Bridging the Attitude-Preference-Gap: A Cognitive Approach To Preference Formation

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By Rebecca Schmitt *

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This paper provides a descriptive decision model that is based on a single behavioral pattern: human beings strive for consistency between what they do, what they think and what they perceive. This pattern manifests in the decision maker’s aim to bring his attitudes, beliefs and behavior into balance.

Drawing principally on the theory of cognitive dissonance by Festinger (1957), the model shows how the concept of attitudes and the concept of preferences are interwoven by the human need for consistency. It closes the conceptual gap between preferences and attitudes.

The model is an alternative approach to additive utility models, such as the one by Fehr and Schmidt (1999). Models of this class are not capable of explaining behavioral discontinuities in the mini ultimatum game. In contrast, the attitude-based model covers this behavioral pattern.

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I. Motivation

Human decision making is studied in many different disciplines. Each discipline has its own focus. Economics and Psychology have independently developed models of decision making with distinct foci and different core concepts. Consequently, their views on how people value things differ.

The core concept in economic theory is a given preference order. Economic theory focusses on the assumptions which are necessary to represent a given preference order by a utility function. It excludes the notion that preference orders are formed, may change, and are influenced by the social environment, e.g., by norms and values.

Psychology focuses on the cognitive aspects of the entire decision process and supplies conceptual tools which can be used to illuminate the process of preference order formation. Such a conceptual tool is the attitude. An attitude is a summary evaluation of a psychological object on a scale (Petty et al, 1997, 611). That is, the evaluation is expressed in so-called attribute dimensions as good-bad, harmful-beneficial, and favorable-unfavorable (Ajzen, 2001, 28). Psychological objects are mental representations of anything a decision maker can sense or imagine (Kahneman et al, 1999, 206). From a psychological perspective, decision makers are not endowed with a given preference order over the outcomes of their alternative behaviors, but with various attitudes towards all elements of the decision situation, including characteristics of other decision makers.

Attitudes can be classified among concepts of psychology and testable propositions which have been incorporated into economic theory like mental accounting and loss aversion (Kahneman et al, 1999, 204). Yet, attitudes have not been fully integrated into economics. There is a conceptual gap between the concept of a preference and the concept of an attitude. The reason for this is that a preference
order is defined within the space of alternatives, while attitudes are related to the
different attributes of alternatives. Moreover, in situations of social interaction,
attitudes are also related to attributes of other decision makers, social norms, and
rules.

Bridging this gap contributes to the theoretical integration among social sciences.
A decision model which is based on attitudes, rather than preferences, can
contribute to this aim and provides a foundation for behavioral patterns shown by
participants in laboratory experiments, which cannot be captured fully by utility
theory’s current paradigms.

One of these patterns is the framing effect. It refers to the phenomenon of
preference reversion due to different descriptions of the decision problem
(Takemura, 2014, 103). We argue that framing can change the mental perception
of any decision situation. As a result of this, the set of psychological objects, which
the decision maker regards as relevant, changes too. As attitudes refer to
psychological objects, the set of relevant attitudes and maybe their strengths are
changed, as well. Therefore a decision theory which is based on attitudes
incorporates framing effects.

Another behavioral pattern is fairness behavior. Although there are plenty of
models which can be calibrated to fit the data of laboratory experiments, they lack
a psychological foundation, while a unifying framework for these models still does
not exist. Hence this class of models is open to further criticism.

Current descriptive fairness models, like the one by Fehr and Schmidt (1999), use
additive separable utility functions for the representation of the agent’s preference
order. This class of fairness models has been invented as an alternative to the model
of Homo Economicus. The model of Homo Economicus has been criticized, not
only because of its methodological individualism, but also because of its reliance
on the axioms of rationality, which are not fulfilled in reality. Now, the crucial point
is: additive separability of utility functions requires the fulfillment of both the axioms of rationality, and of some additional axioms. This has been shown by Fishburn (1992). Hence it can be argued that additive separable utility functions are not very suitable to serve as an alternative to the model of Homo Economicus.

This paper offers an alternative model of human choice behavior which is based on a single basic behavioral pattern identified by Leon Festinger (1957). It is the striving for consistency between what you do, what you think and what you perceive. The idea, that human behavior is a result of the striving for cognitive consistency is the core element of Leon Festinger’s theory of cognitive dissonance. This theory relies heavily on the concept of attitude and is empirically very well verified (Harmon-Jones et al, 2007, 13). This makes it suitable to serve as a base for a decision model which closes the gap between preferences and attitudes.

The model is based on the original theory of cognitive dissonance by Festinger (1957), and on the action-based model of cognitive dissonance by Harmon-Jones et al. (2007, 2009, and 2012). It shows that the concept of attitudes and the concept of preferences are interwoven by the human need for consistency and inner harmony. As we will see, this model can cope with the framing effect and behavioral discontinuities.

The remainder of the paper is organized as follows: the next section provides a short discussion on the related economic literature. Section 3 works out the relevant elements of the theory of cognitive dissonance which can be adapted to our decision model. These provide some basic concepts for the model, which fuse into the model’s core concept: individuals who seek to minimize cognitive dissonance choose an alternative by finding a suitable compromise. A mathematical formulation is carried out in section 3. In section 4 we use the model to explain behavioral discontinuities in the mini ultimatum game. Section 5 comes to an overall conclusion.
II. Related Literature

There are some approaches that already incorporate single aspects of the theory of cognitive dissonance into economic models. Akerlof and Dickens (1982) and Rabin (1994) focus only on aspects of dissonance theory which influence belief formation. Epstein and Kopylov (2005) focus on the post-decision process, which is self-justification on past decisions through changing beliefs. Konow (2000) and Oxoby (2003) relate cognitive dissonance to preferences, but only by adding a dissonance parameter into an additive utility function. We have already outlined the problems which come with additive utility functions.

As noted before, we will apply our model to the mini ultimatum game. In the literature on fairness behavior we find hints concerning the attitudes which are relevant in this game. Although not explicitly mentioned, these models take attitudes into consideration. Indeed, models of distributive fairness, like Fehr and Schmidt (1999), Ottone and Ponzano (2005) and Bolton and Ockenfels (2000) rely on attitudes which are directed towards one’s own monetary payoff, and the differences in players’ monetary outcomes. Other attitudes can be found in interactional fairness models and in procedural fairness models, such as positive attitudes towards a social welfare function (Charness and Rabin (2000)), attitudes towards opponents’ outcomes (Levine (1997)), and attitudes towards the perceived intentions of the opponent (Rabin (1993); Falk, Fehr and Fischbacher (2000)). We will refer to this when explaining behavioral discontinuities in section 4. Next, we derive the basic principles of our model.
III. Basic Premises and the Model’s Core Concept

A. Shaping the theory of cognitive dissonance

In this section, dissonance theory is formulated in such a way that it can be converted into a decision model. We introduce suitable definitions of the theory’s core elements and derive our model’s basic assumptions. Aspects of dissonance theory which are related to ex-post-decision processes, like rationalizing of chosen behaviors, are omitted.

Leon Festinger’s theory of cognitive dissonance states that when a decision process takes place and after the chosen behavior has been carried out, inconsistencies between cognitive elements and one’s behavior can occur. Inconsistencies cause cognitive dissonance. Two cognitive elements are dissonant if they do not fit together (Festinger 1957, 12). If people suffer cognitive dissonance they feel pressure to reduce or eliminate it (Festinger, 1957, 18).

Festinger mentions five different cognitive elements: beliefs, knowledge, attitudes, opinions and values. He does not, however, provide any definitions of these terms. This makes dissonance theory somewhat fuzzy.

The mathematical formulation of our decision model demands precise, and thus quite narrow definitions of the different cognitive elements. We provide definitions which are in line with the standard concepts in social psychology.

Defining Knowledge and Belief

We define “belief” similarly to the standard concept in game theory, but do not refer to strategies only – other components of decision situations are also included.

We define the decision maker’s knowledge and belief in relation to single components: knowledge is perfect information on the nature of a component.
Imperfect information on a component results in a belief as to its nature. A belief is a probability distribution comprising the possible natures of this component.

**Defining Attitude**

In section I, we have defined an attitude as a summary evaluation of a psychological object on a scale, and a psychological object as a mental representation of a thing or of an idea.

Psychological objects can be very concrete such as a person’s hair color, or more abstract such as the idea of sustainability (Petty et al. 2003, 353). In a game theory laboratory experiment, psychological objects may be monetary payoffs, the game structure, or norms which are intertwined with the game structure or with the framing of the experiment. In a decision situation of social interaction, the relevant set of psychological objects is influenced by the situational context. For example, group affiliation can activate two different norms concerning cooperation in the prisoner’s dilemma game. If the other player is a member of an enemy army, the norm not to cooperate with the enemy is activated. If the other player is affiliated to a federate army, then the norm to cooperate is activated.

