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Abstract

In this paper I evaluate the contribution of financial frictions in explaining the drop in aggregate TFP through misallocation during the Great Recession. I build a quantitative model with heterogeneous establishments; with the help of the model I compute the counterfactual drop in misallocation: by how much would aggregate TFP have decreased if the credit crunch had been absent. I find that a "real recession" would have caused a drop of only 0.16 percent, as opposed to 1.04 percent found in the data; therefore financial frictions account for a significant part of the drop in aggregate TFP. The key mechanism is the following: the increase in the cost of external finance affects negatively the reallocation of productive inputs from low to high productivity firms, by dampening the growth of small-highly productive firms.

JEL: D21, D22; E32; G31

Keywords: financing constraints, misallocation, heterogeneous firms, incomplete markets, idiosyncratic shocks.

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1 Introduction

The Great Recession differs from other recessions that happened in the US during the post-war period in terms of both severity and persistence. It has been also characterized by a drop in aggregate TFP and reallocation of unprecedented amount (see Foster et al., 2014). What distinguishes the Great Recession is the disruption in financial markets; it is well understood that households were severely affected by the tightening in credit conditions, but also firms were badly hit: the credit spread between BB and AA corporate bonds, though typically countercyclical, increased much more in the 2007-2009 crisis than in past recessions. If we look at the heterogeneity in financing between firms, it is well documented that the corporate sector as a whole has become a net lender over the recent two decades. Two stylized facts are of paramount importance: the increasing trend in cash holdings by the US corporate sector and the fact that in the aggregate the average firm is able to finance its capital expenditures by internal cash flows. However by looking at disaggregated micro-level data one notices that small and highly productive firms are typically cash constrained: they need to raise external finance for growing up their scale of operations. It is palusible that credit tightening, of the amount seen in the credit recessions, affected more these small and young firms that are more profitable than others. Indeed the measured change in total factor productivity observed during the Great Recession can reflect an increased distortion in the allocation of capital between firms, where difficulty in obtaining credit affects more the growth of more productive but smaller firms.

In this work I document that a significant portion of the measured drop in TFP observed during the Great Recession can be attributed to financial conditions that disrupt the allocation of capital further from that implied by firm productivities. Following the approach of Olley and Pakes (1996), I decompose aggregate productivity in the economy into a technological component and into a second term defined as the covariance between firm size and firm productivity. This second component captures the allocative efficiency in the distribution of production factors between firms with heterogeneous productivity levels. I show that in the Compustat sample roughly 53 percent of the drop in TFP that occurred between the peak (2007:Q1) and the trough (2009:Q2) of the recession is accounted for by a decrease in allocative efficiency. Forthermore, exploiting the time coverage of the Compustat sample, I document that excluding the last two recessions the reallocation in output shares between firms, though still an important driver of TFP fluctuations, accounts for a much lower fraction of TFP changes. Therefore the Great Recession stands out because it witnessed an unprecedented decline in the covariance between firm size (measured by firm's output share) and firm productivity. It is therefore legitimate to consider recessions, and in particular the last episode, times of increased misallocation of resources between heterogeneous production units, rather than times of negative technological shocks.

The aim of this paper is to use an off-the-shelves model of heterogeneous plants with credit market imperfections to quantify the contribution of financial disruption to the dynamics of capital misallocation during the Great Recession. Financial market imperfections are introduced as an external cost function capturing the basic notion that external funds are more costly than internal cash flows. While most of the literature studying the impact of financial frictions on firms' investment focus on debt financing (see for example Kiyotaki and Moore, 1997 or Khan and Thomas, 2013), I explicitely allow for equity financing as well. Considering only debt can be a problem: if firms can avoid a tightening of frictions in debt financing by replacing debt with equity finance, then models that only allow for debt financing could overstate the importance of financial frictions. Moreover, Fama and French (2005) document that equity issuances are quantitatively important in the Compustat sample. A possible concern regarding my modelling choice is that I do not focus on debt and equity separately but I consider their sum, so that in my model debt and equity are perfect subsitutes. Covas and Haan (2011) document that at least for firms up to the 99th percentile of the size distribution both debt and equity issuances are procyclical, suggesting that during recessions firms find it more difficult to raise external finance in either form. Therefore focusing on external finance as the sum between equity and debt financing is not a relevant loss of generality for my purposes.

I use the model (calibrated to the pre-2007 period) to answer the following question: "By how much would misallocation have changed during the recession if borrowing costs had stayed constant to the pre-recession average?". In other words, with the help of the model, I can compute the counterfactual scenario of a recession driven by a productivity shock only, and assess the contribution of the change in financial frictions to the amount of allocative efficiency. To answer the question: "What is the contribution of costly external finance to the fall in misallocation"? I study the transitional path of the economy between two steady states (SS1 and SS2 from here on). The parameters in the initial steady state SS1 are calibrated to match cross-sectional moments of firm distribution in the pre-recession period 1980-2007. I use the Compustat dataset (a large panel of listed firms for the US) to calibrate my model so that is able to reproduce most of the fall in aggregate total factor productivity and misallocation observed from data. More precisely I perform two quantitative exercises: in the *baseline* I compute the transition between SS1 and SS2, using as inputs a deterministic path of aggregate TFP shocks (calibrated to match the observed drop in GDP) and the change in the parameters of the external cost function. In the *counterfactual*, I compute the transition between SS1 and SS2, using as inputs the deterministic path of aggregate TFP shocks but *keeping the parameters in the external cost function fixed at the pre-recession period values*. Using such counterfactual scenario I can answer the following question: "What would have been the fall in capital reallocation between firms if the degree of financial frictions had stayed constant at its pre-recession value?".

In order to highlight the mechanism though which financial frictions affect productivity, it is better to present the flow of funds equation for firms in my model:

$$d = \pi(k, z) - I - AC + e$$

where d denotes dividends, π is operating cash flow, I is investment, AC stands for physical adjustment costs and e stands for external financing. Dividends and external finance cannot be negative by definition. The flow of funds constraint simply states that if firm financing needs for capital expenditures (i.e. $I+AC$) exceed the cash flow generated internally then the firm has to raise additional funds by tapping the financial market (i.e. has to choose $e > 0$). Issuing new shares or borrowing however is costly: the firm pays a fixed cost λ_0 plus a variable cost λ_1. Financial frictions in the model therefore act through this channel: an increase in the cost of raising external finance reduces the share of firms raising external funds. But firms raising external funds are more productive than the rest, as I show in the following part.

Using the Compustat panel, I sort firms according to their finance regimes:

1. Dividends distribution regime ($d > 0$ & $e = 0$)
2. Financial inactivity regime ($d = 0$ & $e = 0$)
3. External finance regime[1] ($d = 0$ & $e > 0$)

Notice, however, that about 20 percent of firms in this sample both raise external finance *and* distribute dividends. This behavior is puzzling given standard corporate finance theory, since it implies that there are profitable opportunities to reduce dividends and equity issuance or debt (remember that in my model, as it is standard in the literature, the cost of external funds is larger than internal funds). I decided to group these firms into the dividend distribution regime (for sure they are not liquidity constrained).

For any year (1980-2007) I compute some statistics for firms in each finance regime. Table 1 summarizes my findings.

[1] I consider that a firm is not raising external finance if the ratio between internal funds and capital expenditues is between 0.95 and 1.05. Perturbing this threshold does not change my results significantly.

1980-2007	**External Finance**	**Fin. Inactive**	**Div.distrib.**
Share of firms	0.23	0.297	0.474
Share of cap	0.028	0.059	0.913
Share of invest	0.039	0.057	0.904
Earnings/cap	0.567	0.275	0.355
Invest/cap	0.29	0.193	0.194
Tobin's q	3.76	1.78	2.83

Table 1: Distribution of Firms across Finance Regimes in the Data (Average over 1980-2007).

Table 1 reveals that about half of the firms pay dividends. Firms paying dividends account for a large share of capital and investment in the sample, they are more productive than liquidity constrained firms but less productive than firms raising external finance. Firms raising external funds are much more productive than the rest, as measured by the earnings-capital ratio. These small firms (measured by capital) with high Tobin's q require external finance to finance investments. Higher costs of raising external funds during the crisis affected most these "growth firms".

2 Literature Review

The present work lies at the intersection between two strands of literature: the empirical literature about TFP growth and reallocation and the theoretical literature about dynamic general equilibrium models with heterogeneous firms and financial frictions.

On the empirical side the growing availability of longitudinal firm-level data has allowed the analysis of reallocation across individual producers and the connection of this reallocation to aggregate productivity growth. Representative work in this area includes Baily, Hulten and Campbell (1992) and Foster et al. (2001). A common theme of these studies is to decompose aggregate productivity growth into several parts to characterize the contributions of within plant productivity growth and reallocation, where the latter includes the contribution of reallocation among continuing establishments and the impact of entry and exit. Despite that their findings vary with the specific data sets and decomposition methodologies used, a uniform finding in these studies is an important role of reallocation in accounting for aggregate productivity growth in the U.S. manufacturing. For instance, Foster et al. (2001) document that reallocation accounts for about half of overall total factor productivity growth in U.S. manufacturing for the period 1977 to 1987. All these empirical studies use the sum of output (or employment) weighted firm/plant level TFP (or labor productivity) to measure the aggregate productivity of an industry. According to Baily, Hulten and Campbell (1992) the definition of aggregate productivity is as follows. Suppose the production function for plant i in period t is:

$$\begin{aligned} y_{it} &= f\left(k_{it}, l_{it}, m_{it}\right) \\ &= k_{it}^{\alpha_k} l_{it}^{\alpha_L} m_{it}^{\alpha_M} \end{aligned}$$

where k, l, m are capital, labor and intermediate inputs, respectively. Then establishment level TFP is computed as:

$$\log TFP_{it} = \log y_{it} - \alpha_k \log k_{it} - \alpha_l \log l_{it} - \alpha_m \log m_{it} \tag{1}$$

where α_k, α_l and α_m are return to scale factors for capital, labor and intermediate inputs. Finally aggregate productivity in period t (at the sector level) is defined as:

$$TFP_t = \sum_i \omega_{it} TFP_{it} \tag{2}$$

where ω_{it} is the output (or labor) weight of plant i in the sector. In this work, in order to measure reallocation of productive inputs I look at the time variation of a measure of allocative efficiency originally proposed by Olley and Pakes (1996), hereafter OP. OP noticed that aggregate productivity at a given point in time (as defined, for example, in 2) can be decomposed as follows:

$$TFP_t = \frac{1}{N_t}\sum_i TFP_{it} + \sum_i \left(TFP_{it} - \overline{TFP}_t\right)\left(\omega_{it} - \overline{\omega}_t\right) \tag{3}$$

where TFP_{it} is firm level productivity, ω_{it} is the share of output (or labor) of the firm, N_t is the total mass of active firms, and a bar over a variable indicates the unweighted average of the firm-level measure. This OP decomposition splits the aggregate productivity TFP_t, defined as the weighted average of firm-level productivity, into an *unweighted firm-level average* and a *covariance term*. The covariance term is a summary measure of the within-industry cross-sectional covariance between size and productivity: it is expected that in a well-functioning market economy such covariance is positive, i.e. firms with higher than average productivity have a larger than average size. A low covariance indicates then that aggregate productivity could improve by reallocating resources towards the most productive firms. This analysis of allocative efficiency by using OP decomposition in (3) has been performed in quite a few studies. In the seminal contribution of Olley and Pakes (1996), the authors found that the covariance term increased substantially in the US telecommunications equipment industry following the deregulation of the sector in the early 1980s. OP argued that this was because the deregulation permitted inputs to be reallocated more readily from less to more productive US firms. In a subsequent study, Bartelsman et al. (2013) found that the OP covariance term for labor productivity averages about 50 log points within US manufacturing industries: this implies that the industry index of labor productivity in the average US manufacturing industry is 50 percent higher than it would be if employment shares were randomly allocated within industries. Bartelsman and his coauthors found however that the OP covariance term reaches only 20-30 log points in Western Europe and it was close to zero, if not negative, in Central and Eastern European countries at the beginning of their transition to a market economy. They documented also that in Central and Eastern European countries the covariance term increased substantially in the 1990s as their transition to a market economy progressed.

