Networks, Frictions, and Price Dispersion

Donna, Javier and Schenone, Pablo and Veramendi, Gregory

The Ohio State University, Arizona State University, Arizona State University

17 July 2015

Networks, Frictions, and Price Dispersion*

Javier Donna — The Ohio State University

Pablo Schenone — Arizona State University

Gregory Veramendi — Arizona State University

First version: January, 2014

This version: November 13, 2015

*We are grateful to Rune Vejlin for the initial conversations which led to this project. Discussions with Matt Backus, David Blau, Hector Chade, Eleanor Dillon, Laura Doval, Domenico Ferraro, John Hatfield, John Kagel, Shachar Kariv, Rasmus Lentz, Dan Levin, Jim Peck, Tiago Pires, Rob Porter, Andrew Rhodes, Huanxing Yang as well as seminar participants at Arizona State University, Cycles, Adjustment, and Policy conference (Aarhus University), the Southwest Search and Matching conference (University of California Riverside), and the 13th International Industrial Organization conference (Boston) have greatly benefited this work. All errors are unintentional.

Abstract

This paper studies price dispersion in buyer-seller markets using networks to model frictions, where buyers are linked with a subset of sellers and sellers are linked with a subset of buyers. Although there is a large search literature studying wage/price dispersion, search models typically restrict the type of competition that can occur between buyers (firms) and sellers (workers) when determining prices or wages. Our approach allows for indirect competition, where a buyer who is not directly linked with a seller affects the price obtained by that seller. Indirect competition generates the central finding of our paper: even when there are significant frictions, price distributions and allocations are close to the perfectly competitive outcome where the law of one price holds. We then investigate the role of indirect competition in a dynamic setting by studying wages in the context of an on-the-job search model. We find that indirect competition has similar effects on wage dispersion and wage dynamics. This leads to two novel predictions relative to the search literature. Lowering frictions (so that workers receive job offers at a higher rate) leads to: (1) *lower* worker mobility and *lower* expected wage growth and (2) *lower* expected wages in markets with high unemployment. We argue that our framework is suited to the analysis of a wide range of real-world markets, such as the labor market and buyer-seller trading platforms like eBay or Amazon.

JEL Codes: L11; J31; J64

Keywords: Indirect Competition; Search Frictions; Wage Dispersion

Javier Donna
Department of Economics
The Ohio State University
1945 N High St
417 Arps Hall
Columbus, OH 43210
Phone: 614-688-0364
Email: donna.1@osu.edu

Pablo Schenone
Department of Economics
W.P. Carey School of Business
Arizona State University
501 E. Orange Street
Tempe, AZ 85287
Phone: 480-965-5596
Email: pablo.schenone@asu.edu

Gregory Veramendi
Department of Economics
W.P. Carey School of Business
Arizona State University
501 E. Orange Street
Tempe, AZ 85287
Phone: 480-965-0894
Email: gregory.veramendi@asu.edu

1 Introduction

Price dispersion is observed in many markets even after accounting for observable characteristics. Examples include labor markets, where similar workers are paid different wages (Mortensen 2005); buyer-seller trading platforms such as eBay or Amazon, where identical goods are sold by the same seller at different prices (Einav, Kuchler, Levin, and Sundaresan 2015); and markets for automobiles, where identical automobiles are sold at different prices by the same dealer (Morton, Zettelmeyer, and Silva-Risso 2001). A common characteristic of these markets is that buyers interact with multiple, but not all, sellers. In the case of labor markets, firms interview multiple applicants. Bidders on eBay bid in multiple auctions for an identical product. Consumers visit multiple automobile dealerships before making a purchase. A natural question arises: When buyers interact with multiple sellers, how does this affect price dispersion?

The central insight of this paper is that "indirect competition" plays an important role in determining price dispersion. For example, consider the case of two eBay auctions where there is only one common bidder participating in both auctions. The bidders in one auction *indirectly compete* with the bidders in the other auction because the two auctions are connected through the common bidder.[1] Indirect competition results in an interdependence in the prices between these two auctions. Even if many sellers are not directly competing for the same buyer, indirect competition can equalize prices across them. How buyers and sellers are connected (linked) determines the extent of the indirect competition. To the best of our knowledge, there has been no attempt to study price dispersion in the presence of indirect competition. See Section 2 for a detailed literature review.

In this paper, we use networks to model buyer-seller markets where a buyer can obtain a good from the seller only if the two are linked. Perfect competition is the special case where all buyers are linked to all sellers in the network, leading to the Walrasian outcome. Frictions are present in the market whenever there is at least one seller that is not linked to every buyer. Hence, the level of frictions in the network is determined by the total number of links (or sparsity) of the network.

The central finding of our paper is that even relatively sparse networks lead to price distributions and allocations that are close to the perfectly competitive outcome.[2] Our analysis has a theoretical and a quantitative component. First, an application of proposition 1 shows that many links in any given network can be redundant (*i.e.* have no effect on prices and allocations).[3] In particular, many links in a perfectly competitive market are redundant. This leads to the following question: How many links can be removed and still obtain prices

[1] See section 3 for an example with buyers and sellers.

[2] Our finding that pricing behavior in networks with frictions closely resembles the perfectly competitive outcome is consistent with the pricing behavior observed in the laboratory experiments in Charness, Corominas-Bosch, and Fréchette (2007) and Gale and Kariv (2009). See Judd and Kearns (2008) for a survey of experiments in networked markets.

[3] See section 3 for examples of redundant links that have no effect on the prices and allocations in a market.

and allocations that are close to the perfectly competitive outcome? We perform a numerical analysis to quantify this effect. For example, in a network of 10,000 sellers with homogeneous goods, over 99% of the sellers are paid the same price when less than 0.1% of the possible links are active. The prediction that even sparse networks lead to price profiles that are close to the perfectly competitive outcome is a consequence of indirect competition. As the number of links increases, indirect competition among buyers rapidly becomes more likely. Indirect competition causes buyers and sellers to behave "as if" there were links between them, making a direct link redundant. Indirect competition makes markets with frictions look like they were perfectly competitive, hence the result.

We then investigate the role of indirect competition in a dynamic setting by studying wages in an on-the-job search model.[4] As in the buyer-seller model, indirect competition pushes wages and wage dynamics toward the Walrasian outcome even when frictions are present (*i.e.* even in sparse networks). This leads to two novel predictions about wage dynamics relative to the search literature. First, reducing frictions (increasing the job offer rate) leads to *lower* expected wages in loose markets (*i.e.* when there are more unemployed workers than open vacancies at firms). This is because in the Walrasian outcome, the less numerous side of the market captures all of the surplus. Second, increasing the job offer rate leads to *lower* wage growth and mobility. Again, in the Walrasian outcome, workers and firms immediately find their best match and so there is no wage growth or mobility. Our model predicts that due to indirect competition workers do not always benefit from reducing frictions. Workers in tight markets will benefit, while workers in loose markets will have lower wages. These predictions give us new insights into the impact of technologies (*e.g.* the internet) and policies (*e.g.* job search assistance programs) that reduce frictions.

In summary, our main contribution is to show the effect of indirect competition on price dispersion. We develop a model using networks where we characterize the set of prices that sustain any pairwise stable matching in an unrestricted network. Although the theory gives us economic intuition for indirect competition, it is uninformative about whether indirect competition is quantitively relevant. To quantify this effect, we simulate a large number of random networks. From a practical standpoint, numerical analysis is necessary for this problem because solving random networks analytically is intractable for all, but the simplest networks.[5] To perform the numerical analysis, we develop a deferred acceptance algorithm for finding pairwise-stable matches and the full set of prices that supports them. Using pairwise stability as our matchmaking criterion implies that the equilibrium of *any* game that is consistent with Pareto efficiency will be included in our set of matchings. Our simulations

[4] Search models can be mapped into the corresponding network using the same firms and workers where each worker receives a link from a firm when they receive a job offer from that firm in the search model. Many search models are in continuous time. Hence, decreasing frictions in these models increases the offer arrival rates to workers. Yet, since these models are in continuous time, at any instant they only receive one offer.

[5] We are not the first to rely on numerical methods for analyzing random networks. See the discussion in Jackson (2008, Chapter 4, Section 2).

show that indirect competition can lead to price distributions that are almost degenerate (*i.e.* very little price dispersion). In the labor market application, we predict that reducing frictions leads to lower expected wages in loose markets, and that increasing the job offer rate leads to lower wage growth and mobility.

Outline of the Paper

The theoretical analysis begins by studying arbitrary buyer-seller networks. These networks are exogenously formed by linking buyers and sellers, but no restrictions are placed on how many sellers a buyer can be linked with. Sellers offer one unit of an indivisible good. Buyers have single unit demand and also differ in their valuation of the good. To keep the model simple, we focus on the homogeneous goods case, where all sellers have the same valuation for the good. Buyers' utility is their valuation less the price if they obtain the good and zero otherwise. Sellers' utility is the price they receive if they sell the good and their valuation otherwise.

In the model, we assume matches form according to pairwise stability restricted to the network. That is, a buyer obtains a good from a seller at a price p if four conditions hold: first, the buyer and the seller are connected in the network; second, there is no other seller linked to this buyer that is willing to sell at a price lower than p; third, there is no other buyer linked to the seller that is willing to pay a price higher than p; finally, the price p lies between the seller's valuation and the buyer's valuation. This is the weakest criterion for matchmaking that is consistent with Pareto efficiency.

The main proposition characterizes the set of all pairwise stable matchings in an arbitrary network and the set of prices that sustain them. To do that we decompose the original network into a network of fully connected subnetworks. With this, we identify two components that jointly determine the prices that sustain pairwise stable matchings: a pure competitive component and an outside option component. Fully connected subnetworks are competitive markets with a unique price. The links between these subnetworks introduce an outside option component because a buyer from one subnetwork might choose to participate in another. The final prices that sustain a stable matching is the result of these two effects.

We use our characterization of the prices that support pairwise stable matchings to design an algorithm that can simulate large markets. For any given pairwise stable matching, calculating the set of prices that sustain it is simple whenever the network is small (*e.g.* 2 sellers and 3 buyers). However, since the applications of interest (the labor market, internet auctions) involve large economies, calculating the set of sustaining prices is intractable. Hence, we design an deferred acceptance algorithm that outputs a matching (*i.e* a complete specification of buyer-seller matches) that is pairwise stable, and the set of prices that sustain it.

The algorithm we use to simulate the model is a deferred acceptance algorithm that works in two stages. In the first stage, one side of the market (*e.g.* sellers) hold "auctions" and the other side (*e.g.* buyers) sequentially "bid" in their linked auctions. Once no bidder wants to make any new bids, the algorithm ends. A corollary of the main proposition is that the outcome of the algorithm is indeed a stable matching. The second stage finds the lower and upper bounds of the set of prices that support the pairwise-stable matching from the first stage.[6]

We simulate a range of networks to obtain predictions about price distributions. We start with a set of heterogeneous buyers and homogeneous sellers. We parameterize the level of frictions by choosing the number of links per buyer in the network. This determines the total number of links in the network. The simulation then randomly draws links between sellers and buyers. After the network is realized, we run the algorithm and generate a price profile. Given that our algorithm can be applied to arbitrary network structures and it is computationally tractable for both small and large markets, our methodology is applicable to many empirical settings such as labor markets, online buyer-seller platforms, and automobile markets, among others.

We adapt the buyer-seller model to the labor market to explore questions about wage dispersion and growth. In this case, workers are sellers and firms are buyers of their services. To study wage growth, we extend our model to accommodate multiple periods. In this extension each period has three stages. In the first stage, J new firms enter the market and links are formed with the employed and unemployed workers. The parameters J and the number of links determine the market tightness (ratio of J to unemployed workers, denoted by θ) and the level of frictions. Firms that are employing a worker from a previous period do not receive any new links but retain the link to their employee. In the second stage, firm-worker matches are formed given the new network as in the basic buyer-seller model. Applying the buyer-seller model implies that workers accept the vacancy that pays the highest wage. Hence, workers do not consider other aspects of the match, such as future wage growth (see section 4.4 for more on this point). Finally, at the end of the period, some matches are randomly destroyed. The firms that are unmatched at the end of a period (either because the match was destroyed or they could not form a match in the first place) exit the market. We interpret "firms" as time sensitive vacancies so that, if by the end of a period, a vacancy is not filled, it disappears from the job market. When the next period starts, J new firms enter.

The rest of the paper is organized as follows. In the next Section we describe the related literature and highlight how our paper contributes to the current body of work. In Section 3, we present two motivating examples. In Section 4 we describe the model. Section 5 presents the deferred acceptance algorithm. In Section 6 we describe the results of the simulation. Finally, in Section 7, we discuss how our results can be interpreted in the context of eBay auctions and labor markets, and how they can be used to develop a framework for robust

[6] Prices that support a matching are a function, mapping each matched pair into a real number. When we say "maximum" and "minimum" one should understand this in the pointwise order of functions.

econometric analysis. All proofs are in the appendix.

2 Contributions and Related Literature

Primary Literature

There is a vast literature in industrial organization (IO) and labor studying price and wage determination, and the allocation of goods and services in markets.[7] Models can be categorized based on how prices are determined: (1) prices are determined *ex ante* by the seller before meeting buyers (*e.g.* wage-posting models in the labor literature and price-setting firms in the IO literature) or (2) prices are determined *ex post* after the seller meets the buyers (*e.g.* wage-bargaining models in the labor literature and auction models in the IO literature).[8] However, none of these works have addressed how prices are determined in the presence of frictions *and* indirect competition. This is due to modeling assumptions that exclude one, or both, of the following features: (1) both buyers (firms) and sellers (workers) receive multiple links, and (2) prices are determined *ex post* after buyers and sellers (workers and firms) meet. Models that omit (1) restrict the network so that indirect competition cannot occur and models that omit (2) do not allow for indirect competition since agents cannot compete on prices after the network has been formed.

The main contribution of this paper is that indirect competition has significant consequences on the predictions of price dispersion, wage inequality, and wage growth (see Section 6 for details). From a normative perspective, ignoring indirect competition could lead to ill-informed policy decisions on fundamental issues such as the analysis of mergers, consumer welfare, minimum wage, and unemployment assistance programs.[9] In this paper we fill this gap by providing: (1) a simple model to study the determination of prices that allows for indirect competition; (2) a proposition that shows that many links in a network can be redundant (due to indirect competition) and, therefore, a fully connected network is not necessary to obtain the Walrasian outcome; and (3) a simulation of the model that quantifies the importance of indirect competition in large buyer-seller and labor markets.

There is an extensive literature in IO that uses models of search to rationalize price dispersion observed in real world markets.[10] Some of these models include fixed sample and sequential search, whereby firms post prices and consumers search for the best price

[7]See Baye, Morgan, and Scholten (2006), Bajari and Hortacsu (2004), Hasker and Sickles (2010); and Alan (2011), Lentz and Mortensen (2010), Eckstein and Van den Berg (2007), Rogerson, Shimer, and Wright (2005) for detailed surveys in the industrial organization and labor literature, respectively.

[8]We follow Rogerson, Shimer, and Wright (2005) in categorizing search-theoretical models in this way.

[9]For example, in Section 6 we show that job search assistance can lead to lower expected wages in labor markets, in contrast with most search models. For antitrust purposes indirect competition may change the relevant definition of a market when, for example, one firm participates in two markets that appear to be separate (see Cooper 1989 for an early investigation of indirect competition with spatial product differentiation).

[10]IO search models can be mapped into the corresponding network using the same firms and consumers where a consumer receives a link from a firm when the consumer searches for a price quote of the firm.

by incurring a cost each time they obtain an additional price quote (*e.g.* Stigler 1961; Rothschild 1973; Reinganum 1979; MacMinn 1980; Burdett and Judd 1983; Carlson and McAfee 1983; Stahl 1989; Janssen and Moraga-González 2004; Janssen, Moraga-Gonzalez, and Wildenbeest 2005; Arbatskaya 2007; Lester 2011) and clearinghouse models, whereby an informational clearinghouse provides consumers with a list of prices and consumers buy at the lowest price listed (*e.g.* Salop and Stiglitz 1977; Rosenthal 1980; Varian 1980; Baye and Morgan 2001; Baye, Morgan, and Scholten 2004).[11] In IO models, price dispersion typically arises as an equilibrium outcome of the behavior of firms and consumers (*e.g.* as a way to price discriminate among consumers with different search costs). In these models there is *ex ante* competition among firms for consumers, but there is no *ex post* competition among firms for consumers (because prices are posted), thus restricting indirect competition. In the competitive auctions literature, either buyers are allowed to bid in only one auction (*e.g.* McAfee 1993; Wolinsky 1988; Julien, Kennes, and King 2000) or there are no frictions (*e.g.* Peters and Severinov 1997, 2006). Restricting buyers to bid in only one auction restricts indirect competition in the market. By allowing buyers to be linked to many sellers, our model generates competition among sellers absent in competitive auctions models with frictions.[12]

There is a large literature in labor that uses on-the-job search models to understand wage dispersion in labor markets (see *e.g.* Mortensen 2005 and Postel-Vinay and Robin 2002). In wage posting and competitive search models (*e.g.* Burdett and Mortensen 1998 and Moen 1997), there is *ex ante* competition among firms for workers. Firms must post and commit to wages before coming into contact with workers. Models of directed search with multiple applications (*e.g.* Kranton and Minehart 2001; Albrecht, Gautier, and Vroman 2006; Kircher 2009; Galenianos and Kircher 2009; Walthoff 2012; Albrecht, Gautier, and Vroman 2014) are related to our model in that workers can apply for multiple jobs in a period. In these models, firms also post wages, but workers can apply to more than one vacancy. Since firms post wages, workers are not able to negotiate wages *ex post* between different firms, disallowing indirect competition. A central question analyzed in this literature is whether the level of entry is efficient. Although an important question, the efficiency of the entry decision is not the focus in our paper. In Albrecht, Gautier, and Vroman (2014), workers post selling mechanisms and firms choose one worker to interact with, thus ruling out indirect competition. Albrecht, Gautier, and Vroman (2014) allows for wage competition between firms but not between workers. Models with Bertrand competition between firms (*e.g.* Postel-Vinay and Robin 2002) have recently become popular in the empirical labor

[11] We follow Baye, Morgan, and Scholten (2006) in categorizing IO search models in this way.

[12] One way to interpret our model is in terms of a competing auctions environment similar to Peters and Severinov (2006), but where buyers are linked with a subset of the sellers (*i.e.* when there are market frictions). The environment of Peters and Severinov (2006) is frictionless in the sense that any buyer may participate in any auction. The bidding rule proposed by Peters and Severinov (2006) for their frictionless competing auctions environment is not a Perfect Bayesian Equilibrium when frictions are present (Donna, Schenone, and Veramendi 2015).

search literature and are also closest to our model.[13] On one hand, these models have *ex post* competition between firms by allowing a firm to make counter-offers when their employees come into contact with rival firms. On the other hand, there is no *ex post* competition between workers. In contrast to the labor literature, we allow both firms and workers to compete *ex post* on wages, which is necessary to generate indirect competition.

Other Related Literature

Since we model markets of buyers and sellers, our paper relates to other fields such as network theory, matching, financial networks, and computer science. Here we briefly relate our paper to these other fields.

Although there are papers in the network theory literature that analyze static models where multiple buyers negotiate with multiple sellers, none of them study price dispersion.[14] Fainmesser and Galeotti (2015) develop a framework for studying optimal pricing of a monopolist in the presence of indirect competition. Kranton and Minehart (2001), Gautier and Holzner (2013), and Elliott (2014) study efficiency of these markets.[15] Corominas-Bosch (2004) examines the equilibrium payoffs of an alternating offers game to answer two questions: what is the set of networks that supports a specific allocation (similar to what we call the "Walrasian outcome"), and what are the networks that only support this allocation.[16] Manea (2011) investigates bilateral bargaining in networks. Polanski and Vega-Redondo

[13]See *e.g.* Bagger, Fontaine, Postel-Vinay, and Robin (2014).

[14]There is a vast literature in economics and sociology that studies information transmission in social networks (for example, see Myers and Shultz 1951; Rees 1966; Montgomery 1991; Calvó-Armengol and Jackson 2004, 2007; and the references there). Pairwise stability has been used to study network formation (*e.g.* Jackson and Wolinsky 1996). We do not study information transmission nor network formation. In contrast, we use pairwise stability as the matchmaking criterion in a network with frictions. We characterize pairwise stable allocations and the set of prices that sustain them in networked markets. We then use this characterization to study price dispersion in these networks. See Jackson (2008) for a detailed review of the literature on social and economic networks.

[15]Kranton and Minehart (2001) study efficiency when using a public ascending auction to clear the market. They do this in two steps: first, for each exogenously given network, they study conditions under which the equilibrium outcome is efficient; second, they study endogenous network formation and prove that the endogenously formed networks satisfy their conditions for efficiency. Gautier and Holzner (2013) study efficient allocations in arbitrary bipartite graphs by studying the set of maximal matchings. Elliott (2014) studies the efficiency of the entry decisions of firms and workers in a labor market model. Elliott (2014) shows that, for his particular labor market game, there exists a perfect Bayes Nash equilibrium (PBNE) where the payoffs in a sparse network are the same as the payoffs in the perfectly competitive outcome. In contrast, our results are game-free: the only constraint we impose on allocations is that they be pairwise stable. So we remain agnostic about the mechanism (game) that generates such allocations. By implication, we are also free of equilibrium assumptions (Nash Equilibrium, PBNE, rationalizability, *etc.*). The results are complementary since our model does not nest his and vice-versa. Workers in his model choose which firms to apply to, while workers in our model receive links exogenously. Our assumption is similar to the assumption made in the random search literature, where workers and firms meet randomly; Elliott's assumption is similar to the assumption made in the directed search literature, where workers choose which firms to meet (see Rogerson, Shimer, and Wright 2005 for more on modeling decisions in search models). We have workers and firms meet exogenously since our goal is not to develop a theory of labor market participation, but a parsimonious model of wage dispersion.

[16]A comparison between our "abstractions" and Corominas-Bosch's decomposition construction can be found in Section C of the online appendix.

(2013) study efficiency in a pairwise stable trading network where buyers and sellers meet in pairs at every point in time and bargain over their bilateral surplus. Elliott (2015) extends the Kranton and Mineheart model to consider different levels of bargaining power, different cost shares, negotiated investments, and ex ante heterogeneous gains from trade.

Moreover, the network literature typically proposes a concrete game to be played within these networks and focuses under which conditions (if any) the equilibria of these games are Pareto efficient.[17] However, to an econometrician, the concrete mechanism through which the goods are allocated is rarely observable. Hence, it is useful to impose a minimal set of restrictions on this allocation mechanism. Remaining agnostic with respect to the details of the game allows the researcher to weaken the behavioral assumptions that would be specific to the game otherwise imposed; for example, whether buyers are submitting simultaneous (or sequential) bids to sellers, whether sellers are proposing simultaneous (or sequential) prices to the buyers, whether they are alternating bids and price propositions, *etc.* The weakest criterion for matchmaking that is consistent with Pareto efficiency and agnostic regarding the game details is pairwise stability. For this reason, we focus on pairwise-stable matchings and characterize the prices that sustain them.

