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Chen, Hung-Ju

National Taiwan University

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Hung-Ju Chen*

ABSTRACT

This paper develops an overlapping generations model with endogenous retirement to examine the effect of fertility on long-run pay-as-you-go (PAYG) pensions. We find that pensions may not necessarily increase with the fertility rate. An increase in the fertility rate will raise pensions if the output elasticity of capital and the tax rate are sufficiently low, but such a change will reduce pensions if the fertility rate is sufficiently high. Our results also indicate that raising the fertility rate is more likely to reduce pensions in developing countries than in developed countries, while such a change tends to raise pensions for countries in which the costs of raising children are low.

Keywords: Fertility; Retirement; OLG, PAYG pensions.

JEL Classification: H55, J13, J26.

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* Department of Economics, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Taipei 10617, Taiwan; Tel.: 886-2-33668416; Fax: 886-2-23657957; e-mail: hjc@ntu.edu.tw.
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1. INTRODUCTION

The provision of child allowances has become a popular policy nowadays in many developed countries such as Australia, Japan and Sweden. On the one hand, the steady decline in the mortality rate raises the burden of social security systems, particularly the pay-as-you-go (PAYG) pension system. On the other hand, the decline in the fertility rate leads to fewer workers to support social security systems. With the fear that the government may not be able to maintain current social security system, there is a huge amount of literature devoted to possible solutions to the problem of the increasing pension burden caused by aging populations.

The literature related to pension crisis essentially follows two major lines of research. The first research line focuses on examining the impact of fertility on pensions. For example, van Groezen, Leers and Meijdam (2003) study how child allowances affect fertility decision and pensions. It is commonly believed that providing child allowances reduces the cost of raising children, and parents then have stronger incentives to have more children, inducing more workers to support old people and reducing the burden placed upon government budgets. The second research line investigates if delaying retirement can mitigate the burden of pensions. The postponement of retirement means that workers work longer and benefit from pensions for a shorter length of time. The influences of retirement on pensions in a model with exogenous and endogenous retirement time are respectively examined in Miyazaki (2014a) and in Miyazaki (2014b). Thus, in prior pension crisis studies there has been a tendency to study the effects of fertility and retirement time on pensions separately, thereby ignoring the interaction between fertility and retirement.

The consensus that the fertility rate leads to a positive impact on pensions has been questioned by Cigno (2007), who argues that "The combined effect of fewer births, longer lives and sluggish retirement age is putting public pension system, all essentially pay-as-you-go, under increasing strain." Recently, Fanti and Gori (2012) analytically examine the effect of fertility on pensions based on an overlapping generations model with a PAYG pension system, but they do not take retirement time into consideration. Chen (2015) also studies the same issue and considers official pension age in the model. These studies all find that pensions do not necessarily increase with the fertility rate. Although increasing fertility raises the labor force and generates a positive effect on pensions, it also causes a negative effect due to the general equilibrium feedback of the wage rate.

In this study we endogenize retirement time to examine the influence of fertility on long-run PAYG pensions. Since our focus is the influence of fertility on retirement and pensions, we follow Fanti and Gori (2012) and Chen (2015) and assume that the fertility rate is exogenous. The endogeneity of retirement time allows the change in fertility to affect retirement and this will in turn affect pensions. Agents need to work in order to consume and to raise children. Workers are composed of adults and old agents. Adults supply labor inelastically, while old agents can choose how long they want to work; that is, they can choose the time to retire. Government implements the PAYG social security system. All workers need to pay an income tax to support the pensions. Old agents enjoy pension benefits after they retire.

We find that pensions do not necessarily increase with the fertility rate. Although an increase in fertility causes a positive direct effect due to more workers contributing to pensions, it also induces two negative effects due to decreases in the elderly's working time and the equilibrium wage. An increase in the fertility rate (tax rate) reduces the elderly's working time. A higher fertility rate will raise pensions if the output elasticity of capital and the tax rate are sufficiently low, but such a change will reduce pensions if the fertility rate is sufficiently high. Our results also indicate that raising the fertility rate is more likely to reduce pensions in developing countries than in developed countries, while such a change tends to raise pensions for countries in which the costs of raising children are low.

