The limits of wise exploitation in dependent and independent symmetric ensembles

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Abstract

Selfish motivated violence and deception are considered as harmful to the performance of a group; therefore, should have negative effects in animal and human societies and are thus condemned as amoral in human societies. Here, I investigate more deeply a recently discovered ethos (i.e. the characteristic spirit of a culture) called wise exploitation. In an ensemble with this ethos violence and deception are an essential part of the success, explaining the organized occurrence of both. The transfer of substrate from source to sink induced by violence and deception will be superadditive within certain limits. In case this superadditivity is able to pay all investments like brute force, deception, reciprocity, information costs, transfer costs, at least one surviving offspring and a premium, the transfer is called wise and will be better in comparison to no transfer and simple additivity. In comparison to the weaker and peaceful ensembles, wise exploitation is active in a different part of substrate availability. This ethos will not end in the Nash equilibrium because biologic systems are open systems continuously powered by the sun. The exploited party will not be lost as offspring will continue the function as a source of benefits or as a sink of costs. In case there is no genetic tradition there will be a tradition of functionality organized.

Keywords: source, sink, ensemble, brute force, deception, superadditivity, subadditivity, wise exploitation, prudent master, benefit, cost, net profit, reciprocity, reward, Nash equilibrium, Pareto efficiency, Homo Economicus, social dilemma, Snowdrift game, Stag-Hunt game, Prisoner’s dilemma, cooperation
Introduction

The idea of a wise exploitation in ensembles has been described earlier (1). However, my past investigation, based on findings and mathematical treatment by Turner and Chao ($s_1$, $s_2$ and $c$, in 2), was simple and did not include a careful net profit analysis of the ensemble within the transfer space.

In wise exploitation the redistribution of substrate from source to sink will increase the efficiency of the ensemble as a whole on cost of the source and/or the sink in a long lasting (wise) manner. In addition, there will be superadditivity. Although the source may give a benefit dominated substrate ($b-c>0$, $b/c>1$) the sink is able to overcompensate the loss of benefit in source by a higher benefit ($b-c>>0$, $b/c>>1$) in sink. On the other hand the source may give a cost dominated substrate ($b-c<<0$, $b/c<<1$) to the sink where it will be still a cost dominated substrate. And yet, it will be less costing there ($b-c<0$, $b/c<1$), reducing the total cost for the ensemble. In the end, the net profit ($b_e-c_e$) of such ensembles will increase due to increased benefit or reduced cost.

The ensemble may be a dependent ensemble. Within a dependent ensemble an external instance (master) is responsible for the transfer of substrate from source to sink (3). This may be peaceful (source $b-c<0$, sink $b-c>0$) but in case the source ($b-c>0$, $b/c>1$) will not give or the sink ($b-c<0$, $b/c<1$) will not take, this external instance will enforce a transfer. This enforcement will be a cost to the external command. Under wise condition the exploitation will be long lasting (offspring will be produced) and the net profit for the ensemble with forced transfer will pay through better efficiency. The investment of the external master including a reward to this authority will be paid.
Within an independent ensemble source and sink regulate the redistribution of substrate on their own without a master (4). The source will always stop to give at b-c=0 (b/c=1) and the sink will always stop to take at b-c=0 (b/c=1). This ensemble is characterized by a completely peaceful ethos. But independent ensembles may also use brute force or deception to further increase the amount of substrate transferred. Here, the source may want to increase the amount of benefit dominated substrate (b-c>0, b/c>1) and thus is not willing to give or the sink is full of cost dominated substrate (b-c<0, b/c<1) and may not be willing to take. In an independent ensemble a transfer of substrate will be forced by the other side not yet at b-c=0 (b/c=1). The source is forced under such conditions by the sink to give or the sink is forced by the source to take (4). Under wise conditions all investments of source or sink like force, deception, information, reciprocity, a reward and a new generation of source and sink are covered by the increase in efficiency of the ensemble.

In addition, I am going to analyse a prudent ethos in dependent and independent ensembles. Some important questions have jet to be addressed and will be investigated within the transfer space.

1. Who is in control of the increased efficiency?
2. What ethos will have the largest increase in efficiency?
3. Where are the limits to exploitation?
4. When will productive exploitation turn into consumptive exploitation and when will the ensemble better fall apart?
5. Why has the Nash equilibrium not to be considered in wise exploitation?
6. How expensive may reciprocity, force, deception and reward become?
Considerations, Calculations and Definitions

A substrate is Janus-headed thing. A cost attribute is contributed by the purchase of the total amount in the past and a benefit attribute exists as the catalysis will produce a benefit in the future. Moreover, the pure ownership confers cost. Condensed: the cost (dimensional unit c) of a substrate will increase in a linear fashion while e.g. in enzymes the product of this substrate - the benefit (dimensional unit b) - will appear with a saturating kinetic. Therefore, we observe regions where b-c>0 (b/c>1) or b-c<0 (b/c<1) and a point where b-c=0 (b/c=1) (figure 1).

The substrate S has two characteristics: benefit (b) and cost (c). The cost increases linearly (orange) and the benefit saturates (olive). Fix costs are generally not considered. The character of the substrate (b-c, brown dots; b/c, light green dots) is the result of the two characteristics. Increasing the amount and concentration of a substrate will change the character of the substrate from benefit domination to cost domination in the eye of the substrate owner.
I assume that benefits are produced in a saturating manner by enzymes. According to Michaelis-Menten the reaction velocity $v$ of an enzyme is:

$$v = ([S]/K_m+[S])*V_{max} \quad (v \text{ in } \mu M/\text{min}, \text{in my example } V_{max} = 5 \mu M/\text{min})$$

$[S] = \text{substrate concentration in mM (0-1mM in source } [S]_{so} \text{ and sink } [S]_{si})$

The cost is not calculated according to the consumed substrate but according to the amount of substrate per volume, $[S]$. $K_m = \text{Michaelis-Menten constant in mM, at } K_m \text{ (a specific and characteristic concentration for an enzyme, here 0.5mM) } v = V_{max}/2$

$V_{max} = \text{maximal reaction velocity, } \mu M/\text{min} \text{ (characteristic for an enzyme)}$

I introduce a benefit factor $bf; bf=b*\text{min}/\mu M$ and a linear cost factor $k; k=c/\text{mM}$. The dimensional unit behind $b$ and $c$ could be kilojoule (kJ) or euro (€)

Source and sink produce in steady state equilibrium e.g.:

$$1bf(b*\text{min}/\mu M)*5(\mu M/\text{min})*([S]/K_m+[S]); [S]=K_m=0.5\text{mM and}$$

$$5k(c/\text{mM})*[S]; [S]=K_m=0.5\text{mM}$$

Here, at $2.5b-2.5c=0$ the substrate has a neutral character according to benefit and cost (at $b-c$ the quality of $b$ and $c$ is identically; at $1b/c$ we have a different quality of $b$ and $c$).

Now, the use of the term “net profit” in this text in comparison to preceding texts has to be examined:

In economics net profit is maximized (net profit ($) = sales revenue ($) - total costs ($)). In older text I used the word “net profit” also for the difference of benefit and cost in source and sink. That was more than unfortunate. The substrate has a cost attribute; cost should be avoided.
or reduced. The substrate has a benefit attribute as a benefit will be produced from the substrate. Benefits should be sought and increased. At \( b-c=0 \) or \( b/c=1 \) a stable equilibrium between both types of aims is reached. With that idea in mind \( b-c \) in source and sink is minimized. In source and sink \( b-c \) is a measure which character dominates the substrate. The character of the substrate will determine the motivation of source and sink to give, give not, take, and take not. However, the character of the substrate is not an intrinsic feature of the substrate; it is a perceived, subjective feature. The character of new substrate is decided by the character of the substrate already owned by source and sink. The usage of the term “net profit” is saved for the ensemble balance \( (b_e-c_e) \).