Our paper is related to the literature on the matching role of markets (*e.g.* Gale and Shapley 1962; Shapley and Shubik 1972; Shapley and Scarf 1974; Crawford and Knoer 1981; Kelso and Crawford 1982; Ausubel and Milgrom 2002; Hatfield and Milgrom (2005); and Hatfield and Kojima 2008, 2010).[18] We follow the matching literature by developing a deferred-acceptance algorithm that picks specific stable matchings. The algorithm has two stages. The first stage outputs an allocation and is motivated by the wage adjusting process in Crawford and Knoer (1981) and Kelso and Crawford (1982). This allocation has the property that there exist prices for which it is pairwise stable. The second stage outputs two prices: the pointwise minimum price at which the stage 1 allocation is stable, and the pointwise maximum price at which the stage 1 allocation is stable.[19]

There is also a growing literature that uses networks to study trading in financial settings such as over-the-counter (OTC) markets (*e.g.* Gofman 2011; Malamud and Rostek 2013; Babus and Kondor 2013; and Alvarez and Barlevy 2014). They use concrete games to investigate OTC markets where dealers trade with other dealers. In contrast, we study markets where the set of sellers and buyers belong to two disjoint sets: sellers can only trade with buyers while buyers can only trade with sellers (*i.e.* bipartite networks as defined in Section 4).

In the computer science literature, Kakade, Kearns, and Ortiz (2004) study trade using an Arrow-Debreu economy (without firms) where consumers trade goods with other con-

[17] Elliott (2015) is the only paper we are aware of that uses pairwise stability as the matchmaking criterion in a network with frictions, but he does not investigate price dispersion.

[18] Roth (2008) discusses recent progress in the study of deferred-acceptance algorithms. See Roth and Sotomayor (1990) for a comprehensive survey of the two-sided matching literature.

[19] See section 5 for details.

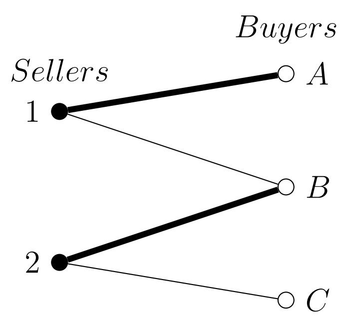
sumers. Kakade, Kearns, Ortiz, Pemantle, and Suri (2004) use a concrete game to study the interaction between the statistical structure of the underlying network and the variation in prices at equilibrium.[20] Our model also relates to the classical assignment problem. The goal of this literature is to develop efficient algorithms (*e.g.* Hungarian algorithm, auction algorithms, simplex algorithms, *etc.*) to maximize total output in a setup similar to ours (*i.e.* obtain the Pareto efficient matching). The first stage of our algorithm is similar to the auction algorithm which is known to perform well with sparse matrices (see Bertsekas 1992 for a survey). The goal of the algorithms literature is to find one allocation and possibly one price per match that maximizes output. In contrast, our goal is to find the *set* of prices that support pairwise stable allocations and study the relationship between frictions and price dispersion.

3 Two Motivating Examples

The following two examples illustrate indirect competition and the usefulness of our theoretical tool for characterizing pairwise stable matches in networks. In both examples, we assume that sellers sell identical goods.

We use Example 1 to get intuition about indirect competition. Indirect competition is a feature of the structure of the network, whereby buyers that are not connected to the same seller compete with each other. Likewise, indirect competition between sellers occurs when sellers that are not connected to the same buyer compete with each other. The simplest example that includes this feature involves two sellers and three buyers.

Example 1. Assume that buyers A, B and C are ordered in their valuations ($\mu(A) > \mu(B) > \mu(C) > 0$) and sellers 1 and 2 have the same valuation (normalized to 0). Consider the following network, where thick lines indicate the pairwise stable matches:

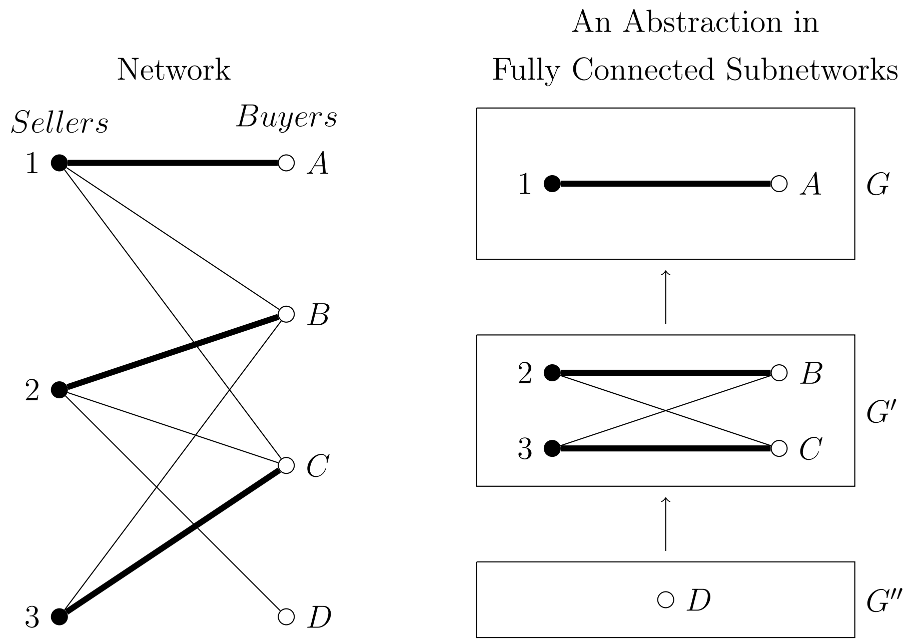


[20] Kakade, Kearns, Ortiz, Pemantle, and Suri (2004) analyze networked markets where the numbers of buyers and sellers are equal. They show that, for their particular game, there is no equilibrium price dispersion when the following conditions hold: (1) the number of buyers and sellers go to infinity, (2) the links are formed uniformly at random, and (3) the probability of forming a link is high enough. In their model there is limiting price dispersion (as the number of buyers and sellers go to infinity) when the network is formed via preferential attachment. In contrast, the only constraint we impose on allocations is that they be pairwise stable. So our results are game-free. In addition, in our simulations we study price dispersion in bipartite networks varying arbitrarily the number of buyers, the number of sellers, and the number of links per seller or buyer.

The buyer-preferred stable match is for buyer A to pay $\mu(C)$ to seller 1 and buyer B to pay $\mu(C)$ to seller 2. Buyer B cannot pay less than $\mu(C)$ because buyer C will poach seller 2. Likewise, buyer A cannot pay less than $\mu(C)$ because buyer B will poach seller 1. In this example, buyer A is indirectly competing with buyer C. Indirect competition forces buyer A to pay $\mu(C)$ even though buyer C is not linked to seller 1. For this reason, adding a link between buyer C and seller 1 is redundant (*i.e.* does not affect prices or allocations). To further highlight the role of indirect competition, notice that if buyer C dropped out of the market, then both buyer A and buyer B paying zero is the buyer-preferred pairwise stable match.

Example 2 demonstrates how we use a network decomposition (which we call *abstractions*) to highlight the importance of indirect competition and characterize the prices that sustain pairwise stable matches. An abstraction in fully connected networks is a decomposition of a network into fully connected subnetworks that satisfy the following properties: (1) each node in the abstraction is a subnetwork of the original network, (2) each link in the original network is either a link within a subnetwork in the abstraction or a link that connects two distinct nodes in the abstraction. This construction uses that fully connected subnetworks are competitive markets with no price dispersion. The following example demonstrates one possible abstraction of a network.

Example 2. Consider a market with three sellers and four buyers. Assume buyers and sellers are connected as in the network below. Assume that the buyers are ordered in their valuations ($\mu(A) > \mu(B) > \mu(C) > \mu(D) > b$) and the sellers have the same valuation (*i.e.* $b(1) = b(2) = b(3) = b$). Thick lines indicate a pairwise stable matching.

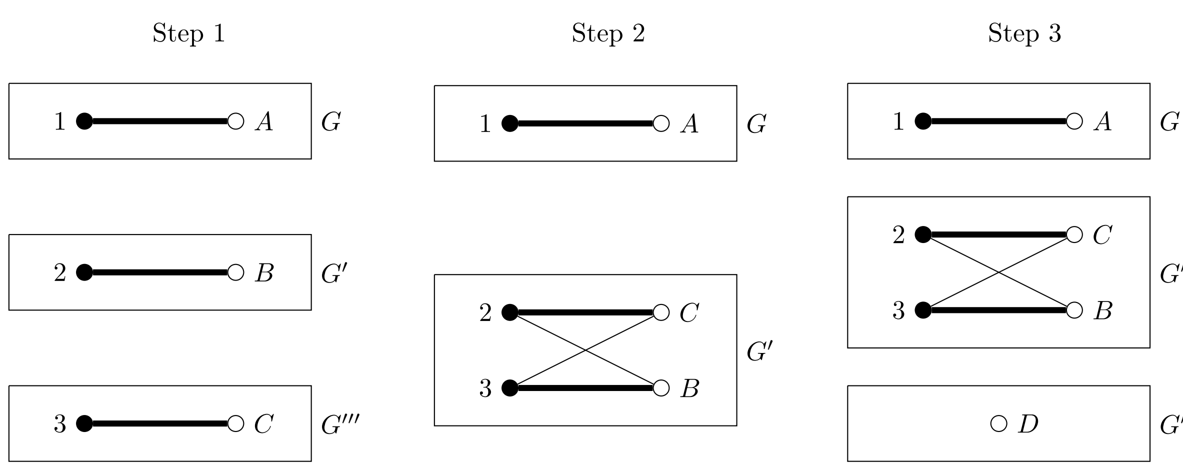


Even though many prices sustain it, there is an essentially unique pairwise stable match: Buyer A buys from seller 1, buyer B buys from seller 2, and buyer C buys from seller 3.[21]

Abstractions are useful to highlight how indirect competition affects price formation.[22] We proceed with 3 observations. (1) Consider subnetwork G' as an independent subnetwork. In this case, a pairwise stable matching corresponds to a perfectly competitive allocation: both B and C buy a good from sellers 2 and 3, and pay a price between b and $\mu(C)$. (2) However,

[21] Nothing changes if buyers B and C are switched, so that B buys from 3 and C buys from 2.

[22] One way to construct an abstraction is to follow four steps: (1) Form a subnetwork around each stable match, (2) combine subnetworks that are fully-connected, (3) form a separate subnetwork for each unmatched buyer (seller), and (4) form a directed link between subnetworks if there is a buyer in one subnetwork that is connected to a seller in another subnetwork. The direction of the link will point from the subnetwork that contains the buyer to the subnetwork that contains the seller. Although there may not be a unique assignment in step 2, any assignment will characterize the same set of pairwise stable matches and their supporting prices.



subnetwork G' is not an independent subnetwork. Since buyer D is linked to seller 2, then sellers C and B must pay at least $\mu(D)$. (3) Similarly, in subnetwork G buyer A will have to pay a price between the price in subnetwork G', and $\mu(A)$. Note that, even though A is not linked to D, indirect competition makes it such that A must pay a price larger than $\mu(D)$. More generally, the direction of the link between subnetwork G' and subnetwork G describes the relationship of the prices that prevails in each subnetwork, namely $p(G') \leq p(G)$.

We call this an abstraction because the identity of the buyer in subnetwork G' linked to the seller in subnetwork G is irrelevant; similarly, the identity of the seller to which D is connected is irrelevant. Abstractions also help clarify which links are redundant. For example, the link between buyer C and seller 1 establishes a link between G' and G, so the link between buyer B and seller 1 is redundant. Similarly, the link between buyer B and seller 1 establishes a link between G' and G, so the link between buyer C and seller 1 is redundant. The important point is that only one of these links is needed, but which one in particular is not relevant. Likewise a link between buyer D and seller 1 is redundant, because there is already a directed link from G'' to G' to G. In other words, buyer D is indirectly competing with buyer A through the links: D – 2 – B – 1 – A. In this way, abstractions facilitate our understanding of the competition in the network. What is relevant is the existance of the links between subgraphs, but not the specific identity nor the number of buyers and sellers that generate those links.

4 The Model

4.1 Buyer-Seller Model

We consider buyers and sellers that wish to match pairwise. Sellers differ in their valuation and offer a homogeneous good. For simplicity, we assume that sellers have no idiosyncratic preferences over the buyer they sell to. Buyers differ in their valuation and have single unit demand. A buyer with valuation μ that buys from a seller at price p has utility $\mu - p$ and 0 otherwise. The sellers' utility is the price, p, if they sell the good, and their valuation, b, if they do not. Single unit demand implies we focus on pairwise matching.

Matching takes place in exogenous buyer-seller networks. Each buyer is linked with a subset of sellers. That a buyer is not (necessarily) linked to all possible sellers captures search frictions in the environment.

The formal model we use to capture these interactions is a graph-theoretic model. A graph is a set of nodes connected by links (or edges). We say the graph is undirected if the direction of the link does not matter. Otherwise, we say the graph is directed. We say that the graph is bipartite if the set of nodes can be partitioned into two sets such that no two nodes in the same set are connected to each other. In our framework, buyers and sellers constitute a bipartite undirected graph: first, the set of nodes is partitioned into buyers and sellers; second, a buyer is linked to a seller if and only if that seller is linked to that buyer; and third, no buyer (respectively seller) is connected to another buyer (respectively seller). For ease of exposition, we leave formal definitions for appendix A.

Since graphs tell us which buyers are connected to which sellers, but they do not tell us the valuation of buyers nor the valuation of the sellers, we extend the definition of the graph to the definition of a network. Intuitively, a network is a graph where each node is given a numerical value. This value is interpreted as the "valuation" of the buyer or seller. For the rest of the paper, even if not explicitly mentioned, $\mathcal{J}$ denotes the set of buyers, and j indexes buyers. Similarly, $\mathcal{I}$ denotes the set of sellers, and i indexes sellers.

Given a network, a matching (typically denoted M) is any subset of the set of links such that three properties hold: first, each buyer is matched to at most one seller (recall that buyers have unit demand); second, each seller is matched to at most one buyer; and finally, if a seller is matched to a buyer then the buyer is matched to the seller. When a seller-buyer pair (i, j) is in matching M, we say that i and j are *matched*. Moreover, given a matching M, we define $i^* : \mathcal{J} \to \mathcal{I} \cup \{\emptyset\}$ as the function that maps each buyer to the seller with whom it is matched, or to the symbol $\emptyset$ if the buyer is unmatched. Likewise, $j^* : \mathcal{I} \to \mathcal{J} \cup \{\emptyset\}$ is the function that maps each seller to the buyer with whom it is matched, or to the symbol $\emptyset$ if the seller is unmatched. Moreover, given a set of links (say, E), a function that maps these links into real numbers (denoted p_E) is called a price function. This real number is interpreted as the price that would prevail if the buyer was to buy the good from the seller. The price function is individually rational if, for each buyer-seller edge, it specifies a price that lies between the seller's valuation and the buyer's valuation. Finally, given a matching M and a price function p_M, the function v summarizes the virtual price each agent (buyer or seller) pays or is getting payed, without having to explicitly distinguish if the they are matched or not. We call these functions v the *payment functions*. In symbols: for each j and i,

$$v(j) = \begin{cases} \mu(j) & \text{if} \quad i^*(j) = \emptyset \\ p_M(i^*(j), j) & \text{if} \quad i^*(j) \neq \emptyset, \end{cases}$$

and

$$v(i) = \begin{cases} b(i) & \text{if} \quad j^*(i) = \emptyset \\ p_M(i, j^*(i)) & \text{if} \quad j^*(i) \neq \emptyset. \end{cases}$$

Next, we define pairwise stability of a matching M with respect to a price function p. Pairwise stability means that the edges in M are priced such that individual rationality holds, and there are no mutually benefit matches by agents that are linked but are not matched (*i.e.* agents linked by an edge $e \in E \setminus M$). In other words, any extension of p_M to all edges cannot yield Pareto improvements over the match M executed at prices p_M. Note that pairwise stability only requires that an agent is able to observe the prices of his linked counterparts, but not who they are linked to.

Definition (block). *Let M be a matching and p be a price function. Suppose a seller i is linked to a worker j, but i is not matched to j. We say the pair (i,j)* blocks *(M, p_M) if $v(i) < v(j)$.*

Definition (pairwise stability). *Given a network $\mathcal{N}$ and a matching M, we say M is* pairwise stable *in $\mathcal{N}$ at prices p_M if the following hold:*

- No blocking: *no seller-buyer pair (i,j) blocks (M, p_M),*
- Individual rationality: *for all seller-buyer pairs (i,j) that are matched, $p_M(i,j) \in [b(i), \mu(j)]$.*

When the identity of the network is clear, we simply say M is stable at prices p_M (omitting "in $\mathcal{N}$").

In fully connected networks it is simple to characterize stable matchings. Indeed, if M is stable at prices p_M, then all prices must be the same. To see this, assume i is matched to j, i' is matched to j', and let p be any individually rational price function defined over all seller-buyer pairs. Then, $p(j,i) \leq p(j,i') \leq p(j',i') \leq p(j',i) \leq p(j,i)$, where all these terms are well defined because the network is fully connected. As a corollary, all stable matchings can be characterized by whether there are more buyers than sellers or vice versa. Intuitively, stable matchings are those matchings which are maximal and can be sustained by individually rational prices that price out the side of the market (sellers or buyers) that is in excess. In this regard, the matchings and prices we obtain from pairwise stability in fully connected networks are those that would prevail if this was a perfectly competitive economy. We summarize this in the following remark.

Remark 1. *Let $(\mathcal{J}, \mathcal{I}, E; \mu, b)$ be a fully connected network, where E is the set of edges. Let $J = \#\mathcal{J}$, $I = \#\mathcal{I}$. Assume that $\bar{b} = \max\{b(i) : i \in \mathcal{I}\} \leq \min\{\mu(j) : j \in \mathcal{J}\} = \underline{\mu}$. Let $M \subset E$ be a matching.*

- *If $I > J$, M is stable if, and only if,*
  - *All buyers are matched: For each $j \in \mathcal{J}$ there is $i \in \mathcal{I}$ such that $(j,i) \in M$.*
  - *Only lowest valuation sellers are matched: If $i \in \mathcal{I}$ is such that $\#\{i' : b(i) > b(i')\} \geq J$ then there is no $j \in \mathcal{J}$ such that $(j,i) \in M$.*
  - *Seller valuations determine matching prices: For each $(j,i) \in E$, $p(j,i) = p$ where $p \in [\max\{b(i) : (\exists j \in \mathcal{J})$ such that $(j,i) \in M\}, \{\min\{b(i) : (\nexists j \in \mathcal{J})$ such that $(j,i) \in M\}]$.*
- *If $I = J$, M is stable if, and only if,*
  - *All buyers are matched: For each $j \in \mathcal{J}$ there is $i \in \mathcal{I}$ such that $(j,i) \in M$.*
  - *All sellers are matched: For each $i \in \mathcal{I}$ there is $j \in \mathcal{J}$ such that $(j,i) \in M$.*

– *Sellers sell at an intermediate price: For each* $(j,i) \in E$*,* $p(j,i) = p$ *where* $p \in [\bar{b}, \underline{\mu}]$*.*

- *If* $I < J$*,* M *is stable if, and only if,*

  – *Only highest valuation buyers are matched: For each* $j \in \mathcal{J}$ *if* $\{j' : \mu(j') > \mu(j)\} \geq I$ *then there is no* $i \in \mathcal{I}$ *such that* $(j,i) \in M$*.*

  – *All sellers are matched: For each* $i \in \mathcal{I}$ *there is* $j \in \mathcal{J}$ *such that* $(j,i) \in M$*.*

  – *Buyer valuations determine matching prices: For each* $(j,i) \in E$*,* $p(j,i) = p$ *where* $p \in [\max\{\mu(j) : (\nexists i \in \mathcal{I}) \text{ such that } (j,i) \in M\}, \min\{\mu(j) : (\exists i \in \mathcal{I}) \text{ such that } (j,i) \in M\}]$*.*

4.2 Two Propositions

In this section we present two propositions which we use to understand how indirect competition affects the degree of price dispersion in the market. Given a buyer-seller network and a stable matching in such network, Proposition 1 states that only a subset of links is relevant for determining the prices that sustain that matching. To identify this set of links, we define the *abstraction* of a network. As the name suggests, this is a construction that abstracts away from links that are irrelevant for determining the prices that sustain a given stable matching. Although useful for understanding which links are relevant in a network, it does not fully characterize the set of prices that support a pairwise stable matching. We then define maximal abstractions and use them to characterise the full set of prices that support any given pairwise stable match. The characterization is provided in Proposition 2.

We start by defining the abstraction of a network. Slightly abusing notation (see remark 5 in appendix A) an abstraction of a network is a directed graph with nodes and edges as follows: each node is a subnetwork of the original network, and each edge in the original network is either (i) an edge within a subnetwork in the abstraction or (ii) connecting two distinct nodes in the abstraction. We now present a formal definition for the abstraction of a network.

Definition (abstraction). *Let* $\mathcal{N}$ *be a buyer-seller network. From* $\mathcal{N}$ *construct a directed graph,* H*, as follows:*

- *Each node in* H *is associated with a fully-connected subnetwork of* $\mathcal{N}$*. These subnetworks should be disjoint, in the sense that if a seller (or buyer) belongs to a subnetwork associated to a node in* H*, it cannot belong to a subnetwork associated with another node.*

- *A node* h *in* H *is linked to a node* h' *in* H *if the subnetwork associated to* h *contains a buyer that, in* $\mathcal{N}$*, is linked to a seller that belong to the subnetwork associated with* h'*.*

Remark 2. *Since a graph that consists of a single node is fully connected, an abstraction in fully connected networks always exists.*

Remark 3. *Abstractions in fully connected networks will not necessarily be unique.*

Remark 4. *Consider a network $\mathcal{N}$, a matching M, and a price function p_M. Let H be an abstraction in fully connected networkss. If p_M is such that M is stable in $\mathcal{N}$ at prices p_M, then p_M induces prices p^* for each subnetwork in the abstraction as follows:*

- *If h contains only seller j, $p^*(h) = \mu(j)$,*
- *If h contains only buyer i, $p^*(h) = b(i)$,*
- *If h contains a matched pair $(i, j) \in M$, then $p^*(h) = p_M(i, j)$.*

This is well defined because subnetworks are fully connected. Conversely, given a function p^* that prices each subnetwork in the abstraction, this function induces prices p_M in the original network as follows:

- *If $(i, j) \in M$ and link (i, j) is contained in subnetwork h, then $p_M(i, j) = p^*(h)$.*

Given an abstraction of a network, a price function for the abstraction is a function p^* that assign a number (*i.e.* a price) to each node in the abstraction. We say a matching $M \subset E$ is stable with respect to an abstraction at prices p^* when three conditions hold. First, the abstraction does not break M: each buyer-seller match in M belongs to the same subnetwork in the abstraction. Second, prices $p^*(\cdot)$ induce pairwise stability in each subnetwork. Third, if node h in the abstraction is linked to node h', then $p^*(h) \leq p^*(h')$. That h is linked to h' implies that some buyer in the subnetwork associated to h is linked to some seller in the subnetwork associated to h'; thus, the "price" that prevails in h should be lower than the price that prevails in h'. We call this last condition the "cheapest sorting" condition.

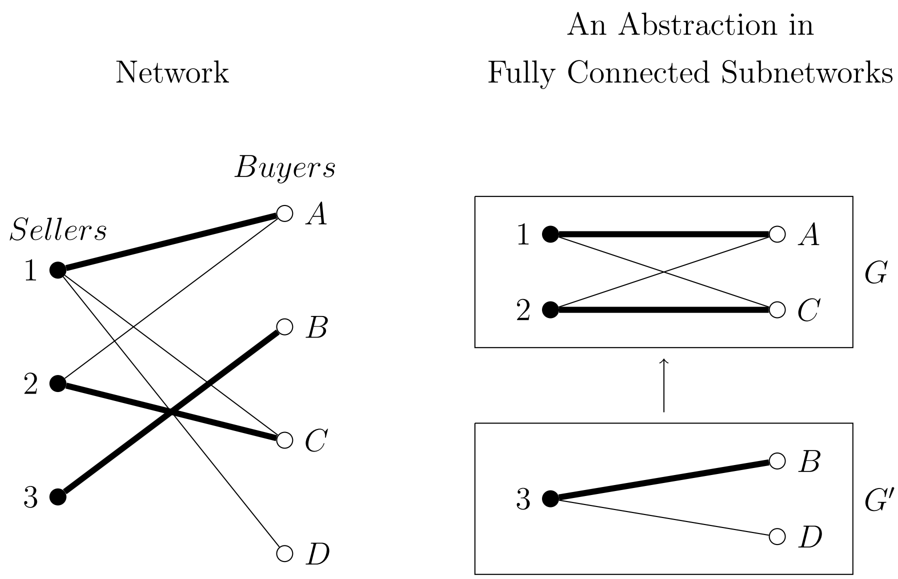
With these definitions we can state our first proposition.

