Graphic explanation for welfare economic foundation of hoarding loss

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Abstract

Saving brings an economic loss. The author intends to publish a paper, which gives a foundation of this paradox of thrift by connecting money circulation analysis and welfare economics in the case where saving is limited to hoarding. As an introduction of the intended paper, this paper provides a simple explanation for hoarding loss using some graphs.

Under certain conditions, the representative agent hoards money in order to increase utility, but the hoarding actually decreases it against agent's rational intention. This irrationality of rationality occurs because the agent maximizes their utility while lowering the budget of the entire relevant term. This conclusion is derived from the agent making the decision with an ignorance of the whole expenditure reflux. Since the interest of a selfish agent is limited to their private range, the agent ignores the objective truth.

Keywords: Money Circulation, Welfare Economics, Under-Consumption, Paradox of Thrift, Intertemporal Choice.

1. Introduction

Saving brings an economic loss even though it is often regarded as a virtue. This proposition, known as the paradox of thrift, is one of main elements of the under-consumption theory.[1]

Before the Second World War, studies to understand the causes of under-consumption while connecting it with the money circulation structure were performed by some economists including Nicholas August Ludwig Jacob Johannsen,[2] the pair of William Trufant Foster and Waddill Catchings,[3] and two German economists, Ferdinand Grünig and Carl Föhl.[4]

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[1] Cf. Haberler [1964] pp.118-141, Klein [1966] pp.124-152, Nash & Gramm [1969], Bleaney [1976], Dimand [1990], Allgoewer [2002], Clark [2008], Schneider [2008] for a history of the under-consumption theory.

[2] Cf. Hegeland [1954] pp.5-14, Schneider [1962] pp.131-134, Marget [1964] pp.329-336, Allsbrock [1986], Hagemann & Rühl [1990], Rühl [2000].

[3] Cf. Gleason [1959], Carlson [1962], Tavlas [1976], Dimand [2008a], Dimand [2008b].

We support such attempts, but they lack a clear evaluation criterion even though under-consumption is an ethical problem of economic society. A clear ethical standard should be introduced to evaluate it. Here, we adopt an evaluation criterion based on individual utility, imitating the new welfare economics after Vilfredo Pareto.

New welfare economics has usually been connected with the general equilibrium theory. This connection derives the first fundamental theorem of welfare economics, which insists that a market economy is realized as a Pareto efficient state.[5] We should not neglect a critique against the general equilibrium theory.

The dual decision hypothesis by Robert Wayne Clower is remarkable as one such critique.[6] It suggests that realized revenue used in expenditure optimization should not be regarded as being decided by the level of commodity supply. Although this is a valid critique of the general equilibrium theory, its weak point is that it does not clarify the decision principle of the realized revenue in lieu of commodity supply.

In a monetary economy, realized revenue is decided by money flow from expenditure, whereas expenditure is affected by money flow from revenue through a decision-making process under budget constraints. Therefore, the money circulation structure is composed of these bidirectional flows between expenditure and revenue. We ought to construct a theory of expenditure optimization in which bidirectional money flow is reflected. This is the money circulation optimization theory.

The author intends to publish a paper, which shows the welfare economic foundation of the paradox of thrift using the money circulation optimization in the case where saving is limited to hoarding. This paper provides a simple explanation for hoarding loss using some graphs. We hope this simple explanation promotes understanding of the intended paper.

2. Budget Constraint of a Monetary Economy

We first make an assumption of this paper clear. In reality, there are many economic agents. However, in order to simplify the explanation, this paper supposes that there exists only a single individual agent in the relevant society. This supposition of the so-called representative agent has a risk, which could cause a misunderstanding of the following description. We will

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The author owes knowledge regarding Foster and Catchings largely to Sasahara [1981], written in Japanese.

4 Cf. Pedersen [1954], Pedersen [1957], Schneider [1962] pp.156-159, Rothschild [1964] pp.8-11, Ambrosi [1996]. The author owes knowledge regarding Föhl largely to Ito [1952], and Nagasawa [1968] written in Japanese.

5 Cf. Arrow [1951], Debreu [1959] pp.90-97, Stiglitz [1991], Blaug [2007], Feldman [2008].

6 Cf. Clower [1965].

explain this risk later.

In this paper, we discuss a primitive monetary economy. This primitive economy is not a faithful description of our current monetary economy, but the working of a more realistic monetary economy is too complicated to understand. Hence, we think that the assumption of a primitive economy is an appropriate primary approach.

One assumption of the primitive economy is that money is neither produced nor disappears in the relevant space-time, which refers to the sphere which satisfies both the relevant society and the relevant term. In addition, it is assumed that money is not transferred between the relevant society and its outside.

A problem of the general equilibrium theory is mainly caused by the budget constraint used in the theory. We must use an appropriate budget constraint of a monetary economy in which time irreversible disposal is considered.

We define expenditure as transferring money to the relevant space-time, and revenue as money being transferred from the relevant space-time. Agents in a monetary economy can expend money they receive, but we must note that revenue can be expended only time irreversibly. In other words, money cannot be disposed as expenditure before or exactly at the time it is received. We call this the disposal irreversibility principle.

Based on this principle, if the relevant term is divided into infinitesimally short terms, the revenue of each divided short term cannot be expended in the same term. We call such a short term a basic-term. For the sake of simplicity, this paper assumes that the relevant term consists of only two basic-terms.

Hoarding is defined as money which is expendable but is not expended in a basic-term. Note that non-expendable money in a basic-term is not called hoarding even if it exists in the basic-term. Accordingly, revenue in a basic-term does not become hoarding in the same term because it is not expendable within the term based on the definition of the basic-term.

In order to connect expenditure optimization with money circulation analysis, it is indispensable to construct budget constraints in keeping with the disposal irreversible principle. The budget constraint used in this paper is a special case of the irreversibility budget constraint reported in Miura [2015b].[7] We will explain a reason of this indispensability of the irreversible principle later.

As it is assumed that money does not disappear and is not transferred to the outside, the budget is either being expended or being hoarded. Therefore, $X_t+H_t=B_t$ holds where B_t refers to the budget of Basic-term t, X_t refers to expenditure in Basic-term t, and H_t refers to hoarding in Basic-term t. This

[7] The idea of the irreversibility budget constraint was primitively suggested by Dennis Holme Robertson, and was developed by Sho Chieh Tsiang and others. Cf. Robertson [1933], Keynes & Hawtrey & Robertson [1933], Metzler [1948], Tsiang [1966], Kohn [1981], Kohn [1988], Laidler [1989], Van Eeghen [2014].

formula is the basis of the budget constraint of Basic-term *t*.

Next, we consider the background of the budget. Due to the definition of the basic-term, a candidate of the budget in a basic-term is limited to the money available at the beginning of the basic-term. Therefore, $B_1=\Psi$ holds where Ψ refers to money possessed at the beginning of the relevant term. Since we assume that money is not produced or transferred from the outside, Ψ also represents the money stock in the relevant space-time. Eventually, the budget constraint of the first basic-term is $X_1+H_1=\Psi$.

Money expended in the first basic-term (X_1) is lost, but money hoarded in the first basic-term (H_1) is still available at the beginning of the second basic-term. Therefore, the latter becomes an element of the budget in the second basic-term (B_2). Moreover, due to the definition of the basic-term, the revenue of the first basic-term cannot be disposed in the same term and is retained entirely at the beginning of the second basic-term. Therefore, the revenue also becomes an element of the budget. Candidates of B_2 are limited to these hoarding and revenue because we assume that money is neither produced nor transferred from the outside. Let Y_t be revenue of Basic-term *t*. From the above, we can derive $B_2=H_1+Y_1$. Eventually, the budget constraint of the second basic-term is given by $X_2+H_2=H_1+Y_1$.

Hereby, the budget constraints of each basic-term have been determined. We will also clarify how money possessed at the end of the relevant term is decided. Whereas expended money of the second basic-term (X_2) is lost by the agent, hoarded money of the second basic-term (H_2) is still possessed at the end of the term. Furthermore, the revenue from the second basic-term (Y_2) adds to the term-end possession. Hence, if we let Ω be the quantity possessed at the end of the relevant term, $\Omega=H_2+Y_2$ holds. We call this the term-end settlement formula.

The set of these budget constraints of the two basic-terms and the settlement formula taken together is the irreversibility budget constraint. However, it is not perfect as a monetary budget constraint of society as a whole because it reflects only the money flow from revenue to expenditure. In order to express money circulation completely, the money flow from expenditure to revenue must also be considered. Such a consideration is indeed the essence of our money circulation optimization.

Note that someone must receive money expended by another. Therefore, the quantity of expenditure and that of revenue are always equal in the whole society. That is, $X_t=Y_t$ holds for any Basic-term *t*. We call this the law of transfer equality. This law also expresses that expenditure of the whole society refluxes to its own revenue. In other words, the law of transfer equality is a quantitative expression of the whole expenditure reflux, which represents the money flow from expenditure to revenue in the whole society.[8] By incorporating the law into the irreversibility budget constraint, the constraint of the whole society can reflect the two money flows, which

[8] Cf. Miura [2015b] p.98.

compose the money circulation. We call such a constraint the irreversibility reflux budget constraint of the whole society. For the sake of brevity, we call it the whole budget constraint hereafter. This is an authentic monetary budget constraint of the whole society.

We aim to derive the whole budget constraint concretely. Since the irreversibility constraint of the first basic-term ($X_1+H_1=\Psi$) does not include revenue, the law of transfer equality cannot have an effect on it. Therefore, the whole budget constraint of the first basic-term is the same as its irreversibility budget constraint. On the other hand, the irreversibility constraint of the second basic-term includes revenue, thus the law has an effect on it.

As derived above, the budget of the second basic-term is equal to H_1+Y_1. Considering the law of transfer equality in the first basic-term ($X_1=Y_1$), the budget is equal to X_1+H_1. This is also equal to Ψ due to the constraint of the first basic-term. Hence, the budget of the second basic-term becomes Ψ. Substituting this for the constraint of the second basic-term, it can be rewritten as $X_2+H_2=\Psi$. This is the whole budget constraint of the second basic-term.

Therefore, the whole budget constraint of each basic-term can be denoted as $X_t+H_t=\Psi$. Since Ψ represents the money stock in the relevant space-time, we can see that the whole budget constraint is nothing but a constraint for expenditure by money stock.

This constraint is essentially the same as the cash-in-advance constraint applied to the whole society.[9] However, it must be noted that it is not an axiom, but a theorem derived from the irreversibility budget constraint and the law of transfer equality.

