Endogenous Defaults in the Business Cycle

Ferrari, Massimo

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Massimo Ferrari
Catholic University of Milan

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Abstract

We developed a new-Keynesian DSGE model with heterogeneous agents and an active interbank market, characterized by an endogenous default probability. Banks are heterogeneous in the sense that they face, each period, different liquidity shocks and may or not be constrained in the total amount of credit that they can extend to the private sector. Banks can invest in loans to firms, risk less assets or lend one another. The key feature of the analysis is that the probability of default of banks evolves endogenously and is explicitly taken into account by other banks in their investment decisions. In each period, only a fraction, or even none, of banks' surplus funds is invested on loans to other financial institutions. If the probability of default is high enough, they shift their portfolio choices to risk-less assets and the interbank market freezes. This affects the total supply of credit to firms and, through it, the total level of investments, output and employment.

The model is than estimated using the Bayes technique and several test are carried on to verify the robustness of the estimation. Our findings show that indeed the default probability plays a crucial role in the decision of banks directly affects the economy; in addition we show how the stability of the financial market affects the the real economy and is connected with real and financial variables. In times of financial turmoil, banks reduce their exposure towards other financial institution, reducing the total supply of loans to the private sector and worsening the crisis. In this context, standard inflation targeting, that seems adequate as a response to standard shocks, is not sufficient to counterbalance negative shocks and may even have a negative effect on the economy, leaving room for unconventional tools. In addition, following real shocks, we have identified an additional channel of transmission of monetary policy, through the resiliance of banks.

1 Introduction

The last financial crisis has shown how even financial markets are far from the "perfect competition" paradigm and how they are influenced by friction just as any other type of market. Economic theory was, in some case, unable to predict the real extent of the crisis and, at the beginning, to formulate effective policy measures to offset its effect on the real economy (Bernanke [2013], Guillen [2011], Thornton [2012]).

In the aftermath of the crisis the key questions were to understand why some of the policy measures carried out by policy makers were so ineffective (Claessens et al. [2010]) and which tools were more unadeguate to the complex economic scenario we were facing[1].

In this context, one of the key issue to analyze is the relationship between financial markets and the real economy, trying to understand, from a macroeconomic point of view, which is the role played by the probability of default and by financial frictions. In the past, this topic was more popular in the literature on developing economies, for example in Ordonez [2009], but now we know how it is important also for advanced economies after the contagion and spillover effects that took place on financial markets.

During the crisis the interbank market froze up, leading to an increasing pressure over institutions already in distress and a reduction of the credit to the real economy. What was surprising at that time was the velocity with which the crisis spreaded and its strength, considering its relatively modest nature at the origin[2]. Suddenly, financial intermediaries stopped to lend money each other, tightening the liquidity constraint that each of them was facing and reducing even more the credit supplied to firms[3]. It is clear that an important role was played by the increased default probability[4] and by the reduction of trust between financial agents.

The aim of this paper is to investigate the relationship between riskiness and portfolio choices of banks. In order to do so the economy is modeled with an active banking sector populated by heterogeneous[5] banks that can allocate their funds between: i) direct lending to firms, ii) interbank lendings to other financial institutions, iii) risk-free assets.

This stylized balance sheet presents the basic problem of allocation of resources for a banker. Interbank lendings provide higher returns than risk-free assets, but may not be rapid. When bankers choose how to allocate excess resources, they try to balance

[1]See Friedman [2012], Krugman [2012], Stiglitz [2010], Turner [2012].

[2]See Delli Gatti [2012], Guillen [2011], Allen et al. [2010].

[3]On this topic, can be seen Brunnermeier [2009] and Gorton [2009].

[4]See De Socio [2011].

[5]Heterogeneity in macro models has been shown to play a very important role in understanding business cycle facts. In particular, results that are valid in the "representative agents" paradigm may not be robust to the diversification of economic actors. On this topic it is possible to see Branch and Evans [2005], Delli Gatti and Longaretti [2006], Hommes [2006], de Grave et al. [2010] and Motta and Tirelli [2010].

these two aspects and, as we will discuss in later parts of this work, when a crisis hit the system they shift their portfolio choice to risk-free assets. Doing so, the final credit to the economy is reduced and the crisis worsened; in addition banks already in distress are more financially constrained and are forced to refuse to finance valuable projects, reducing profits and dealing the strengthening of their net worth.

This behavior of the model resembles what happened in the economy between 2007 and 2010. The contribution of the paper is to investigate the reasons for this behaviors of bankers. Bankers optimize considering the probability of default of their counterpart. In particular, ceteris paribus, they know that each additional unit of credit granted will make their debtor more leveredged and, therefore, more exposed to the risk of default. So, when the *expected returns* (defined as the gross rate weighted for the default probability) of interbank lendings equal the risk-less rate, bankers choose to not issue additional units of credits to other banks.

In this model, in addition, it is shown how there are also reasons exogenous from the debtor-creditor relation that influence the leverage of banks. As pointed out before, when the leverage increases, ceteris paribus, interbank credit decreases and less loans are issued to firms. An addition contribution of this paper is to investigate this second channel of crunch in the interbank market and the relation between real and financial shocks and default probability on the interbank markets.

The role of the default probability was analyzed in the past years by several empirical works[6], but has received moderate attention in theoretical researches.

On this topic, the larger part of the studies come from the microeconomics of finance literature[7], although in the recent years there have been some attempts to incorporate it into macroeconomic models[8]. Beside those attempts, however, I think that there is still the necessity to develop a more comprehensive and complete framework that could allow us to define the default probability as an endogenous variable. This is what I will try to do in the following pages.

On top of the complex interbank market, we propose, a second source of friction between banks. Basically, banks' credit relations are affected by moral hazard[9] given the inability of creditors to control the action of the bank's manager. This second channel affects the total supply of credit and inserts additional frictions in the model. To model

[6]For example by Hong et al. [2009], Heider et al. [2009] or Bracke [2010].

[7]For example Leland and Toft [1996] or Diamond and Rajan [2001].

[8]For example in Stiglitz and Greenwald [2003], Angeloni and Faia [2009] or Dib [2010].

[9]Game theory, in particular agency and moral hazard problems applied to macroeconomics are an increasingly significant field of research. On this topic, in particular, it is possible to see the already cited Gertler and Kiyotaki [2010], Phelan and Townsend [1991], Dib [2010] and Meeks et al. [2013].

this behavior of bankers we propose a set up similar to that of Gertler and Kiyotaki [2010] and more recently in Gertler and Kiyotaki [2015]. Despite the abbundancy of the literature, that bloomed after the crisis[10], we believe that this approach combines flexibility with consistent and meaningful results.

In this paper we will incorporate the complex interbank market described before into a new-keynesian DSGE model; this is something new to the literature given the endogeneity of the default probability in the model. Previous attempts, such as Dib [2010], proposed alternative approaches, for example defining the default probability as a choice variable for bankers. Our choice, however, is preferable given the explicit relation between the default probability and the balance sheet of banks and the possibility to track its movements along the business cycle.

In addition to the original configuration of the interbank market, we have incorporated classic market friction in the model. As has been pointed out in the literature, starting from the pioneering work of Bernanke et al. [1999], financial accelerator plays an important role in the economy [11] with the more complex lender-borrower dynamic and overcoming the assumption of the validity of the Modigliani-Miller Theorem[12]. Additionally we have inserted standard market frictions in the goods market, trough the Calvo formalism[13] and monopolistic competition following Dixit and Stiglitz [1977].

Our contribution, at last, differs from the network literature on financial markets such as Allena and Gale [2000], Delli Gatti et al. [2006] and Battiston et al. [2012][14] for several reasons. While in that field researchers try to exploits the proprieties of networks to understand the connection between economic agents and, in particular, the mechanisms behind the transmission of shocks and the effects of monetary policies we have constructed an environment of heterogeneous agents in which banks are linked together by borrowing relationships that evolves and changes trough time[15]. In contrast to the network literature the dynamics do not arise as property of the underline network but as behaviors and strategical choices of the agents.

[10] For example, it is possible we can remember Bernanke and Gertler [1989], Krishnamurthy [2003], Angeloni and Faia [2009], Gertler and Kiyotaki [2010], Brunnermeier et al. [2012], Brunnermeier and Sunnikov [2014].

[11] Just to make a couple of example, it is possible to see Gilchrist and Zakrajsek [2012] and Hammersland and Trae [2012].

[12] Modigliani and Miller [1958] and Modigliani and Miller [1963].

[13] Calvo [1983]

[14] Additional references may be given by Beltratti et al. [1996], Gale and Kariv [2007] and Elliott et al. [2014].

[15] In particular, we allow for banks to become lenders or borrowers on the interbank market conditioned on the realization of an exogenous shock.

The model will be finally estimated with the Bayes rule in order to obtain impulse response functions augmented with confidence intervals. We will present the policy implication of their analysis in the relative section.

The paper will be developed as follows: section 2 presents the model, section 3 the data, section 4 the Bayesian estimation of the model with robustness checks, section 5 the Bayesian impulse response functions and section 6 the final conclusions.

2 The Model

The economy of the model is populated by households, firms, banks, a government and a Central Bank.

Households owns firms, consume differentiated goods and save trough deposits. Within each household, a fraction of its member is composed by workers and another by bakers, with the fraction of bankers and workers constant trough time. Workers supply labor to firms and earns wages; bankers run banks and return their profits[16] to households as dividends that are paid only when a bank ends its activity. To avoid that banks accumulate enough capital to overcome any kind of financial constraint, I assume that in each period there is a probability that a bank ends its activity. In that case, the banker transfer all his remaining assets to households and becomes a worker while households provide to new bankers an initial endowment of capital. In each household there is perfect consumption insurance, therefore the returns on banking activity and the wages earned are pulled together to finance the household's expenses. Households use their incomes to finance consumption, to pay taxes and to save, in the form of the purchase of risk-less deposits.

The real sector of the economy is composed by goods producers, retailers and capital producers.

Good producers combine capital and labor to produce, in perfect competition, an undifferentiated good that is sold to retailers. They do not accumulate capital, so in each period, they need to finance the already installed capital and investments borrowing funds from the financial system. In addition, each firm is located on a different area and can borrow money only from banks located in the same area. In each period, finally, there is an exogenous probability that in each area emerge new investments opportunities. I am following here what proposed by Gertler and Kiyotaki [2010], and I will call this probability π^i and the opposite probability $\pi^n = 1 - \pi^i$.

[16] As we are in perfect competition, profits are defined as the returns on the invested capital.

Retailers acquire the output of good producers, differentiate it with negligible costs and sell it on a national market to households. Because they sell a differentiated good they enjoy some degree of market power and, therefore, can charge a price higher than the marginal cost. In addition, following the standard idea of Calvo prices[17], they are not able to update the price they charge in each period.

Finally, capital producers combine undepreciated capital and a fraction of final goods to produce new capital.

The financial sector is composed by banks that are finitely lived and raise liquidity from deposits and a national interbank market. Each bank may use its funds to finance interbank loans, loans to firms or to acquire risk-less assets. In each period there is a probability that a bank ends its activity; in that case the entire value of the equity is transferred to households. Banks are run by bankers, who optimize the capital structure in order to maximize the value of the bank's capital at the end of the period and, in that way, maximize the transfer that is made to households in case the bank closes. Because they run the bank, they can also divert a fraction of the total volume of funds intermediated and transfer it to households. If in a given period the value of the bank is lower than the value of divertible assets, the banker will divert funds from the bank and transfer them to the households. In that case, the bank defaults[18]. Each bank, at last, is located on a specific area and while it can borrow on national markets, it can lend money only to firms located in its same area. Bankers know if in their area there are new investment possibilities only after that the period is already started, but they have to decide the level of deposit at the beginning of each period, before that the shock is revealed. For this reason, some banks will have excess funds while others will have a deficit of liquidity. Following Stiglitz and Greenwald [2003] I assume that only banks on areas without new investments acquire risk-less assets, this assumption will be proved endogenously later on. What is crucial for the development of the model is the way in which the credit cycle works: bankers choose the level of deposits before the beginning of the period; then they know if there are new investment possibility on their area and, then, they make their investment choices. I assume also that there is some degree of uncertainty on the outcome of new investments projects, for the reason that they are new and more risky.

For what concerns the last two agents of the model, the Central Bank and the

[17]See Calvo [1983].

[18]This dynamic leads to the definition of an agency problem between lenders and borrowers, following, between the others, Kiyotaki and Moore [1997], Krishnamurthy [2003] and Fostel and Geanakoplos [2009].

Government, I assume that the Central Bank sets the risk-less rate and the Government chooses the level of its consumption of final goods, that is treated as an exogenous variable. We will see in the next section how we can model a more active role of both these agents.

2.1 Households

The member of each household may be either workers or bankers. Workers supply labor to firms in exchange for wages, while each banker runs a bank. Wages and profits from the banking activity are transferred to households and within each family there is perfect consumption insurance. The fractions(f) of workers and $(1-f)$ of bankers within each household are constant through all the periods.

Because banks are finitely lived[19], in each period a fraction $1-\sigma$ of banks closes. Therefore, the bankers who run them become workers and, on the opposite, the same number of workers becomes bankers[20]. When a bank is closed, the banker transfers all the remaining equity to household that will provide the starting capital for new bankers. Following standard assumptions, then, firms are entirely owned by households.

Households get utility from consumption and leisure, are characterized by internal habits and can save acquiring bank's deposits and pays lump sum taxes.

Their objective function, therefore, is defined as:

$$E_t \sum_{t=0}^{\infty} \beta^t e_t \left[\ln \left(C - \varphi C_{t-1} \right)_t - \nu \frac{L_t^{1+\varepsilon}}{1+\varepsilon} \right] \tag{1}$$

with C consumption and L the fraction of time devoted to work. v the weight of labor dis-utility equal to the elasticity of leisure, ε the inverse of Frish elasticity to labor supply, φ the habits parameter and β^t is a discount factor. Finally e_t is a preference shock, that follows an AR(1) process.

If we define D_h the deposits in one type of bank, R_t^D the interest rate granted on them in each period[21], T the taxes paid[22], W the nominal wage, Π the profits of firms

[19] In each period there is a probability σ that a bank closes.

[20] The workers becoming bankers are, following the assumptions, $(1-\sigma) f$.

[21] It will be proved that the interest rate on deposits is equal to the risk-less interest rate in the equilibrium.