The remainder of this paper is organized as follows. Section 2 develops the model. Section 3 analyzes the effects of fertility on long-run pension benefits. Section 4 concludes.

2. THE MODEL

We consider an overlapping generations model where agents live for three periods: childhood, adulthood (parenthood), and old age. In each period, every agent is endowed with one unit of time. Adults with population (N_t) spend all their time working to earn wages w_t. When they become old, they can choose to work at a fraction of time $l_{t+1} \in (0,1)$ to earn wages w_{t+1}. After they retire, they receive pensions provided by the government. Agents care about their consumption in adulthood (c_t) and in old age (c_{t+1}) as well as leisure in old age $(1 - l_{t+1})$. The utility function, which is identical for all agents, is defined as:

$$u_t = \ln c_t + \beta \ln c_{t+1} + \gamma \ln(1 - l_{t+1}), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, and $\gamma \in (0,1)$ is the weight of leisure in the utility function.

Following Fanti and Gori (2012), we assume that the number of children (*n*) is exogenous. It takes a fixed proportion ($q \in (0,1)$) of each adult's wage to raise a child (see, e.g., Wigger 1999; Boldrin and Jones 2002; Fanti and Gori 2012). In order to pay for pensions for the elderly, the government levies an income tax at the rate $\tau \in (0,1)$. The budget constraint for an adult is:

$$c_t + s_t + nqw_t = (1 - \tau)w_t, \tag{2}$$

where s_t represents savings.

The budget constraint for an old agent is:

$$c_{t+1} = R_{t+1}s_t + (1 - \tau)\theta l_{t+1}w_{t+1} + (1 - l_{t+1})P_{t+1}, \tag{3}$$

where R_{t+1} is the gross rate of return of savings, P_{t+1} is the pensions per unit of time, and $\theta > 0$ is the productivity of an old agent.[1]

The government runs a balanced budget. Since the tax revenue is used to provide pensions for old agents who retire, this implies that:

$$(1-l_{t+1})N_tP_{t+1}=\tau w_{t+1}(N_{t+1}+\theta l_{t+1}N_t). \tag{4}$$

Output is produced by using capital (K_t) and effective labor ($L_t=N_t+\theta l_t N_{t-1}$) and is based on the Cobb-Douglas production function $Y_t=AK_t^{\alpha}L_t^{1-\alpha}$, where $A>0$ is the total factor productivity, and $\alpha\in(0,1)$ is the output elasticity of capital. We define $k_t=K_t/L_t$, and then the gross rate of the return on capital and the real wage rate are respectively:

$$R_t=\alpha Ak_t^{\alpha-1}, \tag{5}$$

$$w_t=(1-\alpha)Ak_t^{\alpha}. \tag{6}$$

2.1 Equilibrium and steady state

The optimal decisions of c_t and l_{t+1} are:

$$c_t=\frac{(1-\tau-qn)w_t}{1+\beta}+\frac{(1-\tau)\theta l_{t+1}w_{t+1}+(1-l_{t+1})P_{t+1}}{(1+\beta)R_{t+1}}, \tag{7}$$

$$l_{t+1}=\frac{\theta\beta(1-\tau)(1-\alpha)-[\alpha\gamma+\tau(1-\alpha)(\beta+\gamma)]n}{\theta[(1-\alpha)\beta+\gamma]}=l. \tag{8}$$

Equation (8) indicates that working time for the elderly is constant. Note that $l<1$. In order to guarantee that $l>0$, we assume that $\theta>\frac{[\alpha\gamma+\tau(1-\alpha)(\beta+\gamma)]n}{\beta(1-\tau)(1-\alpha)}$. From (8), we have:

$$\frac{dl}{dn}=-\frac{\alpha\gamma+\tau(1-\alpha)(\beta+\gamma)}{\theta[(1-\alpha)\beta+\gamma]}<0. \tag{9}$$

$$\frac{dl}{d\tau}=-\frac{(1-\alpha)[\theta\beta+(\beta+\gamma)n]}{\theta[(1-\alpha)\beta+\gamma]}<0. \tag{10}$$

