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Abstract:

This paper develops a dynamic stochastic general equilibrium (DSGE) model for analyzing the impact of various capital income tax policies in a small open economy that is populated by households possessing endogenous time preferences. We contribute to the literature by studying the impacts of: i) anticipated tax shocks under stochastically growing output, ii) stochastic tax shocks under deterministic output, on our dynamic general equilibrium framework. With our model's specifications, this is the first attempt to integrate uncertainty in the study of taxation and welfare. Our results suggest that only under certain conditions welfare paradoxes may exist, in the sense that increases in tax instruments may improve welfare.

Key words: Endogenous time preference, adjustment costs, perturbation methods, stochastic shocks.

JEL classification: F4, E62

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1 Introduction

"It is often said that nothing is certain in life except death and taxes. While death is undoubtedly certain, there is, in fact, considerable uncertainty with respect to tax rates." As Sialm (2006) points out, there has been a significant fluctuation in marginal income tax rates in United States from 1913 to 1999 which implies that there is tax policy uncertainty.

This paper is the first attempt to include stochastic taxation in a dynamic general equilibrium framework with endogenous rates of time preference, even though there are many studies existing in both finance and public economics literature. For instance, Stiglitz (1982) discusses the welfare impacts of random taxation. According to Skinner (1988), "tax policy is often unpredictable because of factors beyond the control of government". He shows the considerable variability of tax rates in US during the period 1929-1975. By studying the additional excess burden of uncertain tax policy, he computes that uncertainty in tax policy led to \$12 billion extra burden for US economy in 1985. In addition, Hassett and Metcalf (1999) use a model with an uncertain investment tax credit to study the effects of tax policy uncertainty on aggregate investment. They find that, under a continuous time random walk, tax policy uncertainty causes a delay in investment. This is also in line with the findings of Agliardi (2001). Yet, by extending Hassett and Metcalf's (1999) model, Bohm and Funke (2000) demonstrate that the effects of tax policy uncertainty must be very limited because of various modelling assumptions.

On the other hand, recent literature on fiscal economics points towards the effects of different types of government spending versus tax shocks on GDP and its components. Alesina, Ardagna, Perotti and Schiantarelli (2002) investigate the effects of a change in fiscal policy on private investment using a panel of OECD countries. They find that taxes do have negative impact on output, as do Blanchard and Perotti (2002). Moreover, the latter concludes that private consumption increases follow an increase in tax rates. In both these studies, it is shown that any increase in taxes will crowd out private investment. In addition, Perotti (2004) points out that the impact of any change in tax policy on GDP and its components have become weaker over time. Mountford and Uhlig (2008) try to distinguish the effects of fiscal policy shocks for US economy between 1955-2000.

The objective of this paper is to contribute to the literature by focusing on the impacts of anticipated tax shocks under stochastically growing output, and stochastic tax shocks under deterministic output, in a dynamic general equilibrium framework with endogenous rates of time preference. As pointed out above, the empirical studies indicate that tax rate changes are frequent and, in most cases, future tax changes cannot be predicted. This basically implies that tax changes can be an element of uncertainty. For instance, since the 1990s, there has been a tax competition in the sense that countries cut tax rates to attract investment which makes firms operate in a tax-cut scenario because of the possibility of further future reductions. Thus, whatever the sign of the tax rate change is, tax rate uncertainty is an important issue that must be analyzed (Fedele, Panteghini, Vergalli, 2009). Or to put it differently, it is important to see how the agents react if there is an uncertainty about tax levels in the economy. It is worth noting that over the last two decades, most of the studies in the literature use either real option models to study the effects of tax rate uncertainty on investment or econometric/numerical techniques to see the impact and/or

²Hassett and Metcalf (1999).

³That is also valid for OECD countries. For more information see OECD tax database.

international transmission of any shock in an economy. Here, we deviate from these studies by setting up a stochastic dynamic general equilibrium model to focus on the welfare effects of various capital tax instruments in a small open economy populated by infinitely-lived households possessing endogenous time preference that is in line with Epstein and Hynes (1983)⁴. The model also assumes that the households can borrow or lend freely in the world capital market. Perfectly competitive firms produce one good that can be used for consumption and investment which is subject to adjustment costs in the sense that the firm has to pay an installation cost. We, then simulate the model and show that: (i) under a deterministic setup, a permanent expected increase in any of capital income tax will lead to a change in welfare that depends on whether the marginal productivity of capital exceeds or falls short of the real rate of interest; (ii) when output is stochastically growing, any increase in tax levels causes a decrease in welfare regardless of where the economy stands; (iii) when there is a stochastic change in tax levels, the change in welfare will depend on the change in the type of the capital income tax, in the sense that a stochastic negative change in corporate income tax and tax on capital gains will lead to an increase in welfare because of the increase in wealth.

The model we present in the next section also predicts the effects of any change in tax levels on consumption, capital stock, shadow price of capital and the current account. Unlike most intertemporal equilibrium models, our model leads to non-monotonic adjustment of the current account in response to various policy changes. This implies that, if adjustment costs are beyond a certain threshold, there will be an initial decrease in both savings and investment when investment is at its lowest level. Thereafter, savings will increase parallel to low levels of disinvestment⁵.

There are three theoretical papers closely related to our paper in terms of the questions having light shined on them: Bizer and Judd (1989), Nielsen and Sorensen (1991) and Karayalcin (1995). Bizer and Judd (1995) made a seminal contribution by highlighting the uncertainty in tax policy under a dynamic general equilibrium framework by implementing Markov process. Their model is relatively straightforward since only two types of taxes (investment tax credit and income tax rate) are included.

To examine the dynamic macroeconomic effects of capital income taxation, Nielsen and Sorensen (1991) develop a small open economy with perfect mobility of financial capital as possessing time-additive (exogenous) preference. Although various forms of capital income taxation are included in the model, the changes in taxes are deterministic. Therefore, here we depart from Nielsen and Sorensen (1991) in extending the model by including both endogenous time preference and adding a stochastic framework. The former is adopted because, if the constant-discount rate does not match with the parametric world interest rate, a stationary equilibrium does not exist. On the other hand, if that rate diverges from the parametric world interest rate, the time additive preferences will cause hysteretic adjustment towards the steady state⁶, rendering the analysis dependent on initial conditions.

