Fair Threats and Promises

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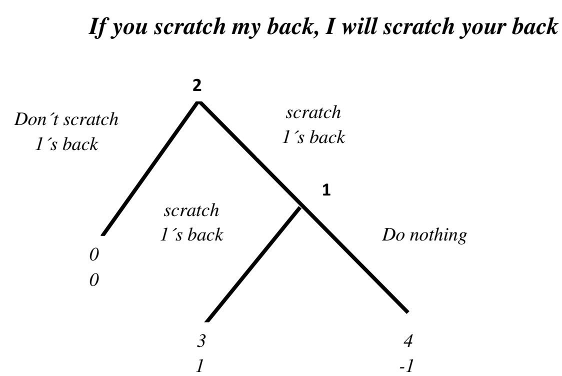
1 Introduction

The role of threats and promises is fundamental in Game Theory, as the predictions of the outcomes of many dynamic games depend on their credibility. For example, in Figure 1 from Klein, player 1 can promise to scratch 2´s back if player 2 scratched 1´s back before. The result of the game depends on the credibility of the promise of player 1 to scratch player 2´s back if she scratch player 1´s back.



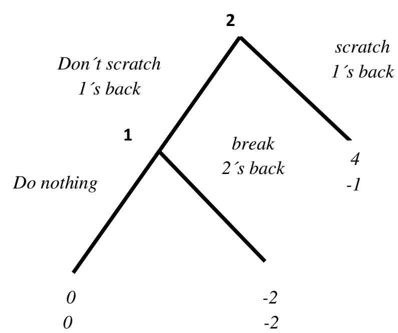
Game 1

And in Figure 2, from Klein, player 1 can threat to break player 2´s back if he does not scratch player 1´s back. The result of the game depends on the credibility of the threat of player 1 to break player 2´s back if he doesn´t scratch his back.

Game 2

As Klein and O'Flaherty (1993) and Schelling (1960) show, a threat or a promise has to have a commitment in order to be credible. They argue that this commitment can be a psychological commitment to keep our own word. However, the emotion of reciprocity can be a powerful commitment too. For

Scratch my back or else I will break your back



example, in Game 1, the emotion of positive reciprocity can make credible the promise of player 1 to scratch 2's back if player 2 scratch player 1's back and in Game 2, the emotion of negative reciprocity can make credible the threat of player 1 to break player 2's back if player 2 don't scratch player 1's back.

In this paper I argue that the concept of sequential reciprocity of Dufwenberg and Kirchsteiger (2004) doesn't take correctly into consideration the role of threats and promises and I develop a solution concept of reciprocity which I call Fair Threats Equilibrium, that I believe is better at evaluating the fairness of threats and promises.

The literature on reciprocity on Game Theory begins with Rabin (1993). In his seminal paper, Rabin introduced his solution concept Fairness Equilibrium (FE from now on), which is defined for static games. Dufwenberg and Kirchsteiger (2004) extended Rabin´s model to dynamic games with their solution concept Sequential Reciprocity Equilibrium (SRE from now on).

The main difference between a static and a dynamic game is that some strategies (some of which can be interpreted as promises and threats) that are optimal in a static game, are no longer optimal (credible) in a dynamic game, where a player is able to reconsider its play as the game advances. As Subgame Perfect Nash Equilibrium eliminates non-credible threats for standard dynamic games, SRE eliminates non-credible threats for dynamic games with reciprocity emotions.

The SRE defines the kindness of an strategy as a function of the strategies of the other players of the game and requires that each player plays the optimal action at each history of the game.

However, in equilibrium, the SRE evaluates the kindness of each strategy by evaluating the actions it prescribes only at the equilibrium path, without taking in consideration the actions that are prescribed off the equilibrium path. Since promises and threats include the actions in both at the equilibrium path and off the equilibrium path I argue that to evaluate correctly the role of promises and threats, the strategies have to be evaluated as a whole.

We believe that most individuals evaluate the kindness of a strategy independently of the strategies of other players. For example in the Ultimatum Game, an offer of 100% of the money should always be evaluated as kind, independently of the receiver´s strategy. The SRE however, would evaluate an offer of 100% of the money as kind, unkind or neutral, depending of the receiver´s strategy[1]. We believe this does not make sense.