Let us suppose that we use a budget constraint which permits time reversible disposal such as $X_1+H_1=\Psi+Y_1$ or $X_2+H_2=H_1+Y_2$. If we substitute the law of transfer equality to these constraints, they become $H_1=\Psi$ or $H_2=H_1$. Accordingly, expenditure disappears from them. If expenditure disappears from the budget constraint, the agent can expend infinitely. This infinite expenditure qualitatively expresses that money circulates infinitely in a temporally closed place. Therefore, money stock cannot become a constraint for expenditure if money can be disposed time reversibly. A constraint by money stock is a product of time-irreversible disposal in a money circulation structure.[10]

In addition, from the term end settlement formula ($\Omega=H_2+Y_2$) and the law

[9] The cash-in-advance constraint was originally suggested by Karl Brunner, and was developed further by Mario Henrique Simonsen, Robert Wayne Clower and others. Cf. Brunner [1951] pp.167-171, Clower [1967], Boianovsky [2002].

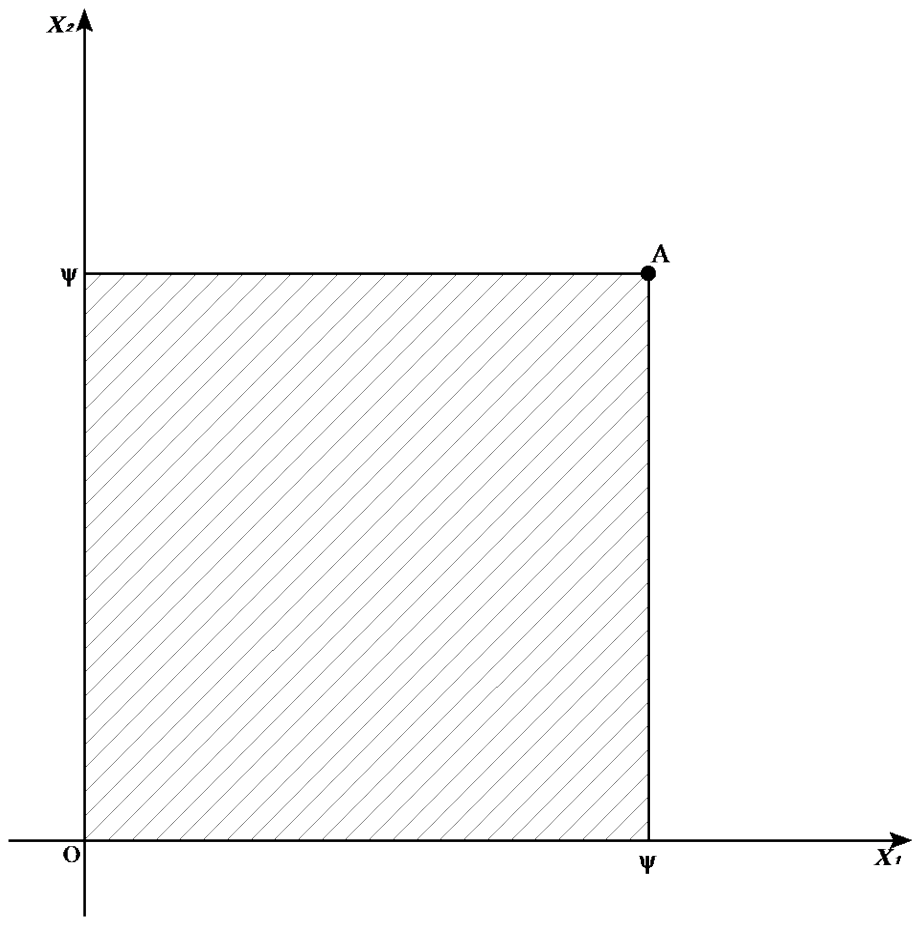
[10] This is an example that the disposal irreversibility principle can solve the second missing problem of the monetary budget constraint shown in Miura [2015b]. A simple explanation to understand this truth using an analogy is provided by Miura [2015c]. The discussion regarding the jinn particle in a physics context may promote the understanding of this issue. Cf. Lossev & Novikov [1992], Gott [2002] pp.20-24.

of transfer equality in the second basic-term ($X_2=Y_2$), we can derive $\Omega=H_2+X_2$. Since $X_2+H_2=\Psi$ holds by the whole budget constraint, $\Omega=\Psi$ is satisfied. This shows that the money stocks are equal at the beginning and the end of a relevant term. We call this the law of money conservation.

Thus, we can conclude $\Psi=B_1=B_2=\Omega$ holds. This conclusion indicates that the quantity of money stock in the relevant space-time is constant. It is a natural conclusion from the assumption that money is not produced, does not disappear, and is not transferred between the outside of the relevant society.

Expenditure and hoarding must be non-negative by their economic meaning, thus $X_t\geq 0$ and $H_t\geq 0$ hold. Considering these with the whole budget constraint, we can derive $0\leq X_1\leq\Psi$ and $0\leq X_2\leq\Psi$. We sometimes indicate this as the whole budget constraint hereafter.

The whole budget constraint can be illustrated as the following graph.



Following this, the axis of expenditure in the first basic-term (X_1) is taken to be horizontal, and the axis of expenditure in the second basic-term (X_2) is taken to be vertical. A shaded area including boundary lines is the range of the whole budget constraint. $X_1=\Psi$ holds if hoarding is not executed in the first basic-term, and $X_2=\Psi$ holds if hoarding is not executed in the second basic-term. Therefore, money is not hoarded at all in Point A(Ψ, Ψ). We call this point the non-hoarding state.

3. Feasible Expenditure Set

The whole budget constraint is not a unique constraint for feasible expenditure. In order to explain another constraint, we first have to clarify why money is expended.

In reality, money is transferred for various reasons, but the main reason is for exchanging commodities. Further, consumption commodities are fundamental commodities. Our primitive economy assumes that money is transferred only for trading with consumption goods. This assumption can also be expressed so that the representative agent is an individual proprietor who sells only consumption goods.

Note that the method of saving is limited to hoarding in our primitive economy by this assumption. In reality, there exist other methods of saving including deposit, loan and equity investment. Since they are transferred from their owner to another agent while saving, we call them transfer saving collectively. Even though transfer saving has a large importance in a modern economy, it is not considered in a primary approach of this paper.

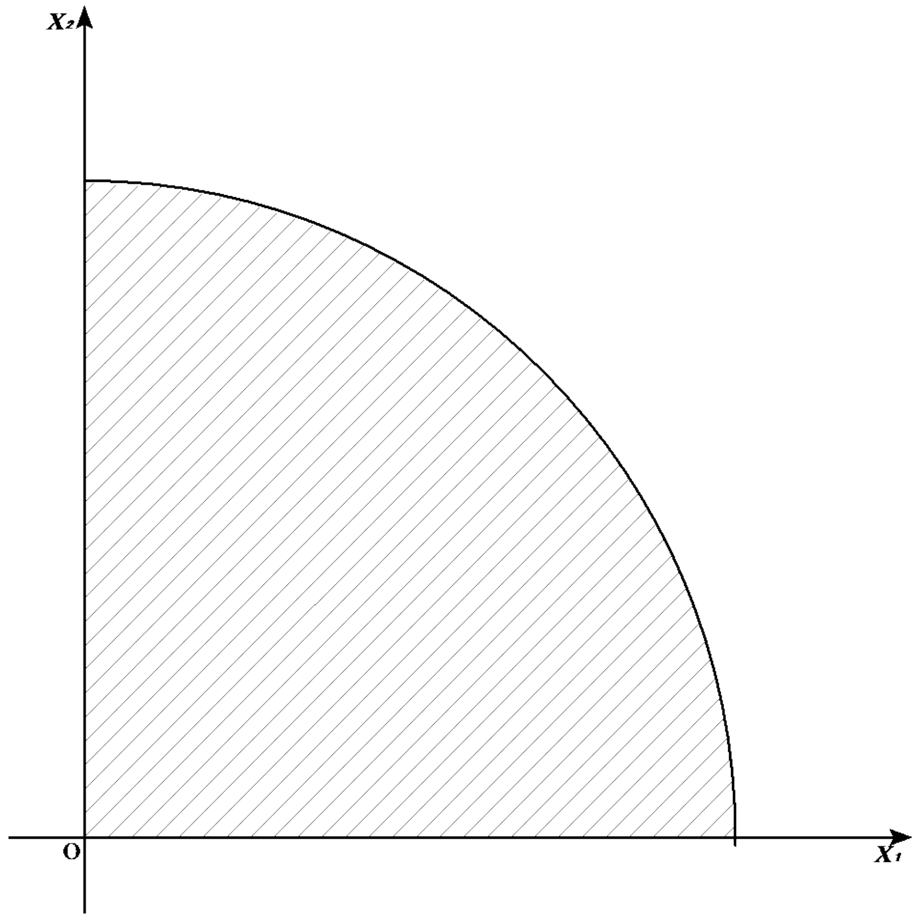
Moreover, we assume that traded consumption goods are of only one kind. Then, the price of a basic-term is defined as the expenditure of the basic-term per unit of consumption goods exchanged with the expenditure. Let C_t be a quantity of consumption goods and P_t be price level in Basic-term *t*. Due to the definition of price, $P_t=X_t/C_t$ holds.

Then, we classify features of price change caused by change in expenditure. If the changing rate of price is less than that of the expenditure, we call that price as sticky. If the changing rate of price is equal to that of expenditure, we refer to that price as flexible. If the changing rate of price is more than that of expenditure, we call that price as hypersensitive.

Hoarding loss is a phenomenon which occurs in the case of price stickiness. In order to simplify the following explanation, we hereafter assume that price is fixed, which is defined as the price of a basic-term that never changes even if the expenditure of the same term changes. Price fixedness is a special case of price stickiness. There may be a possibility that the price of a basic-term is affected by expenditure in another basic-term, but we assume that such an effect does not exist to simplify the explanation.