Finally, by combining endogenous time preference and adjustment costs, Karayalcin (1995) builds a model that focuses on the welfare effects of capital tax instruments in a small open economy. In that framework, due to the adjustment costs, he ended up with a lower degree of

⁴For empirical findings, also see Obstfeld (1990).

⁵See Karayalcin (1994).

⁶ See Karayalcin (1995), Sen and Turnovsky (1990).

consumption smoothing since agents will no longer be able to undertake a frictionless adjustment in the capital stock. Another significant difference is that his study concentrated on unanticipated tax shocks and their welfare effects. Here, we deviate from Karayalcin (1995) in enhancing the model by involving both stochastically growing output and stochastic taxation. Also, we differ from all three papers cited above by incorporating numerical analysis which measures the magnitudes of effects due to variation in capital income taxes.

The paper proceeds as follows. The next section sets up a model with stochastically growing output in a small open economy with perfect capital mobility of financial capital. The setup is one with infinitely-lived households possessing endogenous rate of time preference and adjustment costs in investment. The effects of changes in tax policy instruments will be studied in section three and section four concludes the paper.

2 The Model

Consider an economy that produces a single good that may be used for consumption as well as investment. For simplicity, it is assumed that firms produce in a competitive market. The economy is populated by infinitely-lived households possessing endogenous time preferences. The number of households are normalized to one without loss of generality. Government collects taxes by following a balanced budget policy in the sense that the tax revenue is used to finance the households via transfers.

2.1 Households

The welfare of households is taken to depend on consumption of the good in the market. Also, the households receive the wage w_t by inelastically supplying one unit of labor services per unit of time. We denote the world interest rate by r, the interest income tax rate by t_r and the household's net assets by a_t , that is the sum of the value of domestic equity and the value of foreign assets. Thus, $r(1-t_r)a_t$ is the net interest income. According to equations (1)-(3) below, the consumer is maximizes expected discounted lifetime welfare subject to the constraint that the current increase in non-human wealth is equal to the sum of after tax income from wealth, from labor and from government transfers minus consumption.

We also adopt the endogenous rate of time preference structure proposed by Epstein and Hynes (1983). Other than the time-additive models, in this setting, time preference imply a well-defined long-run target level. Therefore, lifetime welfare, U, is maximized over consumption path, C, by

$$U(C) = -E_t \sum_{t=0}^{\infty} exp(-z_t) \frac{1}{1 + (1 - t_r)r}$$
 (1)

subject to

$$z_{t+1} - z_t = u(c_t) - r, (2)$$

$$a_{t+1} - a_t = (1 - t_r)ra_t + w_t - c_t - t_t,$$
(3)

$$z_0 = 0, (4)$$

where u(c) > 0 is required to be a strictly increasing and strictly concave felicity function in the sense that u' > 0 and u'' < 0 and t_t is the lump-sum government transfer.

Following Obstfeld (1990) and Epstein and Hynes (1983), one can argue that the lifetime welfare functional U deviates from the time-additive utility functionals by its recursivity. As is widely known, additivity implies that the marginal rate of substitution between times t_1 and t_2 is independent of consumption at any $t \neq t_1$, t_2 . However, here, recursivity allows this marginal rate of substitution to be independent of consumption before t_1 but not after t_2 in order to make future consumption weakly seperable from past consumption levels. Therefore, the variable rate of time preference Ω at time s has the following form:

$$\Omega_s = \left\{ \sum_{s=v}^{\infty} exp \left[-\sum_{s=v}^{t} u(c_s) \right] \right\}^{-1}.$$
 (5)

 Ω at time s is the following function of the utility functional U(C)

$$\Omega(\phi_s) = -\phi_s^{-1} \tag{6}$$

and

$$\phi_{s} = U(_{s}C) \tag{7}$$

where U represents positive discounting of future consumption and ϕ_s denotes aggregate future consumption (or lifetime welfare at time s). It should be noted that the rate of time preference is not constant as will be the case in the additive model. On the other hand, as in the steady state, if the consumption path is globally constant, $c_{t+1} = c_t$ for all t and $c_t = \overline{c}$. In this case, the rate of time preference is given by

$$\overline{\Omega}(\overline{\phi}) = u(\overline{c}) \tag{8}$$

where upper-bars describe long-run equilibrium.

Even though there is significant dispute on whether impatience to consume has to increase or decrease as current consumption goes up, since it is assumed that u'(c) > 0, in our framework increasing marginal impatience will take place as in Lucas and Stokey (1984) who basically emphasize that a kind of diminishing private returns to saving is required to have local stability. Therefore, the felicity function is specialized to:

$$u(c_{\star}) = lnc_{\star} + \omega \tag{9}$$

where ω stands for a parameter to measure generalized time preference.

By using equations (1-4) and (9), the standard solution of the life-time welfare maximization problem yields:

$$E_{t}(c_{t+1}) = [(1-t_{r})r + 1 - \Omega(\phi_{t})]c_{t}.$$
(10)

On the other hand, by differentiating (1) with respect to time, we can obtain the dynamics of lifetime welfare:

$$E_{t}(\phi_{t+1}) = 1 + \phi_{t}(u(c_{t}) + 1), \tag{11}$$

2.2 Firms

Competitive firms employ capital, k_t , and labor to produce the single good which is used for both consumption and investment. The production function is

$$f(k_t) = \theta_t^y k_t^\alpha \tag{12}$$

where θ_t^y is the aggregate productivity shock which follows a stochastic autoregressive process⁷ with the disturbance term ε_t assumed to be normally distributed with mean zero and variance σ_y^2 i.e. $\varepsilon_t : N(0, \sigma_y^2)$.

We assume that the firm finances a fraction $(1-\varepsilon)$ of new investment by debt issues and the remaining fraction, ε , by retained earnings as:

$$b_t^c = (1 - \varepsilon)k_t \tag{13}$$

and

$$b_{t+1}^{c} - b_{t}^{c} = (1 - \varepsilon)(k_{t+1} - k_{t})$$
(14)

After corporate income tax is applied, the remaining profits are distributed as dividends to equity holders. Therefore, before personal tax, total dividends, π_t , are:

$$\pi_{t} = \left[f(k_{t}) - w_{t} - rb_{t}^{c} - T \right] (1 - t_{c}) + b_{t+1} - b_{t} - (1 - t_{t})i_{t}$$
(15)

where t_c and t_l stand for the corporate income tax rate and the rate of investment tax credit respectively and T denotes the adjustment cost. If the firm changes its capital, it is subject to adjustment costs in the sense that the firm has to pay a deadweight installation cost other than the actual cost i_s . The installation cost 8 must be an increasing function of i in relation to k, which should have the following properties:

$$T(0) = 0, \quad T' > 0 \quad ,2T' + (i/k)T'' > 0.$$
 (16)

In our framework, the installation cost function is specialized to $T(i_t/k_t) = (\chi/2)(i_t/k_t)$ so that, in order to increase the capital stock by i units, the representative firm needs to pay $i_t[1+(\chi/2)(i_t/k_t)]$ units of output.