[22] Following the well established literature on this topic, started with Ramsey [1927], taxes are assumed to be a lump-sum.

and banks transferred to families[23] and P_t the price level, it is possible to define the budget constraint as:

$$C_t + D_{h,t+1} + \frac{T_t}{P_t} \leq \frac{W_t}{P_t} L_t + \frac{\Pi_t}{P_t} + \frac{R_t^D D_{h,t}}{P_t} \tag{2}$$

from the solution of the households decision problem, it is possible to derive the following equilibrium conditions:

$$\frac{e_t}{C_t - \varphi C_{t-1}} - \varphi\beta \frac{e_{t+1}}{C_{t+1} - \varphi C_t} = \lambda_t^C \tag{3}$$

$$vL_t^{\epsilon} = \frac{W_t}{P_t} \lambda_t^C \tag{4}$$

$$E_t \beta^t \frac{\lambda_{t+1}^C}{\lambda_t^C} \frac{R_t^D}{P_t} = 1 \tag{5}$$

$$C_t + D_{h,t+1} + \frac{T_t}{P_t} = \frac{W_t}{P_t} L_t + \frac{\Pi_t}{P_t} + \frac{R_t^D D_{h,t}}{P_t} \tag{6}$$

with $\left\{\lambda_t^C\right\}_{t=0}^{\infty}$ the sequence of Lagrangian multipliers associated to the optimization problem, that can also be used to define a stochastic discount factor $\Lambda_t^C = \beta^t E_t \left(\frac{\lambda_{t+1}^C}{\lambda_t^C}\right)$.

Equations (3) and (4) describe the optimal choice of consumption and leisure, and link together the choice of consumption and hours devoted to work. They also describe the demand of goods and the supply of labor. Equation (5) describes the Euler condition on deposits and says that the risk-less rate must be such that it equalities the present discounted marginal utility of future consumption with the marginal utility of present consumption. The last equation allows us to determine the value of deposits in t+1 given the optimal choice of all the other variables and the constants.

2.2 Firms

The real sector of the economy is populated by good producers, retailers and capital producers. Good producers operate in perfect competition and produce homogenous

[23] $\Pi_t = \Pi_t^B + \Pi_t^F$, with Π^B the dividends of banks and Π^F the profits of firms.

goods that sell to the retailers. Retailers, with negligible costs, differentiate the goods and sell them to consumers. However, following the Calvo formalism[24], only a fraction $(1-\theta_R)$ of retailers is able to rest its optimum price at each time t, the remaining retailers θ_R will keep the price of the previous period. I define $X_t = \frac{P_t}{P_t^W}$ the mark-up of retailers price over good producers price. Capital producers produce capital using undepreciated capital and a fraction of total output as input for a capital producing technology.

2.2.1 Good Producers

Good producers operate in perfect competition and produce homogeneous goods that sell at a price P_t^W to retailers. In order to produce, they combine labor and capital with a Cobb-Douglass production function:

$$Y_t = A_t K_t^{\alpha} L_t^{(1-\alpha)} \quad 0 < \alpha < 1 \tag{7}$$

with A, the total factor productivity, that evolves following an AR(1) process. Firms acquire labor from households and loans to banks in order to acquire capital from capital producers. Goods producers, in addition, are not able to accumulate capital so they need to completely refinance it in every period.

The total cost function is given by:

$$TC = \frac{W_t}{P_t} L_t + Z_t K_t \tag{8}$$

with Z the gross profits for unit of capital. From the firm's optimization problem it is possible to derive the following first order conditions:

$$\frac{1}{X_t}(1-\alpha)\frac{Y_t}{L_t} = \lambda_t^f \frac{W_t}{P_t} \tag{9}$$

$$\frac{1}{X_t}\alpha\frac{Y_t}{K_t} = \lambda_t^f Z_t \tag{10}$$

[24]Calvo [1983].

with $\left\{\lambda_t^f\right\}_{t=0}^{\infty}$ the sequence of Lagrangian multipliers associated to the optimization problem. Equation (9) defines the demand for labor, while equation (10) defines the gross profits per unit of capital. Firms do not accumulate capital, so in each period they have to finance their investment with loans acquired from the banking system. As long as they are able to obtain that funds, they do not face any other friction and commit to pay to the creditor bank the gross profits per unit of capital. Each unit of equity, in other words, is a claim to the future returns on one unit of investments:

$$Z_{t+1}, \left(1-\delta_K\right) Z_{t+2}, \left(1-\delta_K\right)^2 Z_{t+3} \ldots$$

δ_K describes the rate of depreciation of capital. Because good producers operate in perfect competition, they earn 0 profits in the equilibrium. Given this assumption, it is possible to define the rate of returns on each unit of financed equity, that is given by:

$$R_t^{h,K} = \frac{\left[Z_t + \left(1-\delta_K\right) Q_t^h\right]}{Q_t^h} \tag{11}$$

with $h = i, n$ that defines if a firm is operating on an area with new investment opportunities or not. The rate of returns on each unit of capital, therefore, is given by the gross profits plus capital gains or losses.

In each period, as we know, there is a probability π^i that firms on an area have new investment opportunities and, on the contrary, a probability π^n that they have not. It is possible, then, to define a law of motion of capital as follows:

$$K_{t+1} = \psi_{t+1}\left\{\left[I_t + \pi^i\left(1-\delta_K\right) K_t\right] + \pi^n\left(1-\delta_K\right) K_t\right\} = \psi_{t+1}\left\{I_t + \left(1-\delta_K\right) K_t\right\} \tag{12}$$

with I the value of new investments. Therefore, the aggregate demand for loans on each type of area is given by:

$$S_t^h = \begin{cases} \pi^n (1-\delta_K) K_t \quad for\ h=n \\ \pi^i (1-\delta_K) K_t + I_t \quad for\ h=i \end{cases} \tag{13}$$

firms on "i" areas will need funds to refinance the already existing capital plus new investment projects that they are able to undertake. On the contrary, firms on "n" areas have no new investment projects and will need liquidity only to refinance the depreciated capital of the previous period.

2.2.2 Retailers

Retailers acquire undifferentiated wholesale goods and transform them into a differentiated final good. Their marginal cost of production is equal to the price of the undifferentiated good, therefore it is P_t^W.

Differentiated goods, then, are bundled together so that they can be sold to consumers at the price P_t.

Retailers, therefore, face a standard Dixit-Stiglitz[25] demand function that can easily be computed given a CES aggregator of output:

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{14}$$

with ϵ the elasticity of substitution between different final goods and the aggregate price level given by: $P_t = \left[\int_0^1 P_{j,t}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$.

Therefore the demand function is given by:

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} Y_t \tag{15}$$

In each period there is an exogenous probability that a retailer is able to reset its price. If that is the case he will set the new price $P_{j,t} = P_{j,t}^*$ in order to maximize future expected profits. On the contrary he will keep the same price of the previous period ($P_{j,t} = P_{j,t-1}$). More formally he solves:

$$\sum_{i=0}^{\infty} E_{t-1}\left[\theta_R^i \Lambda_t^C \left(\frac{P_{j,t}^* - P_{t+i}^W}{P_{t+i}}\right) Y_{j,t+i}\right]$$

subject to the demand function given by equation (15).

The first order condition is:

[25] Dixit and Stiglitz [1977].

$$\sum_{i=0}^{\infty} E_{t-1}\left\{\theta_R^i \Lambda_t^C \left(\frac{P_{j,t}^*}{P_{t+i}}\right)^{-\epsilon} Y_{j,t+i}\left[\frac{P_{j,t}^*}{P_{t+i}} - \frac{\epsilon}{\epsilon-1}\frac{P_{t+i}^W}{P_{t+i}}\right]\right\} = 0 \tag{16}$$

with the aggregate price level given by:

$$P_t = \left[\theta_R P_{t-1}^{1-\epsilon} + (1-\theta_R)\, P_{j,t}^{*1-\epsilon}\right]^{\frac{1}{1-\epsilon}} \tag{17}$$

given that all firms are equal, they will choose the same price, so we can drop the index and call $P_t^* = P_{j,t}^*$. Combining (16) with (17) and log-linearizing around the zero inflation steady state of the model, we get a New-Keynesian Phillips curve that has the form of:

$$\pi_t = E_{t-1}\left(\beta\pi_{+1} - kx_t\right) \tag{18}$$

with π_t the inflation rate and $k \equiv \frac{1-\theta_R}{\theta_R}\left(1-\theta_R\beta\right)$.

2.2.3 Capital goods producers

Capital goods producers operate on a national market and make new capital using old (undepreciated) capital and a fraction of final output as input of their production process. They, therefore, are able to produce new capital using a technology described by:

$$f\left(\frac{I_t}{K_t}\right)K_t$$

this function can be seen also as the description of physical adjustments cost in the production process of new capital, and idea that is present in the literature since Kiyotaki and Moore [1997][26], with $f'(\bullet) > 0$, $f''(\bullet) < 0$ and $f(0) = 0$. In this way the production function has decreasing returns to scale in the short run and constant returns to scale in the long run. The price at which these agents sell the new capital is driven to Q_t^i[27] by the perfect competition assumption. Capital goods producers choose I_t in order to maximize expected profits. Their problem is given by:

[26] The presence of physical adjustment costs is widespread in the literature, for example can be seen Friedman and Woodford [2011], Dib [2010] and Angeloni and Faia [2009].

[27] New capital can be sold only on areas with new investments and the price of capital, due to the perfect competition assumption, is driven to its market value.

$$\max E_t \sum_{t=0}^{\infty} \beta^t \Lambda_t^C \left\{ Q_t^i f\left(\frac{I_t}{K_t}\right) K_t - I_t - \sum_{h=i,n} \left[\left(\overline{Q_t^h} - Q_t^h \right) (1-\delta) \pi_h K_{t-1} \right] \right\} \tag{19}$$

because households are the only owners of firms, the discount factor is given by Λ_t^C[28]. $\sum_{h=i,n} \left[\left(\overline{Q_t^h} - Q_t^h \right) (1-\delta) \pi_h K_{t-1} \right]$ defines the undepreciated capital acquired by capital producers and is the sum of the capital existing on each area in the previous period; $\overline{Q_t^h}$ is the price at which old capital is acquired on each area, that, by perfect competition, is driven to the price of capital on each area.

From the solution of this problem, it is possible to derive the following equilibrium condition:

$$Q_t^i = \left[f'\left(\frac{I_t}{K_t}\right) \right]^{-1} \tag{20}$$

the above equation has the role of a Tobin's Q equation[29] for the model and allows to endogenize the price of equity. It's log-linearized version is given by:

$$q_t^i = f\left(i_t - k_{t-1}\right) \tag{21}$$

2.3 Financial system

The financial system is populated by finitely lived banks each of them run by a banker. Each bank is endowed with an initial capital from households and collects deposits on a national market. If needed, banks can also access a national interbank market where they exchange funds between them.

With the funds raised each bank can provide loans to firms, acquire risk-less assets or supply loans to other banks on the interbank market.

It is worth remembering that banks can lend to firms only at local level, so each bank can lend only to firms located on its same area, while they can access national

[28] $\Lambda_t^C = \beta E_t \left(\frac{\lambda_{t+1}^C}{\lambda_t^C} \right)$ defines the households discount factor and is a function of the real marginal utility of consumption.

[29] Brainard and Tobin [1968], Tobin [1969].

markets for deposits and interbank loans[30].

Therefore, we can describe the assets of a bank as the sum of loans to firms, bonds and interbank loans; on the liability side, we have bank's capital, deposits and interbank loans:

Assets	Liabilities
Loans to firms	Capital
Interbank loans	Deposits
Risk-less Assets	Interbank debts

note that interbank loans appear on both sides of the balance sheet. In fact, they are an asset for banks with surplus of funds, that lend on the interbank market and a liability for those banks that borrow on the interbank market.

We also know that in each area of the economy there is an exogenous probability π^i that new investment opportunities arises for firms. If there are new investment possibilities on the area where the bank operates, it will face an higher demand for loans because firms, on top of refinancing already installed capital, want to borrow to invest.

However, the type of area on which a bank operates is revealed after that banks have chosen the optima level of deposits. Therefore, because banks will choose an average level of deposits, some banks (those on areas of the "i" type) will need more funds, while others (those on areas of "n" type) will have an excess of liquidity. This hypothesis reflect the fact that banks collect deposits before knowing the exact amount of the demand they will face and this is one of the reasons why exists an interbank market. On this market banks will be able to exchange funds to overcome their shortage or excess of liquidity.

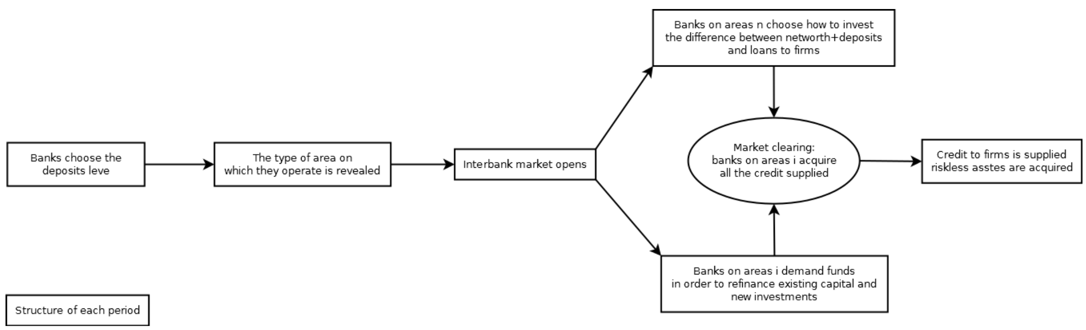
In a perfect, frictionless, market banks with excess of liquidity will borrow to banks on "i" areas and the total credit supplied by the banking system will be equal to the sum of bank's net worth and deposit, reaching the maximum level possible.

However, as we have seen in the last years, banks with excess of liquidity may decide not to borrow on the interbank market. Their behavior will be a key aspect of the model. As we will see later, lending banks on the interbank market can decide to invest their excess of liquidity on interbank lendings or on risk-less assets; when the economy faces a crisis, lending banks may decide to invest all their excess liquidity on

[30]This assumption follows what proposed in Gertler and Kiyotaki [2010].

risk-less assets, not financing any interbank loan and causing a reduction of credit to firms and so a worsening of the crisis.

What is important to understand now, is that only banks on areas without new investment decide to invest part of their funds on the interbank market or on risk-less assets, because, as long as there are investment projects to be financed, it is more convenient to invest on them[31]. This result will be proven later in the model. Before moving on, I believe, it might be useful to provide a description of the time line of the model:



Finally I assume that there is some degree of uncertainty on the returns of loans issued to firms on new investment island. What I am saying is that those returns are subject to a shock. This shock takes the form of a stochastic variable $x_{i,t}$ with $E(x_{i,t}) = 0$, $Var(x_{i,t}) = \sigma_x > 0$ that spans on an interval $[-h; h]$ with a uniform distribution and probability $\frac{1}{2h}$[32].