Proposition 1. *Let $\mathcal{N}$ be a network and M be a matching. Then the following are equivalent:*

1. *There exists a price function, p_M, such that M is stable in $\mathcal{N}$ at prices p_M.*
2. *There exists an abstraction in fully connected networks, and a price function p^* for the abstraction, such that M is stable with respect to the abstraction at prices p^*.*

While most of the intuition for our results comes from Proposition 1, this proposition does not characterize the full set of prices that support any given pairwise stable match. Indeed, Proposition 1 tells us that any matching that is stable in a network $\mathcal{N}$ at some prices p_M is also stable with respect to some abstraction of $\mathcal{N}$. Let H denote one such abstraction. Then, through stability with respect to each subnetwork and cheapest sorting, H imposes a set of constraints on price functions for the original network with the following property: if a different price function (say, p'_M) satisfies the constraints imposed by H, then M is also stable in $\mathcal{N}$ at p'_M. In this sense, abstractions identify sets of prices (for the original network) that make a matching stable. The problem is that, in general, the set of conditions on prices imposed by any one abstraction are sufficient, but not necessary, for stability. The example below shows this.

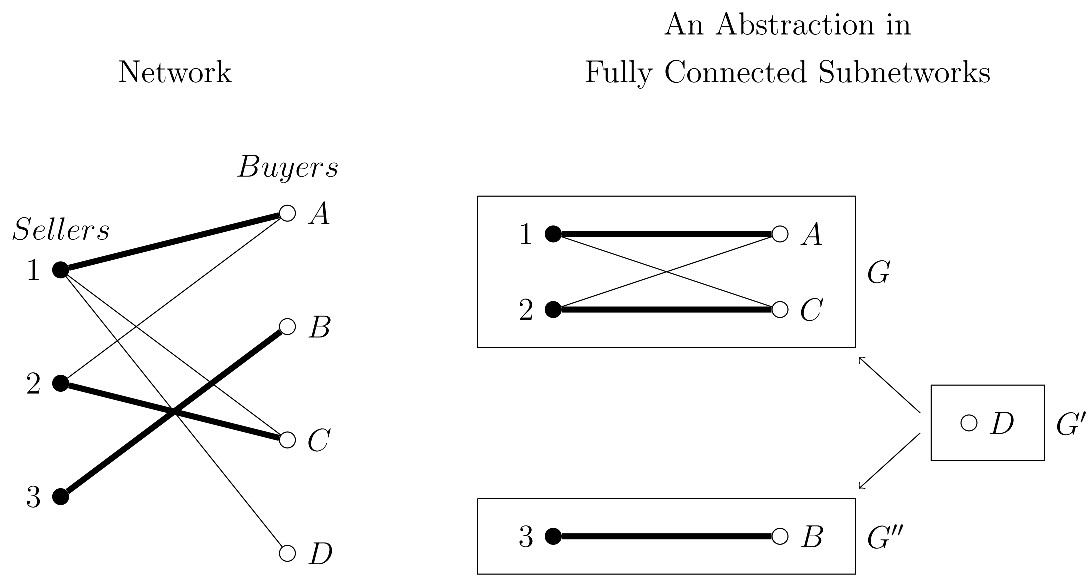
Example 3. The abstraction on the right imposes two constraints that prices need to satisfy in order to make the match stable with respect to the abstraction: stability in every subnetwork implies $p^*(G) \in [b, \mu(C)]$, $p^*(G') \in [\mu(D), \mu(B)]$; cheapest sorting implies $p^*(G') \leq p^*(G)$. Notice that p^* induces prices in the original network (say, p_M) that make M stable. However, there are other prices p'_M that also make M stable in the original network but induce prices p^{**} on the abstraction that do not satisfy constraints mentioned above. For example, $p'_M(1, A) = p'_M(2, A) = \mu(C)$ and $p'_M(3, B) = \mu(B)$ are such that M (in the original network) is stable at p'_M, but induce prices (p^{**}) in the abstraction that violate cheapest sorting (they induce $p^{**}(G) = \mu(C) < \mu(B) = p^{**}(G')$).



Proposition 2 identifies a class of abstractions, which we call *maximal*, such that the constraints imposed by these abstractions are not only necessary but also sufficient for stability. After defining maximal abstractions we present an example and the statement of proposition 2.

Definition (maximal abstraction). *Let $\mathcal{N}$ be a network and M be a matching. We say H is a* maximal abstraction *for M if for every unmatched buyer j, the subnetwork of H that contains j, contains only j.*

Example 4. The abstraction on the right is maximal for M. This abstraction places the following constraints on prices that make M stable with respect to the abstraction: $p^*(G) \in [\mu(D), \mu(C)]$, $p^*(G'') \in [\mu(D), \mu(B)]$. Since this abstraction is maximal for M, then p^* induce prices p_M in the original network such that M is stable with respect to p_M. Conversely, any price function p'_M that makes M stable in the original network induces a price function p^{**} in the abstraction that satisfies the above constraints. This is in contrast to example 3, where the abstraction was not maximal.



Proposition 2. *Let $\mathcal{N}$ be a network and M be a matching. Let H be a maximal abstraction for M. Then M is stable with respect to some price function p_M if and only if p_M induces prices p^* in the abstraction such that M is stable with respect to H at prices p^*.*

4.3 Proposition Application

In this subsection we illustrate four ways in which abstractions and the propositions are useful to understand pairwise stable matches and their supporting prices. For this we use Example 2.

First, propositions 1 and 2 are useful to decompose the prices that sustain pairwise stable matches into a *competitive component* and an *outside option component*. We proceed in three steps. (A) By proposition 2, any price function that makes M stable in the original network induces prices p^* that make M stable with respect to the maximal abstraction. Moreover, consider nodes G' and G as independent networks and note that they are fully connected networks. (B) We use step (A) and Remark 1 to conclude three things: that buyers A, B and C should match to sellers, that buyers B and C should pay a price between b and $\mu(C)$, and that buyer A should pay between b and $\mu(A)$. Steps (A) and (B) imply that we would observe these matches and supporting prices if these were two independent, perfectly competitive economies. This is the competitive component of prices that sustain pairwise stable matchings. (C) Note that buyer D is linked to seller 2, as indicated by the edge in the abstraction that links G'' to G'. Thus, any price that sustains a pairwise stable matching must also satisfy that buyers B and C pay sellers 2 and 3 no less than $\mu(D)$. Similarly, buyer A pays to seller 1 no less than what B and C are paying to sellers 2 and 3. This is reflected by the cheapest sorting condition, and is what we call the outside option component. Therefore, Proposition 2 implies that all prices that support pairwise stable matchings (in the original network) can be decomposed into competitive and outside option components.

Second, a corollary of the above decomposition is that, to calculate the prices that sustain a particular stable matching, only the subnetworks and the links connecting the subnetworks in a maximal abstraction are relevant. In particular, neither the identity nor the number of buyer-seller pairs that generate those links is relevant. For example, consider a modified network where the edge linking D to 2 is replaced by an edge linking D to 3. The original matching is still stable in the abstraction at the original prices. Thus, this matching is also stable in the modified network. Moreover, if the network had D linked to both 2 and 3, one of those links would be redundant given the existance of the other. The relevant aspect of these networks that sustains the proposed matching at the proposed prices is that at least one buyer in $\{B, C\}$ is connected to seller 1, and that D is linked to at least one seller in $\{2, 3\}$. However the exact identities of these buyers and sellers is irrelevant. More generally, given a pairwise stable matching, and a maximal abstraction for that matching, any network obtained by drawing edges in a manner consistent with the abstraction will support the given matching. Proposition 4 in the appendix formalizes this point.

Third, the Proposition 2 is also useful to characterize the set of all prices that can sustain any given stable matching. In Example 2, a stable matching is $M = \{(C, 3), (A, 1), (B, 2)\}$, but there are many prices that can sustain M. Proceeding as we did before, we can conclude that a price function p sustains M if and only if it satisfies the following properties:

- $p(C, 3) = p(B, 2) \in [\mu(D), \mu(C)]$,
- $p(A, 1) \in [p(B, 2), \mu(A)]$.

Fourth, Proposition 2 is also useful to prove that the algorithm in Section 5 finds pairwise stable matchings in any given network, and the upper and lower bounds of the set of prices that sustain those matchings. In Section 5 we present a description of the algorithm and its properties. In Appendix C we present the formal algorithm and formal proofs.

4.4 Labor Market Model with On-the-job Search

In this section we adapt the buyer-seller model to the labor market, where workers are sellers and firms are buyers. We assume workers do on-the-job search and that firms have single unit demand, so a firm is equivalent to a vacancy.

At the beginning of period 1, a finite bipartite graph is randomly drawn between workers and firms. We use $(\mathcal{I}_1, \mathcal{J}_1, E_1)$ to denote the time 1 graph, where $\mathcal{I}_1$ is the set of period 1 workers, $\mathcal{J}_1$ is the set of period 1 firms, and E_1 is the set of links between buyers and sellers. Let I and J denote the respective number of workers and firms, and we define market tightness as the ratio of J to unemployed workers, denoted as θ_1. Conditional on the graph we assign productivities to firms, denoted with $\mu(\cdot)$, drawn *i.i.d.* from a continuous distribution F with support in the interval $[\underline{\mu}, \overline{\mu}]$. To keep the model simple we assume that all workers have the same reservation wage. Using the notation of the buyer-seller model, $b(i) = b$ for each $i \in \mathcal{I}_1$.

After the period 1 graph has been realized, we pick a specific pairwise stable matching M_1^0 at wages $w_{M_1^0}$ using the algorithm that we describe in Section 5. Matched firms receive a period utility of $\mu(j) - w_{M_1^0}(j, i^*(j))$, matched workers receive a period utility of $w_{M_1^0}(j^*(i), i)$, unmatched workers receive their reservation wage b, and unmatched firms receive utility 0 and leave. After these utilities are realized, there is an exogenous job destruction shock. This means that each match $m \in M_1^0$ is dissolved with probability $\delta > 0$. Firms whose links are dissolved become unmatched and leave the market. We denote with M_1 the period 1 matching after the exogenous job destruction.

Now consider the beginning of period $t \geq 2$. Given the matching M_{t-1} of period $t-1$, we add J new firms and no new workers (*i.e.* $\mathcal{I}_t = \mathcal{I}_{t-1}$). We draw the productivities for these new firms from the same distribution F. We randomly draw links between workers and firms that satisfy the following three conditions. First, positive probability is assigned only to graphs with vertices in $\mathcal{I}_t \cup \mathcal{J}_t$, where these denote the set of period t workers and firms, respectively. Second, matches from period $t-1$ are not dissolved (formally, $M_{t-1} \subset E_t$, where E_t is the set of period t edges). Finally, matched firms receive no new applications, but matched workers may apply to new firms because they can do on-the-job search (that is, if $(j, i) \in M_{t-1}$ then $\{i' : (j, i') \in E_t\} = \{i\}$, but no constraints are placed on $\{j' : (j', i) \in E_t\}$). We denote the corresponding graph with $(\mathcal{I}_t, \mathcal{J}_t, E_t)$. The reservation wage of workers who were not matched in $t-1$ is b; the reservation wage for workers who were matched in $t-1$ is the worker's wage, $w_{M_{t-1}}$. As before, the period utility for matched firms are their profits, the period utility for matched workers are their wages, the period utility of unmatched workers are their reservation wages, and the period utility of unmatched firms are 0 and these leave. Finally, period t utilities are discounted at a rate β^t, with $\beta \in (0, 1)$.

We are applying the buyer-seller model within each period, so pairwise stable matchings are independently formed period by period. Determining the matches in this way implies that workers accept the vacancy that pays the highest wage and hence, do not consider other aspects of the match, such as future wage growth. From the firm's perspective this is without loss of generality: if they are unmatched at the end of a period they leave, so their static and dynamic problems coincide. From the workers perspective, however, there is a loss of generality. To see this consider worker i that is matched to firm j at wage w at the end of period t, and assume that in period $t+1$ firm j' will only be linked with i. To rule out the trivial case, assume that $\mu(j') > w$. Then worker i will expect a wage in period $t+1$ that belongs to $[\min\{\mu(j), \mu(j')\}, \max\{\mu(j), \mu(j')\}]$. Hence, worker i is willing to work for a more productive firm in period t even if that firm offers slightly lower salaries than the competitors. To the best of our knowledge, there is no standard solution concept for dynamic matching markets when matching opportunities arrive over time. For a more thorough discussion of the complications that arise in dynamic matching models see, for example, Doval (2014). For this reason, relaxing this assumption is left for future work.

5 A Deferred Acceptance Algorithm

We now present the deferred acceptance algorithm. We describe the algorithm as a first-price auction to give intuition of how the algorithm works. A formal description of the algorithm can be found in Section C in the appendix. We denote the agents on the side of the market that are holding the "auctions" as sellers and the agents on the other side that are "bidding" as bidders. Recall that we are approaching this problem from the matching perspective, so we are not making any statement about the actual economic mechanisms or incentives of the agents that determine prices and matches. Bidders bid in increments of $\frac{\Delta}{2}$. The value of Δ is set so that the productivity of firms lie in a Δ grid. Formally, for all j, $\mu(j) = b + k_j\Delta$ for some integer k_j that is randomly drawn at the start of the algorithm. We describe the algorithm for the case where the sellers hold the auctions. When buyers hold the auctions, the bidding starts at their valuation and prices decrease.

The algorithm has two stages. The first stage outputs an allocation and is motivated by the wage adjusting process in Crawford and Knoer (1981) and Kelso and Crawford (1982). (See Section C in the appendix for a detailed comparison of the first stage of our algorithm and the algorithms in Crawford and Knoer and Kelso and Crawford.) This allocation has the property that there exist prices for which it is pairwise stable. The second stage outputs two prices: the pointwise minimum price at which the stage 1 allocation is stable, and the pointwise maximum price at which the stage 1 allocation is stable.

Stage 1: The Matching Determination Program

The algorithm starts in round $t = 1$ when none of the sellers has received any bid. All bidders are placed into a queue and arrive sequentially. The entering order of the bidders is determined randomly. The *standing bid* of a seller is the last bid accepted by the seller or b if the seller has not received any bids. The *winning bidder* is the bidder who placed the last standing bid.

This is round t of the matching determination program.

1. Take the first bidder in the queue (for concreteness, call it bidder j). Bidder j selects the seller with the lowest standing bid among the linked sellers. If there is more than one such seller, the bidder selects one of these sellers at random. Call it seller i. If

the lowest standing bid is greater than $\mu(j) - \frac{\Delta}{2}$, bidder j does nothing and leaves the queue. Otherwise, bidder j bids the standing bid of seller i plus $\frac{\Delta}{2}$.

2. If bidder j makes a bid, seller i accepts the bid from bidder j. The new standing bid of seller i is now the previous standing bid plus $\frac{\Delta}{2}$. Bidder j leaves the queue. If there was a bidder j' who was the winning bidder (before bidder j bid), bidder j' is placed at the end of the queue.

3. The algorithm continues from step 1 with the next bidder in the queue. The algorithm stops when there are no bidders left in the queue. In this case, each seller is matched to the winning bidder.

We now present the second stage, the price determination program. The key insight of this stage is that, if a seller i is matched to a buyer j, and is also linked to an unmatched buyer j', then the price j pays i must price j' out of the market. That is, $p_M(i,j) \geq \mu(j')$. Moreover, if seller i is matched to buyer j, and seller i is also linked to a buyer j' who is also matched (say, to a seller i') then i must be getting payed at least what i' is getting payed. Otherwise, j' would like to block with i.

Stage 2: The Price Determination Program (I)

The program starts in round $t = 1$ with $M \subset E$ produced from stage 1 as its input.

1. Set the "price" of all unmatched sellers to b.

2. For matched sellers, set the price of seller i for buyer j to the maximum $\mu(j')$ amongst all j' that are linked to i but are not matched.

3. We call these prices $(\rho_i^1)_{i \in \mathcal{I}}$.

This is round $t > 1$ of the price determination algorithm. We take $(\rho_i^{t-1})_{i \in \mathcal{I}}$ as inputs for this round.

1. Set the "price" of all unmatched sellers in round t to b.

2. For matched sellers, set the price of each seller i for buyer j to the maximum price in round $t-1$ of the matched buyers that are linked to i. That is, amongst all matched j' that are linked to i, set ρ_i^t to the maximum $\rho_{i^*(j')}^{t-1}$. Note that one such j' is j itself, so these prices form a non-decreasing sequence.

3. If $\rho_i^t = \rho_i^{t-1}$ for all i, stop the algorithm and output these prices. Otherwise, start step $t + 1$.

As formally stated in Proposition 3, the Price Determination Program (I) captures the pointwise minimum price function at which M is stable. A modified version of this program, which we call Price Determination Program (II), generates the pointwise maximum price function at which M is stable. Rather than starting with ρ^1 at a low value, with successive iterations iterations rising it, the modified program starts with ρ^1 at high values and successive iterations lower it. Section C contains the formal algorithm, including both versions of the Price Determination Program.

Proposition 3. *The deferred acceptance algorithm has the following properties:*

1. *It stops after a finite number of rounds.*
2. *It outputs a pairwise stable allocation.*
3. *Price Determination program (I) outputs the pointwise minimum price function at which M is stable.*
4. *Price Determination program (II) outputs the pointwise maximum price function at which M is stable.*

The proof of Proposition 3 is in section C in the appendix.

6 Results

In the next two subsections we document the results from simulations of the buyer-seller and the labor-market models. We use the results from the simulation to obtain predictions about the population distribution of prices and the matching process.

6.1 Buyer-Seller Model

6.1.1 Simulation

We now describe the simulation of the buyer-seller model.

There are three parameters in the buyer-seller model: the number of buyers (J), the number of sellers (I), and the expected number of links per buyer (ELB). Every seller begins with one unit of a good (so the number of goods is I). The market tightness, θ, is the ratio of the number of buyers to the number of sellers, $\theta = \frac{J}{I}$. The market tightness is exogenous. We start the baseline simulation with $I = 10,000$ identical sellers and $J = 10,000 \times \theta$ heterogeneous buyers.[23] We consider markets with $J \in [1000, 50000]$, so $\theta \in [0.1, 5]$. We also consider markets with ELB $\in [1, 10]$.[24] The higher the ELB, the lower the search frictions in the market. The product of the number of buyers and the ELB determines the

[23]The results do not change substantially using 1,000 or 100,000 sellers. Results are available upon request.
[24]We obtain similar results by varying expected links per seller (ELS) in the simulations.

number of active links in the market. The total number of possible links in the market is $J \times I$. The proportion of active links relative to the total number of possible links in a network is a measure of the sparsity of the network. Given the parameters J, I, and ELB, a network is formed by randomly drawing buyers and sellers to form links. Once the network is constructed, we apply the algorithm from Section 5 to the network. The "bids" in the first stage of the algorithm take place on a grid of possible prices with $2J$ grid points.

Buyers' valuation is normalized to range between 0 and 100 which bounds the minimum and maximum prices between those values. One can interpret the reported prices as if the buyers' valuations are drawn from a uniform distribution, whose support is normalized between 0 and 100. Alternatively, one can interpret the prices or valuations as percentiles of any cumulative distribution function of the buyers' valuations. The second interpretation is possible because the allocations and prices only depend on order statistics and not the actual valuations.

We compare the price distributions to the Walrasian outcome, when each buyer is linked to every seller. The Walrasian outcome price, p_{walras}, is given by:

$$p_{walras} = \begin{cases} 0 & if \quad \theta \leq 1 \\ (1 - \frac{1}{\theta}) \times 100 & if \quad \theta > 1. \end{cases}$$

Recall that the Walrasian outcome has a unique price (see Remark 1). When $\theta \leq 1$, there are more sellers than buyers and so there is always a seller who is indifferent between selling the good at 0 or not selling it at all. In other words, the reservation price of the marginal seller is zero, which is what determines the market price. When $\theta > 1$, there are more buyers than sellers. Only $\frac{1}{\theta}$ of the buyers will buy the good. Hence the valuation of the marginal buyer will be $(1 - \frac{1}{\theta}) \times 100$. This buyer will be indifferent between paying $(1 - \frac{1}{\theta}) \times 100$ and leaving the market, and so the market price will be $(1 - \frac{1}{\theta}) \times 100$.

6.1.2 Results

Distribution of Prices. Figure 1 displays the distribution of prices for the buyer-preferred match by market tightness (horizontal axis in each panel) and ELB (different panels). Each vertical box corresponds to a simulated market characterized by those parameters. Each panel shows the population distribution of prices for different levels of search frictions in different markets. The top-left panel shows the price distribution for high frictions, where ELB equals 1. The top-right and bottom panels show what happens in markets with lower frictions (when ELB equals 2, 3, and 5, respectively). At low levels of θ there are many sellers for each buyer. So low numbers for θ indicate "loose" seller markets where sellers are at a disadvantage. In addition, each panel displays the Walrasian outcome.

For market tightness less than one, the market looks like a monopsony and nearly all sellers are paid their valuation (recall that, for simplicity, all sellers are identical, so we normalized their valuation to zero). This is because it is unlikely for a seller to receive multiple links. Even if a seller receives two links, it is likely that at least one of the buyers has an outside option of zero. This happens if the buyer is also linked with another seller who has no other links.

On the other hand, as market tightness is increased the market becomes more competitive between buyers and more favorable for sellers. The median price increases as does price dispersion. There are now many buyers linked to each seller and the buyers have worse outside options. Even if a buyer is linked to a second seller, it is likely that the second seller is linked to many other buyers. In markets with lower frictions, competition between buyers increases, thus increasing prices until they reach the Walrasian outcome.

Figure 2 shows that similar results to the ones in Figure 1 are obtained using the seller-preferred match. Figure 2 displays, for each market tightness, the distribution of prices using both the seller- and the buyer-preferred match. (For the buyer-preferred match, each vertical box in Figure 2 is identical to the corresponding vertical box in Figure 1.) When ELB equals 5, the 95th and 5th price percentiles coincide with the Walrasian outcome for both the buyer- and the seller- preferred match. The prices in the buyer-preferred matching represents the lower bound of the set of prices that support each match. Likewise, the prices in the seller-preferred match represents the upper-bound of the set of prices that support each match. Since both the seller-preferred and buyer-preferred price distributions mimic the Walrasian outcome when ELB=5, it must be true that the price distribution in any allocation that supports a pairwise stable match must also mimic the Walrasian outcome.

Price Dispersion and the Walrasian Outcome. Price dispersion decreases when search frictions decrease. There are many buyers linked to each seller, but there are also many sellers linked to each buyer, improving the outside options of both parties. These improved outside options reduces price dispersion (*i.e.* the likelihood that a seller has to take a low price is low, but at the same time the probability that a buyer has to pay a high price is also low). Figure 3 shows the evolution of the price distribution for the buyer-preferred match. In the top panel, the figure displays the difference between the 95th and the 5th price percentiles. In the bottom panel, the figure displays the difference between the 99.5th and 0.5th price percentiles. All sellers are paid the same price at the Walrasian outcome, so both differences equal zero at the Walrasian outcome. We are interested in answering the following two questions: How sparse can the network be while 90% and 99% of sellers are paid the same price? While there is price dispersion when there are fewer than four ELB, the price distribution begins to collapse for more ELB. When there are five ELB, there is nearly no difference between the price at the 95th and 5th percentiles. Likewise, when there are eight ELB, there is almost no difference between the price at the 99.5th and 0.5th percentile. In other words, at least 90% or 99% of the sellers are paid the same price when the number of active links relative to the total number of links is only 5/10,000 or 8/10,000, respectively.