In an inactive ensemble (red surfaces) no substrate is transferred.

\[
b_{so} = bf_{so} \cdot v_{so} = \left(\frac{[S_{so}]}{Km_{so} + [S_{so}]}\right) \cdot V_{max_{so}} ; c_{so} = k_{so} \cdot [S_{so}]
\]

\[
b_{si} = bf_{si} \cdot v_{si} = \left(\frac{[S_{si}]}{Km_{si} + [S_{si}]}\right) \cdot V_{max_{si}} ; c_{si} = k_{si} \cdot [S_{si}]
\]

The net profit of such an ensemble is: \( b_e-c_e = b_{so} - c_{so} + b_{si} - c_{si} \)

Within an active ensemble (green or blue surfaces) substrate is removed from the source \((-\Delta S)\) and transferred to the sink \( (+\Delta S) \).

\[
b_{so} = bf_{so} \cdot v_{so} = \left(\frac{[S_{so}-\Delta S]}{Km_{so} + [S_{so}-\Delta S]}\right) \cdot V_{max_{so}} ; c_{so} = k_{so} \cdot [S_{so}-\Delta S]
\]

\[
b_{si} = bf_{si} \cdot v_{si} = \left(\frac{[S_{si}+\Delta S]}{Km_{si} + [S_{si}+\Delta S]}\right) \cdot V_{max_{si}} ; c_{si} = k_{si} \cdot [S_{si}+\Delta S]
\]

The net profit of an active ensemble is again: \( b_e-c_e = b_{so} - c_{so} + b_{si} - c_{si} \). However, now the benefit and cost in source is reduced \((-\Delta S)\) and the benefit and cost in sink is increased \( (+\Delta S) \) by the same amount of substrate.
For every possible pair of concentrations within the upper and lower limit (0-1mM) in source and sink at first the substrate character of each side separately is calculated and then added as ensemble net profit ($b_e - c_e$, simple additivity). A value for a red surface is generated for each such pair of concentrations. Then the size of $\Delta S$ (the size of the transfer) is calculated as the concentration difference to a border (e.g. a wise limit or $b-c=0$). The necessary amount of substrate for source and sink to become a concentration pair on this border is moved from source to sink (only this direction) and the substrate character and the net profit of the ensemble is calculated again. A value for a green or blue surface is generated but in the same pair of concentration coordinates as for the red surface (the old concentration coordinates). The result is a comparison of the ensemble net profit ($b_e - c_e$) of a pair of concentrations before and after a transfer.

In the individual point of view the source wants to get rid only of a cost dominated substrate and the sink only takes when substrate is still benefit dominated. That is the basic motivation to give or give not, to take or take not. The loss of a cost dominated substrate is a gain as well as the gain of a benefit dominated substrate (superadditivity, green surface, below). The ensemble is non-linear. Therefore, the loss of a benefit dominated substrate may be overcompensated by the gain of a benefit dominated substrate (superadditivity, blue surface over red surface, below). But subadditivity is also possible (blue under red surface). There, the loss of a benefit dominated substrate is only partially compensated by the gain of a benefit dominated substrate. Or the loss of a cost dominated substrate will be exceeded (or not) by the gain of a cost dominated substrate in the other side. The worst case is when source loses a benefit dominated substrate and sink gains a cost dominated substrate.
The substrate concentration in source and sink and the net profit of the ensemble (or the benefit to cost ratio of the ensemble) form the transfer space (figure 2).

Figure 2

The red arrows stretch out a three dimensional space (transfer space) with the axes substrate concentration in source, substrate concentration in sink and net profit of the ensemble \( \text{be-c_e} \). The ensemble is not yet visible. On the two sides we see the two dimensional projections of source or sink alone according to benefit (saturating), cost (linear) and substrate character (dotted, with an optimum). The space can be divided into 4 regions of distinct economic logic (on top): area I: source \( b-c<0 \), sink \( b-c>0 \); area II: source \( b-c<0 \), sink \( b-c<0 \); area III: source \( b-c>0 \), sink \( b-c>0 \); area IV: source \( b-c>0 \), sink \( b-c<0 \).
Source and sink are two dimensional entities. The ensemble within the transfer space is a three dimensional entity. If we enter the space from one side we will observe a discontinuity. Four areas with different economic logic are observable on top.

In area I (source b-c<0 and sink b-c>0) it is reasonable for the source to give a cost dominated substrate and for the sink to take a benefit dominated substrate. This will be always superadditive. In area II (source b-c<0 and sink b-c<0) and area III (source b-c>0 and sink b-c>0) it depends on the relative sizes. This will be carefully investigated later. In area IV (source b-c>0 and sink b-c<0) source would give a benefit dominated substrate and sink would take a cost dominated substrate. This is an irrational transfer that should not be observable in rational and independent acting entities. Several types of irrationality have been already discussed (4). Irrationality is observed when subadditivity of the ensemble is the result of a transfer. Here, in area IV not only one side acts irrationally but the whole ensemble. No transfer (simple additivity) would be a rational behaviour then.

Brute force and deception change the perception of cost and benefit in source and sink (4). The external observer may notice an irrational behaviour of a single party. However, in consideration of additional cost (additional cost through force) or delusion about the real cost the forced or deceived party still acts rational confined to the own perception (4). The ensemble may be rational if superadditive but on cost of one party (area II and III).

Glossary and Definitions:

Source: a productive entity where substrates may come from.

Sink: a productive entity where substrates may go to.
*Ensemble*: a unity; a productive entity consisting of source, sink, a cluster of equally likely concentrations and maybe a master.

*Inactive ensemble*: an ensemble with no transfer of substrate.

*Active ensemble*: an ensemble with transfer of substrate.

*Symmetric ensemble*: an ensemble with identical Km, Vmax, bf and k values in source and sink.

*Asymmetric ensemble*: an ensemble with different values of Km, Vmax, bf and k in source and sink.

*Master*: the master is a third party and can force and deceive source or sink or both to cross the border b-c=0 (b/c=1). The source will give then a benefit dominated substrate and end in b-c>0. The sink will take then a cost dominated substrate and end in b-c<0. Both end-conditions are not favourable for the respective party. The master does not contribute directly to the net profit. The masters range from free apps connecting source and sink to complete parasitic elites – with the very best of intentions. The master may be real or a useful fiction set into the world by real, true masters.

*Prudent master*: a prudent master knows that in a symmetric ensemble equal distribution of substrate has the highest net profit and he will achieve this either in peace or with force and deception. In an asymmetric ensemble this would not be a prudent master. Although the master may feel just or morally superior, such a treatment would start irrationality (4). The prudent master tries to move source and sink to the line of strict equivalence. This is comparable to mixing of the different concentrations in source and sink (3). In contrast, this time the cost stays with the substrate as the master does not use destructive mixing but a prudent transfer.
Exploitation: the act of employing a source or a sink or both to the greatest possible advantage. This will be possible only for a very short time as the Nash equilibrium will be reached.

Wise exploitation: the balance of wise exploitation may be superadditive and pay a reward, force, deception, information, a transfer cost and reciprocity. In addition, source or sink are exploited in a way that they are able to regenerate in an open system on a short time scale or on a long time scale. The essence of the long time scale is that the exploited party is able to produce one offspring to finally replace this party. A lack of choice is the price. As we are in an open system a Nash equilibrium will not be reached. The essence of the shorter time scale is the fact that source and sink regenerate daily in their lifetime but no longer breed.

Modest wise limit: a long term strategy. The source loses an amount of substrate with beneficial character. With the residual beneficial amount of substrate it is still possible for the source to reproduce once. The sink gains a load, a substrate with a cost dominated character, but is still able to reproduce once. The exploitation will last many generations (figure 3).

Bold wise limit: a short term strategy. The source loses a substrate which is benefit dominated but is still able to recover for the next round of exploitation with the residual substrate and replace the loss. The sink gains a substrate which is cost dominated. Although this is a burden, sink is able to stay alive and regenerate for the next round of exploitation. Exploitation will last only one generation (figure 3).

Superadditivity: the net profit of the active ensemble is larger than of the inactive ensemble; synonym to rationality. Superadditivity will appear in area I to III.
Subadditivity: the net profit of the active ensemble is smaller than of the inactive ensemble; synonym to irrationality. Subadditive productivity has still residual productivity above zero. The residual productivity of the ensemble could be larger (simple additivity) when the ensemble would become inactive. Then, however, one side will have an even smaller productivity than under subadditivity. Subadditivity will appear in area II to IV. (An exception is the basically violent ensemble; there, subadditivity may appear in all four areas.)