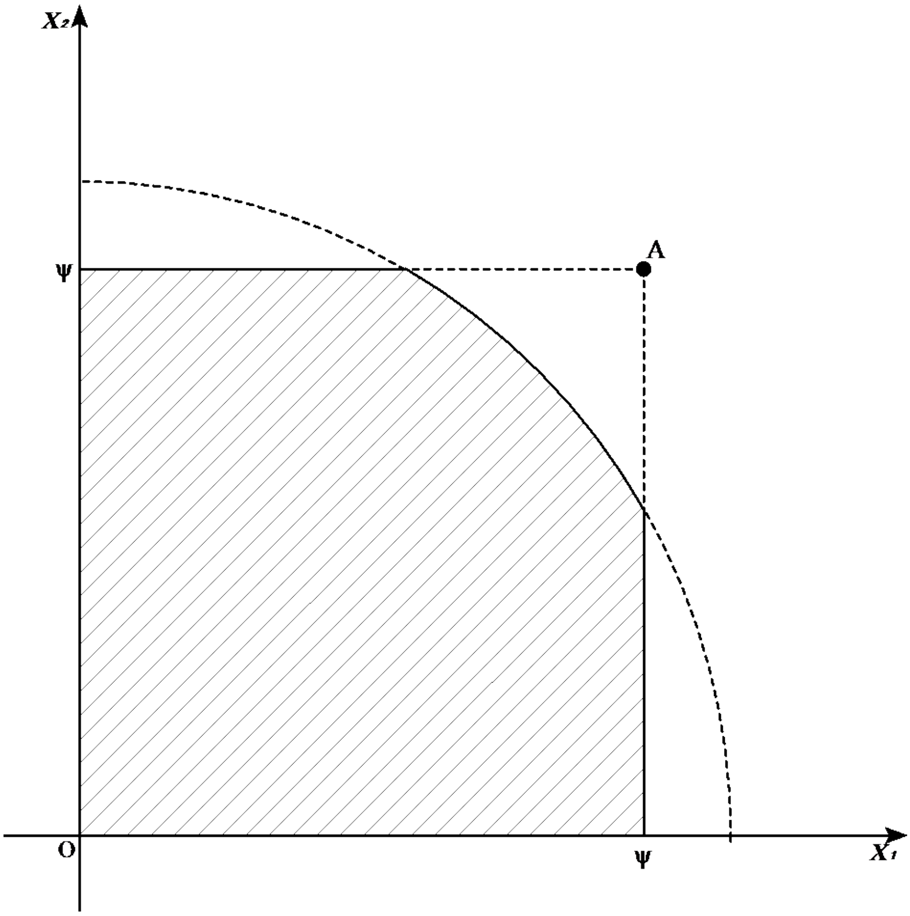
Thus, we define the real commodity supply set as the combination of commodities in each basic-term, which can be supplied. Moreover, in order to enable comparison to nominal expenditure, we define a nominal commodity supply set as follows. A combination of expenditure (X_1, X_2) belongs to the nominal commodity supply set if and only if a combination of consumption goods $(X_1/P_1, X_2/P_2)$ belongs to the real commodity supply set. The nominal commodity supply set represents the expenditure quantity, which is needed to purchase supplied commodities.

The nominal commodity supply set can be illustrated as follows.



The shaded area including boundary lines is a range of the nominal commodity supply set. Although this set is illustrated in a convex shape, this shape was prepared only to easily distinguish it from the whole budget constraint. Note that we do not set any special suppositions regarding the shape of the nominal supply set.

Expenditure which does not belong to the nominal commodity supply set cannot be realized because commodities that ought to be exchanged with the expenditure are not supplied. Accordingly, the feasible expenditure of the whole society must belong to both the whole budget constraint and the nominal supply set. Based on this, we call the common part of the two a feasible expenditure set. The feasible expenditure set is illustrated as follows.

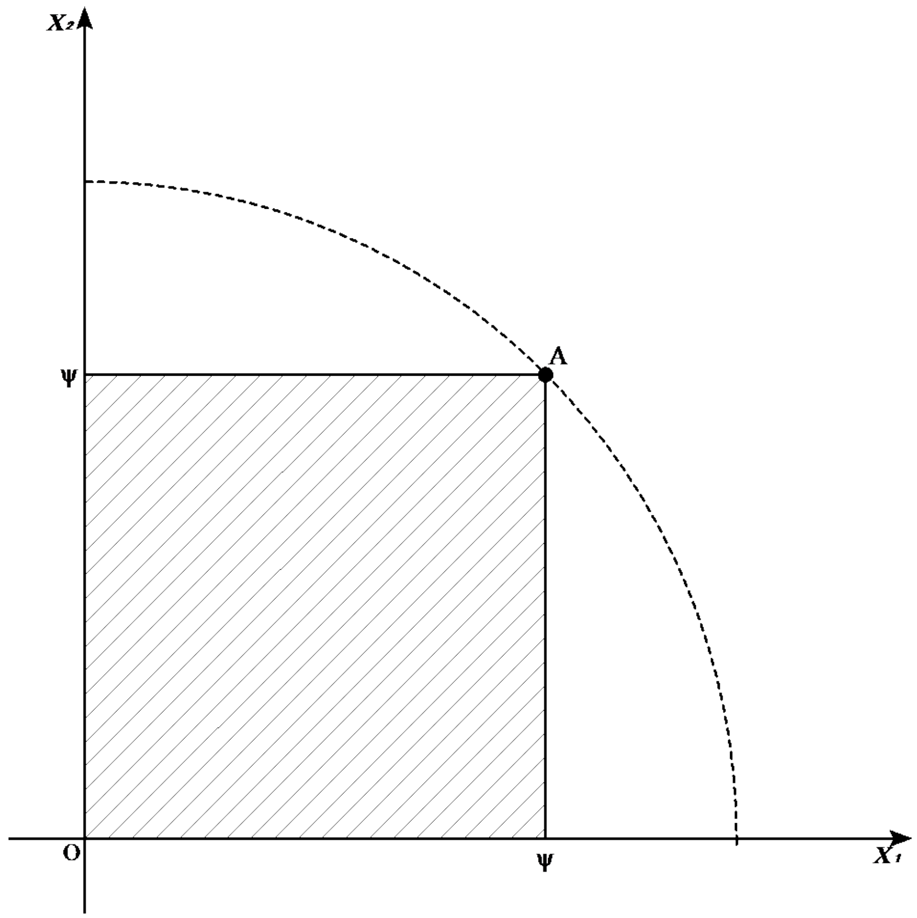


A shaded area including boundary lines is a range of the feasible expenditure set. In this graph, Point A, which refers to the non-hoarding state, does not belong to the feasible expenditure set. This situation shows that money must be hoarded because commodities are sold out even given low expenditure. We call this undersupplied hoarding.

Considering $X_t = P_t C_t$ holds, the nominal supply set becomes small if price becomes low. In this case, undersupplied hoarding occurs easily because commodities are easily sold out.

Even if undersupplied hoarding occurs, all supplied commodities can be consumed. Therefore, this is not a hoarding which brings an economic loss. We should pay attention that all types of hoarding do not cause a loss.

Hereafter, we assume that the whole budget constraint is included in the nominal supply set. It is a situation as the following figure shows.



In this case, Point A is always included in the feasible expenditure set, thus the undersupplied hoarding does not occur. Further, the feasible expenditure set accords with the whole budget constraint. Hence, we identify the whole budget constraint with the feasible expenditure set hereafter.

4. Social Optimal Solution

We assume that the agent obtains utility from consumption goods in two basic-terms. By this assumption, the utility function is denoted as $U[C_1, C_2]$. Further, we assume that the utility is not satiated by consumption. That is, the more commodities are consumed in a basic-term, the more the utility increases, provided that the consumption does not vary the other conditions. Mathematically, this assumption is denoted as $\partial U/\partial C_t>0$.

For the sake of convenience in our money circulation optimization, we transform the utility of real consumption into that of nominal expenditure. We call this transformed utility ($U[X_1, X_2]$) the nominal utility function.

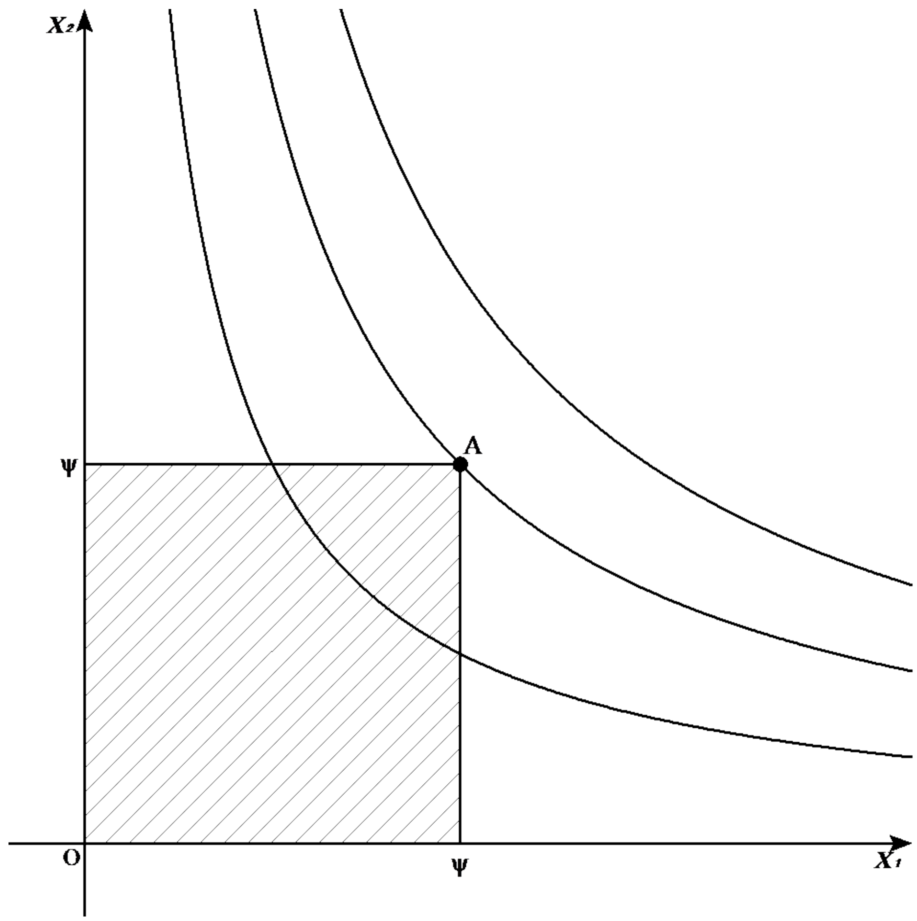
We will clarify how the utility varies by a variation of expenditure. The following equation holds by the chain rule in the differential calculus.

$$\partial U/\partial X_1=(\partial U/\partial C_1)(\partial C_1/\partial X_1)+(\partial U/\partial C_2)(\partial C_2/\partial X_1),$$

Since $C_1=X_1/P_1$ holds by the definition of price, we can derive $\partial C_1/\partial X_1=1/P_1$ under the supposition of price fixedness. Moreover, $\partial C_2/\partial X_1=0$ holds because the price of a basic-term is assumed not to be affected by expenditure of another basic-term. Therefore, $\partial U/\partial X_1=(\partial U/\partial C_1)/P_1$ is satisfied. Note that $\partial U/\partial C_1>0$ holds for the assumption of non-satiation and $P_1>0$ also holds. Hence, $\partial U/\partial X_1>0$ is derived. Similarly, we obtain $\partial U/\partial X_2>0$. This conclusion represents that, the more money is expended in a basic-term, the more the utility increases, provided that the expenditure does not change the other conditions. We call this the monotonicity of the nominal utility function.

