A Discontinuity Model of Technological Change: Catastrophe Theory and Network Structure

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26 February 2015

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Abstract

Discontinuities as a crucial aspect of economic systems have been discussed both verbally - particularly in institutionalist theory - and formally, chiefly using catastrophe theory. Catastrophe theory has, however, been criticized heavily for lacking micro-foundations and has mainly fallen out of use in economics and social sciences. The present paper proposes a simple catastrophe theory model of technological change with network externalities and reevaluates the value of such a model by adding an agent-based micro layer. To this end an agent-based variant of the model is proposed and investigated specifically with regard to the network structure among the agents. While the macro level of the model produces a classical cusp catastrophe - a result that is preserved in the agent-based form - it is found that the behavior of the model changes locally depending on the network structure, especially if networks with features that resemble social networks (low diameter, high clustering, power law distributed node degree) are considered.

While the present work investigates merely an aspect out of a large possibility space, it encourages further research using agent-based catastrophe theory models especially of economic aspects to which catastrophe theory has previously successfully been applied; aspects such as technological and institutional change, economic crises, or industry structure.
Keywords:
network structures; agent-based modeling; catastrophe theory; information and communication technology; preferential attachment networks; technological change

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1. Introduction

Path dependence and discontinuity have been central issues for institutional and evolutionary economics since at least Gunnar Myrdal's introduction of the idea of Circular Cumulative Causation (see, e.g., [16]), possibly much longer (specifically since Thorstein Veblen's [28, 29] writings on Cumulative Causation and path dependence in history, institutions, and fashion goods). A formal framework to capture these concepts presented itself with René Thom's [27] analysis of catastrophe theory. It has since been applied to different aspects of economic systems [32, 14, 15, 19, 10] both with and without the framework of institutionalist theory and evolutionary economics. Catastrophe theory models in economics do, however, generally operate at an aggregate level only; in fact, they became controversial in the 1990s (for an overview, see, e.g., [20]). Consideration of, for instance, group or network structures would transform the model into a set of distinct but overlapping models, thereby increasing the system's complexity by orders of magnitude.

The present paper reevaluates a simple catastrophe theory model of technological change with network externalities by adding an agent-based micro layer. To this end an agent-based variant of the model is proposed and investigated with specific regard to the network structure among agents. The technological-institutional layer of the model is very simple; a base technology is contrasted with a new innovative technology that they may adopt. The latter generates network externalities for the adopters but they also incur (fixed) costs that arise periodically. The model uses a simple evolutionary mechanism (a replicator dynamic with a capacity boundary) for defining the probabilities for agents to adopt the new technology. The network externality enters this function as a standard intensity term while the periodic costs are applied additively. This yields a polynomial of degree 3 which can be reduced to the canonical equation of a cusp catastrophe (for overviews on cusp catastrophes, see, e.g., [23, 21, 20]) by applying the linear Tschrinhaus transformation. While the macro level of the model thus produces a classical cusp catastrophe - a result that is preserved in the agent-based form - it is found that the behavior of the model changes locally with the network structure. The network structure affects the distribution of the agent's neighborhood sizes (number of directly connected agents). The agent will then base her decisions to adopt or abandon the new technology on her neighborhood rather than the entire population. Depending on the network structure, different local neighborhoods may persist in different adoption states (specifically for small world networks) and the theoretical equilibria of the underlying macro-level equation may exhibit different degrees of stability and sensitivity depending on the network structure. In turn, the network structure affects the general catastrophic or non-catastrophic outcomes as well when the slow variables (the catastrophe parameters, namely the cost and the capacity boundary) are included in the simulation, i.e. when the catastrophe is allowed to happen.

While the present work investigates merely an aspect out of a large possibility space, it encourages further research using agent-based catastrophe theory models especially of economic aspects to which catastrophe theory has previously successfully been applied; aspects such as technological and institutional change, economic crises, or industry structure.

In section 2, a more detailed overview of the literature on both network externalities and catastrophe theory in economic systems will be given. Section 3 proceeds with a simple theoretical model of an industry subject to network externality that results in a classical cusp catastrophe. This model will then be extended to an agent-based version and the role of the network structure and other aspects on the behavior of the model will be studied in section 4. section 5 concludes.

2. Literature Review

Catastrophe theory was established and developed as a field of mathematics in the 1970s, mainly by René Thom [27]. It immediately enjoyed some popularity and applications to many fields including economics and social sciences (see, e.g. Woodcock and Davis [32]) were developed but it was heavily criticized and has largely fallen out of use. One of the main critiques focusses on it being rooted in simple macro-level dynamic systems without relating to a micro level (microfoundation). For a brief overview, see Rosser [19] However, this should neither disqualify an entire class of models if it is able to describe or approximate observed phenomena nor is it generally impossible to add micro-foundations to such a model.