We assume that for any specific decision situation, the set of psychological objects is finite and discrete. We denote this set with \( \mathcal{O} \), so that: \( \mathcal{O} = \{1, \ldots, j, \ldots, J\} \). Each psychological object \( j \) has characteristic values. The set of characteristic values of psychological object \( j \) is denoted with \( X_j \). It is either finite and discrete or a closed interval in \( \mathbb{R} \). Each alternative \( s \in S \) is characterized by its specific values of the \( J \) objects. Or, seen from another perspective, we can say that the characteristic value of a psychological object is determined by the choice of the alternative \( s \in S \). Hence, \( x_j(s) \in X_j \) and \( x_j: S \rightarrow X_j, s \mapsto x_j(s) \).
The evaluation of psychological objects is subjective (Olson and Maio, 2003, 299). Attitudes can be weak or strong. Given the vast number of definitions on attitude strength, we follow the definition by Miller and Peterson. “Attitude strength is defined as the extent to which an attitude is stable, resistant to change, impacts information processing, and guides behavior” (Miller and Peterson, 2004, 847).

As our focus is on decision making for given attitudes and beliefs, we can omit the fact that attitudes can change and that they influence the process of information gathering and belief formation. This enables us to simplify the concept “attitude strength”. We define attitude strength as the extent to which the related attitude guides an agent’s behavior. The strength of attitude $j$ is represented by parameter $\beta_j \in [0,1]$.

We introduce the expression “neutral attitude” for objects which the decision maker considers to be irrelevant in a specific decision situation and for attitudes which currently do not have any strength. Finally, we define an attitude as a negative, neutral or positive relation to a psychological object $j$. In the following, an attitude is represented as a combination of a function which represents the attitude’s valence, and a parameter which represents the attitude strength.

**Definition: Attitude**

The attitude on object $j \in O$ is represented by $(a_j, \beta_j)$, where $a_j: X_j \rightarrow V, x_j \mapsto a_j(x_j)$ is the valence function, and $\beta_j \in [0,1]$ is the parameter of attitude strength. The codomain $V$ depends on the attitude valence. If the attitude on object $j$ is positive, then $V \subseteq \mathbb{R}^+$. If it is neutral, then $V = \{0\}$ and if it is negative, then $V \subseteq \mathbb{R}^-$. Those psychological objects towards which the decision maker has a neutral attitude do not play any role in determining his decision. Hence, only psychological
objects towards which the decision maker holds a negative or positive attitude become decision criteria. That is, all decision criteria are psychological objects, but not all psychological objects are decision criteria.

We assume that, given a single attitude, the decision maker ceteris paribus seeks to behave in accordance with this single attitude. This assumption manifests in the following property of the valence function: the number, which it assigns to a characteristic value of a psychological object, is the greater the more the characteristic value is compatible to the underlying (non-neutral) attitude. This assumption implies that the decision maker, ceteris paribus, seeks to maximize the valence function $a_j$ over the set of alternatives.

Hitherto, we have defined “believe” and “attitude,” and have argued why knowledge can be subsumed under the concept of belief. Next, we argue why values are attitudes and why we can also subsume opinions under the concept of belief.

**Value and Opinion**

Concerning the relation between a value and an attitude, we need to differentiate whether a value is held on an individual level or on a societal level. Values, which are held by the society, are institutional in nature. Like norms and rules, which are also institutional, these values can become the object of a decision maker’s attitude. If the attitude is positive and the attitude’s valence is very strong, this societal value becomes an individual value. An individual value is an attitude with high importance, so that it becomes a guiding principle in life. Such attitudes are more resistant to change than other attitudes. An opinion is a nexus between a psychological object and an attribute which is non-judgmental. This means opinions do not include any positive or negative attributions and hence do not evaluate the object. Thus, an opinion essentially differs from an attitude in that it does not involve any value judgement. For example, the opinion, “Berlin is a
cultural city” does not include a negative or positive attribution. Opinions are closely related to beliefs. For convenience, we will not differentiate between opinions and beliefs, but interpret opinions as beliefs.

Reducing the Cognitive Elements in Dissonance Theory

Based on the considerations just made, we can reduce the number of cognitive elements which were mentioned in Festinger’s theory from five to two, namely attitudes and beliefs. Hence, we build our decision model on these two different kinds of cognitive elements.

In the following section we assume that in the moment of decision the decision maker’s attitudes and beliefs are given.

B. Solving Dissonance Theory’s Measurement Difficulties

Festinger states that his theory has some serious measurement difficulties, due to the conceptual definitions of dissonance and consonance (Festinger, 1957, 15). Nevertheless, he specifies the crucial factor which influences the magnitude of dissonance. It is the importance of the cognitive elements which are involved in the specific relation. He states that the magnitude of the dissonance increases with the importance of the involved cognitive elements (Festinger, 1957, 18), and emphasizes that the total magnitude of dissonance is determined by the weighted proportion of all relevant relations (Festinger, 1957, 262). Concerning the concept of attitude, we can identify the attitude strength as a measurement of its importance.

Festinger does not address the issue of how to measure dissonance. As cognitive dissonance can have many different sources like logical inconsistencies, cultural mores, past experience or specific opinions (Festinger, 1957, p. 14), neither the units nor the scale of dissonance are obvious. The action-based model of
dissonance, a recent interpretation of the theory of cognitive dissonance (by Harmon-Jones et al. 2009, 128), gives a hint as to how to measure dissonance. This hint is given by the answer to the question, “what exactly causes dissonance?” Harmon-Jones identifies inconsistencies between important action tendencies as the cause for dissonance (Harmon-Jones, 2012, 546). An action tendency is a motivational mental state. It is considered as the desire to carry out a specific action. (Reisenzein, 2014, 1). Here, in the context of dissonance theory, an action tendency is related to a specific attitude. If there is a single alternative which fits best to the related attitude, then this alternative and the action tendency are congruent. But often several different alternatives fit equally well with the related attitude. Then, the decision maker has the desire to carry out each of these alternatives. This unfeasible desire describes the mental state of indecisiveness.

Next, we conceptualize action tendencies. Representing alternatives as unit vectors allows for an uncomplicated conceptualization.

If the set of alternatives is finite and discrete, so that $S = \{s_1, s_2, \ldots, s_m\}$, then:
$s_1 = (1,0,0,\ldots,0)$, $s_2 = (0,1,0,\ldots,0)$, \ldots, $s_m = (0,0,0,\ldots,1)$.

If the set of alternatives a closed interval in $\mathbb{R}$, we can normalize, so that $S = [0,1]$. Since we can represent the upper bound as a unit vector in the one-dimensional space, an alternative $s \in S$ serves as a convex combination of the unit vector and the zero vector. Note, that in the case of discrete alternatives convex combinations of the unit vectors are not feasible.

In the following, we denote the set of valence maximization alternatives of the attitude to object $j$ with $\hat{S}_j$. It is defined as follows: $\hat{S}_j = \{ s \in S \mid a_j(x_j(s)) \text{ is maximal} \}$. We can represent an action tendency as a convex combination of all favored alternatives, whereas they are weighted equally. We normalize the weights so that they add up to 1.
Definition: Action Tendency

Let the number of elements in \( \hat{S}_j = \{ s \in S \mid a_j(x_j(s)) \text{ is maximal} \} \) be \( n \), i.e. \( \hat{S}_j = \{ s_1, ..., s_i, ..., s_n \} \). The action tendency \( \hat{s}_j \) which has been generated by the attitude to object \( j \) is the convex combination of the \( n \) different attitude maximizing alternatives: \( \hat{s}_j = \frac{1}{n} \sum_{i=1}^{n} s_i \).

In the following \( \Delta \) denotes set of action tendencies. For each of the \( J \) different objects the related attitudes generate action tendencies. There results a set of action tendencies, \( \hat{S} = \{ \hat{s}_1, \hat{s}_2, ..., \hat{s}_J \} \).

Now, we can link to the core proposition of the action-based model and give a solution to the measurement problem. The action-based model states that cognitive elements can activate action tendencies which, in turn, are cognitive elements and that “inconsistency between [cognitive elements] makes persons uncomfortable because inconsistency has the potential to interfere with effective action” (Harmon-Jones, 2012, 546). Cognitive elements, which imply different and thus inconsistent action tendencies cause dissonance. This means, dissonance occurs whenever it is not possible to carry out an alternative which is in line with each of the inconsistent cognitive elements that are very important in regard to the specific decision situation (Harmon-Jones, 2012, 546).