We also believe that the Dictator should think as unkind threats from the receiver if it is trying to force her to give a higher offer than what she considers a fair offer, while threats to reject low offers would be considered as kind, if they are only trying to ensure the Dictator gives a fair offer (I consider a fair offer 50% of the money). The SRE however, evaluates as kind any threat that fulfills its purpose and forces a high offer from the dictator, given that the only action to be evaluated is the acceptance of the equilibrium offer.

In this paper I develop a concept of reciprocity for dynamic games that evaluates the kindness of complete strategies, by defining the kindness of a strategy not as a function of the strategies of other players, but as a function of the maximum payoff the opposite players can receive (if he plays the strategy that maximizes his "material" payoffs). This approach has three advantages. The first is that it is a concept of sequential reciprocity that better takes in consideration the role of promises and threats. The second is that it is a simpler concept of reciprocity than FE and SRE, one that doesn't evaluate the fairness of a strategy as a function of the other players' strategies. The third is that it is closer to the concept of reciprocity that most individuals use in reality and its predictions are closer to the experimental results.

[1] An offer of 100% of the money would be considered as unkind in the case the receiver´s strategy is to reject an offer of 100% of the money, and accept any offer lower than 100% of the money. In this case an offer of 100% would give the receiver a payoff of 0, which compares negatively with a payoff of 99% of the money if the dictator had offered this amount. We believe this does not make sense given that it is the same receiver that is rejecting the offer.

Survey

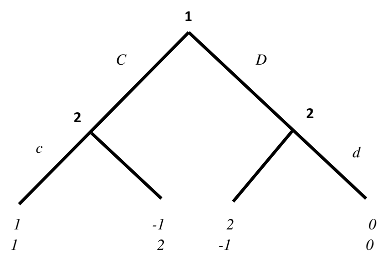
In a small survey conducted at the University of Guanajuato, I asked students a few questions about the Ultimatum game. In the Ultimatum game there are two players. At the beginning of the game, the first player (Proposer) choses the division of an amount of money between himself and the another player. The second player (Responder) chooses to accept the division or not. If he accepts both players are paid the division chosen by the Proposer. If the Recipient rejects the offer, both receive zero.

I asked students what would be a fair division for part of the Proposer. I did not state what was the strategy of the Responder. 14 of 17 students answered that the 50-50 division was fair. One student answered a division of 80-20 in favor of the Proposer. Only one student stated the fair offer was the highest the Responder whould accept, as long as it was less than 50%. One student answer that he did not believed there was a fair division. These answers are consistent with the concept of fairness of a strategy as independent of the strategy of the receptor.

For the Responder, I asked students for the strategies of the form accept any offer that higher or equal than a constant k and reject any offer lower than k. I asked what would be the fair value of k. 9 of 17 students gave a value of k of 50%. 4 gave a value equal or higher than 50% and one gave the highest possible value of k. This suggests that individuals evalute the strategies of the Responder independly of the strategies of the Proposer and that they evaluate the fairness of threats, as these strategies can be interpreted as threats.

I asked a couple of questions about the credibility of two threats. In the first one the Responder threatens the Proposer to reject any offer lower than 80% of the money and the second one it threatens to reject any offer lower than 20% of the money. While 15 of 17 students found incredible the threat of rejecting any offer lower than 80% of the money, 12 found credible the threat when the threshold was 20% of the money. This suggest that what gives credibility to a threat is the fairness of it.

Sequential Prisoners' Dilemma



The Model

In the sequential Prisoners Dilemma in Figure 3, Dufwenberg and Kirchsteiger (2004) argue that the emotion of reciprocity cannot make plausible unconditional cooperation by player 2, as player 2's strategy includes the promise to play cooperate even as player 1 plays defect.

Figure 3

To eliminate this incredible promise, Dufwenberg and Kirchsteiger (2004) develop their solution concept SRE. The SRE requires that each player optimizes at every history. In the sequential Prisoners Dilemma above, the SRE eliminates the incredible promise of unconditional cooperation by requiring that when player 1 defects, player 2 optimizes by also defecting.

