Evolution-Based Approaches in Economics and Evolutionary Loss of Information

Heinrich, Torsten

Institute for Institutional and Innovation Economics (IINO),
University of Bremen, Bremen, Germany

14 December 2015

Evolution-Based Approaches in Economics and Evolutionary Loss of Information

Torsten Heinrich[*]

December 15, 2015

Abstract

Evolutionary economics provides a self-organizing stabilizing mechanism without relying on mechanic equilibria. However, there are substantial differences between the genetic evolutionary biology, and the evolution of institutions, firms, routines or strategies in economics. Most importantly, there is no genetic codification and no sexual reproduction in economic evolution, and the involved agents can interfere consciously and purposefully. This entails a general lack of fixation and perhaps the quick loss of information through a Muller's ratchet like mechanism. The present contribution discusses the analogy of evolution in biology and economics and considers potential problems resulting in evolutionary models in economics.

1 Introduction

From Thorstein Veblen to Alfred Marshall to Joseph Schumpeter and others, economists have called for dynamic theories in economics that would be able to consider evolutionary approaches. While many of the analytical tools were not available at the time, both evolutionary biology and a slowly developing field of evolutionary and dynamic economics (Nelson and Winter (1982), Hayden (1982), Bush (1987), Silverberg et al. (1988), Elsner (2012), Gräbner (2015)) have made tremendous progress.

Providing a mechanism of stabilization without having to assume mechanic equilibria, evolutionary dynamic approaches provide a very powerful modeling tool. Being able to function with very few additional assumptions, they model economic entities like institutions or firms illustratively and represent processes of competition realistically as a selection device.

Yet there are both differences to evolutionary systems in biology and lessons to be learned from evolutionary biology, particularly in relation to the retention of information and the possible danger of its successive loss and catastrophic destruction of the population.

The paper will proceed to discuss the history of economic thought on evolutionary approaches in the following section before reviewing recent advances on evolutionary loss of information in evolutionary biology and considering their application to economics.

---

[*] IINO, University of Bremen, Bremen, Germany; torsten.heinrich@uni-bremen.de

2 Why is Economics not an Evolutionary Science?!

In 1898, Thorstein Veblen (1898) contemplated that economics was not an evolutionary science - in stark contrast to other political and social sciences. His famous phrase had a double meaning: economics was neither open to change as a system of theories nor was it willing to consider evolutionary processes as part of its theories. Even with insights about more complex views of human society, development and decision making ripening in anthropology, psychology, and sociology, Veblen could find but a very static conception of actors in economics, "passive and substantially inert and immutably given" (Veblen (1898)).[1]

The following century saw considerable advances not only in anthropology, psychology, and sociology, on which economics could draw, but also in many subdisciplines of economics. Original institutional economics emphasized the role of human instincts in shaping social and economic structure (Veblen (1899)), the possibility of self reinforcing mechanisms in the form or Gunnar Myrdal's circular cumulative causation (Myrdal (1957), Berger and Elsner (2007)), and the interaction between socio-economic systems and their environment (Georgescu-Roegen (1975)), to name a few among many other dynamic and evolutionary aspects discussed in this community. Drawing on some of this research, Schumpeterian evolutionary economics, starting with Richard Nelson and Sidney Winter's contribution (Nelson and Winter (1974, 1982)) introduced formal models with extensive analogies from evolutionary biology. A very active and diverse tradition has emerged from this line of research.Safarzyńska and van den Bergh (2010) At the same time, Post-Keynesian, Kaldorian traditions (Kaldor (1940), Goodwin (1967)) have added further examples of dynamic economic models that defy the traditional quest for static systems and static equilibrium metaphors. It has further been shown that such economic models can, without great difficulty, be developed into systems with strange attractors (chaotic dynamics) (Lorenz (1987), Keen (1995)) thus begging the question how we are able to make reasonable analyses and predictions about the past, present, and future of real economic systems at all. How do those systems gain the (admittedly not abundant) stability they have? - A question that will be taken up again below.

