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Factor Price Differences in a General Equilibrium Model of Trade and Imperfect Competition

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Abstract

Except for the famous Dornbusch-Fischer-Samuelson (DFS) models, most general equilibrium models of trade rely on factor price equalization. The DFS models demonstrate the gains from trade without factor price equalization under perfect competition. This paper employs a general equilibrium model of oligopolistic competition which implies distortions both at the intensive and extensive margin. If factor prices do not equalize, imperfect competition will not reverse the specialization pattern. However, mutual gains from trade are not guaranteed, but one country may be worse off by trade.

JEL-Classification: F12, D50.
Keywords: Oligopolistic competition, general equilibrium, international trade, factor price differences.

1 Introduction

Traditional models of trade discuss mostly the incentives for and the outcomes of trade between relatively similar countries, an implication of which is that factor prices across countries are equalized. While this seems to be consistent with the observation of significantly large intra-industry trade volumes between countries with similar factor endowments, factor price equalization has found little empirical support. Once the trading partners are sufficiently different (in terms of size, in terms of factor endowments, or in terms of production technology, etc.), it is a well-established result that factor prices will no longer be equalized.[1] The consequences of trade when factor prices are different across countries are, however, not well scrutinized in the trade literature. This paper hence contributes to the literature by delineating the implications of different factor prices across countries, originating from factor endowment differences across countries, on the patterns of and gains from trade, in a multi-industry general equilibrium model of oligopolistic competition with free market entry and exit of firms. The model features two factors of production: capital which is used to establish firms, and labor which is used for production.

It is mainly the literature on the economics of multinational firms and vertical integration that focuses on the implications of different factor prices across countries. Helpman (1984), for example, shows in a general equilibrium framework that when countries are sufficiently asymmetric in terms of their relative factor endowments, factor prices will not equalize in equilibrium, which generates incentives for firms to vertically integrate. Also he finds that the extent of multinationality increases with an increase in the difference between relative factor endowments of the trading partners. In his model, the relative factor prices across countries tend to converge, especially with sufficiently large number of multinational firms: the existence of multinationals puts a downward (upward) pressure on the wage-rental ratio in the capital-abundant (labor-abundant) country. Similarly, in a general equilibrium framework, Feenstra and Hanson (1997) study the labor market consequences of vertical integration between North (the USA) and South (Mexico). They show that different factor prices across countries lead firms to vertically integrate, and that capital flows from a capital- (and skilled-labor-) abundant (North) country to an unskilled-labor-abundant (South) country increase demand

[1] For instance, by introducing differences in country size as measured by labor endowments, the only factor of production, Krugman (1980) shows that the wage rate is necessarily higher in the larger country.

for skills and the average skill intensity in both countries, which can explain the increase in the skill premium in both the USA and Mexico in the late 1980s.[2]

In this paper, we do not focus on multinational activities, but on the effects of trade when factor prices do not equalize across countries. The model we employ can be seen as an extenstion of the famous Dornbusch-Fischer-Samuelson (DFS) model, Dornbusch *et al.* (1980), who demonstrate the gains from trade without factor price equalization under perfect competition. We depart from the DFS model, and from the standard approach to imperfect competition and trade (e.g., a general equilibrium model of monopolistic competition as in Krugman, 1980), by considering a model in which all commodity markets are imperfectly competitive, and there are economies of scale in all industries. The modeling approach in this paper is also different from Markusen (1981), who employs a two-sector model that features one monopoly and one perfectly competitive industry, and focuses on the implications of trade that originates from country size differences. He shows that factor prices do not equalize across countries of different size, and that welfare in a smaller country unambiguously increases, whereas the welfare implications of trade are ambiguous in a larger country.

Our model setup in this study is similar to Koska and Stähler (2014), building on the famous Dornbusch-Fischer-Samuelson (DFS) model, but we now accommodate factor price differences. Koska and Stähler (2014) focus only on trade equilibria with equal factor prices across countries, while the model here scrutinizes the role of different factor prices across countries: we look at inter-industry trade between countries that are sufficiently different in their factor endowments (e.g., North-South or South-North trade).[3] In this setup, domestic and foreign firms cannot coexist in the same industry. However, oligopolistic competition does not reverse the specialization patterns as they are well known from classical trade models: the capital-abundant country is a net exporter of capital services and a net importer of labor services as embodied in trade. Unless one country specializes in production of a sufficiently small range of goods, the wage-rental ratio increases in both countries. If the countries are sufficiently asymmetric in terms of their capital endowments, then a beneficial rationalization effect of free trade

[2]Feenstra and Hanson (1997) discuss that similar results hold also with an increase in capital endowments of both countries as long as the increase is more in South, or with technological progress in both countries as long as it is at a faster rate in South.

[3]Volumes of trade between countries with sufficiently different factor endowments are sginificantly large. UNCTAD (2013), for instance, reports that North-South and South-North trade comprises, approximately, 40 per cent of global trade.

under different factor prices will be materialized unambiguously (via a decrease in the number of firms) in industries that are hosted by the relatively labor-abundant country, whereas the trade-induced changes in firm size and/or per-capita consumption are ambiguous. According to our simulation results, a sufficient increase in the capital stock of the capital-abundant country - so that the countries become sufficiently asymmetric in terms of factor endowments - reduces welfare in the capital-abundant country, while increasing welfare in the labor-abundant country.

The remainder of the paper is organized as follows. Section 2 introduces the model and discusses the autarky equilibrium. Section 3 scrutinizes the equilibrium with no factor price equalization (NFPE) and discusses the implications of inter-industry trade. Section 4 delineates trade-induced changes in per-capita consumption, discusses welfare implications, and presents simulation results. Section 5 offers some concluding remarks. For convenience, we have relegated most proofs and technical details to the Appendix section.

2 The model

We consider two countries: Home (North) and Foreign (South). All variables that are specific to Foreign are presented with ($*$). Each country is endowed with labor (L or L^*) and capital (K or K^*). Factor endowments are fixed in each country: there is no factor mobility across countries. In each country, capital is used to establish a firm, and labor is used for producing output. There is a continuum of goods, indexed by $z \in [0, 1]$. Households are symmetric and have Cobb-Douglas-type preferences, represented by the utility function $U = \int_0^1 \ln(\hat{y}(z))dz$ where $\hat{y}(z)$ denotes per-capita consumption of commodity z. In autarky, the aggregate output of industry z in Home is $Y(z) = L\hat{y}(z)$, and in Foreign is $Y^*(z) = L^*\hat{y}^*(z)$, leading to an inverse demand function $p(z) = I/Y(z)$ in Home, and $p^*(z) = I^*/Y^*(z)$ in Foreign, where I and I^* denote, respectively, Home and Foreign income, and $p(z)$ and $p^*(z)$ denote the price of good z in Home and Foreign, respectively. The profit of firm i in industry z that is hosted by Home, denoted $\Pi_i(z)$, and by Foreign, denoted $\Pi_i^*(z)$, is respectively, equal to

$$\Pi_i(z) = (p(z) - \lambda(z)w)y_i(z) - r\kappa(z), \;\; \Pi_i^*(z) = (p^*(z) - \lambda(z)w^*)y_i^*(z) - r^*\kappa(z), \quad (1)$$

where w and w^* denote, respectively, Home and Foreign labor wages; and r and r^* denote, respectively, the rental rate in Home and in Foreign. Firm-level outputs are denoted by $y_i(z)$ in Home, and by $y_i^*(z)$ in Foreign. The aggregate output of industry z is equal to $Y(z) = \sum_i y_i(z)$ in Home and $Y^*(z) = \sum_i y_i^*(z)$ in Foreign. In each country, all firms within industry z are symmetric: $Y(z) = n(z)y_i(z)$ in (Home) equilibrium, and $Y^*(z) = n^*(z)y_i^*(z)$ in (Foreign) equilibrium, where $n(z)$ and $n^*(z)$ denote, respectively, the number of active Home and Foreign firms in industry z. For a given industry z, the labor input requirement, denoted $\lambda(z)$, is the same in Home and in Foreign. Also, in each country, firms have to make an investment of size $\kappa(z) > 0$ that allows for entry into industry z. It is the cost of establishing a firm in industry z.