However, the SRE has the disadvantage of evaluating the kindness of a player only by her actions at each history. For example, the SRE evaluates the strategy cd as kind if C is played or unkind if D is played. Which action is actually played depends on the strategy of player 1. I argue that for taking in consideration correctly threats and promises, we should evaluate the kindness of a strategy as a whole and independently of other players' strategies. In figure 3, I believe that player 1 should evaluate as kinder the strategy cc than cd, that is unconditional cooperation should be evaluated as kinder than conditional cooperation, independently of the strategy of player 1. In order to accomplish this I define the kindness of a strategy as a function of the maximum payoff that it can give to the opposite players.

Let me give an example of an unreasonable SRE solution that is sustained by unreasonable beliefs. Assume a very expensive Dictator game where the Dictator divides one million dollars. For players that care much about fairness considerations, there is a SRE where the Dictator offers the complete one million dollars to the Receptor and the Receptor rejects. This equilibrium can be sustained if the Dictator beliefs that the Receptor only accepts half a million dollars and rejects any other offer. Given these beliefs, the SRE would evaluate the offer as unkind, given that it is giving the Receptor a payoff of zero (even if it is the same Receptor that is rejecting the offer). The Receptor will reject the offer, given that he believes it is unkind. However, given that the Receptor is rejecting the offer, the Dictator would also believe that the Receptor is unkind and will offer one million dollars in order to be unkind in response. I believe this equilibrium doesn´t make sense and an offer of one million dollars should always be evaluated as kind. I propose a solution concept that eliminates these unreasonable equilibria.

I analyze the case of finite multi-stage games with observed actions and without nature. A_i is the set of (possible mixed) strategies for player i, $a_i \in A_i$ is a strategy for individual i, $b_{ij} \in B_{ij}$ are the beliefs of individual i regarding the strategy of individual j, and $c_{ijk} \in C_{ijk}$ are the beliefs of individual i about the beliefs of the individual j concerning the strategies of player k (second order beliefs). The space of actions is the same as the space of beliefs and the space of second order beliefs. Therefore we have that $A_i = B_{ji} = C_{jki}$. $\pi_i : A \to \mathbb{R}$ are individual i's material payoffs given that $A = \prod_{i \in N} A_i$. $a_i(h)$ is the action that the strategy a_i prescribes at history h for player i. $a_i|h$ is the same as strategy a_i, but playing the history h with probability one.

I define first what is an equitable payoff and use it as a reference point to evaluate the kindness of a strategy. I propose that strategies that give a potentially higher payoff to opposite players (if they play the strategy that maximize their own "material" payoffs) than the equitable payoff be evaluated as kind and strategies that give a potentially lower payoff to opposite players than the equitable payoff be evaluated as unkind.

We only look for the equitable payoffs in the set of efficient strategies. A payoff that is not efficient cannot be equitable given that the player is giving a lower payoff to the opposite player and to himself.

I define a player's strategy as efficient, if there is no other strategy that always gives every player a higher or equal payoffs, with strict inequality for at least one player.

$$E_i = \begin{array}{c} \{a_i \in A_i \mid \text{there exists no } a_i' \in A_i \text{ such that for all } j \in N \text{ and for all } a_{-i} \in A_{-i} \\ \text{we have that } \pi_j(a_i', a_{-i}) \geq \pi_j(a_i, a_{-i}), \\ \text{with strict inequality for some } j\} \end{array}$$

For the FE and SRE the equitable payoff for a player is the average between the highest and the lowest payoff of the efficient payoffs, given her own strategy.

For example, in the Dictator game, the equity payoff is for the Recipient to get 50% of the money. However, in the case of the Ultimatum game, the equity payoff is not necessarily 50% of the money, given that the payoffs for the Recipient depend not only in the Dictator offer, but also in the Recipient's decision to accept or reject any offer. (Any offer would pay zero to the Recipient if he were to reject the offer.) I believe that an offer of 50% of the money should be seen as the equitable payoff in the Ultimatum game, independently on the strategy of the Recipient. If the Dictator offers a division to the Recipient, she is giving him the possibility of accepting the offer. I believe that the kindness of a strategy should be evaluated in function of the potential payoffs it gives to a player.