We know that banks pay dividends to households only when they end their activity, that can occur in each period with a probability σ[33]. When a bank exits the market, the banker transfers all the accumulated net worth to households; in all the previous periods, it is optimal for him to accumulate profits[34] to increase the bank's net worth. The objective of the banker, then, is to maximize the sum that he can transfer to households at the end of each period. Because he runs the bank, I assume that he can divert to households a part of the total funds intermediated by the bank that are equal to the total value of assets $\left(A_t^{h,B}\right)$ except for those acquired on the interbank market[35], leading to the default of the bank.

[31]This particular result is in line what with discussed in Stiglitz and Greenwald [2003].
[32]For the modelization of the shock I followed Angeloni and Faia [2009] and Diamond and Rajan [2001].
[33]This assumption is necessary to avoid that banks accumulate enough capital to become financially unconstrained.
[34]The banking system operates in perfect competition, for profits I intend the returns on the invested capital.
[35]I am assuming that private banks are more efficient in monitoring their counterparts than depositors.

The banker, in each period, decides to divert assets if, doing so, the value of the transfer he makes to household is higher than the value of the bank, given by the expected net worth at the end of the period$\left(n_{t+1}^h\right)$. We can simply define their objective function of a manager as:

$$V_t^h = \max\left\{E_t\left(n_{t+1}^h\right); \theta\left(A_t^{h,B}\right)\right\} \tag{22}$$

with

$$A_t^{h,B} = \begin{cases} Q_t^i s_t^i - b_t \quad for\ h = i \\ Q_t^n s_t^n + b_t + f_t \quad for\ h = n \end{cases}$$

with s_t^h the volume of loans to firms emitted by each type of bank whose value is Q_t^h, b_t the value of interbank loans (that are an asset for lending banks and a liability for borrowing banks), f_t the value of risk-less assets and θ the fraction of assets that bankers can divert.

$A_t^{i,B}$ defines the total divertible funds for banks on area "i", that are equal to the value of funds they intermediate minus interbank loans. As will be proven later on, those banks have on the assets side only loans to firms and will not invest on risk-less assets. In addition, given the assumption on the structure of the interbank market, banks can not divert the funds borrowed on it, so the divertible assets in this case are equal to the total value of loans minus the interbank loans acquired.

$A_t^{n,B}$ defines the values of divertible funds from bank on area "n", equal to the amount of funds intermediated. These banks use their funds to provide loans to firm, to acquire risk-less assets and to lend on the interbank market. Therefore, the total value of funds they intermediate is equal to the tonal value of their assets that is $Q_t^n s_t^n + b_t + f_t$. Finally, there is not index of areas on b_t and f_t. This is due to the fact that interbank loans are considered a liability for banks on areas "i" and an asset for those on areas "n", but they are the same asset, so there is no need for any index. Similarly, as I will prove, only banks on areas "n" will acquire risk-less assets, so there is no need, in this case, to use an index.

The expected net worth of each type of bank, at the end of the period, is given by the sum of the net worth at the beginning of the period plus the returns on the banking activity.

From equation (22) it is straightforward to see how arises an agency problem. In fact, the bank's creditors will not lend to the banker an amount of funds high enough to make $E_t\left(n_{t+1}^h\right) < \theta\left(A_t^{h,B}\right)$, because, in this case, the banker will divert a fraction θ of the active, the bank will default, and they will lost the loan. For this reason, we end up having an additional constraint on the bank's activity, that states that in each period the volume of funds "divertible" by the banker must be smaller than the expected value of the bank's net worth at the end of the period. As long as it holds, bankers will not divert any part of the funds they are intermediating. It is possible, so, to define two incentive constraints, one for each type of bank:

$$E_t\left(n_{t+1}^i\right) \geq \theta\left(Q_t^i s_t^i - b_t\right) \tag{23}$$

$$E_t\left(n_{t+1}^n\right) \geq \theta\left(Q_t^n s_t^n + b_t + f_t\right) \tag{24}$$

the last thing to do before formally analyze the decision problem of banks in each of the two possible state of the world is to formally define the bank's budget constraint, that is given by:

$$Q_t^i s_t^i = n_t^i + d_t + b_t \tag{25}$$

$$Q_t^n s_t^n + b_t + f_t = n_t^n + d_t \tag{26}$$

again, note that interbank loans are a liability for one type of banks and an asset for the other. Again, I could have allowed for risk-less assets in the budget of both banks but, as I said, it will be proven later on that only banks on areas "n" acquire those assets.

2.3.1 Banks on areas with new investment possibilities

Banks on these areas do not have enough funds to supply all the credit demanded by firms, and so they have to ask for interbank loans. We also know that there is a stochastic variables that affects the effective returns on loans to firms. As long as the constraint given by equation (23) is satisfied, bankers that operate on this type of areas will have the objective to maximize the expected net worth at the end of each period. We can, finally, formally define the net-worth at the end of each period for this type of banks as:

$$E_t\left(n^i_{t+1}\right) = E_t\left[\left(R^{i,K}_t + x_{i,t}\right)Q^i_t s^i_t - R^B_t b_t - R^D_t d_t + n^i_t\right] \tag{27}$$

with $R^{i,K}_t$ the expected returns on loans to firms between t and t+1 defined as $\frac{\left[Z_t+(1-\delta_K)Q^i_t\right]}{Q^i_t}$, d_t the deposits acquired by each bank and R^B_t the interbank interest rate between t and t+1. The objective function of this type of banks can be defined as:

$$E_t\sum_{t=0}^{\infty}\Lambda^B_t n^i_{t+1} \tag{28}$$

$$\Lambda^B_t \equiv (1-\sigma)\,\sigma^t\Lambda^C_t$$

$$E_t\left(n^i_{t+1}\right) = R^{i,K}_t Q^i_t s^i_t - R^B_t b_t - R^D_t d_t + n^i_t$$

because banks are owned by households, the discount factor of bank's profits is given by Λ^C_t.

The decision problem of the representative bank can be defined as the maximization of equation (28) under the constraints given by equations (23) and (25).

The solution to that problem leads to the following equilibrium conditions[36]:

$$R^{i,K}_t - R^B_t = 0 \tag{29}$$

$$R^B_t - R^D_t = \frac{\lambda^i_t\theta}{\Lambda^B_t\left(1+\lambda^i_t\right)} \tag{30}$$

[36] If we take the expectation it is true that $E_t\left(R^{i,K}_t + x_{i,t}\right) = R^{i,K}_t$.

$$R_t^{i,K}Q_t^i s_t^i - R_t^B b_t - R_t^D d_t + n_t^i \geq \theta\left(Q_t^i s_t^i - b_t\right) \tag{31}$$

with $\{\lambda_t^i\}_{t=0}^{\infty}$ the sequence of Lagrangian multipliers associated to the problem. It is trivial to show how the solution to the problem is a corner solution. Equation (29) defines an Euler condition for banks on areas with new investments, and defines the interest rate on the interbank market. As long as it is satisfied, banks on this type of areas will acquire all the interbank credit supplied. Equation (30) defines the riskless rate as a mark down on the interbank market rate. Combined with the previous equation it is also defined as a mark down on the real returns on investments. The last relation is the incentive constraint that holds with the equality if $\lambda_t^i > 0$. It is possible to show how that relation must hold with equality, in order to maximize banker's profits.

Proposition 1. *The incentive constraint must hold with equality to maximize profits.*

Proof. Assume that equation (31) holds with inequality. In this case, we have that $R_t^{i,K} = R_t^B$ and $R_t^B = R_t^D$ from equations (29) and (30). It is possible to compute profits for the bank as:

$$\Pi^i_{t|\lambda_t^i=0} = R_t^{i,k}Q_t^i s_t^i - R_t^B b - R_t^D d_t = R_t^{i,k}Q_t^i s_t^i - R_t^B\left(b_t + d_t\right)$$

using the budget constraint we get:

$$\Pi^i_{t|\lambda_t^i=0} = R_t^{i,k}Q_t^i s_t^i - R_t^B\left(Q_t^i s_t^i - n_t^i\right) = R_t^{i,K}n_t^i$$

On the contrary, if we assume that $\lambda_t^i > 0$, we have that $R_t^{i,K} = R_t^B$ but $R_t^B > R_t^D$. So, profits become (substituting the budget constraint into the profit function):

$$\Pi^i_{t|\lambda_t^i>0} = R_t^{i,k}Q_t^i s_t^i - R_t^B b - R_t^D d = R_t^{i,k}Q_t^i s_t^i - R_t^D d_t - R_t^B\left(Q_t^i s_t^i - n_t^i - d_t\right)$$

that can be simplified into:

$$\Pi^i_{t|\lambda_t^i>0} = \left(R_t^B - R_t^D\right)d_t + R_t^{i,K}n_t^i$$

because, according to equation (30), $R_t^B > R_t^D$ if $\lambda_t^i > 0$, $\Pi^i_{t|\lambda_t^i>0} > \Pi^i_{t|\lambda_t^i=0}$. So the incentive constraint must be biding in order to maximize profits. This proof is in line with the standard result of agency theory. □

Proposition 2. *Banks on area with new investment possibilities do not buy risk-less assets*

Proof. Consider the equilibrium conditions described by equations (29) and (30). They state that $R_t^{i,k} = R_t^B > R_t^D$. Therefore, banks on areas with new investment opportunities will not use their initial funds to acquire risk-less assets, because the expected returns on loans to firms are higher. Similarly, they won't borrow from the interbank market to buy risk-less assets, because they will incur in losses. Therefore banks on this area won't invest in risk-less assets, in line with the conclusions of Stiglitz and Greenwald [2003]. □

We can plug equation (31) into it the budget constraint to get to:

$$\tau_t^i n_t^i + \varpi_t^i b_t = \phi_t^i Q_t^i s_t^i \tag{32}$$

with $\tau \equiv \left(R_t^D + 1\right)$, $\varpi \equiv \left(R_t^D + \theta - R_t^B\right)$, $\phi \equiv \left(R_t^D + \theta - R_t^{i,K}\right)$. The previous equation defines the supply of loans to firms on areas with new investments. As it can be seen, it depends positively from the total amount of interbank borrowing received by banks on this type of areas. If $b_t = 0$, it is just defined as a mark-up on the bank's net worth at the beginning of each period.

2.3.2 Banks on areas without new investment possibilities

The analysis of the choice for banks that end up operating on areas without new investments is a bit more complicated.

In fact they can invest in loans to firms and banks or in risk-less assets. In addition, there is the effect of the stochastic shock that influences, as it will be explained later on, the returns on interbank loans. If the realization of the shock is sufficiently small, the debtor bank is not able to refund the total value of the loans received and it defaults. In that case, of course, the creditor will receive less or nothing of the original value of the loan. We can start analyzing how the variable $x_{i,t}$ influences the choice of lending banks. As long as the realized value of the shock is positive, so it falls in the interval $[0; h]$, banks on investment areas have no problem to pay back their debts. On the contrary, if $x_{i,t} \in [-h; 0)$, "borrowing" banks on the interbank market may not be able to refund all their creditors. If the shock falls in that interval, banks have to use part of the net worth accumulated in the previous periods to pay back their debts. As always, depositors are refunded first and only after them, with what is left, are refunded interbank loans.

From this assumption we can define a critical values of the shock. It defines the lowest value of $x_{i,t}$ that sill allows the debtor bank to refund all its creditors using all the net worth accumulated in the previous periods. We can define the first critical value of the shock as that value that drives to zero the bank's assets after that the creditors are paid:

$$0 = \left(R_t^{i,K} + x_{i,t}\right) Q_t^i s_t^i + n_t^i - R_t^D d_t - R_t^B b_t$$

we can solve for $x_{i,t}$ and call that value a_t:

$$a_t = \frac{b_t^i R_t^B - n_t^i + R_t^D d_t}{Q_t^i s_t^i} - R_t^{i,K} \tag{33}$$

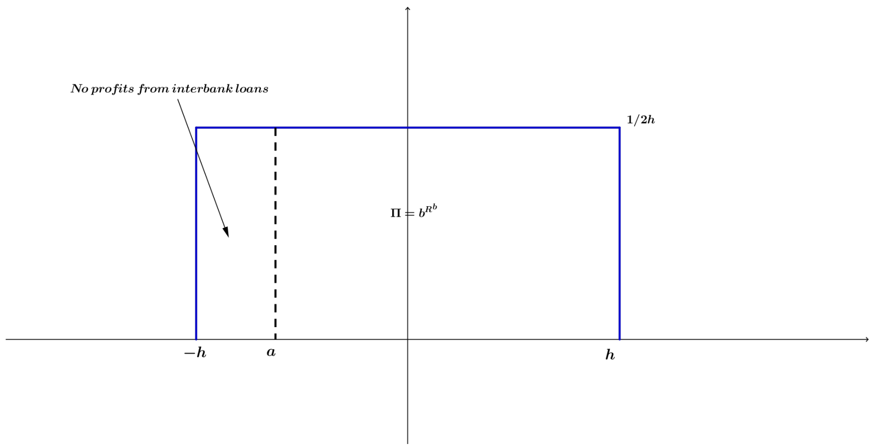
it is possible to see how this value grows as the cost of borrowing grows and decreases in the value of the assets of banks on investment areas and in the returns on the firm's capital.

Intuitively, banks with larger net worth and higher average returns are more robust to adverse shock, so the value of the shock a that leads to a default is lower. In parallel, the more a bank is leveraged (so the higher its deposits and interbank loans) the more easily also limited shocks can erode its net worth and lead to a default.

In case the realization of the shock is lower than a_t, the lending bank gets 0 and the borrowing bank defaults.

It is straightforward to define a payment distribution function for the lending bank, that describes the expected returns on interbank loans:

$$\mathcal{F}_t = \frac{1}{2h} \int_{-h}^{a} 0 dx_{i,t} + \frac{1}{2h} \int_{a}^{h} R_t^B b_t dx_{i,t} \tag{34}$$



Distribution of returns on interbank loans

We can now define the value of the net worth at the end of each period, for a representative bank on an area without new investment possibilities that is given by:

$$E_t n_{t+1}^n = n_t^n + R_t^{n,K} \psi_t Q_t^n s_t^n + \mathcal{F}_t + R_t^F f_t - R_t^D d_t \tag{35}$$

with R_t^F the risk-less interest rate that is equal to the interest rate on deposits.

Before moving on to analyze the optimization problem, it is useful to prove that banks on areas without new investment opportunities do not borrow money from the interbank market. This will allow us to clarify and simplify the optimization problem that will be presented later on. In fact, if a bank on these areas was interested in borrowing from the interbank market, it would contradicting equations (25) and (26).

