Is potential output growth falling?

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Abstract

We estimate the rate of growth consistent with a stable unemployment rate for four advanced economies (Canada, Germany, the United Kingdom and the United States) using a Heckman-type two step estimation procedure that deals with the issue of endogenous regressors in a time-varying parameter model. The results show that this rate of growth of potential output has been falling in the four countries of study, so that these countries have been systematically loosing capacity to grow during the postwar era.

JEL Classification: C50, O40, O49
Keywords: potential output growth rate, constant unemployment rate, time-varying parameter models

1 Introduction

The Great Recession has raised concerns about the possibility that advanced economies are entering an era of secular stagnation. However, different papers have also found that potential output growth has slowed prior to the Great Recession because of a variety of factors that have hampered its evolution. Fernald (2014) and Fernald and Wang (2015) have documented reductions in labour and in total factor productivity growth in the economy of the United States (US); whereas Gordon (2014) mentions the following headwinds: demographic shifts (as a result of the retirement of the baby boom generation and of the exit from the labour force of both young and prime-age adults) that have reduced hours worked per capita, lower levels of educational attainments, and higher levels of inequality and government debt. Likewise, the ageing population and the decline in labour productivity in the Euro area (ECB , 2011; Klump et al. , 2008) also point out the possibility that other advanced economies have been systematically loosing capacity to grow.

As Basu and Fernald (2009: 205) explain, "[e]stimating potential output growth is one modest example and relatively transparent example of [the] interplay between theory and measurement". In this note we estimate a simple measure of potential output growth by

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identifying the latter with that rate of output growth consistent with a stable unemployment rate. We study the evolution over time of the latter in four advanced economies (Canada, Germany, the United Kingdom and the US) during the postwar era using both a conventional Time-Varying Parameter Model (henceforth TVPM) and a Heckman-type two step estimation procedure that deals with the possible endogeneity problem. The results suggest that this measure of potential output growth has been falling during the respective periods of study, so that these four economies have been experiencing reductions in their productive capacity during the post-war era.

The next section presents the theory discussion. Section 3 presents the econometric models and techniques; section 4 presents the main results; and the main conclusions are presented in Section 5.

2 Theory

Suppose that output Y is produced via the following aggregate production function with disembodied technological progress (Huang and Lin , 2008):

$$Y = AF(K,L) = AF(kc,nh) \tag{1}$$

where K is the capital input, L is the labour input, k is the number of capital stock, c is the utilization rate, n denotes the number of workers employed, h is the number of working hours, and A is a measure of the ability of the economy to transform inputs into output.

Equation (1) can be expressed in growth rates as follows:

$$g_t = a_t + \alpha\,(k_t + c_t) + \theta\,(n_t + h_t) \tag{2}$$

where g_t, a_t, k_t, c_t, n_t, and h_t denote the rates of growth of Y, A, k, c, n and h, respectively; whereas $\alpha = (\delta F/\delta K)(K/F)$ and $\theta = (\delta F/\delta L)(L/F)$ represent the elasticities of Y with respect to K and L, respectively.

It is possible to employ the following approximation: $\Delta u_t \approx l_t - n_t$, where Δu_t denotes the change in the unemployment rate (u_t) and l_t is the rate of growth of the labour force. Therefore, equation (2) can be expressed as follows:

$$g_t = a_t + \alpha\,(k_t + c_t) + \theta\,(l_t - \Delta u_t + h_t) \tag{3}$$

Both multifactor productivity and average working hours tend to be pro-cyclical variables because of practices associated with labour hoarding (Huang and Lin , 2008): 1) during recessions, firms maintain excess labour to avoid the firing and retraining costs associated with new employees, so that productivity falls because there is more labour than the one necessary to produce the level of output actually produced; and 2) firms choose to spread the lower work load during recessions more thinly among existing workers instead of reducing the level of employment. Similarly, different processes relating to the Kaldor-Verdoorn mechanism also explain the pro-cyclical behaviour of both variables (León-Ledesma and Thirlwall , 2002): 1) static and dynamic returns to scale associated with increases in the volume of output and the technical progress embodied in capital accumulation; and 2) the learning-by-doing process (Arrow , 1962), so that productivity is a function of cumulative output: the more output produced, the more adept labour becomes at producing it.