On the theoretical side there are several studies that analyse an economy with heterogeneous production units, noting that aggregate TFP depends not only on the TFP's of the individual firms but also on how inputs are allocated across firms. These papers focus on distortions in product, labor or credit market and policies that can all slow down aggregate productivity growth by hindering the reallocation process among heterogeneous producers. A seminal contribution in this field is Hopenhayn and Rogerson (1993): using the Hopenhayn (1992) model of firm dynamics they quantify the aggregate TFP loss due to firing costs. A non-exhaustive list of more recent works comprises Buera and Shin (2013), Buera et al. (2013), Guner et al. (2008), Hsieh and Klenow (2009), Restuccia and Rogerson (2008) and Midrigan and Xu (2014). Much of this literature however emphasizes the role of frictions and policies in the cross-country difference in long-run TFP and, therefore, abstracts from the cyclical dynamics of misallocation. For example, Hsieh and Klenow (2009) build on the key insight that misallocation can result as lower aggregate TFP and using data on manufacturing try to measure the extent of misallocation in China and India compared to US (they need US as a control group that takes into account model misspecification and measurement error). They interpret the gap in marginal revenue product of capital between different establishments as evidence of misallocation; their calculations imply that if capital and labor were hypothetically reallocated to equalize marginal products to the extent observed in the US, manufacturing TFP would increase by 30-40% in China and by 40-60% in India. Restuccia and Rogerson (2008) explore the quantitative impact of *policy distortions* on aggregate productivity in a stationary equilibrium with heterogeneous plants. They show that policy distortions that create heterogeneity in the prices faced by individual producers lead to misallocation of resources across heterogeneous plants, and as a result can lead to sizable decreases in output and measured TFP. However, differently from my work, Restuccia and Rogerson (2008) focus their attention only on the steady-state distribution of firms and therefore are silent about the impact of policy distortions on reallocation during economic downturns.

Midrigan and Xu (2014) also study the impact of financing frictions on misallocation and focus in particular on two distinct channels: borrowing costs distorting the entry decision of firms and borrowing costs distorting the allocation of capital among firms with different productivities. They find that only the first channel is quantitively relevant. Compared to my work, their main task is to explain cross-country differences in TFP whereas I focus my attention on the cyclical variation of TFP in the US economy during the recent recession.

My paper also contributes to the literature exploring the impact of financial shocks on business cycle fluctuations. Jermann and Quadrini (2012) document the behavior of debt and equity financing over the business cycle using aggregate data. They furthermore develop a *representative firm model* in which investment is financed using both debt and equity and costs of adjusting dividends prevent the avoidance of paying financial frictions. Jermann and Quadrini find that credit shocks have been an important source of business cycles. However the representative firm setting that they employ prevents them from studying the impact of financial shocks on resource misallocation.

Perhaps the work closest to mine is Khan and Thomas (2013) who study the cyclical implications of credit market imperfections in a quantitative dynamic general equilibrium model in which firms are subject to two frictions: collateralized borrowing and partial investment irreversibility. Collateral constraints limit the firm's investment behaviour and partial irreversibilities in investment lead firms to follow (S,s) rules with respect to their capital. The presence of these real and financial frictions slow down the reallocation of capital across firms. Since reallocation is essential in determining aggregate TFP, they show that a financial crisis (originating as a sudden shock to the firms' collateral constraint) can generate a large and protracted drop in aggregate TFP. Therefore the drop in TFP following a financial shock is endogenous because it is a consequence of the change in the distribution of firms. They study the behavior of aggregate quantities after a negative shock to borrowing conditions (in the spirit of Guerrieri and Lorenzoni (2011)) and find that their model predicts aggregate changes resembling those from the 2007 US recession. However they are not able to quantify the aggregate productivity loss due to the impact of financial frictions in the form of higher cost of external finance; indeed they assume that the only source of external funds is debt, subject to a collateral constraint. They hence rule the possibility of financing investment by issuing new equity; however it is important to include equity finance, Fama and French (2005) document that firms frequently issue equity, and equity issuances are quantitatively important. Finally, another recent paper investigating the link between credit market imperfections and misallocation is Azariadis and Kaas (2012), who propose a sectoral-shift theory of TFP. They build a model in which sectors are hit with different productivity shocks and limited enforcement in loans prevents reallocation of capital towards more productive sectors. The result is that the level and growth of aggregate TFP is negatively correlated with the dispersion of sectoral TFP growth rates.

The rest of the paper is organized as follows. The next section explains the empirical findings regarding misallocation. In section 4 I set out the model and characterize the optimal decision rules of the firms. In Section 5 I explain the calibration and simulation of the model and in Section 6 I analyze the results. Section 7 concludes the paper.

3 Measuring Misallocation over the Business Cycle

As discussed in the previous section, I look at the time variation of a cross-sectional measure of allocative efficiency (the Olley-Pakes gap) to assess the cyclical properties of capital reallocation. Some empirical studies tend to confirm the procyclical nature of reallocation (contrary to the *cleansing view* of recessions, in which more capital should be liquidated in recessions). Among these, Eisfeldt and Rampini (2006) document that *flows of capital* among firms decrease during downturns. In Figure 1 I plot the series for reallocation using updated data to 2012. The authors define capital reallocation as the sum of acquisitions plus sales of PP&E (property, plant and equipment). Their measure focuses hence only on instances when existing capital is sold or acquired but they are not able to tell whether such transfers of ownership are productivity enhancing or not. In other words, they are silent about the allocative efficiency of capital.

As I argued above, a more informative way to measure reallocation of capital and to assess whether it is productivity enhancing or not is to analyze the covariance of firm level multifactor productivity and firm size. If the creative destruction theory were true, this covariance should sharply increase during economic downturns, reflecting the fact that firms with productivity below the average become smaller since resources are shifted away from them.

Production function estimates. The first step in the computation of the OP covariance term, defined as the second term in the right-hand side of equation (3), is the estimation of the (log) firm-level total factor productivity, which requires the estimation of a production function. Using the Compustat panel, I estimate the production function given by:

$$y_{it} = \alpha_0 + \alpha_j^k k_{it} + \alpha_j^l l_{it} + z_{it} + \varepsilon_{it} \tag{4}$$

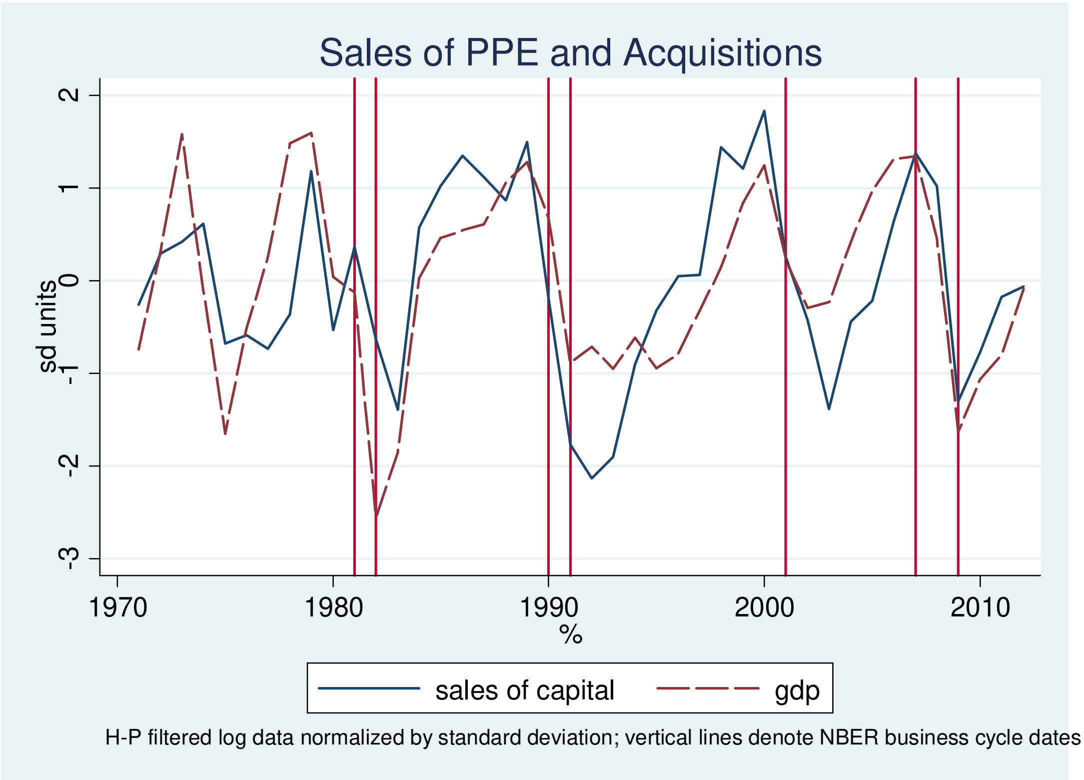


Figure 1: Reallocation over the cycle

where y_{it} is the log of value added[2] for firm i in period t, k_{it} is the log of capital and l_{it} is the log of labor inputs. In the baseline exercise the input elasticities $\{\alpha_k, \alpha_l\}$ are the same for all sectors; as a robustness check I allow them to vary across 2-digit sectors, indexed by j. The error term is composed by two parts: a pure shock ε_{it} that is not observed by the firm nor by the econometrician and a productivity shock observed by the firm but not by the econometrician. The most straightforward way to estimate (4) is by OLS. However the problem with estimating a production function using OLS is that firms that have a large productivity shock may respond by using more inputs, which would yield biased estimates of the input coefficients and hence biased measure of TFP (*simultaneity bias*). Since traditional estimators used to overcome endogeneity issues (fixed effects, instrumental variables) have not proven satisfactory for the case of production function, a number of semiparametric alternatives have been proposed. Both Olley and Pakes (1996) and Levinsohn and Petrin (2003) have developed a semiparametric estimator that addresses the simultaneity bias. The key difference between the two methods is that Olley and Pakes (1996) use investment whereas Levinsohn and Petrin (2003) use materials used in production as a proxy for TFP. Since data on investment is readily available and often non-zero at the firm level but data on materials is not, I follow Olley and Pakes (1996) to estimate the production function. Once I estimate the production function parameters I obtain the level TFP by

$$TFP_{it} = \exp\left(y_{it} - \widehat{\alpha}_0 - \widehat{\alpha}_k k_{it} - \widehat{\alpha}_l l_{it}\right) \tag{5}$$

In the estimation of (4) I use industry specific time dummies, hence my measure of tfp is free of the effect of industry or aggregate growth in any year.

Table 2 reports the estimates for the production function parameters and their standard errors using the entire sample period for manufacturing and non-manufacturing firms. The results for all the firms combined, presented in the second column of the table indicate a labor share of 0.74 and a capital share of 0.29. The estimates for the persistence and conditional volatility of TFP (not reported in the table) are 0.69 and 0.30 respectively.

Validation of my TFP estimates. In order to gauge the sensibility of my TFP measure, I contrast some of its properties with those obtained from studies that use longitudinal micro-level datasets different from Compustat.

[2]Value added is defined as sales - materials, or, equivalently, as operating income before depreciation and amortization plus labor expenses. Unfortunately in COMPUSTAT only information about the number of employees is available, therefore I approximate labor expenses by multiplying the number of employees by average wages from Social Security Administration. See appendix for more on data construction.