The price distribution in the model collapses with less than 0.1% of the possible links in the network.

The Effect of Frictions on Mean Prices. Figure 4 displays the evolution of mean prices (where the expectation is taken relative to the population distribution of prices) over ELB for different market tightness. Mean prices represent the buyers' and sellers' *ex ante* expected prices before the network is drawn. The figure shows how mean prices vary with ELB (*i.e.* frictions) in a given market (holding fixed the market tightness), so mean prices are normalized by the mean price when ELB equals 1. Increasing ELB may increase or decrease mean prices, depending on market tightness. For example, consider markets where there are many sellers for each buyer ($\theta \leq 1$, so that sellers are at a disadvantage). When ELB is low, price dispersion is high, even when there are more sellers than buyers (top-left panel in Figure 1), resulting in relatively high mean prices. As ELB increases, the price distribution collapses to the Walrasian outcome (Figure 3). The Walrasian outcome is zero when there are more sellers than buyers. Thus, when there are more sellers than buyers ($\theta \leq 1$), lowering frictions results in *lower* mean prices as a consequence of indirect competition. Intuitively, since there are more sellers than buyers, increasing ELB improves the outside option of the buyers who now talk to relatively more sellers, even when sellers expect to talk to more buyers.[25]

Distribution of Matched Buyers. Figure 5 shows the distribution of matched buyers for different markets. In loose markets ($\theta < 1$), the probability of finding a match does not depend on the buyer's valuation. Prices are low and there are many unmatched sellers, so buyers have a roughly equal chance of finding a match. The ELB does not change the distributions of matched buyers in loose markets.

As markets become tighter ($\theta > 1$), competition between buyers becomes more important. In these markets, prices are higher and some buyers are priced out of the market. Buyers with high valuations (*e.g.* above the Walrasian price) are more likely to buy goods than buyers with low valuations. When ELB is low, buyers with high valuations may be linked to a seller with another high-valuation buyer. Since they have few links, they are priced out of the market. For markets with higher ELB, buyers have better outside options and the probability that a high-valuation buyer is priced out of the market decreases. When ELB=5, the distribution of matched buyers looks close to the Walrasian outcome, where all buyers with $\mu(j) > p_{walras}$ are matched and all buyers with $\mu(j) < p_{walras}$ are priced out of the market.

Welfare. Results on the welfare in the buyer-seller model are in the online appendix (see Figure A1). We analyze welfare using the labor market model on page 30, where the results

[25] Same results are obtained using expected links per seller (ELS) instead of ELB. When there are more sellers than buyers ($\theta \leq 1$), increasing ELS results in lower mean prices. Results are available upon request.

are similar to the buyer-seller model.

6.2 Labor Market Model

6.2.1 Simulation

We adapt the buyer-seller model to the labor market to explore questions about wage dispersion and growth. In this case, workers are sellers and firms are buyers of their services. Firms have single unit demand for labor and cannot dismiss their employee. We assume that workers are homogeneous and firms are heterogeneous in their productivity ($\mu(j)$).[26] We normalize the reservation wage of the worker to zero. If a worker and firm match at wage w, the worker's utility is w and the firm's profit is $\mu(j) - w$. Wages and firm productivities are normalized to range between 0 and 100 as in the buyer-seller model. To study wage growth, we extend our model to accommodate multiple periods.

In the first period all workers start unemployed and the simulation is identical to the basic buyer-seller model (see subsection 6.1). At the end of the period, some matches are randomly destroyed at rate $\delta \in (0, 1)$. The firms that are unmatched at the end of a period (either because the match was destroyed or they could not form a match in the first place) exit the market. We interpret "firms" as time sensitive vacancies. So if by the end of a period a vacancy is not filled, it disappears from the job market.

At the beginning of the next period, some fraction of workers are employed by old firms and the rest are unemployed. The same number of firms are created (J) and a new network is drawn between all the workers (employed and unemployed) and the new firms. Firms from previous periods maintain the link to their employed worker and lose all other links from previous periods. New firms are placed into the queue and old firms start off as the highest bidders in their employee's "auction". So from the standpoint of the algorithm, the standing bid (or reservation wage) in an employed worker's auction is the wage from the previous period. The new firms either "bid" for unemployed workers or try to poach employed workers from old firms by bidding in the auctions of the employed workers. Firms from previous periods can only bid in their employee's auction. When firms arrive at the front of the queue, they bid in their subnetworks according to the algorithm (see Section 5). The bidding process ends when there are no more firms in the queue. Matched firms produce with the hired workers and the workers receive their wage. At the end of the period matches receive a job destruction shock at rate δ.

We consider markets that are in steady state. The market is in steady state when the flows into unemployment equal the flows out of unemployment. We find the steady state by

[26] Worker heterogeneity is important for understanding wage dispersion in the data and it is straightforward to add worker heterogeneity to our model. However, not including worker heterogeneity makes the exposition of our results more clear as the Walrasian outcome has a unique wage.

simulating the economy for enough periods until the average unemployment remains stable.[27] Then we record 400 periods.

The labor-market model has five parameters: the number of workers (I), which is constant for all periods, the number of firms (J) that enter in each period, the expected number of links per firm (ELF), the job destruction rate (δ), and the relative probability of receiving a link between employed and unemployed workers (λ). The market tightness, θ, is defined as the ratio of firms to unemployed workers, $\frac{J}{U}$. In contrast to the basic buyer-seller model, θ is now endogenous as it depends on the number of unemployed workers (U).

Both the number of workers and the job destruction rate are fixed for all simulations. Following Shimer (2012), the monthly employment to unemployment rate is set at 2% (prime age men, Figure 3), which translates to a quarterly $\delta = 0.06$. We use 5,000 workers for the simulation of the dynamic labor market model.[28] A market is a combination of J and ELF.

The relative probability of receiving a link between employed and unemployed workers (λ) is an important determinant of the structure of the network. Most empirical studies find different job offer rates between employed and unemployed workers.[29] This is important for understanding allocations and wage growth, since indirect competition is substantially diminished when $\theta < 1$ and $\lambda = 1$. To understand why, recall that when $\theta < 1$, there are more unemployed workers (U) than vacancies entering the market (J). Even when the unemployment rate is relatively high (10%-20%), firms have a low probability of linking with an unemployed worker when $\lambda = 1$. So even in markets that appear unfavorable for the workers (high unemployment and low market tightness), it is difficult for the firms to link to unemployed workers when $\lambda = 1$. This implies that in these markets, unemployed workers do not have to compete with each other. If we follow the empirical literature by setting $\lambda < 1$, then firms have a higher probability of linking with an unemployed worker and indirect competition again becomes important even when market tightness is low.

To make the comparison to labor-search models, we use a model of Bertrand competition between two firms as our benchmark. For example, Postel-Vinay and Robin (2002) (henceforth PVR) allows two firms to compete over the wages (*à la* Bertrand) of an employed worker and is closest to our model. The PVR framework in terms of networks is as follows. Given a set of firms, workers, and a matching technology *à la* PVR, construct the corresponding network with the same set of firms and workers, where a worker is linked to a firm if, and only if, the worker and the firm are matched by the matching technology. An important assumption in most search models, including PVR, is that a firm can negotiate with at most

[27]We compute the average unemployment over 10 consecutive periods and compare it to the average unemployment 10 periods before. We find the steady state when this difference is less than the tolerance level. The convergence is relatively fast. It takes between 30 to 100 periods (depending on the values of the parameters) to find the steady state. Let U_t be the unemployment in period t in a specific market and let $\bar{U}_t = \frac{1}{10}\sum_{j=t-9}^{t} U_j$ be the average unemployment of the 10 periods ending in t. We find the steady state, t_{SS}, as follows: $\mid \bar{U}_{t_{SS}-10} - \bar{U}_{t_{SS}} \mid < \epsilon$, where ϵ is a tolerance level.

[28]The results do not change much when using 1,000 or 10,000 workers. Results are available upon request.

[29]See Holzer (1987) and Blau and Robins (1990) for an investigation on different search intensities and offer rates for employed and unemployed workers.

one worker and, in PVR, a worker can negotiate with at most two firms. This limits the type of networks that are realized and drives most of the differences between search models and our model.

We also compare our results to a perfectly competitive model (Walrasian outcome) in steady state. Recall that a market is in steady state when the flows into unemployment equal the flows out of unemployment. In the Walrasian outcome, all unemployed workers find jobs when $\theta \geq 1$ (*i.e.* there are more vacancies than unemployed workers). So the number of employed workers in steady state at the end of a period is the total number of workers in the market adjusted by the separation rate, $I \times (1-\delta)$. When $\theta < 1$, the fraction of unemployed workers in steady state, u_{SS}, is given by equalizing the flow of workers out of and into unemployment:

$$\big[u_{ss} + \delta(1-u_{ss})\big]\theta = \delta(1-u_{ss}),$$

where u_{ss} is the unemployment rate before the job separation shock.[30] The left-hand side represents the number of firms that hire workers (θ times the fraction of unemployed at the beginning of a period). The right-hand side represents the number of workers that lose their jobs. Thus, $u_{SS} = \frac{\delta(1-\theta)}{(1-\delta)\theta+\delta}$ when $\theta < 1$. Then, the number of employed workers in the Walrasian outcome, E_{SS}, is given by $E_{SS} = I \times (1-u_{ss})$. Due to the multi-period aspect of the model, the number of firms employing a worker in steady state, E_{SS}, is greater than the number of firms that enter in each period, $E_{SS} > J$.

6.2.2 Results

Distribution of Wages. Following the empirical results in Bagger, Fontaine, Postel-Vinay, and Robin (2014), we set $\lambda = 0.05$. This implies that an unemployed worker is twenty times more likely to receive a link than an employed worker. Setting λ below one concentrates the network between the new vacancies and the unemployed workers. Figure 6 displays the distribution of wages in different markets when $\lambda = 0.05$.[31] (This figure is similar to Figure 1 but for the dynamic labor market model.) All workers now compete with unemployed workers and the average wage is close to the Walrasian outcome when ELF equals five.

Worker Mobility and Wage Growth. To analyze worker mobility and wage growth we keep $\lambda = 0.05$. Figure 7 shows the wage profiles of workers after an unemployment spell. In loose markets, workers start out with low wages and wages grow very slowly. In tight markets with low frictions, workers attain a high wage immediately out of unemployment and then their wages grow very little over their career. This leads to the result that as you reduce

[30] We use lowercase letters for rates (*e.g.* the unemployment rate is u) and uppercase letters for levels (*e.g.* the number of unemployed workers is U).

[31] Figure A2 in the online appendix displays the distribution of wages in different markets when $\lambda = 1$. As discussed in the previous section, $\lambda = 1$ is inconsistent with the empirical literature and suppresses indirect competition. Comparing figures 6 and A2 shows the effect that including indirect competition has on wage dispersion and average wages.

frictions, workers have lower median wage growth. Figure 8 (top panel) shows the median wage growth after twenty periods of employment. As a benchmark, we also display the wage growth for the Bertrand competition benchmark (see subsubsection 6.2.1). When $\theta < 1$, wage growth is reduced because firms have better outside options. When $\theta > 1$, reducing frictions causes firms to compete strongly for workers. This drives initial wages up leaving little room for wage growth.

The lower panel of Figure 8 displays the worker mobility, which is defined as the fraction of workers that make a job-to-job transition in a period. The intuition for why worker mobility decreases as frictions are lowered is similar to wage growth. When $\theta < 1$, firms are less likely to poach workers from another firm because they have better outside options that likely include an unemployed worker. When $\theta > 1$, workers are less likely to move from one firm to another because when they enter the labor market they immediately find a job that pays a high wage. Our results indicate that empirical researchers should be cautious when using worker mobility to make inference about the level of frictions in a market where indirect competition is present.

Welfare. To analyze welfare we need a welfare criterion. In the labor market model, the utility function of the firms that hire a worker is the production function minus the wage, $\mu(j) - w(j)$, and the utility function of the workers that are hired is the wage, $w(j)$. We use the utilitarian welfare criterion, Ω, that is sum of these utilities and corresponds to $\Omega = \sum_{j=1}^{\tilde{J}} \mu(j)$, where $j = 1, \dots, \tilde{J}$ index the firms that hire a worker. The unconstrained first-best allocation is the one that maximizes the welfare in the absence of frictions. It corresponds to the Walrasian outcome.[32]

Following Bagger, Fontaine, Postel-Vinay, and Robin (2014), we use the following cumulative distribution function for the productivity of firms:

$$F(\mu) = 1 - exp\Big(- \big[c_1(\mu - c_0)\big]^{c_2}\Big),$$

where $c_0 = 5$, $c_1 = 8$, and $c_2 = 0.7$. Thus, the utilitarian welfare in each market, Ω, represents the aggregate production in that market.

Figure 9 displays the aggregate production (welfare) and average labor productivity for different levels of frictions and for the Walrasian outcome (*i.e.* unconstrained first-best allocation).[33] The top panel in Figure 9 shows the ratio of the aggregate production in our model relative to the aggregate production in the Walrasian outcome.[34] As ELF is increased, the aggregate production in our model is close to the aggregate production in the Walrasian

[32]The same welfare analysis holds for the buyer-seller market using buyers, sellers, prices, and a single period. We obtain similar results. See Figure A1 in the online appendix.

[33]All specifications display the aggregate production in steady state at the end of the period after the destruction shock occurred. So, $E_{SS} \leq I \times (1-\delta)$ in our model, and $E_{SS} = I \times (1-\delta)$ in the Walrasian outcome. The results do not change substantially if the welfare is calculated before the destruction shock occurs.

[34]Figure A5 in the online appendix shows the total aggregate production.

outcome. For example, when $\theta = 0.5$ and ELF=5, the aggregate production is approximately 97.5% of the aggregate production in the Walrasian outcome. So even in sparse networks, markets are close to the unconstrained first-best allocation.

We also investigate the allocation of workers to firms. The bottom panel of Figure 9 displays the average productivity of jobs of employed workers. We find that lowering frictions can improve or worsen the allocation of workers to more productive firms. In loose markets (lower θ), reducing frictions lowers average labor market productivity. The intuition is that when frictions are high, firms are less likely to have an outside option when linked with an employed worker. This leads to competition between firms. When frictions are lower, firms are more likely to have another link to an unemployed worker. Hence, there is less competition between firms. Recall that when there are no frictions and $\theta < 1$, there is no competition between firms since there are more unemployed workers than firms. In tight markets (high θ), lower frictions leads to the allocation of workers to the more productive firms as in standard search models.

7 Concluding Remarks

The defining characteristic of markets with frictions is that similar goods or services are traded at different prices, resulting in price dispersion. In this paper we use networks to characterize pairwise stable allocations and their supporting prices in buyer-seller markets with frictions. The central insight of the paper is that including indirect competition causes markets with frictions to have prices and allocations that look close to the Walrasian outcome. To study prices in large networks, we develop an algorithm that finds the upper and lower bounds of the set of prices that sustain any pairwise stable match. Indirect competition reverses the relationship between the level of frictions and many economic outcomes. We find that lowering frictions leads to: 1) lower wage growth, 2) lower worker mobility, and 3) lower expected prices in loose markets ($\theta < 1$).

The main finding of our paper, that sparse networks look as if they were perfectly competitive, might seem inconsistent with the price dispersion observed in eBay, labor markets, automobile markets, *etc.* There are three dimensions in which our model provides a richer understanding of frictions and price dispersion. Consider the case of eBay, where search frictions arise because its search engine is quite sensitive to the buyers' search inquiry and to the sellers' title for the product listing. In addition, buyers most likely do not compare all the listings for a given product at a given time. First, one possible explanation for the price dispersion is that search frictions are relatively high (*i.e.* ELB$<$ 3 or buyers participate in less than three auctions on average). But this is unlikely to be the whole answer. Second, the structure of the network is an important factor in the formation of prices. Although our networks are generated by randomly forming links, this is clearly not the case at eBay. All buyers who make the same search inquiry receive the same list of items or products. Price dispersion in eBay may be more about the structure of the network and less about the absolute level of frictions. Third, our model makes clear predictions on the distribution of prices for any network. Given information on the participation of buyers in auctions, the actual network can be constructed. This can be used to decompose the sources of price dispersion into frictions and other factors, such as unobserved heterogeneity.

Econometric methods where identification is based on minimal assumptions provide a robust structural framework for inference improving credibility and robustness of the empirical analysis.[35] In this context, using pairwise stability as our matchmaking criterion can be used to develop an empirical framework in the spirit of credible econometrics.[36] Since pairwise stability is the weakest criterion for matchmaking that is consistent with Pareto efficiency, not specifying the game details allows the econometrician to weaken the behavioral assumptions that would otherwise be imposed by a specific game.

Our model sheds some light on a puzzle about the recent recovery from the Great Recession in the US. Empirical studies show that wage growth is lower compared to previous recoveries at the same unemployment rate (Yellen, 2014). Our model predicts that lower frictions imply lower wage growth. If we assume that search frictions in the labor market have been decreasing over time due to new technologies, our model provides a possible explanation for this puzzle.

When considering empirical applications, there are a number of ways the model could be enriched. For example, an application to eBay might consider endogenous search intensity. An application to the labor market might consider a more realistic production function including heterogeneity of both workers and firms, endogenous search intensity and endogenous entry of firms. The goal of this paper is to construct a parsimonious model that demonstrates the importance of indirect competition in price dispersion, wage growth, allocations of goods and workers, *etc.* Enriching the model in other dimensions is an avenue for future research.

References

Agarwal, N. (2015): "An empirical model of the medical match," *American Economic Review*, 105(7), 1939–78. (Cited on page 32)

Alan, M. (2011): "Imperfect competition in the labor market," *Handbook of labor economics*, 4, 973–1041. (Cited on page 5)

Albrecht, J., P. Gautier, and S. Vroman (2006): "Equilibrium Directed Search with Multiple Applications," *Review of Economic Studies*, 73(4), 869–891. (Cited on page 6)

——— (2014): "Effcient Entry in Competing Auctions," *American Economic Review,* forthcoming. (Cited on page 6)

Alvarez, F., and G. Barlevy (2014): "Mandatory Disclosure and Financial Contagion," *Working Paper, Federal Reserve Bank of Chicago.* (Cited on page 8)

[35]For example, see Manski (2003), Tamer (2010), Pakes, Porter, Ho, and Ishii (2014).

[36]For example, see Fox (2010a), Fox (2010b), Agarwal (2015), and the references there. See Fox (2009) for a survey.

Arbatskaya, M. (2007): "Ordered search," *The RAND Journal of Economics*, 38(1), 119–126. (Cited on page 6)

Ausubel, L. M., and P. R. Milgrom (2002): "Ascending Auctions with Package Bidding," *Frontiers of Theoretical Economics*, 1(1), 1. (Cited on page 8)

Babus, A., and P. Kondor (2013): "Trading and Information Diffusion in Over-the-Counter Markets," *Working Paper*. (Cited on page 8)

Bagger, J., F. Fontaine, F. Postel-Vinay, and J.-M. Robin (2014): "Tenure, Experience, Human Capital, and Wages: A Tractable Equilibrium Search Model of Wage Dynamics," *American Economic Review*, 104(6), 1551–96. (Cited on pages 7, 29, 30, and 45)

Bajari, P., and A. Hortacsu (2004): "Economic Insights from Internet Auctions," *Journal of Economic Literature*, pp. 457–486. (Cited on page 5)

Baye, M., J. Morgan, and P. Scholten (2004): "Price Dispersion in the Small and Large: Evidence from an Internet Price Comparison Site," *Journal of Industrial Economics*, 52, 463–496. (Cited on page 6)

Baye, M. R., and J. Morgan (2001): "Information gatekeepers on the internet and the competitiveness of homogeneous product markets," *American Economic Review*, pp. 454–474. (Cited on page 6)

Baye, M. R., J. Morgan, and P. Scholten (2006): "Information, search, and price dispersion," *Handbook on economics and information systems*, 1. (Cited on pages 5 and 6)

Bertsekas, D. P. (1992): "Auction algorithms for network flow problems: A tutorial introduction," *Computational Optimization and Applications*, 1(1), 7–66. (Cited on page 9)

Blau, D. M., and P. K. Robins (1990): "Job search outcomes for the employed and unemployed," *Journal of Political Economy*, pp. 637–655. (Cited on page 28)

Burdett, K., and K. L. Judd (1983): "Equilibrium price dispersion," *Econometrica*, pp. 955–969. (Cited on page 6)

Burdett, K., and D. T. Mortensen (1998): "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 39, 257–73. (Cited on page 6)

Calvó-Armengol, A., and M. O. Jackson (2004): "The effects of social networks on employment and inequality," *American economic review*, pp. 426–454. (Cited on page 7)

——— (2007): "Networks in labor markets: Wage and employment dynamics and inequality," *Journal of economic theory*, 132(1), 27–46. (Cited on page 7)

Carlson, J. A., and R. P. McAfee (1983): "Discrete equilibrium price dispersion," *The Journal of Political Economy*, pp. 480–493. (Cited on page 6)

Charness, G., M. Corominas-Bosch, and G. Fréchette (2007): "Bargaining and Network Structure: An Experiment," *Journal of Economic Theory*, 136, 28–65. (Cited on page 1)

Cooper, T. E. (1989): "Indirect competition with spatial product differentiation," *The Journal of Industrial Economics*, pp. 241–257. (Cited on page 5)

Corominas-Bosch, M. (2004): "Bargaining in a Network of Buyers and Sellers," *Journal of Economic Theory*, 115, 35–77. (Cited on page 7)

Crawford, V. P., and E. M. Knoer (1981): "Job Matching with Heterogeneous Firms and Workers," *Econometrica*, 49, 437–450. (Cited on pages 8, 21, and 52)

Donna, J., P. Schenone, and G. Veramendi (2015): "Frictions in Internet Auctions with Many Traders: a Counterexample," *Working Paper*. (Cited on page 6)

Doval, L. (2014): "A theory of stability in dynamic matching markets," *Working Paper*. (Cited on page 21)

Eckstein, Z., and G. J. Van den Berg (2007): "Empirical labor search: A survey," *Journal of Econometrics*, 136(2), 531–564. (Cited on page 5)

Einav, L., T. Kuchler, J. Levin, and N. Sundaresan (2015): "Assessing Sale Strategies in Online Markets using Matched Listings," *American Economic Journal: Microeconomics,* forthcoming. (Cited on page 1)

Elliott, M. (2014): "Heterogeneities and the Fragility of Labor Markets," *Mimeo.* (Cited on page 7)

——— (2015): "Inefficiencies in networked markets," *American Economic Journal: Microeconomics,* forthcoming. (Cited on page 8)

Fainmesser, I. P., and A. Galeotti (2015): "Pricing Network Effects," *Review of Economic Studies*, 1, 1–36. (Cited on page 7)

Fox, J. T. (2009): "Structural empirical work using matching models," *New Palgrave Dictionary of Economics. Online edition.* (Cited on page 32)

——— (2010a): "Estimating matching games with transfers," Discussion paper, National Bureau of Economic Research. (Cited on page 32)

——— (2010b): "Identification in matching games," *Quantitative Economics*, 1(2), 203–254. (Cited on page 32)