Industries are ranked, without loss of generality, according to the capital input requirements, such that z decreases strictly with the capital input requirement κ: $\kappa'(z) < 0$. That said, the model puts no restriction on the behavior of labor input requirements, nor does it put any restriction on firm entry or on firm output. In particular, the model considers a two-stage game, in which firms compete by strategic substitutes in the sense of Bulow *et al.* (1985). In the first stage, firms decide on their entry into industry z. In the second stage, they decide on their outputs. This game structure is strategically equivalent to one in which firms, after having made their market-entry investments, choose their capacities and hire labor to produce maximum outputs before they will compete by prices (see, for example, Kreps and Scheinkman, 1983). We solve the game backwards. Firms are risk-neutral and maximize their profits, given by eq. (1), for a given wage and rental rate, and for a given income level.[4] In the autarky equilibrium, a firm, operating in industry z that is hosted by Home and by Foreign, maximizes its profit by producing, respectively,

$$y_i(z) = \frac{I}{\lambda(z)w} \frac{n(z) - 1}{n(z)^2}, \quad y_i^*(z) = \frac{I^*}{\lambda(z)w^*} \frac{n^*(z) - 1}{n^*(z)^2}.$$

As there is free entry/exit of firms, all firms make zero profits.[5] In equilibrium, ignoring the integer constraint, the zero-profit condition leads to

$$n(z) = \sqrt{\frac{I}{r\kappa(z)}}, \quad n^*(z) = \sqrt{\frac{I^*}{r^*\kappa(z)}}.$$

[4]Firms compete for resources without considering their influence on factor prices (and thus on national income). That is, firms can exercise market power only in their respective commodity markets, while they have no influence on factor markets, because there is a sufficiently large number of industries. For discussions of this, see Neary (2007; 2009), and Neary and Tharakan (2012).

[5]Positive profits in an industry would rise capital demand, reducing industry profits due to firm entry into that industry, whereas negative profits would reduce capital demand, increasing industry profits due to firms exiting that industry. In response to changes in capital demand, also the rental would change, but this effect is not taken into account by potential entrants.

The model focuses on oligopolistically competitive commodity markets, which warrants $n(z) \geq 2$ and $n^*(z) \geq 2$. That is, $\sqrt{r\kappa(z)/I} \leq 1/2$ and $\sqrt{r^*\kappa(z)/I^*} \leq 1/2$. We shall note that the market for capital is perfectly competitive, so firm ownership is diversified and capital owners take r as given. They have no influence on the behavior of firms, and thus the price normalization problem cannot arise as in Dierker *et al.* (2003), and Dierker and Dierker (2006). To ensure that at least two firms will be established in each industry, we assume, throughout the paper, that each country is endowed with sufficient capital, such that $K > 2\kappa(0) \geq 2\Omega^2$ and $K^* > 2\kappa(0) \geq 2\Omega^2$, where the last inequality follows from the ranking of industries. Note that $\Omega \equiv \int_0^1 \sqrt{\kappa(z)}dz$. This assumption also guarantees that firm and industry outputs in equilibrium, which are given (respectively for Home and for Foreign) by eq. (2), are strictly positive:

$$
\begin{aligned}
y_i(z) &= \frac{r\kappa(z)}{\lambda(z)w}\left(\sqrt{\frac{I}{r\kappa(z)}}-1\right), \qquad Y(z) = \frac{\sqrt{Ir\kappa(z)}}{\lambda(z)w}\left(\sqrt{\frac{I}{r\kappa(z)}}-1\right); \\
y_i^*(z) &= \frac{r^*\kappa(z)}{\lambda(z)w^*}\left(\sqrt{\frac{I^*}{r^*\kappa(z)}}-1\right), \quad Y^*(z) = \frac{\sqrt{I^*r^*\kappa(z)}}{\lambda(z)w^*}\left(\sqrt{\frac{I^*}{r^*\kappa(z)}}-1\right).
\end{aligned} \tag{2}
$$

By using eq. (2), labor and capital demand of industry z, respectively for Home and for Foreign, can be expressed as

$$
\begin{aligned}
L(z) &= \lambda(z)Y(z) = \frac{I}{w} - \frac{\sqrt{Ir\kappa(z)}}{w}, \qquad K(z) = \kappa(z)n(z) = \sqrt{\frac{I}{r}}\sqrt{\kappa(z)}, \\
L^*(z) &= \lambda(z)Y^*(z) = \frac{I^*}{w^*} - \frac{\sqrt{I^*r^*\kappa(z)}}{w^*}, \quad K^*(z) = \kappa(z)n^*(z) = \sqrt{\frac{I^*}{r^*}}\sqrt{\kappa(z)}.
\end{aligned} \tag{3}
$$

Throughout out the paper, the ratio between the establishment and operating inputs, denoted $\mu(z)$ and $\mu^*(z)$, is referred to as the capital intensity, which is endogenous in the model. We can use eq. (3) and show that the capital intensity of industry z, hosted by Home and by Foreign, are, respectively, given by eq. (4):

$$
\mu(z) \equiv K(z)/L(z) = \frac{\frac{w}{r}}{\sqrt{\frac{I}{r\kappa(z)}}-1}, \quad \mu^*(z) \equiv K^*(z)/L^*(z) = \frac{\frac{w^*}{r^*}}{\sqrt{\frac{I^*}{r^*\kappa(z)}}-1} \tag{4}
$$

Eq. (3) and eq. (4) show that, irrespective of the labor input requirement, more capital-intensive industries demand more capital ($K'(z) \leq 0$ and $K^{*\prime}(z) \leq 0$) and less labor ($L'(z) \geq 0$ and $L^{*\prime}(z) \geq 0$), and that the capital intensity increases with the capital input requirement ($\mu'(z) \leq 0$ and $\mu^{*\prime}(z) \leq 0$).