Likewise, capacity utilization and labour force growth are also pro-cyclical. Labour force participation rates rise as the unemployment rate falls mainly via the discouraged-worker effect (when unemployment is high, discouraged workers may simply not look for job or may give up looking; however, some of these workers typically re-enter the labour force when the unemployment rate falls) and via labour immigration towards booming labour markets (Huang and Lin , 2008; León-Ledesma and Thirlwall , 2002).

In this sense, if a_t, h_t, c_t, and l_t vary positively with respect to g_t, then $a_t = \gamma_0 g_t$; $h_t = \gamma_1 g_t$; $c_t = \gamma_2 g_t$, and $l_t = \gamma_3 g_t$; where $\gamma_0 > 0$, $\gamma_1 > 0$, $\gamma_2 > 0$, and $\gamma_3 > 0$. Thus:

$$g_t = \beta_0 - \beta_1(\Delta u_t) + e_{1,t} \tag{4}$$

where $\beta_0 = \alpha k_t/(1-\gamma_0-\theta\gamma_1-\alpha\gamma_2-\theta\gamma_3)$; $\beta_1 = \theta/(1-\gamma_0-\theta\gamma_1-\alpha\gamma_2-\theta\gamma_3)$; and $e_{1,t}$ represents the stochastic disturbance term that satisfies the standard statistical properties.

It is possible to say that g_t is growing at its potential (henceforth g_p) when $\Delta u_t = 0$, so that $g_p = \beta_0$. The latter represents the minimum level of output growth needed to reduce u_t given labour force and labour productivity growth (IMF , 2010; Thirlwall , 1969). In other words, β_0 corresponds to the threshold growth rate, which, on a balanced growth path with no changes in unemployment, would equal the sum of growth of the labour force and the labour-augmenting technical progress (Klump et al. , 2008).[1]

3 The time-varying parameter model and a Heckman-type two step estimation procedure

If the different components of β_0 are pro-cyclical variables, then it is necessary to consider the possibility that g_p fluctuates over time. In the same vein, different studies have shown that the relationship between unemployment and output might change over time, so that it is also necessary to incorporate the possibility of a time-varying Okun coefficient on unemployment.

Thereby, we reformulate equation (4) using a TVPM composed of the observed variables Δu_t and g_t, and the unobserved parameters $\beta_{0,t}$ and $\beta_{1,t}$:

$$g_t = \beta_{0,t} - \beta_{1,t}(\Delta u_t) + e_{2,t}, \quad e_{2,t} \sim i.i.d.\mathrm{N}(0, \sigma_e^2) \tag{5}$$

$$\beta_{i,t} = \beta_{i,t-1} + \varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim i.i.d.\mathrm{N}(0, \sigma_{\varepsilon,i}^2), \quad i = 0,1 \tag{6}$$

where $e_{2,t}$ is the error term. Thus, a time-varying potential output growth rate ($g_{p,t} = \beta_{0,t}$) and a time-varying Okun coefficient on unemployment ($\beta_{1,t}$) can be retrieved from the estimation of equations (5) and (6).

Nevertheless, one problem with the estimation of this TVPM is that the regressor Δu_t may be correlated with $e_{2,t}$ since both output and unemployment are endogenous variables to a complex system. The estimation of equations (5) and (6) through the conventional Kalman filter via Maximum Likelihood (ML) cannot be performed because a successful application of the latter critically depends upon the assumption that the regressors are uncorrelated with the disturbance

[1] See also the references in Mendieta-Muñoz (2014) for other papers in which this measure of potential output has been employed.

terms (Kim , 2006). In other words, the Kalman filter provides us with invalid inferences of the model if the regressors are endogenous.

In order to correct the endogeneity problem, we employ the Heckman-type two-step approach developed by Kim (2006), which allows us to obtain consistent estimates of both hyper-parameters and time-varying coefficients.[2] Hence, we use Instrumental Variables (IVs), assuming that the relationship between the endogenous regressor Δu_t and the vector of IVs (z_t) is given by:

$$\Delta u_t = \delta(z_t) + \mu_t, \quad \mu_t \sim i.i.d.\mathrm{N}(0, \sigma_\mu^2) \tag{7}$$

where δ is a vector of constant parameters.[3]

It is possible to decompose Δu_t into its predicted component ($\mathrm{E}[\Delta u_t | \psi_{\mathbf{t-1}}]$) and its prediction error component (v_t):

$$\Delta u_t = \mathrm{E}[\Delta u_t | \psi_{\mathbf{t-1}}] + v_t \tag{8}$$

$$v_t = \sigma_v v_t^*, \quad v_t^* \sim i.i.d.\mathrm{N}(0,1) \tag{9}$$

where $\psi_{\mathbf{t-1}}$ denotes the available information in $t-1$; σ_v is the standard deviation of v_t; and v_t^* is the standardized prediction error of v_t.