Despite the emergence of evolutionary approaches to economics and social sciences and in spite of very promising results (Nelson and Winter (1982), Silverberg et al. (1988), Safarzyńska and van den Bergh (2010)) achieved in recent years, economics must still be considered a non-evolutionary science. Economics textbooks show mostly convergence towards a canon of equilibrium models (Elsner (2013)), the bulk of the profession continues to follow a set of established theories and methods that have extensively been developed with incredible amounts of research effort by incredible numbers of scholars over the years. This has certainly not been without results - in the absence of crises and crashes, the profession's forecasting methods work exemplary, though this is not much reconciliation when faced with the dragon king event they failed to predict. With more effort in both pluralism of theories and theories that explicitly consider evolutionary and dynamic approaches, this might in the future be

---

[1] In contrast to other contemporary social scientists Veblen did notably not call for Darwinian approaches in economics or social sciences. For an overview, see Geoffrey M. Hodgson (2005)

alleviated.

Why should economic theories take evolutionary approaches into account? Beyond what Veblen outlined almost 120 years ago, evolutionary models have considerable advantages over equilibrium models. They provide a mechanism of self-organization and self-stabilization of dynamic systems that is both credible and illustrative. A force of selection driven by current environmental conditions works on a diversity of routines or businesses or strategies which in turn is maintained by either creative decision-making or random trial and error. The current level of adaptation to the environment generically limits the extent of successful economic activity. Environmental conditions themselves may, of course, also be subject to change.

Scholars have warned about the danger of drawing unwarranted analogies to evolutionary biology (Witt (2005), Hodgson (2001)): There are no obvious analogies to a genotype-phenotype distinction in economic systems. Diversity generation likely works in a completely different way as contrary to genetic evolution, conscious actors are able to purposefully steer and influence their businesses and strategies, their routines and technologies (Witt (2005)). Economic entities, businesses, or routines, do not normally undergo reproduction. Even the analogy of the evolutionary vehicle may be questionable; how much resembles a routine or business an organism in their evolutionary function.

What remains is a system capable - under most conditions for most of the time - of self-organization and self-stabilization. And yet, viewed from an information theoretic perspectives, biological and socio-economic evolution may not be so different at all, as long as the analogies of the evolutionary vehicle and of selection pressure are maintained. The evolutionary vehicle is characterized by a distinct set of information - genetic or otherwise - that drives its dynamic success. And this success depends as much on the quality of that information as on the entity's ability to reproduce or retain it intactly - save for a warranted amount of diversity necessary for the functioning of the system. It does not matter whether the information is transferred to increasing numbers of child entities, whether it is maintained in a business experiencing massive growth, whether it is coded in a formal alphabet or whether it exists in a diffuse institutional arrangement or behavioral patterns. In fact, in the case of tacit knowledge, it may even be possible that economic firms are unable to codify or modify some of their crucial evolutionary information, in which case the biological genotype-phenotype analogy may still be instructive.

Beyond the abstract information theoretic nature of the process, the similarities between these different possible evolutionary models are quickly exhausted. Maintenance of the integrity of an information set and its sexual and asexual reproduction are profoundly different. Consequently the evolutionary model chosen to model certain aspects of society or economics will have consequences for the nature of the evolution possible in that model and for the results generated by it. A brief look at properties of evolutionary systems is hence warranted.

3 The Evolution of Nature and the Nature of Evolution

Genetic evolution relies on a distinction between the actual genetic code - mostly stored and expressed in long chains of nucleotides arranged as a stable (and in case of DNA redundant, double) molecule around histone proteins - and the organism that is generated from it. This genotype-phenotype separation does not only remove the genetic code from the direct influence of the organism but lends considerable stability to it. Genetic coding relies on just four distinct nucleotides (though partly different for DNA and RNA), essentially a four letter alphabet, which means that information is spread out across a considerable number of letters, thus further reducing the danger of random errors. Most information in most organisms is present as DNA chains, while the less stable and redundant RNA is used for other purposes. Many organisms are further diploid, i.e. every DNA chromosome is present twice, ensuring the presence of a functional copy even in the event of a deleterious mutation. Reproduction may or may not include recombination (sexual or asexual reproduction) which would add further stability (Higgs (1994)).[2]

The genetic code is subject to evolutionary changes, diversity is generated by way of mutations on which selection can act benefiting those varieties the phenotype of which is better adapted to the current environmental conditions. The evolutionary process would fail if either - diversity generation or selection - would not work.