Gale, D., and S. Kariv (2009): "Trading in Networks: A Normal Form Game Experiment," *American Economic Journal: Microeconomics*, 1(2), 114–132. (Cited on page 1)

Gale, D., and L. S. Shapley (1962): "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69, 9–15. (Cited on page 8)

Galenianos, M., and P. Kircher (2009): "Directed search with multiple job applications," *Journal of Economic Theory*, 144(2), 445–471. (Cited on page 6)

Gautier, P. A., and C. Holzner (2013): "Simultaneous Search and Network Efficiency," CESifo Working Paper Series 3522, CESifo Group Munich. (Cited on page 7)

Gofman, M. (2011): "A Network-Nased Analysis of Over-the-Counter Markets," *Working Paper.* (Cited on page 8)

Hasker, K., and R. Sickles (2010): "eBay in the economic literature: Analysis of an auction marketplace," *Review of Industrial Organization*, 37(1), 3–42. (Cited on page 5)

Hatfield, J. W., and F. Kojima (2008): "Matching with Contracts: Comment," *American Economic Review*, 98, 1189–94. (Cited on page 8)

——— (2010): "Substitutes and Stability for Matching with Contracts," *Journal of Economic Theory*, 145, 1704–1723. (Cited on page 8)

Hatfield, J. W., and P. R. Milgrom (2005): "Matching with Contracts," *American Economic Review*, 95, 913–935. (Cited on page 8)

Holzer, H. J. (1987): "Job Search by Employed and Unemployed Youth," *Industrial and Labor Relations Review*, 40(4), pp. 601–611. (Cited on page 28)

Jackson, M. O. (2008): *Social and Economic Networks*, vol. 3. Princeton University Press Princeton. (Cited on pages 2 and 7)

Jackson, M. O., and A. Wolinsky (1996): "A strategic model of social and economic networks," *Journal of economic theory*, 71(1), 44–74. (Cited on page 7)

Janssen, M. C., and J. L. Moraga-González (2004): "Strategic pricing, consumer search and the number of firms," *The Review of Economic Studies*, 71(4), 1089–1118. (Cited on page 6)

Janssen, M. C., J. L. Moraga-Gonzalez, and M. R. Wildenbeest (2005): "Truly costly sequential search and oligopolistic pricing," *International Journal of Industrial Organization*, 23(5), 451–466. (Cited on page 6)

Judd, J. S., and M. Kearns (2008): "Behavioral experiments in networked trade," in *Proceedings of the 9th ACM conference on Electronic commerce*, pp. 150–159. ACM. (Cited on page 1)

JULIEN, B., J. KENNES, AND I. KING (2000): "Bidding for Labor," *Review of Economic Dynamics*, 3(4), 619–649. (Cited on page 6)

KAKADE, S. M., M. KEARNS, AND L. E. ORTIZ (2004): "Graphical economics," in *Learning Theory*, pp. 17–32. Springer. (Cited on page 8)

KAKADE, S. M., M. KEARNS, L. E. ORTIZ, R. PEMANTLE, AND S. SURI (2004): "Economic properties of social networks," in *Advances in Neural Information Processing Systems*, pp. 633–640. (Cited on page 9)

KELSO, A. S., AND V. P. CRAWFORD (1982): "Job Matching, Coalition Formation, and Gross Substitutes," *Econometrica*, 50, 1483–1504. (Cited on pages 8, 21, and 52)

KIRCHER, P. (2009): "Efficiency of Simultaneous Search," *Journal of Political Economy*, 117, 861–913. (Cited on page 6)

KRANTON, E., AND D. F. MINEHART (2001): "A Theory of Buyer-Seller Networks," *American Economic Review*, 91, 485–508. (Cited on pages 6 and 7)

LENTZ, R., AND D. T. MORTENSEN (2010): "Labor market models of worker and firm heterogeneity," *Annu. Rev. Econ.*, 2(1), 577–602. (Cited on page 5)

LESTER, B. (2011): "Information and prices with capacity constraints," *American Economic Review*, 101(4), 1591–1600. (Cited on page 6)

MACMINN, R. D. (1980): "Search and market equilibrium," *The Journal of Political Economy*, pp. 308–327. (Cited on page 6)

MALAMUD, S., AND M. J. ROSTEK (2013): "Decentralized Exchange," *University of Geneva*. (Cited on page 8)

MANEA, M. (2011): "Bargaining in Stationary Networks," *American Economic Review*, 101(5), 2042–80. (Cited on page 7)

MANSKI, C. F. (2003): *Partial Identification of Probability Distributions*. Springer, New York. (Cited on page 32)

MCAFEE, R. P. (1993): "Mechanism Design by Competing Sellers," *Econometrica*, 61(6), 1281–1312. (Cited on page 6)

MOEN, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105(2), 385–411. (Cited on page 6)

MONTGOMERY, J. D. (1991): "Social networks and labor-market outcomes: Toward an economic analysis," *American Economic Review*, pp. 1408–1418. (Cited on page 7)

MORTENSEN, D. T. (2005): *Wage Dispersion: Why Are Similar Workers Paid Differently?* The MIT Press, Cambridge. (Cited on pages 1 and 6)

MORTON, F. S., F. ZETTELMEYER, AND J. SILVA-RISSO (2001): "Internet car retailing," *The Journal of Industrial Economics*, 49(4), 501–519. (Cited on page 1)

MYERS, C. A., AND G. P. SHULTZ (1951): *The dynamics of a labor market: a study of the impact of employment changes on labor mobility, job satisfactions, and company and union policies*. Prentice-Hall. (Cited on page 7)

PAKES, A., J. PORTER, K. HO, AND J. ISHII (2014): "Moment Inequalities and Their Application," *Econometrica,* forthcoming. (Cited on page 32)

PETERS, M., AND S. SEVERINOV (1997): "Competition among Sellers Who Offer Auctions Instead of Prices," *Journal of Economic Theory*, 75(1), 141–179. (Cited on page 6)

——— (2006): "Internet auctions with many traders," *Journal of Economic Theory*, 130(1), 220–245. (Cited on pages 6 and 50)

POLANSKI, A., AND F. VEGA-REDONDO (2013): "Markets, bargaining, and networks with heterogeneous agents," *Available at SSRN 2218605.* (Cited on page 7)

Postel-Vinay, F., and J.-M. Robin (2002): "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 70, 2295–2350. (Cited on pages 6 and 28)

Rees, A. (1966): "Information networks in labor markets," *American Economic Review*, pp. 559–566. (Cited on page 7)

Reinganum, J. F. (1979): "A simple model of equilibrium price dispersion," *The Journal of Political Economy*, pp. 851–858. (Cited on page 6)

Rogerson, R., R. Shimer, and R. Wright (2005): "Search-Theoretic Models of the Labor Market: A Survey," *Journal of Economic Literature?*, XLIII, 959–988. (Cited on pages 5 and 7)

Rosenthal, R. W. (1980): "A model in which an increase in the number of sellers leads to a higher price," *Econometrica*, pp. 1575–1579. (Cited on page 6)

Roth, A. (2008): "Deferred Acceptance Algorithms: History, Theory, Practice, and Open Questions," *International Journal of Game Theory*, Special Issue in Honor of David Gale on his 85th birthday, 537–569. (Cited on page 8)

Roth, A., and M. Sotomayor (1990): *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*. Econometric Society Monograph Series , Cambridge University Press. (Cited on page 8)

Rothschild, M. (1973): "Models of market organization with imperfect information: A survey," *The Journal of Political Economy*, pp. 1283–1308. (Cited on page 6)

Salop, S., and J. Stiglitz (1977): "Bargains and ripoffs: A model of monopolistically competitive price dispersion," *The Review of Economic Studies*, pp. 493–510. (Cited on page 6)

Shapley, L., and H. Scarf (1974): "On Cores and Indivisibility," *Journal of Mathematical Economics*, 1, 11–26. (Cited on page 8)

Shapley, L., and M. Shubik (1972): "The Assignment Game I: The Core," *Econometrica*, 1, International Journal of Game Theory. (Cited on page 8)

Shimer, R. (2012): "Reassessing the ins and outs of unemployment," *Review of Economic Dynamics*, 15(2), 127–148. (Cited on page 28)

Stahl, D. O. (1989): "Oligopolistic pricing with sequential consumer search," *American Economic Review*, pp. 700–712. (Cited on page 6)

Stigler, G. J. (1961): "The economics of information," *The journal of political economy*, pp. 213–225. (Cited on page 6)

Tamer, E. (2010): "Partial Identification in Econometrics," *Annual Reviews of Economics*, 2. (Cited on page 32)

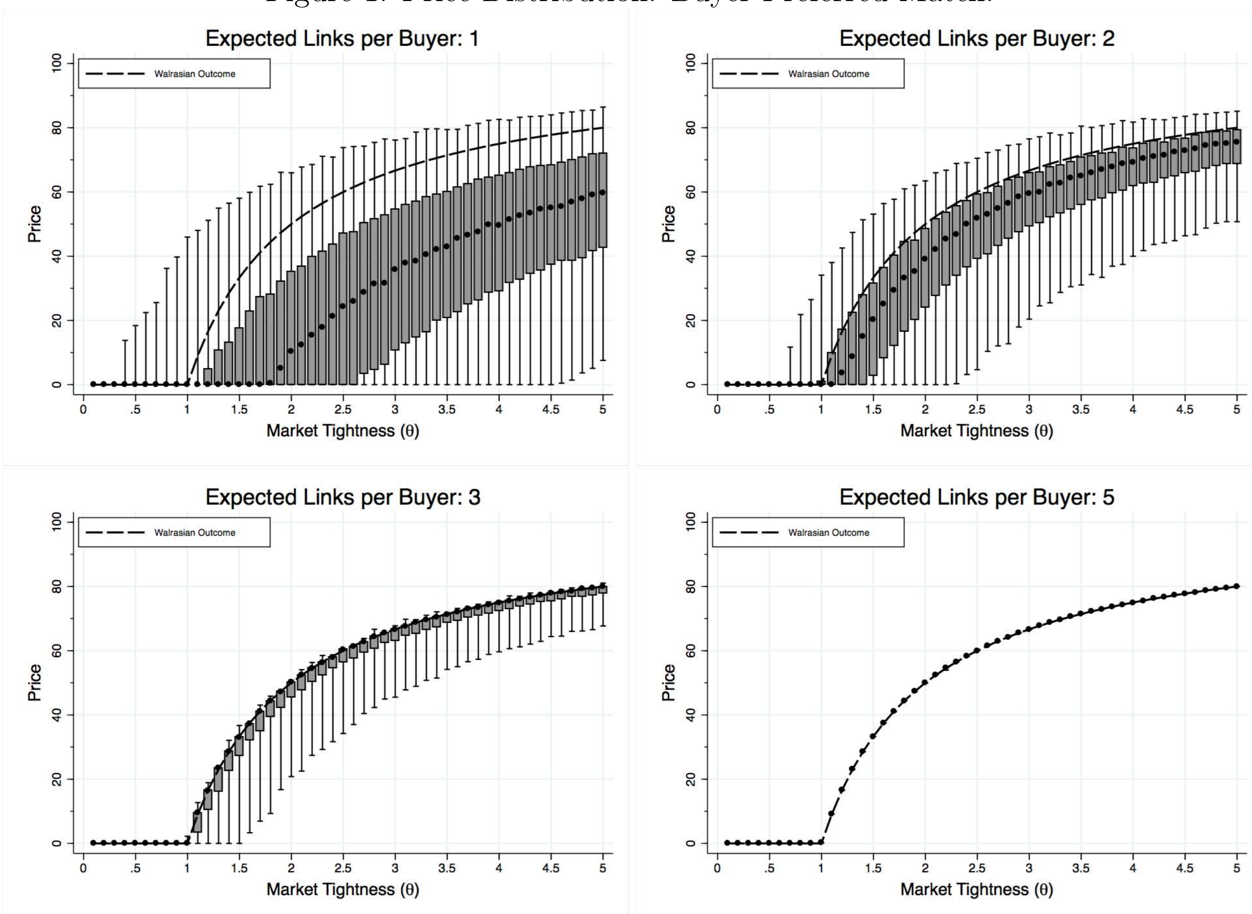
Varian, H. R. (1980): "A model of sales," *American Economic Review*, pp. 651–659. (Cited on page 6)

Walthoff, R. (2012): "Applications and Interviews: A Structural Analysis of Two-Sided Simultaneous Search," *Working Paper*. (Cited on page 6)

Wolinsky, A. (1988): "Dynamic Markets with Competitive Bidding," *Review of Economic Studies*, 55(1), 71–84. (Cited on page 6)

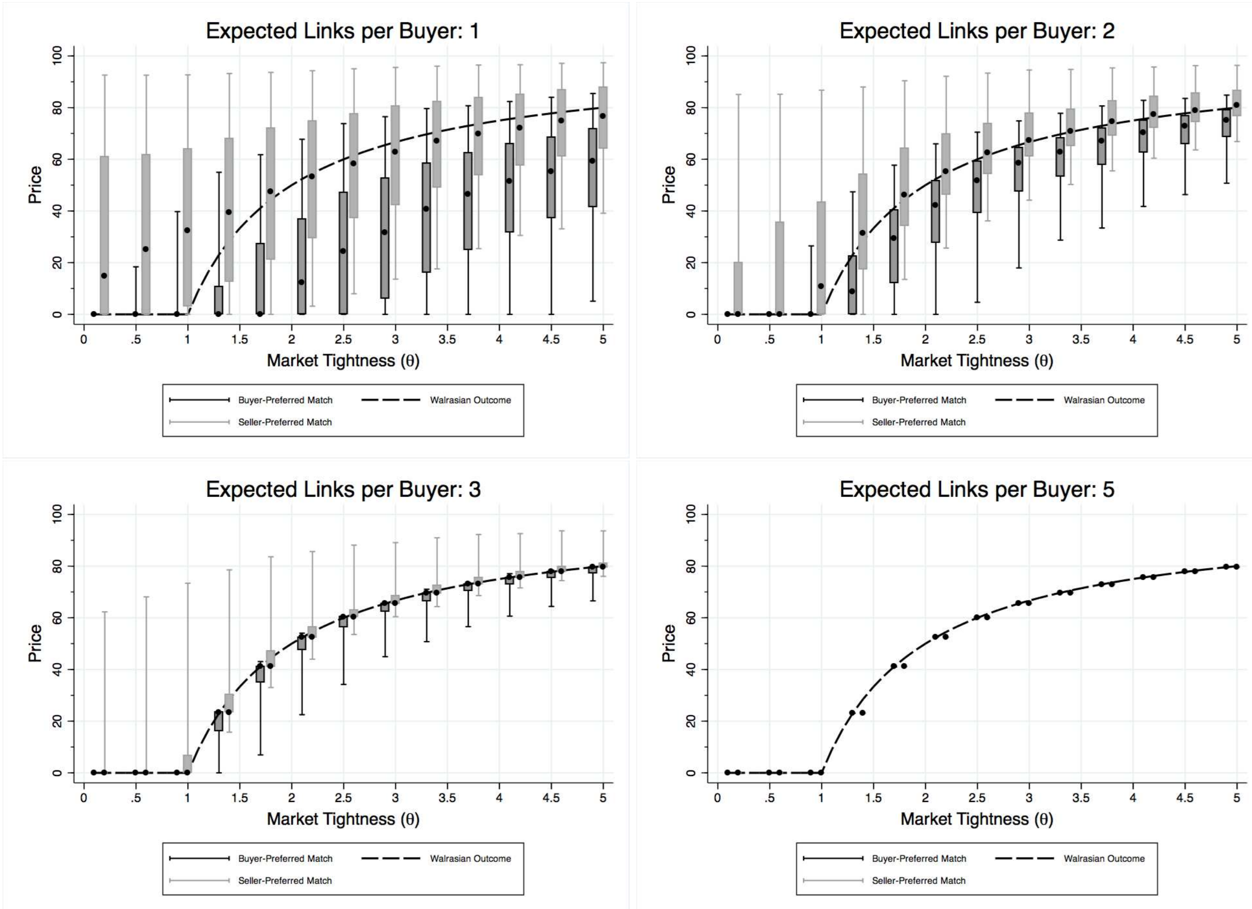
Yellen, J. L. (2014): "Labor Market Dynamics and Monetary Policy," Remarks by Chair Janet Yellen at the Federal Reserve Bank of Kansas City Economic Symposium, Jackson Hole, Wyoming. Available online at http://www.federalreserve.gov/newsevents/speech/yellen20140822a.htm [Accessed: 2015 03 19]. (Cited on page 32)

Figure 1: Price Distribution: Buyer-Preferred Match.



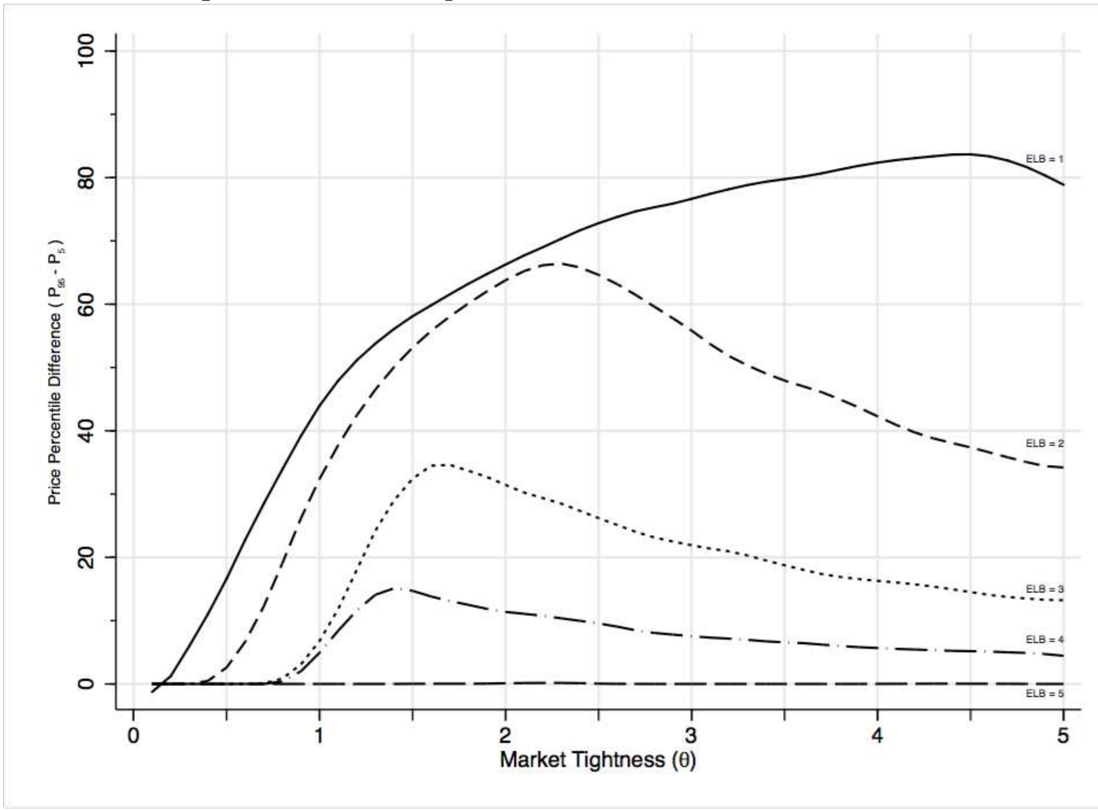
Notes: Starting in the top left, panels 1 to 4 figure display the empirical distribution of prices from the model using the buyer-preferred match disaggregated by: i) Market Tightness (which ranges from 0.1 to 5 in the horizontal axis in each graph) and ii) Expected Links per Buyer (1, 2, 3, and 5). Each vertical box corresponds to a simulated market characterized by those parameters. Each vertical box displays the 95th percentile (upper whisker), 75th percentile (upper hinge), median (black circle marker), 25th percentile (lower hinge), and 5th percentile (lower whisker). Note that buyers' valuation is normalized to range between 0 and 100 which, in turn, bounds the minimum and maximum prices between those values. If the 95th percentile coincides with the 5th percentile, then the figure shows only a dot (which corresponds to the median too). In addition, each panel displays the Walrasian outcome, p_{walras}. We describe how to calculate the Walrasian outcome in subsection 6.1.

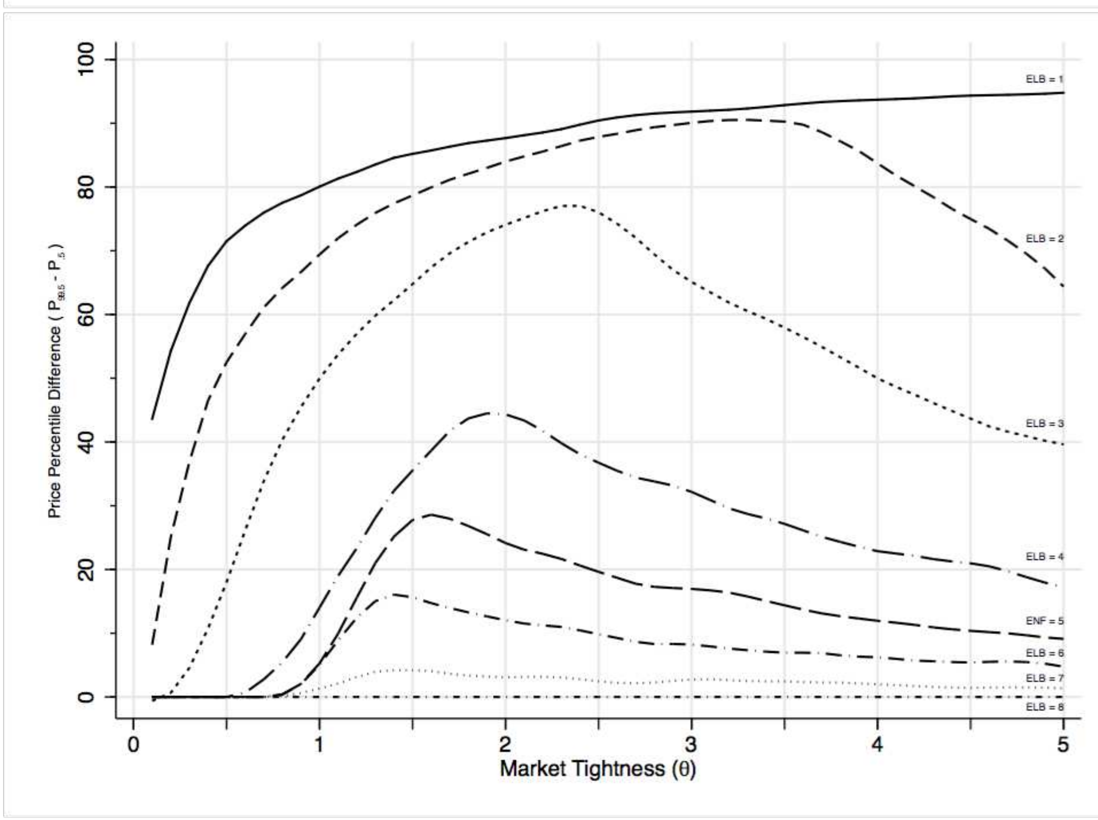
Figure 2: Price Distribution: Buyer- vs. Seller-Preferred Matches.



Notes: At each market tightness, panels 1 to 4 display the distribution of prices in the model using the sellers and the buyer-preferred match. For the seller-preferred match, each vertical box in this figure is identical to the corresponding vertical box in Figure 1. In addition, each panel displays the Walrasian outcome, p_{walras}. We describe how to calculate the Walrasian outcome in subsection 6.1. See the notes in Figure 1 for a description of the vertical boxes.