The current paper attempts to describe technological change with network externalities using a catastrophe model (and to add proper microfoundations with an agent-based version in section 4). This field has long been recognized as falling out of the domain of most earlier models in economics since there are increasing returns to scale driven by and increased willingness to pay higher prices for access to more well-established networks. This leads to path dependent technology choice processes as extensively analyzed by, e.g., Arthur and Ermoliev, and Kaniovski [2, 1], as well as (in a less formal-mathematical way) David [7]. The basic argument is that switching of technologies is costly and can, if it depends on others switching as well only be done with coordination - even if the intent is to switch to a new technology that would clearly be better than the old one. It was, however, long before this that it was recognized that technological change occurs in waves, as paradigm change rather than as a continuous process. The original idea is mainly due to Schumpeter, but the field was advanced greatly in the 1980s by, among others, Dosi [9] and Freeman and Perez [12] whose papers also include discussions of the earlier literature. More recent research has brought these two approaches (path dependent technology choice ans technological change) together [31, 25, 8] and has applied this to the research on and explanation of growth cycles [24].

While the discontinuous nature of this phenomenon is obvious, catastrophe models have rarely been applied to the field of technological change. Herbig [14] proposed that this could be done without, however, providing a model. Two different later approaches, those by Lange et al. [15] and by Dou and Ghose [10], followed Herbig's idea and added catastrophe models for specific cases, software adoption (Dou and Ghose) and online retail trade (Lange et al.) respectively. The two models are, however, not particularly close to the present one and use on different effects to derive the catastrophe equation. Both also remain focused on their particular case studies and neither comment on potential further uses of catastrophe models in the field of technological change or in social and economic systems as a whole nor do they address the earlier criticism of catastrophe models lacking microfoundation (which is why they do not attempt to proceed to replicate the macro-level catastrophe model in an agent-based approach).

3. A Simple Model

Similar to Dou and Ghose's approach [10], the basis of the present model is the canonical replicator dynamic equation for population dynamics with capacity boundary (for a general introduction and detailed explanation of this approach, see e.g. Nowak [18])

$$\frac{dS}{dt} = \alpha S \left(1 - \frac{S}{z}\right)$$

Here, S is the size of the current user base, z ist the capacity of the industry (i.e. the size of the potential user population, the maximum number of users), and α is the growth rate (or rather the fitness term that affects the growth rate as long as the capacity boundary is not approached) of the user base.

Different from Dou and Ghose, the present model introduces network externalities by making the growth rate dependent on the population size, here simply $\alpha = S$. The result is the dynamic system described by the first order differential equation (consisting of a third order polynomial)

$$\frac{dS}{dt} = S^2 \left(1 - \frac{S}{z}\right) = -\frac{1}{z}S^3 + S^2$$

Here, two simple fixed points (only the second one stable) can be found at

$$S = 0$$

$$S = z$$

But so far, the system does not involve catastrophic transitions. It is well known that the bifurcations become supercritical (see figure 1) if a negative S-intercept is added [23, 21, 20]. Other than in Dou and Ghose, this is not introduced as a market interaction polynomial with a denominator of order S^2,[2] but in a more straightforward and simple way as the periodic costs of using a technology. This modification is plausible since costs of infrastructure, maintenance, and expert knowledge increases linearily in the number of different technologies an agent uses while it the additional costs from more intensive use of an existing technology (that is, the respective infrastructure) are decreasing, possibly even zero.

The resulting equation is

[2]Dou and Ghose [10] use such a term to model network effects (and network externalities). The equilibrium set of this model is identical to one given by a polynomial of order S^4 (and without negative powers of S) which they then reduce to cubic order by dividing by S to develop it into the classical cusp model using the Tschirnhaus transformation in much the same way as outlined below.

$$\frac{dS}{dt} = \alpha S\left(1 - \frac{S}{z}\right) - \beta = -\frac{1}{z}S^3 + S^2 - \beta \tag{1}$$

Since it is the fixed points, the stability and potential catastrophic change that is under investigation here, the equilibrium set (i.e. $\frac{dS}{dt} = 0$) is what is of primary interest in this system. The equilibrium set is

$$\left(\frac{dS}{dt} =\right) 0 = -\frac{1}{z}S^3 + S^2 - \beta$$

$$0 = S^3 - zS^2 + z\beta \tag{2}$$

Using a Tschirnhaus transformation

$$S = x + \frac{z}{3}$$

the equation becomes

$$0 = +\left(x + \frac{z}{3}\right)^3 - z\left(x + \frac{z}{3}\right)^2 + z\beta$$

$$0 = x^3 - \frac{z^2}{3}x - \frac{2z^3}{27} + z\beta$$

which can be developed into[3]

$$0 = x^3 + ax + b,$$

the classical cusp catastrophe equation [23, 21, 20].