1. Missing selection pressure would lead to an explosion of population size and variety, and thus to speciation, the development and separation of new species.
2. Excessive selection pressure - hostile or rapidly and persistently changing environmental conditions would lead to the destruction of all varieties of the population.
3. Insufficient diversity generation compared to selection pressure would lead to the existence of but one dominant trait and likely poor resilience towards sudden environmental changes.
4. Excessive mutation, particularly in small non-recombining populations, would lead to the accumulation of deleterious mutations and the loss of the non-mutated master sequence as a result of random fluctuations, followed by the loss also of the least-mutated sequences, etc., leading to - in Hermann Joseph Muller's words - "a ratchet mechanism" (Muller (1964)) that periodically irrevocably cuts away the fittest part of the population (Muller's ratchet). The phenomenon was termed error catastrophe. (Eigen et al. (1988))

Biological populations exist in groups of traits that are distinguished only by a few mutations with back and forth mutations occurring regularly thus generating a steady population flow between the traits. These groups are termed

---

[2] Note that recent evidence points to a variety of other elements that also play a role in genetic evolution, including, most importantly protein folding in histones around which DNA is arranged (for an overview, see Campos et al. (2014)) but also, for instance prions.

quasi-species as they do not share a common genome in the strict sense but do nevertheless genetically evolve together. A mutation rate that is fairly high but does not transgress the threshold to error catastrophe may add significant resilience to a quasispecies in that it becomes much more capable of dealing with changing environmental conditions. The error threshold depends positively (of the order $O\sqrt[n]{0 < const. < 1}$) on the length of the genome; the strategy is thus particularly feasible for entities with particularly short genomes; the objective of achieving a high mutation rate would further make RNA-based, non-recombining reproduction desirable. One particularly well-studied example are RNA viruses (Eigen et al. (1988), Escarmís et al. (2006), Domingo (2006)) such as HIV, poliovirus, hepatitis C virus; particularly with HIV, one difficulty is tat the virus generates resistances faster that drugs can be developed and deployed. A promising treatment relies on artificially putting the virus population into error catastrophe mode (Mullins and Jensen (2006)).

Evolutionary systems in economics lack most of the stabilizing features present in genetic evolution (coding not limited to four letter alphabet, thus fewer redundancy, no diploidy, and arguably neither recombining reproduction nor genotype-phenotype separation, hence the possibility of conscious interference with the evolutionary material). It would therefore be appropriate to consider the role of phenomena known from high-mutation-rate systems in evolutionary biology in evolutionary systems in economics. Both excessive rigidity (insufficient mutation) and evolutionary loss of information in error catastrophe like processes may be genuine concerns. In parts, economics could rely on methods developed in mathematical biology, the threshold to error catastrophe can be determined fairly accurately and given sufficient information about existing traits and mutation rates between them, stable quasispecies distributions could be determined.

4 Cultural, Social, Economic Evolution

While the application of evolutionary or even Darwinian concepts to economics or society is not new (cf. Hodgson (2001)), the influential modern theory of cultural evolution is due to Richard Dawkins (1976): Ideas ("memes") serve as equivalent for genes in the cultural sphere, being exchanged between people whose behavior under the influence of those ideas is then the cultural equivalent of the phenotype.

Evolutionary economic models typically work along the same lines, considering routines, strategies, or technologies (Nelson and Winter (1982), Silverberg et al. (1988), Safarzyńska and van den Bergh (2010)) as gene equivalent, on which evolutionary selection operates. The phenotype, the determinant of evolutionary success, would be the firm itself, potentially including its profits, shareholder value, or public image. As with Dawkinsian cultural evolution, reproduction may include imitation in that firms may try to copy both routines and technologies they deem better. Economic evolutionary dynamics does, however, also work through growth and expansion of successful firms. Ulrich Witt (2005) points out that this entails many differences to genetic evolution, especially that firms are able to access and modify their routines or technologies and that firm expansion itself leads to issues routed in the interaction with the underlying human evolution - anthropological properties of interaction in groups of different sizes (Witt (2005), Cordes et al. (2011)). It is quite possible, that a firm's success and growth will lead to destructive internal developments - an issue not found in biological evolution. An extensive discussion also unfolded around the question whether evolutionary economics is Darwinian or Lamarckian, whether conscious decisions enter the domain of evolutionary information (see Hodgson (2001)). But no matter which one it is, the question of sufficient resilience on the one hand and retention of information, retention of traits, on the other, remains.