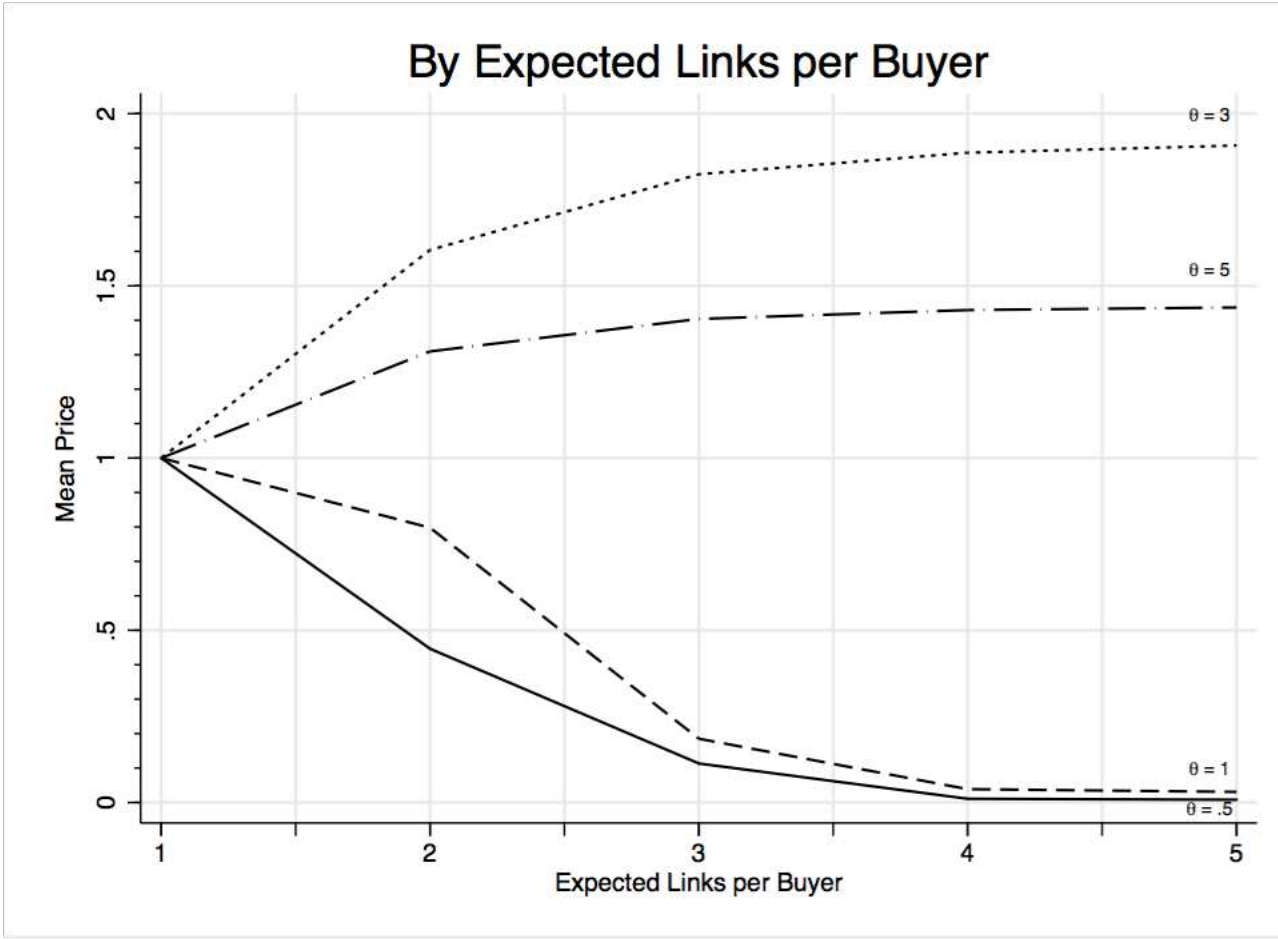
Figure 3: Price Dispersion and the Walrasian Outcome.





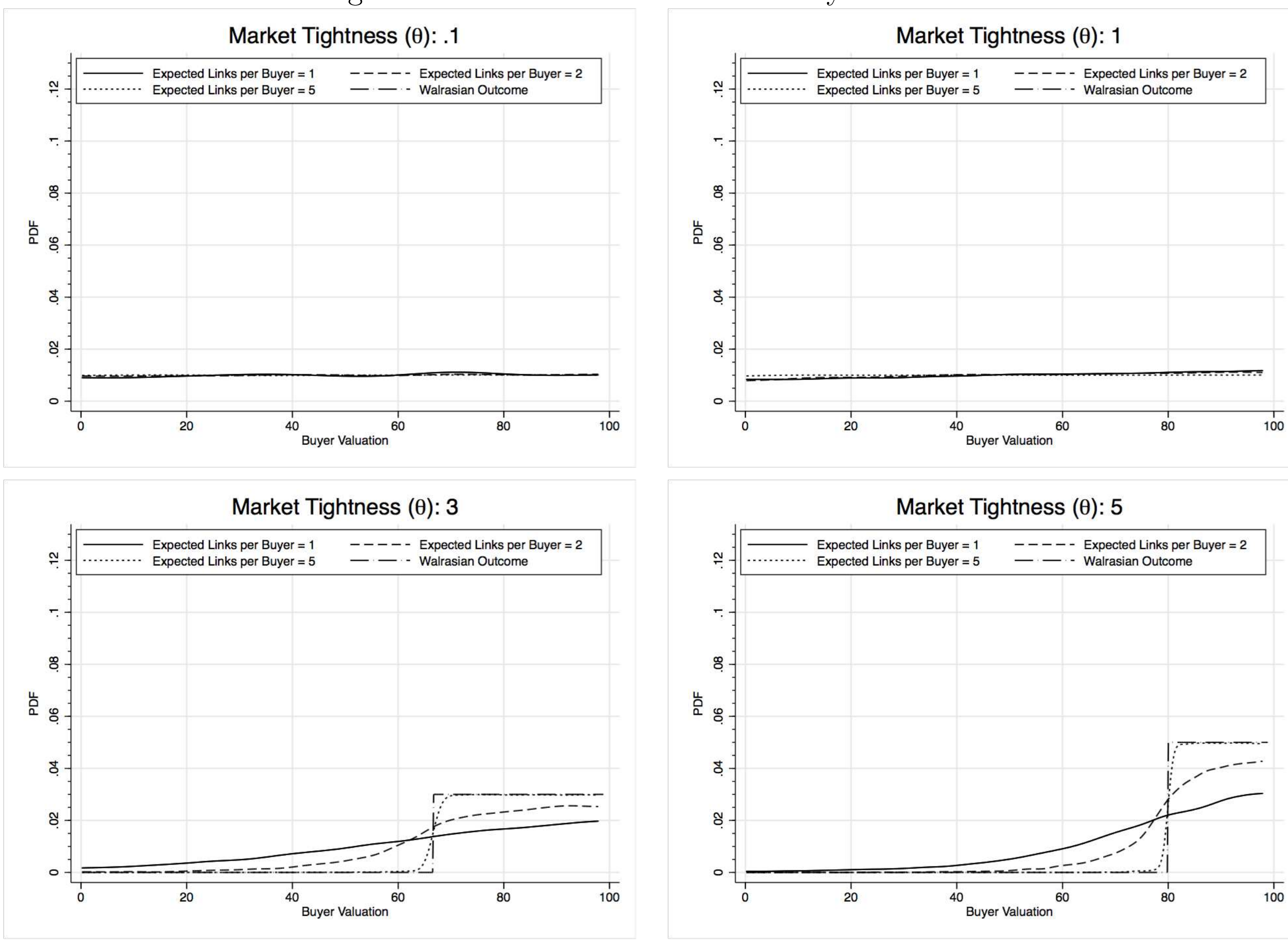
Notes: The top (bottom) figure displays the difference between the 95th (99.5th) price percentile and the 5th (0.5th) price percentile for different market tightness and expected links per buyer using a Nadaraya Watson kernel regression (of the percentile difference on market tightness) with an Epanechnikov kernel with bandwidth selected by cross validation. Price distributions are generated using the buyer-preferred match in a market with 10,000 sellers.

Figure 4: The Effect of Frictions on Mean Prices.



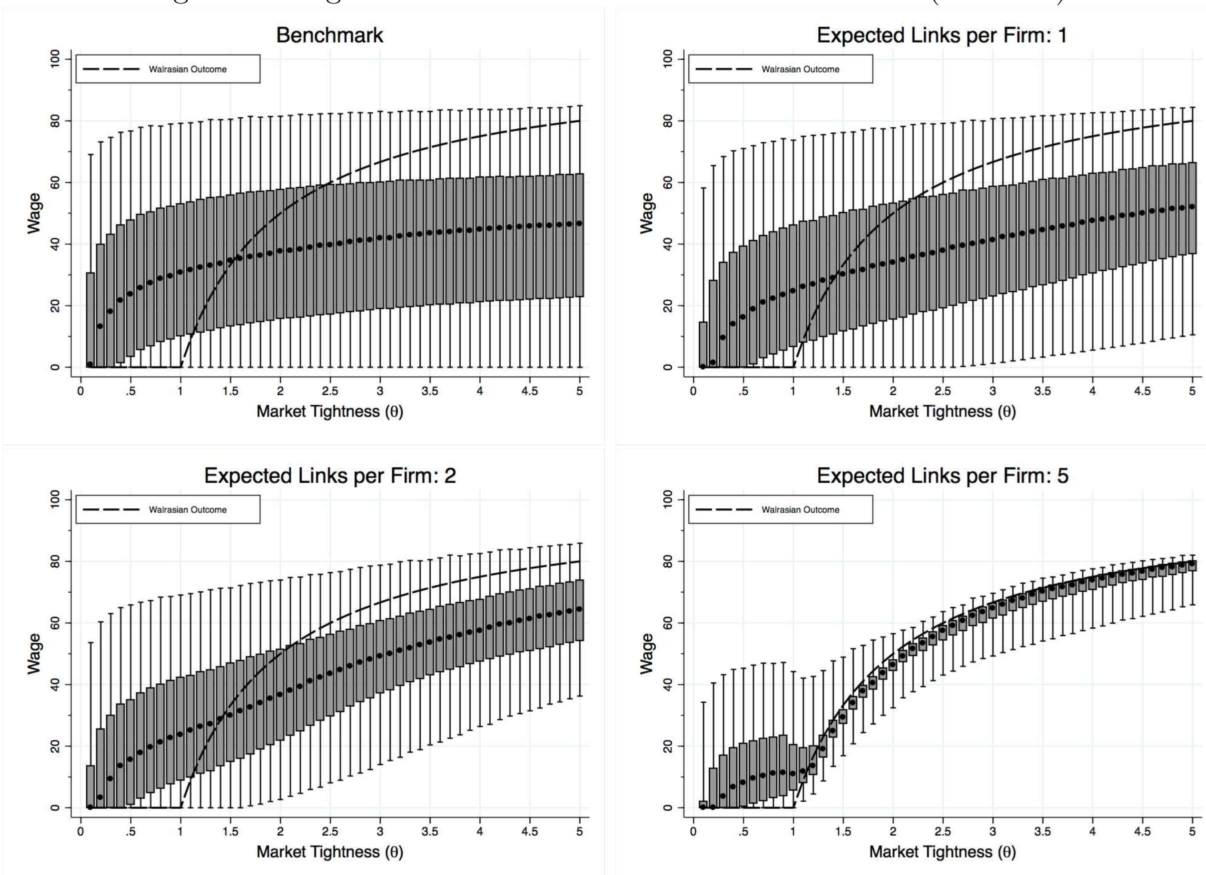
Notes: The figure displays the evolution of mean prices over expected links per buyer for different market tightness. For each market tightness, mean prices are normalized by the mean price when the expected links per buyer is one. So, by construction, mean prices for each market tightness coincide when the expected links per buyer is one.

Figure 5: Distribution of Matched Buyers.



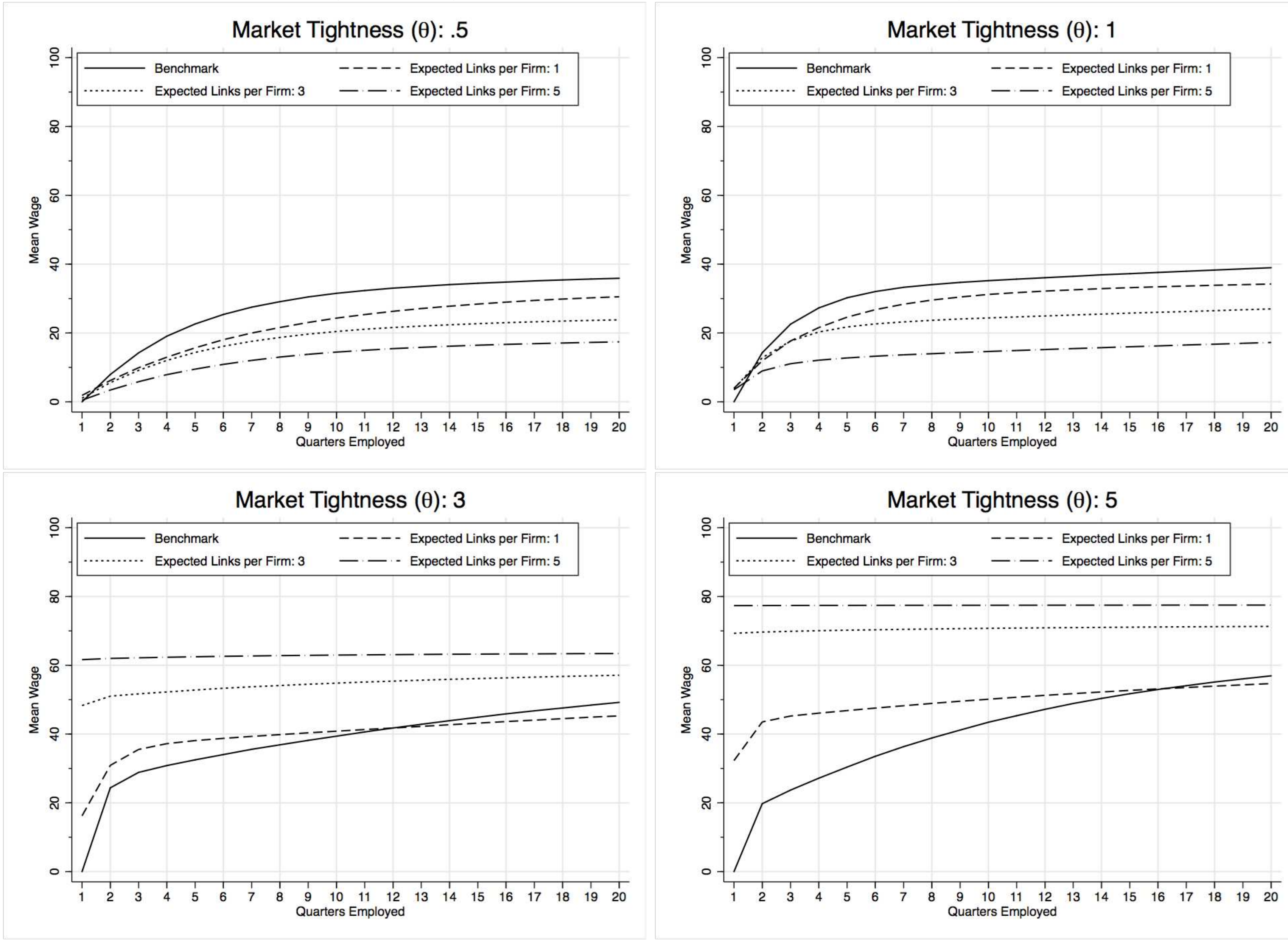
Notes: Each of the four panels in the figure displays the univariate kernel density estimation of the buyers' valuations distribution (buyers who bought a good from a seller) for three markets that differ in the ELB (1, 2, and 5) and for a given market tightness. In addition, each panel displays the distribution of matched buyers in the Walrasian outcome. We describe how to calculate the Walrasian outcome in subsection 6.1. Let μ denote buyers' valuation in the market. We estimate the probability density function in each market, $f(\mu)$, as: $\hat{f}_K(\mu; h) = \frac{1}{N\,h}\sum_{j=1}^{N} K\left(\frac{\mu-\mu(j)}{h}\right)$, where $K(z)$ is a standard univariate gaussian kernel function, h is the bandwidth that we choose by cross validation, and $\mu(j)$, $j = 1, \ldots, N$ are the valuations of the buyers who bought a good in each market. Note that we normalize buyers' valuation to range between 0 and 100 which, in turn, bounds the minimum and maximum prices between those values. Each valuation value between 0 and 100, can be interpreted as the percentile for any distribution of buyers' valuations. Given that the price distribution has its domain bounded we use a renormalization method to deal with the boundaries when estimating the productivity probability density function.

Figure 6: Wage Distribution in the Labor Market Model ($\lambda = 0.05$).



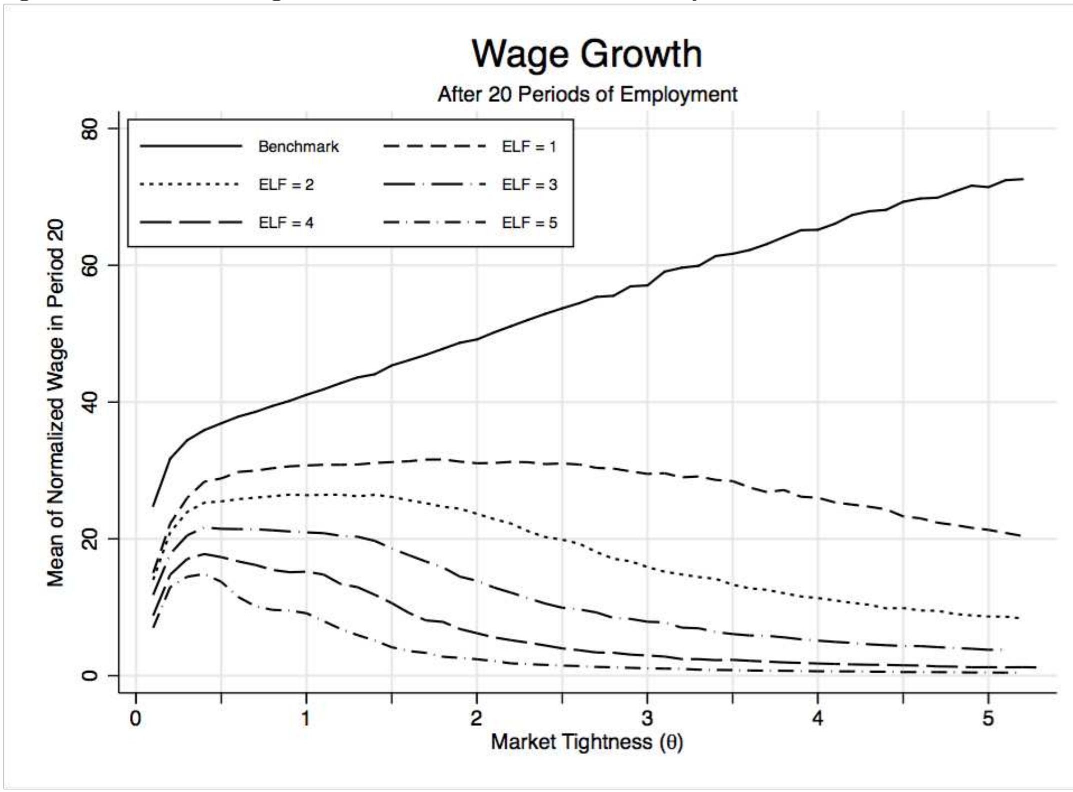
Notes: The figure displays the distribution of wages in the labor market model with $\lambda = 0.05$ (see subsection 6.2.1). All figures display the firm-preferred match. Starting in the top left, panel one shows the empirical distribution of wages for our benchmark, a model with Bertrand competition between at most two firms (see subsection 6.2.1). Panels 2 to 4 figure display the empirical distribution of wages from the model using the firm-preferred match disaggregated by: i) Market Tightness (which ranges from 0.1 to 5 in the horizontal axis in each graph) and ii) Expected Links per firm (1, 2, and 5). Each vertical box corresponds to a simulated market characterized by those parameters. Each vertical box displays the 95th percentile (upper whisker), 75th percentile (upper hinge), median (black circle marker), 25th percentile (lower hinge), and 5th percentile (lower whisker). Note that firms' productivity is normalized to range between 0 and 100 which, in turn, bounds the minimum and maximum wages between those values. If the 95th percentile coincides with the 5th percentile, then the figure shows only a dot (which corresponds to the median too). In addition, each panel displays the Walrasian outcome, w_{walras}. We describe how to calculate the Walrasian outcome in subsection 6.1, where in the case of the labor market $w_{walras} = p_{walras}$.

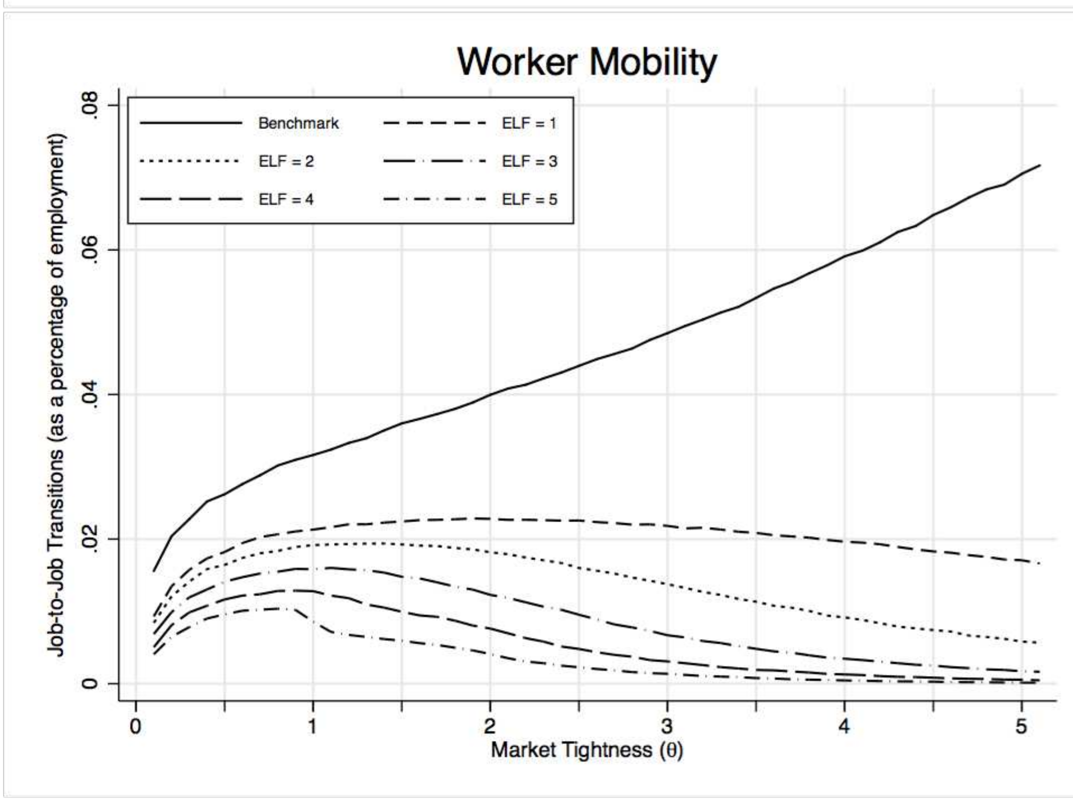
Figure 7: Wage Profile in the Labor Market Model: Mean Wages.



Notes: Each figure displays the wage profile in the labor market model with $\lambda = 0.05$ and the firm-preferred match (see subsection 6.2.1). The horizontal axis shows the number of periods that the worker has been employed. Mean wages are computed by period. The sample is the set of workers that are employed at least 20 consecutive periods in steady state. Each figure shows the wage profile for a given expected number of links per firm (1, 3, and 5) for different market tightness (0.5, 1, 3, and 5). As a benchmark, we also display the wage profile for a model with Bertrand competition between at most two firms (see subsection 6.2.1).

