Problems of utility and prospect theories. A “certain–uncertain” inconsistency within their experimental methods

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In random–lottery incentive experiments, the choices of certain outcomes are stimulated by uncertain lotteries. This "certain–uncertain" inconsistency is evident, but only recently emphasized. Because of it, conclusions from a random–lottery incentive experiment that includes a certain outcome cannot be unquestionably correct. Well-known experimental results and purely mathematical theorems support this. The main result presented here is: The usual experimental systems of utility and prospect theories may need additional independent analyses in the context of the "certain–uncertain" inconsistency.

Contents

Introduction

The present short article considers a potential problem of the usual incentive system of experiments in utility and prospect theories. The problem is an inconsistency in the stimulation of certain (sure) outcomes by uncertain lotteries.

The purpose of the article is to investigate the inconsistency. This purpose is new, so, in the first stages of its study, the general methods of the present research are mainly qualitative.

There are a number of theories concerned with one or another concept of utility. They include, e.g., Bernoullian expected utility, von Neumann–Morgenstern expected utility, subjective expected utility, subjectively weighted utility theories (see, e.g., a review by Schoemaker, 1982); prospect theory (see Kahneman and Tversky, 1979) and cumulative prospect theory (see Tversky and Kahneman, 1992) or, in other terminology, original prospect theory and prospect theory; the salience theory of choice under risk (see Bordalo, Gennaioli, Shleifer, 2012); etc.

Here these theories are referred to as utility and prospect theories.

The article develops the report Harin (2014).

The need for considering the subject of the article is grounded on the prevalence and usefulness of the random–lottery incentive systems of experiments and on the importance of the zone $p \sim 1$.

1. An analysis of a detail of the experiments

1.1. Random incentives

Let us analyze one usual feature of experiments in utility and prospect theories. Let us consider some typical descriptions of the utility experiments. One can see in the literature (the **boldface** is my own):

Loewenstein and Thaler (1989), page 188: "The students … were told that the experimenter would select and implement one of their choices **at random**."

Baltussen et al. (2012), page 424: "In the WRIS treatment, subjects play the game ten times, **one of which** for real payment. In the BRIS treatment, subjects play the game only once with a **one-in-ten** chance of real payment." and page 425: "In both RIS treatments, a **ten-sided die** was thrown individually by each subject to determine her payment."

Other sources such as Kahneman, Knetsch and Thaler (1991), Vossler, Doyon and Rondeau (2012), etc. give similar descriptions.

Such a procedure can be seen not only in the utility and prospect theories but also in other fields of the economics, see, e.g., Larkin and Leider (2012), page 193: "Subjects made fifteen choices between a lottery and a fixed payment. … Subjects were paid for one **randomly** selected decision."

So, subjects are stimulated by random incentives. This is a well-known feature of the experiments, including in the field of utility and prospect theories.

1.2. Random incentives and certain outcomes

Let us consider this feature more closely. One can see a detail in the literature (the **boldface** and **underlining** is my own):

Starmer and Sugden (1991), page 974: “subjects in groups B and C knew that they were taking part in a **random**–lottery experiment in which questions 21 and 22 had equal chances of being for real.” and “One problem, which we shall call P', required a choice between two lotteries R' (for "riskier") and S' (for “**safer**”). R' gave a 0.2 chance of winning £10.00 and a 0.75 chance of winning £7.00 (with the residual 0.05 chance of winning nothing); S' gave £7.00 **for sure**.”

Andreoni and Sprenger (2012), page 3365: “One choice for each subject was selected for payment by drawing a numbered card **at random**. Subjects were told to treat each decision as if it were to determine their payments.” and page 3366: “Section I provided a testable hypothesis for behavior across **certain** and uncertain intertemporal settings.”

Other sources such as Holt and Laury (2002), Harrison et al. (2005), Abdellaoui et al. (2011), etc. give the same detail.

So, the random incentive procedures are used not only in the uncertain but in the certain situations too. Let us consider this detail more closely.

1.3. An inconsistency between the certain outcomes and uncertain incentives

So, a well-known feature of the experiments, including in the field of utility and prospect theories, is that subjects are stimulated by random incentives.