Strict symbiosis: the transfer of substrate from source (at b<c) to sink (at b>c) leads to a superadditive increase \((b_e-c_e)_{\text{transfer}}> (b_e-c_e)_{\text{no transfer}}\) in net profit and comes to exactly 50% from source and to 50% from sink. This is a symmetric win-win situation (area I, figure 3). Both parties contribute identically to the superadditive net profit increase. This contradicts the idea of Pareto efficiency; both parties are better off in area I.

Strict antibiosis: the transfer of substrate from source (at b>c) to sink (at b<c) leads to a subadditive decrease \((b_e-c_e)_{\text{transfer}}< (b_e-c_e)_{\text{no transfer}}\) in net profit and comes to exactly 50% from source and to 50% from sink. A symmetric lose-lose situation. The loss of net profit in the ensemble is generated equally in both parties and defined as irrationality due to subadditivity (area IV, figure 3). No Pareto efficiency in area IV; both parties are worse off.

Strict equivalence: a borderline. Every transfer leading to concentration pairs on this line will be maximal and results in the same net profit as the inactive ensemble there. The net profit of the inactive ensemble is maximal along this line and within the active ensemble the loss of net profit by source is compensated exactly by the gain of net profit in sink. Left and upwards to the line of strict equivalence (blue line in figure 3) we observe productive (superadditive) exploitation, right and downwards to
the line of strict equivalence we observe consumptive (subadditive) exploitation.

*Productive exploitation*: located in area II and III left of the blue line in figure 3. Transfer of substrate results in more net profit of the ensemble gained in sink than lost in source – superadditive.

*Consumptive exploitation*: located in area II and III right of the blue line in figure 3. Transfer of substrate results in less net profit of the ensemble gained in sink than lost in source – subadditive.

*Transfer space*: this space has the three coordinates substrate concentration in source $[S]_{so}$, sink $[S]_{si}$, and net profit $(b_e-c_e)$ of the ensemble (figure 2) or the benefit to cost ratio $(b_e/c_e)$. There, ensembles without or with transfer of substrate are compared. When benefit and cost have the same quality net profit $(b_e-c_e)$ is used; when quality of benefit and cost are different, the quotients $(b_e/c_e)$ should be used. Exactly the second possibility would be an adequate treatment in Biology as e.g. glucose spent ($c$) will be compared to protein earned ($b$) in a hunting ensemble (e.g. a wolf). In the following text the net profit representation is preferred as in the quotient version all values of $b/c<1$ are squeezed between 1 and zero (figure 1).

*Ensemble path*: an ensemble path connects the substrate distribution of source and sink in a specific concentration coordinate before a transfer with the concentration coordinates after the substrate transfer. The ensemble path starts at any specific pair of concentrations in source and sink. In case source and sink become active both concentrations will change; an exemplary green arrow (peaceful) or blue arrow (force, deception) points to the coordinate where the new concentrations would be. However, the resulting net profit remains with the origin coordinate where it can be compared to the net profit of the inactive ensemble.
A schema of the ensemble space with respect to borders and limits is shown in a top down view like in all following figures. The numbered orange arrows identify the following lines:

1. upper limit of source (1mM); 2. lower limit of sink (0mM); 3. line of strict symbiosis; 4. line of strict equivalence separating productive exploitation from consumptive exploitation; 5. line of strict antibiosis; 6. a modest wise limit in source at 0.4mM; 7. a bold wise limit in source at 0.3mM; 8. lower limit of source (0mM), at this concentration the source will immediately collapse due to lack in substrate; 9. upper limit of sink (1mM), at this maximal load the sink will immediately collapse per definition due to overloading; 10. a wise modest limit in sink at 0.6mM; 11. a wise bold limit in sink at 0.7mM. In addition we again observe the four areas of figure 2:

- area I: source b-c<0, sink b-c>0  
- area II: source b-c<0, sink b-c<0  
- area III: source b-c>0, sink b-c>0  
- area IV: source b-c>0, sink b-c<0

The four areas are separated by two red lines forming a cross. There, at 0.5mM substrate in source we find $b_{so} - c_{so} = 0$ and at 0.5mM substrate in sink we find $b_{si} - c_{si} = 0$. 
Results

At first we look at a dependent ensemble with a third party as master. This master starts peacefully (figure 4a, 4b) as an agent and then becomes increasingly demanding (figure 5a, 5b) and finally becomes a predator (figure 8) taking all away from source resulting in a break down there and loading it as burden onto sink leading to a complete breakdown there, too.

Figure 4a

**Dependent ensemble, peaceful master:** We look at the schema of dependent ensemble controlled by a peaceful master. The master will not force source or sink to cross the limit b-c=0. The line of strict equivalence (blue line) is reached only in one point; b_{so}-c_{so}=0 and simultaneously b_{si}-c_{si}=0. The ensemble exists only in area I. The green arrows highlight single exemplary and peaceful ensemble paths (of many parallel ones) along which substrate is transferred from source to sink. As there are many combinations of concentration in source and sink there are many shorter arrows within each green arrow all ending at b-c=0. The absolute maximal length of a green arrow will reach from 1mM in source at 0mM in sink to 0.5mM in sink at 0.5mM in source. There is only one such long arrow following the line of strict symbiosis.
**Dependent ensemble, peaceful master:** The calculated surfaces in a top down (bird’s eye) perspective of the dependent ensemble in figure 4a controlled by a peaceful master in the borders of 0mM to 1mM substrate in source and 0mM to 1mM substrate in sink. The red surface represents the inactive ensemble and the green surface is the active ensemble always on top of the red surface. The blue line of strict equivalence and the purple lines of strict symbiosis and strict antibiosis are added for orientation. The volume between the green and red surface is +0.094205 net profit*mM^2. Side views of similar ensembles are given in literature 4 or in figure 6b.

An ensemble is a bent plane in three dimensions. Between the red and the green surface (or later blue) emerges a volume with the dimension net profit*mM^2. In figure 4b the calculations of the inactive (red surface) and active (green surface) ensemble are shown. The volume of superadditivity (green over red) is +0.094205 net profit*mM^2. The results of all other determinations of super- and subadditivity are summarized in table 1 and 2. In this ensemble we do not observe any subadditivity (red over green or blue over green) which would be necessary to subtract. The peaceful ensemble needs not to be really peaceful but the transfer
ends at b-c=0. A peaceful ethos is rational as it will not waste net profit with useless force or deception.

In the following figure we look at two dependent ensembles again controlled by a third party as master. After the transfer comes to a halt (b-c=0) the master pushes both ensembles by force and deception (blue arrows) to limits beyond the border of b-c=0 at 0.5mmol/l. An additional amount of substrate is transferred (source gives and sink takes additional 0.1mmol/l – 5a left L or 0.2mmol/l – 5a right R).

**Figure 5a**

Dependent ensemble, controlled by a master, modest (L) and bold (R), wise exploitation: We look at the schema of dependent ensemble controlled by a master using force and deception. The master starts peacefully (green arrows) and when source or sink stop to give or take the master will use force and deception (blue arrows) to reach a certain size (modest or bold wise limits) of transfer.

In the left picture the master uses force or deception to push source (0.4mM) and sink (0.6mM) to the modest limit (black line). This ensemble is active in all 4 areas. In the right picture the master uses force or deception to push source (0.3mM) and sink (0.7mM) to the bold limit (black line). This ensemble is also active in all 4 areas. The line of strict equivalence (blue line) is exceeded from the productive to the consumptive side in both pictures.
This situation, as in all following examples of forced transfer, differs from an older approach (4). There, 25% of substrate was more (source) or less (sink) transferred to pay the force. This time it is assumed that the superadditivity obtained is sufficient to pay the whole bill including force.

In the next figure (5b) we look at the calculated surfaces of the two dependent ensembles with two different borders of exploitation (modest wise limit at 0.4mM in source and 0.6mM in sink (5b left) or bold wise limit at 0.3mM source and 0.7mM in sink (5b right)). Below the modest limit the ensemble has no long term perspective and below the bold limit the ensemble has no short term perspective.