In equilibrium, all commodity markets and the two factor markets clear, leading to

$$L = \int_0^1 \left(\frac{I}{w} - \frac{\sqrt{Ir\kappa(z)}}{w} \right) dz = \frac{I}{w} - \frac{\sqrt{Ir}}{w}\Omega, \qquad K = \sqrt{\frac{I}{r}}\Omega,$$
$$L^* = \int_0^1 \left(\frac{I^*}{w^*} - \frac{\sqrt{I^* r^* \kappa(z)}}{w^*} \right) dz = \frac{I^*}{w^*} - \frac{\sqrt{I^* r^*}}{w^*}\Omega, \quad K^* = \sqrt{\frac{I^*}{r^*}}\Omega.$$

The model is closed by the income definition: $I = wL + rK$ and $I^* = w^* L^* + r^* K^*$. Without loss of generality, let the rental rate be a numeraire in the model. Using Walras' Law and the equilibrium conditions for the factor markets, we can show that

$$K = \sqrt{wL + K}\Omega \;\Leftrightarrow\; w = \frac{K}{L}\left(\frac{K}{\Omega^2} - 1\right), \tag{5}$$
$$K^* = \sqrt{w^* L^* + K^*}\Omega \;\Leftrightarrow\; w^* = \frac{K^*}{L^*}\left(\frac{K^*}{\Omega^2} - 1\right)$$

We can substitute each country's autarky wage-rental ratio, given by eq. (5), into the income definition ($I = wL + K$ and $I^* = w^* L^* + K^*$) and show that each country's autarky income increases more than proportionately with its capital stock: $I = K^2/\Omega^2$ and $I^* = K^{*2}/\Omega^2$. Moreover, we can show that the autarky wage-rental ratio in the country that has a larger capital stock (in North) is higher than that in the other country (in South), even if the two country's per-capita capital endowments ($k \equiv K/L$ and $k^* \equiv K^*/L^*$) are the same.

3 Factor price differences and international trade

In this section, we allow for trade between Home (North) and Foreign (South). When the two countries are sufficiently asymmetric in terms of their factor endowments, the factor prices differ across the countries. Factor price equalization is closely related to the existence of a cone of diversification: whether or not all goods are produced within a country. Hence, we look at first the existence of active firms in the two countries where factor prices are different. Different factor prices imply that firms are not indifferent where to locate, and on aggregate their location decisions have an impact on equilibrium factor prices. We find that

Proposition 1 *There is no coexistence of domestic and foreign firms in industry z if factor prices differ.*

Proof: See Appendix A.1.

A simple interpretation of Proposition 1 is that factor price equalization is crucial for diversification. Different factor prices imply different cost structures, and thus different cost structures imply that one country will not host firms of an industry due to higher costs, and firms of that industry will be geographically concentrated in the other country.[6] Of course, due to factor market clearance in both countries, this cannot be true for all industries so that each country will attract some industries.[7] In what follows, we will scrutinize an equilibrium in which factor prices continue to differ after trade liberalization, and we will look at the welfare implications of inter-industry trade. The analysis of trade without factor price equalization is usually not on the agenda if an FPE equilibrium exists. However, we will show in this study that differences in factor endowments between the trading countries play an important role in mutual welfare gains.

We know from Proposition 1 that different factor prices imply geographical concentration of industries. Thus, we are now interested in the possible specialization patterns for the equilibrium factor prices. Given Proposition 1, an industry will be hosted by the country having the lowest price for the equilibrium factor prices. Let $p(z)$ and $p^*(z)$ denote the price if, respectively, the domestic country and the foreign country hosts industry z. We can replace the income definition for each country with the definition of world income, $I^w = wL + rK + w^*L^* + r^*K^*$, in the expressions $Y(z)$ and $Y^*(z)$, given by eq. (2), and in the price definition, such that $p(z) = I^w/Y(z)$ and $p^*(z) = I^w/Y^*(z)$, and we can express the prices as

$$p(z) = \frac{\lambda(z)w}{\Psi(z)}; \qquad \Psi(z) = 1 - \sqrt{\frac{r\kappa(z)}{I^w}},$$
$$p^*(z) = \frac{\lambda(z)w^*}{\Psi^*(z)}; \quad \Psi^*(z) = 1 - \sqrt{\frac{r^*\kappa(z)}{I^w}},$$

if production takes place in Home or in Foreign, respectively. Since all factor markets have to be cleared, there must be at least one indifferent industry $\hat{z}$ for which $p(\hat{z}) = p^*(\hat{z})$. Thus, we find that

$$\frac{\lambda(\hat{z})w}{\Psi(\hat{z})} = \frac{\lambda(\hat{z})w^*}{\Psi^*(\hat{z})} \quad \Leftrightarrow \quad \frac{w}{\Psi(\hat{z})} = \frac{w^*}{\Psi^*(\hat{z})}. \tag{6}$$

[6] Helpman and Krugman (1985, p. 103) discuss that the scale of production may be different across countries when factor prices do not equalize, and when the production technology is not homothetic so that firms of different scale may coexist in a free entry trade equilibrium. The specific structure of the DFS model does not include this case.

[7] Proposition 1 is not completely new; see for example Helpman and Krugman (1985) for the case of homothetic production functions.

The pattern of specialization is then straightforward. Given $\kappa' < 0$ (the behavior of the capital input requirements across the whole range), we find that

Proposition 2 *In an equilibrium in which factor prices continue to differ after trade liberalization, one country has the higher equilibrium wage and the lower equilibrium rental compared to the other country. This country hosts industries in the range* $z \in [0, \hat{z}]$*, and the other country hosts industries in the range* $z \in [\hat{z}, 1]$*.*

Proof: See Appendix A.2.

We can also identify which country is more capital-abundant such that these patterns may emerge. We have already shown that the capital intensity increases with the capital input requirement, which is negatively related to z, thus the capital intensity decreases with z. This will also be true along the range of commodities produced within each country. Assume that Home (North) produces in the range $z \in [0, \hat{z}]$; in this case, $\mu'(z) < 0$ for $z \in [0, \hat{z}]$ and $\mu^{*\prime}(z) < 0$ for $z \in [\hat{z}, 1]$. Using the definition of μ, given by eq. (4), and the definition of $\hat{z}$, given by eq. (6), we find that

$$\mu(\hat{z}) = \sqrt{\frac{r^*}{r}}\mu^*(\hat{z}) > \mu^*(\hat{z}).$$

Proposition 2 has shown that North must have a lower rental than South if it produces in the capital-intensive range, and this is the reason why $\mu(\hat{z}) > \mu^*(\hat{z})$. Therefore, the capital intensity in both countries behaves like in Figure 1.

Factor market clearance implies that relative factor demand for capital is a weighted average of capital intensities. Thus, it follows immediately from the Intermediate Value Theorem that the weighted average will be higher in the country hosting the capital-intensive industries, whereas it will be lower in the country hosting the labor-intensive industries, and this must be matched by the per-capita capital endowment. In our case, it follows that $k > k^*$. Additionally, there are clearly lower bounds on the difference in relative factor endowments. In order to make Figure 1 consistent with the relative factor endowments, a sufficient (but not yet necessary) requirement is that $k \geq \mu(\hat{z})$ and $k^* \leq \mu^*(\hat{z})$. Therefore we conclude that

Proposition 3 *An equilibrium in which factor prices continue to differ after trade liberalization exists only if the countries are sufficiently different with respect to their relative capital endowments. If the countries are sufficiently asymmetric in their relative factor endowments, then the capital-abundant country produces the capital-intensive commodities in equilibrium.*