Table 2: Production function parameters

lnreal_va VARIABLES	(1) All sample	(2) Manuf	(3) Non-manuf
lnlabor	0.736***	0.824***	0.699***
	(0.00147)	(0.00239)	(0.00196)
lnreal_capital	0.292***	0.213***	0.330***
	(0.00131)	(0.00207)	(0.00178)
Constant	-2.653***	-2.827***	-2.582***
	(0.0205)	(0.0210)	(0.0363)
Observations	204,158	108,199	95,959
R-squared	0.932	0.944	0.921

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Total Factor Productivity		
	75th/25th	90th/10th
US Census	1.56	2.68
Compustat	1.47	2.41

Table 3: TFP Dispersion - Comparison

Please notice that labor has a higher coefficient in the manufacturing subsample and the opposite is true for the capital coefficient. This is consistent with the findings of Foster et al. (2001) who examine manufacturing data and service sector data. There is significant dispersion in firm level TFPs. In an important contribution investigating the productivity distribution in the US manufacturing sector, Syverson (2004) finds out that the interquartile range (i.e. ratio between 75th and 25th percentile) of tfp is around 1.56; moreover including more of the tails amplifies the heterogeneity: the ratio between the 90th and the 10th percentile is as much high as 2.68. Using Compustat, I find similar results, as Table 3 summarizes. Another robust finding is that labor productivity is more dispersed than total factor productivity. The interquartile range is 2.53 whereas the 90th/10th ratio is 7.28.

TFP dispersion and allocation efficiency over time.

In Figure 2 I plot firm-level TFP dispersion, computed as follows. First I compute total factor productivity by estimating the production function and taking the exponent of the predicted residuals (see equation (5)).

$$y_{it} = \alpha_0 + \alpha_k k_{it} + \alpha_l l_{it} + z_{it} + \varepsilon_{it}$$

$$TFP_{it} = \exp\left(y_{it} - \widehat{\alpha}_0 - \widehat{\alpha}_k k_{it} - \widehat{\alpha}_l l_{it}\right)$$

Then I define TFP shocks (e_{it}) as the residual from the following first-order autoregressive equation for firm-level log TFP:

$$\log TFP_{it} = \rho \log TFP_{it-1} + \delta_t + e_{it}$$

where δ_t is a year fixed effect (to control for cyclical shocks). Since this residual will also contain firm-level demand shocks that are not controlled for by 2-digit price deflators, my measure will combine both demand and technological shocks[3].

Then I compute the cross-sectional standard deviation and the interquartile range (IQR) of tfp shocks for each year across firms. Finally, I take a simple average across all years in the sample. In Figure 2, I report the IQR since it is more robust to outliers, however the results change little if I use the standard deviation.

In Figure 2, the blue shaded columns represent the share of quarters in recession within a year. It is apparent the negative correlation between cross-sectional tfp dispersion and gdp growth: interquartile range of TFP (IQR) spikes up during recessions, displaying a clearly countercyclical behavior. The findings delivered by Figure 2 confirm that during recessions an increase in cross-sectional heterogeneity is observed, which means that, ceteris paribus, there are more benefits to reallocate resources to more productive firms. The thesis of this paper is that instead during recessions frictions to capital liquidity increase and this dampens a reallocation process between firms that would be otherwise productivity enhancing. This is even more apparent from the Great Recession which has been characterized by a huge financial turmoil which greatly increased the cost of borrowing for firms; this reduced access to market for equity to more productive and small firms, dampening their growth and contributing to the fall in the covariance between firm productivity and firm size. From Figure 3 it is possible to notice that such covariance (Olley-Pakes gap) is generally procyclical and dropped the most in the 2007-2009 recession.

After estimating the total factor productivity series as I explained above, I compute

[3] This is a common feature of TFP estimates, since firm-level prices are very difficult to obtain. A relevant exception is Foster, Haltiwanger and Syverson (2008).

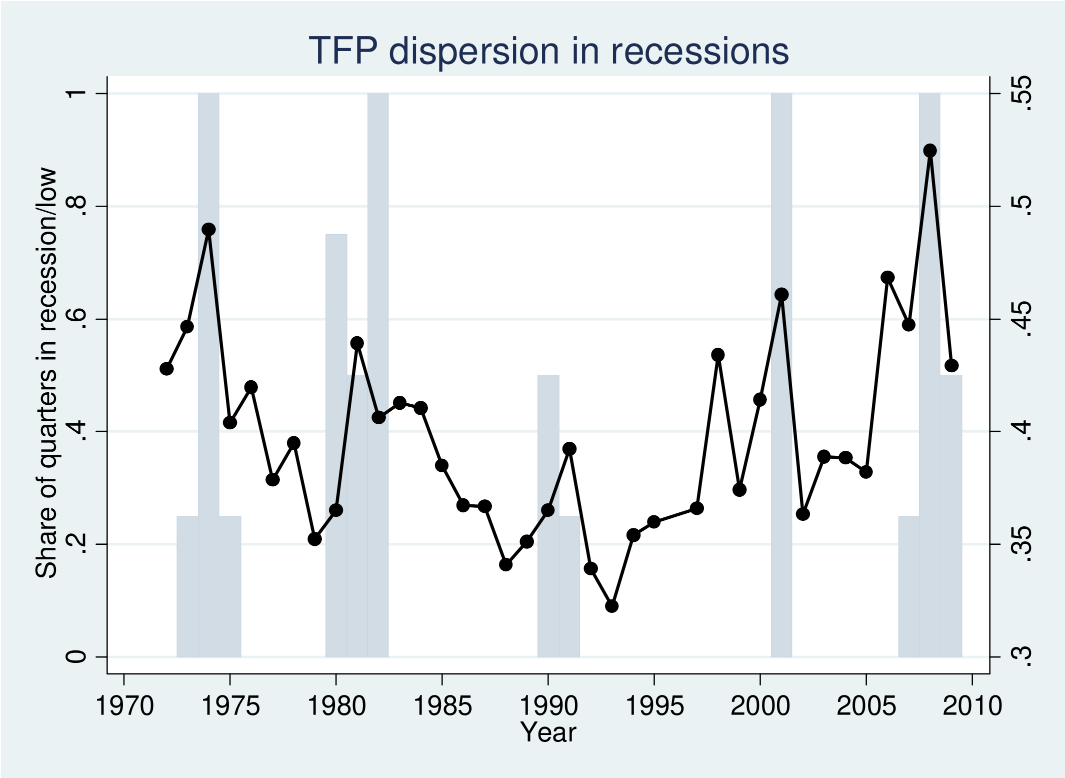


Figure 2: TFP dispersion in recessions

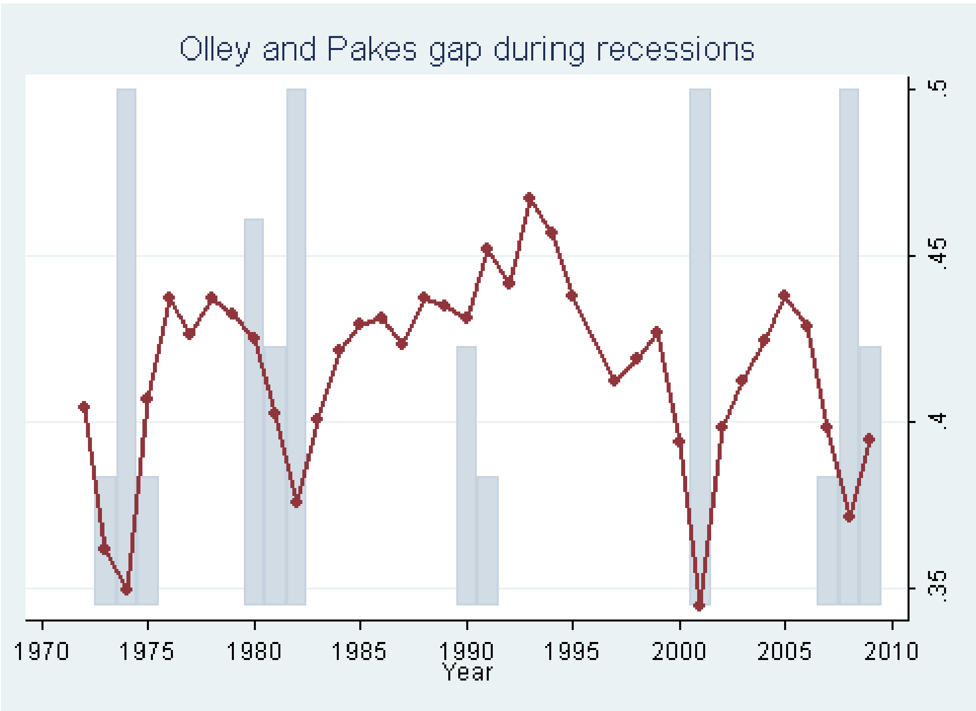


Figure 3: Covariance between size and productivity

the output-weighted TFP for the firms in the Compustat panel:

$$TFP_t = \sum_i \omega_{it} z_{it}$$

I decompose the change in TFP during the Great Recession following the methodology of Olley and Pakes (1996) as the sum of unweighted component and a covariance term between size and total factor productivity:

$$\Delta TFP_{2009:2007} = \Delta UNW_{2009:2007} + \Delta COV_{2009:2007}$$

As I detailed above, the covariance between size and productivity is typically procyclical; it is striking however that in the Great Recession an unprecedented drop in this covariance term was observed. Since the Great Recession has been characterized by an unprecedented contraction in financing conditions, it is relevant to quantify the impact of financial frictions on the covariance term. In the real scenario I simulate the model by feeding only the aggregate shock A. Notice that the unweighted term in the model is given by

$$UNW = A \int \exp(z)\, \mu(dk, dz) = A \int \exp(z)\, \mu^z(dz)$$

hence I calibrate the shock A so to reproduce exactly the drop in UNW. Result: the aggregate shock A alone generates a small drop in COV. In the second scenario, on top of the aggregate shock A, I add the financial shock (modelled as a sudden and unexpected increase in the cost of raising external finance). By construction the drop in UNW is the same as before (and the same as what is observed in the data) but the drop in the COV is much higher than in the "real shock only" case. Hence I claim that the contribution of the financial crisis to reallocation is given by:

$$\Delta COV(\text{real} + fin \text{ shock}) - \Delta COV(\text{real shock}).$$

Compustat vs Census of Manufacturers. A key advantage of using the Compustat database is that I can decompose the change in allocative efficiency over time across both manufacturing and service, whereas previous studies that rely on the Longitudinal Research Database, LRD (from the US Census of Manufacturers) were limited on the manufacturing sector. Figure 4 and 5 plots the evolution of the share of manufacturing in terms of sales and employment over time. Given the decreasing importance of the manufacturing sector in terms of both output and employment, the advantage of a dataset like Compustat that covers all the sectors (though only for listed firms) is clear.