Foreign bonds, b_t^f , and corporate bonds, b_t^c , are treated to be perfect substitutes. Thus, they have to pay the same expected after tax return. If V_t denotes the market value of outstanding

⁷The details can be seen in the next section.

⁸Note that adjustment cost depends on gross investment rather than net investment. Yet, since we ignore depreciation, those terms can be treated as same.

equity, t_r stands for the personal interest income tax rate and t_g is the capital gains tax rate, for all t the arbitrage condition therefore will be:

$$r(1-t_r) = \frac{\pi_t}{V_t} + \frac{(1-t_g)[E_t(V_{t+1}) - V_t]}{V_t}$$
(17)

where the term on the left-hand side is the after tax income on foreign bonds, whereas the right-hand side delineates the after tax equity return, consisting of current yield and capital gains. The market value of equity at time zero will therefore be:

$$V_{s} = E_{s} \sum_{s=1}^{\infty} \theta_{g(s)}^{-1} \pi_{s} \left(\frac{\theta_{g}}{\theta_{g} + r\theta_{r}} \right)^{s-1}$$

$$(18)$$

where $\theta_i = 1 - t_j$, j = c, g, r. Firms choose to maximize the present discounted value of V_s subject to the constraint $i_s = k_{s+1} - k_s$. The solution yields,

$$E_{s}(q_{s+1}) = \frac{q_{s}(r\theta_{r} + \theta_{g})}{\theta_{g}} - \frac{\theta_{c}}{\theta_{g}} \left[E_{s}(f'(k_{s+1})) - r(1 - \varepsilon) + E_{s} \left[(\frac{i_{s+1}}{k_{s+1}})^{2} T'(\frac{i_{s+1}}{k_{s+1}}) \right] \right]$$
(19)

$$q_s = \frac{1}{\theta_g} \left\{ (\varepsilon - t_l) + \theta_c (T + (\frac{i_s}{k_s})^2 T'(\frac{i_s}{k_s}) \right\}$$
 (20)

$$w_s = f(k_s) - f'(k_s)k_s \tag{21}$$

where the shadow price of capital at time s is given by q_s . Following Hayashi (1982), we can define q_s as the ratio of market value of firm's equity to the replacement cost of capital. Thus, q_s in eq(20) can be considered as a variant of Tobin's Q. Equation (21) is the equilibrium condition for the labor market while the law of motion for the shadow price of capital is given by (19).

Equation (20) can be used to denote the rate of investment, i_t/k_t , as the following function of q_t :

$$i_t = k_{t+1} - k_t = k_t \gamma(q_t)$$
 (22)

where $\gamma'(q_t) = (\theta_g/\theta_c)\chi > 0$. Equation (22) simply expresses investment as an increasing function of the shadow value of capital, q_t . Here, it should be noted that both q and i are independent of the consumption and saving decisions of the households.

2.3 The Current Account

In this part, by recalling the government's balanced budget policy, in order to acquire the dynamics of the current account, let us use (3), (15)-(22) and $a_t = b_t^f + q_t k_t + b_t^c$. We therefore obtain

$$b_{t+1}^f - b_t^f = rb_t^f + f(k_t) - i_t(1+T) - c_t$$
(23)

which states that the increase in foreign asset holdings equals the difference between the sum of output and interest earnings of the representative household less the sum of consumption and investment expenditures.

2.4 Characterization of the Equilibrium

It is convenient to describe the economy's steady state before characterizing the equilibrium behavior of our model. The market clearance and optimality conditions will give the long-run relations in this economy. We can easily obtain the steady state value of the shadow price of capital, \bar{q} , by using (19) and (22), and imposing the stationarity conditions. At steady state

$$\frac{-}{q} = \frac{\mathcal{E} - t_l}{\theta_g}, \quad q = q^*(t_g, t_l), \quad q_1^* > 0, q_2^* < 0, \tag{24}$$

where, in order to guarantee a positive value of \bar{q} , it is assumed that, the replacement cost of capital is higher than the debt issue per unit of capital, $1-\varepsilon < 1-t_l$, which indicates that the firm does not overfinance its investment⁹.

Similarly, it yields¹⁰

$$f'(\overline{k}) = r \left[(1 - \varepsilon) + \frac{\theta_r(\varepsilon - t_l)}{\theta_c \theta_g} \right] \quad \overline{k} = k^*(t_g, t_c, t_r, t_l), \tag{25}$$

$$k_1^* < 0, k_2^* < 0, k_3^* > 0, k_4^* > 0,$$

$$r\bar{b}^f = \bar{c} - f(\bar{k}) \quad \bar{b} = b^*(t_g, t_c, t_r, t_l),$$
 (26)

$$b_1^* > 0, b_2^* > 0, b_3^* < 0, b_4^* < 0,$$

⁹For more information, see Nielsen and Sorensen, 1991.

¹⁰The derivatives can be seen in appendix.

$$u(\bar{c}) = r\theta_r, \quad \bar{c} = c^*(t_r), c^* < 0,$$
 (27)

$$\overline{\phi} = -\left(\frac{1}{r\theta_r}\right), \quad \overline{\phi} = \phi^*(t_r), \phi^* < 0 \tag{28}$$

Thus, one can obtain the steady state values of $(\bar{q}, \bar{k}, \bar{b}, \bar{c}, \bar{\phi})$ by using equations (24)-(28).

Since we do have the steady state values of all the variables, it is convenient to follow Schmitt-Grohe(2005) and solve our system by perturbation methods. Before doing so, let us briefly discuss this method. The first-order perturbation method is similar to linearization of the Euler equations around steady state. Essentially, with the help of the techniques for forward looking rational expectations, the linear model is solved. One of the main requirements we need to satisfy is the Blanchard-Kahn (1980) condition: that is the number of the roots larger than 1 in modulus has to match the number of forward looking variables in the model¹¹. Or, to put it differently, for the system to be locally saddlepath stable, it has to be the case that three of the eigenvalues have to be larger than 1 in modulus, since the system has three predetermined (backward looking) (k, θ^y, b) and three control (foreward looking) variables (c, ϕ, q) . It is straightforward to show that this is the case here.