If we denote the correlation between v_t and $e_{2,t}$ by the constant correlation coefficient ρ, the joint distribution of v_t^* and $e_{2,t}$ is the following:

$$\begin{bmatrix} v_t^* \\ e_{2,t} \end{bmatrix} \sim i.i.d.\mathrm{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho\sigma_e \\ \rho^{-1}\sigma_e & \sigma_e^2 \end{bmatrix} \right) \tag{10}$$

Therefore, a Cholesky decomposition of the covariance matrix results leads us to decompose $e_{2,t}$ in equation (10) into:

$$e_{2,t} = \rho\sigma_e v_t^* + \omega_t^*, \quad \omega_t^* \sim i.i.d.\mathrm{N}(0, \sigma_\omega^2), \quad \sigma_\omega^2 = (1-\rho^2)\sigma_e^2 \tag{11}$$

Equation (11) shows the two components of $e_{2,t}$: the v_t^* component, which is correlated with Δu_t; and the ω_t^* component, which is not correlated with Δu_t. Substituting equation (11) into equation (5) results in:

$$g_t = \beta'_{0,t} - \beta'_{1,t}(\Delta u_t) + \rho\sigma_e v_t^* + \omega_t^* \tag{12}$$

where ω_t^* is not correlated with v_t^* or Δu_t; and both the new time-varying potential output growth rate ($g'_{p,t} = \beta'_{0,t}$) and the new time-varying Okun coefficient on unemployment ($\beta'_{1,t}$) can be generated assuming that the time-varying coefficients follow a random walk:

$$\beta'_{i,t} = \beta'_{i,t-1} + \varepsilon'_{i,t}, \quad \varepsilon'_{i,t} \sim i.i.d.\mathrm{N}(0, \sigma^{2'}_{\varepsilon,i}), \quad i = 0,1 \tag{13}$$

Thus, the standardised prediction errors v_t^* enter in equation (12) as bias correction terms in the spirit of Heckman (1976)'s two-step procedure for a sample selection model. To summarise, the TVPM with bias correction terms is estimated via ML in two steps:

[2] Kim and Nelson (2006) implemented this approach in the context of the estimation of a forward-looking monetary policy rule using ex post data for the US economy.

[3] We also estimated equation (7) for all countries assuming that δ follows a random walk. However, it was not possible to find a global solution for these models—that is, the solution went singular. We also employed different starting values (using the OLS estimates as initial parameters); however, the results did not change.

1. Equation (7) is estimated through ML and the standardised one step-ahead forecast errors are obtained.

2. Equations (12) and (13) are estimated based on the prediction error decomposition and the Kalman filter.[4]

4 Results

With the exception of the US, the g_t series for all countries were extracted from the Total Economy Database (TED) of the Groningen Development Centre; whereas the u_t series were extracted from the OECD electronic database.[5] Both the g_t and the u_t series for the US were obtained from the Federal Reserve Bank of Saint Louis (Fred) electronic database, which correspond to the rate of growth of GDP (in billions of chained 2009 dollars) and to the civilian unemployment rate, respectively.[6]

In all cases we proceeded as follows. We first ran the Kalman filter in order to obtain the respective innovation variances and the initial values of the parameters to be estimated in the different equitations. In the subsequent step, the Kalman filter was run again with the preceding estimates of the innovation variances, the initial values of the parameters and their respective variance-covariance matrices in order to obtain the evolutionary coefficients of the models.

Table 1 below presents the estimates of the innovation variances of the different state-space models. We employed the first two lags of Δu_t as IVs in the estimation of equation (7) for all countries.[7] Following Kim and Nelson (1999), we have corroborated the appropriateness of the specified models checking for the lack of serial correlation and of heteroskedasticity in the standardized one-period-ahead-forecast errors. These results are presented in Table 2.