Applying this argument to our interpretation of dissonance theory uncovers two sources of dissonance. One results from the differences of distinct action tendencies and the second from the difference between action tendencies and alternatives. The last source is only relevant if \( S \) is discrete.

This means that dissonance occurs if either there is at least one \( \hat{s}_j \) that is not a unit vector and \( S \) is discrete, or \( \hat{S} \) contains at least two different action tendencies,
because in these cases the decision maker has a wish to carry out different alternatives at the same time – which is impossible.

In the following section we measure the amount of imposed dissonance between any pair of action tendencies by the square of their Euclidian distance. Hence, we define the overall amount of cognitive dissonance $D$ which has been imposed by $J$ different attitudes:

**Definition: Overall Amount of Cognitive Dissonance**

The overall amount of cognitive dissonance $D$, which has been imposed on a decision maker whose decision making is guided by $J$ different attitudes, is measured as follows:

$$D = \sum_{j=1}^{J} \beta_j \cdot \left( \| \hat{s}_j - \sigma \|_2 \right)^2.$$

In the next section we derive the model’s core concept.

**C. The Model’s Core Concept: The Compromise**

We derive the model’s core concept by isolating the methods mentioned by Festinger (1957), with which a decision maker seeks to reduce cognitive dissonance.

Festinger states that cognitive dissonance is a motivating factor because its existence is psychologically uncomfortable, so that people seek to reduce it and avoid situations and information which might contribute towards an increase of dissonance (Festinger, 1957, 3). Three methods of reducing dissonance are specified: “changing one or more of the elements involved in dissonant relations [... and] adding new cognitive elements that are consonant with already existing cognition [... and] decreasing the importance of the elements involved in the dissonant relations” (Festinger, 1957, 264).
Changing the behavioral cognitive element can be performed by taking a specific action which better suits the environmental cognitive element than any other action. Hence, this method of reducing dissonance is a way to make a decision. Therefore, this aspect of dissonance theory can serve as a core assumption of our model.

Cognitive elements are resistant to change, which implies that changing cognitive elements causes psychological cost. There are different sources of resistance, namely the responsiveness to reality, the extent of pain or loss which is inherent in the change, the degree of satisfaction obtained from present behavior and the relationships of all other elements with the element which is considered to be changed (Festinger, 1957, 24f) That is, changing one element in order to eliminate some dissonance “may create a whole host of new ones” (Festinger, 1957, 19).

Here, the responsiveness to reality is a very important point. It indicates that changing beliefs tendentially is done in accordance with (perceived) reality. Therefore, changing beliefs is, to a vast extent, similar to a learning process. But note: we have assumed that the decision maker’s attitudes and beliefs are given. From this, it follows that in the pre-decision stage, cognitive dissonance is changed by adapting behavior to given beliefs and attitudes. Changing beliefs and attitudes is part of the post-decision process, except the decision maker would stay undecided without any change of these cognitive elements. Hence, for given beliefs, the decision maker needs to find an alternative which reduces the overall amount of cognitive dissonance which has been imposed by competing attitudes respectively by the related action tendencies.

The action-based model refers to this by proposing two different motivations for reducing dissonance: a proximal motivation and a distal motivation. The term “proximal motivation” refers to the negative emotion of dissonance and “distal motivation” refers to the decision maker’s need for a non-conflicted action. (Harmon-Jones et al, 2009, 128).
Both motivations induce the wish to carry out an alternative which balances the decision maker’s attitudes by minimizing the amount of cognitive dissonance. The action tendency which minimizes the amount of cognitive dissonance is called “dissonance minimizing action tendency” and is denoted with $\sigma^*$.

The decision maker chooses the alternative which imposes the least amount of additional cognitive dissonance on him in comparison to his dissonance minimizing action tendency.

Due to our measurement of cognitive dissonance, we specify $\sigma^*$ as the action tendency which minimizes the weighted sum of the distances between itself and each of the competing action tendencies. Thereby, each dissonant relation between an action tendency and the dissonance minimizing action tendency is weighted with the parameter which represents the strength of the underlying attitude. In the following, we normalize the weights, so that $\sum_{j=1}^{J} \beta_j = 1$.

**Definition: Dissonance Minimizing Action Tendency**

Let $\| \cdot \|_2$ denote the Euclidian distance. The dissonance minimizing action tendency is defined by

$$\sigma^* = \arg \min_{\sigma \in \Delta} \sum_{j=1}^{J} \beta_j \cdot \left( \| \hat{s}_j - \sigma \|_2 \right)^2$$

By using weights, we assume that reducing the dissonance, which has been caused by a strong attitude, is more important to the decision maker than reducing the dissonance which has been caused by a weak attitude.

Usually, the dissonance minimizing action tendency turns out to be a convex combination of the alternatives. This dissonance minimizing action tendency is what the decision maker actually desires to do. $\sigma^*$ brings the different and thus competing attitudes into balance. It is a mental equilibrium in form of a cognitive compromise between the divergent action tendencies $\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_J$. 
Only if all cognitive elements lead to the same action tendency, will the dissonance minimizing action tendency be congruent to an action. But usually $\sigma^*$ is not feasible as it usually entails tendencies towards different alternatives. In other words it entails the desire to carry out different alternatives at the same time.

Hence a compromise between desire and reality is necessary for making a decision. It is choosing the alternative $s^*$ which imposes as little additional inconsistency as possible on the decision maker. We call $s^*$ the “dissonance minimizing alternative.” If the desire is realizable, then $\sigma^* = s^*$.

**Definition: Dissonance Minimizing Alternative**

The dissonance minimizing alternative is defined by

$$s^* = \arg \min_{s \in S} \|\sigma^* - s\|_2.$$  

We summarize the process of decision making by the following core assumptions: First, attitudes lead to action tendencies. Second, competing attitudes can lead to competing action tendencies. Third, individuals seek to minimize the amount of cognitive dissonance by choosing an alternative which is closest to the dissonance minimizing action tendency.

In the next section we provide the compact mathematical description of our model.
IV. Resume: The Cognitive Decision Model

Here, we provide a resume in the form of a compact mathematical description of the decision model.

Let \( O \) denote the set of psychological objects and \( X_j \) the set of characteristic values, which is bounded above and below. \( S \) is the set of alternatives, and \( \Delta \) is the set of action tendencies. \( s \in S, \sigma \in \Delta \) and \( S \subseteq \Delta \). If \( S \) is discrete and \( S = \{1, \ldots, m\} \), then \( \Delta \) is the 1-m dimensional unit simplex and \( S \) is the set of its vertices. Let \( \| \|_2 \) denote the Euclidean distance.

The characteristic value of a psychological object is a function:

\[
(1) \quad x_j: S \rightarrow X_j, s \mapsto x_j(s).
\]

A decision maker is endowed with \( J \) different attitudes \( (a_j, \beta_j)_{j=1, \ldots, J} \), where

\[
(2) \quad a_j: X_j \rightarrow V, x_j \mapsto a_j(x_j)
\]

represents the attitude's valence and \( \beta_j \in [0, 1] \) represents the attitude strength. \( V \subseteq \mathbb{R}^+ \) if the attitude is positive. \( V = \{0\} \) if the attitude is neutral. \( V \subseteq \mathbb{R}^- \) if the attitude is negative.

We normalize the attitude strengths, such that:

\[
(3) \quad \sum_{j=1}^J \beta_j = 1.
\]

The decision problem can be formulated as a problem of maximizing \( j = 1, \ldots, J \) different objective functions \( a_j \left(x_j(s)\right) \) over argument \( s \).

From the theory of cognitive dissonance, we have derived the idea that the solution concept of this decision problem is similar to compromise programming. Dissonance theory states that the solution is found within the following two mental
steps. In the first step the decision maker’s action tendency is determined. In the second step a feasible alternative is determined.