In order to make the equitable payoff independent on the strategy of the receiver, my definition of equitable payoff is based on the maximum payoff a player can potentially receive (by playing the strategy that maximize its own payoff).

The equitable payoff that player j believes is fair for player i (given what player j believes everybody else is playing) is defined as

$$\pi^e_{ji}((b_{jk})_{k\neq i,j}) = \begin{array}{l} \frac{1}{2}[\max_{a_j \epsilon A_j} \max_{a_i \epsilon A_i} \{\pi_i (a_i, a_j, (b_{jk})_{k\neq i,j})\} + \\ \min_{a_j \epsilon E_j} \max_{a_i \epsilon A_i} \{\pi_i (a_i, a_j, (b_{jk})_{k\neq i,j})\}] \end{array}$$

I believe the kindness of a player toward another player (let me call him the receiver) has to be evaluated independently on the strategy of the receiver. In order to do this, I evaluate the kindness of a strategy as the difference of the maximum payoffs a receiver can get with that strategy, with respect to the equity payoffs.

Definition: the kindness of player i towards player j is given by:

$$f_{ij} (a_i, (b_{ik})_{k\neq i,j}) = \max_{a_j \epsilon A_j} \pi_j (a_i, a_j, (b_{ik})_{k\neq i,j}) - \pi^e_{ij}((b_{ik})_{k\neq i,j})$$

Definition: player *i's* beliefs about how kind player j is with him is given by:

$$\widetilde{f}_{iji} (a_i, (b_{ij}, (c_{ijk})_{k\neq i,j})_{i\neq j}) = \max_{a_i \epsilon A_i} \pi_i (a_i, (b_{ij}, (c_{ijk})_{k\neq i,j})_{i\neq j}) - \pi^e_{ji}((c_{ijk})_{i\neq j, k\neq i,j})$$

For example, in the Ultimatum game, an offer higher than 50% of the money would be seen as kind, given that the maximum amount of money the Recipient can get (by accepting) is higher than 50% of the money. And offers lower than 50% of the money would be seen as unkind, given that the maximum amount of money a Recipient can get is lower than 50% of the money. My definitions have the advantage of been simpler, as the kindness of a strategy is not evaluated as a function of the strategy of the receiver.

Note that the kindness of a player toward the receiver may depend on the strategies, and the beliefs of strategies of the rest of the players, as the intended payoffs may change with their strategies. However, I argue that has to be independent of the strategies of the receiver.

Once I have completed the definitions of kindness and belief as regards kindness I can define an individual's utility function.

Definition: The utility of individual i is given by:

$$\begin{aligned} & U_i\left(a_i, (b_{ij})_{i \neq j}, (c_{ijk})_{k \neq j}\right) \\ = \; & \pi_i\left(a_i, (b_{ij})_{i \neq j}\right) - \sum_{j \neq i} \lambda_i \left(f_{ij}\left(a_i, (b_{ij})_{i \neq j}\right) - \widetilde{f}_{iji}\left(b_{ij}, (c_{ijk})_{k \neq j}\right) \right)^2 \end{aligned}$$

where λ_i is a measure of how much importance the individual i gives to the emotions of reciprocity. In the utility function above, the fairness payoff enter the utility function as a subtraction of the absolute value of the difference of the fairness of the players. I make this assumption for two reasons. One is for simplicity. The second reason being to represent that individuals want to reciprocate in the same magnitude the kindness and unkindness (according to FE and SRE individuals want to reply any offense with the most severe punishment possible).

In order to eliminate the possibility of incredible promises and threats I require that in equilibrium, the actions be optimal at each history, as the SRE does.

A complication of evaluating the kindness of a strategy as a whole is that a player may want to change her strategy at different histories. I will assume that at each history, every player plays the action that is part of his optimal strategy and that the credible strategy is the union of the actions that belong to the optimal strategies at every history. For example, in the game in figure 3, player 2 may want to play unconditional cooperation (cc) at history C and may want to play unconditional defection (dd) at history D. I will assume that the credible strategy is cd, as player 2 plays c as part of the strategy cc at history C and plays d as part of the optimal strategy dd at history D. Although in this example I get the same credible strategy as SRE in general my results will differ from it.