First, let us note that the stimulation (incentive) by a random payment selected from two or more alternatives may be called a random, uncertain stimulation. One may refer to it also as a stimulation by an uncertain incentive.

Further, let us consider a stimulation by this uncertain incentive separately for uncertain and certain choices.

Suppose, that subjects choose an uncertain choice, that is, a choice whose probability is strictly less than *1* (and strictly more than *0*). In this case, the choice and the incentive are of the same type.

Suppose, that the subjects choose a certain choice, that is, a choice whose probability is strictly equal to *1*. In this case, the choice and the incentive are of the essentially different types. The choice is certain but the incentive is uncertain.

Moreover, one should emphasize: this uncertain incentive can call into question the certain outcome.

Therefore, there is an evident inconsistency between the certain type of the choice and the uncertain type of the incentive.

Therefore, the correctness of the use of uncertain incentives for certain outcomes cannot be unquestionable. One may call this problem the “certain–uncertain” inconsistency.

This inconsistency is evident but the author of this article has found no mention about it in the literature: see, e.g., Andreoni and Sprenger (2012); Vossler, Doyon and Rondeau (2012); Baltussen et al. (2012); see also all issues of RePEc's "New Economics Papers. Utility Models & Prospect Theory" at http://econpapers.repec.org/scripts/nep.pf?list=nepupt for the period 2005–2015. The inconsistency was revealed in the report Harin (2014). The present article develops this report.

1.4. The role of the incentives

Incentives have been widely discussed in economics (see, e.g., Starmer and Sugden, 1991; Fehr and Falk, 2002; Holt and Laury, 2002; Baltussen et al., 2012: Larkin and Leider, 2012). Do incentives influence the choices made by the subjects in utility and prospect theories?

The correct answer to this question needs a special investigation. However, one may be sure that if incentives did not have any influence on the choices made by the subjects, then there would be no reason to use such incentives.

Therefore, one may not exclude that an incentive can influence the choice made by a subject, at least partially.

Therefore, one may not exclude that an uncertain incentive can call into question the certain outcome, at least partially.

2. The "certain–uncertain" inconsistency of the random–lottery incentive system

2.1. The random–lottery incentive system

The above discussed random incentive procedure is usually referred to as the random–lottery incentive system (or the random lottery incentive system or random incentive system (RIS), etc.).

The random–lottery incentive system is a widely used experimental procedure employed in the utility and prospect theories (see, e.g., Starmer and Sugden, 1991, Starmer, 2000, Andreoni and Sprenger, 2012, Baltussen et al., 2012, etc.).

For example, Starmer (2000), page 371: "… the most common reward mechanism is the random lottery incentive system."

Moreover, we can see in Baltussen et al. (2012), page 419: "If a subject performs multiple tasks in an experiment where each task is for real, then income and portfolio effects will arise … The RIS is the only incentive system known today that can avoid such effects. In addition, for a given research budget and with the face values of the monetary amounts kept the same, RISs allow for a larger number of observations."

So, the random–lottery incentive system is, at least, the usual experimental system (procedure) employed in the utility and prospect theories.

2.2. The “certain–uncertain” inconsistency of the system

In many works (see, e.g., Andreoni and Sprenger, 2012; Vossler, Doyon and Rondeau, 2012; Baltussen et al., 2012, Vrijdags and Marchant, 2015) one can find the elaborated investigations of the correctness of the random–lottery incentive system. The author of this article has found, however, no mention of the “certain–uncertain” inconsistency or similar questions.

So, one may conclude:

- The random–lottery incentive system is widespread in the utility and prospect theories. There are no wide discussion about differences between the results of the random–lottery incentive system and other systems.
- The essence of the random–lottery incentive system corresponds to the random, uncertain name of the system.
- The considered specific “certain–uncertain” inconsistency of the random–lottery incentive system has not yet been mentioned in the literature.

So, the random–lottery incentive system is concerned with the “certain–uncertain” inconsistency. This inconsistency means that the certain choice is stimulated by the uncertain incentive. Because of this evident “certain–uncertain” inconsistency, the deductions from a random–lottery incentive experiment, that includes a certain outcome, cannot be unquestionably correct.