First step: Every single objective function is maximized. This results in a solution set for each attitude:

\[ \hat{S}_j = \left\{ s \in S \mid a_j \left( x_j(s) \right) \text{ is maximal} \right\} \]

We gave this set a psychological interpretation: The decision maker has the same mental motivation to carry out any of the alternatives in the solution set \( \hat{S}_j = \{ s_1, \ldots, s_i, \ldots, s_n \} \). This leads to the action tendency:

\[ \hat{s}_j = \frac{1}{n} \sum_{i=1}^{n} s_i. \]

The \( J \) objective functions lead to the set of action tendencies:

\[ \hat{S} = \{ \hat{s}_1, \hat{s}_2, \ldots, \hat{s}_J \} \]

These different, and thus competing, action tendencies impose cognitive dissonance on the decision maker which we measure with:

\[ D = \sum_{j=1}^{J} \beta_j \cdot \left( \left\| \hat{s}_j - \sigma \right\|_2 \right)^2 \]

Dissonance is minimized by \( \sigma^* \) which is defined as:

\[ \sigma^* = \arg \min_{\sigma \in \Delta} \sum_{j=1}^{J} \beta_j \cdot \left( \left\| \hat{s}_j - \sigma \right\|_2 \right)^2 \]

**Proposition 1:**

\( \sigma^* \) is a convex combination of the action tendencies \( \hat{s}_1, \ldots, \hat{s}_j, \ldots, \hat{s}_J \). More precisely

\[ \sigma^* = \sum_{j=1}^{J} \beta_j \cdot \hat{s}_j. \]

**Proof of Proposition:** see Appendix.
Second step: The dissonance minimizing alternative is determined. The dissonance minimizing alternative $s^*$ is the alternative whose unit vector has minimal distance to the dissonance minimizing action tendency $\sigma^*$:

\[ s^* = \arg\min_{s \in S} \|\sigma^* - s\|_2 \]

Let the set of alternatives be continuous. Then, obviously $s^* = \sigma^*$. Let the set of alternatives be discrete and the k-th component of $\sigma^*$ be the largest component. We distinguish two cases.

Case 1: All other components are smaller than the k-th component.

Case 2: There is at least one other component which has the same magnitude as the k-th component.

In case 1, $s^*$ is the alternative which is represented by the k-th unit vector. Hence, it follows: $s^* = s_k$. In case 2, at least two unit vectors have the same (minimum) distance to $\sigma^*$. Hence, the decision maker is indifferent between these alternatives and favors each of them equally. He is undecided. As long as the decision maker stays in the mental state of undecidedness, he cannot carry out any alternative. The decision maker will engage in psychological effort to get rid of this indifference. This might involve searching for new information or changing the strengths of the less important attitudes. As we have assumed that beliefs and attitudes are given, the mechanisms with regard to attitude change and belief change are beyond the scope of this model.

Of major interest here is the notion that the components of the vector $\sigma^*$, which represents the dissonance minimizing action tendency, yield a preference order over the alternatives. If $\sigma^*_k > \sigma^*_l$ then the decision maker prefers alternative $s_k$ over alternative $s_l$ and if $\sigma^*_k = \sigma^*_l$ then they are indifferent between alternative $s_k$ and $s_l$. The preference rank of each alternative is given by its distance to the dissonance
minimizing action tendency $\sigma^*$. Hence attitudes are the foundation of preference orders.

Up to now, we have not clarified how constraints are incorporated into the psychological model. Clearly a constraint, like a budget constraint, can shift the decision maker’s focus to certain psychological objects and related attitudes. A decision maker’s awareness of a budget constraint might shift his focus to pecuniary objects or norms which are related to austerity. So, in contrast to normative decision models, constraints do not only influence the set of feasible alternatives, but also the set of decision criteria. But this does not change anything of our mathematical representation. Neither does the constraint affect the second step, as this step determines the decision maker’s preference order. Hence, the decision maker will take the alternative which is highest in rank and not excluded by constraints.

The human need for “internal harmony, consistency, or congruity among his opinions, attitudes, knowledge, and values” (Festinger, 1957, 260) makes humans strive for the ideal cognitive compromise. This ideal compromise balances the divergent attitudes. Reality forces people to choose an alternative which differs from this cognitive compromise. The desirability of the alternatives is given by their distance to the cognitive compromise. A preference order is defined. The most preferred alternative is not what the decision maker desires most. It is only the best compromise between wish and reality.

In the next section we apply our model to the ultimatum game in order to explain behavioral discontinuities in mini ultimatum games. A behavioral discontinuity means that small payoff changes can reverse the behavior of decision makers (Güth, Huck and Müller, 1998, 5).
V. Discussion

Our model is an alternative approach to the class of additive utility models which were invented to explain prosocial behavior, such as the model by Fehr and Schmidt (1999).

In contrast to these models which cannot cope with the framing effect, as they are context free (Binmore and Shaked, 2010, 91), our model captures this effect by the vector of attitude strengths. In our model, attitude strengths are influenced by framing. Some attitudes are neutral in one situation but not neutral in another situation.

Our idea of attitudes being triggered and attitude strength being changed by the framing process is in line with the idea of Binmore and Shaked (2010, 88), who state that social norms are triggered by the framing of the laboratory game. Likewise, our idea is consistent with the contingent focus model by Takemura (1994), which states that framing effects emerge, not due to a shift of the reference point, but due to a change of the decision makers’ foci on the decision situation (Takemura, 2014, 118).

Although most of the models of the class of additive utility models are based on experimental evidence on the ultimatum game, without further ado they are not capable of explaining behavioral discontinuities in the mini ultimatum game. In contrast, our model covers this behavioral pattern, which we show in the following. We focus on proposer behavior only.

Our decision model allows for factoring in different attitudes at the same time. Therefore it is important to think which of the vast amount of possible attitudes are most relevant in specified decision situations. The easiest way to find out decision makers' sets of relevant attitudes is asking them directly. Here we point to further
research. Meanwhile let us consider the experimental evidence on fairness behavior.

As most games were played in an anonymous setting, the set of known attitudes is not very large. In the relevant literature we find attitudes towards the material payoff of the decision maker, material payoffs of the other decision makers (Levine, 1997), payoff-inequity (Fehr and Schmidt, 1999; Bolton and Ockenfels, 2000), payoff efficiency and loss aversion (Engel and Zhurakhovska, 2013) and the norm of reciprocity (Rabin, 1993; Falk and Fischbacher, 2000). Other social norms such as “you shall share”, “do not exploit others” or “do not let yourself be exploited” can influence decisions, too.

The most prominent model, the Fehr-Schmidt model, omits social norms and explains fairness behavior only with “inequity aversion.” Inequity aversion predicts behavior in the ultimatum game, but not in the dictator game. (Fehr and Schmidt, 1999, 848). If we look carefully at the Fehr-Schmidt model we can find, that their concept of inequity aversion relies only on two different attitudes: a negative attitude towards advantageous inequity and a negative attitude towards disadvantageous inequity. The latter attitude is similar to envy; the former is similar to compassion.

Another attitude is presented by Kirchsteiger (1994), who argues that the empirical results of behavior in the ultimatum game can be explained by the proposers’ fear that their offer might be rejected by an envious responder. We can translate the proposer’s fear into the negative attitude against one’s offer being refuted with the consequence of receiving nothing in the end.

People can also have positive attitudes towards “advantageous inequity.” This attitude corresponds to spitefulness. Spitefulness has been considered to contribute to fairness behavior for example in the model by Levine (1997). Levine based his
model on a second attitude, namely altruism. The attitude altruism is a positive attitude towards the payoff of someone else.

Another explanation for behavioral patterns in ultimatum and dictator games is the influence of norms of fair sharing. Experiments on the sharing behavior of young children support an explanation by fairness norms rather than by inequity aversion, because young children at the age of 3-6 years are willing to accept advantageous inequity (McAuliffe, Blake, Warneken, 2015, 1), but refuse disadvantageous offers. The aversion to disadvantageous outcomes is observed in decisions of children at the age of 4. (McAuliffe, Blake, Warneken, 2015, 1) Showing concern for disadvantageous inequity can be caused by psychological spitefulness. At the age of 4, children behave spitefully. That is, they are willing to sacrifice a small pay-off to prevent [others] from receiving more” (McAuliffe, Blake and Warneken, 2015, 2). The co-occurrence of accepting advantageous offers and refuting disadvantageous offers cannot be explained by a general desire to reduce inequality (McAuliffe, Blake and Warneken, 2015, 4).

Children at the age of 3 - 6 are aware of the validity of sharing norms (Smith, Blake and Harris 2013, 9). They apply the norm to themselves and to others, but do not follow the norm themselves (Smith, Blake and Harris, 2013, 9). Young children forego equal sharing although they do not expect their peers to engage in unfair resource distribution and do not deny the applicability of the norm of equal sharing. They are aware of their norm-deviant behavior (Smith, Blake, and Harris 2013, 8). The sharing behavior of children at the age of 8 is more in line with the norm of equal sharing (Smith, Blake and Harris 2013, 8). At this age, egalitarian motives in sharing behavior emerge too (McAuliffe, Blake and Warneken, 2015, 4). This indicates that sharing behavior is rather motivated by norms which are fully internalized at the age of about 8 and not by inequity aversion. We think that envy and spitefulness are overlaid by an internalized norm of sharing.
Fairness behavior can be induced by the interplay of all of the attitudes which we have found in the literature so far. It is important to note that our model does not require the exclusion of any attitude on an ad hoc basis. But it seems reasonable to assume that the positive attitude to one’s own payoff, refusal aversion and norm compliance play a more dominant role, whereas spitefulness, envy and altruism play a less important role for the proposer’s decision making. Next we analyze the proposer’s behavior in the ultimatum game.