For the original system (equation 1), the equilibrium surface as stated above (equation 2) yields two supercritical bifurcations (together called the bifurcation set). They are obtained as the solution of[4]

$$\begin{aligned} 0 = & \ -\tfrac{1}{z}S^3 + S^2 - \beta \\ 0 = & \ -\tfrac{3}{z}S^2 + 2S \end{aligned}$$

as

[3]By substituting

$$a = -\frac{1}{3}z^2$$

and

$$b = -\frac{2z^3}{27} + z\beta$$

.

[4]That is, the intersection of the equilibrium surface $\frac{dS}{dt} = 0$ with the set of marginal stability $\lambda = \frac{\partial(dS/dt)}{\partial S} = 0$. The bifurcation set therefore contains all points on the equilibrium surface that are marginally stable, indicating that the system's stability properties change in the vicinity of the set (or rather at this very point) in the control space (i.e. the z-β-plane.)

$$\begin{array}{ll} \beta = 0 & z = S \\ \beta = \frac{1}{3}S^2 & z = \frac{3}{2}S \end{array},$$

more conveniently written (with S eliminated) as $\beta = 0$ and $4z^2 = 27\beta$.

The system is shown in figures 1 (transition for changing β), 2 (bifurcation set in control space) and 3 (equilibrium surface). Note that for the cusp area (between the two bifurcation sets), there are three equilibria, the lower and the upper of which are stable while the middle one is not. Outside the cusp area, there is only one equilibrium. Consequently, if an additional dynamic which modifies the parameters b and Z is introduced, catastrophic changes occur, when either of the bifurcations is reached from the stable equilibrium plane which does not continue on the other side of the bifurcation. Specifically, the interpretation of the two bifurcations is

1. $b = 0$: The size of the user base is zero (lower stable equilibrium) and the costs become zero - since, e.g. the vendor is supplies the technology for free to the first couple of users. The system will then jump to the upper stable equilibrium.

2. $4z^2 = 27\beta$: The system is in the upper stable equilibrium and either the costs increase beyond the threshold (for given capacity z) or the

population shrinks (for given β) below the bifurcation threshold (or a combination of the two). In this case, using the technology becomes uneconomic fore all its users and the network will collapse (to the lower stable equilibrium, i.e. zero size).

It should be noted, that many modifications to the fitness (growth rate) or network externality term $\alpha = S$ leave the properties of the model intact, i.e. that this model is generalizable at least to some degree. For instance, if the term is a positive linear function of S (with factor f), $\alpha = fS$, the system results in the same pattern of equilibrium surfaces and bifurcations with the specific bifurcation set (1) $\beta = 0$ and (2) $4z^2 = 27\frac{\beta}{f}$.

It is nevertheless obvious, that this is too simple a model to capture interactive usage decisions and other socio-economic system processes relevant to network technologies. It certainly can serve for identifying a potential key mechanism that may arise in and be relevant to the economics of innovation and technology. However, it must now be shown that the effect is preserved in systems which take the micro-layer into account, i.e. agent-based models. In that case, not only the network externality and the technology diffusion (following for instance the above population dynamics) are important factors, but the network structure and its properties as well as potential heterogene-

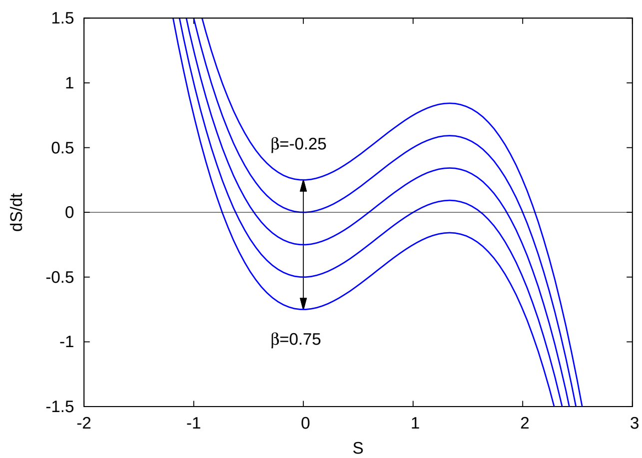


Figure 1: The cusp catastrophe: Bifurcations of the system with $z = 2$, for increasing β. For $0 < \beta < \frac{16}{27} \left(= \frac{4}{27}z^2\right)$ there are three equilibria ($dS = 0$), above and below this interval, two of the equilibria vanish.

ity of agents become relevant to the dynamic properties and the outcome of the resulting system. This will be the focus of section 4.