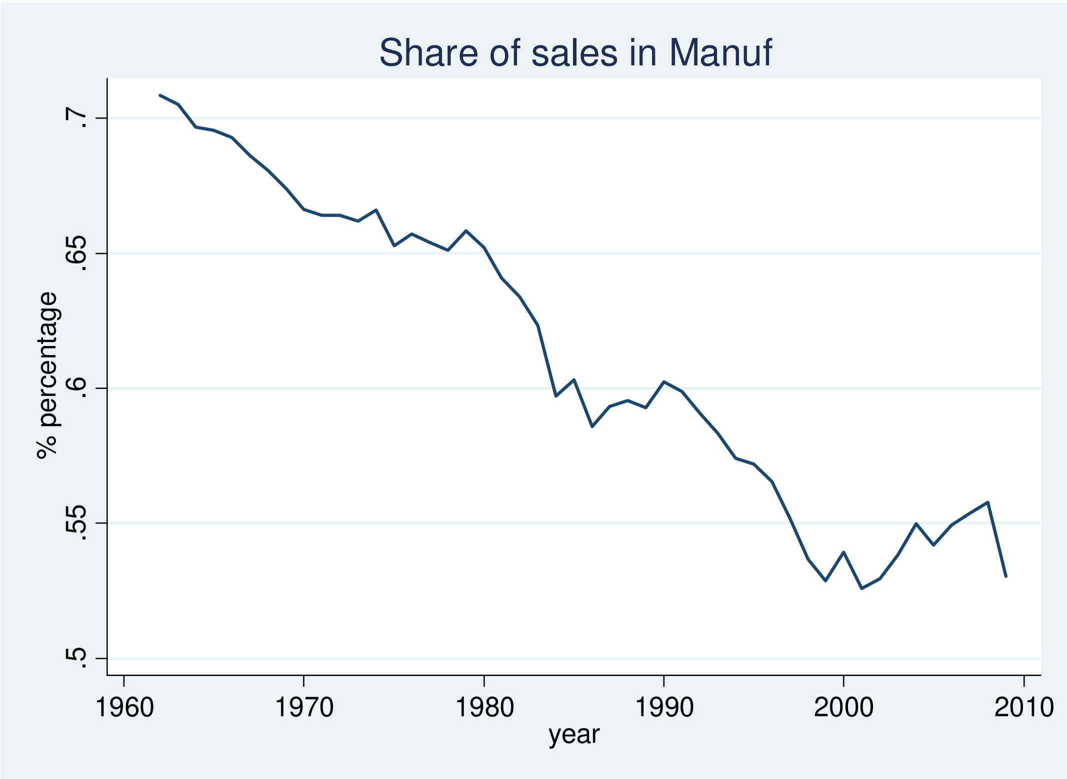


Figure 4: Share of sales in manufacturing over all sectors

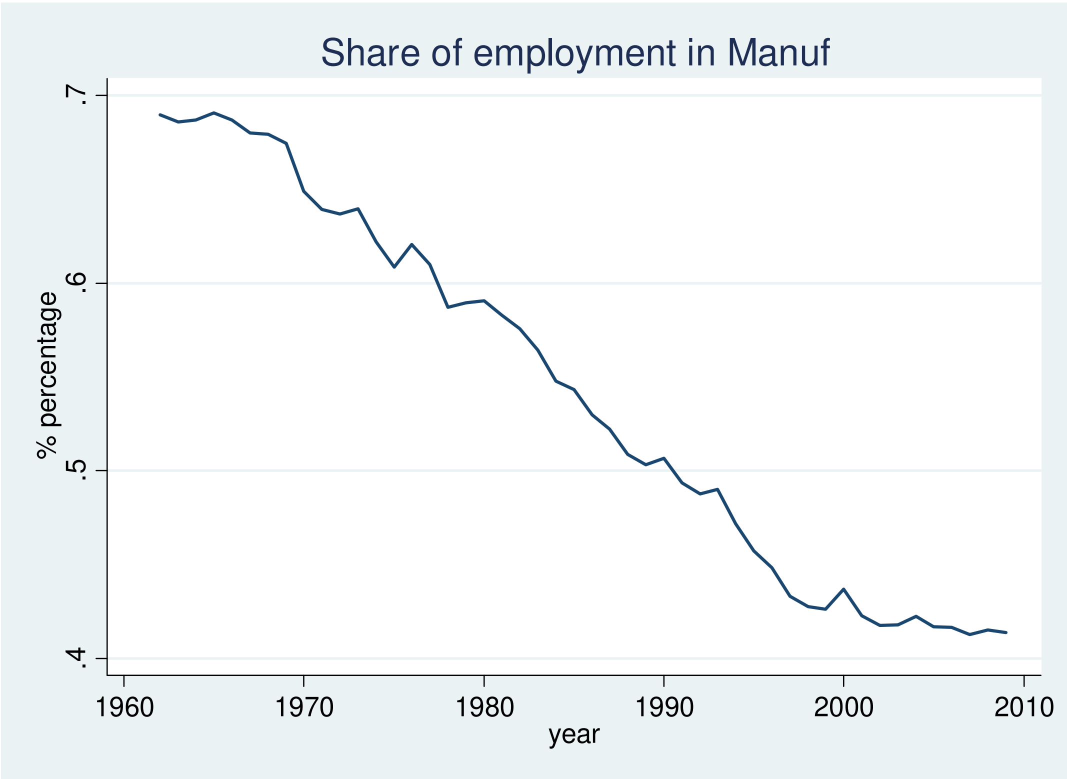


Figure 5: Share of employment in manufacturing sector over all sectors

4 The Model

The aim is to use my model to infer the impact of financial frictions on TFP through the reallocation channel. I therefore need a model that contains the following features:

- Firms with heterogeneous productivity levels, to make reallocation of capital among firms meaningful.
- Imperfect credit markets: I assume that in case firms want to raise additional funds (in excess over operating cash flow) they need to pay an additional cost.
- Shock to aggregate TFP to generate the recession; for the sake of tractability I model the negative aggregate shock to technology as a deterministic sequence that is unforeseen by economic agents[4].

4.1 Firms

I begin with describing the economic problem of the firms. Firms are ex-ante identical and are subject to an exogenous TFP shock A_t that is common across all firms and to an idiosyncratic productivity shocks denoted by z_{it}[5]. Since in the data firm-level productivity shocks show a high degree of persistence (as documented, among others, by Foster et al. (2001) and Bloom et al. (2012)) I assume that these shocks are generated by an autoregressive process with persistence ρ:

$$\log z_{it} = \rho \log z_{it-1} + \varepsilon_{it} \tag{6}$$

where ε_{it} is distributed as a $N(0, \sigma_\varepsilon^2)$. As it is standard in the literature I discretize the continuous time process described in (6) as a first-order Markov chain with transition matrix Q using Tauchen (1986) procedure. I assume $\Pr\{z' = z_j | z = z_i\} = Q_{ij} \geq 0$ and $\sum_j Q_{ij} = 1$ for each $i = 1, \ldots, N_z$. The sequence of aggregate shocks A_t is known with perfect foresight. Even though firms are ex-ante identical they differ ex-post since they experience different histories of idiosyncratic productivity shocks.

[4]The alternative way would be to introduce explicitely aggregate uncertainty in the model, along the lines of Krusell and Smith (1996).

[5]The shocks z_{it} could in principle capture any shock affecting firm's revenues, hence not only shocks to technical efficiency but also (idiosyncratic) demand shocks.

Firms use capital and labor as factor inputs and produce output by operating a decreasing returns to scale production function; the operating profit function (whose counterpart in the data is cash flow from operations) is:

$$\pi\left(A_t, k_{it}, z_{it}\right) = \max_{l_{it}\geq 0}\left\{A_t z_{it} F\left(k_{it}, l_{it}\right) - w l_{it}\right\} \tag{7}$$

Notice that $\pi(\cdot)$ is the operating profit function that is obtained after solving for the static labor choice, therefore it is a function of k and the shocks only. Denoting by I_{it} the investment made by firm i in year t, capital obeys the following law of motion

$$k_{i,t+1} = (1-\delta)\,k_{it} + I_{it},$$

where $\delta \in (0,1)$ denotes the depreciation rate. It is well-known that a model in which it is costless to adjust the capital stock delivers a time series for investment rates that is far too volatile; I therefore assume that the firm incurs quadratic adjustment costs when investing. It is also well-know, since at least Caballero et al. (1995), that plant-level investment is characterized by periods of inactivity followed by large spikes in investment; while it is hard to match this type of evidence with a quadratic cost of adjustment, I chose to adopt the quadratic specification for his computational tractability.

Firms can finance investment either with internal funds or borrowing from the financial market (by raising new equity or issuing debt). By raising external finance the firm incurs a variety of additional costs going from flotation costs to adverse selection premia. As in Gomes (2001) and Hennessy and Whited (2007) I do not model explicitly a setting with asymmetric information but I attempt to capture the simple fact that external funds are more costly than internal funds in a reduced form way. In particular, I assume that the additional cost of raising external finance is given by

$$c(e) = \lambda_0 + \lambda_1 \cdot \text{amount of external funds}$$

In other words there is a fixed cost λ_0 and a per unit cost λ_1 associated with external finance. A large body of empirical research provides detailed evidence regarding underwriting fees (see, among others, Altinkilic and Hansen (2000)) finding that there are significant economies of scale: this is why a cost function with decreasing average cost seems most appropriate. Cooley and Quadrini (2001) use a slightly different formulation which omits the fixed cost. However the fixed cost formulation is needed in my framework in order (i) to rationalize the presence of economies of scale and (ii) to match the degree of financial inaction that I documented in the Compustat sample (see Table 1).

The firm problem is to choose investment and financial policy to maximize net payments to its shareholders[6], taking as given the real interest rate and the wage rate:

$$V(k,z) = \max_{d,e,I,k'} \left\{ d - e - c(e) + \frac{1}{1+r} \sum_{z'} V(k',z') \, Q(z'|z) \right\} \tag{8}$$

s.t.

$$d + I + \frac{\psi}{2} \frac{I^2}{k} = \pi(k,z,A) + e, \tag{9}$$

$$k' = (1-\delta) k + I, \tag{10}$$

$$c(e) = (\lambda_0 + \lambda_1 e) \, 1_{\{e>0\}}, \tag{11}$$

$$d \geq 0, \tag{12}$$

$$e \geq 0. \tag{13}$$

Equation (9) describes the flow of funds condition for the firm. The sources of funds (on the right-hand side) consists of operating cash-flows π, and external funds, e. The uses of funds (on the left-hand side) consist of capital expenditures, adjustment costs and dividend payments. Please notice that in this setting the only way firms can save is by accumulating capital: I choose to rule out firm savings in cash holdings or other financial assets. Adding financial savings or debt would make the problem more realistic but would also increase considerably the computational burden: with the additional debt choice there is a cross-sectional distribution of firms over three states capital, debt and idiosyncratic productivity (k, b, z) that is more difficult to handle with[7].

Equation (11) describes the external finance cost function: these costs are positive and increasing if the firm uses external funds. If no external funds are required, these costs are zero. This formulation is consistent with the Pecking Order Hypothesis (Myers and Majluf (1984)): firms first use internal finance and if they do not have enough, then issue debt, and as a last resort equity. The pecking order hypothesis can account for the stylized facts that retentions and then debt are the primary sources of finance. Notice furthermore that it is never optimal to raise external finance and at the same time distribute dividends. Indeed

Lemma 1 *It is never optimal for the firm to choose $e > 0$ and $d > 0$.*
Proof. *Suppose by contradiction that the firms chooses $e > 0$ and $d > 0$. Then the firm can decrease both e and d by a small amount $\varepsilon > 0$, which induce a change in profits given by $\lambda_1 \varepsilon > 0$* ■

[6] See appendix D for a derivation of the optimal value maximization problem of the firm.
[7] However, I'm currently working on an extension to incorporate debt into the firm's problem.

4.2 Household

Since I am mainly interested in reallocation of capital among firms with heterogeneous productivities, on the household's side I can focus on a representative agent formulation. The representative agent has preferences over consumption and labor that are summarized by the following utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{14}$$

Household's income comes from wages and dividends. In order to write its budget constraint, I must aggregate all firm-level quantities. To this end, let me define μ_t as the cross sectional distribution of firms over the individual state (k, z) in period t. The budget constraint can then be stated as:

$$C_t + \int P_t \theta_{t+1}(k, z) \mu_t(dk, dz) + b_{t+1} - (1 + r_t) b_t = w_t N_t + \int (d_t + P_t - e_t - c(e_t)) \theta_t(k, z) \mu_t(dk, dz) \tag{15}$$

where θ_t denotes the shares owned by household, b_t denotes bond holdings. In equilibrium $\theta_t = 1$ and $b_t = 0$ for all t since all households are equal.