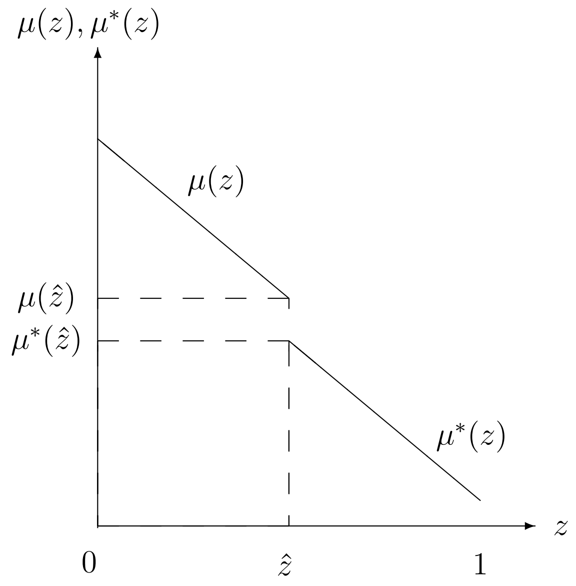

Figure 1: Capital intensities in the NFPE case

In the trade equilibrium with different factor prices across countries, we find that oligopolistic competition cannot reverse the specialization patterns as they are well known from classic trade models. Furthermore, a direct implication of Proposition 3 is that the capital-abundant country will be a net exporter of capital services and a net importer of labor services as embodied in trade.[8]

If we maintain our assumption that Home (North) produces goods in the range $[0, \hat{z}]$, and Foreign (South) produces goods in the range $[\hat{z}, 1]$ then labor and capital markets clear in Home and in Foreign if, respectively,

$$L = \int_0^{\hat{z}} \left(\frac{I^w}{w} - \frac{\sqrt{I^w r \kappa(z)}}{w} \right) dz; \qquad K = \sqrt{\frac{I^w}{r}} \int_0^{\hat{z}} \sqrt{\kappa(z)} dz,$$
$$L^* = \int_{\hat{z}}^1 \left(\frac{I^w}{w^*} - \frac{\sqrt{I^w r^* \kappa(z)}}{w^*} \right) dz; \qquad K^* = \sqrt{\frac{I^w}{r^*}} \int_{\hat{z}}^1 \sqrt{\kappa(z)} dz,$$

where I^w is world income. Rearranging the capital market condition and substituting it into the labor market condition yields (respectively for Home and for Foreign):

$$L = \frac{I^w \hat{z} - rK}{w}; \quad L^* = \frac{I^w(1-\hat{z}) - r^* K^*}{w^*}.$$

[8]See Brecher and Choudhri (1982) for this result in a Heckscher-Ohlin model without factor price equalization.

Rearranging these equations yields the trade balance condition, that is, each country's income should be equal to its production such that

$$wL + rK = I^w \hat{z}; \quad w^* L^* + r^* K^* = I^w (1 - \hat{z}). \tag{7}$$

So we have three equations, the two equations clearing the capital markets - one for each country - and the trade balance condition. Furthermore, eq. (6) determines $\hat{z}$. In summary, we have four independent equations, and, after using the domestic rental rate as a numeraire again, that is, $r = 1$, four unknowns, w, w^*, r^* and $\hat{z}$, and we can show:

Proposition 4 *If an NFPE equilibrium exists, it is unique.*

Proof: See Appendix A.3.

Using the conditions clearing capital markets, and given the numeraire $r = 1$, we can show that, in equilibrium,

$$I^w = \frac{K^2}{\zeta(\hat{z})^2} = \frac{(K^*)^2 r^*}{(\Omega - \zeta(\hat{z}))^2}; \quad \zeta(\hat{z}) = \int_0^{\hat{z}} \sqrt{\kappa(z)} dz. \tag{8}$$

We can solve equation (8) for r^*, and show that

$$r^* = \left(\frac{K}{K^*}\right)^2 \left(\frac{\Omega}{\zeta(\hat{z})} - 1\right)^2. \tag{9}$$

Using the trade balance condition for each country, given by equation (7), world income, given by equation (8), and r^*, given by equation (9), and given $r = 1$, we can derive the equilibrium wage-rental ratio, respectively, in North and in South:

$$w = \frac{K}{L}\left(\frac{K\hat{z}}{\zeta(\hat{z})^2} - 1\right), \quad \frac{w^*}{r^*} = \frac{K^*}{L^*}\left(\frac{K^*(1-\hat{z})}{(\Omega - \zeta(\hat{z}))^2} - 1\right). \tag{10}$$

We can now compare the wage-rental ratio in each country before and after trade liberalization (given by equations (5) and (10), respectively), and we can show that when the two countries are sufficiently different in terms of factor endowments - so that factor prices continue to differ after trade liberalization - trade increases the wage-rental ratio in both countries if each country hosts a sufficiently large range of industries, such that

$$\underline{z} < \hat{z} < \overline{z}\ ; \text{ where } \quad \underline{z} \equiv \frac{\zeta(\hat{z})^2}{\Omega^2} < \overline{z} \equiv \frac{\zeta(\hat{z})}{\Omega}\left(2 - \frac{\zeta(\hat{z})}{\Omega}\right) < 1.$$

If the range of industries located in North, $[0, \hat{z}]$, is significantly small, such that $\hat{z} < \underline{z}$ – if North specializes too much in a few substantially capital-intensive industries –

then the wage-rental ratio decreases in North, whereas South experiences an increase in the wage-rental ratio. Similarly, if the range of industries located in South, $[\hat{z}, 1]$, is significantly small, such that $\hat{z} > \overline{z}$ – if South specializes too much in a few substantially labor-intensive industries – then the wage-rental ratio in South decreases, whereas North experiences an increase in the wage-rental ratio. It is clear that the wage-rental ratio cannot decrease in both countries: there will be an increase in the wage-rental ratio in at least one country. This leads to

Proposition 5 *The wage-rental ratio will increase in both countries with trade under different factor prices if each country hosts a significant range of industries, following the sepacialization pattern, given by Propositions 2 and 3. If a country hosts only a too small range of industries, then it may experience a decrease in the wage-rental ratio, whereas the other country will definitely experience an increase.*

A decrease in the wage-rental ratio implies that domestic income decreases, that is, the country experiencing a decrease in the wage-rental ratio becomes less influential in terms of its share in world income, which coincides with $\hat{z}$ for North, and with $(1 - \hat{z})$ for South; see equation (7).[9] If the wage-rental ratio increases in only one country, this country will become a significantly dominant player in a larger (integrated) market than before. An increase in the wage-rental ratio, however, implies also an increase in production costs, and so does a change in commodity prices. Therefore, a thorough analysis of potential rationalization effects is warranted to find out about welfare effects of trade under different factor prices.