So, such deductions need an additional proof or an amendment, or a new approach.

3. Purely mathematical support of the possibility of existence of the inconsistency

Kahneman and Thaler (2006) pointed out that the basic problems of utility and prospect theories have not yet been adequately solved.

One possible way to solve these problems is widely discussed, e.g., in Schoemaker and Hershey (1992), Chay et al (2005), Butler and Loomes (2007). The essence of this way consists in a proper attention to noise, uncertainty, imprecision, etc.

Another possible way to solve these problems is to consider the vicinities of the borders of the probability scale, e.g. at $p\sim 1$. Steingrimsson and Luce (2007) and Aczél and Luce (2007) emphasized a fundamental question: whether Prelec’s weighting function (see Prelec, 1998) is equal to 1 at $p=1$.

In any case, one may suppose that a synthesis of the above two ways can be of interest.

Purely mathematical theorems (see, e.g., Harin, 2012) were proved indeed for the mean and probability near the borders of intervals. The theorems state, in particular, that, in the presence of a non-zero dispersion of data (e.g., due to noise), the probability cannot attain $p=1$. It follows from the theorems that the “certain–uncertain” inconsistency is at least not excluded.

4. Experimental evidence of the “certain–uncertain” inconsistency

4.1. Conditions

One can see the following in the description of the well-known experiment of Starmer and Sugden (1991):

Page 974: “For groups A and D, this page began with an underlined text stating that question 22 would be played for real. For groups B and C, the corresponding text stated that one of the two questions would be played for real and that which question was to played out would be decided at the end of the experiment in the following way. The subject would roll a six-sided die. If the number on the die was 1, 2, or 3, then question 21 would be played; if the number was 4, 5, or 6, question 22 would be played.

…

One problem, which we shall call P', required a choice between two lotteries R' (for "riskier") and S' (for "safer"). R' gave a 0.2 chance of winning £10.00 and a 0.75 chance of winning £7.00 (with the residual 0.05 chance of winning nothing); S' gave £7.00 for sure.”

4.2. Results

So, in the R'-S' problem, R' gives $£10.00\times0.2+£7.00\times0.75 = £7.25$. S' gives $£7.00\times1 = £7.00$. Here $R' = £7.25>S' = £7.00$.

Let us consider the results from table 2 on Page 976, those are of interest here (the **boldface** is my own):

- Group = B, Incentive = **Random lottery**, R':S' = **19:21**
- Group = C, Incentive = **Random lottery**, R':S' = **22:18**
- Group = D, Incentive = **P' real**, R':S' = **13:27**

So, the results for **P' real** incentive (**13:27**) differ evidently and essentially from the results for random lottery incentive (19:21 and 22:18).

Let us evaluate the percentage of the subjects choosing the uncertain outcome and the direction of the modification of $W(p)$. The total number of the subjects in each group is equal to $40=19+21=22+18=13+27$. So, the percentage is equal to $19/40\sim48\%$, $22/40=55\%$ and $13/40\sim33\%$. One may see that the modification of $W(p)$ by the random lottery incentives is directed from $13/40\sim33\%$ to $19/40\sim48\%$ and $22/40=55\%$. That is it is directed from 0 to 1.

4.3. Deductions

The modification of $W(p)$ by the "certain–uncertain" inconsistency:

One can easily see the experiment shows that the random lottery incentives can essentially modify subjects' choices in comparison with the real incentives, when these choices include certain outcomes and the probability $(p = 0.2 + 0.75 = 0.95 \sim 1)$ of the uncertain choices is near the border of the probability scale.

The direction of the modification of $W(p)$:

The modification of $W(p)$ by the random lottery incentives is directed from 0 to 1.

Therefore, the real unbiased probability weighting function $W(p)$, at $p \sim 1$, is located farther from 1 and nearer to 0 than the function biased by the random lottery incentives.