Proposition 3. *Banks on areas without new investment do not borrow from the interbank market.*

Proof. Assume that a bank borrows from the interbank market. Its objective function would be:

$$\max E_t \sum_{t=0}^{\infty} \Lambda_t^B n_{t+1}^n \tag{36}$$

$$\Lambda_t^B \equiv (1-\sigma)\, \sigma^t \beta^t \Lambda_t^C$$

with

$$E_t n_{t+1}^n = n_t^n + R_t^{n,K} Q_t^n s_t^n + R_t^F f_t - R_t^D d_t - R_t^B b_t \tag{37}$$

we know that $n_t^n + d_t \leq Q_t^n s_t^n$. This bank, therefore, has sufficient funds in each period to finance loans to firms; in addition, given that in these areas there are not additional investment opportunities, the bank can not supply more credit to firms so is not interested in acquiring additional resources for that on the interbank market. Additional funds, so, can only be invested in risk-less assets.

In this case, the bank pays R_t^B *for sure* on each dollar acquired on the interbank market and gets R_t^F from its investment in bonds. But given equation (30) we know that $R_t^B < R_t^F$, therefore banks on these areas do not borrow from the interbank market. □

Coming back to the original problem, the objective function of this type o banks is given by:

$$E_t \sum_{t=0}^{\infty} \Lambda_t^B n_{t+1}^n \tag{38}$$

this function is to be maximized under the constraints given by equations (24) and (26). The first order conditions of the problem are:

$$R_t^{n,K} - R_t^F = 0 \tag{39}$$

this equation states that as long as the real interest rates paid on loans to firms on areas without new investments is at least as large as the risk-less rate, banks will supply all the credit demanded from the private sector. This means that as long as there is demand for loans, banks will use their liquidity to finance them and not interbank loans. The optimum condition on deposits is given by:

$$R_t^F - R_t^D = \frac{\lambda_t^n \theta}{\Lambda_t^B \left(1 + \lambda_t^n\right)} \tag{40}$$

with $\{\lambda_t^n\}_{t=0}^{\infty}$ the sequence of Lagrangian multipliers associated to the problem. This equation describes the relation between the risk-less rate and the deposit rate. In order to verify the non arbitrage condition, it follows that $\lambda_t^n = 0 \, \forall t$.

Until now we have derived the supply of loans, that is equal to the demand, for areas without new investments and the condition at which banks acquire deposits from households. It is necessary now to find what use banks on this type of areas make of their excess liquidity, that remains them after that they have served all firms. It is possible to derive the optimum condition for the supply of interbank loans, for a generic bank, that is given by:

$$\frac{1}{2h}\left[R_t^B h - R_t^B a_t - b_t R_t^B \frac{R_t^B}{Q_t^i s_t^i}\right] - R_t^F = 0 \tag{41}$$

from which follows that:

$$b_t^* = \frac{Q_t^i s_t^i}{2hR_t^B}\left[R_t^B\left(h - a_t\right) - R_t^F\right]$$

interbank loans are a fraction of total loans on areas with new investments. Their amount decrease as risk increases (the parameter a gets closer to h) and in the risk-less interest rate that is the opportunity cost of each dollar invested in the interbank market.

We can now compute the equilibrium value of risk-less assets held by each bank of this type from the budget constraint, that is given by:

$$f_t^* = n_t^n + d_t - Q_t^n s_t^n - b_t^* \tag{42}$$

at last, it is also possible to define the probability of a default(δ) to occur[37], simply as the probability of a shock to fall in the interval $[-h; a]$:

$$\delta_t = \frac{1}{2h}\int_{-h}^{a} 1 dx_{i,t} = \frac{1}{2h}\left(a_t + h\right) + \varepsilon_d \tag{43}$$

[37]For a default I intend a default on a loan, so also the case that a bank defaults on a loan but uses its net worth to refund the creditor.

as it can be seen, because a_t is a negative number, the probability of default increases as the dispersion of revenues increases. In addition, we can see how in each period the probability of default is defined by the financial conditions of debtor banks. ε_d is a shock on the probability of default that follows an AR(1) process.

2.3.3 Aggregation

Because the condition of banking and the choices of banks are different depending on the type of area on which they operate in each period, it is not possible to aggregate through all the areas, but we can do so only between banks on the same area.

We can start from banks on areas without new investments (capital letters define aggregate variables). As they are all equal between them, it is possible to aggregate through them, to get to the supply function of loans to firms, that is given by:

$$S_t^n = \frac{1}{Q_t^n} \left(N_t^n + \pi^n D_t - B_t - F_t \right) \tag{44}$$

with, according to equation (13), $S_t^n = \pi^n \left(1 - \delta_K\right) K_t$ and $N_t^n = \pi^n N_t$.

The total supply of interbank loans is given by:

$$B_t^* = \frac{Q_t^i S_t^i}{2hR_t^B} \left[R_t^B \left(h - a\right) - R_t^F \right] \tag{45}$$

and the total value of risk-less assets by:

$$F_t^* = N_t^n + D_t - Q_t^n S_t^n - B_t^* \tag{46}$$

on the other type of areas, the total supply of loans to firms is given by:

$$\tau_t^i N_t^i + \varpi_t^i B_t^* = \phi_t^i Q_t^i S_t^i \tag{47}$$

with, according to equation (13), $S_t^i = \pi^i (1-\delta_K) K_t + I_t$ and $N_t^i = \pi^i N_t$. It is easy to see how the value of the previous relationship is influenced by equation (45). In this way, the choices of lending banks directly influences the supply of credit to firms and, because firms relay completely on bank's loans to finance their activity, the level of output, employment and consumption of the system.

Before moving on to the next section, we must define a low of motion for bank's capital. In each period, on each type of areas, the bank's net worth at the end of the period is equal to the sum of the capital accumulated in the previous periods plus the gain or losses from the banking activity. At the end of the period, however, a fraction of banks quits and, therefore, the value of their capital is transferred to households who supply the initial capital for new banks. Therefore, at the beginning of the new period, the aggregate bank's net worth is given by the sum of the bank's capital at the end of the period, minus the capital of the banks that have quit plus the transfers from households to new banks. We can set up the equality:

$$N_t = N_{t-1}^o + N_{t-1}^y \tag{48}$$

with N_t the bank's capital at the beginning of each period, N_{t-1}^o the bank's capital of surviving banks and N_{t-1}^y the bank's capital of new banks, that is equal to the transfers received from households. It is straightforward to define the transfer as a fraction of total loans intermediated:

$$N_t^y = \xi \left\{ \pi^i \left[Z_t + (1-\delta_Q) Q_t^i \right] S_t^i + \pi^n \left[Z_t^n + (1-\delta_Q) Q_t^n \right] S_t^n \right\} \tag{49}$$

with ξ a positive parameter smaller than 1.

The capital of surviving banks is equal to the value of the capital of banks that survives from t-1 to t, that is given by $N_t^o = \sigma \left(N_t^{o,i} + N_t^{o,n} \right)$. The previous relation simply equals the value of net worth of surviving banks to the sum of the net worth on each area. These values, of course, are influenced by the effective returns on investments, and so by the realization of the stochastic shock. So, the net worth of banks on areas "i" is given by:

$$N_t^{o,i} = \frac{1}{2h}\int_{-h}^{a} 0 dx_{i,t} + \frac{1}{2h}\int_{a}^{h} \left[R_t^{i,K} + x_{i,t}\right] Q_t^i S_t^n dx_{i,t} + \pi^i N_t - \pi^i R_t^F D_t - R_t^B B_t \tag{50}$$

the first integer describes the case in which the bank defaults as a consequence of the realization of the shock. Similarly, it is possible to define $N_t^{o,n}$ as the sum of the capital at the beginning of the period plus the returns on the investments. Therefore, we can set up the equality:

$$N_t^{o,n} = R_t^{n,k} Q_t^n S_t^n - \pi^n R_t^F D_t + \Pi_t^B + R_t^F F_t + \pi^n N_t \tag{51}$$

with Π_{t-1}^B the profits on interbank loans that are defined as:

$$\frac{1}{2h}\int_{-h}^{a} 0 dx_{i,t} + \frac{1}{2h}\int_{a}^{h} R_t^B B_t dx_{i,t} \tag{52}$$

the value of bank's capital at the beginning of each period on each type of area is given by $\pi^i N_t = N_t^i$ for areas with new investment opportunities and $\pi^n N_t = N_t^n$ for areas without new investment opportunities.

2.4 Equilibrium

Equilibrium on the goods market is given by the well-known relationship:

$$Y_t = C_t + I_t + G_t + \varepsilon_g \tag{53}$$

if we plug in the optimal values of consumption and investments, we obtain an IS curve augmented for the adjustment costs. On the supply side of the good market, the supply is defined by the firms production function, given by equation (7), after having plugged in the optimal values of labor and capital. ε_g is a government spending shock that follows an AR(1) process.

The equilibrium on the credit market is granted by equations (13), (20), (45), (46), (44) and (47); while on the labor market by equations (4) and (9). The total volume of risk-less assets in the economy is given by the sum of deposits and government securities acquired by banks:

$$D_t^T = D_t + F_t \tag{54}$$

Finally, total deposits are given by:

$$D_t = \sum_{h=i,n} \left(Q_t^h S_t^h - N_t^h \right) \tag{55}$$

while the Central Bank follows a Taylor-type rule:

$$r_t^f = \psi_\pi \pi_t + \psi_y Y_t + \varepsilon_r \tag{56}$$

with ψ_π and ψ_y the sensitivity of the monetary policy rule to inflation and output and ε_r a monetary policy shock that follows an AR(1) process.

3 Data

To estimate the model we use a data set composed by 5 key macroeconomic variables. The data are U.S. quarterly time series for: the log difference of real GDP, real consumption, real investment, interbank lendings and inflation. All data are detrended and are related to the model by the measurement equations:

$$\begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlINT_t \\ dlP_t \end{bmatrix} = \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ b_t - b_{t-1} \\ \pi_t \end{bmatrix}$$

All data are obtained from the U.S. Department of Commerce - Bureau of Economic Analysis databank.

Real GDP Product is expressed in Billions of Chained 1996 Dollars. Nominal Personal Consumption Expenditures, Fixed Private Domestic Investment and Interbank Loans are deflated with the GDP-deflator. These variables are expressed per capita by dividing for the population over 16 and in 100 times log, following the procedure adopted also in Smets and Wouters [2003] and Smets and Wouters [2007]. All series are detrended and seasonally adjusted.

The inflation rate is expressed on a quarterly basis corresponding with their appearance in the model.

4 Estimation

The log-linearized version of the model can be written as:

$$AE_t\left(Y_{t+1}\right) = BY_t + CE_t$$

where A, B and C are matrices of parameters, Y_t a vector of endogenous variables and E_t a vector of the shocks of the autoregressive processes.

Using a Schur decomposition, it is possible to solve the model and write it in its state space form, that is:

$$X_{t+1} = \Lambda_1 X_t + \Lambda_2 \varepsilon_t \tag{57}$$

with Λ_1 and Λ_2 matrices of combinations of the deep parameters of the model, X_{t+1} the law of motion of states variables and ε_t a vector of shocks.

Finally the measurement equations can be defined as:

$$Y_t = \Lambda_0 X_t \tag{58}$$

Combining (57) and (58) it is possible to set up a Kalman filter to recover the likelihood of the model.

4.1 Bayesian Estimation

Following Gerali et al. [2010] and Darracq Paries et al. [2011], we estimate a subset of the deep parameters of the model.

We calibrate the households discount factor β to its standard value in the literature of 0.99. Capital depreciation δ is set to 0.025 while, following Gertler and Kiyotaki [2010], π^i is set to 0.25. The steady state default rate on the interbank market is set to 0.003 according to the long run average bank defaults provided by Moody's[38] while the

[38] For more details see Moody's [2009].

variance of the returns on investment projects h is set to 0.5 following the calibration proposed in Angeloni and Faia [2009] that is based on the estimation of returns carried on in Bloom et al. [2012]. ν is set to 0.6 following Gertler and Kiyotaki [2010]. σ, the surviving probability of banks, is set to 0.972 leading to an average surviving time of 10 years. The steady state ration of $\frac{C}{Y}$, $\frac{I}{Y}$ and $\frac{G}{Y}$ are calibrated to 0.62, 0.18 and 0.2 respectively. The steady state output to capital ration is set to 3, that is the average value in advanced economies. θ is set to 0.129 after Gertler and Kiyotaki [2010] and the Calvo parameter θ_R to 0.75 following the New Keynesian literature.

The remaining deep parameters of the model are estimated following the procedure proposed in Smets and Wouters [2007]. In particular, through the Bayes rule, the posterior distribution of the model's parameters Ψ is approximated to the likelihood times the prior distribution, according to the well-known relation:

$$p\left(\Psi \mid Y\right) \propto \mathcal{L}\left(Y \mid \Psi\right) p\left(\Psi\right) \tag{59}$$

with $\Psi = \left[\sigma^{\varepsilon_e},\, \sigma^{\varepsilon_A},\, \sigma^{\varepsilon_d},\, \sigma^{\varepsilon_r},\, \sigma^{\varepsilon_g},\, \rho_e,\, \rho_A,\, \rho_d,\, \rho_r,\, \rho_d,\, \alpha,\, \varphi,\, \varepsilon,\, f,\, \theta,\, \theta_R,\, \psi_y,\, \psi_\pi\right]^T$. Given that does not exists a close form solution for (59), the equation is evaluated with a MCMC algorithm repeated for 2 chains with 1'000'000 draws each. Convergence diagnostics can be found in the appendix.

4.2 Prior Choices

Following Smets and Wouters [2007], the standard errors for the shocks processes are assumed to follow an inverse gamma distribution with mean of 0.01 and a standard error of 2.

We select a beta distribution for the autoregressive components of the shock process with mean 0.5 and a standard error of 0.2. The technology parameter α is assumed to follow a beta distribution with mean 0.33 and a 0.05 standard error.

The parameters of the utility function, φ and ε, follow a beta distribution with mean of 0.5 and 0.1 and standard errors of 0.1 and 0.01.

f follows a beta distribution with mean 0.25 and standard error of 0.1. The mean for f is chosen according to the steady value of the same parameter proposed in Bernanke et al. [1999].

Finally the two policy parameters follows a gamma distribution with mean 0.8 and 1.5 and standard error of 0.2 and 0.5 respectively.

4.3 Estimation Results

Estimation results are reported in table 2 of the appendix. Results are generally in line with the previous literature. The sensitivity of the policy function to inflation is larger than to output; the technology and preference parameters are in line with other results in the literature and autocorrelation coefficients are generally high as in Smets and Wouters [2003] and Smets and Wouters [2007].

Surprisingly, f, the parameter associated with the log-linearized Tobin's Q equation, is much lower than what calibrated in Bernanke et al. [1999]. To check this result we performed a specific test on the identification strength that shows that, actually, that parameter is between the best identified in the model and strengthen our conclusion. The test is presented in the next section and its results are displayed in the appendix.