I assume that in equilibrium, individuals' beliefs and beliefs regarding beliefs have to be correct, and therefore an individual's beliefs have to match both their beliefs regarding beliefs and their actual strategies.

Definition: a Fair Threat Equilibrium of an extensive game with perfect information is a strategy profile a^* such that for every player $i \in N$ and every non terminal history $h \in H \setminus Z$ for which $P(h) = i$ where $a_i^* = \prod_{h \in H} \widetilde{a}_i^*(h)$ where

1) $\widetilde{a_i} \in \arg\max_{a_i \in A_i} U_i \left(a_i, \left(b_{ij}|h, \left(c_{ijk}|h\right)_{k \neq j}\right)_{j \neq i}\right)$
2) $b_{ij} = a_j^*$
3) $c_{ijk} = a_k^*$

I name this equilibrium Fair Threat Equilibrium (FTE) as it incorporates the idea that individuals not only want to reciprocate against kind or unkind actions of other players, but the also want to reciprocate against kind or unkind promises and threats.

One drawback of using this specific utility function is that I cannot prove the existence of the Fair Threat Equilibrium as the utility function is not quasiconcave. However, I use this utility function because the solutions obtained from this model represent more closely the experimental results. So far, I have found an equilibrium in every example I have tried. In the Appendix I present a variation of this utility function for which it can be proved that it has a Fair Threat Equilibrium.

2 Examples

2.1 Dictator Game

Let's consider the case of the Dictator game. A player, the Dictator, chooses to divide an amount of money between himself and another player, let's say the Receptor. After his decision, the game ends and both players are paid what the Dictator decided.

The prediction of the SRE is that the Dictator keeps all the money for himself. Given that the Receptor does not have any choice to make, his kindness toward the Dictator is zero. Therefore, the reciprocity part of the utility function is zero for the Dictator and he should only maximize her own material payoff and keep all the money for herself.

The Fair Threat Equilibrium prediction. In my model, the equitable payoff for the Receptor is 50% of the money. Every offer lower than that would be evaluated as unfair. Given that the Receptor does not take any decision, his kindness function is zero. The Dictator maximizes her emotional utility by being as kind to the Receptor as she is to her, making her kindness function zero. The utility function of the Dictator is: $U_D = \pi - \lambda(f_D - f_R)^2$. By solving the First Order Conditions we get that the Dictator maximizes when $f_D = -\frac{1}{2\lambda_D}$. The higher is the importance of the emotion of reciprocity for the Dictator (λ_D), the smaller she wants her unkindness to the Receptor.

I believe that my model's results represent more accurately the experimental results. According to the Dictator Games: A Meta Study, by Christof Engel, the Dictators on average give the 28% of the money. The 36% give nothing to the Recipient, but the majority give something. 16% split in half the money. A player that gives the average and has my utility function has a λ_D of 2.27. In order to represent a Dictator that does not share any money, we need that $1 > \lambda$. In order to represent a Dictator that splits the money, we need that λ is arbitrarily large, which I recognize is not possible.[2]

[2] In order to represent that some people want to be nice to others, even when they are not nice to them, we could modify their utility function, adding a constant to the emotional part. For example, if the utility function of the Dictator were $U_D = \pi - \lambda(f_D - f_R - k)^2$ where k is a positive constant, some individuals with a high λ and high k would want to split the money with the Receiver.

2.2 The Ultimatum game

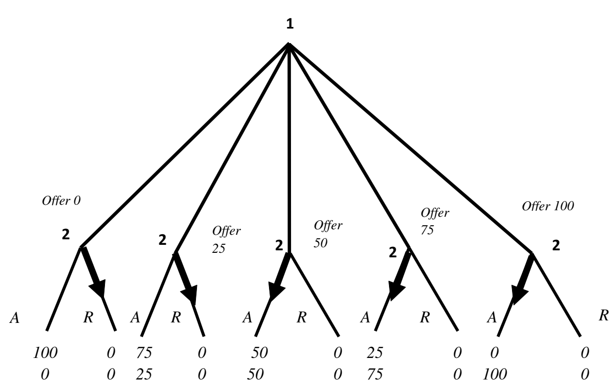
2.2.1 The strategy from the Responder

The only Responder's strategies that can be part of an FTE are of the form: accept any offer that is higher or equal than a constant n, reject any offer that is lower than n. The FTE only allows strategies where players optimize in every history. If it is optimal to accept an offer, then it is optimal to accept a higher offer. Note that even if several strategies give the same maximum payoff to the Proposer, the Responder would choose to accept higher offers because the material payoffs.