**Figure 5b**

**Dependent ensemble, controlled by a master, modest (L) and bold (R), wise exploitation:** The calculated surfaces of two ensembles: an inactive ensemble (red) and an active ensemble (peaceful part green, transfer by force and deception blue) with a modest wise limit (left) and a bold wise limit (right) in a top down perspective. A detailed description is given in figure 5a. At concentrations near 0.5mM parts of the blue surface are under the red surface and are thus subadditive (table 1). The size of this area is indicated by blue dots within the red surface, a useful bug of the program.
A master as third party may be a prudent master. A prudent master knows that in a symmetric ensemble symmetric distribution of substrate will give the highest net profit (figure 6a and 6b). A prudent master has to use force and deception to cross the border $b-c=0$ in source and sink. He will stop at line of strict equivalence.

**Figure 6a**

**Dependent ensemble, prudent master:** In this picture we observe a dependent ensemble with a prudent master. The prudent master will take an amount of substrate from source that is necessary to make the substrate concentration in this symmetric ensemble identical on both sides. This is done also with force and deception by crossing $b-c=0$ and enter area II and area III. The line of strict equivalence is never crossed; the basic principle of prudence.

Although the prudent master will make the most out of the unused potential to produce net profit in source and sink, he will cross in the lower concentrations of source and in the higher concentrations of sink the modest and/or bold limit of wise exploitation. There he will harm both
parties and the ensemble will lose a long or short term perspective. He may be prudent but his exploitation then is not wise and therefore will not last long. In figure 6b we look at the calculated surfaces.

Dependent ensemble, prudent master: The calculated surfaces of a dependent ensemble with a prudent master: inactive ensemble (red) and an active ensemble (in green the peaceful part; in blue the part with violence and deception); no subadditivity observable. The small insets on both sides are added to give at least in this figure an impression of the complex three dimensional look. The black arrows indicate the viewpoint from the side.

Finally, the prudent master may wisely exploit source and sink. He may stop at the modest (figure 7a left, 8a left) or at the bold limit (figure 7a right, 8a right).
Dependent ensemble, a prudent master in modest (L) and bold (R) wise exploitation: schema of a prudent master observing wise limits (modest, left; bold, right).

Dependent ensemble, prudent master, modest (L) and bold (R), wise exploitation: The calculated surfaces viewed in top down perspective. Red: inactive ensemble, green: active and peaceful, blue: active with force and deception. Subadditivity is not observable.
The result will be that source and sink are not destroyed on the long (modest) or on the short (bold) run. In addition, the ensemble avoids subadditivity in the central part of the concentrations around 0.5mM substrate.

The predatory master immediately consumes source and sink. There is no medium or long term perspective for the ensemble (figure 8). The advantage of such a master is that he does not have to wait for a complete live cycle or many generations. He takes all and now, then he looks for the next victims. The break down is immediately.

Figure 8

**Dependent ensemble, predatory master:** We observe a predatory master exploiting a source and a sink both to completeness. The source is reduced to a concentration of 0mM and will collapse instantly and the sink is overloaded to 1mM and will collapse there per definition, too. The blue surface is everywhere under the red surface; a sign of a complete irrationality.
The behaviour of the predatory master seems to be completely irrational. Does the model lead to an unrealistic result with no counterpart in the real world; have we found a weak point in the model?

The predatory master withdraws at first the superadditive net profit (volume between green and red surface. And then he pushes the ensemble further beyond b-c=0. There, the ensemble is irrational (red over blue) but still producing net profit (blue surface over ground). This net profit can be harvested by the predatory master and is in addition to the superadditive net profit. The additional net profit comes from destructive harm to source and sink.

The master does not participate in the direct production of net profit. Net profit is only produced in source and sink. The master acts as an agent. There are many ways to measure the performance of an agent and determine the size of the payment for his service. One possibility is that the master is paid according to the success of the ensemble (additional net profit, superadditivity, quality). This is difficult to determine when there will be a time lag. In contrast, it is easy to determine the amount transferred; the quantity. In that case the master has an interest to transfer as much as possible. The whole setting is well known to everybody: socialise the losses; privatise the gains. Source and sink may be lost after this transfer. In case the probability is high for the master to find soon again a new source and a new sink he is able to repeat his strategy. This behaviour can be very effective if nobody keeps a record on the fate of the ensembles the master was in charge of.
Now we are going to explore the independent ensemble. It can be a peaceful ensemble (figure 9a, 9b) stopping to give or to take when at least one side has reached $b-c=0$. This implies many results where one side will not reach the equilibrium between cost and benefit. There is only one point when both simultaneously stop to give and stop to take.

At this point the line of equivalence is reached and strict symbiosis would change immediately to strict antibiosis. The concentrations in this example are 0.5mM in source and 0.5mM in sink, right in the centre of the picture. Source and sink are only there at $b-c=0$ simultaneously.

**Figure 9a**

**Independent ensemble, peaceful:** This is the schema of an independent, peaceful ensemble and looks like a dependent ensemble with a peaceful master (4a). Source gives to get rid of cost dominated substrate until $b-c=0$ and sink takes benefit dominated substrate until $b-c=0$. 
Independent ensemble, peaceful: Here the calculated surfaces of a peaceful and independent ensemble are presented in a top down view; it looks like 4b. The superadditive volume between the green and red surface is as always +0.094205 net profit*mM^2.

The result of the calculations looks like a dependent ensemble with a peaceful master (figure 4b). Only area I is accessible. The calculated surfaces appear within the borders of 0mM to 1mM substrate in source and 0mM to 1mM substrate in sink. The red surface represents the inactive ensemble and the green surface is the active ensemble always on top of the red surface. The blue line of strict equivalence and the purple lines of strict symbiosis and strict antibiosis are added for orientation. The volume between the green and red surface is +0.094205 superadditive net profit*mM^2, too (table 1).

In the next figure we look at an independent ensemble where either source forces sink to take until the modest wise limit is reached (10a left,
On the left the source is in control. The source will use force or deception to push sink only to the modest wise limit of 0.6mM. Source will stop at $b_{so}-c_{so}=0$ in its own interest. The line of strict equivalence is crossed near 0.5mM. Area I and partially area II are accessible. On the right side the sink is in control. The sink will stop at the modest wise limit of source (0.4mM). The own limit is also followed; sink stops at $b_{si}-c_{si}=0$ in its own interest. Area I and partially area III are accessible. The line of strict equivalence is crossed from the productive to the consumptive side in both pictures near 0.5mM substrate. The volumes of superadditivity and subadditivity are given in table 1.

**Independent ensemble, source as master, modest wise exploitation (L), sink as master, modest wise exploitation (R):** We look at the schema an independent ensemble in wise exploitation. There is no third party. Source or sink are in control.
Independent ensemble, source as master, modest wise exploitation (L), sink as master, modest wise exploitation (R): Top down view of the calculated surfaces. Parts of the blue surface are under the red surface and are thus subadditive (table 1).

The modest limit is long lasting as one offspring will be produced. However, the price is a lack of choice and in case this offspring is a mutation and will not fit completely into the task the ensemble will face problems. Additional investments may be necessary to adjust task and ability. This investment will reduce the reward of the exploiting party.

While the modest wise limit appears to be a long term perspective for the independent ensemble, the bold wise limit (figure 11a and 11 b) has only a short term perspective – one lifetime. Within this timeframe the ensemble is stable as the exploited source or the exploited sink is able to regenerate. The ensemble will slowly decay below this limit. This is still an extended time frame in comparison to a predatory behaviour (figure...
15) with immediate consumption. However, the Nash equilibrium looms ahead.

**Independent ensemble, source as master, bold wise exploitation (L), sink as master, bold wise exploitation (R):** We observe a wise independent ensemble with source or sink in control. There is no third party as master but source or sink is the master.