Equation (9) indicates that an increase in fertility reduces working time in old age. On the one hand, the increase in the fertility rate will result, *ceteris paribus*, in a higher expenditure on raising children and lower savings. This will motivate old agents to work longer. On the other hand, the increase in the fertility rate means higher pensions due to more workers contributing to pension funds, which will cause old agents to retire earlier. As the fertility rate increases, the second effect will dominate the first effect and the overall working time for the elderly will decrease. Concerning the effect of the tax rate on the elderly's working time, we find that although an increase in the tax rate lowers the motivation to work and reduces the working time for the elderly, the lower after-tax income induces the elderly to work longer. Equation (10) shows that an increase in the tax rate

[1] Sala-i-Martin (1996) argues that old workers are less productive than young workers due to the depreciation of human capital.

makes old agents retire earlier.

Using (2), (4), and (5)-(8), we can derive savings as:

$$s_t = \frac{1-\alpha}{1+\beta+\gamma}\left[(\beta+\gamma)(1-\tau-qn)Ak_t^{\alpha} - \frac{(1-\tau)\theta}{\alpha}k_{t+1}\right]. \qquad (11)$$

Market clearing in the capital market indicates that $(n+\theta l_{t+1})k_t = s_t$. This implies the following law of motion of k_t:

$$k_{t+1} = \frac{\alpha A(1-\tau-qn)[(1-\alpha)\beta+\gamma]}{(1-\tau)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)]}k_t^{\alpha}. \qquad (12)$$

Equation (12) implies that there exists a unique, globally stable-steady state k^*:

$$k^* = \left[\frac{\alpha A(1-\tau-qn)[(1-\alpha)\beta+\gamma]}{(1-\tau)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)]}\right]^{\frac{1}{1-\alpha}}. \qquad (13)$$

To ensure $k^* > 0$, we assume that $q < \frac{1-\tau}{n}$, so that $(1-\tau-qn) > 0$. Differentiating (13) with respect to the fertility rate yields:

$$\frac{dk^*}{dn} = -\frac{[\alpha(1-\tau)(1+\beta+\gamma)+q\theta(1+\alpha\beta)]k^*}{(1-\alpha)(1-\tau-qn)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)]} < 0. \qquad (14)$$

Equation (14) indicates that an increase in the fertility rate reduces savings and the accumulation of capital, resulting in a lower k^*.

3. PENSIONS

Let w^* represent the steady-state wage rate; that is, $w^* = (1-\alpha)(k^*)^{\alpha}$. Equation (4) implies that the steady-state pensions per efficiency unit of labor can be re-written as:

$$p^* = \frac{\tau w^*(n+\theta l)}{1-l}. \qquad (15)$$

Substituting (8) and (13) into (15) yields:

$$p^* = p^*(n, l(n), k^*(n)). \qquad (16)$$

Differentiating (16) with respect to n gives:

$$\frac{dp^*}{dn} = \underbrace{\frac{\partial p^*}{\partial n}}_{+} + \underbrace{\underbrace{\frac{\partial p^*}{\partial l}}_{+}\underbrace{\frac{\partial l}{\partial n}}_{-}}_{-} + \underbrace{\underbrace{\frac{\partial p^*}{\partial w^*}}_{+}\underbrace{\frac{\partial w^*}{\partial k^*}}_{+}\underbrace{\frac{\partial k^*}{\partial n}}_{-}}_{-}. \qquad (17)$$

Equation (17) shows that an increase in the fertility rate will cause three effects on pensions: one positive direct effect and two negative indirect effects. First, a higher fertility rate means that more workers contribute to pensions p^* (direct effect). Second, the higher fertility rate causes old agents to retire earlier and reduces pensions (indirect effect). Third, the lower equilibrium wage rate caused by the increasing fertility rate reduces pensions (indirect effect). Depending on the magnitude of these effects, the fertility rate may increase or decrease per efficiency unit of pensions.