3 Various Fiscal Shocks and Their Effects on Economy

In this section, we examine the outcomes of the model under different types of shocks. Before moving onto that part, although actual tax systems differ, let us define the case where $t_g = t_r$ and $t_c = t_l = 0$; in other words, an economy under a uniform, comprehensive income tax, no investment subsidies and with fully integrated corporation taxes. Therefore, in this case we will end up with $f'(\overline{k}) = r$ equivalence confirming the Schanz-Haig-Simons result which argues for the neutrality of income tax with respect to investment.

Yet, in a real world tax system, \overline{q} and \overline{k} will be affected by tax factors because generally neither the investment tax credit is equal to zero, and nor is the effective tax rate on interest earnings less than the tax rate on accrued capital gains. Thus, there occurs a distortionary wedge between the world interest rate and the marginal productivity of capital in the home country i.e $r^{\mathbf{p}} f'(\overline{k})$. Suppose, initially, that the former exceeds the latter. In this case, any policy that shrinks the home capital stock will, by increasing its marginal productivity, reduce the distortion and raise lifetime welfare. Thus, in order both to analyze our model more concretely and to investigate the outcomes of different scenarios more profoundly, two countries, Canada and Sweden, are included. Among our countries, Canada represents the $f'(\overline{k}) < r$ case, whereas Sweden embodies $f'(\overline{k}) > r$ case given the initial values¹².

¹¹For details, see Judd (1998), Sims (2002), Judd (1996) Kim and Kim (2003), Schmitt-Grohe&Uribe (2004) and Collard and Juillard (2001).

¹²See appendix B.

3.1 Deterministic Tax Shocks and Their Effects on Welfare

Since the welfare effect is going to play an important role in the next section, in order to be able to compare and contrast, we will start out considering the effects of an expected permanent increase in the corporate income tax rate. As seen in eq(25), this will reduce the long-run capital stock of the economy and therefore the effect on consumption on impact will be positive. However, since the long-run foreign asset holdings increase, there is expected to be an increase in savings as well, which causes a negative effect on consumption on impact. The net effect is determined by the inital condition of the economy i.e whether $f'(\overline{k}) > r$ or $f'(\overline{k}) < r$. Following Karayalcin (1995), we can infer that ϕ on impact yields the present discounted value of the future felicity stream as of time t = 0. Thus, the difference between ϕ on impact and initial $\overline{\phi}$ is a measure of the welfare effects of the policies under consideration. To summarize, if, initially $f'(\overline{k}) > r$, we can expect a drop in consumption on impact so will lifetime welfare because the decline in capital stock caused by the rise in t_c will accentuate the distortion by increasing the marginal product of capital.

Figures (1)-(4) show the simulation results for an increase in t_c and t_g both for Canada and Sweden. We find that under an expected permanent increase in the corporate income tax or tax on capital gains, lifetime welfare increases in Canada, whereas Sweden will suffer from the rise in taxes due to the reasons mentioned above. To place the discussion in a familiar setting, let us examine the Canadian, $f'(\bar{k}) < r$, case. An expected permanent rise in the corporate income tax¹³ will cause an increase in welfare. This is mainly because of the required long-run decrease in the domestic capital stock and the long-run increase in foreign asset holdings (see equations (25) and (26)). If the interest earned on foreign bonds, r, is less than the long-run effect of the decline in the capital stock on income $(i.e.f'(\bar{k}) < r)$, the decrease in the capital stock caused by the rise in t_c will reduce the distortion and increase lifetime welfare t_c

Let us turn to the effects of an increase in the investment tax credit. As seen in eq(26), there will be a decrease in long-run foreign asset holding and a rise on the long-run capital stock because of the decline in the replacement cost of capital for both countries. Thus, for Canada, the distortion will be accentuated and there will be a decrease in welfare on impact as seen in figures (5)-(6). On the other hand, since $f'(\bar{k}) > r$ holds for Sweden, the lifetime welfare will increase because of the reduction in the distortion.

Finally, we will consider the effects of a rise in the tax rate on interest income, t_r . Such a policy will make ownership of real capital more attractive relative to the ownership of bonds. Thus, there is expected to be an increase in the long-run level of capital and a decrease in the steady state level of foreign asset holdings. It should also be noted that the increase in k will take place until the equity price returns to its initial level¹⁵. As can be seen in figure (8), lifetime welfare

 $^{^{13}}$ Same result is valid for the tax on capital gains.

¹⁴On the other hand, if $f'(\overline{k}) > r$ initially holds, an increase in t_c will give rise to a fall in capital stock which will accentuate the distortion and reduce lifetime welfare.

¹⁵The details can be seen in the next section.

increases in Sweden because of the rise in capital stock which will lead to reduce the distortion. On the other hand, as the increase in tax rate discussed above indicate, if initially $f'(\overline{k}) < r$ holds, the same policy will accentuate the distortion and reduce lifetime welfare.

3.2 Mixed Shocks and The Model

In this section¹⁶, we begin analyzing the model by only under a stochastic productivity shock and considering that model as the benchmark. Thereafter, the shocks will be mixed in the sense that the stochastic productivity shock will be kept while the anticipated permanent tax shocks will be added to see how the model reacts if agents began expecting higher tax rates.

In a stochastic framework, since the model needs to be made stationary around a steady state, permanent shocks cannot be placed. Moreover, shocks can only hit the system today and the expectation of future shocks has to be zero. However, by adding a latent shock variable (which has a normal distribution with zero mean), ε_t , it is possible to make the effect of the shock disperse slowly throughout the economy. Here, basically, ε_t is going to affect the model's true exogenous variable θ_t^y that is itself an AR(1) process (Griffoli, 2007).

$$\theta_t^y = \rho \theta_{t-1}^y + \mathcal{E}_t \tag{29}$$

Our results suggest that, under stochastically growing output any increase in tax level will worsen the economy and reduce welfare regardless of whether $f'(\overline{k})\mathfrak{L}r$. For both countries, the results are in tables (1) and (2) and the impulse response analyses can be seen in figures (11)-(18).