On the left the source is in control. The source will use force or deception to push sink to the bold wise limit of 0.7mM. Source stops at \( b_{so} - c_{so} = 0 \), the line of strict equivalence is crossed. Area I and larger parts of area II are accessible. In case the sink is in control (right) the sink will stop at the bold wise limit of source (0.3mM). The internal limit is also followed - sink stops at \( b_{si} - c_{si} = 0 \). Area I and larger parts of area III are accessible. The line of strict equivalence is crossed from the productive to the consumptive side in both pictures to a larger extend than in figure 10.
Independent ensemble, source as master, bold wise exploitation (L), sink as master, bold wise exploitation (R): The calculated surfaces of a wise independent ensemble with one side in control stopping at a bold limit of exploitation. We look top down on the surfaces of an inactive (red), active but peaceful (green) and violent ensemble (blue). Parts of the blue surface are subadditive and therefore under the red surface (simple additivity, inactive ensemble).

An ensemble respecting the modest or bold wise limit will have a perspective. However, to follow the modest wise limit and the bold wise limit will produce a considerable amount of subadditivity as the line of strict equivalence is crossed. The prudent master showed a way to avoid subadditivity – never cross the line of strict equivalence; that is the essence of prudent behaviour. This will be observed also in the independent ensemble in the following figures.
At first we observe a prudent source (12a and b; left L) and a prudent sink (12a and b; right R). The prudent ethos brings all concentration pairs left of strict equivalence to the line of equivalence. There, in source and sink we have identical concentrations and the maximal productivity. On the left in 12a the source exploits the sink to take substrate up to the border of strict equivalence. The source will not cross $b_{so} - c_{so} = 0$. Area I and 50% of area II is accessible. The line of strict equivalence is not crossed. On the right the sink takes away from source until the line of strict equivalence is reached or sink stops at $b_{si} - c_{si} = 0$. Area I and 50% of area III is accessible.

**Figure 12a**

Independent ensemble, source as prudent master (L); sink as prudent master (R): a schema of source or sink in prudent exploitation of the other side. No wise limits are included.
Independent ensemble, source as prudent master (L); sink as prudent master (R): a top down view on the calculated surfaces. Green: the peaceful part of the ensemble. Blue: source or sink in prudent exploitation of the other side. On the left the source exploits the sink to take substrate up to the line of strict equivalence. On the right the sink exploits the source to give up to the line of strict equivalence.

The pure prudent master source or sink will not have a long term perspective in certain concentrations; the ensemble will slowly decay.

In figure 13 and 14 we observe a prudent ethos respecting long term (modest) or short term (bold) limits of wise exploitation. The prudent ethos will lead to the largest net profit in this symmetric ensemble including force and deception. The source as master will not cross $b_{so}-c_{so}=0$; the sink as master sink will not cross $b_{si}-c_{si}=0$, both acting in own interest. In addition, source will not cross the modest limit in sink of 0.6mM and the sink will not cross the modest limit in source of 0.4mM. As both are prudent masters they will not cross the blue line of strict equivalence. The source controlled ensemble is active in area I and II and the sink controlled ensemble is active in area I and III.
Independent ensemble, source as prudent master, modest wise exploitation (L); sink as prudent master, modest wise exploitation (R): One side is in control respecting the modest wise limit, the own limit \((b-c=0)\) and the prudent limit.

Independent ensemble, source as prudent master, modest wise exploitation (L), sink as prudent master, modest wise exploitation (R): Here we look at the calculated surfaces of a prudent and wise independent ensemble. The active ensemble is in green and blue, the inactive ensemble is in red depicted.
The prudent master source (left) or the sink (right) respecting the bold limit is depicted in figure 14. Again the prudent source will give substrate to sink in a way that the concentrations are equalized on both sides. The prudent sink will take away substrate from source in the same way. This will lead to the largest net profit including the use of force and deception. The source as master will not cross $b_{so}-c_{so}=0$ and the sink as master will not cross $b_{si}-c_{si}=0$ in own interest. In addition, source will not cross the bold limit in sink of 0.7mM and the sink will not cross the bold limit in source of 0.3mM. As both are prudent masters they will not cross the blue line of strict equivalence. The source controlled ensemble is active in area I and II, the sink controlled ensemble is active in area I and III. (All calculated volumes in table 1.)

Figure 14a

Independent ensemble, source as prudent master, bold wise exploitation (L), sink as prudent master, bold wise exploitation (R): In this schema we observe an independent and prudent ensemble in bold wise exploitation.
Independent ensemble, source as prudent master, bold wise exploitation (L), sink as prudent master, bold wise exploitation (R): Here we look at the calculated surfaces of a prudent independent ensemble in bold wise exploitation. Red inactive, green active and peaceful, blue active with force and deception.

The volumes of super and subadditivity are given in table1. The size of super- or subadditivity for sink as master is always much larger.

Finally we look at a predatory source or a predatory sink in an independent ensemble. On the left in 15a and 15b the source completely consumes (exploits) the sink by overloading. The sink is loaded with a cost dominated benefit up to the upper limit (1mM) but the source will not cross $b_{so} - c_{so} = 0$. Area I and II are completely accessible. The line of strict equivalence is crossed, too. The ensemble is in area II completely subadditive (red over blue, 15b left). On the right the sink completely consumes the source. All substrate is taken away from the source (rest 0mM) or the sink stops at $b_{si} - c_{si} = 0$. Area I and III are completely accessible. The line of strict equivalence is crossed. The ensemble is in area III completely subadditive (red over blue, 15b right).
Figure 15a

**Independent ensemble, predatory source (L), predatory sink (R):** A predatory independent ensemble is depicted. On the left the source completely consumes (exploits) the sink by overloading or stops at $b_{so}-c_{so}=0$. On the right the sink completely consumes the source or the sink stops at $b_{si}-c_{si}=0$.

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Figure 15b

**Independent ensemble, predatory source (L), predatory sink (R):** The calculated surfaces of 15a but the blue surfaces in area II and III hide under the red surface.
In the last figure (16) I briefly touch an ensemble not covered by the basic definition. The basic definition is that a source will give a cost dominated substrate voluntarily or a sink will take a benefit dominated substrate voluntarily until $b-c=0$. There, at the limit $b-c=0$ the transfer will stop. Force and deception are only necessary for the region beyond that limit. However, the master can use also force in area I to increase the total amount already there. This master is a basically violent master (figure 16).

**Dependent ensemble, basically violent master, modest wise exploitation:** The violent master does not wait with the use of brute force or deception until source or sink have reached the limit of peaceful transfer ($b-c=0$). He takes in one step to a limit beyond $b-c=0$.

Schema on the left: The master takes already in area I the complete amount of substrate bringing the ensemble to area II, III and IV with a modest wise limit (no prudence). The price the ensemble pays is subadditivity in area I. Now subadditivity appears on the productive side of the line of strict equivalence in area I. This is not the case in all other examples. The superadditivity (+0.110484) comes from 3 areas and is larger than in a peaceful ensemble confined to area I. This superadditivity has to be corrected for the appearing subadditivity in all 4 (!) areas (-0.000675). The residual superadditivity is +0.109809 net profit* mM$^2$. 
In the basically violent ensemble with a basically violent master irrationality is also observable in area I. It is notably that a basic violent ensemble with the same biochemistry like a peaceful ensemble creates about 16.56% more superadditivity for the price of 0.72% irrationality (table 2). However, this superadditivity is created in a larger concentration range including area II and III ending at the modest wise limit. Thus, a direct comparison with the peaceful ensemble is not possible. With good reason we may compare the basically violent ensemble with the ensemble of figure 5left (5L, table 1). This ensemble is peaceful in area I with an identical superadditivity like the peaceful ensemble. Using force and deception to reach the modest wise limit, this master creates an additional 5.87% superadditivity at the price of 0.49% subadditivity (irrationality) in area II and III. The complete balance results in 90.4% superadditivity in comparison to the basically violent ensemble (master). This may disappoint, but I speculate a little on the internal distribution of the net profit. A basically violent master has to use force against source and sink in all concentrations including concentrations where the master of figure 5L has no extra expenditures. Therefore, the internal balance of the partially peaceful ensemble according to the reward for the master may look differently. This may be especially true as we assume equal probability of all concentrations. The basically violent master will have much higher expenditures for force and deception, diminishing his rewards probably far below the rewards of the partially peaceful master. In case of conflict between the masters of a basically violent ensemble with a modest wise limit and a partially peaceful ensemble with a modest wise limit, what will be critical? Will it be the size of the force or the size of the rewards? What is the trade-off between force and rewards; what leverage effect can be observed when the rewards are invested into research and development?
Discussion

Dependent ensembles with a master and independent ensembles on own authority appear within the three dimensional transfer space as curved surfaces. The master may be peaceful, wise, prudent, wise and prudent or predatory. The single party (source or sink) in independent ensembles may also be peaceful, wise, prudent, wise and prudent or predatory. All ensembles are completely superadditive in area I with the exception of the basically violent ensemble (figure 16) which does not follow the general setting of a voluntary transfer from source to sink in the limits of source b-c<0 and sink b-c>0.

