Vertical Mergers and Downstream Spatial Competition with Different Product Varieties, Revised and Corrected

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Vertical mergers and downstream spatial competition with different product varieties, revised and corrected

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Abstract

The aim of this paper is to revise and correct the results obtained in Beladi et al. [Beladi, H., Chakrabarti, A., Marjit, S., 2008. Vertical mergers and downstream spatial competition with different product varieties. *Economics Letters* 101, 262–264]. Specifically, we prove that in the pre-merger case, Nash equilibrium locations are socially optimal, whereas a vertical merger will relocate downstream firms by making them move to the right of their socially optimal positions while keeping their in-between distance intact.

*JEL classification:* L13, L42, D43, R32  
*Keywords:* Price discrimination; Spatial competition; Merger

1 Introduction

In their paper Beladi et al. (2008) attempt to extend the results in Braid (2008) by evaluating the equilibrium locations of two firms selling different varieties of a product following a vertical merger. In a market with upstream monopoly, they claim that a vertical merger will "exacerbate the distance of the downstream firms from the socially optimal location: the firm outside the merger will deviate more than the firm that is part of the merger". The precision of this result has been compromised by both technical and conceptual flaws. Specifically, we prove:

(i) in the pre-merger case, the Nash equilibrium locations are socially optimal, and

(ii) in the post-merger case, the downstream firms relocate by both moving to the right of their socially optimal locations maintaining their relative distance intact.
2 Results

The model is as in Beladi et al. (2008). To the end of facilitating the presentation we make use of the exact same notation as in Beladi et al. (2008), with $M$ the upstream monopolist and $R_1$ and $R_2$ the two downstream competing firms. It is straightforward to see that absent a merger the monopolist $M$ is charging a uniform wholesale price $w$ which being equal for both firms affects equally their delivered cost at location $z$ [see Braid (2008), p. 345]. As a result, $w$ is cancelled out from the profit both firms have selling the common good ($W$) to the consumer located at place $z$. Therefore, their respective profit functions are

$$
\Pi_1 = \left( c(k - w) - \frac{ct}{2} \left[ x^2 + (1 - x)^2 \right] \right) + \left( \int_0^x b[t(y - x)]dz + \int_{x}^{(\frac{x+y}{2})} b[t(x + y)]dz \right) - F
$$

(1)

$$
\Pi_2 = \left( c(k - w) - \frac{ct}{2} \left[ y^2 + (1 - y)^2 \right] \right) + \left( \int_0^y b[t(2z - x - y)]dz + \int_{y}^{1} b[t(y - x)]dz \right) - F
$$

(2)

and the Nash equilibrium locations are given by

$$(x, y) = \left( \frac{1}{2} - R, \frac{1}{2} + R \right)$$

(3)

where $R = \frac{b}{4(b+c)}$. This leads to the following proposition.

**Proposition 1** Absent a merger, the Nash equilibrium locations of the two downstream firms, in a vertically related industry are socially optimal ($\frac{1}{2} \pm R$).

In case of a merger taking place between the upstream firm $M$ and $R_1$, the profit functions
of the downstream firms become

\[
\Pi_1 = \left( c(k) - \frac{ct}{2} \left[ x^2 + (1 - x)^2 \right] \right) \\
+ \left( \int_0^x b[t(y - x) + w_2]dz + \int_x^{(\frac{x+y}{2} + \frac{w_2}{2t})} b[t(x + y - 2z) + w_2]dz \right) 
\] (4)

\[
\Pi_2 = \left( c(k - w_2) - \frac{ct}{2} \left[ y^2 + (1 - y)^2 \right] \right) \\
+ \left( \int_y^{\frac{x+y}{2} + \frac{w_2}{2t}} b[t(2z - x - y) - w_2]dz + \int_y^{1} b[t(y - x) - w_2]dz \right) - F_2 
\] (5)

with \( \frac{x+y}{2} + \frac{w_2}{2t} \leq y \) and \( w_2 > w \).

Notice the difference of equations (4) and (5) and the corresponding equations in Beladi et al. (2008). The integration threshold \( s \) is the outcome of the comparison of the two delivered costs for the common good \( W \):

\[ t(y - s) + w_2 = t(s - x) \Rightarrow s = \frac{x+y}{2} + \frac{w_2}{2t}. \]

Solving (4) and (5) for \( x, y \), we get the following Nash equilibrium locations

\[
(x, y) = \left( \frac{1}{2} - R + \omega, \frac{1}{2} + R + \omega \right) 
\] (6)

where \( \omega = \frac{bw_2}{2t(b+2c)} \). This leads to the next proposition

**Proposition 2** When one of the downstream firms merges upstream, the Nash equilibrium locations of the two downstream firms, in a vertically related industry, are \( \left( \frac{1}{2} - R + \frac{bw_2}{2t(b+2c)} \right) \) for the integrated firm and \( \left( \frac{1}{2} + R + \frac{bw_2}{2t(b+2c)} \right) \) for the un-integrated firm.

One deduces from Propositions 1 and 2 that after the merger, both downstream firms move to the right of their socially optimal locations. The intuition behind this result is that, following the merger, the un-integrated firm having lost its competitive edge is forced to give away part of its market share. It is remarkable, although perhaps natural, that the distance

\[ \frac{x+y}{2} + \frac{w_2}{2t} \leq y. \]

\[ 1 \] If \( \frac{x+y}{2} + \frac{w_2}{2t} > y \), both firms are reduced to spatial-price discriminating monopolists where the pre-merger common good \( W \) is now provided only by the integrated firm. It can easily be shown that in such case, both firms will locate in the center (i.e., \( x = y = 1/2 \)) replicating the equilibrium à la Hotelling (Hotelling, 1929). We consider this case trivial and focus only on the case where \( \frac{x+y}{2} + \frac{w_2}{2t} \leq y. \)
between the two firms is \( 2R = \frac{b}{2(b+c)} \) which means that the relocation of the two firms is taking place with no effect on their relative distance.

3 Conclusion

We have shown that the existence of an upstream monopolist will not affect the social optimality of the Nash equilibrium locations for the downstream firms. Moreover, in the event of a vertical merger, compared to the pre-merger situation, the integrated firm will move closer to the center, never exceeding it, while the un-integrated firm will move an equal distance away.

References