[INSERT TABLE 1 ABOUT HERE]
[INSERT TABLE 2 ABOUT HERE]

Firstly, from Table 2 it is possible to observe that the estimations of the time-varying parameters do not present problems of serial correlation (up to order 3) or heteroskedasticity at the 1% level of significance, which suggests no evidence of model misspecification.[8]

[4]The state-state representations of the TVPM without bias correction terms —equations (5) and (6)— and of the TVPM with bias correction terms —equations (12) and (13)— are presented in appendix A.

[5]This explains the different periods of time considered for each country: the u_t series provided by the OECD database are available only for the period 1955-2014 for Canada, 1962-2014 for Germany, and 2000-2014 for the UK. The u_t series for the UK during the period 1950-1999 was extracted from the Bank of England "300 Years of Data" dataset.

[6]We also estimated the models using the OECD's u_t series and the g_t series obtained from the TED. The results obtained were fairly similar.

[7]We also estimated equation (7) including the first lag of Δu_t and the first lag of labour productivity growth (measured as the rate of growth of output per number of hours worked, which was obtained from the TED) as instruments. However, the F-statistics for the estimated regressions in this first stage were always lower than the F-statistics obtained from the estimations that employed the first two lags of Δu_t, which suggests that the latter are more relevant instruments than the former in all countries. Moreover, the estimation for Germany using the the first lag of Δu_t and the first lag of labour productivity growth as IVs presented problems of heteroskedasticity.

[8]We also corroborated the lack of autocorrelation and heteroskedasticity in the IVs estimations. The results show that the models are correctly specified at the 10% level of significance.

Secondly, the estimates of the standard errors associated with $g_{p,t}$ and $g'_{p,t}$ (that is, $\sigma_{\varepsilon,0}$ and $\sigma'_{\varepsilon,0}$) are statistically significant in the majority of countries (the only exception being the UK), thus suggesting evidence of a temporary variation in the potential rate of growth. The latter is corroborated by the Likelihood Ratio (LR) tests calculated assuming the respective models with constant parameters (H_0: $\sigma_{\varepsilon,0} = \sigma_{\varepsilon,1} = 0$ or H_0: $\sigma'_{\varepsilon,0} = \sigma'_{\varepsilon,1} = 0$), which are also presented in Table 2. The different LR tests show a rejection of the null hypothesis of constant parameters at the 1% level in the majority of cases (the only exception being the UK, where the null hypothesis is rejected at the 10% level).

Thirdly, regarding the endogeneity problem of the regressor Δu_t, it is possible to observe that the estimated coefficient of the correction term bias ρ is significant in all countries, the only exception being Germany. Hence, it is not possible to ignore the endogeneity problem in Canada, the UK and the US.

Finally, figures 1 to 4 present the smoothed estimates of the $g_{p,t}$s together with their respective 90% confidence intervals.[9] As mentioned above, the $g_{p,t}$ shown for Canada, the UK and the US correspond to the TVPMs with bias correction terms; whereas the $g_{p,t}$ for Germany (Figure 2) corresponds to the TVPM without bias correction term.[10] For the US economy (Figure 4) we have also plotted the rate of growth of potential output estimated by the Congressional Budget Office (CBO), which was extracted from the Fred database. It is worth noting the CBO's estimate of $g_{p,t}$ lies within our estimated 90% confidence interval during most of the period of study.

[INSERT FIGURE 1 ABOUT HERE]
[INSERT FIGURE 2 ABOUT HERE]
[INSERT FIGURE 3 ABOUT HERE]
[INSERT FIGURE 4 ABOUT HERE]

Therefore, from the figures above it is possible to conclude that the rate of growth consistent with a constant unemployment rate has been falling in the four countries of study. Germany is the country that has experienced the largest reduction in this measure of potential output growth (-4.3 percentage points (pp)), followed by the US (-2.9 pp), Canada (-1.9 pp), and the UK (-1.2 pp).

5 Conclusions

This note has estimated the rate of growth consistent with a stable unemployment rate in Canada, Germany, the United Kingdom and the United States using time-varying parameter models. We have employed both the conventional Kalman filter via Maximum Likelihood and a Heckman-type two step estimation procedure that deals with the issue of endogenous regressors in a time-varying parameter model. Our results show that the minimum rate of growth required to reduce unemployment has been falling in the four countries of study, thus suggesting that these four economies have experienced a reduction in their productive capacity during the post-war era.

[9] Depending upon the information set used, it is possible to find the basic filter and smoothing filter. The former refers to an estimate of the time-varying coefficients based on information available up to time t; whereas the latter refers to an estimate of the time-varying coefficients based on all the available information in the sample through time T. The smoothed values provide a more accurate inference about the time-varying parameters (see Kim and Nelson (1999) and Kim (2006) for a description.)