A. Proposer Behavior in the Ultimatum Game

In the ultimatum game the proposer’s belief as to the responder’s threshold for acceptance $s_{\text{min}}$ of an offer is a crucial factor. If the proposer’s offer $s_P$ exceeds $s_{\text{min}}$, then the responder accepts, otherwise she refutes the offer. In the standard game theory setting, the proposer knows that the responder is a payoff maximizer. The acceptance threshold of a payoff maximizer is the smallest possible share of money. In reality, the proposer is aware that social norms are valid, that attitudes such as envy, spite and altruism guide decisions. Hence the proposer is aware that there is uncertainty concerning the acceptance of his offer. A belief with regard to the acceptance threshold is formed. Here, we formalize such a belief as a probability distribution over the possible shares of money. We do not discuss how such a belief is formed.

Let $s_{\text{min}} \in [0; 1]$ denote the acceptance threshold of the responder, $\bar{F}(s_{\text{min}})$ the cumulative distribution function and $\hat{f}(s_{\text{min}})$ the density function which represent the proposer’s belief on $s_{\text{min}}$. The proposer’s awareness on the validity of a sharing norm and on envy and spitefulness may lead to a belief over the responder’s action tendency which takes into account that higher offers are more often accepted than lower offers, that most people would refute very low offers and would accept offers
above the sharing norm. In an anonymous setting, this may lead to a density function which has a modal value near the sharing norm.

The proposer’s belief with regard to the responder’s action tendency $\tilde{f}(s_{\min})$, respectively $\tilde{F}(s_{\min})$, results in a belief as to the responder’s acceptance level $\tilde{s}_{\min}$. The believed acceptance level is the smallest offer for which the distribution function $\tilde{F}(s_{\min})$ takes on the value 1. Hence: $\tilde{s}_{\min} = \min\{s_{\min} | \tilde{F}(s_{\min}) = 1\}$.

Next, we represent the proposer’s different attitudes as functions which lead to competing action tendencies. We normalize the stake to size 1. Let $s^P \in S^P = [0; 1]$ denote a strategy of the proposer, $s^R \in S^R = \{accept, refuse\}$ a strategy of the responder, and $\tilde{s}^R$ the best response of the responder. The payoff of the proposer is denoted by $\pi^P(s^P, s^R)$. The payoff of the responder is $\pi^R(s^R, s^P) = 1 - \pi^P(s^P, s^R)$ if they accept, and zero if they do not accept. From the viewpoint of the proposer the best response of the responder depends on the belief on the acceptance level, so that:

$$\tilde{s}^R(s^P, \tilde{s}_{\min}) = \begin{cases} 
accept & \text{if } s^P \geq \tilde{s}_{\min} \\
refuse & \text{if } s^P < \tilde{s}_{\min}
\end{cases}$$  

(10)

The belief contingent action tendency $\tilde{s}^P_1$ which is most compatible to the proposer’s positive attitude towards his own payoff is the maximum of the attitude function:

$$a^P_1 = -|\pi^P(s^P, \tilde{s}^R(s^P, \tilde{s}_{\min})) - \tilde{s}_{\min}|$$

(11)

Hence:

$$\tilde{s}^P_1 = \arg\min_{s^P \in S^P} |\pi^P(s^P, \tilde{s}^R(s^P, \tilde{s}_{\min})) - \tilde{s}_{\min}|$$

(12)
It easy to see \( \hat{S}_1^P = \{ \hat{s}_{min} \} \). That is, a decision maker who only seeks to maximize his own payoff offers the acceptance level to the responder: \( \hat{s}_1^P = \hat{s}_{min} \). The action tendencies which are most compatible to the proposer’s negative attitude towards refusal of his offer are determined by the attitude function:

\[
(13) \quad a_2^P = -(F(\hat{s}_{min}) - F(s^P))
\]

As \( F(\hat{s}_{min}) = 1 \) we can write:

\[
(14) \quad a_2^P = -(1 - F(s^P))
\]

The set of attitude maximizing action tendencies is:

\[
(15) \quad \hat{S}_2^P = \arg \min_{s^P \in \hat{S}_P} \{ 1 - F(s^P) \}
\]

\( \hat{S}_2^P \) entails all offers which are equal to or greater than \( \hat{s}_{min} \). Hence \( \hat{S}_j = [\hat{s}_{min}; 1] \), that is, each element in the interval is an attitude maximizing alternative. According to our model, in the case in which an attitude leads to more than one attitude maximizing alternative, the resulting action tendency is the following convex combination of the \( n \) different attitude maximizing alternatives: \( \hat{s}_j = \frac{1}{n} \sum_{i=1}^n s_i \). Therefore, in the case at hand it holds that:

\[
(16) \quad \hat{s}_2^P = \frac{1}{1 - \hat{s}_{min}} \int_{\hat{s}_{min}}^{1} s_i \, ds_i
\]

Hence:

\[
(17) \quad \hat{s}_2^P = 0.5 \cdot (1 + \hat{s}_{min})
\]

The strength of the negative attitude towards refusal is denoted with \( \beta_1 \). We have already stated, that attitudes towards norms play a crucial role in human decision
making. In the ultimatum game, the norm of equal sharing is dominant, especially if the game is not played anonymously. A positive attitude towards the norm of equal sharing is represented by the (negative) attitude towards one’s own norm deviance behavior and hence by

\begin{equation}
a_3^P = |0.5 - s^P|.
\end{equation}

We denote by \( \hat{s}_3^P \) the proposer’s action tendency which is most norm compliant. As we have normalized the game stake to size 1, \( \hat{s}_3^P \) is determined by:

\begin{equation}
S_3^P = \arg \min_{s^P \in S^P} |0.5 - s^P|
\end{equation}

\( \hat{s}_3^P = \{0.5\} \). Hence: \( \hat{s}_3^P = 0.5 \). That is, a decision maker who only seeks for norm compliance only offers half of the stakes to the responder. The attitude strength of the attitude toward the norm of equal sharing is denoted by \( \beta_3 \). Spitefulness, the positive attitude towards advantageous inequity, is an attitude which seeks to avoid equal outcomes and outcomes in which the responder earns more. Spitefulness is represented by the constraint

\begin{equation}
\pi^P(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) \geq \pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}}))
\end{equation}

and the function

\begin{equation}
a_4^P = \pi^P(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) - \pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})).
\end{equation}

Its attitude strength is \( \beta_4 \). This leads to the following set of attitude maximizing action tendencies:

\begin{equation}
\hat{s}_4^P = \arg \max_{s^P \in S^P} \pi^P(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) - \pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}}))
\end{equation}

s. t.: \( \pi^P(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) \geq \pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) \forall s^P \in \hat{s}_4^P \)
The constraint of the attitude function induces the proposer to offer only shares less or equal to 0.5. Hence, the proposer offers 0.5 if $\bar{s}_{\text{min}} > 0.5$ and offers $\bar{s}_{\text{min}}$ if $\bar{s}_{\text{min}} \leq 0.5$. Envy, the negative attitude towards disadvantageous inequity, which has a strength of $\beta_5$, plays a role only if $\pi^R \geq \pi^P$. It is represented by:

$$a_5^P = -\left( \pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) - \pi^P(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) \right).$$

(23)

Envy guides the decision only, if the proposer believes that it leads to the following action tendency:

$$\hat{s}_5^P = \arg \min_{s^P \in S^P} \pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) - \pi^P(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})).$$

(24)

Let us assume the proposer thinks that the acceptance level of the responder exceeds 0.5, so that: $\bar{s}_{\text{min}} > 0.5$. From the proposer’s viewpoint the responder earns more than 0.5 if their offer is in accordance to $\bar{s}_{\text{min}}$. But then:

$$\pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) - \pi^P(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) > 0$$

(25)

But if they offer less than $\bar{s}_{\text{min}}$ to the responder, then the responder rejects the offer and both receive nothing, so that:

$$\pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) - \pi^P(s^P, \bar{s}^R(s^P, \bar{s}_{\text{min}})) = 0$$

(26)