Points 1 and 2 above (missing or excessive selection pressure) are relatively changeable properties of the environment that may affect but are not directly comprised in the evolutionary adaptation mechanisms of competing entities. For very rigid systems (point 3), there are abundant examples from economic history. Joel Mokyr (1990) makes a compelling case that central organization without tolerance for pluralism led to China's stunning lack of progress and the loss of its technological and economic lead during the European early modern age. The same may, however also hold for modern countries - consider the Soviet government's ill-fated support for Trofim Lysenko's unfounded theories (Soyfer (2001)) - and business enterprises - consider that small firms are known to be more innovative than large ones (Nooteboom (1994)).

The most interesting case may, however, be that of excessive diversity generation, failure to retain information, and error catastrophe. Economic entities, lacking many of the stability inducing mechanisms present in biological evolution, may be particularly vulnerable to that. In many economic systems, the population size - the number of firms, for instance - is due to network effects and other causes in many cases extremely small. Examples include all typical oligopolistic sectors from resource extraction to manufacturing to information technology. Consequently, the failure of one firm - even accidental without structural reasons - would lead to the loss of a considerable knowledge base. The same may be true for decisions not to deploy resources in training of new skilled laborers or development of human capital (a task that often falls to the governments). However, even large populations can undergo error catastrophe if the mutation rate is sufficiently large. What if this occurs in an economic sector with large numbers of small firms (such as craft businesses, retail businesses, and other firms with strong local focus)? In ecological terms, that sector would shrink, lose the some of its deep knowledge, and would make way for other sectors, perhaps more modern technologies. Would that, in a world continuous technological change, be clear in retrospect? It is unlikely, but the idea is conceivable. Two questions would in that regard be of importance: What would trigger the transition to error catastrophe (the maximum sustainable mutation rate)? And where is the threshold? From evolutionary biology, it is clear that error catastrophe the threshold declines in the genome length and rises in the number of individuals. Typically, intensifying technological change would increase the economic equivalent to the mutation rate, particularly in traditional sectors. It could, ceteris paribus, with constant population and knowledge base, put the firm population into error catastrophe. Different from biological populations (which would then be destroyed), economic systems would probably be resilient enough to respond with a smaller "genome length", i.e. less deep knowledge.[3] The threshold could even be estimated. Note that it is conceiv-

---

[3] Note that a larger population size would mathematically also work but would not be

able that, akin to RNA viruses, some economic systems exist close to the error threshold to achieve maximum resilience - which is of critical importance in quickly changing environments.

As a potential recent example, consider the development of bookselling since the advent of information technology and the internet. In many countries[4], most small bookshops have been forced out of business and replaced by large retail chains or online bookstores. However, even before the rise of Amazon, they faced difficulties competing with new information sources (the internet), something that might currently also be happening to newspaper journalism. The independent bookshops that remain tend to be specialized, catering to certain communities, sometimes with a second business as antiquarian shops and often still at risk of going out of business.

5 Conclusion: The Last Bookshop in the Information Age

Is the decline of bookselling a case of error catastrophe or just the natural course of Schumpeterian creative destruction? The question of whether to preserve the tacit knowledge of dying sectors is also an ethical one. If technological and structural changes require an adjustment, some of the knowledge in question might soon be functionally obsolete anyway, but much of it - as every book salesperson will confirm - might actually still be useful even in the information age. It is as yet unclear if books will be transformed into decorative commodities, if a limited demand for actual books will persist, and even if the medium will survive as e-book. It is also unclear if, to what extent, and by which trade the tasks of book salespeople will be performed in the future. What is clear is that a considerable part of the diversity is about to be lost - as has happened to other trades before.

Whether of not the phenomenon can - as I would like to argue - be described as an evolutionary error catastrophe, if retention of a part of the diversity was desirable, the task would likely fall to the public. Protecting small businesses, enhancing cooperation between them, and facilitating the perpetuation of knowledge in their trades (professional education) could serve to add stability, lower random variation and delay error catastrophe mechanisms; the active development of interconnections with new technologies and emerging sectors might even transfer a part of the knowledge base into a new, less hostile, environment.

Whether or not the decline of economic sectors can be described as error catastrophe, the possibility of such a phenomenon of successive and catastrophic loss of information in economic systems is present - no matter how such systems are constructed. The present article gave an overview over existing knowledge of importance to this, discussed potential phenomena, and attempted to identify examples.

Many questions remain for future research: Where would the threshold for error catastrophe be in economic evolutionary systems? Could it take the same

---

feasible for a shrinking sector.