5. Possible consequences of the "certain–uncertain" inconsistency

One can specify a value $W_{Certain}$ of the probability weighting function $W(p)$ for the certain outcome. $W_{Certain}$ may be evidently assumed to be equal to 1. One can also specify a value $W(1)$ as the limit of the probability weighting function $W(p)$ for the probable outcome when the probability p tends to 1. If

$$W(1) \neq W_{Certain}$$

then the probability weighting function $W(p)$ is discontinuous at $p = 1$.

Due to the experiment, the real unbiased probability weighting function $W(p)$ is located farther from 1 (at $p \sim 1$) than the function biased by the random lottery incentives. Therefore a question can also arise whether the uncertain incentives can hide a possible discontinuity of $W(p)$ at $p = 1$.

Therefore the experiment of Starmer and Sugden (1991) and the purely mathematical theorems (see, e.g., Harin, 2012) support the feasibility of a possible discontinuity of $W(p)$ at the probability $p = 1$.

Note, a discontinuity and jumps has been already discussed in utility and prospect theories (see, e.g., Masson, 1974, Delbaen, F., S. Drapeau, and M. Kupper 2011.

A discontinuity is not a quantitative but a qualitative, moreover, a topological feature. So, it can qualitatively change the situation in the utility and prospect theories, at least in their mathematical aspects.

It may be supposed that such basic and useful tools as the random incentive systems, the overwhelming majority of the data already obtained by means of them, and the deductions from the data may and should continue to be used.

Apparently, the farther from $p = 1$ the less relevant is a possible discontinuity at $p = 1$ and the smaller can be corrections of the data and deductions. Note, that the experiments (see, e.g., Cubitt, Starmer and Sugden, 1998; Beattie and Loomes, 1997) at the probabilities that are less than 0.9 are not so informative and not so sensitive to the "certain–uncertain" inconsistency as that of Starmer and Sugden (1991).

The following may be supposed:

In the narrow middle of the probability scale (where the probability weighting function intercepts the line $W(p) = p$) and in the obvious cases, the data and deductions may be used "as is". This may be true also for the cases those do not include certain (sure) outcomes.

In the wide middle of the probability scale, the deductions may be the same or slightly corrected. This may be true when the probability p is located sufficiently far from $p = 1\text{-}r_{mean}$, where the restriction r_{mean} on the mean and on the probability is obtained from the theorems of existence of restrictions on the mean and on the probability (see, e.g., Harin, 2012).

When the probability tends to the restriction $p \rightarrow 1\text{-}r_{mean}$, the data should be used with non-linear corrections and the deductions should be recalculated by non-linear functions.

At the probabilities that are in the forbidden zone $1\text{-}r_{mean} \leq p \leq 1$, a new approach may be needed to make the deductions correct.

At first, the simplest possible and very rough correction might be straight-line approximations of the middle (roughly straight) parts of the already existing experimental data curves from the middle to the borders. At that, these straight-line approximations will pass lower than the point $W(p) = 1$.

The further consideration of the idea of the "certain-uncertain" inconsistency may be developed only after its independent confirmations will take place.

Conclusions

The usual experimental procedure in utility and prospect theories is the random–lottery incentive system (see, e.g., Starmer, 2000 and Baltussen et al., 2012; etc.).

The present short article emphasizes that in the random–lottery incentive system, the choices of certain (sure) outcomes are stimulated by uncertain lotteries.

This inconsistency is quite evident but has not yet been mentioned in the literature (see, e.g., Andreoni and Sprenger, 2012; Vossler, Doyon and Rondeau, 2012; Baltussen et al., 2012, Vrijdags and Marchant, 2015). The inconsistency was detected in the recent report Harin (2014). The present article develops this report.

Because of this "certain–uncertain" inconsistency, the deductions from a random–lottery incentive experiment that includes a certain outcome cannot be unquestionably correct, especially at $p \sim 1$.

The well-known experiment of Starmer and Sugden (1991) and purely mathematical theorems (see, e.g., Harin, 2012) evidently support the possibility of the existence of this inconsistency.

So, the main result of this article is that the usual experimental systems of the utility and prospect theories may need additional independent analyses and/or investigations in the context of the "certain–uncertain" inconsistency.

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