[10] Appendix B compares the time-varying potential rates of growth obtained from the models with and without bias correction terms. It is possible to observe that the results are fairly similar in all countries.

Table 1. Estimation of the hyper-parameters for the time-varying parameter models

Hyper-parameters	Canada (1956-2014)	Germany (1963-2014)	United Kingdom (1951-2014)	United States (1951-2014)
Estimation without bias correction terms: equations (5) and (6)[a]				
$\sigma_{\varepsilon,0}$	0.38***	0.40***	0.11	0.24
	(0.12)	(0.11)	(0.15)	(0.15)
$\sigma_{\varepsilon,1}$	0.04	0.48*	0.46**	0.07
	(0.17)	(0.25)	(0.22)	(0.11)
σ_e	0.92***	1.36***	1.31***	0.98***
	(0.11)	(0.14)	(0.10)	(0.09)
L[b]	-106.36	-116.63	-132.77	-113.76
Instrumental variable estimation: equation (7)[a]				
σ_μ	0.93***	0.64***	0.68***	1.04***
	(0.07)	(0.06)	(0.05)	(0.07)
L[b]	-102.05	-74.27	-89.92	-117.24
Estimation with bias correction terms: equations (12) and (13)[a]				
$\sigma'_{\varepsilon,0}$	0.44***	0.46***	0.14	0.27**
	(0.13)	(0.12)	(0.16)	(0.13)
$\sigma'_{\varepsilon,1}$	0.03	0.56*	0.52**	0.09
	(0.26)	(0.31)	(0.22)	(0.07)
σ_ω	0.84***	1.26***	1.25***	0.94***
	(0.09)	(0.15)	(0.13)	(0.10)
ρ	-0.66**	-0.44	-0.56*	0.57**
	(0.30)	(0.37)	(0.31)	(0.28)
L[b]	-108.66	-119.82	-136.13	-117.70

Notes: [a]Standard errors are shown in parenthesis; [b]Log likelihood.
*, **, and *** respectively denote rejection of the null hypothesis at the 10%, 5%, and 1% significance levels.

Table 2. Correct specification tests on the one-period-ahead forecast errors obtained from the time-varying parameter models and likelihood ratio tests

Country	Autocorrelation[a]			Heteroskedasticity[b]	Likelihood ratio test[c]
	Order 1	Order 2	Order 3		
Estimation without bias correction terms: equations (5) and (6)[d]					
Canada	0.03	0.03	1.78	0.33	44.49***
	(0.87)	(0.99)	(0.62)	(0.57)	(0)
Germany	0.25	1.88	1.88	1.76	21.92***
	(0.62)	(0.39)	(0.60)	(0.19)	(0)
UK	2.76	4.11	4.29	0.03	5.08*
	(0.10)	(0.13)	(0.23)	(0.86)	(0.08)
US	0.18	0.26	1.0	0.28	22.33***
	(0.67)	(0.88)	(0.80)	(0.60)	(0)
Estimation with bias correction terms: equations (12) and (13)[d]					
Canada	0.33	0.43	0.54	0.51	41.72***
	(0.57)	(0.81)	(0.91)	(0.48)	(0)
Germany	0.17	1.29	1.32	2.01	20.76***
	(0.68)	(0.52)	(0.73)	(0.16)	(0)
UK	0.01	0.16	1.13	0.01	6.27**
	(0.90)	(0.92)	(0.77)	(0.94)	(0.04)
US	0.62	0.91	0.96	0.10	24.41***
	(0.43)	(0.63)	(0.81)	(0.76)	(0)

Notes: [a]Ljung-Box statistic; [b]ARCH test (using 1 lag): F-statistics are shown; [c]Chi-squared statistics are shown. [d]p-values are shown in parenthesis.
*, **, and *** respectively denote rejection of the null hypothesis at the 10%, 5%, and 1% significance levels.