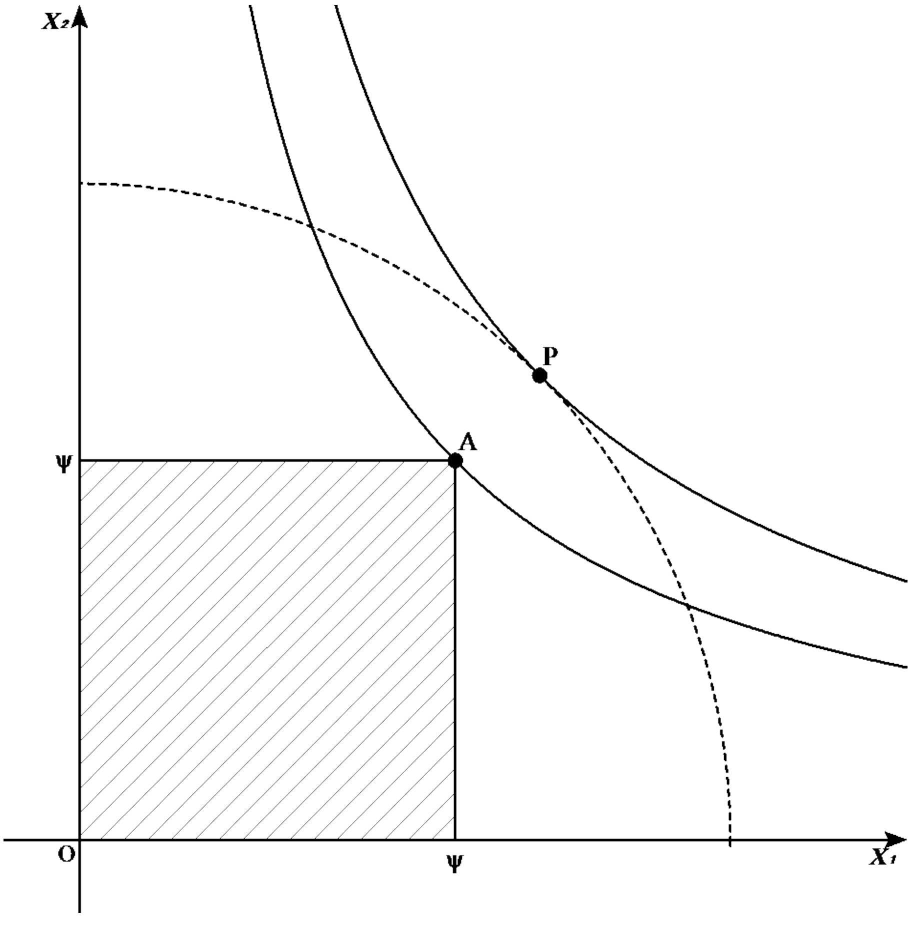
In addition, we introduce the following two suppositions for the nominal utility function. First, we suppose that a nominal marginal utility of the basic-term is not affected by expenditure of another basic-term. As a result, the marginal utility of a basic-term becomes a function of expenditure only in the same term. Based on this independency of nominal marginal utilities, we denote the marginal utility of Basic-term t as $U_t[X_t]$. Secondly, the nominal marginal utility is supposed to diminish as expenditure increases.

Since we assume that only a single agent exists, the utility function and the social welfare function are identified. Based on this, we define a social optimal solution as a state in which the utility is maximized within the range of the feasible expenditure set. We derive the optimal solution while considering that the feasible set agrees with the whole budget constraint.



In this graph, curves falling downward to the right represent indifference curves, which are graphical expressions of the nominal utility function. By the monotonicity of the utility, if X_1 increases, the utility also increases provided that X_2 is constant. Similarly, if X_2 increases, the utility also increases provided that X_1 is constant. Therefore, the social optimal solution is both on $X_1=\Psi$ and $X_2=\Psi$. That is, the solution is Point A, which is a non-hoarding state. We can see that the utility is objectively maximized if the agent does not hoard money at all in our primitive economy.

Assume a situation described as the following graph.



The social optimal solution is Point A, but it is not an optimal solution in the range that real commodities can be supplied. Point P is such an optimal solution where utility is maximized in the nominal supply set. This point is a social optimal solution if we ignore the constraint of money stock. We call it the pure social optimal solution.

In the case of the preceding graph, the feasibility of the pure social optimal solution is blocked by the existence of a money stock. Since this under-consumption occurs even if money is not hoarded at all, we call it a non-hoarding loss.

This economic loss occurs because the money stock is too low to execute the expenditure needed to purchase all supplied commodities. Considering $X_t=P_tC_t$ holds, the nominal supply set becomes large if price becomes high. Hence, the non-hoarding loss easily occurs where the fixed price is high.

However, under-consumption is not caused only by this reason. Even if the pure social optimal solution accords with the non-hoarding state, there is a possibility that the agent hoards money and an economic loss occurs. We will clarify the cause of this hoarding loss.

5. Individual Optimal Solution

In order to judge whether hoarding loss occurs or not, we have to examine expenditure optimization by an individual agent. For the purpose, we will clarify the budget constraint of an individual agent.

Note that this paper supposes that there exists only a single agent in the relevant society. Therefore, some may think that the individual budget constraint ought to be the same as the whole budget constraint. This thought is appropriate if we regard this supposition as literally true.

However, that supposition was introduced only for simplifying an explanation. It does not represent the truth in a monetary economy faithfully. Exchange does not occur if plural agents do not exist, and money cannot exist if exchange does not occur. Plurality of agents is essential for a monetary economy. Accordingly, an individual agent and the whole society must be distinguished to understand a monetary economy. Since this paper aims at a simple explanation, the distinction is expressed only by a difference in the budget constraint. The representative agent should not be interpreted as a literal expression of a single agent but should be interpreted as a collective expression of plural agents. We request readers to read the following description while recognizing this cautionary point.

The individual agent shares the irreversibility budget constraint with the whole society. The irreversibility constraint is $X_1+H_1=\Psi$ and $X_2+H_2=H_1+Y_1$. When we derived the whole budget constraint, we incorporated the law of transfer equality into the constraint. This law is a reflection that expenditure and revenue are the same events in the whole society.

However, if an individual agent expends, their revenue is not guaranteed to increase. Inversely, even if the agent receives money, they do not need to expend it. Therefore, expenditure and revenue are two different events for an individual agent. As a result, an individual agent optimizes expenditure under the assumption that expenditure does not affect revenue.[11]

We intend to eliminate hoarding from the constraint for a convenient

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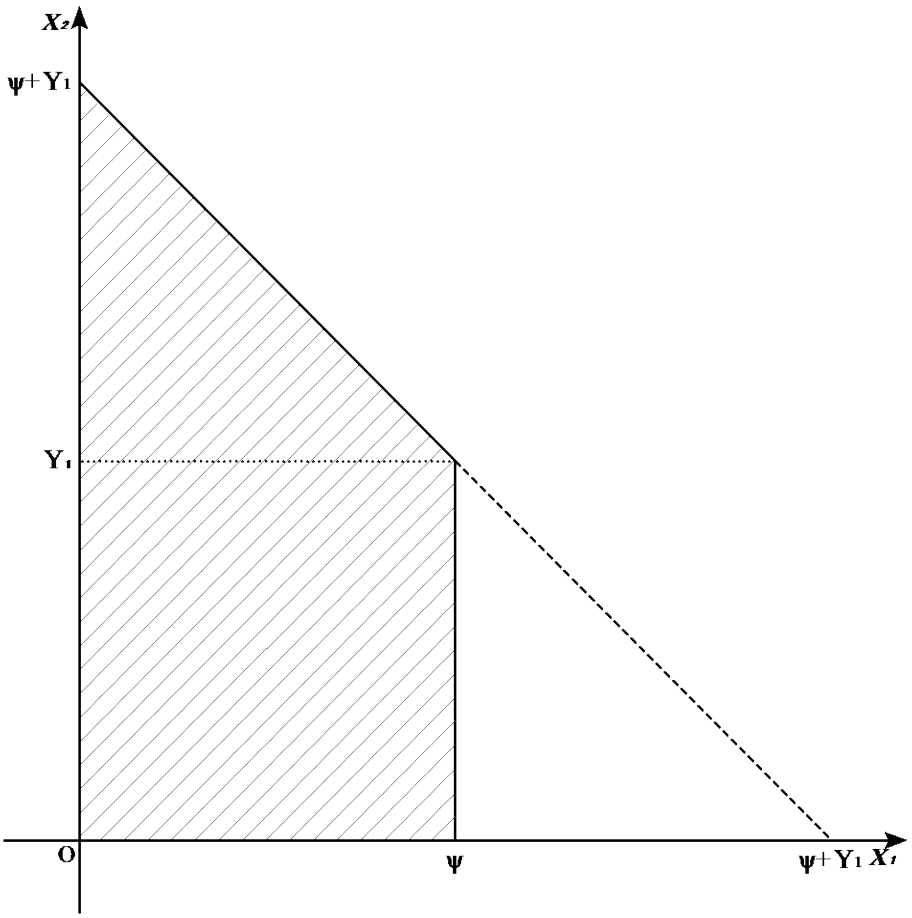
[11] For an individual agent, expenditure can affect revenue by the expenditure reflux (Cf. Miura [2015a], [2015b]). However, based on the disposal irreversibility principle, expenditure affects only revenue of or after the second basic-term. In the framework of this paper, these revenue are not included in the budget, thus the expenditure reflux does not have an effect on individual decision-making.

explanation. H_1 is included in both the budget constraints of the first basic-term and that of the second basic-term. We make one hoarding correspond to one constraint. If we aggregate the budget constraints of two basic-terms and arrange the aggregated formula, we can derive $X_1+X_2+H_2=\Psi+Y_1$. This formula can be regarded as the budget constraint of the entire relevant term. We can substitute this for the constraint of the second basic-term. As a result, the individual budget constraint becomes a set of the following equations.

$$X_1+H_1=\Psi.$$
$$X_1+X_2+H_2=\Psi+Y_1.$$

Thus, we have succeeded in making one hoarding correspond to one constraint. Then, we apply the non-negativity of hoarding ($H_1\geq 0$ and $H_2\geq 0$) to these equations. We can derive $X_1\leq\Psi$ and $X_1+X_2\leq\Psi+Y_1$. We have thus succeeded in eliminating hoarding from the constraints. The individual budget constraint is a common part of these constraints and the non-negativity of expenditure ($X_1\geq 0$ and $X_2\geq 0$). It is illustrated as the following graph.