4.4 Robustness Check

A recurrent issue in the estimation of DSGE models is the weak identification of parameters. Lack of identification is generally due to a flat likelihood caused by a limited or absent curvature of the function in the area or by the fact that the function itself does not changes with the parameters. In order to verify the robustness of the estimation we apply a test for the identification strength based on the Fisher information matrix. The aim of the test is to verify that the likelihood function has indeed a significant curvature in the direction of the parameters.

In order to do so, we use the procedure developed in Andrle [2010], Iskrev [2010] and Iskrev and Ratto [2011]. The measure of sensitivity for a general parameter θ_i is given by:

$$s_i = \sqrt{\theta_i^2 / I_{i,i}\left(\theta\right)^{-1}} \tag{60}$$

with $I\left(\theta\right)$ the Fisher information matrix. If this is exactly equal to 1, it means that the likelihood function is flat and, therefore, the parameter is not well identify. An alternative approach is to normalize the identification measure using the prior standard deviation. This second specification is useful as a second check when the value of the parameter is close to 0 and the previous approach may lead to inaccurate results[39]. In this case the measure of sensitivity becomes:

$$s_i^p = \sigma\left(\theta_i\right) / \sqrt{I_{i,i}\left(\theta\right)} \tag{61}$$

[39] For more details see again Andrle [2010] and Iskrev [2010].

we produce both graphs (in log scale) to show how the parameters we have estimated pass those tests. Figure 3 in the appendix shows a graph with on the bars the absolute values of the test results.

Furthermore it is possible to use a similar procedure to check if the likelihood changes with changes in the parameters. The index we use are given by:

$$\Delta_i = \sqrt{\theta_i^2 I_{i,i}(\theta)}$$

$$\Delta_i^p = \sigma(\theta_i)\sqrt{I_{i,i}(\theta)}$$

the result of this test are reported in figure 3 of the appendix. As it can be seen all parameters are well identified.

Another reason why the parameters in the model may be weakly identified is because of a lack of information in the data. In order to verify the robustness of our findings with respect to this possibility we can exploit the information contained in the Jacobin matrix.

It is possible to measure "locally" the impact of each parameter on each element m_j of the moment vector using the derivatives of the Jacobiam matrix:

$$\frac{\partial m_j}{\partial \theta_i}$$

however, derivatives are not scale invariant, therefore we need to normalize them using the ratio of standard deviations: $\frac{std(\theta_i)}{std(m_j)}$. In this way we can take into account the differences in the uncertainty on each parameter and put more weight on parameters that affect more the likelihood leading to higher changes. In addition in this way we can compare the impact of each parameter on different volatile moments (see Iskrev and Ratto [2011] for an extensive proof). In particular, we will going to compare derivatives of the Jacobin matrices for: the moment matrix, the model solution matrix and the linear rational expectation matrix. Results are reported in figure 4 of the appendix.

Using the moment matrix we can see how well a parameter is identified due to the strength of its impact on observed movements while with the model solution matrix we see how well a parameter *could be* identified if all state variables were observed. Comparing these two results we can be sure of the presence of the information we need in the data set used. Figure 4 in the appendix displays the norms of $\frac{\partial m_j}{\partial \theta_i}\frac{std(\theta_i)}{std(m_j)}$ for each of the three matrices.

It is also possible to look for the best identified parameters in the model. Again we star from the Fisher information matrix of the model. We apply a Singular Value Decomposition of the matrix that leads to[40]:

$$Q_1' I(\theta) Q_2 = \Sigma \tag{62}$$

with Q_1 and Q_2 two square matrices of vectors and Σ a diagonal matrix of singular values σ_i. This decomposition is a standard way to determine the rank of a matrix and delivers us eigenvectors that associate each singular value with the estimated parameters. The rank r is equal to:

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > \sigma_{r+1} = 0$$

we know that if the Fisher information matrix is rank deficient, i.e. $r < \min\{m, n\}$, the model is not identified. Figure 5 and 6 in the appendix show the lowest and highest singular values with associated eigenvectors of parameters. A parameter associated to a singular value of 0 means that it is completely unidentified and, therefore, causes the deficiency in the rank of the matrix. Similarly, parameters associated with low singular values are more close to collinearity and, thus, more weakly identified. On the contrary, parameters associated with high singular values are the best identified in the model. For a more extended proof of these results see Andrle [2010].

Figure 2 and 3 deliver the following conclusion: i) there are not unidentified parameters in the model, ii) the parameter f is between the best identified in the model and, therefore, our result on its estimation is robust, iii) the variance of the exogenous shocks appears to be well identified as well.

5 Shock Responses

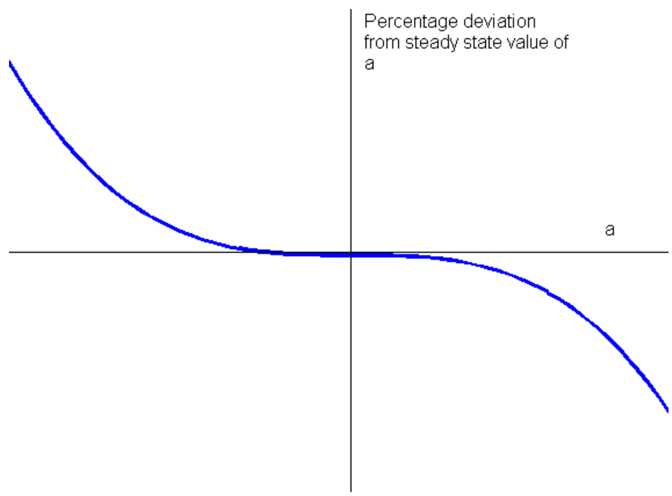
In this paragraph we will describe how the system answers to shocks of various nature focusing, in particular, on the supply of credit, aggregate production, investments, consumption and the stability of the financial system given by the average probability of default.

As in standard DSGE literature, a positive productivity shock boosts production and raises inflation even if, in the immediate, inflation declines due to the decrease

[40] Note that in this special case the Singular Value Decomposition delivers the same results of the Eigenvalues Decomposition.

in the hours worked and the consequent increase of the mark up. The Central Banks reacts accordingly, decreasing the interest rate in the first periods (in which the inflation effect dominates) and increasing it (on to counterbalance the increase in output and in inflation). As expected, the price of capital rises.

On the interbank market, the increase in the average returns boosts loans to firms on both types of areas. In particular, in areas with new investment opportunities this leads to a decrease in the value of a[41] and a consequent decrease in the probability of default.



Percentage deviation from the steady state as a function of a. As it can be seen, the deviation in 0 when a is exactly equal to its steady state value a^*. When $a > a^*$ the deviation from the steady state computed as $\frac{a-a^*}{a^*}$ is negative because a^* is negative. On the opposite, when $a < a^*$ the deviation is positive.

Banks are willing to lend more because, at the same time, their counterparts are less fragile, so the "risk" of lending is lower. This fact leads to a decrease in the interbank interest rate, that is a direct consequence of the decrease of the "zero profits area" described in equation (34). The combinations of all these facts lead to an increase in the supply of credit in the interbank market and a decrease of the interbank interest rate. These events lead to an increase in profitability for banks on areas with new investments and to a decrease of profits (driven by the reduction of the interbank interest rate) for banks on areas without new investment opportunities. Therefore, the

[41] Remember that a is negative in the steady state, so the percentage deviations of a, given by $\frac{a-a_{ss}}{a_{ss}}$ are positive if a is smaller than the steady state value and negative in the opposite case.

first type of bank will increase its net worth above the steady while for the other type of bank we observe a decrease of the net worth. Given the superior number of banks of the second type, the aggregate net worth will decrease.

What is most interesting, is that the effects of the increase average returns on capital and of the decrease of the interbank interest rate dominate the reduction of the aggregate bank's net worth, leading to a significant reduction of the default probability and, therefore, to a more stable financial system. The beneficial effect for the economy is clearly visible in the increase of investments that boost capital and output. However, banks in this context have less capital, so the stability of this particular configuration of the financial system is aleatory. If a shock hits the interbank market or, for any exogenous or unpredictable reason, some banks default, the financial system is less able to absorb losses and, therefore, is still fragile to a purely financial shock. Note that the significant (in magnitude) decrease in the default probability is due to the fact that the steady state for that variable is very small, equal to 0.003.

A consumers' preference shock decreases the marginal value of consumption and, therefore, the actual level of consumption. This dynamic is not compensated by a sufficiently large increase in the investments leading to a fall in aggregate output. At the same time, through the labor demand equation (9), the real wage increases pushing up marginal cost for firms and rising inflation.

However, the change in investment is positive and long lasting, leading to an increase in the price of assets. This affects the total supply of credit, that goes up, thanks to the reduction of a, that captures the riskiness on the interbank market. Again, a goes down affecting equation (34) and banks' profits. The decrease in a increases the supply on the interbank market that boosts the total supply of credit. In addition it also reduces the interbank market interest rate. As observed before, banks on areas with new investment opportunities increase their profits and, consequently, their net worth. The opposite is true for banks operating on the other type of areas. This, on the aggregate, leads to a decrease in the total net worth of banks.

However, the sharp decrease in a reduces the default probability of banks. This dynamic is similar to what observed in the case of a positive total factor productivity shock, with the difference that, in this case, the decrease in the default probability is smaller, as a consequence of a smaller decrease of a.

The consideration expressed in the previous case apply also in the case under consideration.

We can now move to the analysis of the last real shock, an increase of government expenditure. In this case, this model departs slightly from the standard literature featuring a (very) small reduction of inflation following a government spending shock. This fact, that will be explained more in details later on, is due to the combined effect of habits, investments rigidity and of the complex financial system.

A positive government spending shock increases total demand, leading to an increase of production. To increase production firms demand more labor, increasing the total hours worked and decreasing the real wage. This leads to higher profits and to an increase of the marginal utility of consumption. To go back to the steady state, given the habit formation assumption, consumers consume more reinforcing the mechanism. At the same time, the mark up rises and, so, inflation decreases. The combined effect of a decrease in inflation and a rise in output leads to a decrease of the Central Bank's target rate.

At the same time, due to the reduction in the real wage, firms start to substitute capital with labor leading to a decrease in the demand of investments. This decrease leads to a reduction of the price for new capital affecting the value of a. In this scenario a goes above its steady state value, according to equation (33). As a consequence the "zero profit area" described in equation (34) increases. The consequence for the interbank market are easy to predict: the interbank interest rate increases while total supply of interbank loans decreases. As a consequence, the budget constraint for banks on areas with new investment opportunities becomes tighter and, so, the supply of loans to firm decrease. On the opposite with respect to what we have seen in the previous case, here are the banks that operate on areas without new investment opportunities that increase their profits, due to the higher returns they are able to obtain from interbank loans. On the contrary, banks that operate on areas without new investment opportunities are not able to finance enough new loans to the private sectors, due to the tighter liquidity constraint they face. In addition, the decrease in the value of assets decreases their capital gains on loans to firms. However, on the aggregate, total net worth of the banking system increase because, as always, the increase of net worth of banks on "n" areas offsets that of banks on "i" areas, due to the larger number of the firsts.

Finally the default probability increases, even if slightly. This is due, of course, to the increased value of a.

We can now turn to pure financial shocks: an interest rate shock and a default probability shock.

A positive interest rate shock slightly increases λ^C leading to a small increase in consumption that, however, dies out quickly over time. As in standard New-Keynesian literature it reduces output and inflation. Both these phenomenon can be explained by the increase in the mark-up and the decrease in capital, investment and real wages.

In fact, as the risk less rate increases for banks becomes more convenient to invest excess liquidity in risk less asset rather than on the interbank market. Therefore the supply of interbank credit decreases and liquidity constraints become tighter for banks on areas with new investments. This leads to a decrease of loans granted on both types of areas due to the tighter constraints and, therefore, the reduction of available funds. The immediate consequence is the reduction of investments and capital. Following equation (20), a decreased level of investments leads to a lower price of capital, reducing capital gains for banks and, therefore, worsening credit conditions. Again, we see how banks on areas with new investment opportunities face a decrease of their net worth, while the opposite is true for banks on areas without new investment opportunities. The total net worth of banks still increases, even if less than in the previous cases, due to the reduced contribution of banks on areas of type "n".

As a result of the dynamics analyzed so far, the variable a increases above its steady state value. Again the "zero profits area" increases and, therefore, the probability of default increases. What is interesting now is that we have an additional channel with which monetary politics affect the real economy. Through equation (33) the decisions on the risk less interest rate affect the total supply of interbank credit and, therefore, the total amount of loans to firms. An expansionary policy reduces the value of a allowing for a larger supply of credit to the economy and, at the same time, reduces the default probability on the interbank market, contributing to the stability of the system. This, of course, comes at the price of an higher inflation rate and a lower level of banks' capital. The increase in the inflation rate is something standard in this type of literature; in this context the real source of concerning is the lower level of banks' capital induced by an expansionary policy. Despite, in fact, it reduces the probability of default it leaves the system more fragile to additional exogenous financial shocks. However, the evidences of this model suggest that in times of financial turmoil, reducing the interest rate (when possible) may benefit both the real and the financial sectors.

The last type of shock we have considered is a pure riskiness shock affecting equation (33). Namely this shock can be thought as an increase, for any exogenous reason, of the riskiness of banks that access the interbank market to acquire funds. For example it may be seen as a deterioration of their reserves or of the value of their balance sheet assets or any other type of shack that affects their solvency.

A positive shock on the value of a shifts the level of the threshold to the right, making banks more fragile to shocks on the returns of their investments. As a result, the default probability increases, for given levels of the fundamentals of the banking system. On the contrary, a negative shock on a shifts the level of the threshold to the left, decreasing the "zero profits area" and, therefore, making banks more robust to unexpected shocks on the returns. These particular dynamics are a consequence of the fact that a is negative in the steady state and that the fragility of the banking system is negatively correlated with it.

In our model we simulated a negative shock on a. As expected, the default probability decreases significantly; this is not a surprise if we remember how it is defined in equation (43). What is surprising is that, despite it generates volatility in consumption, through the changes in its marginal utility, a decrease in the average default probability is connected to a decrease in output, a reduction in real wages and a limited increase in inflation.

The main reason behind this is related to the drop in investments. In fact, as a increases the returns and the total volume of interbank lending decrease following equations (30) and (40). This fact reduces the total amount of credit extended to firms and, with that, the volume of investments. As investments fall, Q^i decreases, worsening the condition of banks on investment areas and reducing even more the supply of interbank loans. The combine effect of these processes reduce the total demand and, as a consequence, output drops. The Central Bank should than intervene reducing the risk less rate, making than relatively more convenient to invest on interbank loans, however this do not occurs.

The explanation for that may be found on the goods market. As investment drops, firms are forced to increase the labor share in the production process. Doing so, hours worked increase while wages decrease. At the same time, however, firms produce with an inefficient combination of inputs, therefore marginal costs increase and inflation follows. According to the Taylor principle, equation (56), the Central Banks balances output and inflation when choosing the policy rate, in this case the change in inflation offsets the change in the output and, therefore, leads to an increase in the interest rate, worsening the crisis.