4. An Agent-Based Simulation

This section reconsiders the model developed in section 3 above and transforms it into an agent based model. Here, the behavior of the population is not governed any more by macro-level equations, but all agents make their own decisions. These decisions follow the same model (and equations) as above but the reference usage share that the agents base their decision on is (generally) not the entire population but only the immediate neighborhood

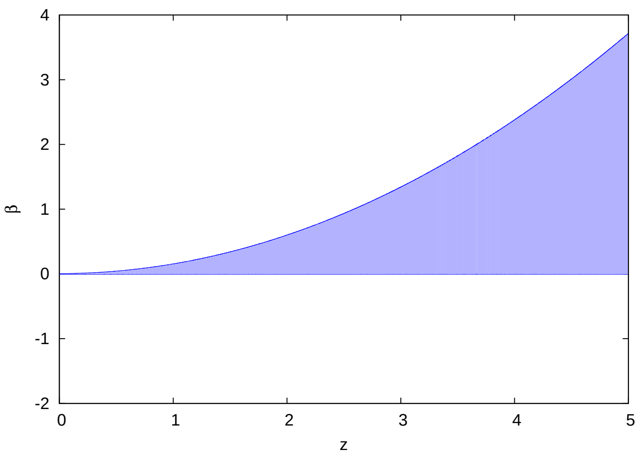


Figure 2: Bifurcation set and cusp area of the model.

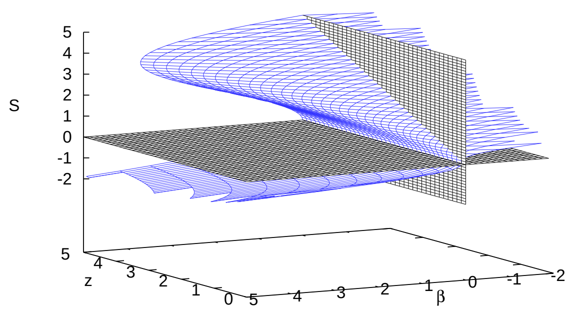


Figure 3: Equilibrium surface of the model including cusp area with three equilibrium surface points for every point in control space. Note that though the lower equilibrium surface is below zero, negative network sizes S are not allowed and the equilibrium is instead at $S = 0$. Also note that other than in figures 6 through 10, the vertical axis is absolute network sizes S.

of the respective agent. As a reference point, the section starts with the complete network (i.e. every agent is direct neighbor to every other agent), a setting that unsurprisingly yields the same result as the analytical macro-level model above, before more complex network topologies are considered. The agent-based version of the technology choice model on non-trivial networks is related to but not the same as the variously studied models of contagion between agents on networks as discussed in the literature review section 2 (of particular interest in this respect, Barash et al. [4] investigate discontinuities, bifurcation points, etc. of contagion models in Barabási-Albert networks).

The agents base their decision on the same polynomial as in equation 1 above,

$$-\frac{1}{z}S^3 + S^2 - \beta,$$

this time, however, the resulting quantity is not directly the dynamic change of the usage share (since the agent does not control this quantity) but the agent's inclination to join the network (if she is not currently a subscriber)[5]

$$\varrho_{+} = \min\left(z, \max\left(0, -\frac{1}{z}S^3 + S^2 - \beta\right)\right) \tag{3}$$

or to leave the network (if she currently is a subscriber).

$$\varrho_{-} = \min\left(z, \max\left(0, -\frac{1}{z}S^3 + S^2 - \beta\right)\right) \tag{4}$$

Three network structures are considered besides the complete network (examples of the three network structures are shown in figure 4):

1. The grid network (figure 4a): Agents are arranged in a 1-dimensional order; they are directly connected with their m predecessors and m successors in this order Grid networks have a high clustering coefficient, but a high diameter (average path length between random nodes). The first property resembles social networks, the second does not.

2. The preferential attachment network (figure 4b): The network is constructed consecutively; every new agent is connected to m agents who are already part of the network; she chooses these agents with a prob-

[5]The min and max operations guarantee that the resulting probability is between 0 and 1, if ϱ already falls into these limits, the probabilities are more conveniently written as ϱ and $1 - \varrho$.

ability proportional to their current degree, i.e. well connected nodes have a much higher probability to gain even more connections (hence, a Barabási-Albert network, [3]). The mechanism is known to lead to a power law degree distribution. This is not not generally a property of social networks as there seems to be a limit to the number of one's stable social relations[6], but it may be a realistic way to model networks among social groups, organizations, or firm networks, especially in network industries.[7] Further, preferential attachement networks are characterized by small diameter[8] but also (if no other modifications have been made to the generating process) by low clustering coefficient. The former (small diameter) is also realistic for networks observed in reality (e.g. in networks among firms) while the latter (low clustering

[6]Dunbar [11] for instance proposed a number of around 300. There are some models that nevertheless propose using Barabási-Albert networks as a model of social networks, e.g. [4] - the argument of networks among social groups (as opposed to within social groups) may be relevant to this discussion.

[7]This has been suggested directly in some models [6, 26], but since both the size of firms (profits, capital, number of employees) and the degrees of the internet (but also the sizes of urban centers and many other quantities related to network technologies in one way or another) are known to be power law distributed [17, 13], this is generally a plausible assumption related to other stylized facts.