[4] Some countries implemented regulations preventing the use of some competition techniques favoring large firms (retail price wars).

catastrophic form as in biological systems or would the system respond by adjusting the size of the evolutionary information downward - creating a less specialized business? Is the importance of resilience in economic systems high enough to facilitate the emergence of evolutionary entities that exist close to the error catastrophe threshold (as with RNA viruses)?

Acknowledgements

The author would like to thank Claudius Gräbner for many helpful comments. The usual disclaimers apply.

References

Berger, S. and Elsner, W. (2007). European contributions to evolutionary institutional economics: The cases of "cumulative circular causation" (CCC) and "open systems approach" (OSA). *Journal of Economic Issues*, 41(2):529–538.

Bush, P. D. (1987). The theory of institutional change. *Journal of Economic Issues*, 21:1075–1116.

Campos, E. I., Stafford, J. M., and Reinberg, D. (2014). Epigenetic inheritance: histone bookmarks across generations. *Trends in cell biology*, 24(11):664–674.

Cordes, C., Richerson, P., McElreath, R., and Stirmling, P. (2011). How does opportunistic behavior influence firm size? an evolutionary approach to organizational behavior. *Journal of Institutional Economics*, 7(01):1–21.

Dawkins, R. (1976). *The Selfish Gene*. Oxford University Press, Oxford, UK.

Domingo, E. (2006). *Quasispecies: concept and implications for virology*. Springer Science & Business Media, Berlin and Heidelberg. editor.

Eigen, M., McCaskill, J., and Schuster, P. (1988). Molecular quasi-species. *The Journal of Physical Chemistry*, 92(24):6881–6891.

Elsner, W. (2012). The theory of institutional change revisited: The institutional dichotomy, its dynamic, and its policy implications in a more formal analysis. *Journal of Economic Issues*, 46(1):1–44.

Elsner, W. (2013). State and future of the 'citadel' and of the heterodoxies in economics: challenges and dangers, convergences and cooperation. *European Journal of Economics and Economic Policies*, (3):286–298.

Escarmís, C., Lázaro, E., and Manrubia, S. C. (2006). Population bottlenecks in quasispecies dynamics. In Domingo, E., editor, *Quasispecies: concept and implications for virology*, pages 141–170. Springer Science & Business Media, Berlin and Heidelberg.

Georgescu-Roegen, N. (1975). Energy and economic myths. *Southern Economic Journal*, 41(3):347–381.

Goodwin, R. M. (1967). A growth cycle. In Feinstein, C., editor, *Socialism, Capitalism and Economic Growth*, pages 54–58. Cambridge University Press, Cambridge, UK.

Gräbner, C. (2015). Formal approaches to socio-economic analysis-past and perspectives. *Forum for Social Economics*. forthcoming.

Hayden, F. G. (1982). Social fabric matrix: from perspective to analytical tool. *Journal of Economic Issues*, pages 637–662.

Higgs, P. G. (1994). Error thresholds and stationary mutant distributions in multi-locus diploid genetics models. *Genetical Research*, 63(01):63–78.

Hodgson, G. M. (2001). Is social evolution lamarckian or darwinian. In Laurent, J. and Nightingale, J., editors, *Darwinism and evolutionary economics*, pages 87–120. Edward Elgar, Cheltenham.

Hodgson, G. M. (2005). Decomposition and growth: biological metaphors in economics from the 1880s to the 1980s. In Dopfer, K., editor, *The evolutionary foundations of economics*, pages 105–148. Cambridge University Press, Cambridge.

Kaldor, N. (1940). A model of the trade cycle. *The Economic Journal*, 50(197):78–92.

Keen, S. (1995). Finance and economic breakdown: Modeling Minsky's "financial instability hypothesis". *Journal of Post Keynesian Economics*, 17(4):607–635.

Lorenz, H.-W. (1987). Strange attractors in a multisector business cycle model. *Journal of Economic Behavior & Organization*, 8(3):397 – 411.

Mokyr, J. (1990). *The lever of riches: Technological creativity and economic progress*. Oxford University Press.

Muller, H. J. (1964). The relation of recombination to mutational advance. *Mutation Research/Fundamental and Molecular Mechanisms of Mutagenesis*, 1(1):2 – 9.

Mullins, J. I. and Jensen, M. A. (2006). Evolutionary dynamics of HIV-1 and the control of AIDS. In Domingo, E., editor, *Quasispecies: concept and implications for virology*, pages 171–192. Springer Science & Business Media, Berlin and Heidelberg.