The representative household's problem is to maximize (14) subject to (15). The first-order conditions with respect to labor supply N_t and bond holdings b_{t+1} are respectively:

$$-\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = w_t,$$

$$U_c(C_t, N_t) = \beta U_c(C_{t+1}, N_{t+1})(1 + r_{t+1}).$$

Since I consider the model in the stationary equilibrium with interest rate r_t, wage rate w_t and aggregate quantities constant over time, the household's problem can be simplified in this following *static* problem:

$$\max_{C,N} U(C, N)$$

s.t.

$$C = wN + \int d(k, z; w) \, d\mu(k, z) - \int e(k, z; w) \, d\mu(k, z) - \int c(e(k, z; w)) \, d\mu(k, z) \tag{16}$$

This is a standard concave problem with interior solutions. In the steady state the Euler equation pins down the interest rate as

$$r = \frac{1}{\beta} - 1 \tag{17}$$

and optimality condition with respect to labor supply becomes:

$$-\frac{U_n(C,N)}{U_c(C,N)} = w \tag{18}$$

Solving the household's problem I get the household's decision rules for consumption $C(w;\mu)$ and labor supply $L^s(w;\mu)$.

4.3 Stationary distribution and Aggregation

Assuming A_t constant, the solution to the firm's optimization problem (8) delivers the *policy functions*

$$k' = g(k,z),\ I(k,z),\ l(k,z),\ y(k,z),\ e(k,z)$$

mapping the firm's state variables k and z into the firm's current choices (please notice that for simplicity I omit the dependence of the policy functions upon the wage w). The vector of individual state variables $x = (k,z)$ lies in $X = [0,\infty) \times Z$, where Z is the discrete set for productivity shocks z, i.e. $Z = \{z_1, z_1, \ldots, z_{nz}\}$. Let $\mathbb{B}$ be the associated Borel σ algebra. For any set $B \in \mathbb{B}$, $\mu(B)$ is the mass of firms whose individual states lie in the set B. The transition function $T(x,B)$ defines the probability that a firm in state $x = (k,z)$ will have a state lying in B in the next period, given the decision rule g for next-period capital. I can define each set B as the Cartesian product $B_K \times B_Z$; then the transition function $T : X \times \mathbb{B} \rightarrow [0,1]$ can be written as:

$$T((k,z), B_K \times B_Z) = \begin{cases} \sum_{z' \in B_Z} Q(z,z') & \text{if } g(k,z) \in B_K \\ 0 & \text{otherwise} \end{cases}$$

where $g(k,z)$ is the policy function for next-period capital. Given the transition function, I can define the probability measure μ as

$$\mu'(B) = \int_X T(x,B)\,\mu(dx) \tag{19}$$

Given the invariant distribution $\mu^*(k,z) = \mu = \mu'$, I can compute the aggregate variables:

- Aggregate investment:

$$I(w;\mu^*) = \int I(k, k'(k,z); w)\,\mu^*(dk,dz)$$

- Aggregate labor demand:

$$L^d(w;\mu^*) = \int l(k,z;w)\,\mu^*(dk,dz)$$

- Aggregate output supply:

$$Y(w;\mu^*) = \int y(k,z;w)\,\mu^*(dk,dz)$$

- Aggregate adjustment costs:

$$AC(w;\mu^*) = \int \frac{\gamma}{2}\frac{I(k,z;w)^2}{k}\mu^*(dk,dz)$$

- Aggregate external finance costs:

$$E(w;\mu^*) = \int c(e(k,z))\,\mu^*(dk,dz)$$

Now I give the definition of equilibrium in my model, focusing for simplicity on the steady-state.

Definition 2 *A stationary recursive competitive equilibrium is a list of value function V, policy functions, invariant measure μ and prices r, w such that:*

(1) Given the prices $\{r, w\}$, the policy functions $d(k,z)$, $e(k,z)$, $I(k,z)$, $k'(k,z)$ solve the optimization problem of the firm in (8)

(2) Factor prices (r, w) are determined by equations (17) and (18)

(3) Markets clear; in particular in the labor market supply equals demand:

$$N^s(w;\mu) = \int_{x=(k,z)} l(k,z)\,\mu(dk,dz) \tag{20}$$

and the good market clears:

$$C(w;\mu) + I(w;\mu^*) + AC(w;\mu^*) + E(w;\mu^*) = Y(w;\mu^*)$$

where the term $E(w;\mu^*)$ represents aggregate costs of raising external finance. Of course by Walras' law this last resource constraint is redundant[8]*: it is implied by combining the firm's flow of funds constraint (9) with the household's budget constraint (16).*

[8] Indeed I don't use it when computing the equilibrium but I verify ex-post that it is satisfied.

4.4 Economic mechanism

Before reporting the results from the simulation, it is useful to look at the steady state distribution. Firms can be in three different finance regimes (this why heterogeneity is important)

1. $d = 0$, $e > 0$: external finance regime
2. $d = 0$, $e = 0$: financial inactivity regime
3. $d > 0$, $e = 0$: dividend distribution regime

Figure 6 illustrates these regimes for the baseline model and reveals a few interesting features. First, firms that are either very small or very productive tap the financial market and do not distribute dividends (top-left region: high z and low k; remember that z and k are the firm's state variables). These firms are in the external finance regime. Second, firms that are either very large or less productive use internal funds to finance investment and also distribute dividends (bottom-right region: low z and high k). They are in the dividend distribution regime. Finally the remaining firms do not distribute dividends and do not raise external finance. They are in the financial inactivity regime. Figure 9 confirms this.

Figure 7 depicts the policy function for external finance, $e(k, z)$. It confirms that large firms, with a high capital stock, generate enough internal cash flow and do not need to raise additional resources from banks or from the equity market. In particular, there exists a capital threshold $\overline{k}(z)$ such that only firms with $k < \overline{k}(z)$ choose a strictly positive value of e; interestingly, such threshold is increasing with respect to productivity: holding the size of the firm fixed, firms that are hit by higher productivity shocks are more likely to raise external finance. This policy rule for external finance shows an inverted U-shape form when external finance is positive. In this model, external finance is a double-edged sword. For a given productivity, a firm needs to borrow to invest and this increases their expected profits. On the other hand, this also increases the cost related to external finance. The policy function reflects these two opposing tendencies creating the inverted U-shape that we observe. Higher values of z increase the future profits, allowing the firm to borrow larger amounts and shift the policy rule up.

Figure 8 plots instead the policy function for dividends $d(k, z)$: small firms tend not to distribute dividends since they need to use all their internal cash flow to finance investment, and this effect is of course more pronounced for higher productivity firms.