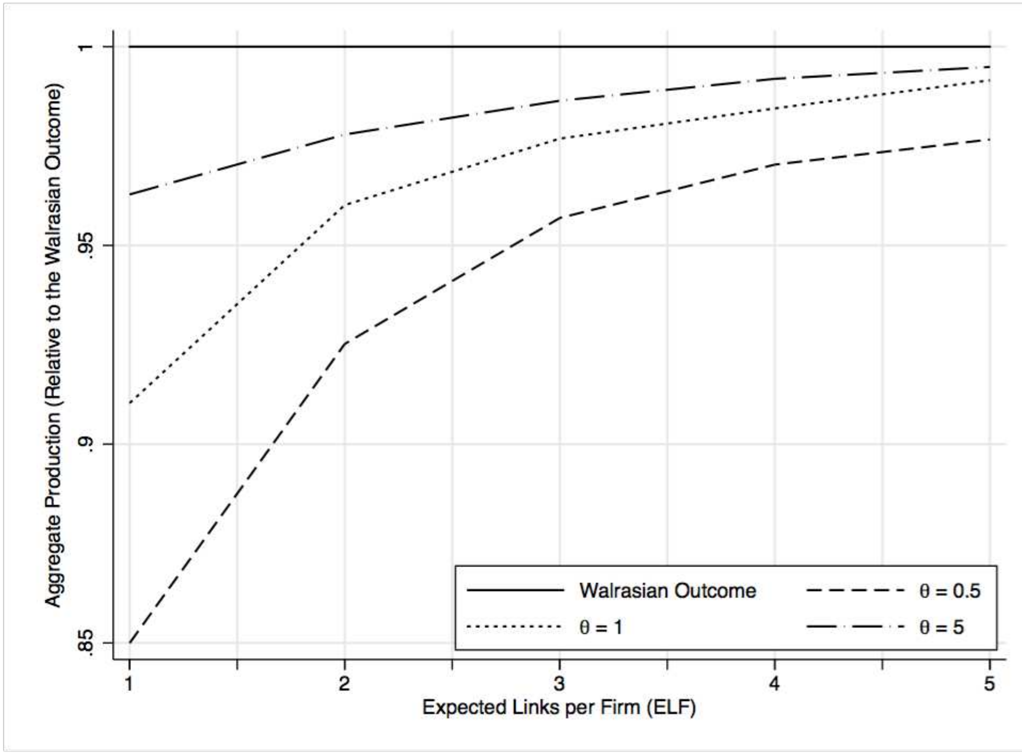
Figure 8: Mean Wage Growth and Worker Mobility in the Labor Market Model.

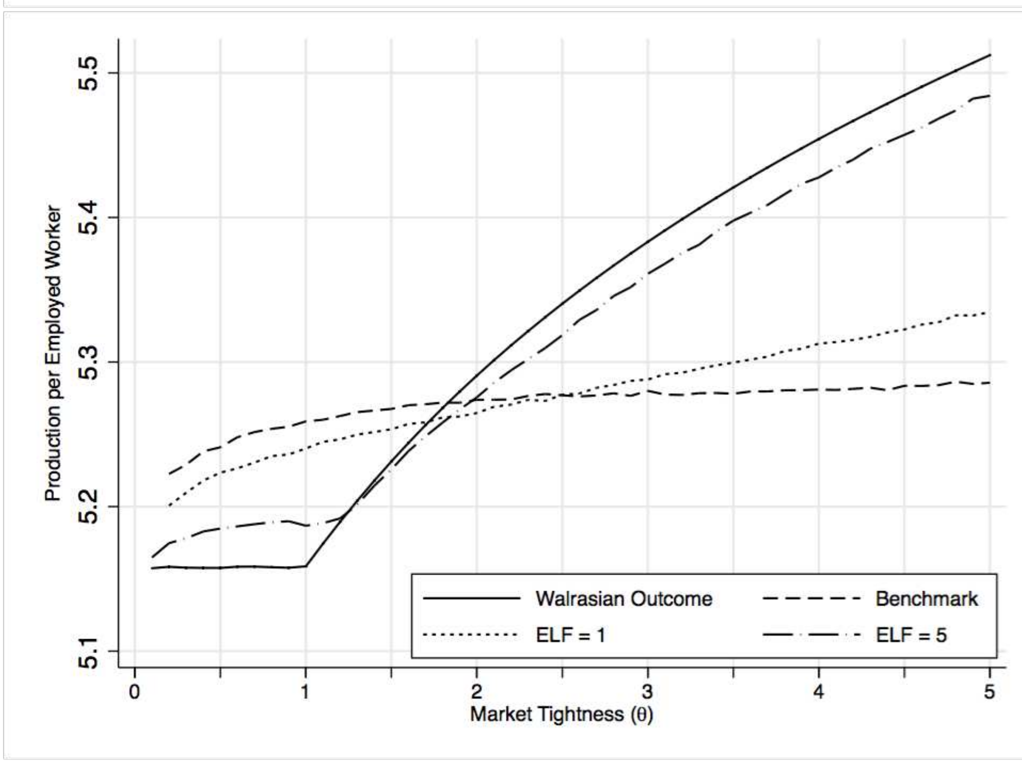




Notes: The top figure displays wage growth by market tightness and expected number of links per firm. The wage growth is defined as the wage of the worker in period 20 minus the wage of the same worker in period 1. We use the sample of workers who have been employed for at least 20 periods. The bottom figure displays the worker mobility by market tightness and expected number of links per firm. Worker mobility is defined as the probability that an employed worker makes a job-to-job transition in a period. For both figures, we set $\lambda = 0.05$ and use the firm-preferred match (see subsection 6.2.1). As a benchmark, we also display the results for a model restricted to have Bertrand competition between at most two firms (see subsection 6.2.1).

Figure 9: Welfare in the Labor Market Model.





Notes: The top panel displays the aggregate production relative to the Walrasian outcome by ELF and by market tightness. For example, when $\theta = 0.5$ and ELF=5, the aggregate production is approximately 97.5% of the aggregate production in the Walrasian outcome. The aggregate production in each market represents the utilitarian welfare in that market. Figure A5 in the online appendix shows the total aggregate production. The bottom panel displays the average productivity per worker (total production divided by number of employees) for different markets. As a benchmark, we also display the results for a model restricted to have Bertrand competition between at most two firms (see subsection 6.2.1). We set $\lambda = 0.05$ and use the firm-preferred match (see subsection 6.2.1). Following Bagger, Fontaine, Postel-Vinay, and Robin (2014), we use the following cumulative distribution function for the productivity of firms $F(\mu) = 1 - exp\Big(-\big[c_1(\mu - c_0)\big]^{c_2}\Big)$, where $c_0 = 5$, $c_1 = 8$, and $c_2 = 0.7$.

Appendix

A Formal Definitions

Definition (graph). *Given a set V of* nodes *and a set $E \subset V^2$ of* edges *we say (V, E) is a* graph*. Moreover,*

- *We say a graph (V, E) is trivial if $E = \emptyset$ and V is a singleton.*
- *We say a graph (V, E) is finite if E is finite.*
- *We say the graph is* undirected *when, for each $v, v' \in V$, $(v, v') \in E$ if and only if $(v', v) \in E$.*
- *We say (V, E) is* a bipartite graph *if there exists two disjoint sets, $V_1, V_2 \subset V$, such that $V = V_1 \cup V_2$ and $(v, v') \in E$ only if $v \in V_i \Rightarrow v' \in V_j$, for $i \neq j$. We write these graphs explicitly as (V_1, V_2, E).*
- *We say a bipartite graph (V_1, V_2, E) is* fully connected *if for each $v_1 \in V_1$, $(v_1, v_2) \in E$ for each $v_2 \in V_2$.*

Definition (networks and prices). *Let $(\mathcal{J}, \mathcal{I}, E)$ be an undirected bipartite graph, and $M \subset E$ be any subset of edges. Let $\mu : \mathcal{J} \to \mathbb{R}$, $b : \mathcal{I} \to \mathbb{R}$, and $p_M : M \to \mathbb{R}$ be functions such that $p_M((j, i)) = p_M((i, j))$ for each $(j, i) \in M$. We say $N = (\mathcal{J}, \mathcal{I}, E; \mu, b)$ is a* buyer-seller network*. We say the function p_M is a* price function *. We say p_M is* individually rational *(IR) if for each $(j, i) \in M$, $p_M(j, i) \in [b(i), \mu(j)]$.*

Definition (matching). *Let $(\mathcal{J}, \mathcal{I}, E; \mu, b)$ be a network. A set $M \subset E$ is a matching if it satisfies:*

- *if $(j, i), (j, i') \in M$, then $i = i'$,*
- *if $(j, i), (j', i) \in M$, then $j = j'$,*
- *$(j, i) \in M$ if and only if $(i, j) \in M$.*

Definition (block). *Let M be a matching and $p_M : M \to \mathbb{R}$. We say an edge $(i, j) \in E \setminus M$* blocks *(M, p_M) if $v(i) < v(j)$.*

Definition (pairwise stability). *Given a non-trivial network $(\mathcal{J}, \mathcal{I}, E; \mu, b)$ and a matching $M \subset E$, we say M is* pairwise stable *at prices p_M if the following hold:*

- No blocking: *no edge $(i, j) \in E \setminus M$ blocks (M, p_M).*
- Individual rationality: *$p_M(i, j) \in [b(i), \mu(j)]$ for all $(i, j) \in M$.*

Definition (graph abstraction). *Let $(\mathcal{J}, \mathcal{I}, E)$ be a buyer-seller graph. We say a directed graph $(\mathcal{G}, E^*)$ is an* abstraction of $(\mathcal{J}, \mathcal{I}, E)$ in fully connected graphs *if the following hold:*

- *Each $G \in \mathcal{G}$ is a fully connected graph $(\mathcal{J}_G, \mathcal{I}_G, E_G)$ such that $\mathcal{J}_G \subset \mathcal{J}$, $\mathcal{I}_G \subset \mathcal{I}$, $E_G = \{e : e \in E \text{ and } e \in (\mathcal{J}_G \times \mathcal{I}_G) \cup (\mathcal{I}_G \times \mathcal{J}_G)\}$,*
- *$\{\mathcal{J}_G : G \in \mathcal{G}\}$ and $\{\mathcal{I}_G : G \in \mathcal{G}\}$ are a partition of $\mathcal{J}$ and $\mathcal{I}$ respectively,*
- *$(G, G') \in E^*$ if and only if there exists $j \in \mathcal{J}_G, i \in \mathcal{I}_{G'}$ such that $(j, i) \in E$.*

Moreover, given any matching $M \subset E$, we say $(\mathcal{G}, E^*)$ is maximal for M *if for each $j \in \mathcal{J}$ such that $i^*(j) = \emptyset$, the subgraph that contains j is trivial.*

Remark 5. *Generally, nodes in graphs are the smallest object in the graph. As such, a more standard definition of abstraction would let the vertices $G \in \mathcal{G}$ be arbitrary objects in an arbitrary set, and would include a bijective mapping between $\mathcal{G}$ and the relevant subgraphs of $(\mathcal{J}, \mathcal{I}, E)$. However, this would imply adding notation that makes the model unnecessarily cumbersome.*

Remark 6. *Since one-to-one graphs and trivial graphs are fully connected, an abstraction in fully connected graphs always exists.*

Remark 7. *Given a graph $(\mathcal{J}, \mathcal{I}, E)$, abstractions in fully connected graphs will not necessarily be unique.*

Definition (network abstraction). *Let $(\mathcal{J}, \mathcal{I}, E; \mu, b)$ be a buyer-seller network. We say $(\mathcal{G}, E^*; p)$ is an* abstraction of $(\mathcal{J}, \mathcal{I}, E; \mu, b)$ *if the following hold:*

- *$(\mathcal{G}, E^*)$ is an abstraction of $(\mathcal{J}, \mathcal{I}, E)$,*
- *If $G \in \mathcal{G}$ satisfies $\mathcal{J}_G = \{j\}$, $\mathcal{I}_G = \emptyset$, then $p(G) = \mu(j)$; if $G \in \mathcal{G}$ satisfies $\mathcal{J}_G = \emptyset$, $\mathcal{I}_G = \{i\}$, then $p(G) = b(i)$; otherwise, $p(G) \in [\min\{b(i) : i \in \mathcal{I}_G\}, \max\{\mu(j) : j \in \mathcal{J}_G\}]$.*

Definition (stability abstraction). *Let $(\mathcal{J}, \mathcal{I}, E; \mu, b)$ be a buyer-seller network and $(\mathcal{G}, E^*; p^*)$ be an abstraction of it in fully connected graphs. We say that M is stable with respect to the abstraction if three conditions hold:*

- *$\mathcal{G}$ does not break M: for each $e \in M$, $e \in E_G$ for some $G \in \mathcal{G}$.*
- *Prices $p(\cdot)$ induce pairwise stability:*
  - *For each non-trivial $G \in \mathcal{G}$, M restricted to G is stable at prices $p(j, i) = p^*(G)$ for all $(j, i) \in M \cap E_G$,*
  - *If $G = (\{i\}, \emptyset, \emptyset)$ for some j, then $p^*(G) = b(i)$,*
  - *If $G = (\emptyset, \{j\}, \emptyset)$ for some j, then $p^*(G) = \mu(j)$.*

- *Cheapest sorting: if $(G, G') \in E^*$ then $p(G) \leq p(G')$.*

Remark 8. *The cheapest sorting condition is defined to be consistent with the construction of the edges in E^*. Indeed, we define E^* so that $(G, G') \in E^*$ whenever a buyer in G is linked to a seller in G'. Since buyers search for the cheapest price, then we define cheapest sorting as $p(G) \leq p(G')$. If we took the opposite convention for the edges in E^*, that $(G, G') \in E$ whenever a seller in G is linked to a buyer in G', since sellers search for the highest price, then cheapest sorting would be defined as $p(G) \geq p(G')$.*

B Proof of Propositions 1, 2, and 4

In this section we prove Propositions 1 and 2. To do this, we define formally what the *induced prices* are. Let Let $\mathcal{N} = (\mathcal{J}, \mathcal{I}, E; \mu, b)$ be a network, M be a matching, and $A = (\mathcal{G}, E^*)$ be an abstraction of $(\mathcal{J}, \mathcal{I}, E)$ in fully connected subgraphs. Given a price function p_M such that M is stable with respect to p_M, the prices p^* induced on A are those that satisfy the following:

- If $G \in \mathcal{G}$ contains only seller j, then $p(G) = \mu(j)$,
- If $G \in \mathcal{G}$ contains only buyer i, then $p(G) = b(i)$,
- If $G \in \mathcal{G}$ contains a matched pair $(i, j) \in M$, then $p(G) = p_M(i, j)$.
- Else, $p(G) = \max\{b(i) : i \in \mathcal{I}_G\}$.

Similarly, given a function $p^* : \mathcal{G} \to \mathbb{R}$ we say the prices induced on A by p^* is the price function p_M defined as follows:

- If $(i, j) \in E_G$ then $p_M(i, j) = p(^*G)$

Lastly, for simplicity we normalize all outside options to 0. If this were not the case we need to redefine maximal abstractions so that unmatched sellers are also isolated in trivial subgraphs. Finally, to rule out trivial cases, assume that $\mu(j) \geq 0$ for all buyers j.

Proposition 1: Let $\mathcal{N} = (\mathcal{J}, \mathcal{I}, E; \mu, b)$ be a network. Let M be a matching. Then the following are equivalent:

1. There exists p_M such that M is stable with respect to p_M,
2. There exists an abstraction in fully connected graphs $A = (\mathcal{G}, E^*; p^*)$ such that M is stable with respect to A.

Proof. 1 implies 2: Let $A = (\mathcal{G}, E^*; p^*)$ be defined as follows: For each $(i, j) \in M$ let $G_{ij} = (\{i\}, \{j\}, \{(i, j), (j, i)\})$. For each $i \in \mathcal{I}$ such that $i^*(j) = \emptyset$ let $G_{i\emptyset} = (\{i\}, \emptyset, \emptyset)$ and for each $j \in \mathcal{J}$ such that $j^*(i) = \emptyset$, let $G_{j\emptyset} = (\emptyset, \{j\}, \emptyset)$. Let $p(G_{ij}) = p_M(i, j)$, $p(G_{i\emptyset}) = b(i)$ and

$p(G_{j\emptyset}) = \mu(j)$. Define E^* so that $(G, G') \in E^*$ if and only, in the original graph, the buyer in G is linked with the seller in G'. These ingredients define an abstraction in fully connected graphs. Moreover, this abstraction is maximal for M and does not break M. Proposition 2 proves that M is stable with respect to $(\mathcal{G}, E^*; p^*)$ where p^* are the prices induced by p_M.

2 implies 1: Let A be as in item 2. Then, M is stable with respect to p_M where p_M are the prices induced by p^*.

□

Proposition 2: Let $\mathcal{N} = (\mathcal{J}, \mathcal{I}, E; \mu, b)$ be a network, M be a matching, and $A = (\mathcal{G}, E^*)$ be an abstraction of $(\mathcal{J}, \mathcal{I}, E)$ that does not break M. Assume that A is a maximal abstraction for M.

- If M is stable with respect to p_M, then M is stable with respect to $(\mathcal{G}, E^*; p^*)$ where p^* are the prices induced on A by p_M.
- If M is stable with respect to $(\mathcal{G}, E^*; p^*)$ for some p^*, then M is stable with respect to p_M, where p_M are the prices induced on $\mathcal{N}$ by p^*.

Proof. Part 1: First, notice that because M is stable with respect to p_M then p^* is well defined. We now check M is stable with respect to $(\mathcal{G}, E^*; p^*)$. By construction A does not break M. Also, because M is stable with respect to p_M then prices p^* induce pairwise stability in each $G \in \mathcal{G}$. We only need to check that cheapest sorting is satisfied. Let G, G' be such that $(G, G') \in E^*$, $j \in \mathcal{J}_G$, $i \in \mathcal{I}_{G'}$ such that $(i, j) \in E$. Notice that $v(i) \leq p^*(G')$. Indeed, if i is unmatched then $v(i) = b(i) \leq p^*(G')$, and if i is matched then $p^*(G') = v(i)$. If G is trivial (i.e. $\mathcal{J}_G = \{j\}$) then $p^*(G) = \mu(j)$. Because M is stable with respect to p_M and $(i, j) \in M$, then $\mu(j) \leq v(i)$. Therefore, $p^*(G) \leq p^*(G')$. Now assume G is not trivial (i.e. $\mathcal{J}_G \neq \{j\}$). Then j is matched, so $p^*(G) = p_M(j, i^*(j))$. By stability of M with respect to p_M, $p_M(j, i^*(j)) = v(j) \leq v(i)$. Thus, $p^*(G) \leq p^*(G')$.
Part 2: Let $(i, j) \in M$. There are four cases:
case 1: no unmatched buyer j' blocks: Let j' be such that $(j', i) \in E$ and $i^*(j') = \emptyset$. By cheapest sorting, $p^*(G') \leq p^*(G)$ where G' is the subgraph that contains j' and G is the subgraph that contains i. Since $\mu(j') = p^*(G')$ then $\mu(j') \leq p_M(i, j)$ so j' does not block.
case 2: no matched buyer j' blocks: Let j' be such that $(j', i) \in E$ and $i^*(j') = i'$. By cheapest sorting, $p^*(G') \leq p^*(G)$ where G' is the subgraph that contains j' and G is the subgraph that contains i. Since $p_M(i', j') = p^*(G')$ then $p_M(i', j') \leq p_M(i, j)$ so j' does not block.
case 3: no unmatched seller i' blocks: Let i' be such that $(i', j) \in E$ and $j^*(i') = \emptyset'$. By cheapest sorting, $p^*(G) \leq p^*(G')$ where G' is the subgraph that contains i' and G is the subgraph that contains j. Since G' contains an unmatched seller then $p^*(G') = 0$. Since $0 = p^*(G')$ then $p_M(i, j) = p^*(G) \leq 0$ so $0 = p_M(i, j)$. Thus, i' does not block.
case 4: no matched seller i' blocks: Let i' be such that $(i', j) \in E$ and $j^*(i') = j''$. By cheapest

sorting, $p^*(G) \leq p^*(G')$ where G' is the subgraph that contains i' and G is the subgraph that contains j. Since $p_M(i', j') = p^*(G')$ and $p_M(i,j) = p^*(G)$ then $p_M(i', j') \geq p_M(i,j)$. Thus, i' does not block. Since no blocks are made, M is stable with respect to p_M. $\square$

Proposition 4. *Let $\mathcal{J}$ and $\mathcal{I}$ be sets of sellers and buyers respectively. Let $M \subset (\mathcal{J} \times \mathcal{I})^2$ be a matching and let $\mathcal{N} = (\mathcal{J}, \mathcal{I}, E; \mu, b)$ and $\mathcal{N}' = (\mathcal{J}, \mathcal{I}, E'; \mu, b)$ be two networks such that $M \subset E$ and $M \subset E'$. Assume there exists a graph $A = (\mathcal{G}, E^*)$ such that A is an abstraction of both $\mathcal{N}$ and $\mathcal{N}'$, and that M is stable with respect to $(\mathcal{G}, E^*; p^*)$ for some p^*. Then M is stable in $\mathcal{N}$ with respecto to p_M, and M is stable in $\mathcal{N}'$ with respect to p'_M, where these are the prices induced by p^* in $\mathcal{N}$ and $\mathcal{N}'$ respectively.*

The proof of this proposition is a straightforward application of proposition 1. If we add the assumption that A is maximal for M, then A completely characterizes the prices that sustain M and the networks that sustain M at those prices. If A is not maximal for M then A only characterizes a subset of the prices that sustain M, as well as the set of networks that sustain M at those prices.

An additional application of Proposition 1

Proposition 1 allows us to rationalize pairwise stable matchings *as if* these where the result of equilibrium bidding strategies in simultaneous auction games. In the example above, we can interpret each node in the abstraction as the following second-price auction. Each seller holds an auction. Buyers can only bid in the sellers' auction that belongs to same node in the abstraction. In the example, buyer A only bids in seller 1's auction, buyers B and C only bids in seller 3's auction and seller 2's auction, buyer D does not bid in any auction. Finally, we assign fictitious values to the buyer with the second highest valuation in each node of the abstraction. This fictitious value is determined by the constraints imposed by the edges in the abstraction. In the example, C has fictitious value $\hat{\mu}(C) = p(C,3) \in [\mu(D), \mu(C)]$, B has ficticious value $\hat{\mu}(B) \in [p(C,3), \mu(B)]$, A has fictitious value $\hat{\mu}(A) = p(A,1) \in [p(C,3), \mu(A)]$, and D has fictitious value $\hat{\mu}(D) = \mu(D)$. Following Peters and Severinov (2006), an equilibrium of these two independent second price auctions is that B and C purchase goods at a price $p = p(C,3)$, and buyer A purchases a good at a price between $p(C,3)$ and $\mu(A)$. By virtue of out previous arguments, these prices make the matching M stable with respect to the abstraction and thus stable in the original network. Alternatively, instead of assigning fictitious values to the buyers with the second highest valuation, we can assign reservation prices to the sellers. Again, these reservation prices are determined by the constraints imposed by the edges in the abstraction. The outcome of the independent second-price auctions with fictitious valuations for the buyers is observationally equivalent to the independent second-price auctions with reservation prices for the sellers. Moreover, the outcome of either of these independent second-price auctions is indistinguishable from the outcome of any other mechanism that generates the same matching at the supporting prices.