We first discuss the simulation results when there is only a stochastic productivity shock. Figure (9) shows the results for Canada and figure (10) for Sweden. For both countries, our indicators show a similar trend in adjustment path when hit by a stochastic productivity shock. A 1% standard error increase in ε_t causes an increase in capital as well as consumption in the short run. The former is caused by the increase in the marginal product of capital whereas the latter is a result of an increase in output. On the other hand, there will be an increase in the rate of return on equity which leads to an immediate jump in the price of equity q. This will also stimulate investment by causing a current account deficit. Since c jumps on impact as well, domestic absorption will increase which deteriorates the current account deficit in the short run. However, in the long run, investment starts to decrease along with the decrease in q which causes an improvement in current account balance. In addition, consumption adjusts much faster than the investment level. Thus, the accentuating effect of consumption on current account will die out, or, to put it differently, the current account balance will improve in the long-run by adjusting non-monotonically.

Next, as discussed above, we will mix the stochastic productivity shock and anticipated permanent tax shocks. In analyzing table 1, let us start out considering the effects of a foreseen rise in the corporate income tax rate under stochastic productivity shocks. Suppose the government

¹⁶It should be noted that the same stochastic productivity shock remains throughout this part although its level is kept small in order to avoid any outweighing effect in the sense that productivity shock never dominates the tax shock. Thus, it must be considering while evaluating the results.

announces that a corporate tax will be implemented at time t=20. Anticipating a future reduction in the rate of return on equity, investors will decrease their demand for equity which will cause a drop in the price of equity q on impact along with a process of capital decumulation. It should be noted that, considering eq(22), as q remains below its long-run level along the adjustment path, investment will decrease (see figures 11-12). In the short-run, the decline in the rate of investment along with the reduction in consumption will improve current account balance. Yet, in the long-run, investment will increase to reach its long-run value in addition to a decline in savings that leads to a deterioration in the current account balance as in both figure (11) and (12). Therefore, the current account will adjust non-monotonically.

Under stochastic productivity shock, a foreseen increase in capital gains tax rate, t_g , will increase long-run foreign asset holdings and reduce the long-run domestic capital stock as well as lead to a rise in the long-run equity price q. Notice that the long-run changes in k and q, by putting opposing pressures on impact, determine the short-run adjustment of q. Although there is a jump in q on impact, there will be follow a process of capital decumulation, or, to put it differently, a decrease in investment occurs because of the increase in t_g which clearly outweighs the jump in q^{17} . Afterwards, the capital stock will adjust to its new long-run level by giving rise to an increase in real investment. On the other hand, consumption drops on impact by strengthening the improving effects of decreasing investment on current account. Thus, current account will have an upward trend until the increasing effects of higher savings die out. Afterwards, the current account surplus starts to decrease along with an increase in investment and adjusts to its new steady-state level (see figure 13-14).

Conversely, under stochastic productivity shocks, an anticipated future increase in t_1 has an expansionary long-run effect because the investment tax credit decreases the effective price of new capital goods relative to initial capital. Therefore, the long-run level of k will increase, whereas there will be a decline in the steady state level of foreign asset holdings. As in the case of an increase in t_o , there will be opposing pressures on the equity price q (see figures 15-16). On the other hand, a foreseen future tax credit is expansionary from the time it is expected until the time it takes effect. This is because of the foreseen reduction in the effective price of new capital goods. As is well-known, a higher rate of investment tax credit makes new capital cheaper relative to initial capital. Thus, there will be an increase in the domestic capital stock up until the implementation of new tax policy. To summarize, analyzing figures (15) and (16), we can say that capital stock per capita is increasing from the time of announcement of the new tax policy until the time it takes effect, whereas a contraction occurs after the new tax policy is introduced. On the other hand, as opposed to the case of an increase in t_g , we see an immediate jump in consumption on impact which will aggravate the domestic absorption, worsening the current account deficit. As the economy starts to increase its holdings of foreign assets (right after implementing the new tax levels), it has to run a current account surplus which implies a nonmonotonic adjustment of the current account balance.

Finally, let us examine the effects of a foreseen rise in the personal interest income tax rate, t_r , under a stochastic productivity shock. Note that an anticipated rise in the tax rate on interest

¹⁷See eq(35) in appendix A.

income will lead to a higher demand for equity since there will be a reduction in the rate of return on foreign bonds. The excess stock demand for equity will be eliminated by an immediate jump in the price of q on impact. As a result, real investment will rise as q remains above its long-run level. Yet, as capital intensity rises we see that its marginal product declines, up until q and the rate of investment go back to their long-run values. On the other hand, since a rise in t_r will reduce the long-run utility target, households will reduce their long-run consumption. As a consequence, current account balance will deteriorate because of the rise in both investment and consumption levels. However, consumption will reach its steady state value sooner, and therefore, the accentuating effects of higher consumption on current account will disappear in parallel to the reduction in investment. Thus, there again occurs a non-monotonic adjustment of current account balance.

Last but not least, we will highlight the impacts of any changes in tax levels under stochastically growing output on welfare. Since the logic is, mutatis mutandis, same for the rest of the exercises, we will examine the effects of an increase in the tax rate on capital gains and investment tax credit. As seen in table 2, an increase in tax rate on capital gains will lead to a decrase in the long-run domestic capital stock and an increase in foreign asset holdings. Because of the reasons discussed in the previous part, this change in tax level is expected to give rise to a reduction in lifetime welfare if, initially, $f'(\overline{k}) > r$. On the other hand, there will be an increase in welfare if $f'(\overline{k}) < r$ initially holds. However, here, for both of the countries, consumption drops on impact, so will lifetime welfare which is because of the uncertainty and the risk averse utility function¹⁸. As is in our model, when the effects of output shocks propagate over time, a risk averse utility function will lead to a cost called the cost of uncertainty¹⁹. That is mainly why we ended up with the following: It is clear from table 1 and 2 that on impact consumption levels (so will lifetime welfare) are lower than the benchmark cases. On the other hand, an increase in the investment tax credit has an expansionary long-run effect on domestic capital stock and a contractionary long-run effect on foreign asset holdings. Therefore, a reduction in life-time welfare is expected for Canada whereas a rise in life-time welfare is anticipated for Sweden. However, in this case, although there is a jump on impact, life-time welfare reduces for both of the countries -when compared to the benchmark- regardless of where the economy stands. For instance, the on impact value of ϕ is -14.75 and -16.92 for Canada and Sweden respectively (see table 1). Yet, as a response to a change in various type of tax instruments, in none of the cases, we are able to have a higher ϕ on impact value. Thus, under stochastically growing output, a welfare paradox does not exist.