A country will benefit from rationalization effects if the number of firms in an industry that it is hosting decreases, and/or if each firm's size increases. Recall that in equilibrium Home (North), the capital-abundant country, produces the capital-intensive goods in the range $[0, \hat{z}]$; and Foreign (South), the labor-abundant country produces the labor-intensive goods in the range $[\hat{z}, 1]$. Let us denote the trade situation and autarky, respectively, by superscript t and a. In equilibrium, the number of firms in industry z is thus denoted by $n^t(z)$ if industry z is hosted by Home ($z \in [0, \hat{z}]$), or denoted by $n^{*t}(z)$ if it is hosted by Foreign ($z \in [\hat{z}, 1]$) under trade. Similarly, in equilibrium, denote by $n^a(z)$ and $n^{*a}(z)$, the number of firms in industry z that is hosted,

[9] We shall note that $\hat{z}$ is determined endogenously by equation (6); it can be computed by substituting equations (8), (9) and (10) back into equation (6), and by solving for $\hat{z}$, which will warrant a specific κ-function that must be consistent with the ranking of industries; see Section 4 for one that is used in simulating the model for a welfare analysis.

respectively, by Home and Foreign under autarky. Given that the rental in each country serves as the numeraire in the autarky equilibrium, and $r = 1$ in the integrated equilibrium, and that $\zeta(\hat{z}) \equiv \int_0^{\hat{z}} \sqrt{\kappa(z)}dz < \Omega \equiv \int_0^1 \sqrt{\kappa(z)}dz$, we can show that

$$\begin{aligned} n^t(z) = \sqrt{\frac{I^w}{\kappa(z)}} &= \frac{K}{\zeta(\hat{z})\sqrt{\kappa(z)}} \\ &> n^a(z) = \sqrt{\frac{I}{\kappa(z)}} = \frac{K}{\Omega\sqrt{\kappa(z)}}, \quad \text{for any } z \in [0, \hat{z}]; \\ n^{*t}(z) = \sqrt{\frac{I^w}{r^*\kappa(z)}} &= \frac{K^*}{(\Omega - \zeta(\hat{z}))\sqrt{\kappa(z)}} \\ &> n^{*a}(z) = \sqrt{\frac{I^*}{\kappa(z)}} = \frac{K^*}{\Omega\sqrt{\kappa(z)}}, \quad \text{for any } z \in [\hat{z}, 1]. \end{aligned} \tag{11}$$

It is straightforward to show that, unlike partial equilibrium Cournot models in which market entry is excessive, the socially optimal number of firms ($K/\int_0^1 \kappa(z)dz$ in North, or $K^*/\int_0^1 \kappa(z)dz$ in South)[10] is less than the autarky number of firms ($n^a(z)$ in North, or $n^{*a}(z)$ in South) only for less capital-intensive industries, for which $\Omega\sqrt{\kappa(z)} < \int_0^1 \kappa(z)dz$, whereas it is more for more capital-intensive industries, for which $\Omega\sqrt{\kappa(z)} > \int_0^1 \kappa(z)dz$. We can conclude from equation (11) that

Proposition 6 *The laissez-faire number of firms can be either below or above the socially optimal number of firms, and free trade under different factor prices, leading countries to specialize in production of a certain range of goods, exacerbates distortions at the extensive margin (firm entry).*

Using equation (11), we can show that the aggregate number of firms in industry z is equal to $n^a(z) + n^{*a}(z) = (K + K^*)/\Omega\sqrt{\kappa(z)}$ under autarky, which is less than $n^t(z)$ for all $z \in [0, \hat{z}]$, or greater than $n^{*t}(z)$ for all $z \in [\hat{z}, 1]$ if

$$\frac{K}{K^*} > \frac{\zeta(\hat{z})}{\Omega - \zeta(\hat{z})}. \tag{12}$$

A sufficient condition for inequality (12) to hold is that North, the capital-abundant country, and South, the labor-abundant country, are sufficiently asymmetric in terms of their absolute capital endowments, leading us to

[10]Note that the output behavior follows profit maximization, and we consider only market entry as in Mankiw and Whinston (1986). See Koska and Stähler (2014) for the derivation of the socially optimal number of firms.

Proposition 7 *A beneficial rationalization effect of free trade under different factor prices will be metarialized unambiguously in industries that are hosted by South if North has a sufficiently larger capital endowment than South, whereas the rationalization effect in industries that are hosted by North can only come about from changes in firm size.*

Firm-level outputs in industry z in North and in South under autarky, which we will denote, respectively, by $y_i^a(z)$ and $y_i^{*a}(z)$, are given by equation (2), where $r = r^* = 1$, $I = K^2/\Omega^2$ and $I^* = K^{*2}/\Omega^2$, in equilibrium, and the wage-rental ratios are given by equation (5). Similar functions will hold for the trade situation, except that autarky income in each country should be replaced by world income, given by equation (8), and that factor prices are given by equations (9) and (10), and $r = 1$. Let $y_i^t(z)$ and $y_i^{*t}(z)$ denote firm-level outputs in the trade situation. The change in firm size with trade in industry z hosted by North or by South is, respectively, given by

$$\frac{y_i^t(z)}{y_i^a(z)} = \sigma(z) \underbrace{\left(\frac{K - \Omega^2}{K\hat{z} - \zeta(\hat{z})^2}\right) \frac{\zeta(\hat{z})}{\Omega}}_{\Upsilon(\hat{z})} \qquad \text{for any } z \in [0, \hat{z}],$$

$$\frac{y_i^{*t}(z)}{y_i^{*a}(z)} = \sigma^*(z) \underbrace{\left(\frac{K^* - \Omega^2}{K^*(1-\hat{z}) - (\Omega - \zeta(\hat{z}))^2}\right)\left(1 - \frac{\zeta(\hat{z})}{\Omega}\right)}_{\Upsilon^*(\hat{z})} \qquad \text{for any } z \in [\hat{z}, 1],$$

where $\sigma(z) \equiv \left(K - \zeta(\hat{z})\sqrt{\kappa(z)}\right) / \left(K - \Omega\sqrt{\kappa(z)}\right) > 1$ for all z because $\zeta(\hat{z}) < \Omega$, and where $\sigma^*(z) \equiv 1 + \left(\zeta(\hat{z})\sqrt{\kappa(z)} / \left(K^* - \Omega\sqrt{\kappa(z)}\right)\right) > 1$ for all z.

We now compare the change in firm size with trade in each country. Not surprisingly, a decrease in the wage-rental ratio in a country (in North if $\hat{z} < \underline{z}$, or in South if $\hat{z} > \overline{z}$; see Proposition 5) leads to an increase in firm size in all industries hosted by this country (as $\Upsilon(\hat{z} \leq \underline{z}) \geq \Omega/\zeta(\hat{z}) > 1$ and $\Upsilon^*(\hat{z} \geq \overline{z}) \geq \Omega/(\Omega - \zeta(\hat{z})) > 1$): production costs decrease with a decrease in the wage-rental ratio and firms face a larger (integrated) market with trade liberalization. If, however, there is an increase in the wage-rental ratio in a country, then each firm estblished in this country faces a larger market, while facing an increase in production costs, which implies that an increase in firm size may not be guaranteed for all industries in a country where the wage-rental ratio increases.[11] In particular, we can find threshold values of $\hat{z} \in [\underline{z}, \overline{z}]$ such that, depending on the equilibrium value of $\hat{z}$, $y_i^t(z)/y_i^a(z) > 1$ for all $z \in [0, \hat{z}]$, or $y_i^{*t}(z)/y_i^{*a}(z) > 1$ for all $z \in [\hat{z}, 1]$. Similarly, we can find threshold values of $\hat{z} \in$

[11]In perfectly competitive models with a single production factor (labor) we would not see such labor market effects.