The main point that can be drawn from this experiment is that the equilibrium is delicate. An excess of stability may reduce the returns on interbank lending, leading to a drop in vestment and output. However, too much riskiness produce a similar effect, but of greater magnitude. Policy maker should consider both side of the problem in the difficult task of searching for the equilibrium between output growth and financial stability.

6 Conclusions

In this paper we integrated into a standard DSGE model with frictions a complex financial system. The economy is populated by banks that are affected by demand shocks, shocks on the returns of investment and are financially constrained. They interact on a complex interbank market to exchange funds between them but doing so they become exposed to the risk of default. The key feature of the model is that banks with excess of liquidity have an alternative option (constituted by the acquisition of risk free assets) to the investment into the interbank market. When the conditions on the interbank market become critical, banks with excess funds simply divert them to assets that grant safe returns, shielding them self from the losses connected to potential defaults. However, this comes at the price of lowering the total amount of funds that the banking system is able to transfer to firms, thus decreasing investment and capital.

The analysis of the behavior of each bank allows to identify the causes of the risk of default and a measure of global financial stability, defined as the average probability of default, for each bank, in a given moment.

With this result it is possible to connect the business cycle and the stability of the financial system trough a micro-founded model and to observe, along with the fluctuations of output or inflation how the financial system changes its average characteristics of risk.

The simulation of different types of shock shows how generally the default probability of each bank and the business cycle comoves. In particular, an increase in the default probability increases the riskiness of interbank lendings and leads banks to divert funds to risk less assets. This triggers a decrease in investment and a fall in output. In addition, a standard Taylor principle might present an additional channel in which it can affect the real economy.

In fact, if output and inflation drop, the Central Bank will try to stimulate the economy lowering the interest rate, this had the (positive) side effect of reducing the opportunity cost of investing on the interbank market and, therefore, causes an increase of interbank loans and credit to firms. This simple principle appears to be on one hand, robust to the expansion of the standard macroeconomic model to the inclusion of a complex financial system and, on the other, seems to suggest the presence of additional channels of transmission of monetary policy not studied so far.

However, our simulations show also how the equilibrium in this type of framework is hard to achieve. In fact, on the contrary, as the monetary policy tries to fight inflation it also generate a worsening of the stability of each bank, increasing the average default probability. In addition, we have shown how shocks on the fundamentals of banks leads into a difficult situation where the economy shrinks but, at the same time, the central bank alone is not able reduce the riskiness but, on the contrary, it might even worsen the situation. This model, therefore, calls for alternative instruments of monetary policy, to be used in extraordinary circumstances, that can achieve the results that, in some cases, a simple monetary policy rule is not able to obtain. For example, in the case under consideration, a temporary increase in of the robustness of banks balance sheets (for example trough refinancement operations) seems more appropriate to increase the value of the assets of banks, reduce their probability of default and increase the amount of credit granted by the financial system.

In conclusion, this model shows the complex interactions that exist between the financial market and the real economy. Financial agents, especially in times of crisis, are particularly sensible to the riskiness of their counterparts. With this model we have analyzed an economy where counterpart risk is take explicitly into consideration and its connection to the real economy is clarified. In this context, the standard Taylor principle seems unadeguate to counterbalance real shocks while it presents some limits when it comes to pure financial shocks. In particular, in the case of riskiness shocks, the application of the Taylor principle may even worsen the crisis and, therefore, in those circumstances our model calls for the implementation of different and unconventional policy instruments.

Appendix A: Parameters Values

Table 1: Calibrated Parameters

Parameter	Name	Value
β	Discount Factor	0.99
δ	Depreciation Rate	0.025
$^{C}/_{Y}$	Consumption to Output	0.62
$^{I}/_{Y}$	Investment to Output	0.18
$^{G}/_{Y}$	Government Spending to Output	0.20
π^i	Probability of new investments	0.25
π^n	Probability of absence of new investments	0.75
σ	Bank's surviving probability	0.972
h	Dispersion of Returns	0.5
$^{Y}/_{K}$	Output to capital ration	3
θ	Degree of financial friction	0.129
θ_R	Calvo parameter	0.75

Table 2: Estimated Parameters

Parameter	Prior Shape	Prior Mean	Post. st. Dev.	Post. Mean	10%	90%
σ^{ε_e}	Γ^{-1}	0.01	2	15.6631	7.1084	24.1765
σ^{ε_A}	Γ^{-1}	0.01	2	5.9798	2.9871	8.8270
σ^{ε_d}	Γ^{-1}	0.01	2	22.3271	19.9628	24.6378
σ^{ε_r}	Γ^{-1}	0.01	2	1.1123	0.6661	1.5642
σ^{ε_g}	Γ^{-1}	0.01	2	2.6648	2.4200	2.9009
ρ_e	β	0.5	0.2	0.9960	0.9929	0.9992
ρ_A	β	0.5	0.2	0.9875	0.9806	0.9947
ρ_d	β	0.5	0.2	0.9843	0.9722	0.9967
ρ_r	β	0.5	0.2	0.9623	0.9384	0.9876
ρ_d	β	0.5	0.2	0.9936	0.9888	0.9987
α	β	0.33	0.05	0.3788	0.2996	0.4571
φ	β	0.5	0.1	0.7420	0.6810	0.8043
ε	β	0.1	0.01	0.1007	0.0842	0.1172
f	β	0.25	0.1	0.0385	0.0230	0.0540
ψ_y	Γ	0.8	0.2	0.2456	0.1346	0.3527
ψ_π	Γ	1.5	0.5	2.5746	1.5167	3.5706

Appendix B: Figures

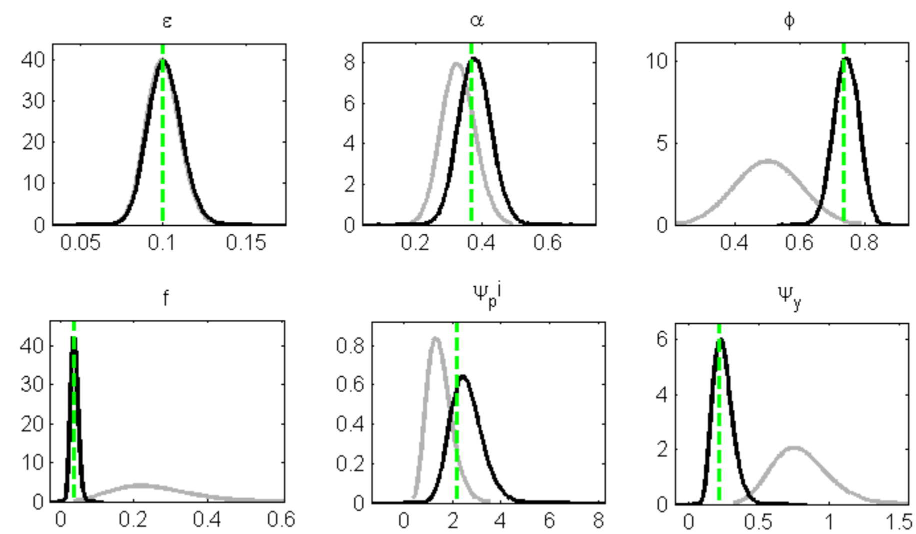


Figure 1: Prior and Posteriors, deep parameters.

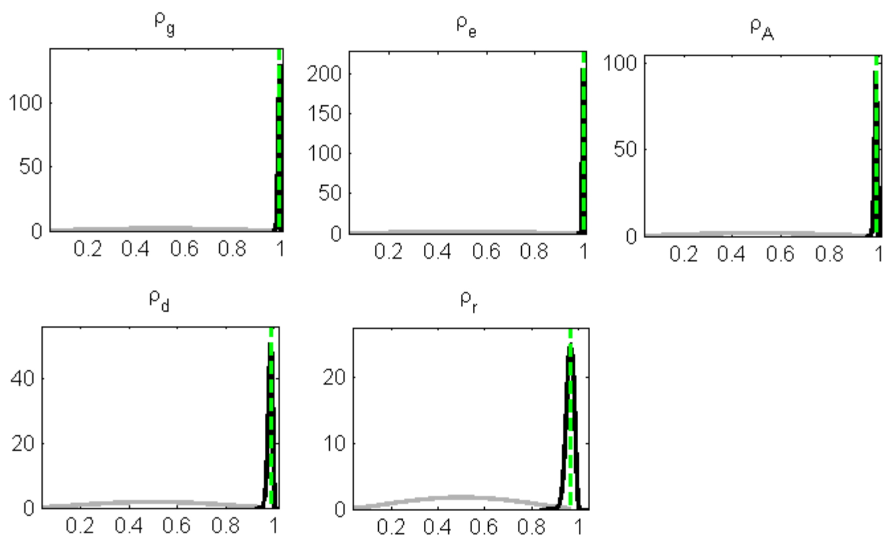


Figure 2: Prior and Posteriors, autocorrelation coefficients.

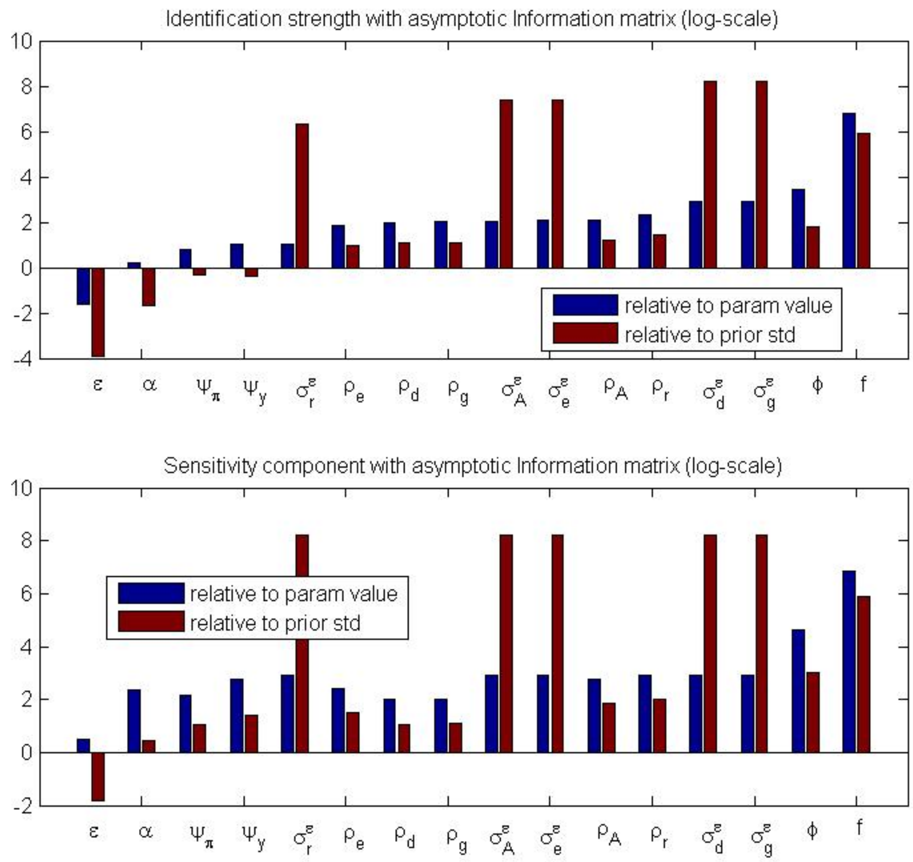


Figure 3: identification tests. Upper panel: results of the test based of equation (60); blue bars represent normalization based on prior mean and red bars on standard deviation at the prior mean. Lower panel: results of the test based on equation (61); blue bars represent normalization based on prior mean and red bars on standard deviation at the prior mean. None of the two tests shows signs of weak identification as the absolute values of the indicators is always positive.

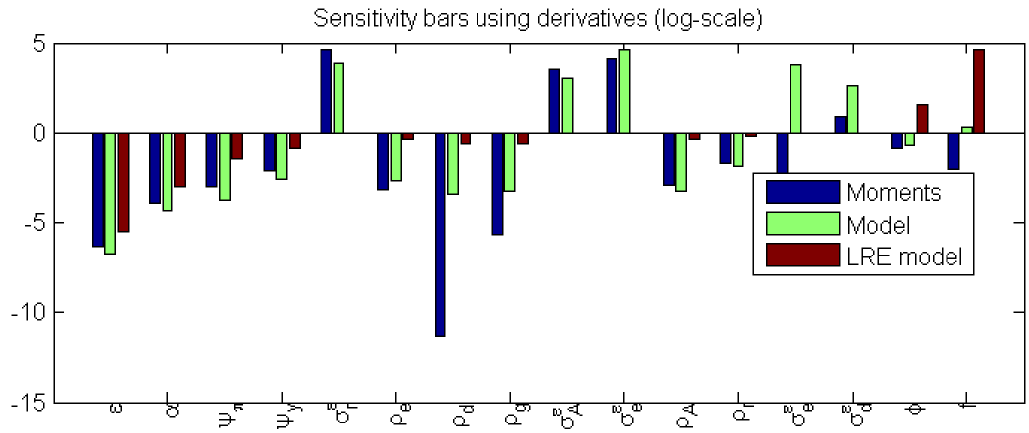


Figure 4: moments derivatives. Bars represent the absolute values of (weighted) derivatives of different Jacobin matrices. They prove how the data are informative for the estimation of the parameters of the model.