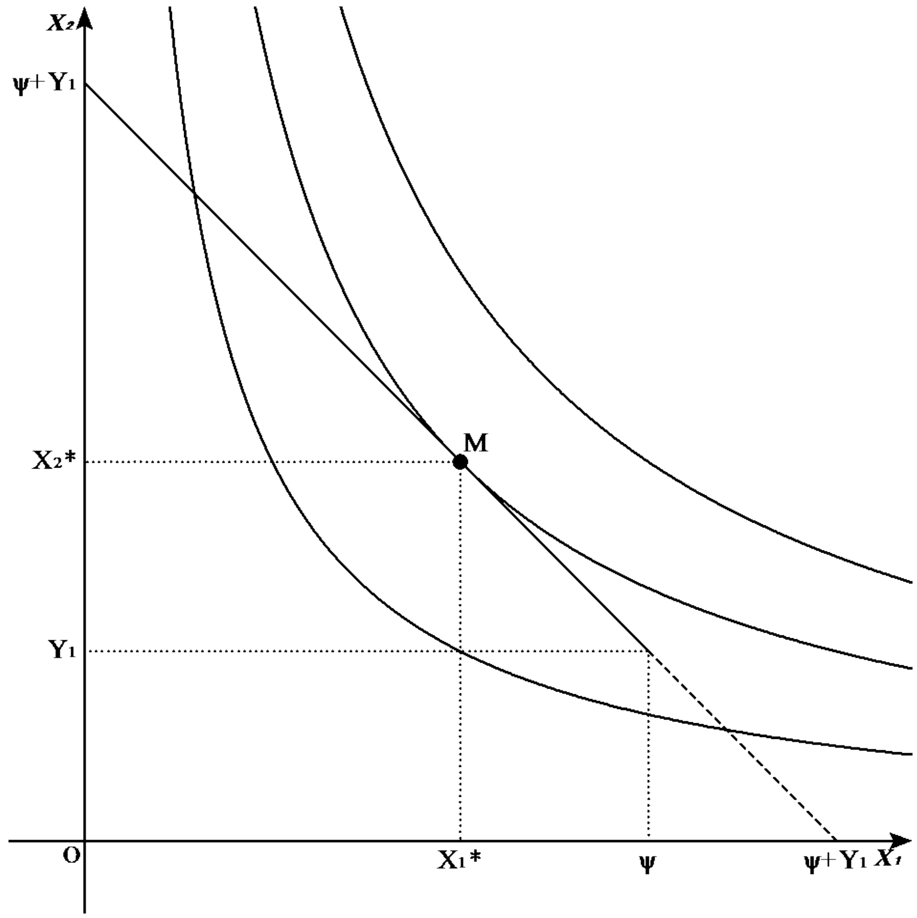
We should pay attention to difference of shapes between the whole budget constraint and the individual budget constraint. Whereas the former constraint is a square, the latter constraint is a trapezoid. This difference reflects the effect of expenditure on revenue.

The shape of the individual budget constraint in simultaneous choice, which as usual economics shows, is a triangle. The reason why the constraint in our temporal choice does not shape into a triangle but shapes into a trapezoid is because time irreversible disposal was considered here.

The individual agent maximizes the nominal utility function under this constraint. By the monotonicity of the nominal utility, if X_2 increases, the utility also increases provided that X_1 is constant. As a result, the individual optimal solution satisfies $X_1+X_2=\Psi+Y_1$. Therefore, the agent does not hoard money in the second basic-term. This is intuitively correct because hoarding in the last basic-term cannot be expended in the relevant term and is therefore ineffective. Hereafter, $X_1+X_2=\Psi+Y_1$ is called the individual budget line.

Next, we will examine an individual optimal condition. We can classify the condition into two cases.

The first case is illustrated as follows.



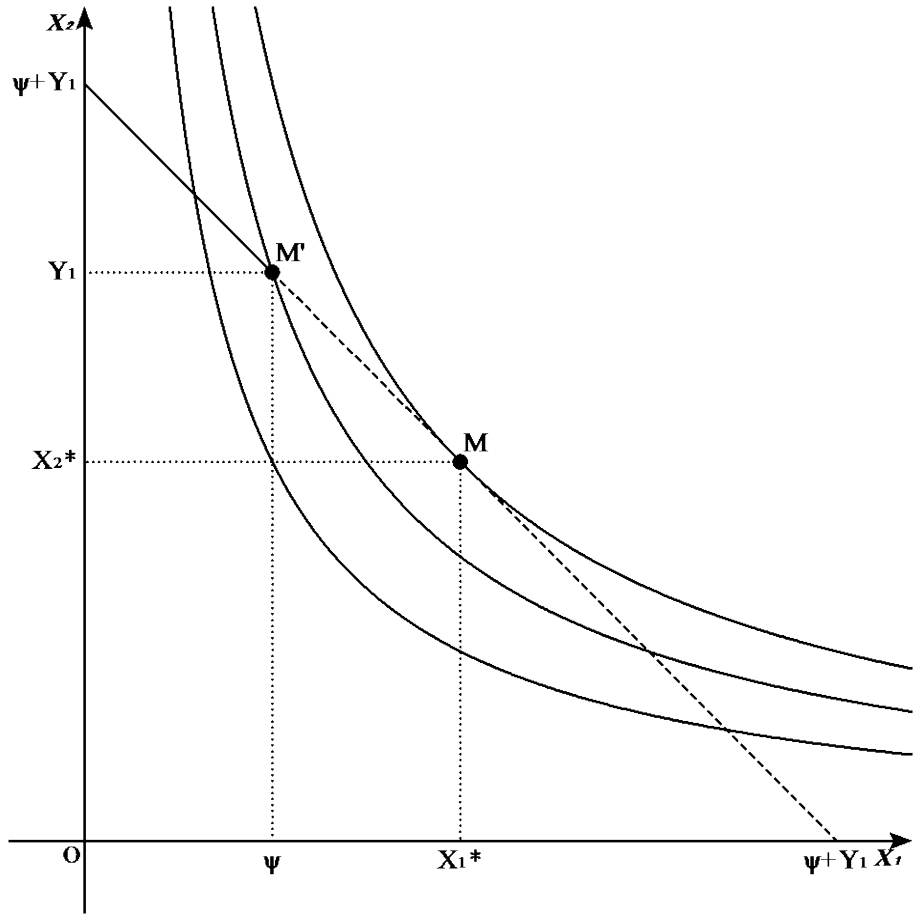
In this case, the individual optimal solution is Point $M(X_1^*, X_2^*)$, in which the slope of the tangent of an indifference curve and the slope of the budget line are equal.

Note that the following equation holds by the formula of the total derivative.

$$dU=U_1[X_1]dX_1+U_2[X_2]dX_2.$$

Since the indifference curve represents a combination of expenditure in which utilities are equal, $dU=0$ is satisfied on the curve. Therefore, we can derive $dX_2/dX_1=-U_1[X_1]/U_2[X_2]$ as a slope of the tangent of the indifference curve. On the other hand, the slope of the budget line ($X_1+X_2=\Psi+Y_1$) is equal to -1. Accordingly, if we let (X_1*, X_2*) be a combination of expenditure which satisfies the individual optimal condition, $X_1^*+X_2^*=\Psi+Y_1$ and $-U_1[X_1^*]/U_2[X_2^*]=-1$ have to hold. The latter condition can be rewritten as $U_1[X_1^*]=U_2[X_2^*]$. This condition is nothing but the law of equi-marginal utility, which was first stated by Hermann Heinrich Gossen.[12]

However, the law of equi-marginal utility is not a universal condition for maximizing utility. If we let (X_1*, X_2*) be a combination of expenditure whose marginal utilities are equalized, $0\leq X_1^*\leq\Psi$ must be satisfied to hold the law. Next, we examine the individual optimal condition in a case where $X_1^*>\Psi$ holds. This second case is illustrated as follows.



In this case, Point M(X_1*, X_2*), in which marginal utilities are equalized, is

[12] Cf. Jolink & Van Daal [1998], Ikeda [2000], Meijer & Vogal [2000], Meijer [2000], Niehans [2008] regarding Gossen.

not an individual optimal solution because the money needed for the realization of X_1* does not exist. A feasible budget line is $X_1+X_2=\Psi+Y_1$ in the range of $0 \leq X_1 \leq \Psi$. An optimal solution of this case is Point M'(Ψ, Y_1), which satisfies the feasible budget line and is the closest point to Point M. In this case, money is not hoarded in the first basic-term.

Additionally, there is a possibility that $X_1{}^*<0$. In this case, the X_1-coordinate of the individual optimal solution becomes zero, but this paper does not consider this case. We assume that an individual optimal solution of expenditure is derived as a positive value.

Thus, we have succeeded in deriving the individual optimal solution provisionally. However, the solution has not been sufficiently clarified yet because it is not obvious how revenue is decided. Note that the above solution reflects only money flow from revenue to expenditure, which is expressed by the irreversibility budget constraint. It has not yet been shown where the expended money goes. A money circulation structure is never expressed entirely unless the money flow from expenditure to revenue is also considered. Revenue is decided by this whole expenditure reflux. This is the core idea of our money circulation optimization. We have to continue our examination to incorporate the effect of the whole expenditure reflux into the individual optimal solution.

5. Time Preference Regarding Expenditure

Whether an individual agent hoards money or not depends on the time preference type with respect to expenditure of the agent. We classify three preference types by comparing nominal marginal utilities of different basic-terms obtained from the same amount of expenditure.

Let X be an expenditure level.