[8]The diameter of a network is the longest distance (shortest path) between two nodes in the network. A small diameter compared to the size of the network (number of nodes) this means that the network are relatively well-connected, more specifically having Watts and Strogatz' small world property [30] (or having a huge number of links compared to the number of nodes, thus being a complete or almost complete network as this would also lead to a small diameter).

coefficient) is not as networks between firms, organizations, but also between individuals are generally highly clustered.[9]

3. A preferential attachment network with triadic closure (figure 4c). In order to obtain a network that does not only have a small diameter and a degree distribution following a power law, but also a generally higher clustering, one can apply start with a Barabási-Albert preferential attachment network and apply triadic closure. After the Barabási-Albert process completes, a predefined number of open triads - groups of three nodes (agents) where one is connected to the other two but the other two are not linked directly - are selected and closed (thereby linking the unconnected nodes of the triad directly). This network type has other interesting properties. It is more likely to contain components that are internally well-connected but poorly connected to other components of the network (though not isolated), something that is also observed in real social networks. Formally, the betweenness centrailty[10] distri-

[9]These two properties (and the conjunction of these two properties) in social networks has received particular attention in the small-world network literature [30]; for an extensive overview, see [22].

[10]The betweenness centrailty of a node i is the share of shortest path $sp_{j,k}$ in the network between any two nodes (j and k) of which it is part (denoted $sp_{j,k}(i)$)

$$bc(i) = \sum_{j \neq k \neq i} \frac{sp_{j,k}(i)}{sp_{j,k}}.$$

bution of the network changes such that some nodes (the ones with a connection to other closely connected components of the network) will have extraordinarily high betweenness centrality while most of the nodes score very low in this measure. In the other types of networks considered here (complete, grid, and preferential attachment without triadic closure) the distributions of betweenness centrailty are substantially more even[11] (See figure 5).[12]

All simulations in this section have been conducted in networks of 1250 agents. Figures 6a through 10a for illustration of the simulations show the entire equilibrium surface as derived in the simulation. For the stable equilibrium surfaces (darker grey), the simulation is straightforward and accurate; for the unstable equilibria (repeller surface, light grey), the location was approximated from the approximate borders of the basins of attraction of the stable equilibria. Note that other than in figure 3 the vertical axis gives the

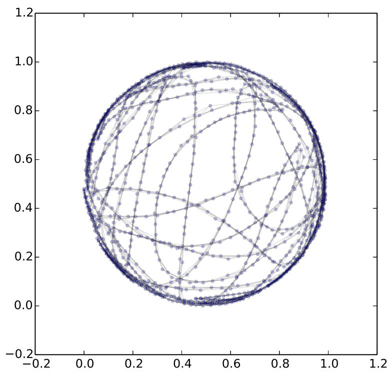
[11] In fact, the distributions of the node's betweenness centrality seem to decay according to a power law for the preferential attachment network and the preferential attachment network with triadic closure, but with different tail slopes.

[12] Note that this module uses a breadth-first search algorithm to obtain a single shortest path between any two nodes. As in regular grids there are always many equally short paths, one is selected randomly, therefore the computed betweenness centralities differ slighly between the nodes; the correct result would be a constant and equal value for all nodes.

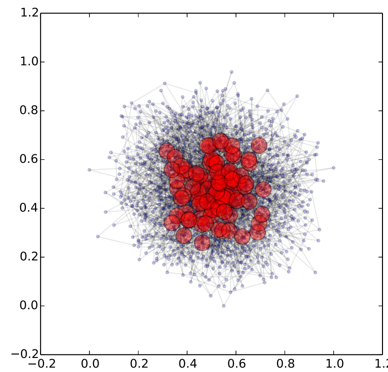
usage network sizes relative to the capacity (population size), $\frac{S}{z}$. To show the effect of the network structure in more detail, the results of simulations with constant z and initial value ($z = 4$ and $S_0/z = 0.5$) are also shown (figures 6b through 10b). This differs from the simulations shown in figures 6a through 10a in that this does not show the entire equilibrium surface but just the convergence point for a specific starting value ($S_0/z = 0.5$).

Unsurprisingly, the simulation of the complete network (figure 6) results in the same picture as obtained from the macro-level development equation (figure 6). Note that the S/z-value of the upper sheet of the equilibrium surface again first declines smoothly in β (like in the theoretical model above); this is because the constant cost (β) makes the participation undesirable for a part of the population. When the bifurcation set from the theoretical model is reached, the triple equilibrium vanishes and just one of the equilibrium surfaces continues to exist (here visible is the part for which the lower equilibrium continues to exist; towards the observer).