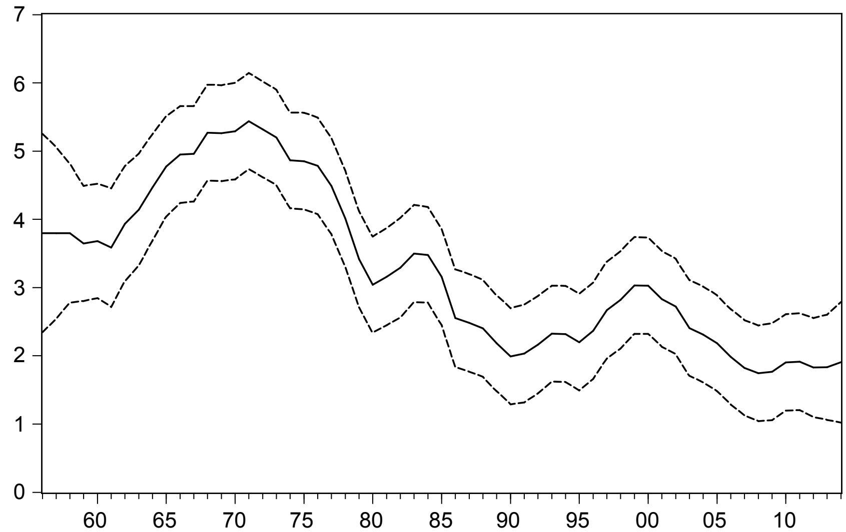

Figure 1. Canada, 1956-2014. Time-varying potential output growth rate (straight line) with 90% confidence intervals (dotted lines)

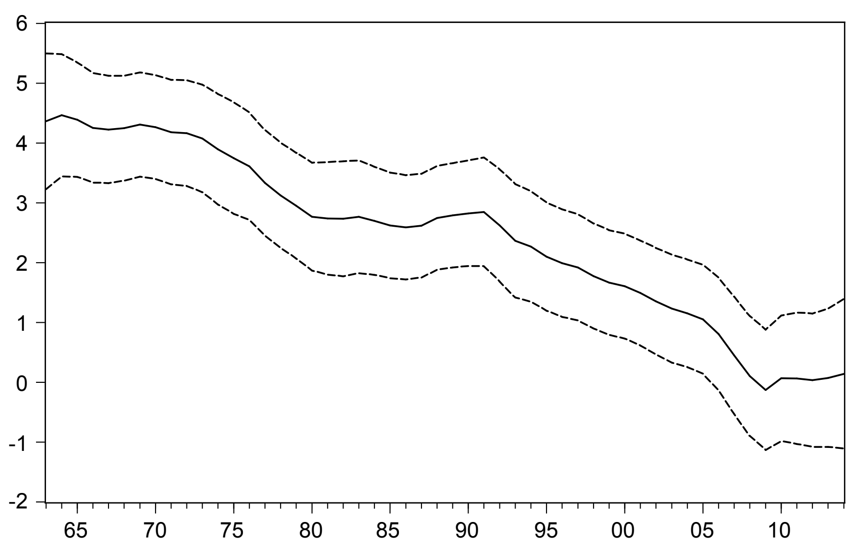

Figure 2. Germany, 1963-2014. Time-varying potential output growth rate (straight line) with 90% confidence intervals (dotted lines)

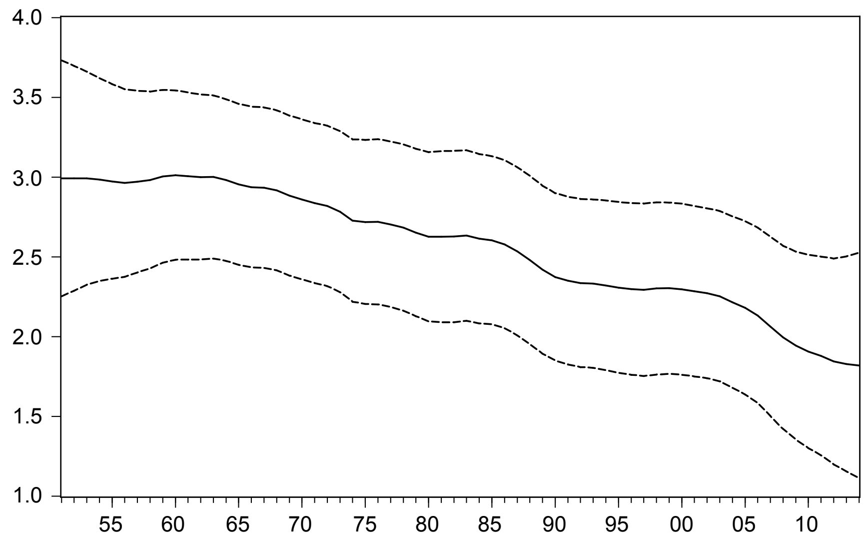

Figure 3. UK, 1951-2014. Time-varying potential output growth rate (straight line) with 90% confidence intervals (dotted lines)

