, 2013. ICC Open Markets Index.

Imai, J., Boyle, P. P., 2001. Dynamic Fund Protection. North American Actuarial Journal 5 (3), 31–49; Disscussion 49–51.

Jermann, U. J., 2015. Financial Markets' Views about the Euro-Swiss Franc Floor. Working Paper, available at SSRN 2490086.

Kifer, Y., 2000. Game Options. Finance and Stochastics 4 (4), 443–463.

Ko, B., Shiu, E., Wei, L., 2010. Pricing Maturity Guarantee with Dynamic Withdrawal Benefit. Insurance: Mathematics and Economics 47 (2), 216–223.

Krugman, P. R., 1991. Target Zones and Exchange Rate Dynamics. Quarterly Journal of Economics 106 (3), 669–682.

Kühn, C., Kyprianou, A., 2007. Callable Puts as Composite Exotic Options. Mathematical Finance 17 (4), 487–502.

Lando, D., 2004. Credit Risk Modeling. Princeton: Princeton University Press.

López, J. R., Mendizábal, H. R., 2003. On the Choice of an Exchange Regime: Target Zones Revisited. Working Paper.

Musiela, M., Rutkowski, M., 2009. Martingale Methods in Financial Modelling. Heidelberg: Springer.

Reiswich, D., Wystup, U., 2010. A Guide to FX Options Quoting Conventions. Journal of Derivatives 18 (2), 58–68.

Satchell, S., 2007. Forecasting Expected Returns in the Financial Markets. London: Academic Press.

Studer-Suter, R., Janssen, A., 2014. The Swiss Franc's Honeymoon. Working Paper, University of Zurich.

Svensson, L. E. O., 1991. The Simplest Test of Target Zone Credibility. International Monetary Fund Staff Papers 38 (3), 655–665.

Svensson, L. E. O., 1992. The Foreign Exchange Risk Premium in a Target Zone Model with Devaluation Risk. Journal of International Economics 33 (1), 21–40.

Swiss National Bank, 2012. Annual report 2012, accountability report.

Veestraeten, D., 2008. Valuing Stock Options when Prices are Subject to a Lower Boundary. Journal of Futures Markets 28 (3), 231–247.

Veestraeten, D., 2013. Currency Option Pricing in a Credible Exchange Rate Target Zone. Applied Financial Economics 23 (11), 951–962.

Wang, Z., Daigler, R., 2011. The Performance of VIX Option Pricing Models: Empirical Evidence Beyond Simulation. Journal of Futures Markets 31 (3), 251–281.

Whaley, R., 1993. Derivatives on Market Volatility: Hedging Tools Long Overdue. Journal of Derivatives 1 (1), 71–84.

Wystup, U., 2010a. Foreign Exchange Symmetries. In: Cont, R. (Ed.), Encyclopedia of Quantitative Finance. Vol. 2. Chichester: John Wiley & Sons, pp. 752–759.

Wystup, U., 2010b. Vanna-Volga Pricing. In: Cont, R. (Ed.), Encyclopedia of Quantitative Finance. Vol. 4. Chichester: John Wiley & Sons, pp. 1867–1874.

A. Identities for the Normal Distribution Function

This appendix contains two identities that are used in this paper to obtain the main results. The identities correspond to Equations A.1 and A.2 in Gerber and Pafumi (2000) for $a = 0$ and Equations 3.4 and 3.5 in Ko et al. (2010):

$$\int_a^{\infty} \exp^{c \cdot x} \cdot n\left(x; \mu, \sigma^2\right) dx = \exp^{\mu \cdot c + \frac{1}{2} \cdot \sigma^2 \cdot c^2} \Phi\left(\frac{-a + \mu + \sigma^2 \cdot c}{\sigma}\right) \tag{A.1}$$

and

$$\begin{aligned} &\int_a^{\infty} \exp^{c \cdot x} \cdot \left[1 - \Phi\left(\frac{x - \mu}{\sigma}\right)\right] dx \\ &\quad = -\frac{1}{c} \cdot \exp^{a \cdot c} \cdot \Phi\left(\frac{-a + \mu}{\sigma}\right) \\ &\qquad + \frac{1}{c} \cdot \exp^{\mu \cdot c + \frac{1}{2} \cdot \sigma^2 \cdot c^2} \cdot \Phi\left(\frac{-a + \mu + \sigma^2 \cdot c}{\sigma}\right). \end{aligned} \tag{A.2}$$

In Equation A.1, a and c are both arbitrary real numbers. Equation B.2 requires the condition $c \neq 0$.