Area I

This is an always peaceful area. In area I source is loaded with cost dominated substrate (b<c) and wants to get rid of this load. The sink could increase the benefit with more substrate as it is still benefit dominated (b>c), lacks additional, still beneficial substrate and therefore wants more substrate. A transfer of substrate from source to sink either induced by accident (collision), by a broker (master, dependent ensemble) or by the free decision of source and sink (independent ensemble) will lead to superadditivity of the ensemble and will end in source and sink at b-c=0; not necessarily simultaneously. Reciprocity here is not necessary because both parties have an advantage (win-win situation). Only exploitation needs reciprocity. In the dependent ensemble the agent must be paid. The question is by whom, how much, and at what proportion if one side is not yet at b-c=0. In the independent ensemble some sort of signalling, a display of information on the status as willing source and sink, has to be paid. On top, maybe some transfer costs in all cases. The net profit in area I is always 0.094205 net profit*mM^2 in the limits of this biochemical setting here. The frayed look
of the colours near borders in general is a problem of the program to depict the right colour of values in close vertical neighbourhood as we look top down on overlapping, three dimensional surfaces. Area I is understood in the two dimensional world of game theory as the area of cooperation (figure 2 in literature 5). However, cooperation is not an aim of evolution. Evolution has no aim besides increasing efficiency; cooperation is just a condition lacking subadditivity and has in addition no other expenses reducing the net profit. Cooperation is also no structure-less area of puppies and kittens or peace, love and harmony. Usually one side will not reach b-c=0. That will be the starting point to go further in case there are ways to do so. Cooperation is not an end point but it may be a starting point.

Area II

In area II source is loaded with a cost dominated substrate (b<c) but sink is also loaded with cost dominated substrate (b<c). Therefore, sink is not willing to accept the substrate of source. In area II it is not preferred to take. The master or the source or a coalition of both will use brute force or deception to make the sink accept additional substrate. The starting point for the use of force and deception is the upper limit of the ensemble (1mM) or the border b-c=0 of sink. In this area cost is rearranged. A considerable part of the ensemble in this area is superadditive. This part is completely on the productive side of the line of strict equivalence. The additional superadditivity however comes only from source. In area II the source will overcompensate the loss in sink. Other parts of the ensemble in this area are already subadditive, although they are on the productive side of the line of strict equivalence. Here the transfer is too large leading onto the consumptive side. This happens in the centre of the ensemble. As soon as the ensemble will cross the line of strict equivalence to
become consuming it is always subadditive and under the red surface of the inactive ensemble.

This differs from older observations (4). There, by force and deception only residual 75% substrate was transferred of 100% substrate for source to reach b-c=0. In the past example 25% of the total amount was already consumed by the measures to transfer by force and deception. Now it is assumed that 100% is transferred and that the bill is paid by the developing superadditivity within the ensemble. Therefore, this time we do not observe consuming superadditivity. Area II is interpreted by others as the area of Snowdrift game. The colourful and smooth transition (figure 2 in literature 5) is owed to nearly linear drop of net profit in area II in my three dimensional model.

Area III

In area III source and sink lack substrate with a benefit domination (source b>c, sink b>c). In area III giving is not preferred. Therefore, source is not willing to give substrate to sink. The master or the sink or a coalition of both will use brute force or deception to make the source give valuable substrate. The starting point for the use of force and deception is the lower limit in sink (0mM) or the border b-c=0 of source. In this area benefit is rearranged. A considerable part of the ensemble in this area is superadditive. This part is completely on the productive side of the line of strict equivalence. The additional superadditivity however comes only from sink. In area III the sink will overcompensate the loss in source. Other parts of the ensemble in this area are already subadditive, although they are on the productive side of the line of strict equivalence. This happens in the centre of the ensemble. Here the transfer is too large leading onto the consumptive side. As soon as the ensemble will
cross the line of strict equivalence to become consuming it is always subadditive and under the red surface of the inactive ensemble.

This differs from older observations (4). There, by force and deception 125% substrate was transferred from source to sink to reach \( b-c=0 \) in sink. 25% of which were consumed by the measures to transfer by force and deception. Now it is assumed that 100% is transferred and that the bill is paid by the developing superadditivity. Therefore, this time we do not observe consuming superadditivity.

Area III is interpreted in the two dimensional models as the Stag-Hunt domain (figure 2 in literature 5). The sharp, colourful transition is owed to the fact that in my three dimensional model the drop of net profit is non-linear steep in this area. The transition border in Santos et. al. (5) follows perfectly the line of strict equivalence. The usage of colours in two dimensional figures is usually a good hint of a hidden dimension.

Area II and III are in direct neighbourhood of area I. If we assume that area I compares to cooperation, it is no wonder that an author (6) has the impression that exploitation can hide behind cooperation. But this author could also have a basically violent ensemble in mind. To all authors it seems inconceivable that exploitation can be superadditive and stable. In addition, exploitation is a successful strategy of an ensemble in comparison to the alternative – an inactive ensemble. Their imagination is blocked by moral judgements and the Nash equilibrium.

Area II and III are the stages for wise exploitation. Here only the exploiting party controls the increased efficiency as only there the efficiency will increase. The two stages are both divided into two zones – productive and consumptive exploitation. Both sides determine the success of exploitation in comparison to another ensemble and are under surveillance of “prudence” and “wisdom” (hopefully). The exploited
party always has a real loss. This loss may be decreased by reciprocity. However, reciprocity is never a complete compensation. In addition, reciprocity may confuse our judgement who is source and who is sink. The exploited party stays alive. Most of the offspring is consumed, heavy burdens are imposed, benefits are removed, freedom is restricted, diseases and predators are kept away, cheap food is provided and ailment will exist up to a level harming the residual productivity. This residual productivity has either a long or a short term perspective. The long term perspective avoids the Nash equilibrium the best.

Area IV

This area of complete irrationality is only accessible to an ensemble with a third party as master. What an interesting observation and prognosis! In the basic violent ensemble of figure 16 the master is responsible that irrationality starts to spread even to area I. In area IV the conflict aggravates: both sides are simultaneously unwilling and completely subadditive. Source has to give a substrate with benefit domination \((b>c)\) and sink has to take it as a substrate with cost domination \((b<c)\). Both actions are not a good idea for the respective party. The complete area is on the consumptive side of strict equivalence. In this area we observe lose-lose situations as a result; one side gives eyesight so that the other side gains blindness. Area IV is interpreted as the area of Prisoner´s dilemma \((5)\). Prisoner´s dilemma is often characterized as lose-lose situation. Now this can be understood, too. Area IV is completely irrational as the inactive ensemble has always a better productivity. But will this fact become aware to source, sink and the master in the absence of an inactive ensemble as competitor? Irrationality will be especially persistent when the knowledge of cost and benefits is befogged by ideology or ignorance. In case the master would be aware
of the real cost and benefit, would he care? An inactive ensemble does not need a master and why should he then receive rewards?