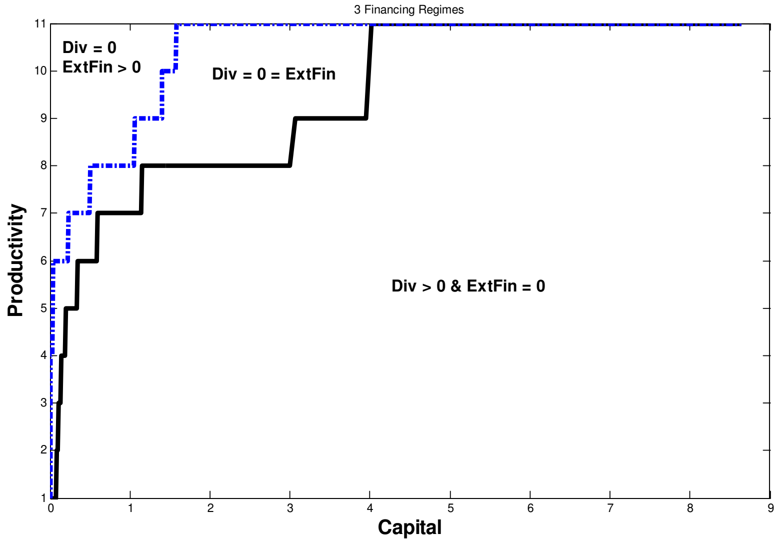


Figure 6: Finance Regimes

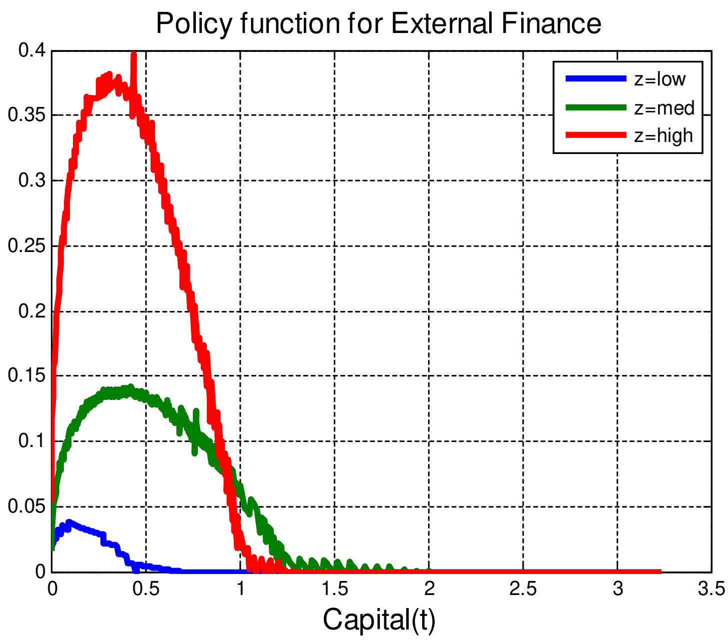


Figure 7: External Finance

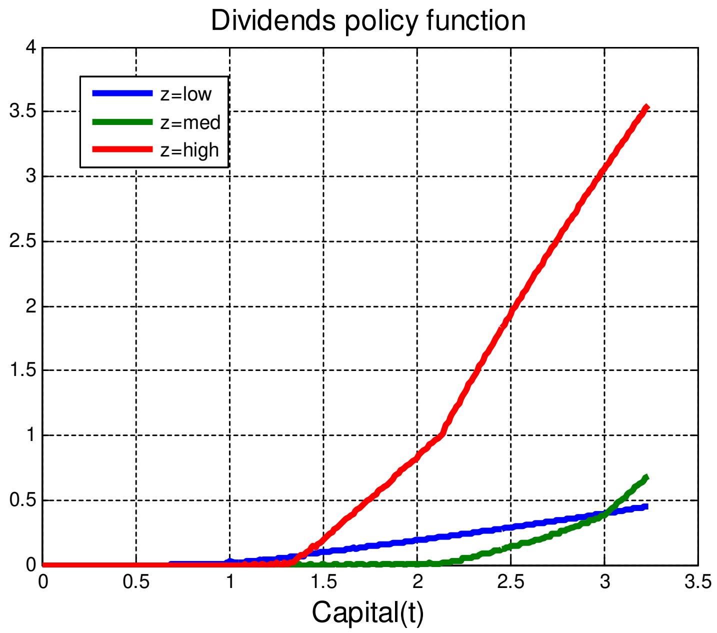


Figure 8: Policy Function for Dividends

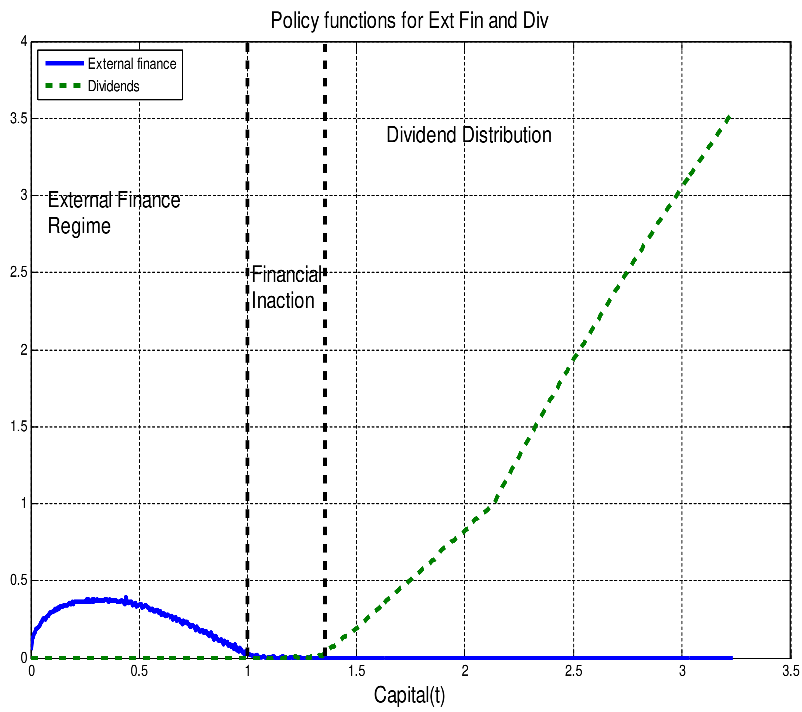


Figure 9: External finance, inaction and dividend distribution

5 Calibration and Quantitative Results

I assume that a time period corresponds to one year. I calibrate the baseline model to match some moments obtained from Compustat. The Longitudinal Research Database (LRD), a large panel dataset of U.S. manufacturing plants developed by the U.S. Bureau of the Census, is another dataset that is widely used in productivity and reallocation studies. One major shortcoming of the LRD for my purposes is that it lacks detailed data on firm's financing choices such as equity issuances, debt, interest expenses, etc. Another shortcoming of the LRD is that it is strictly limited to manufacturing establishments; hence the non-manufacturing sector, which is getting more important over time, is not represented at the LRD (see also discussion in section 3). Consequently I consider Compustat a better choice.

The sample period ranges from 1980 to 2006 which corresponds roughly to the Great Moderation period, before the 2007 recession started. The table below describes the calibration in the baseline scenario (i.e. steady state before the recession)

Parameter	**Symbol**	**Calibration target**
Exponent on capital	α_k	TFP process
Exponent on labor	α_l	TFP process
Depreciation rate	δ	Average I/K
Discount factor	β	Interest rate 4%
Weight on leisure	h	time spent on market work
Adjustment cost	ψ	std I/K
Shock persistence	ρ	TFP process
Shock standard deviation	σ_ε	TFP process
Fixed cost	λ_0	share firms $e > 0$
Marginal cost	λ_1	share firms $e > 0$

Table 4: CALIBRATION

Preferences. Regarding the consumer's side of the economy, as I explained above it is highly stylized (representative agent) since I am mainly interested in firm dynamics. As per-period utility I choose the following functional form:

$$u(C, N) = \log C - \frac{h}{2} N^2$$

where h is the weight on leisure. This utility function has a unit Frisch elasticity of labor supply, which is reasonable for macro models as argued by Hall (2005). I choose the discount factor β such that the interest rate is equal to 4% using equation (17). I choose the parameter h to match the equilibrium labor supply of 0.3, which is the average fraction of time spent on market work.

Technology. I assume firms operate a Cobb-Douglas production function with decreasing returns to scale

$$y_{it} = A_t F\left(z_{it}, k_{it}, n_{it}\right) = A_t z_{it} k_{it}^{\alpha_k} n_{it}^{\alpha_l}$$

with $0 < \alpha_k + \alpha_l < 1$. Productivity shock follows the process

$$\log z_{it} = \rho \log z_{it-1} + \varepsilon_{it}$$

where ε_{it} is independently and identically distributed and normally distributed with mean 0 and variance σ_ε^2. The procedure for calibrating the parameter values α_k, α_l, ρ and σ exploits the micro level information on firm's technology provided by Compustat. As I explained in section 3 of my paper I estimate the following Cobb-Douglas production function in logarithms:

$$y_{it} = \alpha_0 + \alpha_{jk} k_{it} + \alpha_{jl} l_{it} + z_{it} + \varepsilon_{it}$$

allowing the factor elasticities to vary across 2-digit industries (as usual the index i refers to the firms whereas the index j refers to the sector). Then I consider the median across sectors of the α_{jk} and I set the capital coefficient in the model equal to this value. I do the same for the labor coefficient α_l. This procedure delivers a coefficient for capital $\alpha_k = 0.311$ and a coefficient for labor $\alpha_l = 0.65$. Interestingly the micro data do not reject the hypothesis of decreasing returns to scale.

To calibrate the persistence and the standard deviation of the stochastic process for idiosyncratic TFP shocks I first compute TFP in levels from the residual of the estimated equation:

$$\log TFP_{it} = \exp\left(y_{it} - \widehat{\alpha}_0 - \widehat{\alpha}_k k_{it} - \widehat{\alpha}_l l_{it}\right)$$

Then I fit a first-order autoregressive process to $\log TFP_{it}$

$$\log TFP_{it} = \rho \log TFP_{it-1} + \sigma e_{it},$$

where e_{it} is independently and identically distributed across i and t, and drawn from a standard normal distribution. These estimates imply that the parameters of the shock process z in the model are

$$\begin{aligned}\widehat{\rho} &= 0.742\\ \widehat{\sigma} &= 0.275\end{aligned}$$

It is useful to contrast these estimates for the productivity process with the study of Abraham and White (2006) who employ Census data. Their results imply that the persistence of firm-level shocks is surprisingly low: $\widehat{\rho}$ is only 0.37, whereas the standard deviation of the shock is 0.397 (Table 1 in their paper). This striking difference can be partly due to the different size of firms (firms in Compustat are typically bigger than firms in the Census) and to the fact that I consider all sectors in the economy (excluding only financial and government) whereas they can analyze only the manufacturing.

Another possible concern regarding my calibration is the non-standard choice for the Cobb-Douglas parameters α_k and α_l. Typically in the macro literature these parameters are calibrated to match the average labor share in aggregate data; however I find more reasonable to use micro-level estimates, since I do not have a representative firm with an aggregate production function in my model. The average labor share in my model implied by my calibration is 0.54 which is not too far from what reported in the real business cycle literature.

The final parameter to be calibrated is the adjustment cost parameter ψ. Because the volatility of the investment rate is very sensitive to this parameter, I choose a value to match the cross-sectional volatility of the investment rate in my data, which is around 0.16. More specifically, for any given value of ψ, I solve the model numerically and obtain the stationary distribution of firms. Using this stationary distribution, I compute the cross-sectional standard deviation of the investment rate in the model. Without adjustment cost, my model would imply excessive sensitivity of investment to variations in productivity shocks, which is inconsistent with empirical evidence. My calibrated value of ψ is close to the value reported by Cooper and Haltiwanger (2006), who estimate it using indirect inference.

Financing costs. The external cost function

$$c(e) = (\lambda_0 + \lambda_1 e)\, 1_{\{e>0\}}$$

is meant to capture the basic notion that external funds are more costly than internal ones. Broadly speaking, there are two types of costs associated with external finance: (i) informational costs and (ii) transaction costs. Informational costs are related to the bad signal the firm may transmit to the market when trying to raise funds (see agency cost theories, Myers and Majluf (1984)) but these are very hard to quantify. Transaction costs are given by compensation to intermediaries, legal and accounting costs associated to debt or equity issuance.

In order to calibrate the parameters of this external cost function, I need to construct an empirical measure of a firm's external financing needs. The aim is to choose λ_0 and λ_1 so that the model moments referring to external finance closely match the corresponding statistic computed from the data.

Consider the flow budget constraint of a firm in my model:

$$d_{it} - e_{it} = \pi \left(k_{it}, z_{it}\right) - I_{it} - \frac{\psi}{2} \frac{I_{it}^2}{k}, \tag{21}$$

where the left hand side represents the net financial flow out of the firm (if positive) or into the firm (if negative). If the right-hand side of (21) is positive, so that the firms's capital expenditure is less than the cash-flow generated by the firm in t, then funds flow *out* of the firm. In this case the firm is distributing dividends to its shareholders. Conversely, if the right-hand side of (21) is negative, then the firm's investment needs exceeds the available cash-flow, which means that funds flow *into* the firm. Then the firm is raising external funds, i.e. e_{it} is positive. Let me define the following two statistics:

- X_{it}: capital expenditures.
- AF_{it}: available funds. These are cash flow from operations net from interest payments.

Here I follow the standard approach in the literature on external finance dependence (see, among others, Rajan and Zingales (1998)). Since my model does not distinguish between investment in existing assets or acquisition of new assets, I compute the measure of investment as: X_{it}= capital expenditures + acquisition - sale of PPE (property, plant and equipment).

To compute available funds I have two possibilities: (1) Available Funds = Operating activities - net cash flow (OANCF) or Funds from operations (FOPT). (2) Available Funds = Income before extraordinary items (IBC) + depreciation and amortization (DPC). Both methods yield similar results. Then I can compute the share of firms raising external finance in year t as the number of firms whose investment is greater then their available funds in year t over the total number of firms in t:

$$\frac{\sum_{i=1}^{N_t} 1_{\{X_{it} > AF_{it}\}}}{N_t}. \tag{22}$$

I can also compute the fraction of investment that must be financed externally in year t

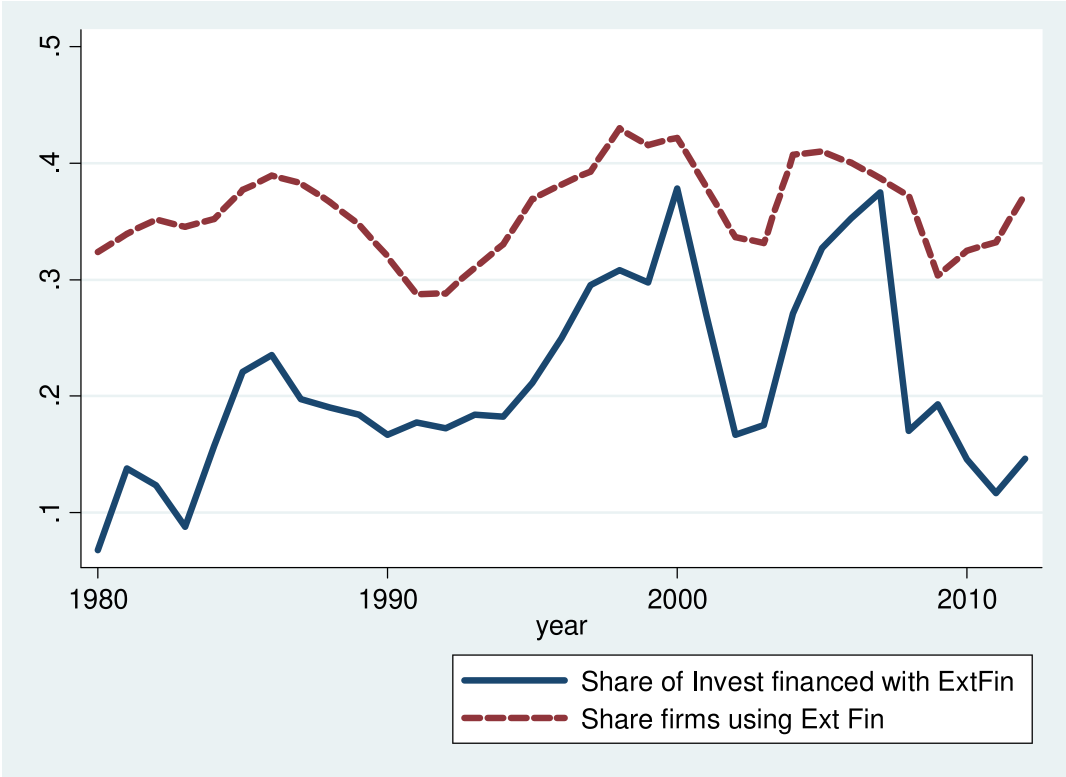


Figure 10: External Finance Flows

Parameter	value	Calibration target	Data moment
λ_0	0.10	Share of firms with $e > 0$	0.36
λ_1	0.28	Ext fin / investment	0.21

Table 5: Steady-state Calibration

as:

$$\frac{\sum_{i=1}^{N_t} (X_{it} - AF_{it})\, 1_{\{X_{it} > AF_{it}\}}}{\sum_{i=1}^{N_t} X_{it}}. \tag{23}$$

In Figure 10 I plot the evolution of (22) and (23) over time.