B. The Costs of a One-Sided Target Zone under RGBM Modeled as a Call Option

In this section, it is first shown how the costs of enforcing a one-sided unilateral exchange rate target zone can be decomposed into the price of a call option when the underlying exchange rate follows a RGBM plus the implemented minimum exchange rate b, minus the value of one unit of foreign currency denominated in domestic currency units (i.e., S_t). Specifically, using a modification of Equation 11 in Veestraeten (2008) or Equation 15 in Veestraeten (2013) and setting the strike price X in these formulas equal to the minimum exchange rate b, the corresponding call option price (as of time t) then equals:

$$\begin{aligned} C_r\left(S_t, \widetilde{T}\right) &\equiv C_r\left(b, r, r^f, \sigma, S_t, \widetilde{T}\right) \\ &= S_t \exp^{-r^f \widetilde{T}} \Phi\left(z_1\right) - b \exp^{-r\widetilde{T}} \Phi\left(z_1 - \sigma\sqrt{\widetilde{T}}\right) \\ &\quad + \frac{1}{\theta}\left\{ S_t \exp^{-r^f\widetilde{T}} \left(\frac{b}{S_t}\right)^{1+\theta} \cdot \Phi(z_2) \right. \\ &\quad \left. - b\exp^{-r\widetilde{T}} \Phi\left(z_2 - \theta\sigma\sqrt{\widetilde{T}}\right) \right\}, \end{aligned} \tag{B.1}$$

with

$$\begin{aligned} \theta &= 2\frac{\mu^*}{\sigma^2}, \\ z_1 &= \frac{\ln\left(S_t/b\right) + \left(\mu^* + \sigma^2/2\right)\widetilde{T}}{\sigma\sqrt{\widetilde{T}}}, \\ z_2 &= \frac{\ln\left(b/S_t\right) + \left(\mu^* + \sigma^2/2\right)\widetilde{T}}{\sigma\sqrt{\widetilde{T}}}, \end{aligned}$$

where the subscript r in $C_r\left(S_t, \widetilde{T}\right)$ is included to distinguish the call option price under RGBM from its natural counterpart under GBM.

Lending $\left\{b\exp^{-rT}\right\}$ units of the domestic currency plus the call option $C_r\left(\cdot\right)$ ensures a payoff of S_T at maturity date. Hence, the value of both equals the discounted expected value of S_T, which according to Equation 7 (using S_t instead of S_0) equals:

$$\exp^{r^f\widetilde{T}}\left[C_r\left(S_t, \widetilde{T}\right) + b\exp^{-r\widetilde{T}}\right] = \exp^{\left(r^f - r\right)\widetilde{T}} \mathrm{E}^{Q^{RGBM}}\left(S_T\right) = V\left(S_t, \widetilde{T}\right) + S_t. \tag{B.2}$$

C. The Maximum Possible Costs of a One-Sided Target Zone Modeled under RGBM

In this section, the condition that implies an upper bound for the costs of enforcing a unilateral one-sided exchange rate target zone $V\left(S_t, \widetilde{T}\right)$ is derived. Taking the derivative of Equation B.2 with respect to S_t and using the derivative of a standard call option under

GBM with respect to S_t gives:

$$\begin{aligned}\frac{\partial V\left(S_t,\widetilde{T}\right)}{\partial S_t} &= \frac{\partial C_r\left(S_t,\widetilde{T}\right)}{\partial S_t}-1,\\ &= \frac{\partial C_r\left(S_t,\widetilde{T}\right)}{\partial S_t}-\left[\frac{\partial C\left(S_t,\widetilde{T}\right)}{\partial S_t}+\Phi\left(-z_1\right)\right]<0,\end{aligned} \tag{C.1}$$

since $\frac{\partial C_r\left(S_t,\widetilde{T}\right)}{\partial S_t} \leq \frac{\partial C\left(S_t,\widetilde{T}\right)}{\partial S_t}$ (Veestraeten, 2013).