1. **Who is in control of the increased efficiency?**

In area I both source and sink are in control and participate rationally and selfishly in the increased efficiency. The source will increase efficiency as cost will decrease faster than benefit. The source gets rid of a burden \((b<c, \frac{b}{c}<1)\). The sink obtains a substrate with a beneficial character \((b>c, \frac{b}{c}>1)\). Efficiency will increase as benefit will increase more than cost. However, in dependent ensembles the master controls the ensemble. He may be a modest middleman and only withdraw a small fee or he takes all of the increased efficiency. In economy it is attempted to maximize net profit. This is achieved by lowering costs or increasing productivity. This can be done until \(b-c=0\) for source and sink without harm. In the independent ensemble only a small price of information cost has to be payed to bring source and sink together. The rest will stay within source and sink but not necessarily to the same size outside of strict symbiosis. But this is also true for the dependent ensemble.

2. **What ethos will have the largest increase in efficiency?**

That is a wrong question. We are dealing with a biochemical model; fixed Km, Vmax, bf, k and a limited, equally probable, and identical concentration range in all ensembles. Within this model a transfer of substrate will result in a definite increase or decrease of net profit in comparison to an inactive ensemble. At a distinct pair of concentrations this will always be the same value for the same amount transferred. It is insignificant how much of this value is spent for the different cost centres
(brute force, deception, reward, reciprocity, transfer etc.). The superadditivity produced by the ensemble in a certain region of concentration pairs has to be corrected by the amount of subadditivity the ensemble produces in other concentration pairs – that is the final net profit of the ensemble in the accessible concentration range. The ensemble is a unity. We are not allowed to pick a smaller (or larger) concentration range or compare parts of the ensemble as if they were independent. This is a modification of my older view of an ensemble. In older papers I look at the single concentration pair. Now I include a set of equally probable concentrations into the “unity” ensemble which have to be taken into account. An ensemble will have besides superadditive regions also subadditive regions – this comes as a package.

A good ethos is not only marked by the largest increase through superadditivity but by the smallest decrease through subadditivity in the areas of exploitation (II and III) or irrationality (IV). The wise ethos only avoids the Nash equilibrium and is responsible for the long term perspective. The prudent ethos acts in a way that the ensemble is kept away from transfers in concentration pairs producing subadditivity. Such concentration pairs fall in two groups. 1. All pairs on the consumptive side of strict equivalence. 2. Pairs on the productive side of strict equivalence in case the size of a transfer will lead to a concentration pair on the consumptive side.

The ensemble must find the narrow path between irrationality leading to harming subadditivity and exhausting superadditivity leading to extinction (Nash equilibrium). This path will become more difficult when the probability of different concentration pairs is no longer equal, when the ensembles are asymmetric, or when not 100% substrate is transferred (4). Let us compare the different ensembles within identical biochemistry and rank them in table 1:
<table>
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<tr>
<th>name</th>
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<th>superadditivity in area I</th>
<th>superadditivity in area II and III</th>
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<th>ensemble balance $(\Delta \text{net profit}\cdot mM^2)$</th>
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Table 2

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<th>superadditivity area I to III</th>
<th>subadditivity area I to IV</th>
<th>ensemble balance (Δ net profit*mM²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>basically violent master, modest wise exploitation</td>
<td>16</td>
<td>+0.110484</td>
<td>-0.000675</td>
<td>+0.109809</td>
</tr>
</tbody>
</table>
The long or short term perspectives (modest or bold wise exploitation) will not have the probability 1 for the next generation or next day. It will be tempting to exploit now with certainty, earning a small though subadditive net profit, than later a larger net profit after several rounds of uncertain repetition. A problem probably best investigated with intertemporal optimization.

The differences in net profit between active and inactive ensembles may seem small. But imagine a small group of masters and an ensemble of millions of sources and sinks – a food chain, an ecological pyramid. Economy of scale is an important feature in exploitation by force and deception. A lash and a rope to control the cattle, a few armed forces patrolling in the streets, or a nice little story on hope and salvation can be cheaply used repeatedly in various circumstances with many addressees. The different behaviours result in different outcomes and an unequal status. Preferential attachment processes will make the ensemble ethos with the best net profit the preferred type in a population of mixed strategies. It could well be that 80% of the ensembles will follow the best 20% in table1.

<table>
<thead>
<tr>
<th>Rank</th>
<th>Type</th>
<th>Net Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank 1.</td>
<td>prudent master</td>
<td>+0.114587</td>
</tr>
<tr>
<td>Rank 2.</td>
<td>prudent sink</td>
<td>+0.111144</td>
</tr>
<tr>
<td>Rank 3.</td>
<td>prudent master, bold wise exploitation</td>
<td>+0.108232</td>
</tr>
<tr>
<td>Rank 4.</td>
<td>prudent sink, bold wise exploitation</td>
<td>+0.105525</td>
</tr>
</tbody>
</table>

This ranking will have consequences: A prudent behaviour will end in the Nash Equilibrium; a bold exploitation has no truly long term perspective. The master or the sink must organize substitutes for the lost parties. The first rank is occupied by a third party master. Only ensembles with a third party master will enter the irrational area IV. All four do not follow a long
term perspective over many generations; a fertility decline should be observable – and that is what we observe in highly productive societies. Substitutes for the lost parties are organized.

3. **Where are the limits to exploitation?**

There are internal and external limits.

External limits: In source the upper limit is 1mM. There is no more substrate available. In case there would be more substrate the net profit could be higher. The upper limit for sink is also 1mM. At this concentration the sink will break down per definition due to overloading. Both limits are arbitrary. The same is true for the lower concentration limits. Source will collapse at 0mM. In reality this might be already at higher amounts. Sink starts at 0mM; that may be too low to get started. The external limits mark the absolute borders of the whole system and for the exploitability. This is only a biochemical model, although I can`t resist to look beyond.

There are two types of internal limits: A first type of internal limit is the modest and bold limit of exploitation. These limits in my examples are even more arbitrary but important and critical. What is most important is the feature that this type of internal limit is not negotiable between the parties. They are a part of the system reality. Modest and bold limits run in the concentration plane of the model. Below the modest threshold there will be no longer reproduction and below the bold threshold there will be exhaustion of source (in sink above modest and above bold limit) and finally a breakdown of the ensemble. Beyond the bold limits the exploited fade away and are lost; the Nash equilibrium will take hold of
the system. These internal limits decide the timescale of exploitation. “Wise” and “prudent” do not refer to the measures but to the results.

The second type of internal limit runs along the net profit axes. The relative sizes are a trade-off. They determine how large a reward or reciprocity may be and whether measures like force or deception can be paid and to what extend and relative size within a fixed amount of net profit. This type of limit is negotiable between source, sink and master. In case the net profit would not be sufficient to pay all the expenditures the ensemble would have to become inactive, too. However, if more net profit can be obtained, the ensemble falls into a different ensemble set with a different limit of the first internal type. Or it may be active now beyond the bold limit and will decay. This second type of limit will not be analysed in detail, neither here nor in the answer to question 6. It is useless to discuss the relative sizes of force and counter force, deception and counter propaganda, transfer costs, information costs and other expenditures. They are only expenditures and not causal connected to the size of the net profit - with the exception of expenditures like research or mutability and recombination. Such expenditures will change the whole setting as they may change e.g. Vmax or cost.

A detailed look: At 0.4mM the source (0.6mM in sink) is able to make a living from the residual net profit and to reproduce once. The single offspring of the source is replacing the source (sink) at the end of the lifetime. This system will not reach the Nash equilibrium as there will be always a new exploitable individual. This is not a “perpetual motion machine” as the sun is powering the system of sources and sinks (the sink being the next source) in a food chain (e.g.: leaf-fungus-leafcutter ant or grass-dear-wolf-fur hunter or grass-cow-farmer-king-Kaiser-pope).
At 0.3mM the source (0.7 in sink) is able to only make a living from the residual net profit. This strong exploitation will end after the source (sink) is lost at the end of the lifetime (or useful life).

The superadditive increase in net profit by force- or deception-induced rearrangement of cost and benefit is large enough to pay the whole bill. The wisdom is realized according to two time horizons. 1: The exploited party is able to reproduce once. This is really wise as the system will continue. 2: The wisdom of the stronger exploitation (0.3mM source, 0.7mM sink) is limited. The bill of exploitation is paid, but only in one time period – a single lifetime.

In case we compare two different but active, symmetric ensembles with different Km, Vmax, bf and k values; we may observe that one ensemble – due to its cost or Vmax – is in a certain concentration range of substrate peaceful where the other ensemble already will use force and deception at that concentration.