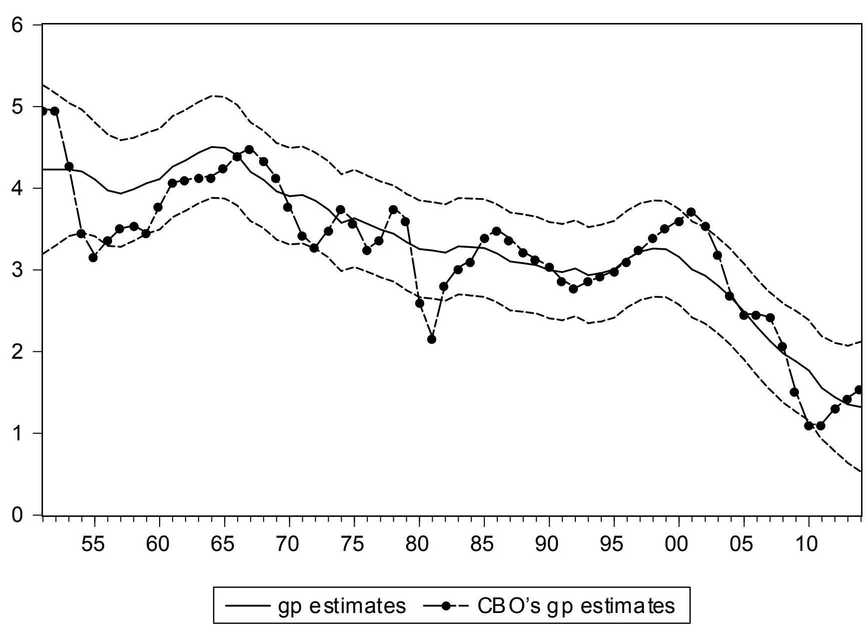

Figure 4. US, 1956-2014. Time-varying potential output growth rate (gp estimates) with 90% confidence intervals (dotted lines) and Congressional Budget Office (CBO)'s potential output growth rate (CBO's gp estimates)

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A State-state representation of the time-varying parameter models

The state-space formulation of the time-varying parameter model without bias correction terms (equations (2) and (3)) is the following:

$$\mathbf{Y_t} = \mathbf{X_t}\mathbf{B_t} + \mathbf{e_t}, \quad \mathbf{e_t} \sim i.i.d.\mathrm{N}(0, \sigma_e^2) \tag{A.1}$$

$$\mathbf{B_t} = \mathbf{B_{t-1}} + \boldsymbol{\varepsilon}_\mathbf{t}, \quad \boldsymbol{\varepsilon}_\mathbf{t} \sim i.i.d.\mathrm{N}(0, \Sigma_\varepsilon) \tag{A.2}$$

where $\mathbf{Y_t} = [g_t]$; $\mathbf{X_t} = [1, \Delta u_t]$; $\mathbf{B_t} = \begin{bmatrix} \beta_{0,t} \\ \beta_{1,t} \end{bmatrix}$; $\mathbf{e_t} = [e_{2,t}]$; $\mathbf{B_{t-1}} = \begin{bmatrix} \beta_{0,t-1} \\ \beta_{1,t-1} \end{bmatrix}$; and $\boldsymbol{\varepsilon}_\mathbf{t} = \begin{bmatrix} \varepsilon_0 \\ \varepsilon_1 \end{bmatrix}$.

On the other hand, the state-state representation of the model with bias correction terms (equations (9) and (10)) is the following:

$$\mathbf{Y_t} = \mathbf{X_t}\mathbf{B'_t} + \rho\sigma_e\widehat{v_t^*} + \omega_t^*, \quad \omega_t^* \sim i.i.d.\mathrm{N}(0, (1-\rho^2)\sigma_e^2) \tag{A.3}$$

$$\mathbf{B'_t} = \mathbf{B'_{t-1}} + \boldsymbol{\varepsilon}'_\mathbf{t}, \quad \boldsymbol{\varepsilon}'_\mathbf{t} \sim i.i.d.\mathrm{N}(0, \Sigma'_\varepsilon) \tag{A.4}$$

where, in addition to the previously defined variables, we now have that $\mathbf{B'_t} = \begin{bmatrix} \beta'_{0,t} \\ \beta'_{1,t} \end{bmatrix}$; $\mathbf{B_{t-1}} = \begin{bmatrix} \beta'_{0,t-1} \\ \beta'_{1,t-1} \end{bmatrix}$; $\boldsymbol{\varepsilon}'_\mathbf{t} = \begin{bmatrix} \varepsilon'_0 \\ \varepsilon'_1 \end{bmatrix}$.