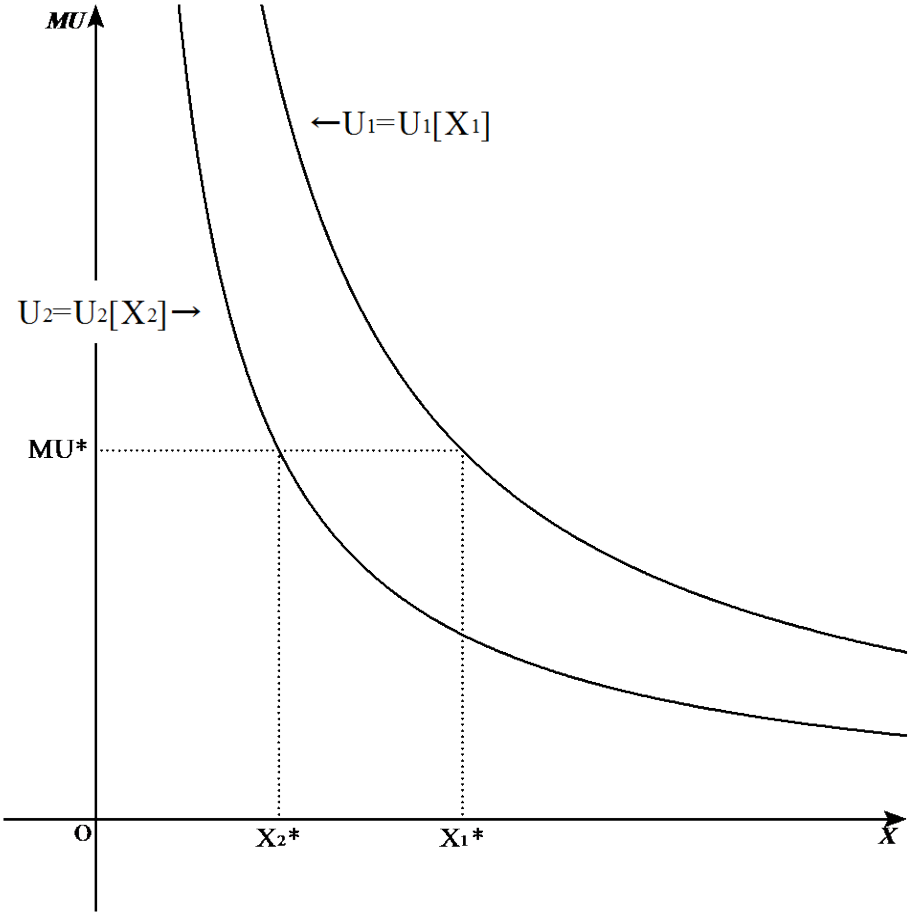
If $U_1[X]>U_2[X]$ is satisfied, past expenditure gives larger marginal utility than future expenditure at an expenditure level X. In this case, it is defined as the agent preferring past expenditure at an expenditure level X.

If $U_1[X]<U_2[X]$ is satisfied, future expenditure gives larger marginal utility than past expenditure at an expenditure level X. In this case, it is defined as the agent preferring future expenditure at an expenditure level X.

If $U_1[X]=U_2[X]$ is satisfied, past expenditure and future expenditure give the same marginal utility at an expenditure level X. In this case, it is defined as the agent preferring expenditure time neutrally at an expenditure level X.

Then, we suppose a uniformity of time preference regarding expenditure level. This uniformity means that time preference does not depend on the expenditure level. If an agent in a basic-term prefers the past expenditure at an expenditure level, the agent also prefers the past expenditure in any another expenditure level. In the case that the agent prefers the future or time neutrality, the same relationship is supposed. Uniformity may not be an appropriate supposition in reality, but this paper supposes to simplify analysis.

We will explain the meaning of this uniformity using the following graphs, which express the relationship between expenditure (X) and the nominal marginal utility (MU).

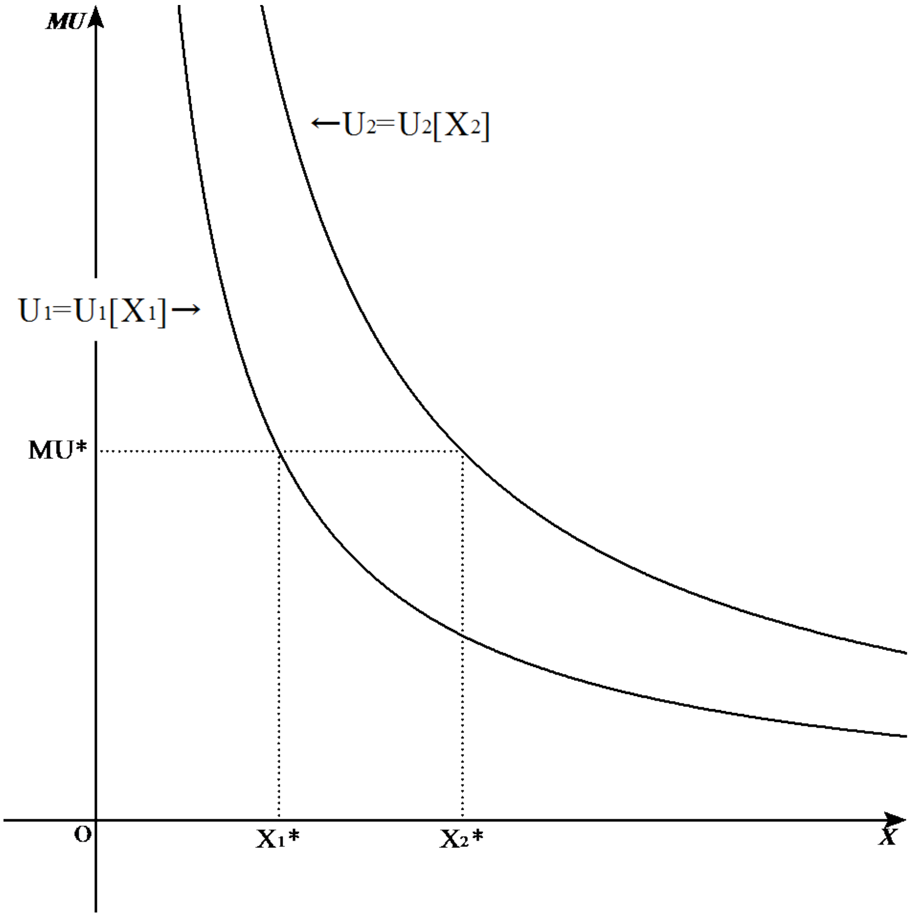


In this graph, curves exist in a positive area of the nominal marginal utility. This reflects the monotonicity of the nominal utility. Further, curves fall downward to the right. This reflects the diminishment of the marginal utility. The curve of a basic-term does not intersect with the curve of another basic-term. This is a graphical representation of the uniformity of time preference.

In the preceding graph, the curve of the first basic-term is always located above the curve of the second basic-term. Hence, this is a graph that represents the case where the agent prefers past expenditure. Based on a supposition of the uniformity, an individual agent is called a past preference type if the agent prefers past expenditure.

X_t* expresses the expenditure of Basic-term *t* in the case where the nominal marginal utility is equal to MU*. Note that X_1*>X_2* holds. This feature is satisfied regardless of MU* level. This suggests that, if marginal utilities are equal, the past expenditure will always be larger than future expenditure. This is a feature of the past preference type.

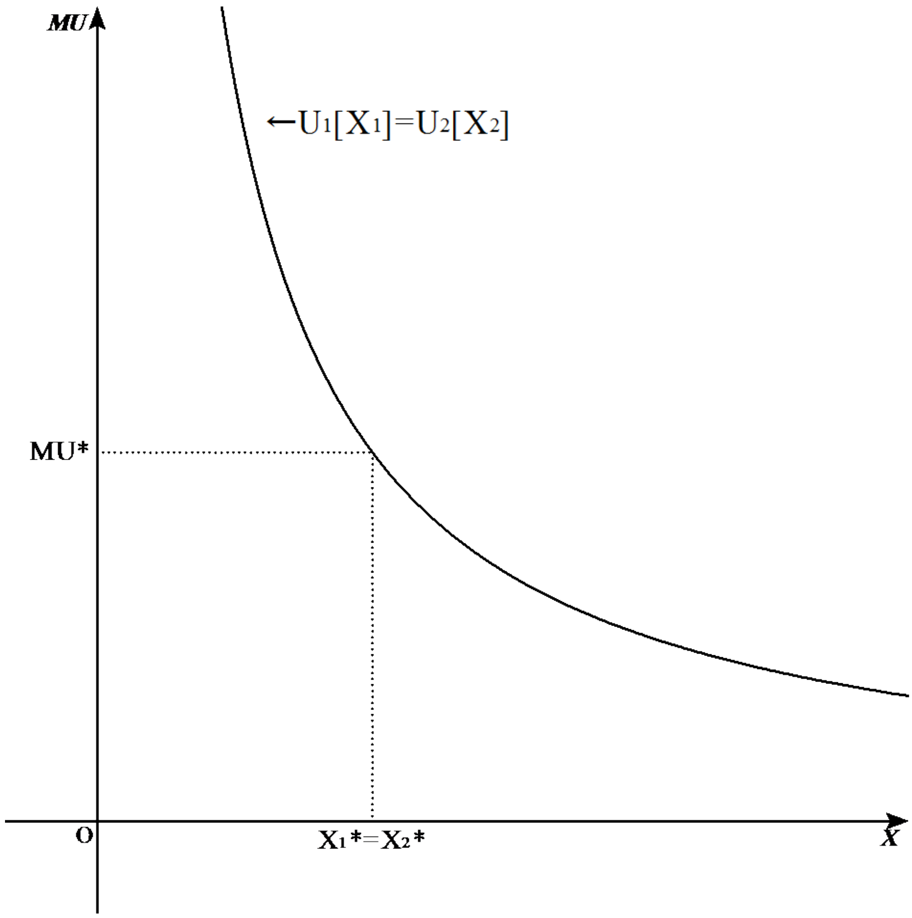
Next, see the following graph.



In this graph, the curve of the first basic-term is always located below the curve of the second basic-term. This graph shows that the agent prefers future expenditure. Based on the supposition of uniformity, an individual agent is called a future preference type if the agent prefers future expenditure.

Note that $X_1^*<X_2^*$ holds irrespective of the level of marginal utility MU*. This indicates that, if marginal utilities are equal, future expenditure is always larger than past expenditure. This is a feature of the future preference type.

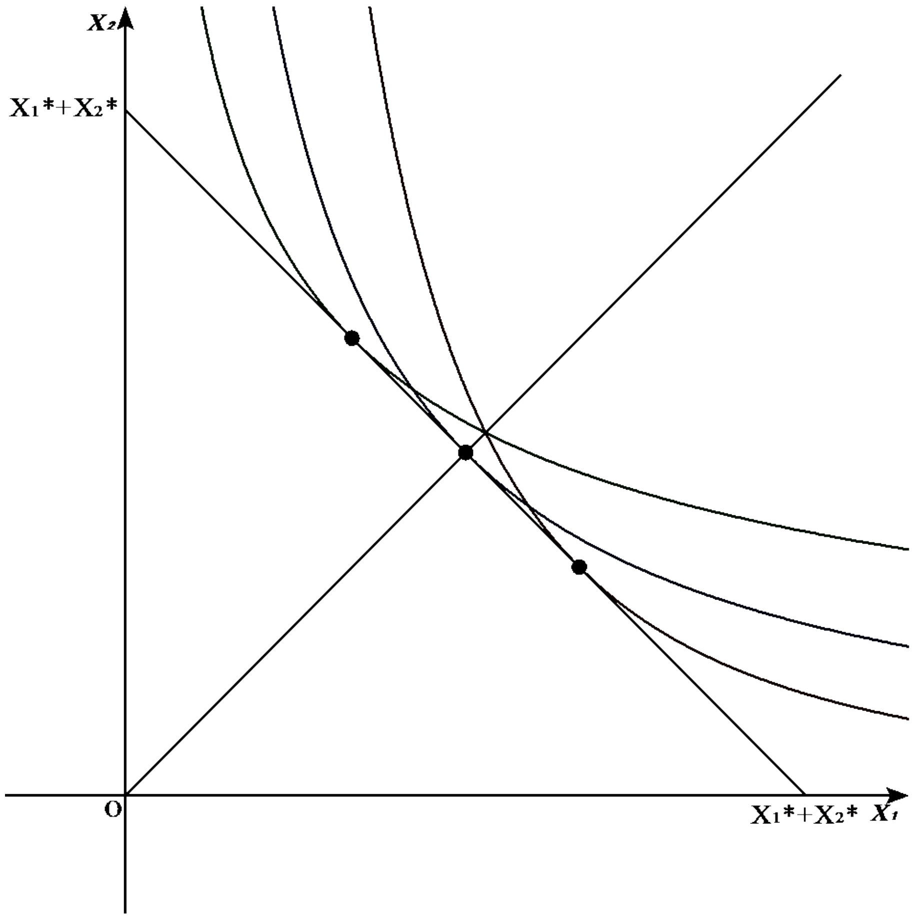
Moreover, see the following graph.