C Formal algorithm and proof of Proposition 3

In this appendix we discuss the formal algorithm used in the main paper and prove some of its properties. We now present the basic notation we use in the match determination program. Let $s^t \in \mathbb{R}^{J\times I}$ be a matrix of prices for each buyer-seller pair. Each element, $s^t_{i,j}$, represents the price that buyer j would have to bid for seller i if j were to bid for i in round t. Vector q^t represents the bidding queue in period t: $q^t_n = j \in \mathcal{J}$ represents that in round t, buyer j is the n-th bidder in the queue. The algorithm ends when $l(q) = 0$, where $l(q)$ indicates the length of q. Quantity $D(j)$ indicates j's demand. Quantities with primes will indicate quantities that will carry over to the next round of the algorithm. Finally, for each seller j, we use the following payoff function to model that a buyer can only buy a good from a seller if the two are linked in the network: $u_j : \mathcal{I} \times \mathbb{R}^{I\times J} :\to \bar{\mathbb{R}}$, $u_j(i, s) = \mu(j) - s_{i,j}$ if $(i, j) \in E$ and $u_j(i, s) = -\infty$ otherwise.

Recall some notational conventions: given a matching M, $i^*(\cdot) : \mathcal{J} \to \mathcal{I} \cup \{\emptyset\}$ satisfies $(i^*(j), j) \in M$ for each M-matched j, and $i^*(j) = \emptyset$ if j is M-unmatched. Analogously, $j^*(\cdot) : \mathcal{I} \to \mathcal{J} \cup \{\emptyset\}$ satisfies $(i, j^*(i)) \in M$ for each M-matched i, and $j^*(i) = \emptyset$ if i is M-unmatched. Also, even if not explicitly stated, the network is denoted $\mathcal{N} = (\mathcal{I}, \mathcal{J}, E; b, \mu(\cdot))$, $I = \#\mathcal{I}$, $J = \#\mathcal{J}$.

Match Determination Program.

Input$= (\mathcal{N}, s^0, (u_1, ..., u_J), h^0, q)$ where:

- $s^0 = (s^0_1, ..., s^0_J) \in \mathbb{R}^{J\times I}$, $s^0_j = (b, ..., b) \in \mathbb{R}^I$,
- For each buyer j, and each $t \in \mathbb{N} \cup \{0\}$, $u_j(i, s^t) = \mu(j) - s^t_{i,j}$ if $(i, j) \in E$ and $u_j(i, s^t) = -\infty$ if $(i, j) \notin E$,
- $h^0 = (0, ..., 0) \in \mathbb{R}^{I\times J}$.
- $q^0 \in \mathcal{J}^J$ such that $q^0_n = q^0_m$ iff $m = n$.

Start step $R(1)$:

R(t). Set $h^t = h$, $s^t = s$, $q^t = q$, $j = q_1$.

1. If $\max\{u_j(i, s) : i \in \mathcal{I}\} < 0$ set $s' = s$ and $h' = h$, $q' = (q_2, ..., q_{l(q)})$.
   a. If $l(q') = 0$, stop, set $M = \{(i, j) : h_{i,j} = 1\}$, and *Output*$= M$.
   b. If $l(q') \neq 0$, set $q^{t+1} = q'$, $s^{t+1} = s'$, $h^{t+1} = h'$ and proceed to $R(t + 1)$.
2. If $\max\{u_j(i, s) : i \in \mathcal{I}\} \geq 0$ let $D(j) \in \arg\max\{u_j(i, s) : i \in \mathcal{I}\}$.
   a. If $\arg\max\{u_j(i, s) : i \in \mathcal{I}\}$ has more than one element, select $D(j) \in \arg\max_{i\in\mathcal{I}}\{u_j(i, s)\}$ randomly.

3. Set the following parameters:

   a. $s'_{D(j),j} = s_{D(j),j}$; for all $j' \neq j$, $s'_{D(j),j'} = s_{D(j),j} + \frac{\Delta}{2}$; $s'_{i'',j''} = s_{i'',j''}$ elsewhere,

   b. If $h_{D(j),j'} = 0$ for all $j' \neq j$, set $q' = (q_2, ..., q_{l(q)})$; if $h_{D(j),j'} = 1$ for some $j' \neq j$, set $q' = (q_2, ..., q_{l(q)}, j')$,

   c. $h'_{D(j),j} = 1$; for all $j' \neq j$, $h'_{D(j),j'} = 0$; $h'_{i'',j''} = h_{i'',j''}$ elsewhere.

4. If $l(q') = 0$, stop. Set $M = \{(i,j) : h_{i,j} = 1\}$. *Output*$= M$.

   If $l(q) \neq 0$ set $h' = h^{t+1}$ and $s' = s^{t+1}$ and $q' = q^{t+1}$. Then start $R(t+1)$.

Although this algorithm is motivated by Crawford and Knoer (1981) and Kelso and Crawford (1982), there are three important differences. The first is that firm productivities increase in increments of Δ whereas bids increase in increments of $\frac{\Delta}{2}$. Since Crawford and Knoer (1981) and Kelso and Crawford (1982) work with a discrete core, the algorithm they run produces a stable match when both bid increments and productivities increase by the same amount. However, since we work with a continuous core, it is not true that the matching generated by such an algorithm is stable. One can construct examples where the matching generated by the algorithms in Crawford and Knoer (1981) and Kelso and Crawford (1982) (say, M) satisfies that there is no price function p_M such that M is stable with respect to p_M. We provide one example in section B (Figure A6) in the online appendix. The modification we introduce, that bids live in a finer grid than firm productivities, helps us bypass this problem. The second difference with the algorithms in Crawford and Knoer (1981) and Kelso and Crawford (1982) is that we only use their program to find the matching, but not the prices that make it stable. The reason is that their algorithm makes prices rise too quickly. While in some networks the price generated by the algorithms in Crawford and Knoer (1981) and Kelso and Crawford (1982) is the pointwise minimum price that makes the matching stable, this is not always guaranteed. This is because in our setting we violate the non-indifference assumptions made in Crawford and Knoer (1981) and Kelso and Crawford (1982). In order to capture, for each matching, the pointwise maximum and minimum prices at which that matching is stable we run two independent programs. We call these the Price Determination Programs, and we describe them below. The first Price Determination Program (I), outputs the pointwise minimum price function at which a matching is stable. The second Price Determination program (II), outputs the pointwise maximum price function at which a matching is stable. The third difference is that, when a seller i accepts a bid from a buyer j, then any future bid buyer j' submits to i must outbid j's bid. In symbols, if in round t seller i accepts bid $s^t_{i,j}$ from j, then at the end of round t all sellers j' linked to i have their bid price raised to $s^{t+1}_{i,j'} = s^t_{i,j} + \frac{\Delta}{2}$. This modification reduces the run time of the algorithm by a factor of four.

Price Determination Program (I).

Input$= (\mathcal{N}, M)$.

1. For each $i \in \mathcal{I}$ such that $j^*(i) = \emptyset$ set $\rho_i^1 = b$.

2. For each $i \in \mathcal{I}$ such that $j^*(i) \neq \emptyset$ set $\rho_i^1 = \max\{\mu(j) : (i,j) \in E \text{ and } i^*(j) = \emptyset\}$.

3. Set $t = 1$. Start step 4(1).

4(t). Given $(\rho_1^t, ..., \rho_I^t)$:

   a. For each $i \in \mathcal{I}$ such that $j^*(i) = \emptyset$ set $\rho_i^{t+1} = \rho_i^t$.

   b. For each $i \in \mathcal{I}$ such that $j^*(i) \neq \emptyset$, let $j \equiv j^*(i)$. Then, set

$$\rho_i^{t+1} = \max\{\rho_{i'}^t : (\exists j')(i', j') \in M,\, (i, j') \in E\}.$$

   c. If for all $i \in \mathcal{I}$ $\rho_i^{t+1} = \rho_i^t$:

      * For each i such that $j^*(i) \neq \emptyset$ set $p_M(i, j^*(i)) = \rho_i^{t+1}$.
      * *Output*$= (p_M(\cdot))$.

   d. Otherwise, start step $4(t+1)$.

Price Determination program (I) outputs the minimum price at which M can be made stable.

Price Determination Program (II).
Input$= (\mathcal{N}, M)$.

1. For each $i \in \mathcal{I}$ such that $j^*(i) = \emptyset$ set $\rho_i^1 = b$.

2. For each $i \in \mathcal{I}$ such that $j^*(i) \neq \emptyset$ set $\rho_i^1 = \mu(j^*(i))$.

3. Set $t = 1$. Start step 4(1).

4(t). Given $(\rho_1^t, ..., \rho_I^t)$:

   a. For each $i \in \mathcal{I}$ such that $j^*(i) = \emptyset$ set $\rho_i^{t+1} = \rho_i^t$.

   b. For each $i \in \mathcal{I}$ such that $j^*(i) \neq \emptyset$, let $j \equiv j^*(i)$. Then, set

$$\rho_i^{t+1} = \min\{\rho_{i'}^t : (i', j) \in E\}.$$

   c. If for all $i \in \mathcal{I}$ $\rho_i^{t+1} = \rho_i^t$:

      * For each i such that $j^*(i) \neq \emptyset$ set $p_M(i, j^*(i)) = \rho_i^{t+1}$.
      * *Output*$= (p_M(\cdot))$.

   d. Otherwise, start step $4(t+1)$.

Price Determination program (II) outputs the maximum price at which M can be made stable.

In Section 5 we claimed our algorithm has four properties: it ends in finite time, it selects a pairwise stable allocation, and for each allocation it selects the pointwise minimum and maximum prices that sustain it.

Proposition 3: The deferred acceptance algorithm has the following properties:

1. It stops after a finite number of rounds.

2. It outputs a pairwise stable allocation.

3. Price Determination program (I) outputs the pointwise minimum price function at which M is stable.

4. Price Determination program (II) outputs the pointwise maximum price function at which M is stable.

We now prove these items one at a time. In what follows, we use MDP and PDP to abbreviate the Matching Determination Program and the Price Determination Program respectively. Finally, if $(x_i)_{i\in\mathcal{I}}$ is a vector indexed by $\mathcal{I}$ we use the convenient shorthand notation $x_\cdot$ to denote the whole vector, whenever ambiguity is unlikely.

We need two lemmas: the first, shows that, given M produced by the MDP, there exist prices p_M such that M is stable with respect to M. The second shows that the prices generated by the PDP are weakly lower than any p_M such that M is stable with respect to M. To prove these Lemmas, recall that $(\rho_i^t)_{i\in\mathcal{I},t\geq 1}$ from the PDP(I) is defined as follows:

- If $j^*(i)=\emptyset$, $\rho_i^t = b$ for all t.

- If $j^*(i)=j$ for some $j\in\mathcal{J}$, $\rho_i^1 = \max\{\mu(j) : (i,j)\in E, i^*(j)=\emptyset\}$ for each $i\in\mathcal{I}$, and $\rho_i^t = \max\{\rho_{i'}^{t-1} : (\exists j', i') : (j',i')\in M,\, (j',i)\in E\}$ for all $t\geq 2$.

The following properties imply that there exists a value T such that, for all i and all $t\geq T$, $\rho_i^t = \rho_i^{t+1}$. That is, $(\rho_\cdot^t)_{t\geq 0}$ is eventually constant. We let $\rho_\cdot^\infty \equiv \lim_{t\to\infty}\rho_\cdot^t$.

1. For all i, $\rho_i^t \leq \rho_i^{t+1}$. This follows because $\rho_i^{t-1}\in\{\rho_{i'}^{t-1} : (\exists j') : (j',i')\in M,\, (j',i)\in E\}$ whenever $j^*(i)=j$ and $\rho_i^t = b$ whenever $j^*(i)=\emptyset$.

2. For all i, $\rho_i^t \leq \max\{\mu(j) : j\in\mathcal{J}\}$.

3. For all i, if $\rho_i^t\neq\rho_i^{t+1}$ then $\rho_i^{t+1}-\rho_i^t\geq\Delta$.

Finally, recall that $\Delta \in \mathbb{R}$ is chosen so that for all $j \in \mathcal{J}$, $\mu(j) = b + k_j\Delta$ for for $k_j \in \mathbb{N} \cup \{0\}$. In particular, $\mu(j) \geq b$ for all j. This normalization only rules out uninteresting cases where a buyer never places a bid and is never matched to a seller.

Lemma 1. *Let M be the matching produced by the MDP. Then, there exists p_M such that M is stable with respect to p_M.*

Proof. For each edge $(i,j) \in M$ define $p_M(i,j) = \rho_i^\infty$ where $(\rho_i^t)_{i\in\mathcal{J},t\in\mathbb{N}\cup\{\infty\}}$ is as defined in the PDP(I). Also, let T be the last round of the MDP and let $[s_{i,j}^T]_{i\in\mathcal{I},j\in\mathcal{J}}$ be the matrix of final prices generated by the MDP. We show that M is stable with respect to p_M. Assume first that $(i,j) \in E$ are such that $j^*(i) = i^*(j) = \emptyset$. Then i received no bids, so $s_{i,j}^T = b$. Since the algorithm ended, it must be that $u_j(i, s^T) < 0 \Leftrightarrow \mu(j) < b$, a contradiction. Thus, there does not exist an edge $(i,j) \in E$ such that $j^*(i) = i^*(j) = \emptyset$ so, a fortiori, no such edge $(i,j) \in E$ blocks M. Now let $(i,j) \in M$. Pick $j' \neq j$ such that $(i,j') \in E$. We show $(i,j') \in E$ does not block M. If $i^*(j') = \emptyset$ then $\mu(j') \leq \rho_i^1 \leq \rho_i^\infty = p_M(i,j)$. If $i^*(j') \neq \emptyset$ then $\rho_i^\infty \geq \rho_{i^*(j')}^\infty$ by construction. Thus, $p_M(i,j) \geq p_M(i^*(j'),j')$. Thus, (i,j') does not block M. Pick $i' \neq i$ such that $(i',j) \in E$. We show $(i',j) \in E$ does not block M. If $j^*(i') \neq \emptyset$ then $\rho_{i'}^\infty \geq \rho_i^\infty$ by construction. Thus, $p_M(i', j^*(i')) \geq p_M(i,j)$. Let $j^*(i') = \emptyset$. Then i' never received a bid. Let t be the last time j bids for i. Since bidders bid for the cheapest seller $s_{i,j}^t \leq s_{i',j}^t = b$. By definition of t, $s_{i,j}^t = s_{i,j}^T$ so $s_{i,j}^T = p_M(i,j) = b$. We use this to argue that $\rho_{\hat{\imath}}^1 = b$ for all matched $\hat{\imath}$ such that $(i, j^*(\hat{\imath})) \in E$ (note that i is one such $\hat{\imath}$). Pick $\hat{\imath}$ such that $j^*(\hat{\imath}) \neq \emptyset$ and $(i, j^*(\hat{\imath})) \in E$. Then, $s_{\hat{\imath},j^*(\hat{\imath})}^T \leq s_{i,j^*(\hat{\imath})}^T$.[37] Since $s_{i,j}^T = b$ and $s_{i,j^*(\hat{\imath})}^T \leq s_{i,j}^T + \frac{\Delta}{2}$, then $s_{\hat{\imath},j^*(\hat{\imath})}^T \leq b + \frac{\Delta}{2}$. If there exists $\tilde{j}$ such that $i^*(\tilde{j}) = \emptyset$ and $(\hat{\imath}, \tilde{j}) \in E$, then it must be that $\mu(\tilde{j}) = b$. Indeed, if $\mu(\tilde{j}) > b$ then $\mu(\tilde{j}) \geq b + \Delta$ which is a contradiction: since $s_{\hat{\imath},j^*(\hat{\imath})}^T \leq b + \frac{\Delta}{2}$ and $i^*(\tilde{j}) = \emptyset$, $u_{\tilde{j}}(\hat{\imath}, s^T) \geq 0$, which contradicts T being the last round of the MDP. Thus, $\mu(\tilde{j}) = b$. Hence, $\rho_{\hat{\imath}}^1 = b$. We now conclude the argument in an inductive manner: if $\rho_{\hat{\imath}}^k = b$ for some k and all $\hat{\imath}$ that satisfy $j^*(\hat{\imath}) \neq \emptyset$ and $(i, j^*(\hat{\imath})) \in E$, then by construction $\rho_i^{k+1} = b$. Thus, $\rho_i^\infty = b = p_M(i,j)$. Thus, (i',j) does not block M. Therefore, M is stable with respect to p_M. $\square$

Lemma 2. *Let M be the matching generated by the MDP. Let p_M be any price function such that M is stable with respect to p_M (which is well defined by our previous lemma) and let v be the associated payment function. Let p_M^* be the price generated by the PDP(I) and v^* the corresponding payment function. Then, $v^* \leq v$.*

Proof. Let M, p_M, v, p_M^* and v^* be as in the statement of the lemma. Then, for all i, $v(i) \geq \rho_i^1$. Indeed, if $j^*(i) = \emptyset$ then $v(i) = b = \rho_i^1$. If $j^*(i) = j$ for some j then, by stability of M with respect to p_M, $v(i) \geq \mu(j')$ for each j' such that $i^*(j') = \emptyset$. Thus, $v(i) \geq \rho_i^1$.

[37] Indeed, let t be the last time $j^*(\hat{\imath})$ bids for $\hat{\imath}$. Then, $s_{\hat{\imath},j^*(\hat{\imath})}^t = s_{\hat{\imath},j^*(\hat{\imath})}^T$, and $s_{\hat{\imath},j^*(\hat{\imath})}^t \leq s_{i,j^*(\hat{\imath})}^t$, where the last inequality holds because buyers always bid for the cheapest sellers. By monotonicity of the matrix of prices, $s_{i,j^*(\hat{\imath})}^t \leq s_{i,j^*(\hat{\imath})}^T$. Thus, $s_{\hat{\imath},j^*(\hat{\imath})}^T \leq s_{i,j^*(\hat{\imath})}^T$.

We now show that if $v \geq \rho^t$ for some k, then $v \geq \rho^{t+1}$. Indeed, for all i such that $j^*(i) = \emptyset$, $v(i) = b = \rho_i^t = \rho_i^{t+1}$. For all i such that $j^*(i) = j$, we have the following:

$$
\begin{aligned}
\rho_i^{t+1} &= \max\{\rho_{i'}^t : (\exists j', i')(i', j') \in M \text{ and } (i, j') \in E\} \\
&\leq \max\{v(i') : (\exists j', i')(i', j') \in M \text{ and } (i, j') \in E\} \leq v(i),
\end{aligned}
$$

where the last inequality follows from stability of M with respect to p_M. Thus, for each t and each i, $\rho_i^t \leq v(i)$. Hence $\rho_\cdot^\infty \equiv v^*(\cdot) \leq v(\cdot)$. □

We now prove items 1 through 4 of Proposition 1.

1. The algorithm ends in finite time.

Proof. By the same arguments as Crawford-Knoer, the matching determination program ends in finite time. Furthermore, let $K \in \mathbb{N}$ satisfy $\max\{\mu(j) : j \in \mathcal{J}\} = b + K\Delta$. Then the price determination program ends in at most $2K$ rounds. □

2. The algorithm outputs a pairwise stable matching.

Lemma 1 already shows that M is stable with respect to p_M when p_M is the price function generated by PDP(I). Lemma 3 below shows that M is also stable with respect to p_M when p_M is the price function generated by PDP(II).

3. Price Determination program (I) outputs the pointwise minimum price function at which M is stable.

Proof. Follows from lemma 2 and that M is stable with respect to p_M, where p_M is the price function generated by the PDP (I). □

4. Price Determination program (II) outputs the pointwise maximum price function at which M is stable.

The proof is analogous to the proof that the PDP(I) outputs the pointwise minimum price function at which M is stable. Reasoning as in Lemma 2, if p_M is such that M is stable at prices p_M, and v is the corresponding payment function, then $v(\cdot) \leq \rho_\cdot^\infty$ (Lemma 4 below). Moreover, M is stable at prices induced by $\rho_\cdot^\infty$ (Lemma 3, below). The result then follows from Lemmas 3 and 4.

Lemma 3. *Let M be the matching generated by the MDP, and let p_M be the prices generated by the PDP(II). M is stable with respect to p_M.*

Proof. Let M be the matching outputted by the matching determination program, and p_M be the prices generated by the price determination program. Assume $(i, j) \in E$, $j^*(i) = i^*(j) = \emptyset$. Since there exists $\hat{p}_M$ such that M is stable with respect to $\hat{p}_M$ then $\mu(j) \leq b$.

Thus (i,j) do not block M at ρ^∞. Now consider $(i,j) \in M$. We show no seller and no buyer wishes to block (i,j):

a. *No Buyer blocks:* Let j' be such that $(i,j') \in E$. If $i^*(j') \neq \emptyset$ then, by construction, $\rho^\infty_{i^*(j')} \leq \rho^\infty_i$, so (i,j') does not block. Assume now that $i^*(j') = \emptyset$. We say a seller i' is indirectly connected to seller j if there exists sequences $(i_1, ..., i_k)$ and $(j_1, ..., j_{k-1})$ such that $(i_1, j) \in E$, $(i_1, j_1) \in E$, $(i_2, j_1) \in E$, ..., $(i_k, j_{k-1}) \in E$, with $i' = i_k$. That is, if a path can be constructed from j to i'. By construction, $\min\{\mu(j^*(i')) : i' \text{ is indirectly connected to j}\} \leq \rho^\infty_i$ where, by convention, $\mu(\emptyset) = b$. Now consider the abstraction used in Proposition 1 item [1]: each matched pair $(\hat{i}, \hat{j}) \in M$ is assigned their own subgraph, and all unmatched buyers/sellers are assigned a trivial subgraph that contains only them. Because there exist prices $\hat{p}_M$ such that M is stable at $\hat{p}_M$, cheapest sorting implies that

$$\begin{aligned} \mu(j') &\leq \hat{p}_M(i,j) \leq \min\{v(i') : i' \text{ is indirectly connected to j}\} \\ &\leq \min\{\mu(j^*(i')) : i' \text{ is indirectly connected to j}\}. \end{aligned}$$

Thus, $\mu(j') \leq \rho^\infty_i$ so (i,j') does not block.

b *No Seller blocks:* Let i' be such that $(i',j) \in E$. By construction, $\rho^\infty_i \leq \rho^\infty_{i'}$. Thus, (i',j) does not block.

$\square$

Lemma 4. *Let M be the matching generated by the MDP. Let p_M be any price function such that M is stable with respect to p_M (which is well defined by our previous lemma) and let v be the associated payment function. Let p^*_M be the price generated by the PDP(II) and v^* the corresponding payment function. Then, $v^* \geq v$.*

Proof. Let M, p_M, v, p^*_M and v^* be as in the statement of the lemma. Then, $v(i) \leq \rho^1_i$ for all i. Indeed, if $j^*(i) = \emptyset$ then $v(i) = b = \rho^1_i$. If $j^*(i) = j$ for some j then, by stability of M with respect to p_M, $v(i) \leq \mu(j) = \rho^1_i$.

We now show that if $v \leq \rho^t$ for some k, then $v \leq \rho^{t+1}$. Indeed, for all i such that $j^*(i) = \emptyset$, $v(i) = b = \rho^t_i = \rho^{t+1}_i$. For all i such that $j^*(i) = j$, we have the following:

$$\begin{aligned} \rho^{t+1}_i &= \min\{\rho^t_{i'} : (i',j) \in E\} \\ &\geq \min\{v(i') : (i',j) \in E\} \geq v(i), \end{aligned}$$

where the last inequality follows from stability of M with respect to p_M. Thus, for each t and eaxh i, $\rho^t_i \geq v(i)$. Hence $\rho^\infty_\cdot \equiv v^*(\cdot) \geq v(\cdot)$. $\square$