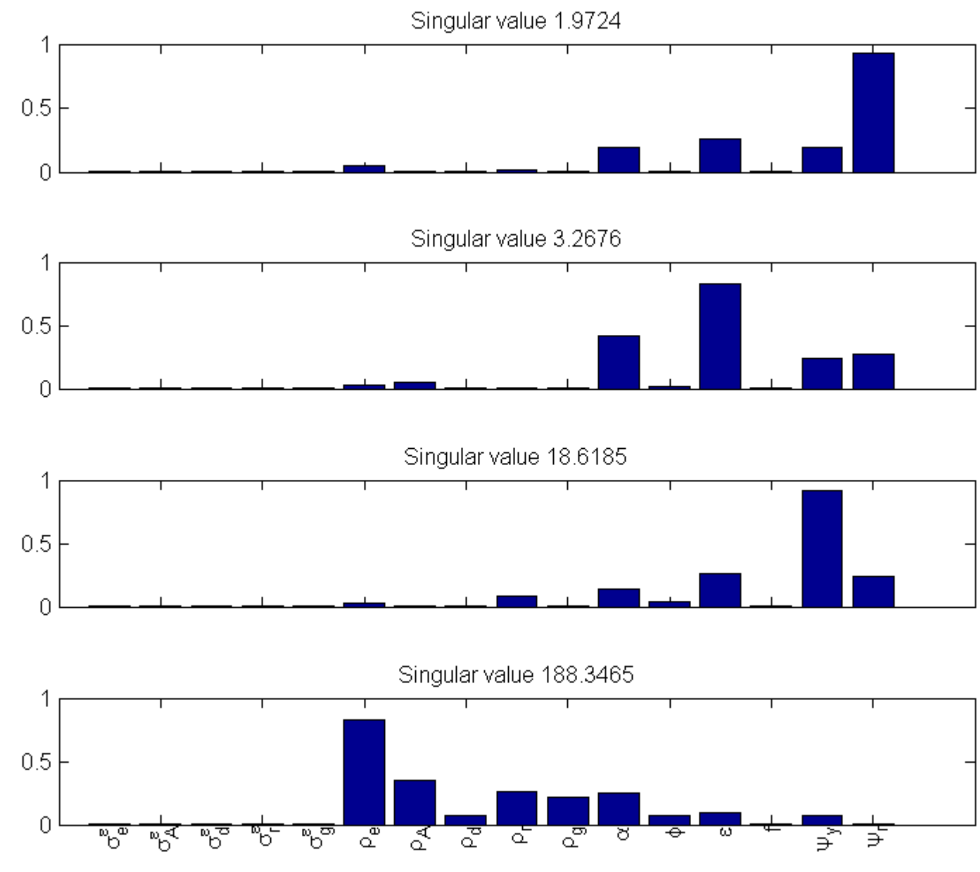


Figure 5: low singular values. 5 lowest singular values computed after equation (62) and associated vectors.

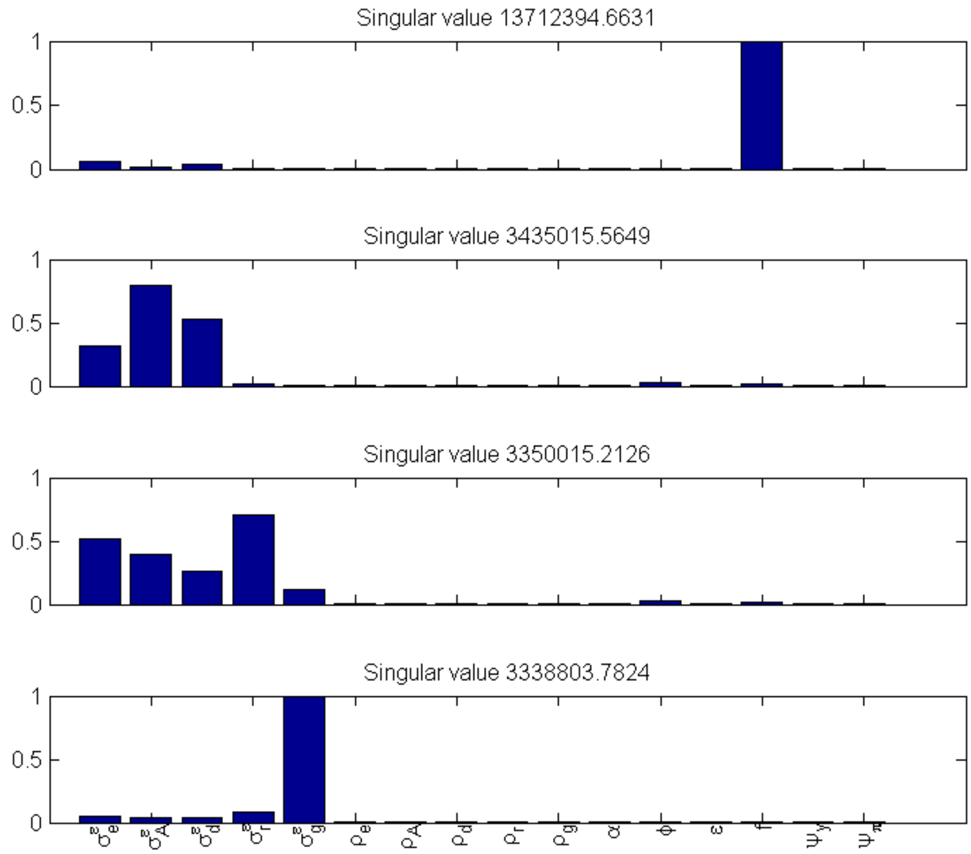
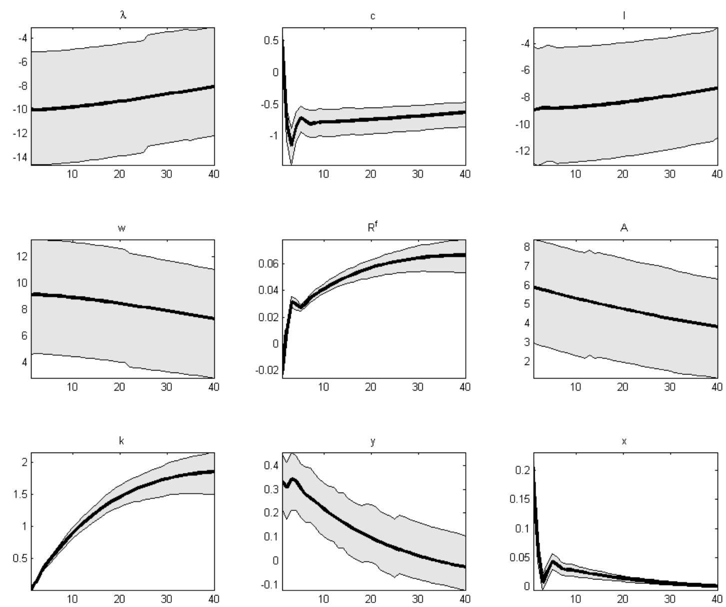
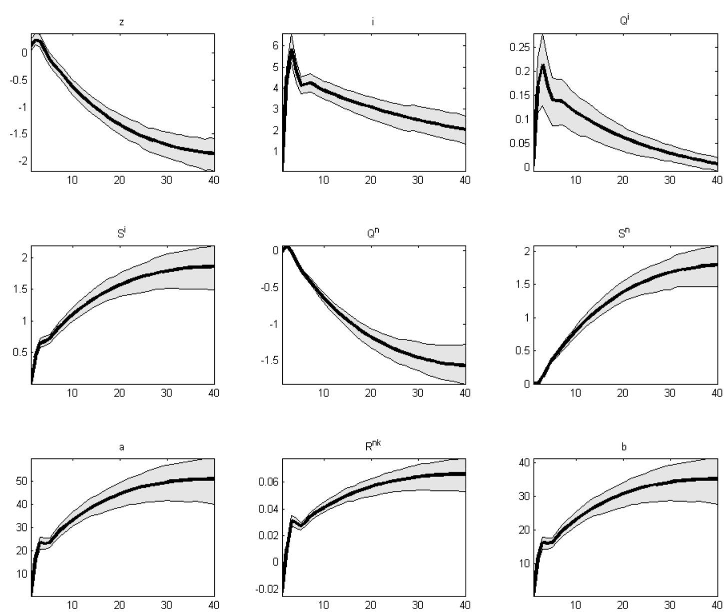
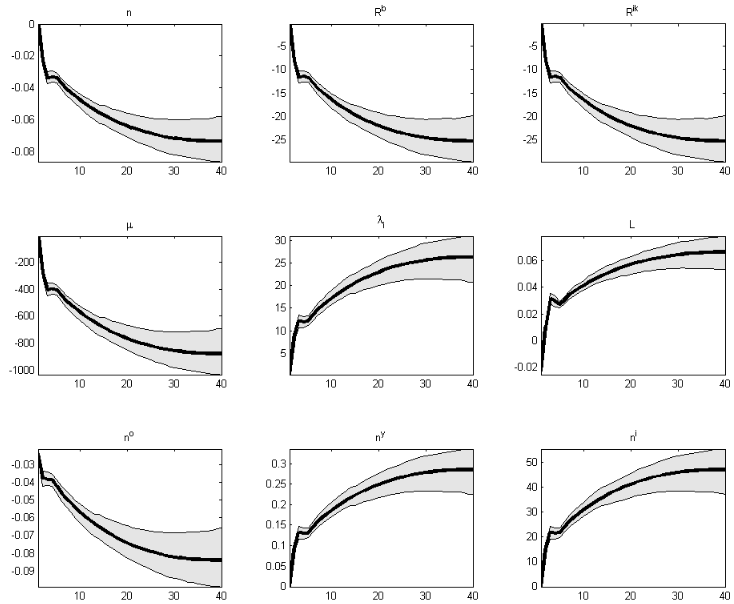


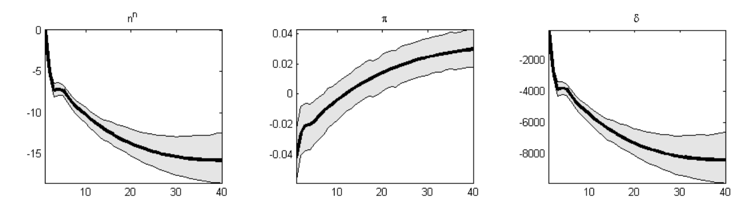
Figure 6: high singular values. 5 highest singular values computed after equation (62) and associated vectors.