$[\underline{z}, \overline{z}]$ such that, depending on the equilibrium value of $\hat{z}$ and for threshold industry $\tilde{z}$, $y_i^t(\tilde{z})/y_i^a(\tilde{z}) = 1$, or $y_i^{*t}(\tilde{z})/y_i^{*a}(\tilde{z}) = 1$, firm size increases in industries that are more capital-intensive than the threshold industry ($z < \tilde{z}$), and it decreases in other industries ($z > \tilde{z}$), because

$$\frac{\partial \sigma(z)}{\partial z} = \frac{K(\Omega - \zeta(\hat{z}))\kappa'(z)}{2\sqrt{\kappa(z)}\left(K - \Omega\sqrt{\kappa(z)}\right)^2} < 0; \quad \frac{\partial \sigma^*(z)}{\partial z} = \frac{K^*\zeta(\hat{z})\kappa'(z)}{2\sqrt{\kappa(z)}\left(K^* - \Omega\sqrt{\kappa(z)}\right)^2} < 0,$$

since $\kappa'(z) < 0$. It is now clear that the effect of trade on firm size depends on the industry's capital intensity and on the range of products that each country specializes in, which is mainly determined by each country's factor endowments. To scrutinize the welfare implications of trade under different factor prices between the trading partners, we next look at the trade-induced changes in per-capita consumption and simulate the model for different constellations of parameter values.

4 Welfare implications

Per-capita consumption of commodity z in North and in South under autarky, which we will denote, respectively, by $\hat{y}_i^a(z) \equiv Y^a(z)/L$ and $\hat{y}_i^{*a}(z) \equiv Y^{*a}(z)/L^*$, can be derived by substituting $r = r^* = 1$, $I = K^2/\Omega^2$ and $I^* = K^{*2}/\Omega^2$, and the wage-rental ratios, given by equation (5), into equation (2). Similar functions will hold for the trade situation, except that autarky income in each country should be replaced by world income, given by equation (8), and that factor prices are given by equations (9) and (10), and $r = 1$. Let $\hat{y}_i^t(z) \equiv Y^t(z)/(L + L^*)$ and $\hat{y}_i^{*t}(z) \equiv Y^{*t}(z)/(L + L^*)$ denote, respectively, per-capita consumption of commodity z in North and in South in the trade situation. The trade-induced changes in per-capita consumption of commodity z can be expressed as

$$\delta(z) \equiv \frac{\hat{y}_i^t(z)}{\hat{y}_i^a(z)} = \sigma(z)\left(\frac{K - \Omega^2}{K\hat{z} - \zeta(\hat{z})^2}\right), \quad \text{for any } z \in [0, \hat{z}],$$
$$\delta^*(z) \equiv \frac{\hat{y}_i^{*t}(z)}{\hat{y}_i^{*a}(z)} = \sigma^*(z)\left(\frac{K^* - \Omega^2}{K^*(1 - \hat{z}) - (\Omega - \zeta(\hat{z}))^2}\right), \quad \text{for any } z \in [\hat{z}, 1].$$

When the capital input requirements strictly decrease with z, the trade-induced changes in per-capita consumption and welfare are ambiguous. Given our specification of utility, trade unambiguously improves welfare in North and in South if, respectively,

$$v \equiv \int_0^{\hat{z}} \ln(\delta(z))dz \geq 0; \quad v^* \equiv \int_{\hat{z}}^1 \ln(\delta^*(z))dz \geq 0.$$

As to analyze the welfare effects, we have simulated the model, assuming labor input requirements are constant across industries, $\forall\ z \in [0,1]\,,\ \lambda(z) = 1$. Also we assign a specific function to $\kappa(z)$, that is, $\kappa(z) = 2 - z$.[12] In this setup, trade would improve welfare if factor prices could equalize as is shown by Koska and Stähler (2014).[13] Table 1 summarizes the results of our simulation.

<Table 1 about here.>

Table 1 shows the factor endowments, the relative factor prices, income levels, the (aggregate) number of firms serving each country, and the welfare change indices v and v^* for both countries. The number of firms is defined as the average number of firms in *all* industries hosted by the respective country, that is,

$$n^a \equiv \int_0^1 n^a(z)dz; \quad n^{*a} \equiv \int_0^1 n^{*a}(z)dz;$$
$$n \equiv \int_0^{\hat{z}} n^t(z)dz; \quad n^* \equiv \int_{\hat{z}}^1 n^{*t}(z)dz.$$

In the first simulation (Sim#1), we look at the outcomes of autarky and trade integration when there is perfect symmetry across countries such that the two countries are perfectly symmetric in their factor endowments: $K = K^* = L = L^* = 30$.

In case of perfect symmetry, in autarky, the two countries have identical relative factor prices, income levels, and number of firms, because not only their per-capita endowments are the same, but also their absolute capital stocks and labor endowments are identical. In case of trade integration under equal relative factor prices, autarky per-capita endowments coincide with the one for the integrated economy such that $((K + K^*)/(L + L^*) = K/L = K^*/L^* = 1)$, although the size of the capital stock in the integrated economy is twice as big as each country's capital stock in autarky, which leads to an increase in the wage-rental ratio and in world income. As the (world) aggregate income - when relative factor prices are equalized - is four times as big as each country's autarky income (each country has doubled its autarky income with

[12]Assigning such a specific function to $\kappa(z)$ is merely a technical simplification. Any functional form that is consistent with the model's setup (e.g., the ranking of industries and differentiability of κ) and that assigns a well-defined value to $\kappa(z)$ for any $z \in [0,1]$ would generate qualitatively similar simulation results.

[13]The reason is that $\kappa'' = 0$ guarantees positive gains from trade in the case of factor price equalization. Nevertheless, we simulate also the case of perfectly symmetric countries, leading to factor price equalization, and include the results in Table 1.

trade integration), the aggregate number of firms serving both markets $(n + n^*)$ is now twice as big as the aggregate number of firms serving each market in autarky $(n^a = n^{*a})$. Although the number of firms located in each country is ambiguous, it is straightforward to show that firm-level output for capital-intensively produced goods and per-capita consumption in these goods increase, whereas firm-level output and per-capita consumption for all other goods decrease.

Our simulation results confirm that, given λ and κ, where $\kappa'' = 0$, each country's welfare (and overall welfare) improves when the two countries that are perfectly symmetric in their factor endowments engage in free trade. The intuition is as follows. The aggregate number of firms in the FPE equilibrium is equal to the sum of each country's aggregate autarky number of firms, although the integrated market is larger and carries more firms as compared to each country's market under autarky: $(n + n^*) > n^a = n^{*a}$. Moreover, trade integration increases production costs relative to establishment costs as the FPE wage-rental ratio is almost twice as high as each country's autarky wage-rental ratio. This has an implication for autarky distortions such that firm-level output increases in industries where firm entry in autarky was too moderate, and it decreases in industries where firm entry in autarky was too excessive. Similarly, per-capita consumption in industries where firm entry in autarky was too moderate increases. Finally, we shall recall that any distortion in this simulation is symmetric across countries.

In all other simulations (Sim$\#2 - 11$), we scrutinize the outcomes of the NFPE case, simply by looking at the two countries that are sufficiently asymmetric in their factor endowments such that there exists an NFPE equilibrium. We shall note that, in Sim$\#2 - 11$, the factor endowments are outside the FPE set.[14] The simulations differ such that we increase the capital stock of the capital-abundant, domestic country by 10. We see that this increase in the domestic capital stock increases the wage-rental ratio in the domestic country both in autarky and under free trade. However, the increase under free trade is much larger. At the same time, it is straightforward to show that the increases in the wage-rental ratios from autarky to trade for both countries, that is, w/w^a and $(w^*/r^*)/w^{*a}$, do not change much across these simulations. This has the implication that $\hat{z}$ is roughly equal to 0.5 across all simulations and decreases only marginally with an increase in K.