According to the FTE, a threat to only accept offers of more than 50% of the money would be seen as unkind, as the maximum amount the Proposer can get is lower than 50% if the Responder follows her threat (the most unfair threat of all is to reject any offer lower than 100% of the money and only accept 100% of the money). A threat to reject any offer lower than 50% of the money, but to accept 50% or more would seem as fair as it gives the possibility to Proposer to get 50% of the money. And threats that include accepting offers lower than 50% of the money would be seen as kind.

Poner gráfica

Reduced Ultimatum Game



In the FTE, the kindness from an offer n from the Proposer is given by: $n - 50$. The kindness of the Responder to the Proposer when she accepts offers higher or equal to a constant k is $100 - k - 50 = 50 - k$. I will find the credible strategy for the Responder by looking for the value of k^* for which the Responder is indifferent between accepting and rejecting the offer. The Responder will accept higher offers and reject lower offers than k^*. The utility function of a Responder that accepts an offer of k^* is $U_R = k^* - \lambda_R 4 \left(k^* - 50\right)^2$. If she rejects, she has the choice of any strategy. Given that she is going to reject the offer of the Responder, the value of k that is going to maximize her utility is the one that makes her emotional payoff equal to zero. Therefore, if she rejects the offer, her utility is zero. If for example, we assume that $\lambda_R = 1/2$, the lowest value of k where U_R is equal to zero is $k^* = 45.24$. As λ_R grows arbitrarily to zero, we get that k^* goes to 50.

2.2.2 The offer from the Proposer

The utility of the Proposer that offers n when he believes the lowest offer the Responder will accept is k^* is: $U_P = 100 - n - \lambda_P \left((50 - k^*) - (n - 50)\right)^2$. If $\lambda_P = 1/2$, we know from above that $k^* = 45.24$. We have that the utility is $U_P = 100 - n - \lambda_P \left(54.24 - n\right)^2$. After taking the First Order Conditions, we get that the offer from the Dictator is $n = 53.76$. That is, he gives a little more than what the Responder is going to accept, in order to be kind to him, given that the Responder is also kind by accepting offers lower than 50. As λ_P becomes smaller, the offer goes to k^*. As λ_P becomes arbitrarily larger, the offers n goes to $100 - k^*$, that is, the Proposer tries to pay all the kindness of the Responder.

2.2.3 SRE

A drawback of the SRE is that it allows a high number of equilibria, many of them unreasonable. For example, for a very large λ_P and λ_R (individuals that only care about their emotional payoffs) a SRE of the Ultimatum Game is for the Proposer to offer 100% of the money to the Responder, and for the Responder to accept any offer except 100% of the money. According to the SRE, the strategy of the Proposer is unkind, given that her offer is the only one the Responder would reject. And according to the SRE the strategy of the Responder is unkind, given that it is rejecting the offer of the Proposer. These strategies are a SRE as both players are maximizing their utility by paying unkindness with unkindness.

2.2.4 Experimental Evidence

My model's predictions are closer to the experimental evidence than those of the SRE. According to Güth and Kocher (2013), the most common offer is 50% of the money. On average players tend to give 40-50% and such offers are almost accepted. For very low offers, the rate of acceptance approach zero.

3 Conclusions

Reciprocity is a complex concept. Dufwenberg and Kirchsteiger (2004) extended Rabin (1993) concept Fairness Equilibrium to dynamic games. Although their solution concept: Sequential Reciprocity Equilibrium eliminates non-credible threats, it does not correctly take in consideration the role of threats and promises.