In the above graph, the curve of the first basic-term is overlaid with the curve of the second basic-term. This graph represents the case where the agent preference is time neutral. We earlier stated that the two curves do not intersect under the supposition of uniformity, but only this case is an exception. Based on the supposition of the uniformity, the individual agent is called a time neutral preference type if the agent prefers expenditure time neutrality.

Note that X_1*=X_2* holds even regardless of the level of nominal marginal utility MU*. This shows that, if marginal utilities are equal, the past expenditure and the future expenditure are always equal. This is a feature of the neutral preference type.

We have thus obtained quantitative relationships between past and future expenditure where nominal marginal utilities are equal. We will illustrate these relationships with a graph of indifference curves.



Let (X_1*, X_2*) be a combination of expenditure whose marginal utilities are equal. As confirmed in the preceding section, a slope of the tangent line of (X_1*, X_2*) is equal to −1. Further, this line passes through (X_1*, X_2*). Hence, it can be expressed as $X_2 - X_2^* = -1(X_1 - X_1^*)$. This can be rewritten as $X_1 + X_2 = X_1^* + X_2^*$. The straight line downward to the right represents this line.

The straight line upward to the right represents the line representing the scenario where expenditures of two basic-terms are equal. Based on the above conclusion, if the agent is a time neutral preference type, the point (X_1*, X_2*) exists on the expenditure equal line. The middle curve represents an indifference curve of this type. If the agent is a past preference type, the point (X_1*, X_2*) exists on the right of the expenditure equal line. The most rightward curve represents an indifference curve of this type. If the agent is a future preference type, the point (X_1*, X_2*) exists on the left of the expenditure equal line. The most leftward curve represents an indifference curve of this type.

6. Foundation of Hoarding Loss

We have at length reached a stage where we can derive an individual optimal solution considering money circulation. However, there is a further difficulty to obtain this derivation. We have to derive expenditure using the budget which includes revenue. Hence, revenue must be decided before expenditure is decided. On the other hand, revenue is decided by expenditure through the whole expenditure reflux. Accordingly, we fall into a circular argument such that expenditure must be decided before expenditure is decided.

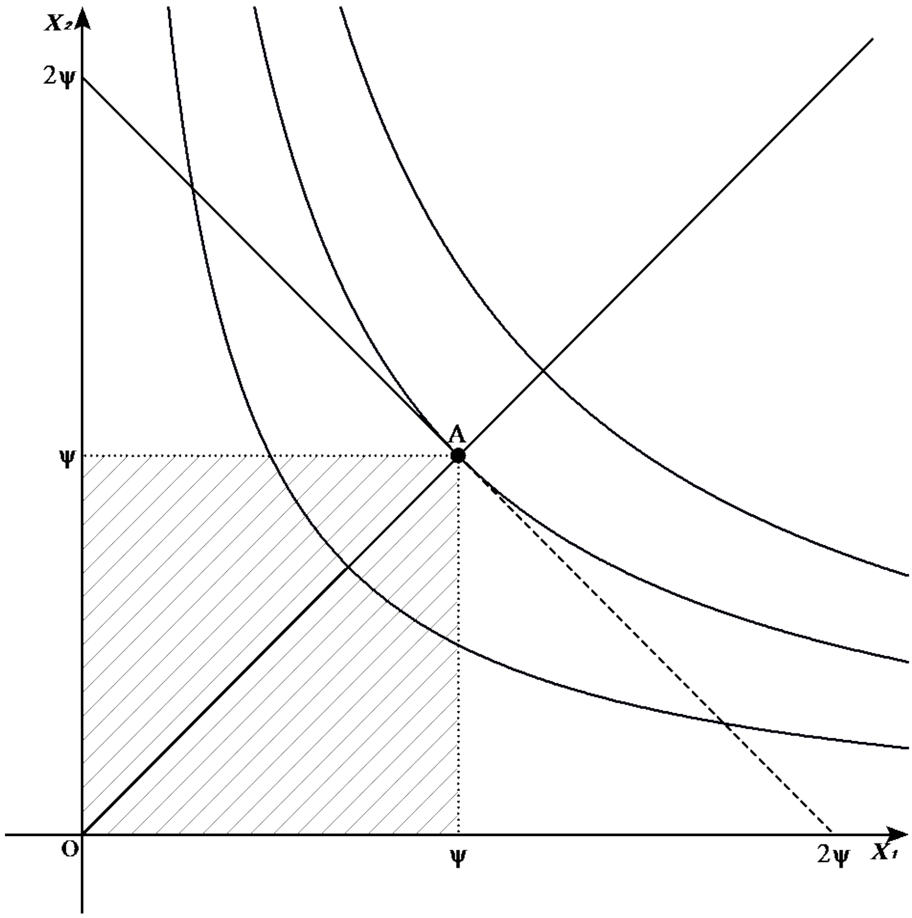
To overcome this difficulty, we adopt the following method. Note that our main concern is whether the non-hoarding state, which is a social optimal solution, is an individual optimal solution or not. We first attempt to judge it.

We initially suppose that the agent forms an expenditure plan to not hoard money at all. Next, we derive revenue according to the initial expenditure plan and the law of transfer equality. Hereby, the budget is determined provisionally. The agent is supposed to optimize their expenditure under the provisional budget. As a result, a provisional solution of expenditure will be derived.

An equivalent condition for the authentic individual optimal solution is that both of the expenditure optimization and the law of transfer equality hold consistently. If the provisional solution agrees with the initial expenditure plan, the optimal solution of expenditure to not hoard money at all is consistent with the law of transfer equality. Therefore, we can conclude that it is an authentic individual optimal solution. If the provisional solution does not agree with the initial expenditure plan, it is impossible that the non-hoarding state cannot satisfy both the expenditure optimization and the law of transfer equality. Hence, we can conclude that it is not an authentic individual optimal solution.

Based on this policy, we initially suppose that the agent forms an expenditure plan such that $(X_1, X_2)=(\Psi, \Psi)$. Due to this expenditure plan and the law of transfer equality, we can derive $Y_1=\Psi$. Since $\Psi+Y_1=2\Psi$ is satisfied, the feasible individual budget line provisionally becomes $X_1+X_2=2\Psi$ in the range of $0\leq X_1\leq\Psi$. We will derive an individual optimal solution under this constraint and examine it with the three time preference types.

First, we examine where the agent is the time neutral preference type. As confirmed in Section 4, the individual optimal solution must satisfy the equality of marginal utilities unless expenditure of the first basic-term exceeds a quantity of the money stock. Let (X_1^*, X_2^*) be a combination of expenditure which satisfies the provisional budget line and whose marginal utilities are equal. Due to the latter condition, this point exists on the expenditure equal line in the case of the neutral preference type. Therefore, (X_1^*, X_2^*) lies on an intersection point of $X_1+X_2=2\Psi$ and $X_1=X_2$. The intersection point becomes $(X_1^*, X_2^*)=(\Psi, \Psi)$, which is the non-hoarding state. The following graph illustrates this.

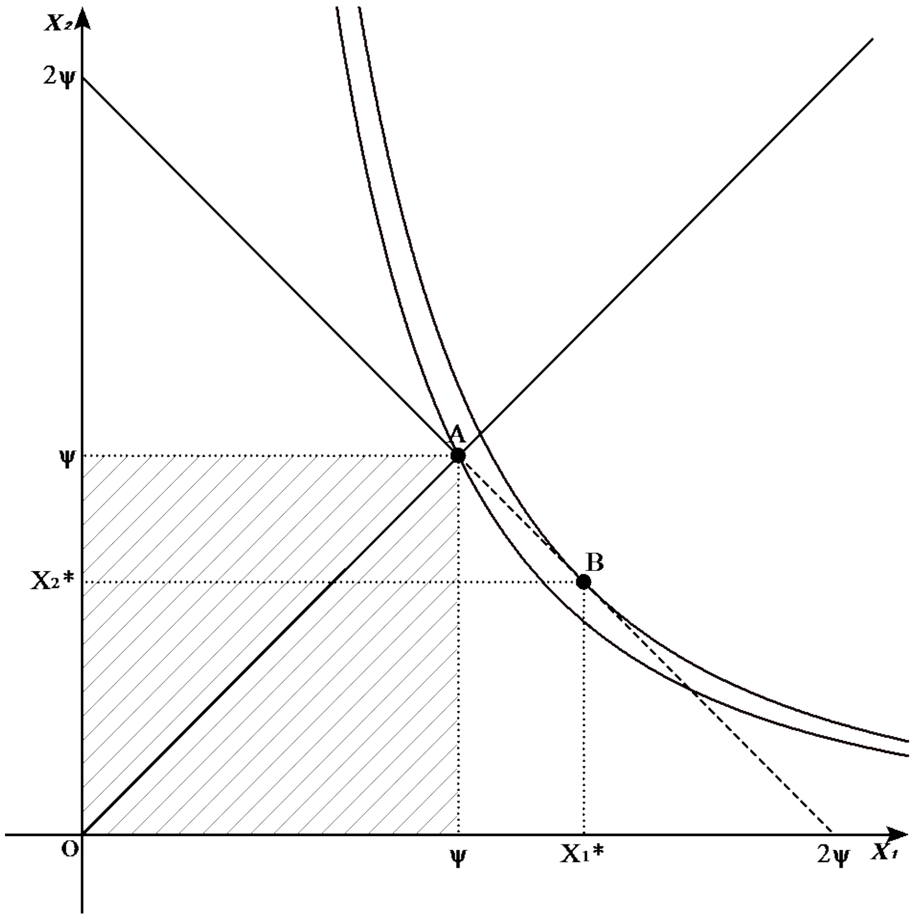


Note that the shaded area including boundary lines refers to the whole budget constraint.