Both countries gain from trade in the first three simulations (Sim$\#2 - 4$), but once $K = 60$ is reached, it is only the foreign country that gains, and these gains increase

[14]The computations of all simulations are available upon request.

when the domestic capital stock is increased further. For the domestic country, the gains from trade become smaller over the whole range and turn negative after Sim#4 which shows that trade may reduce welfare and proves

Proposition 8 *In the case that trade does not equalize factor prices, trade may reduce a country's welfare even if parameters are such that the welfare effects were unambiguously positive in an equilibrium in which relative factor prices had been equalized.*

Proposition 8 is interesting because it is in contrast to the common view that specialization is the key for gains from trade. Why does trade make the domestic country even suffer if it does not equalize factor prices at the same time? The increase in the wage-rental ratio has ambiguous effects: on the one hand, it becomes relatively less expensive to establish a firm, and this reduces entry distortions for the industries hosted by the domestic country; on the other hand, it becomes more expensive to run a firm which leads to a reduction in firm size. The last effect will be pronounced when the capital stock is relatively large compared to the foreign country.

Since the changes in the wage-rental ratio and $\hat{z}$ are not strong, the key for understanding the asymmetric welfare effect is the comparison of the average numbers of firms before and after trade. Due to a nearly unchanged $\hat{z}$, the average number of firms in the foreign country goes up from 13 to 16 from autarky to trade in all simulations. In autarky, however, the foreign country had to host all industries, and thus it is clear that less than 13 were active on average in the range $z \in [0, 0.5]$. Furthermore, this country now exploits its comparative advantage and establishes firms only in labor-intensive industries under free trade. Thus, the average number of firms in the range $z \in [0.5, 1]$ must have been less than 16 under autarky as it also had to host capital-intensive industries under autarky. Hence, this country experiences a strong welfare gain because many more domestic firms serve the range $z \in [0, 0.5]$ under free trade.

The (large) domestic country, however, faces a trade-off. On the one hand, it specializes in the range $z \in [0, 0.5]$ which is clearly a welfare gain. In this range, more firms are active on average because the range $z \in [0.5, 1]$ will not be hosted anymore under free trade. On the other hand, it now imports commodities in this range from the (small) foreign country which establishes on average 16 firms in these industries. If $K = 30$ (Sim#2), $n^a = 20.38$ and since $z \in [0.5, 1]$ does not require much investment, more than 20 firms have been active in these industries under autarky. Thus, trade increases the domestic markup for these commodities, leading to a domestic welfare loss. For a sufficiently small asymmetry, however, the specialization gain dominates this welfare loss. But with an increase in K, the overall welfare effect becomes negative. For example, if $K = 120$ (Sim#11), more than 81 firms have been active on average in the range $z \in [0.5, 1]$ under autarky which are replaced by only 16 foreign firms under free trade, leading to a substantial decrease in domestic consumption of labor-intensively produced goods. In this sense, the foreign country is too small for the domestic country.

Note carefully that the small change of $\hat{z}$ with K implied by our simulation is favorable for domestic welfare gains in case of substantial asymmetries. If $\hat{z}$ dropped substantially, specialization gains would affect a smaller range of industries and the (small) foreign country would have to host even more industries. Last but not least, the overall (net) change in (world) welfare seems to be positive such that the positive change in the foreign country's welfare in our simulations is always larger than the negative change in the domestic country's welfare. Therefore we may argue that the foreign firm could theoretically compensate the domestic firm so that both countries gain from trade.

5 Concluding remarks

This paper has studied the implications of different factor prices on the patterns of and gains from trade in a multi-industry general equilibrium model of oligopolistic competition and free market entry in which all industries are subject to increasing returns to scale. As in Koska and Stähler (2014), building on the famous Dornbusch-Fischer-Samuelson (DFS) model and focusing only on the equilibrium with factor price equalization, differences in both absolute and relative factor endowments give rise to trade in our model. The difference is that we have focused on inter-industry trade. We have considered two countries that are sufficiently asymmetric in terms of factor endowments, two factors of production that play different roles, and free market entry and exit that depends on both absolute and relative factor endowments. Hence, also the capital market is endogenous. We have shown that the general equilibrium effects of factor price changes play a crucial role for the welfare effects of trade.

In particular, we have shown that there is no coexistence of domestic and foreign firms in the same industry if factor prices are different between the countries, and that oligopolistic competition does not reverse the specialization patterns as they are well known from classical trade models. Unless one country specializes in production of a sufficiently small range of goods, the wage-rental ratio increases in both countries. While markets become larger due to trade integration, also production costs increase.

If the countries are sufficiently asymmetric in terms of their capital endowments, then a beneficial rationalization effect of free trade under different factor prices will be materialized unambiguously only in industries that are hosted by South, whereas the trade-induced changes in firm size and/or per-capita consumption are ambiguous. In an equilibrium in which factor prices do not equalize, the general equilibrium effects may make a country worse off. This is especially the case when countries are sufficiently asymmetric in their factor endowments such that the capital stock of one country is sufficiently large compared to that of the other country. In such a situation, when the two countries engage in free trade, the wage-rental ratio in the capital-abundant country substantially increases, with which establishing a firm becomes relatively cheap, reducing entry distortions for the capital-abundant country hosting capital-intensive industries. However, a small country may not be able to host a sufficiently large number of firms in labor-intensive industries, despite the specialization gains. Provided there is sufficient asymmetry in relative factor endowments, this effect is not overcompensated by the capital-abundant country's gains from specialization.

Appendix

A.1 Proof of Proposition 1

In an integrated market: $Y(z) = \sum_i y_i(z) + \sum_i y_i^*(z)$. The first-order conditions are

$$\begin{aligned}
\frac{\partial \Pi_i(z)}{\partial y_i(z)} &= p(z) - \lambda(z)w + p_{Y(z)}(z)y_i(z) \\
&= \frac{I^w}{Y(z)} - \lambda(z)w - \frac{I^w y_i(z)}{Y(z)^2} = 0, \\
\frac{\partial \Pi_i^*(z)}{\partial y_i^*(z)} &= p(z) - \lambda(z)w^* + p_{Y(z)}(z)y_i^*(z) \\
&= \frac{I^w}{Y(z)} - \lambda(z)w^* - \frac{I^w y_i^*(z)}{Y(z)^2} = 0.
\end{aligned}$$

All firms within industry z are symmetric such that $Y(z) = n(z)y_i(z) + n^*(z)y_i^*(z)$. Using symmetry, we find that

$$\begin{aligned}
y_i(z) &= \frac{I^w(n(z)+n^*(z)-1)(n^*(z)w^* - (n^*(z)-1)w)}{\lambda(z)(n(z)w + n^*(z)w^*)^2}, \\
y_i^*(z) &= \frac{I^w(n(z)+n^*(z)-1)(n(z)w - (n(z)-1)w^*)}{\lambda(z)(n(z)w + n^*(z)w^*)^2},
\end{aligned}$$

leading to maximized profits

$$\Pi_i(z) = \frac{I^w(n^*(z)w^* - (n^*(z)-1)w)^2}{(n(z)w + n^*(z)w^*)^2} - r\kappa(z) = 0,$$
$$\Pi_i^*(z) = \frac{I^w(n(z)w - (n(z)-1)w^*)^2}{(n(z)w + n^*(z)w^*)^2} - r^*\kappa(z) = 0,$$

which are equal to zero in equilibrium. Coexistence requires that both equations are independent. However, differentiation of maximized profits w.r.t. $n(z)$ and $n^*(z)$ leads to

$$|J| = \frac{\partial\Pi_i(z)}{\partial n(z)}\frac{\partial\Pi_i^*(z)}{\partial n^*(z)} - \frac{\partial\Pi_i(z)}{\partial n^*(z)}\frac{\partial\Pi_i^*(z)}{\partial n(z)} = 0,$$

which shows that the Jacobian is zero and proves that coexistence is impossible for different factor prices.