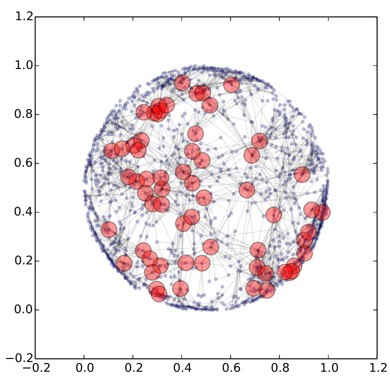
This result is still preserved in the case of the grid network (figure 7) with the slight difference that the upper equilibrium surface is much lower. This is a numerical issue resulting from the low degree of the nodes in this network (shown in figure 7 is a network with even node degree 4): If all



(a) Grid (double ring)



(b) Preferential attachment



(c) Preferential attachment with triadic closure

Figure 4: Networks of the same size (1250 agents, ca. 3750 links) but different types (grid, preferential attachment, and preferential attachment with triadic closure). Nodes (agents) with particularly high betweenness centrality highlighted; illustration generated by the python-networkx module.

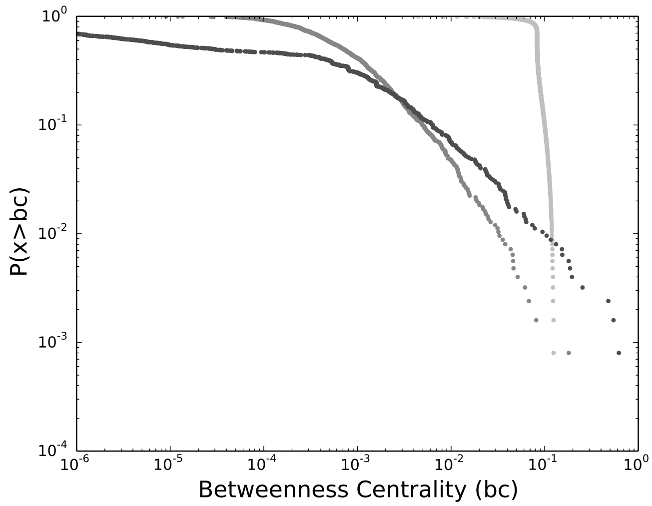


Figure 5: Complementary cumulative distribution of betweenness centrality of the vertices (agents) in the three networks in figure 4 (log-log plot): Grid (double ring, 4a) light grey, preferential attachment (4b), grey, and preferential attachment with triadic closure (4c), dark grey. Betweenness centrality computed with the python-networkx module.

the neighbors of a node are subscribers but the node is not, the dynamic (equations $eq : abm : 1$) yields 0 and the agent will not subscribe; if $\beta > 0$, there is even a positive probability for subscribers to unsubscribe - which is as it should be, but at the macro-level this should result in a very small number of unsubscriptions while here it matches every 3rd agent (who is between 2 subscribers to the left and 2 subscribers to the right). Indeed, this effect is much less prominent with a higher neighborhood degree (see figure 8 for comparison); the such denser networks will, however, increase computation time substantially.[13] In both grid networks, the catastrophe continues to exist and is identical to the theoretical model.

In a preferential attachment network (figure 9), this seems to change slightly. The catastrophe (i.e. the bifurcation, the switch between a regime with 1 equilibrium and one with 3 equilibria) is still present, but it seems to happen slightly less sudden. This is an illusion. It results from the fact that for the area around the bifurcation set in parameter space, different parts of the network may converge to different equilibria (yielding a network average between the upper and lower equilibria). This is even more pronounced in the case of preferential attachment networks with triadic closure (see below). Further, the effect described for grid networks, a shift of the upper equilibrium surface to a lower level) is also present here (and also more pronounced because the median neighborhood size is even smaller than for the grid network).

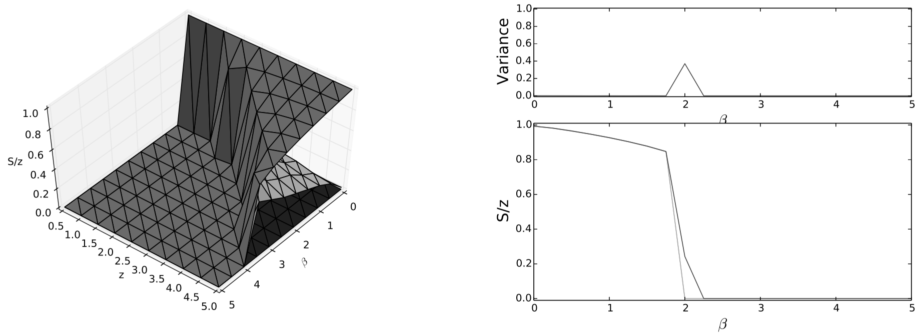
Finally, in a preferential attachment network with triadic closure (figure 10) the transition seems even more smooth as different parts of the network are not strongly connected. This also holds for preferential attachment net-

[13] Further, for the network 7, the number of links is such that the network is directly comparable to a Barabási-Albert preferential attachment network of the same size (figure 9).

works with $m = 1$ which are entirely loop free. For this case it is obvious from figure 10b that even for low values of β there are increasingly large parts of the network that converge to the lower equilibrium. Figure 11 shows how this changes with the amount of triadic closure. While it is clear that if triatic closure were applied permanently for a long time the graph would eventually converge to a complete network (with the associated characteristics shown above), the figure shows that for even a substantial amount (roughly 3 times the number of links present before is added through triadic closure) the characteristics of the process are preserved. While the catastrophe is still present - and the macro-level model shown above is therefore a suitable first approximation of the situation - the network structure may in this case lead to different outcomes in different parts of the network.