Myrdal, G. (1957). *Economic Theory and Under-Development Regions*. Duckworth, London.

Nelson, R. R. and Winter, S. G. (1974). Neoclassical versus evolutionary theories of economic growth: Critique and prospectus. *Economic Journal*, 84(336):886–905.

Nelson, R. R. and Winter, S. G. (1982). *An Evolutionary Theory of Economic Change*. Harvard University Press, Cambridge.

Nooteboom, B. (1994). Innovation and diffusion in small firms: theory and evidence. *Small Business Economics*, 6(5):327–347.

Safarzyńska, K. and van den Bergh, J. C. (2010). Evolutionary models in economics: a survey of methods and building blocks. *Journal of Evolutionary Economics*, 20(3):329–373.

Silverberg, G., Dosi, G., and Orsenigo, L. (1988). Innovation, diversity and diffusion: A self-organisation model. *The Economic Journal*, 98(393):1032–1054.

Soyfer, V. N. (2001). The consequences of political dictatorship for russian science. *Nature Reviews Genetics*, 2(9):723–729.

Veblen, T. B. (1898). Why is economics not an evolutionary science? *The Quarterly Journal of Economics*, 12(4):373–397.

Veblen, T. B. (2008 [1899]). *Theory of the Leisure Class*. Project Gutenberg.

Witt, U. (2005). The evolutionary perspective on organizational change and the theory of the firm1. In Dopfer, K., editor, *The evolutionary foundations of economics*, pages 339–364. Cambridge University Press, Cambridge.

A Threshold to Error Catastrophe Following Eigen et al. (1988)

Consider a set of m information sequences (gene sequences in biological systems) of length v, with population sizes $x_0, x_1, ..., x_{m-1}$ and reproductive fitnesses $a_0, a_1, ..., a_{m-1}$. Let the share of mutations from sequence i to sequence j be denoted Q_{ij} (Q_{ii} denotes the share of identical replicants) and let there be a $m \times m$ matrix W with elements $W_{ij} = a_i Q_{ij}$.[5] The dynamic development of the system is given by

$$\frac{\nabla x}{dt} = Wx.$$

Call matrix W's the eigenvalues-eigenvector pairs (λ_w, w). By definition ($\lambda_w w = Ww$) it follows that the set of eigenvalues w with $\lambda_w \neq 0$ are exactly the vectors multiplying which to the matrix W yield a scaled version (with factor λ_w) of the vector itself, i.e. the set of states x for which the proportions between the components $(x_0, x_1, ..., x_{m-1})$ remain dynamically constant. These states (as far as they are valid, i.e. do not contain negative elements) are termed the quasispecies of the evolutionary system. Since their dynamic development is governed by factor λ_w, the population converges towards the quasispecies corresponding to the dominant (largest) eigenvalue.

Call x_0 the master sequence (unmutated form) and combine all sequences $k \neq 0$ into one set of variables and equations. The system simplifies to

$$\begin{pmatrix} \frac{dx_0}{dt} \\ \frac{dx_{k\neq 0}}{dt} \end{pmatrix} = \begin{pmatrix} a_0 Q_{00} & \overline{a_{k\neq 0}} Q_{k0} \\ a_0(1-Q_{00}) & \overline{a_{k\neq 0}} Q_{kk} \end{pmatrix} \begin{pmatrix} x_0 \\ x_{k\neq 0} \end{pmatrix}.$$

---

[5] The matrix may be extended to include a degradation term D_{ij} (only populated in the main diagonal, otherwise 0) that accounty for the death of individuals representing sequence i, hence $W_{ij} = a_i Q_{ij} - Dij$.

where $\overline{a_{k\neq 0}}$ denotes the average mutant reproductive fitness. Assume that back mutations from mutant sequences back to the master sequence are extremely unlikely, with the probability decreasing quickly for smaller populations, and neglect consequently the upper element of the new transormation matrix, $\overline{a_{k\neq 0}}Q_{k0}$. Thus,

$$\begin{pmatrix} \frac{dx_0}{dt} \\ \frac{dx_{k\neq 0}}{dt} \end{pmatrix} = \begin{pmatrix} a_0 Q_{00} & 0 \\ a_0(1-Q_{00}) & \overline{a_{k\neq 0}}Q_{kk} \end{pmatrix} \begin{pmatrix} x_0 \\ x_{k\neq 0} \end{pmatrix}.$$