Orthogonal shock to A

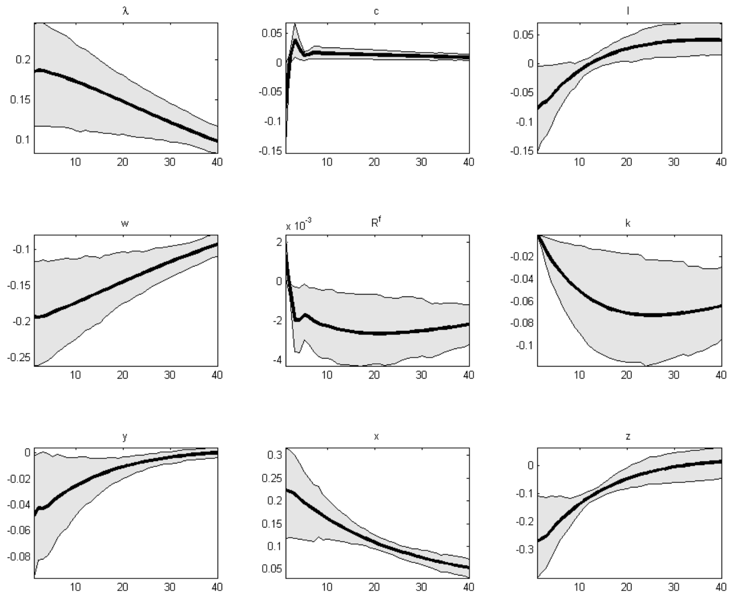


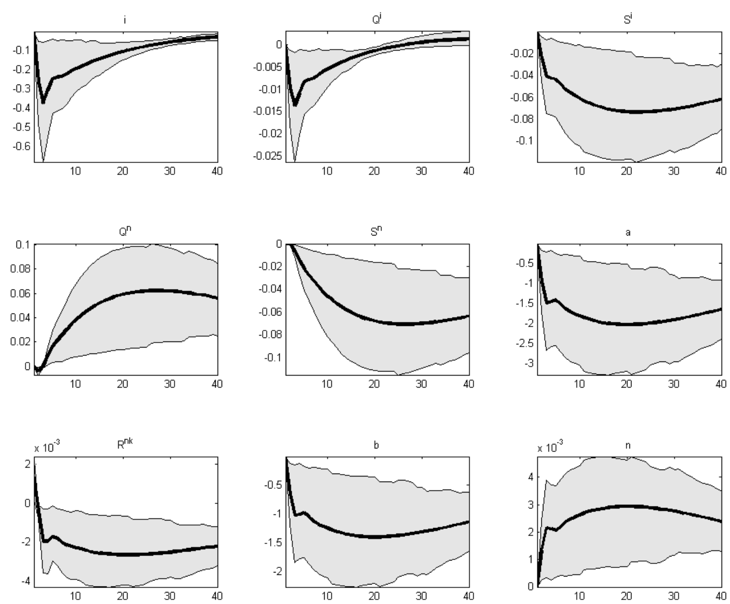


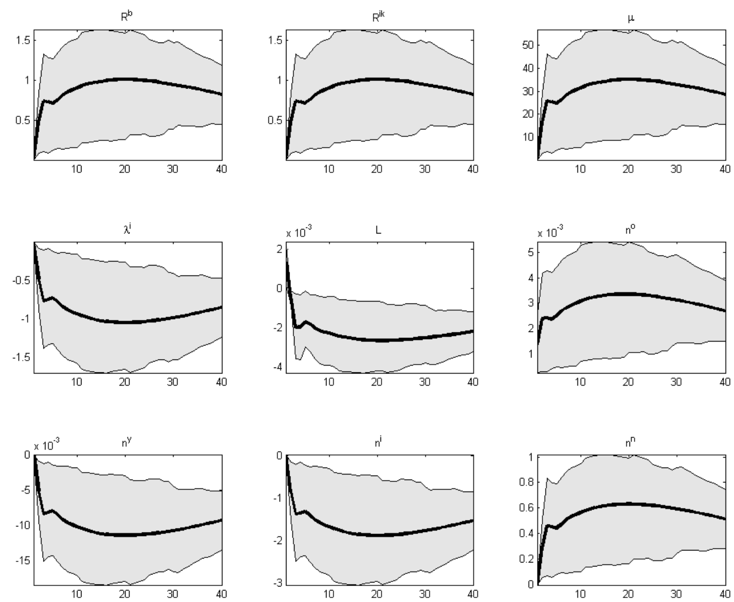

n
R^b
R^lk
μ
λ_l
L
n^o
n^y
n^i


n^n
π
δ

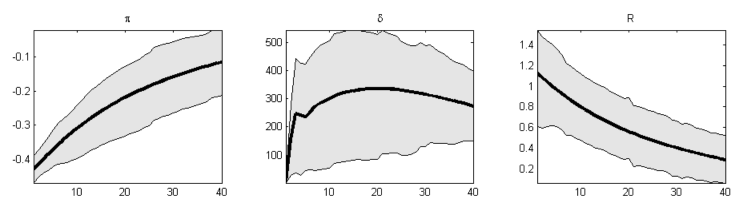
Orthogonal shock to R^f





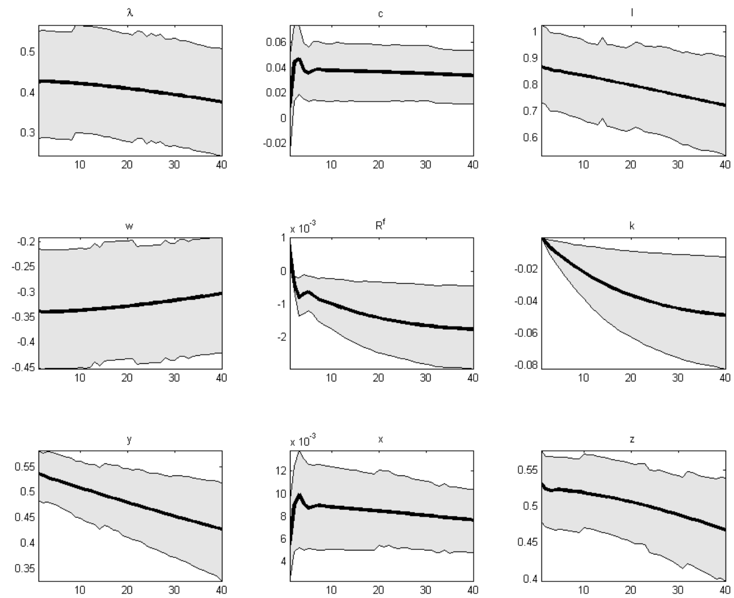


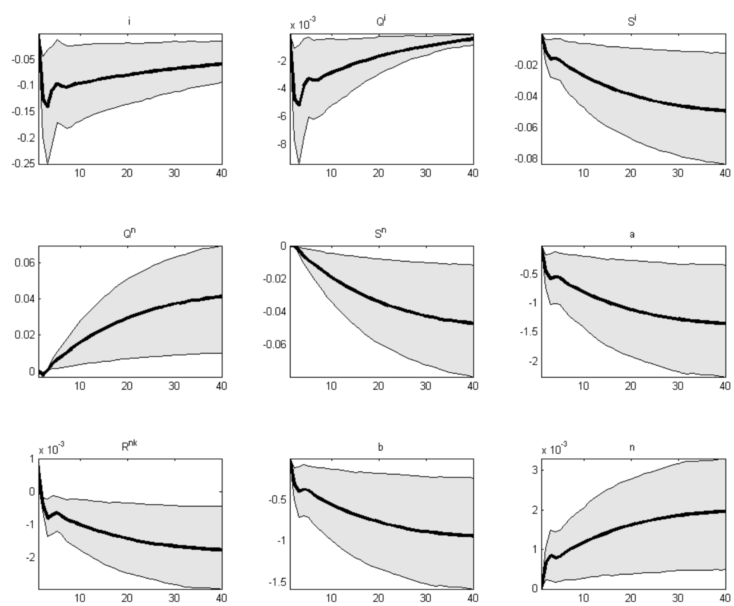
R^b
R^ik
μ
λ^i
L
n^o
n^y
n^i
n^n

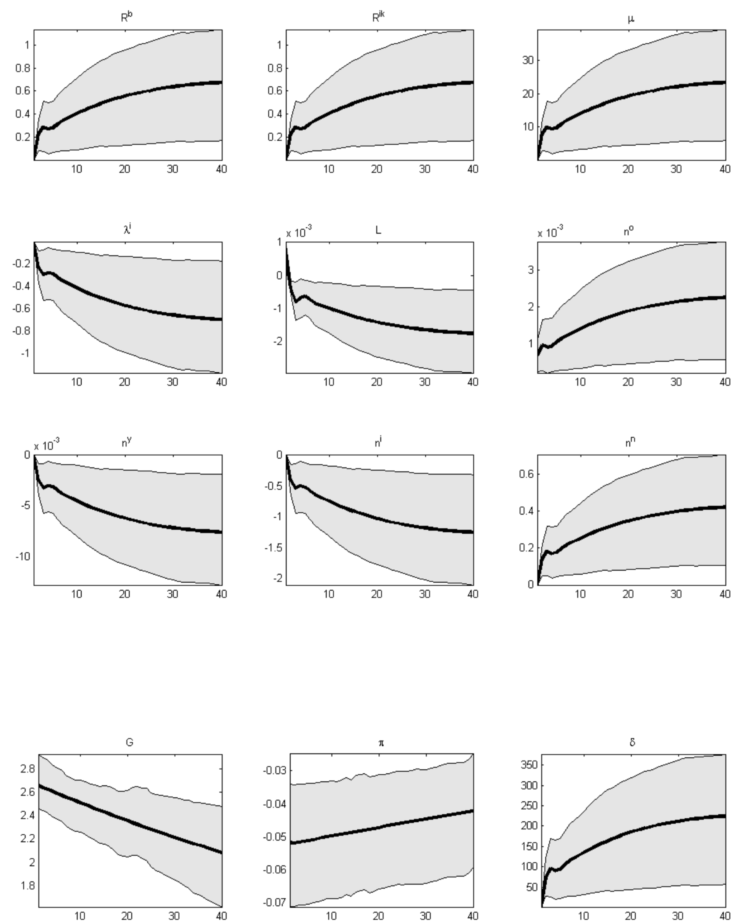


π
δ
R

Orthogonal shock to G

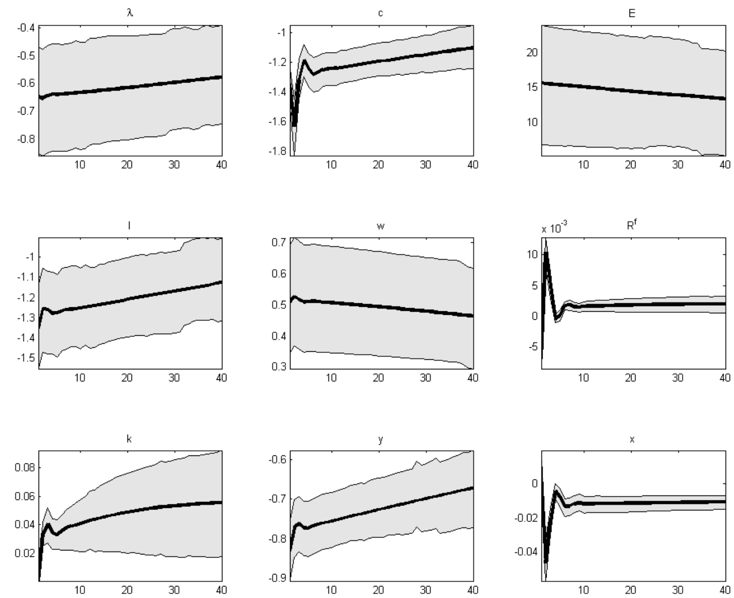


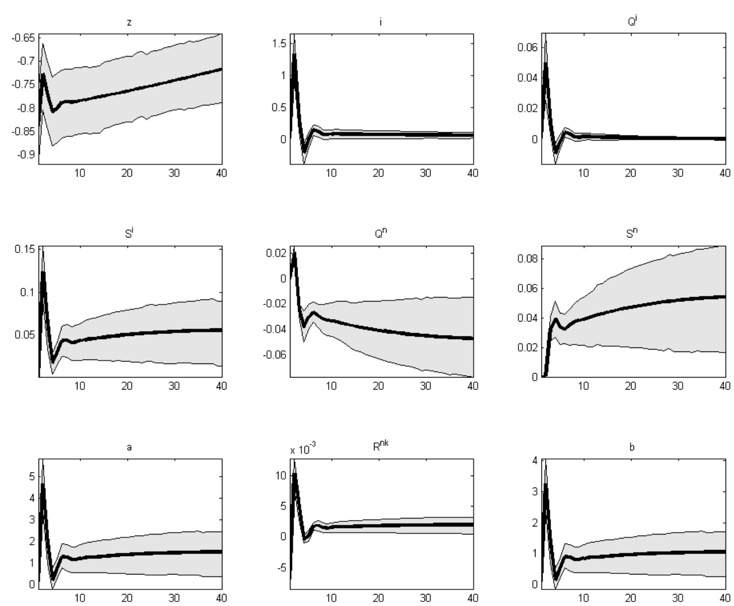


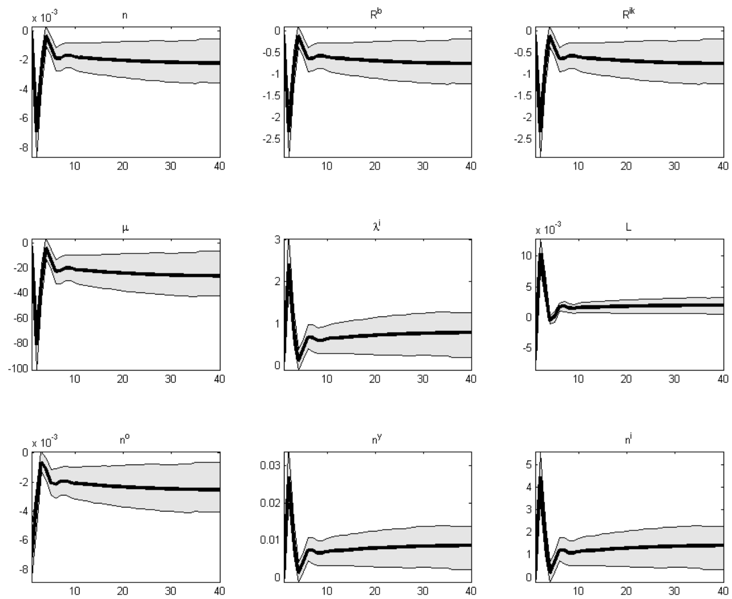


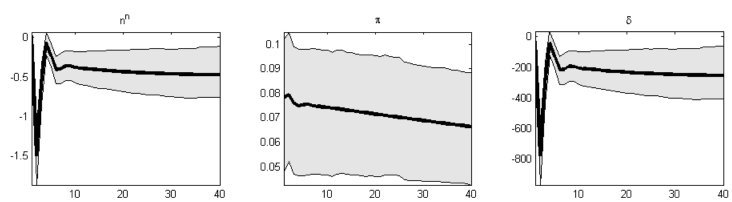
R^b
R^ik
μ
λ^i
L
n^o
n^y
n^i
n^n
G
π
δ

Orthogonal shock to e

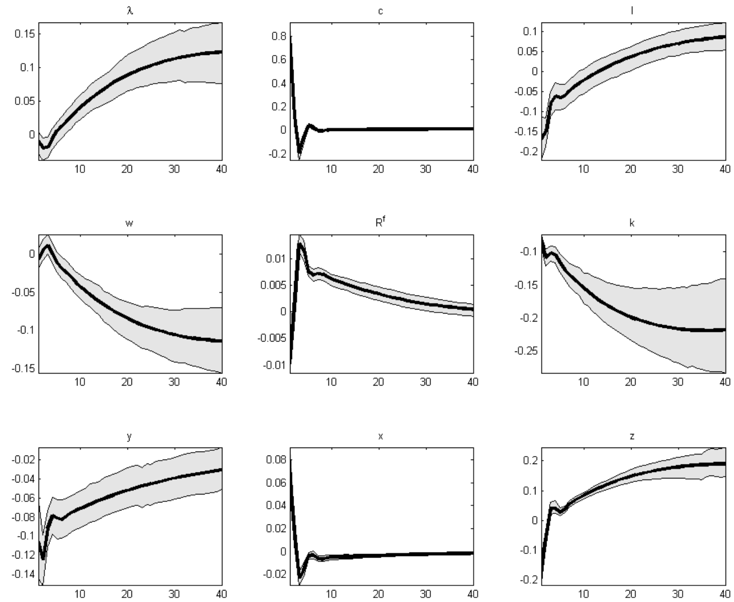


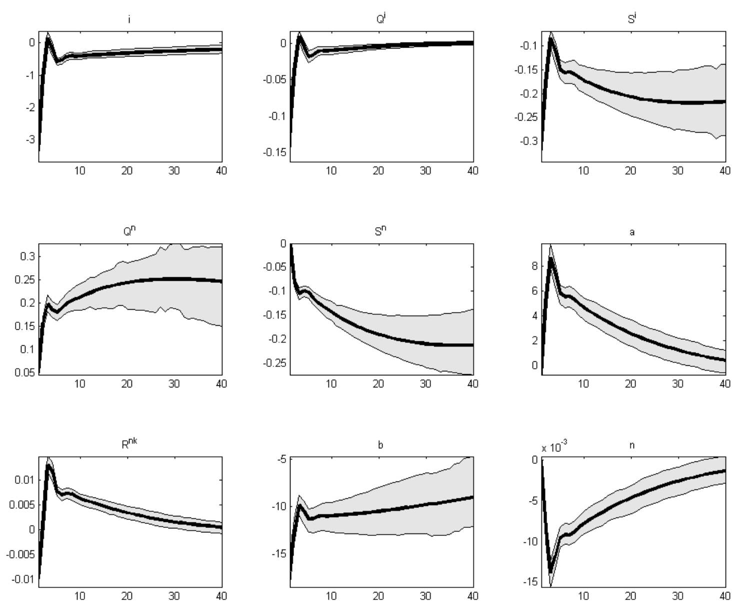


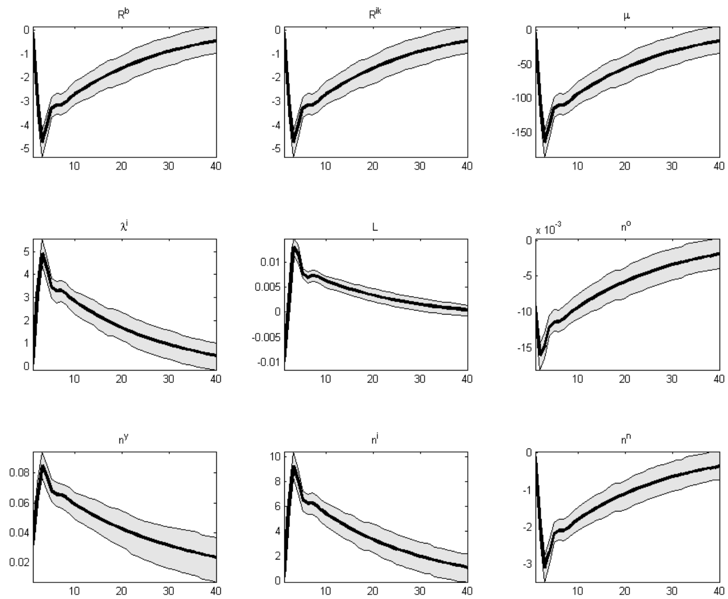

n
R^b
R^ik
μ
λ^i
L
n^o
n^y
n^i


n^n
π
δ

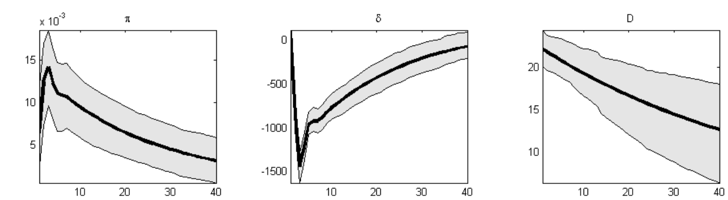
Orthogonal shock to a







R^b
R^lk
μ
λ^i
L
x 10^-3
n^o
n^y
n^l
n^n



x 10^-3
π
δ
D

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