5. Conclusion

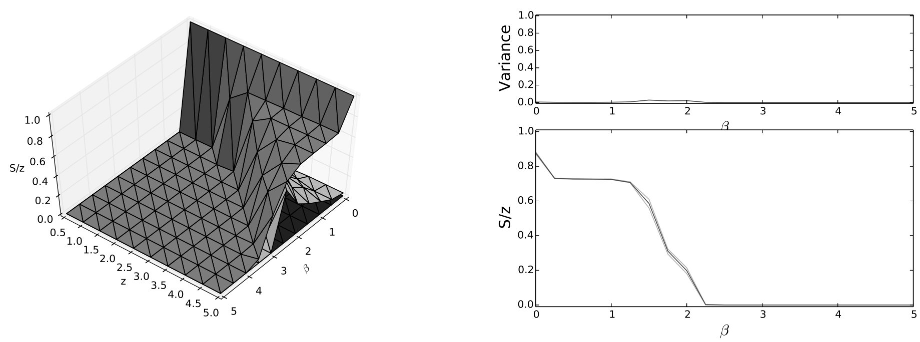
The present work attempted to apply a catastrophe theory model to the problem of network industries. A development equation is defined for the development of the user base of a technology with network externalities. Those not part of the user base are assumed to use a very simple and free standard technology or no equivalent technology at all. With the size of the



(a) Equilibrium surface (dark grey), repeller surface (light grey)

(b) Average outcome across 10 runs for $z = 4$, $s_0 = 2$

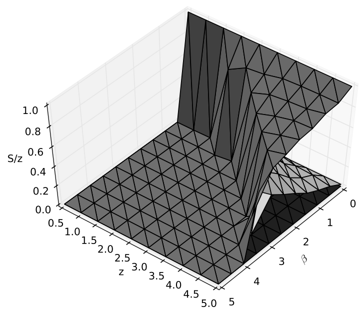
Figure 6: Agent based model on a complete network (1250 agents, $1250 \times 1249 = 1561250$ links).



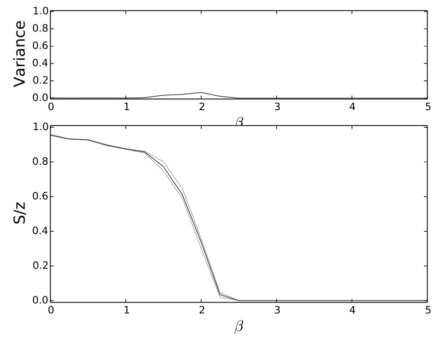
(a) Equilibrium surface (dark grey), repeller surface (light grey)

(b) Average outcome across 10 runs for $z = 4$, $s_0 = 2$

Figure 7: Agent based model on a grid (double ring) network (1250 agents, 2500 links).

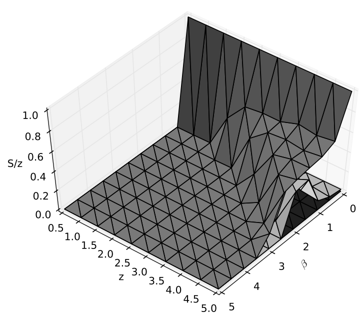


(a) Equilibrium surface (dark grey), repeller surface (light grey)

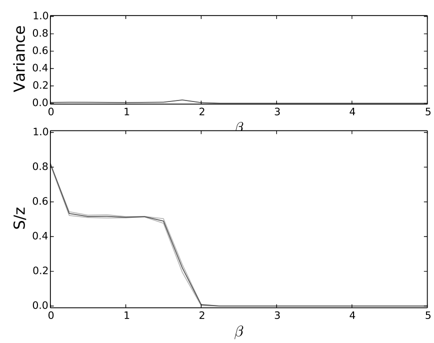


(b) Average outcome across 10 runs for $z = 4$, $s_0 = 2$

Figure 8: Agent based model on a grid (double ring) network (1250 agents, 12500 links).