A.2 Proof of Proposition 2

Define the following functions

$$\theta(z) = w\left(1 - \sqrt{\frac{r^*\kappa(z)}{I^w}}\right), \phi(z) = w^*\left(1 - \sqrt{\frac{\kappa(z)}{I^w}}\right).$$

Using the domestic rental as a numeraire, that is $r = 1$, at $\hat{z} : \theta(\hat{z}) = \phi(\hat{z})$. Differentiation yields

$$\theta'(z) = -\frac{w}{2}\sqrt{\frac{r^*}{I^w\kappa(z)}}\kappa'(z) > 0, \phi'(z) = -\frac{w^*}{2}\sqrt{\frac{1}{I^w\kappa(z)}}\kappa'(z) > 0.$$

At any intersection of $\theta(z)$ and $\phi(z)$ in the $\theta/\phi - z$–space, either $\theta'(z) \geq \phi'(z)$ or $\theta'(z) < \phi'(z)$. If $\theta'(z) \geq \phi'(z)$, prices are (weakly) lower for all $z < (>)\hat{z}$ in the domestic (foreign) country and it must be that $\sqrt{r^*}w \geq w^*$. On the contrary, if $\theta'(z) < \phi'(z)$, prices are lower for all $z < (>)\hat{z}$ in the foreign (domestic) country and it must be that $\sqrt{r^*}w < w^*$. Hence, there can be only one $\hat{z}$ as two intersections would warrant $\sqrt{r^*}w \geq w^*$ and $\sqrt{r^*}w < w^*$ at the same time.

A.3 Proof of Proposition 4

A direct implication of Proposition 2 is that a certain vector of factor prices implies a unique $\hat{z}$. Suppose that $\hat{z}$ is given. Differentiating the equilibrium conditions

$$wL + K - \hat{z}I^w = 0,$$
$$K - \sqrt{I^w}\int_0^{\hat{z}}\sqrt{\kappa(z)}dz = 0,$$
$$K^* - \sqrt{\frac{I^w}{r^*}}\int_{\hat{z}}^1\sqrt{\kappa(z)}dz = 0,$$
$$I^w - wL - K - w^*L^* - r^*K^* = 0$$

yields

$$\begin{bmatrix} L & 0 & 0 & \hat{z} \\ 0 & 0 & 0 & \xi_{24} \\ 0 & 0 & \xi_{33} & \xi_{34} \\ -L & -L^* & -K^* & 1 \end{bmatrix} \begin{bmatrix} dw \\ dw^* \\ dr^* \\ dI^w \end{bmatrix} = 0, \tag{A.1}$$

where

$$\begin{aligned} \xi_{24} &= -\frac{1}{2}\sqrt{\frac{1}{I^w}} \int_0^{\hat{z}} \sqrt{\kappa(z)} dz < 0 \\ \xi_{33} &= \frac{1}{2r^*}\sqrt{\frac{I^w}{r^*}} \int_{\hat{z}}^1 \sqrt{\kappa(z)} dz > 0 \\ \xi_{34} &= -\frac{1}{2}\sqrt{\frac{1}{I^w r^*}} \int_{\hat{z}}^1 \sqrt{\kappa(z)} dz < 0. \end{aligned}$$

The Jacobian determinant of the matrix in (A.1) is equal to $\xi_{24}\xi_{33}LL^*$ and is thus unambiguously negative. The unambiguous sign implies that there is one and only one vector of factor prices for a given $\hat{z}$ and, due to Proposition 2, one and only one $\hat{z}$ for any vector of factor prices, and the equilibrium is unique.

Table 1: Simulation Results

	Factor Endowments				Relative Factor Prices					Income			# Firms				Welfare Change	
Sim#	K	K^*	L	L^*	w^a	w^{*a}	w	w^*	r^*	I^a	I^{*a}	I^w	n^a	n^{*a}	n	n^*	$v \times 10^8$	$v^* \times 10^8$
	Perfectly symmetric countries (FPE)																	
1	30	30	30	30	19.19	19.19	39.38	39.38	1.00	605.71	605.71	2422.87	20.38	20.38	40.77		985.99	985.99
	Asymmetric countries (NFPE)																	
2	30	20	30	30	19.19	8.30	33.26	33.02	1.58	605.71	269.20	2050.53	20.38	13.59	17.20	16.11	813.77	2587.37
3	40	20	30	30	34.56	8.30	59.59	58.76	2.82	1076.83	269.20	3647.28	27.18	13.59	22.95	16.11	317.86	2675.72
4	50	20	30	30	54.41	8.30	93.55	91.85	4.41	1682.55	269.20	5700.64	33.98	13.59	28.69	16.10	101.98	2721.91
5	60	20	30	30	78.76	8.30	135.13	132.31	6.36	2422.87	269.20	8210.60	40.77	13.59	34.43	16.10	−9.62	2749.94
6	70	20	30	30	107.59	8.30	184.33	180.13	8.66	3297.79	269.20	11177.20	47.57	13.59	40.17	16.10	−74.02	2768.64
7	80	20	30	30	140.91	8.30	241.15	235.31	11.31	4307.32	269.20	14600.30	54.36	13.59	45.91	16.10	−114.13	2781.94
8	90	20	30	30	178.71	8.30	305.59	297.85	14.32	5451.45	269.20	18480.10	61.16	13.59	51.66	16.10	−140.56	2791.87
9	100	20	30	30	221.00	8.30	377.65	367.76	17.69	6730.19	269.20	22816.50	67.96	13.59	57.40	16.10	−158.74	2799.55
10	110	20	30	30	267.78	8.30	457.34	445.03	21.41	8143.53	269.20	27609.40	74.75	13.59	63.14	16.10	−171.68	2805.66
11	120	20	30	30	319.04	8.30	544.65	529.66	25.48	9691.47	269.20	32859.00	81.55	13.59	68.88	16.10	−181.13	2810.63

Notes: The superscript a denotes the case of autarky. The numeraire for relative factor prices is the rental, that is, $r^a = 1$ for w^a, $r^{*a} = 1$ for w^{*a} and $r = 1$ for w, w^* and r^*, respectively. The total number of firms in the integrated world $(n + n^*)$ is reported in case of FPE (Sim # 1) as the number of firms in each country is ambiguous. The results are reported only up to two decimals.

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