3.3 The Stochastic Tax Shocks and Their Effects

In this part, we will put the model to work by considering the effects of stochastic tax shocks²⁰. With having the following structure, we assume that the economy has been experiencing a deterministic tax structure and expects it to last forever. Then, we change the tax policy by

¹⁸Risk aversion simply implies that individuals strictly prefers to take the expected value of a lottery to than the lottery itself.

¹⁹See Obstfeld (1994).

²⁰Throughout this part, productivity parameter is normalized to one without loss of generality. In other words, productivity is constant.

introducing some uncertainty but keeping the mean tax rate constant after an initial decrease in tax levels. For this purpose, we have δ 's such that $\delta_t^i = \theta_j e^{z_t}$ where j = c, g, l, r and z_t is the tax shock which follows a stochastic autoregressive process $z_t = \mu z_{t-1} + \zeta_t$ with the disturbance term ζ_t assumed to be normally distributed with mean zero and variance σ_z^2 i.e. $\zeta_t: N(0, \sigma_z^2)$. As in the case of stochastic productivity shock discussed above, similarly, here we do shock the system today by dispersing its effect slowly throughout the economy. It is worth noting that an increase in δ_t^i on impact means a reduction in tax levels. Thus, throughout this part, the shocks hitting the system today are negative tax shocks.

Let us start out by considering the effects of a stochastic change in the corporation tax. On impact, this will increase dividends and the rate of return on equity. The expectations of capital gains will rise and there will be an immediate jump in the price of equity, q, leads to an immediate increase in investment which will cause a current account deficit²¹. Since c jumps on impact, we will end up with an increase in domestic absorption which accentuates the current account deficit in the short run. Towards steady state, this process reverses and continues until the rate of return on foreign bonds equal the rate of return on equity. In other words, right after an initial jump in q, investment starts to decrease along with the shrink in q which causes an improve on current account balance. Moreover, consumption adjusts much quicker, or, to put it differently, the accentuating effect of higher consumption on current account will disappear. Therefore, in the long-run, that will improve the current account balance as well, besides the shrink in investment.

Next, consider the effects of a stochastic change in the capital gains tax. As opposed to the previous case, here, what we see is an initial drop in q along with an increase in domestic capital stock. We can explain this situation by combining 22 eq(18) and eq(19). Depending on the corresponding equation, a decrease in capital gains tax rate, θ_g , will lead to a decrease in q on impact. However, although we expect (and also see) an increase in investment level, here, q declines. This situation can be explained by eq(22) which basically indicates that the change in θ_g outweighs the decline in q and therefore, even though q drops on impact, there is an increase in the invesment level. In the short-run, an increase in the rate of investment along with an increase in consumption will deteriorate current account balance. On the other hand, in the long-run, the accentuating effect of c on current account will disappear in addition to the decrease in investment that leads to an improve in current account balance as seen in figure 20.

Conversely, a stochastic change in invesment tax credit causes a jump in q on impact. Yet, here, investment is decreasing in the short-run which can be explained by eq(20)²³. It is straightforward in the equation that we will end up a decrease in investment level at least in the short-run because the change in t_l has a dominating effect on the increase in q. Therefore, k goes up in the medium-run. However, since the shock on t_l dies out over time, towards steady state the increase in q offsets the change in investment tax credit which will give rise to an

²¹It should also be noted in figure 19 that, k rises/declines as soon as q exceeds/falls behind its steady state value.

²²The corresponding equation can be seen in appendix.

²³Also see appendix A.

increase in k (so will investment). The decrease in investment, along with declining consumption, will improve current account balance in the short-run. However, in the long-run, consumption will adjust before domestic capital stock. Thus, the improving effect of higher savings on current account will disappear in conjuction with the increase in investment level causing a deterioration in current account balance. Here, again, this implies a non-monotonic adjustment of the current account (see figure 21).

Finally, let us consider the effects of a stochastic change in the tax rate on personal interest income. It should be noted that, initially, the rate of return on foreign bonds will increase. On impact, there will be a decrease on demand for equity, which will be absorbed by an immediate drop in the price of equity which increases the yield on it. In the medium run, the investment will decrease caused by the reduction in domestic capital stock. Therefore, the marginal productivity of capital will decrease up until the equity price, q, and the investment level will reach their initial levels in the long-run. On the other hand, since there is a drop in consumption on impact and a decrease in the rate of investment in the short run, the current account balance will improve. However, again, that process reverses in the long-run, indicating a deterioration in the current account balance before it reaches its long-run level, or, to put it differently, an implication of a non-monotonic adjustment of the current account.

Next, we will briefly point out the welfare impacts of stochastic tax changes. As seen from figures (19)-(22), while a stochastic change in corporate income tax and the tax rate on capital gains cause an increase in consumption on impact (so will welfare), a stochastic change in investment tax credit and the tax rate on personal interest income lead to a drop in welfare. Here, since long run values do not change, we need to provide an alternative explanation. As known, wealth is roughly equal to qk. Thus, an increase in wealth level will give rise to an increase in consumption level which will also cause an increase in welfare. For instance, it is obvious from figure (19) that, as a response to a stochastic corporation tax shock, both k and most of the time q remains above their steady-state level that makes individuals wealthier and leads to a rise in consumption. On the other hand, the opposite holds for a stochastic interest income tax shock (see figure (22)). As a response to a capital gains tax shock, while k stays above its long-run level because of the reasons explained above, q is below its steady state level. Here we can conclude that the increase in domestic capital stock, k, outweighs the decline in q. Thus, consumption increases since there is an increase in wealth level.

4 Conclusion

In this paper, we have examined the dynamic macroeconomic effects of different capital income taxation methods in a model of a small open economy with endogenous recursive time preferences and adjustment costs in investment where perfectly competitive firms produce one good that can be used for consumption and investment. We have shown the adjustment paths of consumption, lifetime welfare, equity prices, current account and investment in response to various tax shocks. Our results suggest that welfare paradoxes may exist only under deterministic output and anticipated tax shocks. Unlike most intertemporal equilibrium models, our model leads to non-monotonic adjustment of the current account in response to various policy changes.