This, in turn can be expressed in terms of shares by setting x as the vector of shares instead of absolute population sizes and caring for renormalization to $\sum_{i=0}^{m-1} x_i = x_0 + x_{k\neq 0} = 1$ by subtracting the mean excess reproduction in the development equation

$$\frac{\nabla x}{dt} = Wx - \phi x$$

where $\phi = \sum_{i=0}^{m-1} x_i a_i = x_0 a_0 + x_{k\neq 0} a_{k\neq 0} = x_0 a_0 + (1-x_0) a_{k\neq 0}$ is the averagefitness. Hence, in the reduced system

$$\begin{aligned} \frac{dx_0}{dt} &= a_0 Q_{00} x_0 - \phi x_0 = x_0(a_0 Q_{00} x_0 - x_0 a_0 - (1-x_0)\overline{a_{k\neq 0}}) \\ \frac{dx_{k\neq 0}}{dt} &= a_0(1-Q_{00})x_0 + \overline{a_{k\neq 0}}Q_{kk}(1-x_0) - \phi x_0 \end{aligned}$$

As the two shares sum up to 1, the system can be completely represented by the first equation, the population share of the master sequence. The equilibrium set results (with $\frac{dx_0}{dt} = 0$) as

$$\begin{aligned} x_{0,1} &= 0 \\ x_{0,2} &= \frac{a_0 Q_{00} - \overline{a_{k\neq 0}}}{a_0 + \overline{a_{k\neq 0}}} \end{aligned}$$

with the stability determined[6] by the eigenvalue of the linearized Jacobian of this system, given by the linearization of

$$\lambda = \frac{\partial dx_0/dt}{\partial x_0} = -2x_0(a_0 + \overline{a_{k\neq 0}}) + (a_0 Q_{00} - \overline{a_{k\neq 0}})$$

which for the two equilibria takes the values

$$\begin{aligned} \lambda(x_{0,1} = 0) &= a_0 Q_{00} - \overline{a_{k\neq 0}} \\ \lambda(x_{0,2} = \frac{a_0 Q_{00} - \overline{a_{k\neq 0}}}{a_0 + \overline{a_{k\neq 0}}}) &= -a_0 Q_{00} + \overline{a_{k\neq 0}}. \end{aligned}$$

While the first (trivial) equilibrium is the collapse if the master sequence population (error catastrophe), the second defines a stable share for the master sequence. The system undergoes a phase transition at

$$0 = a_0 Q_{00} - \overline{a_{k\neq 0}},$$

the stability condition for the second (non-trivial) equilibrium is

$$0 < a_0 Q_{00} - \overline{a_{k\neq 0}}$$

---

[6] As with all differential equation systems, the fixed point is stable if and only if the (dominant) eigenvalue is negative.

$$a_0 Q_{00} > \overline{a_{k\neq 0}}$$

$$Q_{00} > \frac{\overline{a_{k\neq 0}}}{a_0} = \frac{1}{\sigma_0}$$

where $\sigma_0 = \frac{a_0}{\overline{a_{k\neq 0}}}$ stands for the relative superiority of the fitness of the master sequence.

Until now, Q_{00} denotes the probability of retention of the entire master sequence, i.e. including all v loci of the sequence. This could also be evaluated position-wise, yielding (if mutations in all positions are uniformly likely)

$$Q_{00} = (q_{00})^v > \frac{1}{\sigma_0}$$

$$q_{00} > \sqrt[v]{\frac{1}{\sigma_0}}$$

$$v \log q_{00} > -\log \sigma_0$$

$$v < -\frac{\log \sigma_0}{\log q_{00}} \approx \frac{\log \sigma_0}{1 - q_{00}}.$$

It can be seen that the survival of the master sequence, in other words the prevention of error catastrophe depends on (1) the population size (the larger the better), (2) the amount of information (genome length in biological systems; the smaller the better), and (3) the mutation/change probability (the smaller the better). As argued above, for social or economic evolutonary systems, the population size tends to be much smaller than for, say a colony of bacteria. With many stabilizing mechanisms in the reproduction mechanism not in place, the mutation rate should equally be higher. The size of the involved information set (which is for typical economic systems not fully codified) is difficult to estimate but perhaps smaller than genome sizes in biology (of the order of thousands or tens of thousands for simple entities such as viruses), hence perhaps working in favor or preventing error catastrophe.