Proposition 1. An increase in the fertility rate will reduce pensions per efficiency unit of labor if the fertility rate is sufficiently large.

Proof: Substituting (9) and (14) into (17) yields:

$$\frac{dp^*}{dn} = \frac{\tau w^*[\alpha(1+\beta+\gamma)\xi_1 + \theta(1+\alpha\beta)\xi_2]}{(1-l)^2(1-\alpha)(1-\tau-qn)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)][(1-\alpha)\beta+\gamma]}. \quad (18)$$

where

$$\xi_1 = (1-\tau)\{-(1-l)\alpha(n+\theta l)[(1-\alpha)\beta+\gamma] + (1-\alpha)^2\gamma(1-\tau-qn)n\},$$

and

$$\xi_2 = -(1-l)\alpha q(n+\theta l)[(1-\alpha)\beta+\gamma] + (1-\alpha)^2(1-\tau)\gamma(1-\tau-qn).$$

Define $\chi = \tau[(1-\alpha)\beta+\gamma] + (1-\tau)\alpha\gamma$. Using (8) to substitute l in ξ_1 and ξ_2, we have:

$$\xi_1 = (1-\tau)(1-\alpha)\left\{\frac{-\theta\{[(1-\alpha)\tau\beta+\gamma]+\chi n\}\alpha(1-\tau)[n(\beta+\gamma)+\theta\beta]}{\theta[(1-\alpha)\beta+\gamma]} + \gamma(1-\alpha)n(1-\tau-qn)\right\},$$

and

$$\xi_2 = (1-\tau)(1-\alpha)\left\{\frac{-\theta\{[(1-\alpha)\tau\beta+\gamma]+\chi n\}\alpha q[n(\beta+\gamma)+\theta\beta]}{\theta[(1-\alpha)\beta+\gamma]} + (1-\alpha)\gamma(1-\tau-qn)\right\}.$$

Using the assumption that $(1-\tau-qn) > 0$, we derive:

$$\xi_1 = (1-\tau)(1-\alpha)\left\{n\left[\gamma(1-\alpha)(1-\tau-qn) - \frac{\chi\alpha(1-\tau)[n(\beta+\gamma)+\theta\beta]}{\theta[(1-\alpha)\beta+\gamma]}\right] - \frac{\theta[(1-\alpha)\tau\beta+\gamma]\alpha(1-\tau)[n(\beta+\gamma)+\theta\beta]}{\theta[(1-\alpha)\beta+\gamma]}\right\}.$$

Therefore, if n is sufficiently large, then $\xi_1 < 0$ and $\xi_2 < 0$. From (18), we have $\frac{dp^*}{dn} < 0$ if n is sufficiently large.

QED.

Recall that an increase in fertility generates three effects on p^*. When the fertility rate is high, an increase in fertility will induce a small positive direct effect on p^*. However, it will cause large reductions in the elderly's working time and wages, resulting in a large decrease in p^*. Proposition 1 demonstrates that if the fertility rate is sufficiently large, then an increase in the fertility rate will reduce p^*. Because developing countries tend to have higher fertility rates, an increase in them is more likely to reduce p^* in these countries than in developed countries.

The following proposition concerns how the tax rate affects the influence of the fertility rate on pensions. Proposition 2 indicates that the influence of the fertility rate on p^* also depends on the tax rate.

Proposition 2. An increase in the fertility rate will raise pensions per efficiency unit of labor if the output elasticity of capital and the tax rate are sufficiently low.