Asymmetric ensembles are a complete new field (4). Here we find new protagonists like the biased master. The case of a master with a bias is not considered in this paper. Such a master would e.g. push the source to the modest wise limit but would push sink to the bold wise limit (or vice versa). Such a case would be interesting in asymmetric ensembles to either compensate or increase the asymmetry.

4. **When will productive exploitation turn into consumptive exploitation and when will the ensemble better fall apart?**

Whether a concentration pair will become superadditive depends on two features. Only concentration pairs left of the line of strict equivalence are at all able to produce superadditivity. However, if the size of transfer in
such a pair exceeds the concentration difference to the line of strict equivalence also such pairs will become subadditive. Therefore, we observe subadditivity in the centre of the ensemble left of strict equivalence in wise exploitation. Only prudent ethos is able to avoid this. On the other hand prudent behaviour must be controlled by wise ethos not to cross modest or bold limits of exploitation.

When superadditive concentration pairs in area I, area II, and area III approach the border of strict equivalence, the local net profit will become small. In that case the exploiting party should use cheap(er) measures. Then the border to subadditivity, the line of strict equivalence, is reached. Here, productive exploitation turns into consumptive exploitation. The essence of this is that more net profit is lost in source than is created in sink. When the line of strict equivalence is crossed the ensemble should better stop to be active and should fall apart – that simple; that difficult.

In my present view the ensemble is more a unity than in my older considerations. Even fully integrated ensembles are quite blind. They are fixed to a genetic, legal, or cultural operational instruction that can’t be changed on the short run. This operational instruction is followed in all single accidental concentration pairs if they are not excluded by a certain ethos. The ensemble has to endure the subadditivity if there are no additional detectors for subadditivity and emergency stop switches. In addition, the subadditivity may have to be tolerated because other more probable (different probabilities are not considered in the present model) concentrations produce lots of superadditivity. The ensemble needs instructions in form of limits to avoid concentration pairs with subadditive features (prudent behaviour). Other limits are necessary to avoid exhaustion (wise behaviour). The wise ethos will prevent even small productive transfers when they have no long term perspective. The prudent ethos will prevent also large transfers when they are consuming,
although they may seem attractive due to a visible yet consuming net profit in the new location.

In man pragmatism could be an emergency switch. Whatever the operational instruction may say in general, pragmatism abrogates the instruction in certain conditions and will not follow blindly.

The ensemble better falls apart when the subadditive volume starts to exceed the superadditive volume. This may occur for example when the probability for concentration pairs on the consumptive side of the line of strict equivalence will increase.

Completely subadditive ensembles like the predatory master (-0.313325) should definitively fall apart. But such irrational ensembles exist. They are glued together in desperation and distress by force and propaganda. Such ensembles may vanish. However, the ensemble could be part of a food chain. In this chain the whole ensemble may be a sink exploiting a source, thereby acquiring huge amounts of substrates to waste irrationally. This ensemble may have so much substrate to waste that the observed subadditivity (irrationality) will still generate more additional net profit than the superadditivity including the residual net profit (simple additivity) in a different, peaceful ensemble with much less substrate. What a tragedy. The basically violent (deceptive) master is not considered within this discussion.

5. **Why has the Nash equilibrium not to be considered in wise exploitation?**

The Nash equilibrium is inspired by classical physics. A resting physical system is charged with energy and becomes a dynamic system, the energy dissipates, everything comes to an end and the system is back in
rest. The balance of the system according to “energy in, energy transformation, energy out” is completed. A party is exploited and either is lost or refuses to participate in the next round; the Nash equilibrium – end of line.

This is not Biology or Economy! Biology and Economy are open systems continuously charged with energy, continuously dissipating energy. In wise exploitation the exploited party either recovers using external sources, disposes the waste into sinks, or is consumed (lost) and replaced by a new generation with the same functionality. The exploited party is replaced or recovers over and over again; the bill is paid by the sun – no end of line. The Nash equilibrium has no meaning here.

6. **How expensive may reciprocity, force, deception and reward become?**

The answer is simple: The upper limit to all expenditures is the achievable and available superadditivity. The difference between the net profit of the active ensemble and the net profit of the inactive ensemble could be used to produce more offspring or can be used for other expenditures. This may include the transformation of biological productivity into economic, military and cultural productivity.

\[ b_e - c_e \text{ (active)} - b_e - c_e \text{ (inactive)} = \text{brute force + deception + reciprocity + reward} \]

How expensive the single measure may be depends. More and better deception may mean less brute force. Reward and reciprocity may also appear as a trade-off. The more reciprocity is necessary for the exploited party the less is remaining as a reward to the exploiting party. This is the place for negotiations, introduction of taboos, the call for frugality, fasting,
and modesty or generosity, altruism, open mindedness and all the other characteristics of good behaviour – of the other side or the whole ensemble when we would ask the master.

A final question:

*Why do we observe in behavioural experiments with humans even in complete anonymity only rarely the pure, unmerciful, and rational “Homo Economicus” described here?*

The reason is that we never make experiments with a Kaspar Hauser. Everybody will feel observed in such a situation beyond the general paranoia. The experiments deal usually with grown, educated personalities. Experiments with children strongly depend on their age for different reasons. These adult personalities have lived a life long within a culture. They have been trained to behave in certain ways. The intuitive and so called altruistic cooperation (7) which is said to be observable in extreme situations is not the result of genetic predisposition towards altruism but is the result of long conditioning best observed in soldiers and bodyguards and their respective drill. (“Our Ethos: …… each Marine is *infused* with an understanding of the deeds of his or her predecessors. Marines undergo a personal *transformation* at recruit training. There, they receive more than just superb training; they are *ingrained* with a sense of service, honor, and discipline.” 8)

In the real world we observe the two dimensional parts of the three dimensional ensemble. Moreover, we only look at an isolated party. This part of the ensemble may act unexpectedly and in a non-economic, non-rational sense to an external observer. That is the moment when we encounter the exploited part of the ensemble. This part carries voluntarily
a heavy load or gives joyful a valuable thing beyond b-c=0. This party may be brutally forced and educated to fear, infused with propaganda, intentionally misinformed, conditioned, mislead, lied to, trained, educated or kindly provided with philosophical, ideological or religious verdicts and worldviews – completely misinterpreting the size of benefits and costs. This is not a good basis for rational decisions. But who cares in a victorious ensemble – besides non-participating, non-benefiting or even competing masters? The superiority of a master-servant setting over tit-for-tat reciprocity has already been proven (9). The experimental observations (10, 11) on the correlation of fast, spontaneous decisions in connection to “cooperative decisions” and sacrifices for a common good in contrast to decisions after a longer cognitive process being more selfish support my interpretation. The single party educated in a community and culture is infused with a-priori operating instructions. To follow them result in the haste of the moment, under the pressure of time and fist, in self-harming behaviour. In case the Homo Economicus has some time to think he comes to different conclusions; more advisable and less harming to him.

It seems the pure Homo Economicus is not suitable for a society or economy with scarce resources or with an overflow of burdens. Strong and well informed he is immune to force and deception or he is at least no easy prey; he will not deliver a positive net profit after force or perceived deception. The society he fits in is restricted to the substrate pairs and transfer sizes of a peaceful ensemble. Such an ensemble is weak due to a lower net profit (table 1). Therefore, beyond his biological basic-equipment, he is now fermented in culture to be more palatable and digestible (… an animal which hath been taught to dance by blows and scanty fare. F. Nitzsche).
The exploited party will be replaced in one or another way. And then we observe wise exploitation in action. In human’s the replacement will often not be of biologic (genetic) tradition. This will harm the source or the sink on a genetic level in its own and direct, genetic tradition. The ensemble will not be affected at all. The superadditive ensemble is completely reasonable with an economic rational and will organize a non-biological tradition – a tradition of functionality. Culture will assign the task and function and those who are about to die salute in pride. Although, cultures differ in their local appearance in otherwise identical environments (12), they are united in the attempt to form a tradition of their own characteristics like the organisms in the level below, recreating similar patterns.

**Literature**

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