Hence, for $\bar{s}_{\text{min}} > 0.5$, any offer $\hat{s}_5^P \leq 0.5$ results in an envy-free outcome. If $\bar{s}_{\text{min}} = 0.5$, offering 0.5 leads to an outcome which is envy-free, whereas both receive 0.5. But offering less also leads to an envy-free outcome, as both receive nothing. As the action tendency is a convex combination of the different attitude maximizing alternatives ($\hat{s}_j = \frac{1}{n} \sum_{i=1}^{n} s_i$), this implies that the action tendency is the convex combination of all numbers in the interval $[0; \bar{s}_{\text{min}}]$ if $\bar{s}_{\text{min}} = 0.5$ and $[0; \bar{s}_{\text{min}}]$ if $\bar{s}_{\text{min}} > 0.5$. Hence:
(27) \[ \hat{s}^P_5 = \frac{1}{s_{min}} \int_{0}^{s_{min}} s_i \, ds_i \] respectively \( \hat{s}^P_5 = 0.5 \cdot \bar{s}_{min} \)

If \( \bar{s}_{min} < 0.5 \), envy does not play any role, so that \( \beta_5 = 0 \). Altruism, the positive attitude towards the responder’s material payoff, is represented by:

(28) \[ a^P_6 = \pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{min})) \]

and leads to the following set of attitude maximizing action tendencies:

(20) \[ \hat{s}^P_6 = \arg \max_{s^P \in S^P} \pi^R(s^P, \bar{s}^R(s^P, \bar{s}_{min})) \]

Hence: \( \hat{s}^P_6 = \{1\} \). Altruism pushes the proposer to the maximal offer \( s_{max} \), which is 1, so that \( \hat{s}^P_6 = 1 \). Altruism has a strength of \( \beta_6 \).

The dissonance minimizing alternative \( s^* \), which has been defined as the alternative whose unit vector has minimal distance to the dissonance minimizing action tendency \( \sigma^* \), depends on \( \bar{s}_{min} \). In the ultimatum game, the responder’s alternative space is continuous, so that \( s^* = \sigma^* \). Hence, \( s^* = \sum_{j=1}^{J} \beta_j \hat{s}_j \). If \( \bar{s}_{min} > 0.5 \), then: \( s^* = \sigma^* = \bar{s}_{min} \cdot \beta_1 + (0.5 + 0.5 \cdot \bar{s}_{min}) \cdot \beta_2 + 0.5 \cdot \beta_3 + 0.5 \cdot \beta_4 + 0.5 \cdot \bar{s}_{min} \cdot \beta_5 + s_{max} \cdot \beta_6 \). If \( \bar{s}_{min} \leq 0.5 \), then: \( s^* = \sigma^* = \bar{s}_{min} \cdot \beta_1 + (0.5 + 0.5 \cdot \bar{s}_{min}) \cdot \beta_2 + 0.5 \cdot \beta_3 + \bar{s}_{min} \cdot \beta_4 + 0.5 \cdot \bar{s}_{min} \cdot \beta_5 + s_{max} \cdot \beta_6 \).

Now, we apply this outcome to the mini ultimatum game in order to explain the behavioral discontinuity of proposer behavior.
B. Explaining Behavioral Discontinuity of Proposer Behavior in the Mini Ultimatum Game

We refer to three mini ultimatum game versions which Güth, Huck and Müller (1998) have used to show that proposer behavior changes, if the equal split is not available. The three games are shown in figure 2:

In the game “Equal,” the equal split is feasible whereas in the games “Prop” and “Resp” only nearly equal splits are feasible. It turned out that fair offers (choosing strategy \( r \)) occurred less often when the equal split was not available. The fraction of proposers who chose the fair offer (strategy \( r \)) was largest in game “Equal”, second largest in game “Prop” and smallest in game “Resp.” That is, a small payoff change turned out to be capable of reversing the behavior of participants (Güth, Huck and Müller, 1998, 5). We can explain this behavioral pattern with our model.

First, in Game “Prop” and game “Equal” envy does not play any role at all, because the responder cannot earn a higher outcome than the responder. Hence:

\[
\hat{\beta}_5 \mathcal{R}_r > \hat{\beta}_5 \mathcal{E}_e = \hat{\beta}_5 \mathcal{P}_p = 0.
\]

Second, in game “Prop” and in game “Resp,” the equal split is not feasible and thus the fulfillment of the norm of equal sharing is not possible. This means that
the proposer is forced to break the norm. Here, the theory of cognitive dissonance states that striving for consistency between behavior and attitudes induces a change of the attitude which is related to the broken norm. On the other hand the game “Equal” puts a decision maker’s focus directly on the norm of equal sharing, as there is only one other alternative available. Therefore, the positive attitude towards the norm of fair sharing plays a more important role in the Game “Equal.” Hence:

(31) \[ \beta_3^{Eq} > \beta_3^{Pr} = \beta_3^{Re} . \]

We denote the unfair strategy \( m \) with \( \begin{pmatrix} 1 \\ 0 \end{pmatrix} \) and the fair strategy \( r \) with \( \begin{pmatrix} 0 \\ 1 \end{pmatrix} \).

First we consider behavioral discontinuities in games “Equal” and “Prop.” \( \epsilon \in ]0,1[ \) denotes the change of attitude strength, so that: \( \epsilon = \beta_3^{Eq} - \beta_3^{Pr} \). As for all \( i = 1,2,4,6 \) it holds that \( \beta_i^{Eq} \leq \beta_i^{Pr} \), we can write: \( \beta_i^{Eq} + \delta_i = \beta_i^{Pr} \), where \( \delta_1 + \delta_2 + \delta_4 + \delta_6 = \epsilon \) and \( \delta_i \in [0, \epsilon] \). In the games “Prop” and “Equal” the feasible offers, are \( \frac{3}{20} \) and \( \frac{9}{20} \) respectively \( \frac{3}{20} \) and \( \frac{10}{20} \). This implies \( s_{min} \leq 0.5 \).

**Proposition 2:**

If the proposer believes, that the responder accepts both, \( r \) and \( l \), and if for any \( \delta_i \in [0, \epsilon] \) and \( \epsilon \in ]0,1[ \), it holds \( \epsilon \neq \delta_6 \) and \( 0.5 - 0.5\delta_2 - \delta_6 + \epsilon > 0.5\beta_2^{Eq} + \beta_3^{Eq} + \beta_6^{Eq} > 0.5 \), then the proposer behaves discontinuous in games “Equal” and “Prop”.

**Proof of Proposition 2** see Appendix.

Proposition 2 states that discontinuity occurs either if the decision maker’s focus is shifted to an attitude which is quite strong or if the decision maker’s tendency towards fair behavior (strategy \( r \)) in the game “Equal” is not very strong.
Now, we consider the case in which the proposer believes that the responder accepts only the fair offer \( r \). Here, we need to take into account that \( \hat{S}_4^{eq} = \{l, r\} \), because if the unfair offer is rejected, this leads to the same outcome for both, namely 0. If the fair offer is accepted, this leads to the same outcome for both, namely 10. Therefore, both offers are equal in regard to spitefulness.

Proposition 3:

If the proposer believes that the responder accepts only the fair offer \( r \), no behavioral discontinuity occurs. The proposer chooses the fair offer in both the games “Equal” and “Prop.”

Proof of Proposition 3 see Appendix.

The intuition for this is as follows: The decision maker chooses the fair offer in the game “Equal.” In the game “Prop,” the unfair offer is rejected and leads to the same outcome for both. But the fair offer is accepted and leads to a larger outcome for the proposer. Hence, the fair outcome is compatible to spite.

Now, we consider behavioral discontinuities in games “Prop” and game “Resp.” We have already stated that envy plays a role in the game “Resp,” but there is also a difference concerning spite.

Let \( \varepsilon \in ]0; 1[ \) and \( \varepsilon = \beta_5^{Re} > \beta_5^{Pr} = 0 \). For all \( i = 1, 2, 3, 4, 6 \) it holds that \( \beta_i^{Re} \leq \beta_i^{Pr} \). We can write: \( \beta_i^{Re} = \beta_i^{Pr} - \delta_i \) for \( i = 1, 2, 3, 4, 6 \), where \( \delta_1 + \delta_2 + \delta_3 + \delta_4 + \delta_6 = \varepsilon \) and \( \delta_i \in [0, \varepsilon] \) for all \( i = 1, 2, 3, 4, 6 \).

Proposition 4:

If the proposer believes that the responder accepts both the fair offer \( r \) and \( l \), and if for any \( \delta_1 \in [0, \varepsilon] \) and \( \varepsilon \in ]0, 1[ \), it holds \( \varepsilon > \delta_1 + 0.5\delta_2 + \delta_4 \) and \( 0.5 > \beta_1^{Pr} + \)
\[ 0.5 \cdot \beta_{2}^{Pr} + \beta_{4}^{Pr} > 0.5 + \delta_{1} + 0.5\delta_{2} + \delta_{4} - \varepsilon \], then the proposer behaves discontinuously in games “Prop” and “Resp.”