The objective of this article is to develop a concept of sequential reciprocity that gives more reasonable predictions when threats and promises are involved. My concept: Fair Threats Equilibrium, evaluates the kindness of whole strategies, not only of actions, what I argue is fundamental for consider correctly threats and promises. My concept has the additional advantage of being a simpler concept than Fairness Equilibrium and Sequential Reciprocity Equilibrium and being closer to experimental results.

I applied my solution concept to the Dictator and the Ultimatum games and show that its predictions are close to the experimental results.

My concept may be useful to analyzed Repeated Games. As the collusion in Repeated Games is sustained by threats, the Fair Threat Equilibrium is a good solution concept to this type of games. It may be useful for narrowing the set of equilibria that are possible according to the Faulk Theorem (Friedman year...).

It may be also useful to analyze the solution when explicit promises and threats allow a commitment by the individuals that promises and threats.

4 Notation

A_i is the set of (possible mixed) strategies for player i,

$a_i \in A_i$ is a strategy for individual i,

$b_{ij} \in B_{ij}$ are the beliefs of individual i regarding the strategy of individual j

$c_{ijk} \in C_{ijk}$ are the beliefs of individual i about the beliefs of the individual j concerning the strategies of player k

$A_i(h)$ is the set of possible actions of player i at history h

$a_i|h$ is the same strategy as a_i, with the exception of history h, which is played with probability of one

$A = \prod\limits_{i \in N} A_i$

$a_i(h)$ is the action that the strategy a_i prescribes at history h for player i
$a_i(h)$ is the action that the strategy a_i prescribes at history h for player i.

$a_i|_h$ is the part of the strategy a_i that follows the history h.

$a_i|h$ is the same as strategy a_i, but playing the history h with probability one.

$(a_i|h)(h)$ is the action that the strategy $a_i \backslash h$ prescribes at history h.

5 Appendix

In this section, we probe the existence of the FTE for a variant of a utility function. Dufwenberg shows that we cannot use an standard proof of existence. The reason being that we are requiring that at equilibrium players optimize at every history of the game and the optimal strategy depends on actions beyond a particular history. We follow Dufwenberg by showing that there is a fixed point in a best reply that includes every history of the game simultaneously.

Definition: The utility of individual i is given by:

$$\begin{aligned} & \widetilde{U}_i\left(a_i, (b_{ij})_{i \neq j}, (c_{ijk})_{k \neq j}\right) \\ = \quad & \pi_i\left(a_i, (b_{ij})_{i \neq j}\right) + \sum_{j \neq i} \lambda_i f_{ij}\left(a_i, (b_{ij})_{i \neq j}\right) \cdot \widetilde{f}_{iji} b_{ij}, (c_{ijk})_{k \neq j} \end{aligned} \tag{1}$$

Theorem: Every finite extensive game with perfect information and utility function given by equation (1) has a Fair Threat Equilibrium.

Proof.

As Dufwenberg and Kirchsteiger (2004) proof of existence of their concept of Sequential Reciprocity Equilibrium we prove that there is an equilibrium where players maximize at every history simultaneously using Kakutani´s fixed point theorem.

The best response for individual i at history h is:
$\widetilde{\beta}_i(a) = \arg\max_{a_i \in A_i} U_i \left(a_i, (b_{ij}|h, (c_{ijk}|h)_{k\neq j})_{j\neq i}\right)$. Let´s define the best response as the set of the actions that these best responses prescribe at every history:
$\beta = \prod_{i\in N, h\in H} \widetilde{\beta}_{i,h}(a)$.

First note that U_i is a continuous function because all components: the absolute value, maximum and the addition are a continuos function. Therefore, $B_j(a_{-j})$ is hemicontinuous. Because U_i is a quasi concave function on a, we have that $\widetilde{\beta}_i(a)$ is convex. Because $\widetilde{\beta}_i(a)$ is continuous, $\widetilde{\beta}_{i,h}(a)$ is also continuous and therefore β is also continuos.

Given that A is a nonempty, compact, convex space and U_i is continuos in a, and quasiconcave in a_i, β is a nonempty, convex-valued and upper hemicountinous correspondence.

Therefore, we can apply Kakutanis fix point theorem. Hence there exists a fixed point in the best response. This fixed point is a Fair Threat Equilibrium.

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