Thus, equations (A.1) and (A.3) represent the measurement equations of the models; whereas equations (A.2) and (A.4) represent the transition equations.

B Time-varying potential output growth rates estimates

Figure B.1. Canada, 1956-2014. Time-varying potential rates of growth

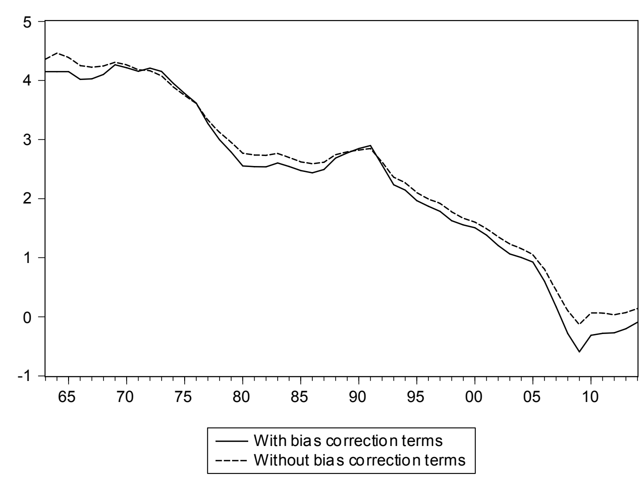

Figure B.2. Germany, 1963-2014. Time-varying potential rates of growth

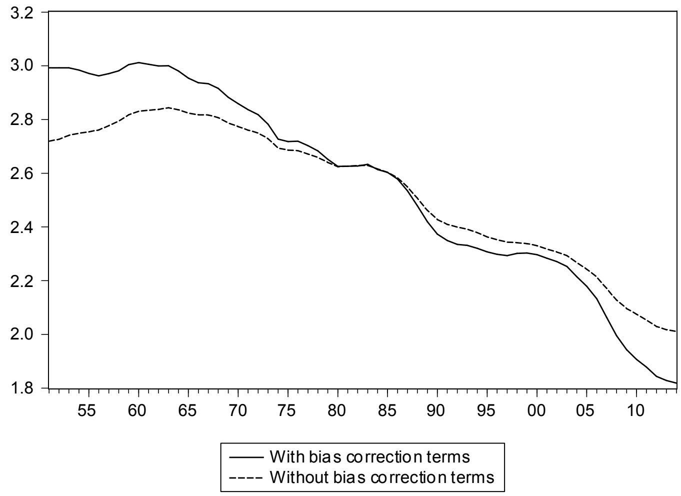

Figure B.3. UK, 1951-2014. Time-varying potential rates of growth

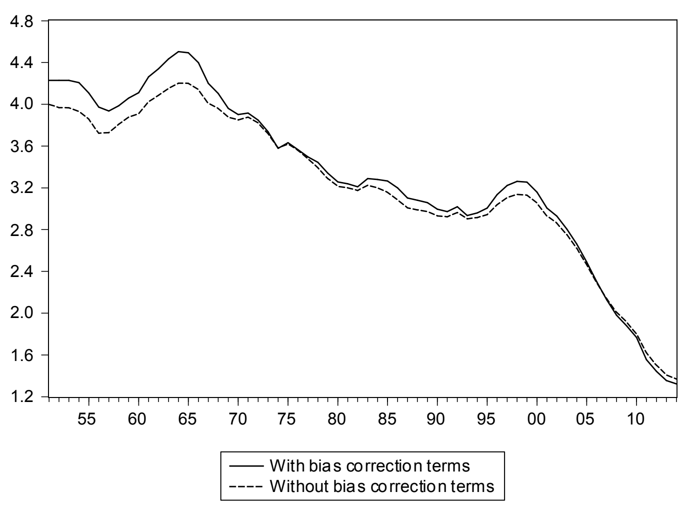

Figure B.4. US, 1951-2014. Time-varying potential rates of growth