Proof of proposition 4, see Appendix.

Discontinuity between “Prop” and “Resp” occurs only if envy plays a role in the game “Resp.” Decision makers who do not have a negative attitude towards disadvantageous inequity do not change behavior across games.

Proposition 5:

If the proposer believes, that the responder accepts only the fair offer \( a \), then the proposer chooses the fair offer in the game “prop” and the unfair offer in the game “Resp”, if:

\[ \beta_{4}^{Re} + \beta_{5}^{Re} > 0.5 \iff \beta_{4}^{Pr} - \delta_{4} + \beta_{5}^{Re} > 0.5. \]

Proof of proposition 5, see Appendix.

Here, a proposer, whose attitude “spite” is very strong, chooses the fair offer in the game “Prop” because, from his point of view, the fair offer ensures that they will receive more than the responder and he believes that the unfair offer is being rejected. Spitefulness is not in conflict with fairness, because envy does not play any role in the game “Prop.” But in the game “Resp,” both spitefulness and envy influence the action tendency and thus behavior. Someone who is sufficiently spiteful and envious will chose the fair offer in the game “Prop” and the unfair offer in the game “Resp.”

In this section, we have shown how our model can be used to explain how small payoff changes reverse fairness behavior of a decision maker. In the next section we draw some pertinent conclusions.
VI. Conclusions

We have developed a decision model in which a decision maker’s driving force is minimizing the amount of experienced cognitive dissonance. We have based this model on Leon Festinger’s theory of cognitive dissonance. The model is built on three core assumptions. First, attitudes lead to action tendencies. Second, competing attitudes can lead to competing action tendencies. Third, individuals seek to minimize cognitive dissonance by choosing the alternative which is closest to the dissonance minimizing action tendency. The dissonance minimizing action tendency is a cognitive compromise between the competing action tendencies, and this provides a useful reference point. By using the dissonance minimizing action tendency as reference point, a preference order over the alternatives can be determined. The shorter the distance of an alternative to this reference point, the higher this alternative is in rank. Hence, attitudes are the basis of preference orders. By showing this, the model states that in most cases people do not necessarily choose what they wish to choose. A pure payoff-maximizer who chooses an alternative in a continuous alternative space likes what he prefers, but a real person who bases the decisions to be taken on more than a single attitude is forced to make a compromise, especially if the alternative space is discrete. It can happen that people do not like what they prefer, because the best compromise is still a compromise. This is why people engage in the reduction of cognitive dissonance after they have carried out their choice.

The model at hand takes into consideration that changes in the decision context can affect the set of triggered attitudes and the attitude strengths and thus the ideal cognitive compromise $\sigma^*$. As we have seen in the discussion of the three mini ultimatum games, the context may shift the ideal cognitive compromise and hence may change the behavior $s^*$, but need not do so. People who have a very strong tendency towards a particular behavior are not affected by small shifts of $\sigma^*$. 

A crucial factor in our model is the measurement of attitude strength. It is measurable on a magnitude scale (Kahneman et al, 1999, 222; Lodge, 1981, 42f). This scale is a ratio scale (Montello 1991, 104). A magnitude scale is characterized by a meaningful zero, and is able to express the ratios of the variable which are measured. It is not interpersonally comparable as individuals differ in the assignment of numbers to stimuli. That is, some people generally assign low numbers, whereas others assign large numbers. (Kahneman et al, 1999, 222). An example for an attitude scale is the dollar scale. People assign sums of dollars to an attitude (Kahneman et al., 1999, 207). For a discussion of this scaling method, see for example Lodge (1981).

Our model is an alternative approach to additive utility models of the Fehr-Schmidt type. These models represent a preference order over multi-attributive alternatives by an overall utility function which is the weighted sum of sub-utility functions. Among others, the models differ with regard to the required scales. The additive utility model requires that the sub-utility functions are cardinally scaled, because otherwise, addition is not possible. Cardinal utility requires a bunch of assumptions on the choice behavior of the decision maker which are formulated as axioms of rationality. Our model does not use utility functions to represent preference order, and does not rely on rationality assumptions which contradict behaviors of real decision makers.

By assuming that people are seeking an inner harmony, and therefore minimize cognitive dissonance, we have defined an idea of man that is rooted in psychology. This model can be used to explain why people act irrationally and inconsistently, ignore maximization principles and do not always show self-interest. Our model can be extended to a model of preference change by incorporating the process of post decisional dissonance reduction. This would be a profitable area for further research.
VII. Appendix

Proof of Proposition 1:

\[ Z(\sigma) = \sum_{j=1}^{J} \beta_j \cdot \left( \| \mathbf{s}_j - \sigma \|_2 \right)^2 ; \mathbf{s}_j = (s_{j_1}, \ldots, s_{j_i}, \ldots, s_{j_m})^T \quad \text{and} \]

\[ \sigma = (\sigma_1, \ldots, \sigma_i, \ldots, \sigma_m)^T \]

Hence:

\[ Z(\sigma) = \sum_{j=1}^{J} \beta_j \cdot \left( \| \mathbf{s}_j - \sigma \|_2 \right)^2 \iff Z(\sigma) = \sum_{j=1}^{J} \beta_j \cdot \left( \sum_{i=1}^{m} (\mathbf{s}_{ji} - \sigma_i)^2 \right) \]

\[ \frac{\partial Z}{\partial \sigma_i} = 0 \iff \sum_{j=1}^{J} \beta_j \cdot (-2) \cdot (\mathbf{s}_{ji} - \sigma_i^*) = 0 \iff \sigma_i^* = \frac{\sum_{j=1}^{J} \beta_j \cdot \mathbf{s}_{ji}}{\sum_{j=1}^{J} \beta_j} \]

\[ \iff \sigma_i^* = \sum_{j=1}^{J} \frac{\beta_j}{\sum_{k=1}^{J} \beta_k} \cdot \mathbf{s}_{ji} \]

With \( \sum_{k=1}^{J} \beta_k = 1 \) it follows:

\[ \sigma_i^* = \sum_{j=1}^{J} \beta_j \cdot \mathbf{s}_{ji} \]

From \( \sigma^* = (\sigma_1^*, \ldots, \sigma_i^*, \ldots, \sigma_m^*)^T \) and \( \sigma_i^* = \sum_{j=1}^{J} \beta_j \cdot \mathbf{s}_{ji} \) for \( i = 1, \ldots, n \) it follows:

\[ \sigma^* = \left( \sum_{j=1}^{J} \beta_j \cdot s_{j_1}, \ldots, \sum_{j=1}^{J} \beta_j \cdot s_{j_i}, \ldots, \sum_{j=1}^{J} \beta_j \cdot s_{jm} \right)^T \]
\[ \Leftrightarrow \sigma^* = \beta_1 \cdot (\hat{s}_{11} \quad \vdots \quad \hat{s}_{1m}) + \cdots + \beta_j \cdot (\hat{s}_{j1} \quad \vdots \quad \hat{s}_{jm}) \]
\[ \Leftrightarrow \sigma^* = \sum_{j=1}^J \beta_j \cdot \hat{s}_j. \]

**Proof of Proposition 2:**

The Action tendency in the game “Equal” is:

\[ \sigma_{Eq}^* = \beta_{1}^{Eq} (1 \quad 0) + \beta_{2}^{Eq} \left( 0.5 \cdot (1 \quad 0) + 0.5 \cdot (0 \quad 1) \right) + \beta_{3}^{Eq} (0 \quad 1) + \beta_{4}^{Eq} (1 \quad 0) + \beta_{6}^{Eq} (0 \quad 1) \]

Hence:

\[ \sigma_{Eq}^* = \left( \beta_{1}^{Eq} + 0.5 \cdot \beta_{2}^{Eq} + \beta_{4}^{Eq} \right) \left( 0.5 \cdot \beta_{2}^{Eq} + \beta_{3}^{Eq} + \beta_{6}^{Eq} \right). \]

The Action tendency in the game “Prop” is:

\[ \sigma_{Pr}^* = \beta_{1}^{Pr} (1 \quad 0) + \beta_{2}^{Pr} \left( 0.5 \cdot (1 \quad 0) + 0.5 \cdot (0 \quad 1) \right) + \beta_{3}^{Pr} (0 \quad 1) + \beta_{4}^{Pr} (1 \quad 0) + \beta_{6}^{Pr} (0 \quad 1) \]

Hence:

\[ \sigma_{Pr}^* = \left( \beta_{1}^{Pr} + 0.5 \cdot \beta_{2}^{Pr} + \beta_{4}^{Pr} \right) \left( 0.5 \cdot \beta_{2}^{Pr} + \beta_{3}^{Pr} + \beta_{6}^{Pr} \right). \]

If \( 0.5 \cdot \beta_{2}^{Eq} + \beta_{3}^{Eq} + \beta_{6}^{Eq} > 0.5 \) the proposer chooses the fair strategy in the game “Equal”, that is \( s_{Eq}^* = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \). If \( 0.5 \cdot \beta_{2}^{Pr} + \beta_{3}^{Pr} + \beta_{6}^{Pr} > 0.5 \) he chooses the fair strategy in the game “Prop”. If \( 0.5 \cdot \beta_{2}^{Pr} + \beta_{3}^{Pr} + \beta_{6}^{Pr} < 0.5 \) the proposer choses the unfair strategy in the game “Prop”, \( s_{Pr}^* = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \). Hence the proposer
behaves discontinuous, if the following necessary and sufficient conditions are fulfilled.