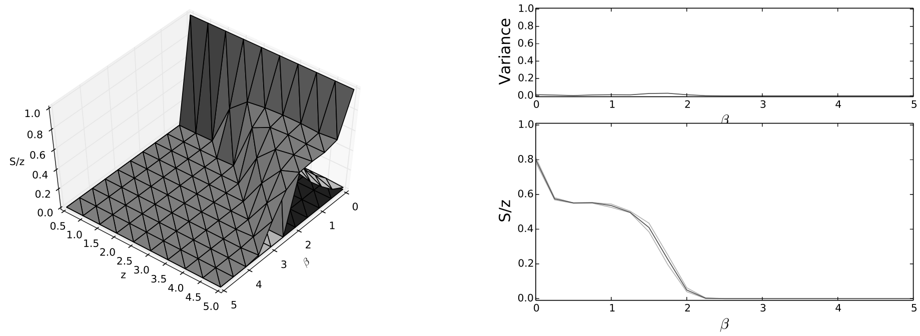


(a) Equilibrium surface (dark grey), repeller surface (light grey)



(b) Average outcome across 10 runs for $z = 4$, $s_0 = 2$

Figure 9: Agent based model on a preferential attachment network (1250 agents, 2497 links; Barabási-Albert with $m = 2$).



(a) Equilibrium surface (dark grey), repeller surface (light grey)

(b) Average outcome across 10 runs for $z = 4$, $s_0 = 2$

Figure 10: Agent based model on a preferential attachment network with triadic closure (1250 agents, 2499 links, Barabási-Albert with $m = 1$, thus 1249 links, and 1250 random open triads closed, thus $1249 + 1250 = 2499$ links).

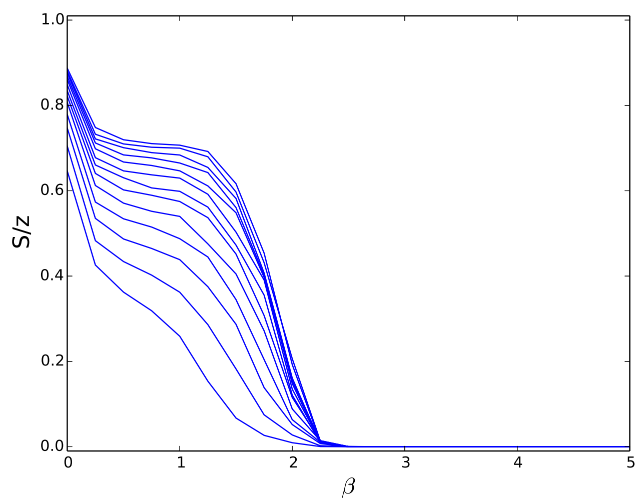


Figure 11: Average outcome across 10 runs for $z = 4$, $s_0 = 2$ for preferential attachment networks (Barabási-Albert with $m = 1$, thus 1250 agents, 1249 links) with different degrees of triadic closure (between 0 additional links, lowest graph, and 3750 additional links, uppermost graph).

user base as the only variable, the characteristics of the model depend on two control variables, the total population size (capacity boundary) and the periodic fixed costs of using the technology.

The model follows a simple but generalizable layout that includes a population dynamic with capacity boundary, the network externality in the form of a growth rate of the user base[14] proportional to the size of the user base, and the cost term as an intercept. As was shown in section 3, the model produces a cusp catastrophe and it is, in fact, possible to transform the model into the classical cusp catastrophe equation. The bifurcations of the model occur (1) at zero costs and (2) at a specific relation of costs and the capacity boundary. It can be shown, that any factors applied to the network externality term would in effect shift this second bifurcation (dividing the cost term).

Further, it was established using an agent-based simulation, that the findings of this model are robust with respect to the introduction of a true micro-level (interdependent agents) for several network structures (grid networks, Barabási-Albert preferential attachment networks, and preferential

[14]Or rather, the fitness term that dominates the growth rate as long as the capacity boundary is not reached.

attachment networks with triadic closure). While the general behavior - the bifurcation, the catastrophe - is preserved, there were some critical changes to the macro-level for some network structures. More realistic approximations of social networks (and specifically firm networks, with small diameter, high clustering, and scale free degree distribution) such as preferential attachment networks with triadic closure found that different parts of the network may converge to different equilibria. The structure of such networks with several well-connected subgraphs that are poorly interconnected through a couple of bottleneck nodes, appears to facilitate this outcome.

As sudden and intense changes in the use of network technologies have repeatedly been observed (so in the case of Microsoft's takeover of the PC operating system market in the 1980s, but also in the case of the rise and fall of MySpace, in the rise of tablet computers, etc.) catastrophe models may indeed be a promising approach to explaining the specific dynamics of these sectors. This is particularly relevant as it is clear that they are subject to increasing returns to the size of the user base (which allows the introduction of many non-trivial strategies on the part of the market participants), that many standard models focusing on equilibrium concepts are therefore not applicable, and that the reasons for the asymmetric industry structure in almost all of these sectors are not entirely understood.[15]

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