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²⁴Again, the opposite holds for a stochastic investment tax credit shock.

Our model can be generalized in numerous ways. For instance, it is worthwhile to generalize to see the outcomes of any change in tax policy or, in other words, how the tax policy might be adjusted in an economy. It may also be interesting to enhance our analysis by allowing the households to supply their labor elastically to study the impact of tax policy changes on wages and consumption. Our model can also be extended to a two-country framework to analyze the international transmission of shocks.

Appendix A. Derivatives

The derivates in (24)-(28) are as follows:

$$q_1^* = \frac{(\varepsilon - t_l)}{\theta_g^2} > 0, \quad q_2^* = -\frac{1}{\theta_g} < 0$$
 (30)

$$k_1^* = \frac{r\theta_r(\varepsilon - t_1)}{\theta_c \theta_g^2 f''(\overline{k})} < 0, \quad k_2^* = \frac{\theta_g k_1^*}{\theta_c} < 0, \tag{31}$$

$$k_3^* = \frac{r(\varepsilon - t_l)}{-\theta_c \theta_g f''(\overline{k})} > 0, \quad k_4^* = \frac{r\theta_r}{-\theta_c \theta_g f''(\overline{k})} > 0,$$

$$b_1^* = -\frac{f'(\overline{k})k_1^*}{r} > 0, \quad b_2^* = -\frac{f'(\overline{k})k_2^*}{r} > 0,$$
 (32)

$$b_3^* = -\left[-\frac{1}{c} + \frac{f'(\overline{k})k_3^*}{r} \right] < 0, \quad b_4^* = \frac{f'(\overline{k})k_4^*}{r},$$

$$c^* = r\overline{c} \tag{33}$$

$$\phi^{*'} = -\left(\frac{1}{r\theta_r^2}\right) \tag{34}$$

The aforementioned equations are as follows:

For section 3.2

Plugging adjustment cost equation into eq(20) and manipulating the obtained one will yield:

$$i_{t} = k_{t} \left(\frac{\theta_{g} (q - \overline{q})}{\chi \theta_{c}} \right)$$
 (35)

For section 3.3

$$q_{0} = E_{t} \sum_{t=0}^{\infty} \left(\frac{1}{1 + \frac{r\theta_{r}}{\theta_{g}}} \right)^{t} \frac{\theta_{c}}{\theta_{g}} \left[f'(k_{t+1}) - r(1 - \varepsilon) + \left[(\frac{i_{t+1}}{k_{t+1}})^{2} T'(\frac{i_{t+1}}{k_{t+1}}) \right] \right]$$
(36)

if capital gains tax rate decreases, $\left(\frac{1}{1+\frac{r\theta_r}{\theta_g}}\right)^t$ will increase which means an increase in q_o . On the other hand, q_o will decline becaues of the decrease in $\frac{\theta_c}{\theta_g}$. Thus, we can conclude that the

latter impact dominates the former.

Once the adjustment cost equation is plugged into eq(20), after some manipulation, we will end up with the following:

$$\frac{i_t}{k_t} = \frac{\theta_g q_t - \varepsilon + t_l}{\chi \theta_c} \tag{37}$$

As investment tax credit declines, t_i , investment level will decrease. Yet, on the other hand, the increase in q will stimulate investment. From figure (21), one can infer that the impacts of the decrease in t_1 dominates the effects of the decrease in q on investment level in the short-run. However, it should be noted that the shock is temporary and dies out over time. Thus, after a certain point, the change in q outweighs by giving rise to an increase in investment as seen in figure.

Appendix B. Parameters

To calibrate the model, we choose the parameters in conformance to the best practice in the literature, so that we have following initial values:

$$r = 0.04$$
, $\alpha = 0.33$, $\rho = 0.9$, $\mu = 0.9$, $w = 0.8$, $\chi = 2.9$.

On the other hand, the country specific tax rate data is obtained from OECD Tax Data Base.

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Appendix C: Tables and Figures

Table 1. Foreseen Tax Shocks under Stochastically Growing Output and Their Impacts (Sweden)

	\bar{c}	$ar{arphi}$	\overline{q}	$ar{k}$	$ar{b}$	c ⁺	$arphi^+$			
A rise										
in										
t_{c}	0.4587	-48.07	0.9285	25.98	-61.78	0.4405	-60.46			
t_{g}	0.4587	-48.07	0.9352	26.18	-61.96	0.4054	-84.17			
t_1	0.4587	-48.07	0.9000	27.29	-62.98	0.4960	-22.93			
$t_{\rm r}$	0.4585	-49.01	0.9285	26.95	-62.67	0.4713	-40.76			
Benchmark for Sweden										
	0.4587	-48.07	0.9285	26.39	-62.15	0.5047	-16.92			

Note: + above a variable indicates on impact value for the parameter.

Table 2. Foreseen Tax Shocks under Stochastically Growing Output and Their Impacts (Canada)

	Ē	$ar{arphi}$	\overline{q}	$ar{k}$	$ar{b}$	c^+	$arphi^+$			
A rise										
in										
t_{c}	0.4617	-36.76	0.7971	23.58	-59.4	0.4408	-46.07			
t_{g}	0.4617	-36.76	0.8092	23.54	-59.36	0.4122	-58.79			
t_l	0.4617	-36.76	0.7681	24.95	-60.73	0.4855	-26.20			
t _r	0.4615	-37.30	0.7971	24.33	-60.14	0.4822	-28.25			
Benchmark for Canada										
	0.4617	-36.76	0.7971	23.94	-59.76	0.5112	-14.75			

Figure 1. The welfare effect of a 3% increase in corporate income tax for Canada

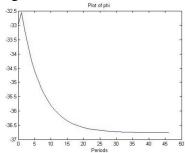


Figure 2. The welfare effect of a 3% increase in capital gains tax for Canada

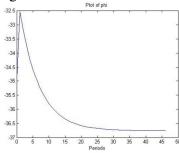


Figure 3. The welfare effect of a 3% increase in corporate income tax for Sweden

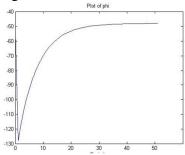


Figure 4. The welfare effect of a 3% increase in capital gains tax for Sweden

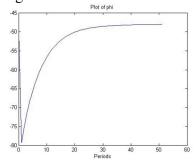


Figure 5. The welfare effect of a 3% increase in investment tax credit for Canada