Necessary condition:

(A1) \[ 0.5 \cdot \beta_2^{Ed} + \beta_3^{Ed} + \beta_6^{Eq} > 0.5 \cdot \beta_2^{Pr} + \beta_3^{Pr} + \beta_6^{Pr} \]

We insert \( \varepsilon = \beta_3^{Eq} - \beta_3^{Pr} \) and \( \beta_i^{Eq} + \delta_i = \beta_i^{Pr} \) for \( i = 1, 2, 4, 6 \) and receive:

(A2) \[ \varepsilon > 0.5 \cdot \delta_2 + \delta_6 \]

Except for \( \delta_6 = \varepsilon \) this inequality can hold.

Sufficient Condition:

(A3) \[ 0.5 \beta_2^{Eq} + \beta_3^{Eq} + \beta_6^{Eq} > 0.5 \]

(A4) \[ 0.5 \cdot \beta_2^{Pr} + \beta_3^{Pr} + \beta_6^{Pr} < 0.5 \]

Insert \( \beta_3^{Pr} = \beta_3^{Eq} - \varepsilon, \beta_2^{Eq} + \delta_2 = \beta_2^{Pr} \), and \( \beta_6^{Eq} + \delta_6 = \beta_6^{Pr} \) into (A4):

(A5) \[ 0.5 \beta_2^{Eq} + \beta_3^{Eq} + \beta_6^{Eq} < 0.5 - 0.5 \delta_2 - \delta_6 + \varepsilon \]

Note that: \(-0.5 \delta_2 - \delta_6 + \varepsilon > 0\). From (A3) and (A5) it follows, that discontinuous behavior occurs, if:

(A6) \[ 0.5 - 0.5 \delta_2 - \delta_6 + \varepsilon > 0.5 \beta_2^{Eq} + \beta_3^{Eq} + \beta_6^{Eq} > 0.5 \]

\[ \Box \]

Proof of Proposition 3:

The Action tendency in the game “Prop” is: \( \sigma_r^* = \beta_1^{Pr} (\frac{0}{1}) + \beta_2^{Pr} (\frac{0}{1}) + \beta_3^{Pr} (\frac{0}{1}) + \beta_4^{Pr} (\frac{0}{1}) + \beta_6^{Pr} (\frac{0}{1}) = (\frac{0}{1}) \). The Action tendency in the game “Equal” is:
\[
\sigma_{Eq} = \beta_1^{Eq}(\frac{0}{1}) + \beta_2^{Eq}(\frac{0}{1}) + \beta_3^{Eq}(\frac{0}{1}) + \beta_4^{Eq}(\frac{1}{0}) + 0.5(\frac{0}{1}) + \beta_6^{Eq}(\frac{0}{1})
\]

Hence: \(\sigma_{Eq}^* = \left(\begin{array}{c}
0.5 \cdot \beta_4^{Eq} \\
\beta_1^{Eq} + \beta_2^{Eq} + \beta_3^{Eq} + 0.5 \cdot \beta_4^{Eq} + \beta_6^{Eq}
\end{array}\right)\). If \(\beta_4^{Eq} < 1\), \(s_{Eq} = (\frac{0}{1})\).

The decision maker chooses the fair offer in the game “Equal”.

**Proof of Proposition 4:**

The Action tendency in the game “Resp” is: \(\sigma_{Re}^* = \beta_1^{Re}(\frac{1}{0}) + \beta_2^{Re}(\frac{1}{0}) + 0.5(\frac{0}{1}) + \beta_3^{Re}(\frac{1}{0}) + \beta_4^{Re}(\frac{1}{0}) + \beta_5^{Re}(\frac{1}{0}) + \beta_6^{Re}(\frac{0}{1})\). Hence:

\[
\sigma_{Re}^* = \left(\begin{array}{c}
\beta_1^{Re} + 0.5 \cdot \beta_2^{Re} + \beta_4^{Re} + \beta_5^{Re} \\
0.5 \cdot \beta_2^{Re} + \beta_5^{Re} + \beta_6^{Re}
\end{array}\right).
\]

The Action tendency in the game “Prop” is:

\[
\sigma_{Pr}^* = \beta_1^{Pr}(\frac{1}{0}) + \beta_2^{Pr}(\frac{1}{0}) + 0.5(\frac{0}{1}) + \beta_3^{Pr}(\frac{1}{0}) + \beta_4^{Pr}(\frac{1}{0}) + \beta_5^{Pr}(\frac{0}{1}) + \beta_6^{Pr}(\frac{0}{1})
\]

Hence: \(\sigma_{Pr}^* = \left(\begin{array}{c}
\beta_1^{Pr} + 0.5 \cdot \beta_2^{Pr} + \beta_4^{Pr} \\
0.5 \cdot \beta_2^{Pr} + \beta_3^{Pr} + \beta_6^{Pr}
\end{array}\right)\).

Necessary Condition:

(A7) \(\beta_1^{Re} + 0.5 \cdot \beta_2^{Re} + \beta_4^{Re} + \beta_5^{Re} > \beta_1^{Pr} + 0.5 \cdot \beta_2^{Pr} + \beta_4^{Pr}\)

Inserting \(\beta_i^{Re} = \beta_i^{Pr} - \delta_i\) for \(i = 1, 2, 4\) and \(\epsilon = \beta_5^{Re}\) leads to:

(A8) \(\epsilon > \delta_1 + 0.5 \delta_2 + \delta_4\)

This condition is fulfilled.
Sufficient Condition:

\[(A9) \quad \beta_1^{Re} + 0.5 \cdot \beta_2^{Re} + \beta_4^{Re} + \beta_5^{Re} > 0.5 \text{ (I)}\]

and

\[(A10) \quad \beta_1^{Pr} + 0.5 \cdot \beta_2^{Pr} + \beta_4^{Pr} < 0.5 \text{ (II)}\]

Inser: \(\beta_i^{Re} = \beta_i^{Pr} - \delta_i\) for \(i = 1, 2, 4\) and \(\epsilon = \beta_5^{Re}\) into (I):

\[(A11) \quad \beta_1^{Pr} + 0.5 \cdot \beta_2^{Pr} + \beta_4^{Pr} > 0.5 + \delta_1 + 0.5\delta_2 + \delta_4 - \epsilon\]

The right side of (A11) is strictly less than 0.5 if the necessary condition is fulfilled. It follows:

\[(A12) \quad 0.5 > \beta_1^{Pr} + 0.5 \cdot \beta_2^{Pr} + \beta_4^{Pr} > 0.5 + \delta_1 + 0.5\delta_2 + \delta_4 - \epsilon\]

**Proof of Proposition 5:**

The Action tendency in the game “Resp” is:

\[\sigma_{Re}^* = \beta_1^{Re} \binom{0}{1} + \beta_2^{Re} \binom{0}{1} + \beta_3^{Re} \binom{1}{0} + \beta_4^{Re} \binom{1}{0} + \beta_5^{Re} \binom{0}{1} + \beta_6^{Re} \binom{0}{1}\]

Hence:

\[\sigma_{Re}^* = \left(\begin{array}{c} \beta_4^{Re} + \beta_5^{Re} \\ \beta_1^{Re} + \beta_2^{Re} + \beta_3^{Re} + \beta_6^{Re} \end{array}\right)\]

The action tendency in the game “Prop” is:

\[\sigma_{Pr}^* = \binom{0}{1}\]

The decision maker choses the fair offer in the game “prop” and the unfair offer in the game “Resp”, if:

\[\beta_4^{Re} + \beta_5^{Re} > 0.5 \iff \beta_4^{Pr} - \delta_4 + \beta_5^{Re} > 0.5\]
VIII. REFERENCES


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