Calibration of cost function

I summarize the calibration in Table 5. The two moments reported in the table are computed taking the average of (22) and (23) across the years from 1980 to 2007; they are meant to capture the average financing needs of firm in the steady state before the Great Recession broke out.

Note: when you read in the table that the external finance over investment ratio is 0.21 it means that on average 21% of investment undertaken by firms is financed externally

Calibration target	**Average 1980-2007**	**value in 2007**	**value in 2009**
Share of firms with $e > 0$	0.36	0.39	0.30
Ext fin / investment	0.21	0.37	0.19

Table 6: Before and After the recession, Calibration

(average across years). Please notice that this value is consistent with the empirical findings of Zetlin-Jones and Shourideh (2012).

As I discussed in the introduction, to simulate the impact of the Great Recession on firms financing environment I set the parameters of the cost function to match the share of firms raising external finance after the Great Recession hit the economy. As it is apparent from Figure (10) during the GR it became more difficult for firms to access credit: indeed the share of firms accessing outside financing dropped from 39% in 2007 to 30% in 2009. Moreover the fraction of investment financed with external funds dropped from 37% in 2007 to 19% in 2009 (see Figure 10 or Table 6).

6 Results

6.1 Steady State

In Table 7 I report the moments of the firm dynamics generated by the model and compare them with the corresponding data from Compustat. I report in italics the moments that are a calibration target, where the match is exact by construction. As I explained in the previous section, I chose the depreciation parameter δ to match the aggregate investment ratio and the adjustment cost parameter ψ to match the volatility of the investment rate. For the other quantities, one can see that my model matches most cross-sectional moments reasonably well. In particular the model slightly overpredicts the autocorrelation of the investment rate that is observed in the data sample and slightly overpredicts the covariance between firm size and firm productivity.

Considering the financing regimes for the firms in the cross-section, the model by construction matches the shares of firms whose capital expenditures are larger than internal funds; however it generates more firms distributing dividends and less firms inactive than what is observed in the data.

Variable	Data	Model
Average I/K	*0.17*	*0.17*
std I/K	*0.156*	*0.156*
Autocorr. of I/K	0.596	0.64
Cov(ω,z)	0.438	0.534
External Finance	*0.36*	*0.36*
Financial Inactivity	0.213	0.147
Dividend distrib.	0.426	0.493

Table 7: Data vs Model, Results

6.2 Great Recession Simulation

As I discussed in section 3, I decompose the total factor productivity index for firms in Compustat as the sum of an unweighted component and a covariance component, following the methodology pioneered by Olley and Pakes (1996) and reprised by Bartelsman et al. (2013):

$$TFP_t = \sum_i \omega_{it} z_{it} = \overline{z}_t + \sum_i \left(\omega_{it} - \overline{\omega}_t\right)\left(z_{it} - \overline{z}_t\right) \tag{24}$$

where $\overline{z}_t$ and $\overline{\omega}_t$ represent unweighted mean productivity and unweighted mean share, respectively. This decomposition is useful to understand if the Great Recession impacted more on the productivity of the average firm or on the covariance between size and productivity. As documented in the first row of Table 8, the output-weighted total factor productivity decreased by 1.97% from 2007 to 2009; of such drop the unweighted term accounted for -0.93% and the covariance for -1.04%. Remember that the lower this covariance, the lower is the share of output that goes to more productive firms and the lower is the weighted productivity. But what is the contribution of the worsening in credit conditions on this covariance, which measures the allocative efficiency in the distribution of production factors across firms? I can evaluate this contribution by simulating the counterfactual scenario of a real recession only using my model.

The production function equation in the model is given by:

$$y_{it} = A_t z_{it} k_{it}^{\alpha_k} l_{it}^{\alpha_l}$$

Hence total factor productivity in logs is equal to:

$$TFP_{it} = A_t z_{it}$$

and it is the product of an aggregate shock times a firm-level idiosyncratic shock. The

	% ΔTFP	% Δunweighted	% ΔCov(k_{it},z_{it})
Data (Compustat)	-1.97	-0.93	-1.04
(I) Real shock	-1.095	-0.93	-0.165^{B}
(II) Real and fin shock	-1.79	-0.93	-0.86^{A}

Table 8: Data and Counterfactual Exercise

output-weighted TFP, which is the model counterpart of (24) is:

$$TFP = A\int z \cdot \omega(k,z)\,\mu(dk,dz) = A \cdot \mathbf{E}(z) + A \cdot COV(z,\omega)$$

where $\omega(k,z)$ is the output weight of a firm with capital k and idiosyncratic productivity z:

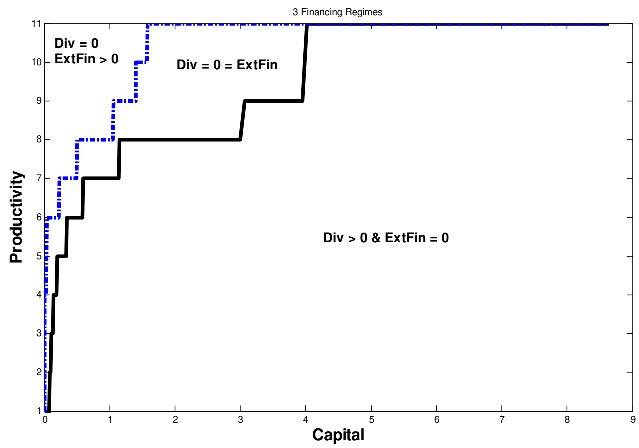
$$\omega(k,z) = \frac{y(k,z)}{\int y(k,z)\,\mu(dk,dz)}$$

This share is the model counterpart of ω_{it} in equation (24). The Great Recession had a negative impact both on the technological term and on the allocative efficiency term. I calibrate the aggregate shock to reproduce exactly the observed drop in the unweighted term.

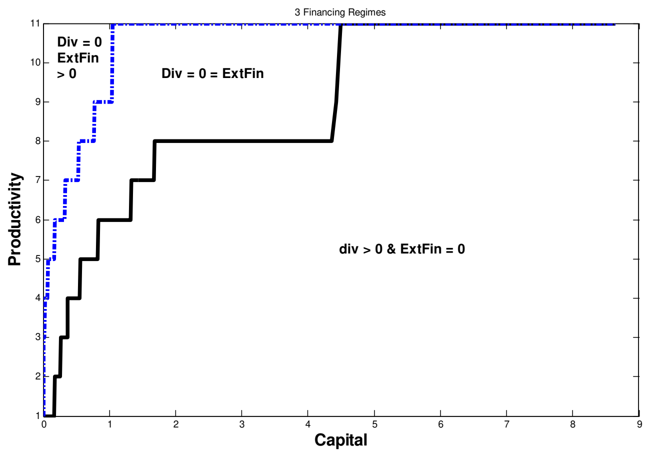
In the first scenario I hit the economy with a real shock only. From the table it is evident that a recession driven by a real aggregate shock only have a modest impact on the cross-sectional efficiency in the allocation of factors. A financial recession, as captured by the second exercise, instead, has a much larger impact on the covariance term. As in a diff-in-diff strategy, I can recover the contribution of financial frictions to the variation in the covariance by taking the difference between the two cells A and B in the table. In other words the impact of the financial shock on the cross-sectional efficiency of resources is

$$\begin{aligned} & \Delta Cov(\text{real+fin shock})\text{-}\Delta Cov(\text{real}) \\ = \quad & -0.86-(-0.165)=-0.695 \end{aligned}$$

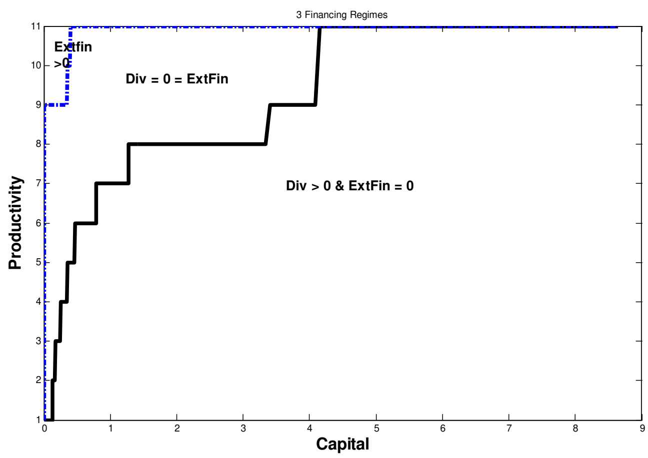
The main channel through which the increase in financial cost affects the covariance term is that it changes the distribution of firms across the three different financing regimes. The picture below show the partition of the state space into the 3 financing regimes



The financing costs are calibrated so that the share of firms in the external finance regime is roughly equal to the corresponding share in the data. In exercise I, when only the real aggregate shock hits the economy, the new distribution is



In exercise II, when also the financial shock hits the economy, the new distribution is:



	% ΔTFP	% Δunweighted	% ΔCov(k_{it},z_{it})
Data (Compustat)	-1.97	-0.93	-1.04
(I) Real shock	-1.67	-0.93	-0.74
(II) Real and fin shock	-5.19	-0.93	-4.26

Table 9: Data and Counterfactual Exercise - Partial Equilibrium

Much less firms can access now external finance; but those firms who were accessing external finance were the most productive. Therefore the financial shock causes a reallocation of productive inputs from high to low productivity firms, decreasing the cross-sectional efficiency, as captured by the covariance term.

General Equilibrium Effect. Since my model is cast in general equilibrium, it is insightful to conduct the hypothetical experiment of shutting down the price feedback mechanism. Specifically, I fix the wage rate at its level in the steady state before the recession. At this wage I use labor demand to determine aggregate employment by ignoring the labor market-clearing condition (20). After solving the firm's problem, I derive aggregate investment and aggregate output. I then use the resource constraint to solve for aggregate consumption. The profit function of the firm, under the parametric assumptions described in section 5, is:

$$\pi\left(A,k,z;w\right)=\left(1-\alpha_l\right)\left(\frac{\alpha_l}{w}\right)^{\frac{\alpha_l}{1-\alpha_l}}A\left(zk^{\alpha_k}\right)^{\frac{1}{1-\alpha_l}}$$

The above equation reveals that the lower wage increases the firms' profits and its return to investment. Moreover, since π also represents operating sales net of labor payments[9], a lower wage increases the firm's internal cash flows. This equilibrium price feedback effect dampens the decrease in investment among firms that are raising external finance and hence the drop in the covariance between size and productivity is smaller in general equilibrium. Table 9 reports the results from the simulations in the partial equilibrium, i.e. the changes in the total factor productivity that I would observe if the wage stayed constant at the level before the recession. In particular the decrease in the covariance between size and productivity is about 4/5 times larger than in general equilibrium.