Proof: Since $0 < l < 1$, we have:

$$\xi_1 > (1-\tau)\{-\alpha(n+\theta l)[(1-\alpha)\beta+\gamma] + (1-\alpha)^2\gamma(1-\tau-qn)n\},$$

and

$$\xi_2 > -\alpha q(n+\theta l)[(1-\alpha)\beta+\gamma] + (1-\alpha)^2(1-\tau)\gamma(1-\tau-qn).$$

Using (8) to substitute l in ξ_1 and ξ_2, we have:

$$\begin{aligned}
&\alpha(1+\beta+\gamma)\xi_1 + \theta(1+\alpha\beta)\xi_2 \\
&> (1-\tau)(1-\alpha)\{-\alpha[\alpha(1-\tau)(1+\beta+\gamma)+\theta q(1+\alpha\beta)][n(\beta+\gamma)+\theta\beta] \\
&\quad +(1-\alpha)\gamma(1-\tau-qn)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)]\} \\
&= (1-\tau)(1-\alpha)(-\tau\xi_3+\xi_4),
\end{aligned}$$

where

$$\xi_3 = (1-\alpha)\gamma[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)] - \alpha^2(1+\beta+\gamma)[n(\beta+\gamma)+\theta\beta],$$

and

$$\begin{aligned}
\xi_4 = &(1-\alpha)\gamma(1-qn)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)] \\
&-\alpha[\alpha(1+\beta+\gamma)+\theta q(1+\alpha\beta)][n(\beta+\gamma)+\theta\beta].
\end{aligned}$$

Note that $\xi_3 > \xi_4$.

We can re-write ξ_4 as:

$$\begin{aligned}
\xi_4 = &\alpha(1+\beta+\gamma)\{(1-\alpha)\gamma(1-qn)n - \alpha[n(\beta+\gamma)+\theta\beta]\} \\
&+\theta(1+\alpha\beta)\{(1-\alpha)\gamma(1-qn) - \alpha q[n(\beta+\gamma)+\theta\beta]\}.
\end{aligned}$$

Thus, $\xi_4 > 0$ if $(1-\alpha)\gamma(1-qn)n > \alpha[n(\beta+\gamma)+\theta\beta]$ and $(1-\alpha)\gamma(1-qn) > \alpha q[n(\beta+\gamma)+\theta\beta]$. Define $\alpha_1 = \frac{\gamma(1-qn)n}{\gamma(1-qn)n+n(\beta+\gamma)+\theta\beta} < 1$ and $\alpha_2 = \frac{\gamma(1-qn)}{\gamma(1-qn)+q[n(\beta+\gamma)+\theta\beta]} < 1$, and then $\xi_4 > 0$ if α is sufficiently small such that $\alpha < \min(\alpha_1, \alpha_2)$. Since $\xi_3 > \xi_4$, then $\xi_3 > 0$ if $\xi_4 > 0$. Define $\tau_n = \frac{\xi_4}{\xi_3} < 1$. Thus, $\frac{dp^*}{dn} > 0$ if $\alpha < \min(\alpha_1, \alpha_2)$ and $\tau < \tau_n$.

QED.

When the output elasticity of capital is low, an increase in the fertility rate causes a small reduction in wages, generating a small reduction in pensions. Concerning the effect of the tax rate, although a lower tax rate directly decreases contributions to p^*, such a change also induces two indirect effects on p^*. First, it raises p^* due to higher after-tax income and savings. Second, as indicated by (10), a lower tax rate makes old agents delay their retirement, leading to an increase in p^* due to more contributions to pensions. We find that if the output elasticity of capital and the tax rate are sufficiently low, then an increase in the fertility rate causes an overall increase in p^*. Notice that $\frac{\partial \tau_n}{\partial q} < 0$. Since a lower q implies a higher threshold τ_n, then for those countries with low costs of raising children, an increase in the fertility rate tends to raise p^*.

4. CONCLUSIONS

In this paper we examine the effect of the fertility rate on PAYG pensions based on an overlapping generations model in which retirement is endogenized. The endogeneity of retirement allows the change in this rate to affect retirement, which will in turn affect pensions. An increase in the fertility rate affects pensions through three channels: a positive direct effect and two negative indirect effects caused by the early retirement of the elderly and the reduction in equilibrium wage rates. The results suggest that when concerning the influence of the fertility rate on PAYG pensions, we need to take economic variables such as the level of the fertility rate, the output elasticity of capital, and the tax rate into consideration.

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