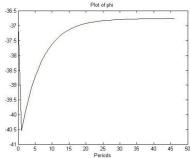


Figure 6. The welfare effect of a 3% increase in investment tax credit for Sweden

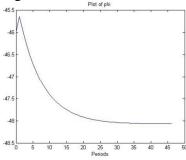


Figure 7. The welfare effect of a 3% increase in interest income tax for Canada

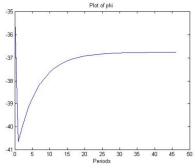


Figure 8. The welfare effect of a 3% increase in interest income tax for Sweden

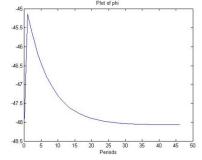
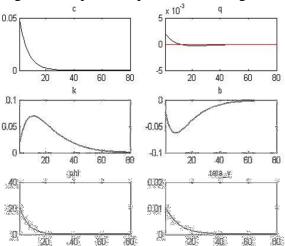
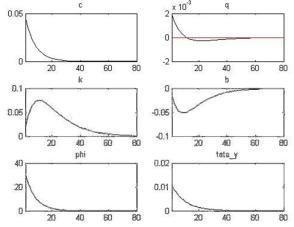


Figure 9. Impulse responses following a stochastic productivity shock for Canada.



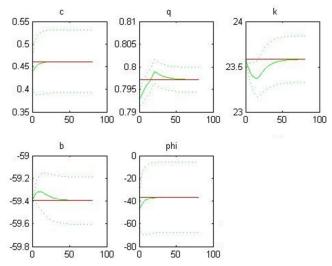
Note: Figure depicts the deviations from steady-state.

Figure 10. Impulse responses following a stochastic productivity shock for Sweden.



Note: Figure depicts the deviations from steady-state.

Figure 11. The effects of anticipated 1% increase in t_c under stochastically growing output for Canada.



Note: Dotted lines in any figure indicate 90% confidence interval around the mean trajectory.

Figure 12. The effects of anticipated 1% increase in t_c under stochastically growing output for Sweden.

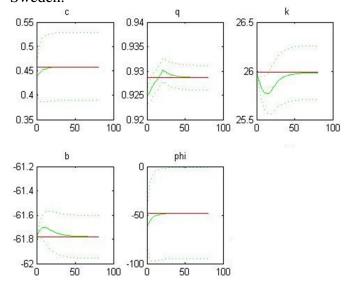


Figure 13. The effects of anticipated 1% increase in t_g under stochastically growing output for Canada.

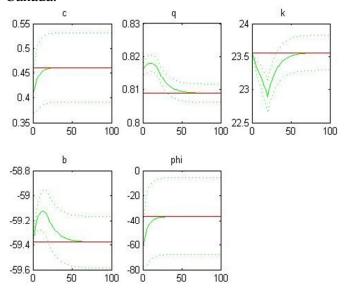


Figure 14. The effects of anticipated 1% increase in t_g under stochastically growing output for Sweden.

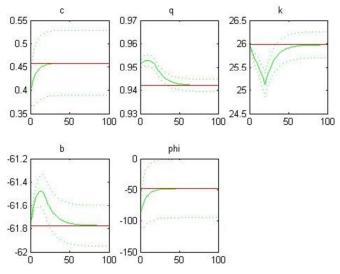


Figure 15. The effects of anticipated 1% increase in t_l under stochastically growing output for Canada.

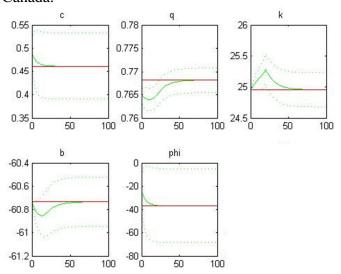


Figure 16. The effects of anticipated 1% increase in t_1 under stochastically growing output for Sweden.

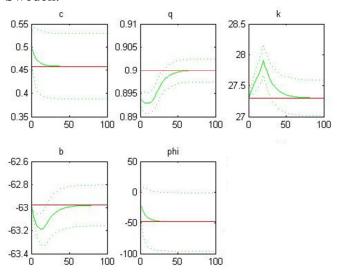


Figure 17. The effects of anticipated 1% increase in t_r under stochastically growing output for Canada.

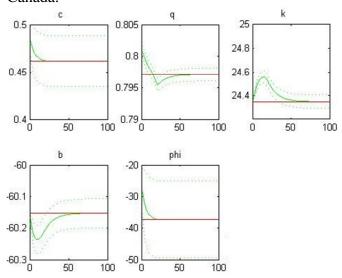


Figure 18. The effects of anticipated 1% increase in t_r under stochastically growing output for Sweden.

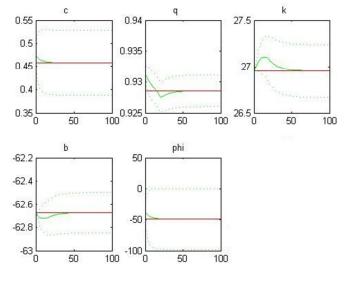
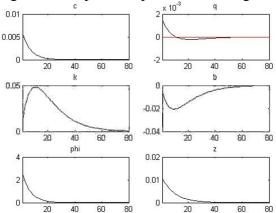
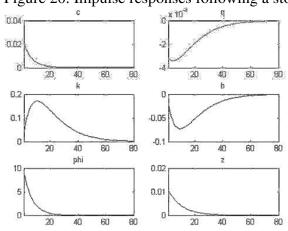


Figure 19. Impulse responses following a stochastic corporation tax shock.



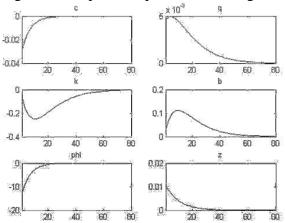
Note: Figure depicts the deviations from steady-state.

Figure 20. Impulse responses following a stochastic capital gains tax shock.



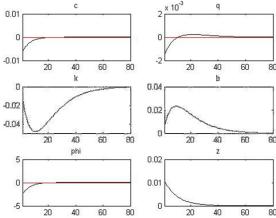
Note: Figure depicts the deviations from steady-state.

Figure 21. Impulse responses following a stochastic investment tax credit shock.



Note: Figure depicts the deviations from steady-state.

Figure 22. Impulse responses following a stochastic interest income tax shock.



Note: Figure depicts the deviations from steady-state.