To sum up, my numerical experiment demonstrates that performing counterfactuals in partial equilibrium can have potentially misleading outcomes.

[9] Indeed $\pi\left(k,z\right)=\max_l\left\{zy\left(k,l\right)-wl\right\}$.

7 Conclusions

In this work I document that in the Compustat dataset (representative sample of listed firms in the US) a significant part of the drop in total factor productivity observed during the Great Recession can be attributed to a decrease in the allocative efficiency of capital among firms (rather than to a technological effect common to all firms). Indeed the decrease in the covariance between size and productivity (a measure of allocative efficiency) is 1.04 percent out of roughly 2 percent decline in total factor productivity. The use of Compustat improves upon previous studies for at least two reasons. First the service sector, extensively represented in Compustat, has become increasingly important in the recent years; second while Compustat does not cover *small* firms (as long as small firms are not listed firms) it offers a very thorough description of *large* firms that account of more than 50 percent of total GDP and more than 30 percent of total employment in the US economy.

The empirical finding that the allocative efficiency of resources among firms worsens during economic downturns sharply contrasts with the *cleansing* view of recessions: according to this theory, that dates back at least to Schumpeter[10], recessions should be times of enhanced reallocation. Since during economic downturns the dispersion in profit growth rates increases (as documented by Eisfeldt and Rampini 2006) there are more benefits of reallocating capital from less to more productive plants; moreover during recessions the opportunity cost of resources are typically low (plants are underutilized); these observations should imply that recessions are times of accelerated productivity enhancing reallocation. However in the Great Recession financial conditions worsened and hence the increase in credit market frictions could have had a negative impact on reallocation. In particular during the Great Recession, reallocation of productive inputs was driven more by frictions in credit markets than by economic fundamentals such as productivity.

With the help of a model with heterogeneous firms I find out that the distribution of firms among financing regime is a crucial determinant of this covariance. Since in the data I see that the 2007-2009 period witnessed a large drop in the firms raising funds from financial market, I relate the increase in the cost of external financing to the misallocation of resources among firms. In the model a reduced-form cost function captures the basic notion than external funds are more costly than internally generated cash-flow. The increase in the cost of external finance affects most firms that are small and highly productive; these firms are growing and are giving a positive contribution to the covariance term. In order to assess the contribution of financial conditions to the covariance I simulate a counterfactual recession where the economy is hit by a technological worsening

[10] The first formalization of the creative destruction theory is by Aghion and Howitt (1992).

only; by construction this shock to the average total factor productivity matches exactly the drop in the unweighted tfp.

To summarize the two critical implications from my study are the following:

(i) variations in measured total factor productivity are only to a small extent variation in the productivity of the average firm. The main part is attributable to a reallocation of market shares between firms with heterogeneous productivity levels.

(ii) The extent to which more productive firms also enjoy a larger market share critically depends on the easiness in accessing financial markets to get external funds for investment.

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A Data Appendix

How to construct investment rates

Since firms record capital stock at book value rather than the more useful economic concept which is replacement value, I use perpetual inventory model (as described in Salinger and Summers 1983 and Gomes 2001) to convert book value of capital into replacement value for every firm-year. First, I set the replacement value of the initial capital stock equal to the book value of gross PPE for the first year that the firm shows up in Compustat. Then I estimate the useful life of capital goods in any year using the formula

$$L_{i,t} = \frac{Bk_{i,t-1} + I_{i,t}}{Depr_{i,t}}$$

where $Depr_{i,t}$ is the reported value of depreciation and amortization, and take the time average of L_{it}, which I call L_i. Finally I compute the series for the capital stock k_{it} (in market value terms) iterating on the following recursive formula:

$$k_{i,t} = \left[k_{i,t-1}\frac{P_t}{P_{t-1}} + I_{i,t}\right]\left(1 - \frac{2}{L_i}\right)$$

for $t = 1, 2, ..$, where P_t is deflator for non-residential investment. and L_i is the time average of $L_{i,t}$.

Derivation of the Olley-Pakes decomposition

The decomposition proposed by Olley and Pakes (equation 3 in the main text) follows after some algebra:

$$\begin{aligned}
TFP_t &= \sum_i \omega_{it} TFP_{it} \\
&= \sum_i \left(\overline{\omega}_t + \omega_{it} - \overline{\omega}_t\right)\left(\overline{TFP}_t + TFP_{it} - \overline{TFP}_t\right) \\
&= N_t \overline{\omega}_t \overline{TFP}_t + \sum_i \left(\omega_{it} - \overline{\omega}_t\right)\left(TFP_{it} - \overline{TFP}_t\right) \\
&= \overline{TFP}_t + \sum_i \left(\omega_{it} - \overline{\omega}_t\right)\left(TFP_{it} - \overline{TFP}_t\right)
\end{aligned}$$

where $\overline{TFP}_t = \frac{1}{N_t}\sum_i TFP_{it}$, where N_t is the number of active firms in period t.

Output weighted TFP in the model

- Let $\mu(k, z)$ denote the stationary distribution of firms over capital and productivity

- The output-weighted productivity in the model is computed as:

$$TFP = \int_z \int_k \omega(k,z) e^z$$

where

$$\omega(k,z) \equiv \frac{y(k,z)\mu(k,z)}{\int_z \int_k y(k,z) d\mu(k,z)}$$

B Computation - Steady State

The algorithm follows Aiyagari(1994) and Huggett(1993). I start by guessing a value for the wage w. For the given wage I solve the firm's decision problem by value function iteration on a discrete grid. Then I compute the invariant distribution of firms over capital and productivity. As a last step I check whether the labor market equilibrium condition holds. If not, I update the wage.

More in detail:

- Step 1 - Make a guess for equilibrium wage w.

- Step 2 - Given w, solve the firm's problem by value function iteration on a discrete grid. Even if slow, it is the most robust method (better to use this because policy functions are non linear due to the fixed equity cost). Get policy function $k' = g(k,z)$ and the other decision rules.

- Step 3 - Using the policy function $g(k,z)$ computed in step 2 and the exogenous Markov chain for productivity shocks, compute the invariant distribution $\mu^*(k,z)$ by iterating on (19)

- Step 4 - Using the stationary distribution $\mu^*(k,z)$ obtained in step 3, compute aggregate labor demand $N^d(w) = \sum_{k,z} n(k,z)\mu^*(k,z)$. Then check if equation

$$-\frac{U_n\left(C, N^d(w)\right)}{U_c\left(C, N^d(w)\right)} = w$$

is satisfied[11]. If it is, stop; otherwise update the wage and go back to step 2.

[11] The function $-\frac{U_n\left(C,N^d(w)\right)}{U_c\left(C,N^d(w)\right)} - w = 0$ is not perfectly continuous given the discretized nature of the algorithm, and it is therefore not always possible to compute a clearing wage level to an arbitrary level of precision. However the problem is generally well-behaved with a tolerance level of 10^{-7} in the baseline simulation.

An alternative way is to compute explicitly the excess demand function for labor: $N^d - N^s$.

- Iterate until convergence.

C Computation - Transition

I describe the algorithm for a transitory shock (i.e. the initial and the final steady state are equal).

1. Compute steady state

2. Make a guess for the wage and the interest rate path along the transition: $\left\{r_t^{old}, w_t^{old}\right\}_{t=1}^{T}$. A good guess is r^{ss}, w^{ss}

3. Solve the firm's problem by backward induction, starting with $V_T = V^{ss}$. Compute policy functions $\{g(k,z)\}_t$ for $t = T-1, T-2, .., 1$

4. Using the exogenous Markov chain and the time-varying policy functions computed in the previous step, iterate forward the distribution starting from $\mu_1 = \mu^{ss}$:

$$\mu_{t+1}(k', z') = \sum_k \sum_z \Pi(z, z') \, 1_{\{k: g(k,z)=k'\}} \mu_t(k, z)$$

5. Using $\{\mu\}_{t=1}^{T}$ and $\{g(k,z)\}_{t=1}^{T}$ compute aggregate variables C_t, N_t, Y_t for each time t

6. Get new sequence of wage and interest rates $\{w_t', r_t'\}$ from the household's first order conditions:

$$w_t' = -\frac{U_n\left(C_t, N_t^d\right)}{U_c\left(C_t, N_t^d\right)}$$

$$r_t' = \frac{U_c(C_t, N_t)}{\beta U_c(C_{t+1}, N_{t+1})}$$

If $\max_t \left\{|r_t^{old} - r_t'| + |w_t^{old} - w_t'|\right\}$ is less than a precision threshold, stop. Otherwise update the prices sequences in this way:

$$\begin{aligned} w_t^{new} &= \phi w_t' + (1-\phi) w_t^{old}, \\ r_t^{new} &= \phi r_t' + (1-\phi) r_t^{old} \end{aligned}$$

and go back to step 2.

D Firm's value problem

Consider the representative household's maximization problem which I re-write below for convenience:

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{25}$$

s.t.

$$C_t + \int P_t \theta_{t+1} d\mu_t + b_{t+1} - \left(1 + r_t\right) b_t = w_t N_t + \int \left(d_t + P_t - e_t - c\left(e_t\right)\right) \theta_t d\mu_t \tag{26}$$

The first-order conditions with respect to bond b_{t+1} and share holdings θ_{t+1} are:

$$U_1\left(C_t, N_t\right) = \beta\left(1 + r_{t+1}\right) U_1\left(C_{t+1}, N_{t+1}\right)$$ [12]

and

$$U_1\left(C_t, N_t\right) P_t = \beta U_1\left(C_{t+1}, N_{t+1}\right) E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

Hence combining the two equations I get the result that the required rate on return on equity must be equal to the real interest rate (in other words, there is no risk premium).

$$\left(1 + r_{t+1}\right) = \frac{E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}}{P_t}.$$

or,

$$P_t = \left(\frac{1}{1 + r_{t+1}}\right) E_t\left\{d_{t+1} + P_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

Iterating forward yields

$$P_t = \sum_{n=1}^{\infty} \prod_{j=1}^{n} \left(\frac{1}{1 + r_{t+j}}\right) E_t\left\{d_{t+1} - e_{t+1} - c\left(e_{t+1}\right)\right\}$$

which corresponds to (**??**) or (8).

[12] Along the transition aggregate variables and prices are deterministic sequences, hence I do not need the expectation operator.

E An alternative formulation of the firm's problem

An alternative formulation of the firm's problem is the following[13]:

$$V(k,z) = \max_{k'\geq 0, I} \left\{ \begin{array}{c} \pi(k,z) - I - \frac{\psi}{2}\frac{I^2}{k} - \lambda_0 1\left\{I + \frac{\psi}{2}\frac{I^2}{k} > \pi(k,z)\right\} - \lambda_1 \max\left\{I + \frac{\psi}{2}\frac{I^2}{k} - \pi(k,z), 0\right\} \\ + \frac{1}{1+r}\mathbb{E}_{z'|z} V(k',z') \end{array} \right\}$$

s.t.

$$k' = (1-\delta)k + I,$$

where $1\{\cdot\}$ is an indicator function. The term in brackets is the sum of current net cash flow and expected discounted continuation value; net cash flow is current profits minus investment spending and financing costs. If current profits are lower than desired capital expenditures then the firm has to pay an additional cost (both fixed and linear). To see the equivalence with (8) it is useful to define the auxiliary variable e:

$$e = \max\left\{I + \frac{\psi}{2}\frac{I^2}{k} - \pi(k,z), 0\right\}$$

The term $I + \frac{\psi}{2}\frac{I^2}{k} - \pi$, if positive, represents the amount of external finance raised by the firm. As it is explained in the main text, the cost function captures the basic fact that external funds are more costly than internal funds.

[13] I would like to thank Matthias Messner for suggesting this equivalent formulation