Point A represents (Ψ, Ψ). Since $X_1^*=\Psi$ holds, this point lies on the feasible budget line. Therefore, Point A qualifies as an individual optimal solution. This provisional solution accords with the initial expenditure plan. We can thus conclude that the non-hoarding state is an authentic individual solution in this case.

Hence, individual rational behavior achieves social optimality if the agent is the time neutral preference type.

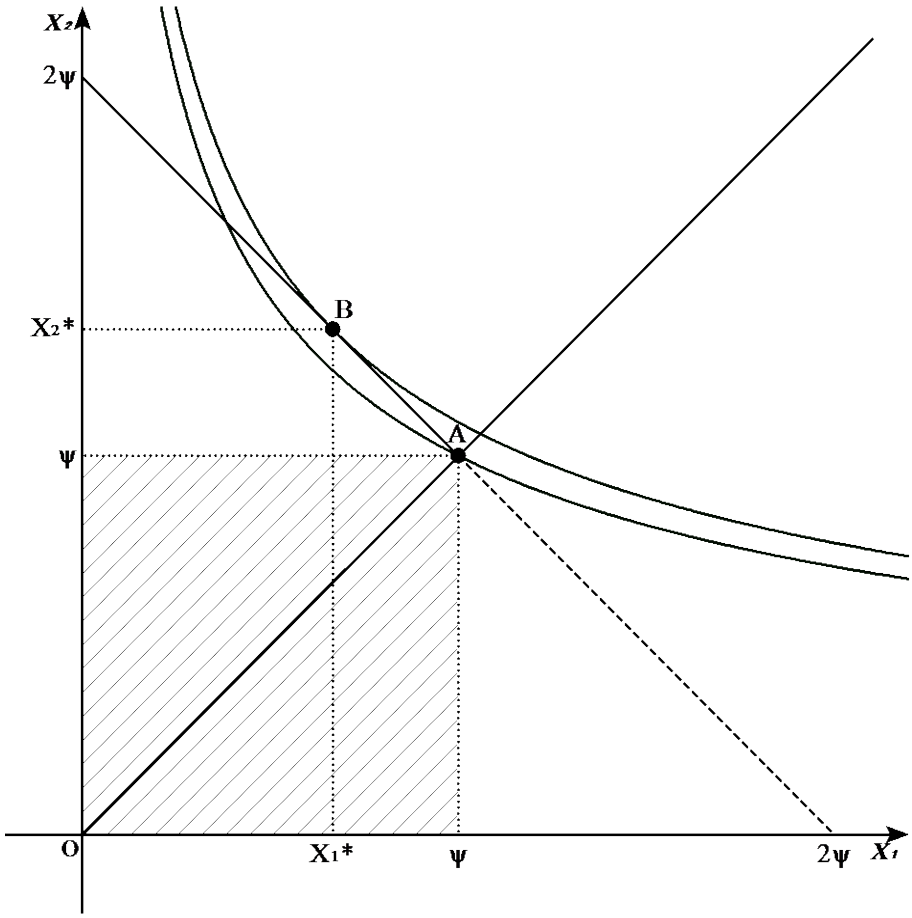
Next, we examine the case where the agent is the past preference type. Let (X_1^*, X_2^*) be a combination of expenditure which satisfies the provisional budget line $(X_1+X_2=2\Psi)$ and whose marginal utilities are equal. As confirmed in Section 5, the latter condition requires that this point exists on the right side of the expenditure equal line $(X_1>X_2)$ in the case of the past preference type. This scenario is illustrated in the following graph.



Point A represents (Ψ, Ψ), and Point B represents (X_1*, X_2*). Since X_1*+X_2*=2Ψ and X_1*>X_2* are satisfied, 2X_1*>X_1*+X_2*=2Ψ holds. Hence, we can derive X_1*>Ψ. This conclusion implies that Point B is not included within the feasible budget line. Therefore, Point B does not qualify as a feasible optimal solution even though marginal utilities are equalized at this point. The authentic solution is Point A, which satisfies the feasible budget line and is the closest to Point B. This provisional solution agrees with the initial expenditure plan. We can conclude that the non-hoarding state is an authentic individual optimal solution in this case.

Similar to case of the neutral preference type, individual rational behavior also achieves social optimality if the agent is the past preference type.

Finally, we examine the case where the agent is the future preference type. Let (X_1*, X_2*) be a combination of expenditure which satisfies the provisional budget line (X_1+X_2=2Ψ) and whose marginal utilities are equal. As confirmed in Section 5, the latter condition requires that this point exists on the left side of the expenditure equal line (X_1<X_2) in the case of the future preference type. This scenario is illustrated in the following graph.



Point A represents (Ψ, Ψ), and Point B represents (X_1*, X_2*). Since $X_1^*+X_2^*=2\Psi$ and $X_1^*<X_2^*$ are satisfied, $2X_1^*<X_1^*+X_2^*=2\Psi$ holds. Hence, we can derive $X_1^*<\Psi$. This conclusion implies that Point B is included in the feasible budget line. Further, as the marginal utilities of this point are equalized, its utility is higher than the utilities of all other points which satisfy the feasible budget line. Therefore, Point B is an optimal solution under the provisional budget. However, this solution does not agree with the initial expenditure plan, which is equivalent to Point A. At this stage, we can conclude that the non-hoarding state is not an authentic individual optimal solution.

Judging simply, Point B seems to the optimal solution. Then, how can the agent fulfill Point B? Let H_1* be a hoarding quantity when the agent expends X_1*. $H_1^*=\Psi-X_1^*$ holds by the budget constraint of the first basic-term. Note that $H_1^*>0$ is satisfied because $X_1^*<\Psi$ holds. From the definition of H_1* and what (X_1*, X_2*) satisfies the individual budget line, we can derive

$$X_2^*=2\Psi-X_1^*=\Psi+(\Psi-X_1^*)=\Psi+H_1^*.$$

This calculation implies that the agent can change their expenditure plan from Point A to Point B by hoarding some money in the first basic-term and adding the hoarded money to expenditure of the second basic-term. That is, the agent can obtain higher utility by hoarding than by not hoarding at all. We can see that the agent hoards money if they behave rationally as maximizing their utility.

However, is Point B really an individual optimal solution? Since this point maximizes utility under the individual budget line, it seems to be qualified as the solution. Nevertheless, if it can be realized, a strange situation occurs.

Since $X_2^*=\Psi+H_1^*$ and $H_1^*>0$ hold, $X_2^*>\Psi$ ought to be satisfied. This concludes that $B(X_1^*, X_2^*)$ is not included within the whole budget constraint as illustrated in the preceding graph. This means that expenditure which exceeds money stock must be executed in a basic-term. How does the agent expend exceeding the money stock? Do they use the same money repeatedly? It is impossible to do so in a single basic-term if we recall the definition of basic-term. The whole budget constraint is a reflection of this impossibility.

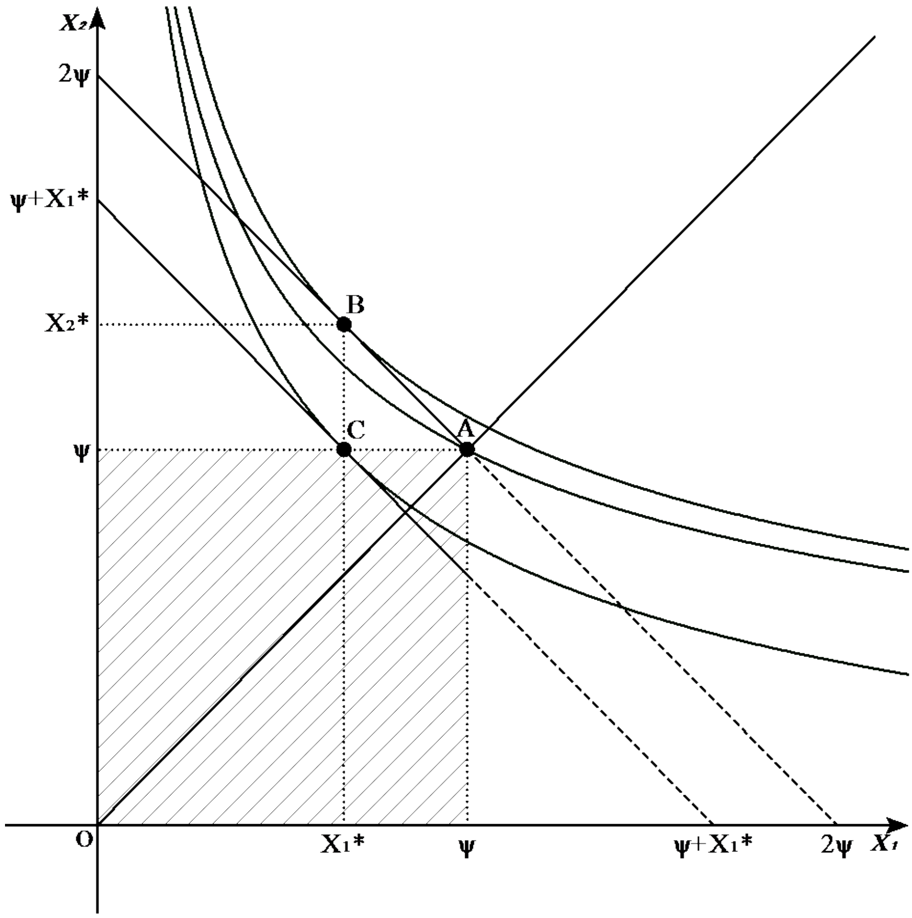
On the other hand, money is hoarded and sent to the second basic-term. As a result, the agent in the second basic-term can expend more money at Point B than at Point A, can't they? If they can't do so, where does the hoarded money disappear? It is unlikely that the hoarded money disappears.

Then, what happens? We will clarify the answer.

Point B is impossible. We should pay attention to the relationship between expenditure and revenue in the first basic-term. In Point B, $X_1^*<\Psi$ holds as confirmed above. However, $Y_1=\Psi$ was assumed when the budget constraint was set. Therefore, expenditure and revenue are not the same quantity in the first basic-term. This contradicts the law of transfer equality, which is an objective law in a monetary economy. This is a definite reason why Point B cannot be realized.

We ought to consider an effect of the law of transfer equality in the case where money is hoarded. When we derived Point B as an optimal solution, the agent optimized their utility under the budget line $X_1+X_2=2\Psi$. We should recall that the original budget line is $X_1+X_2=\Psi+Y_1$. The former budget line is derived because we assume $Y_1=\Psi$. However, when the agent decides to hoard H_1^*, expenditure of the first basic-term is decreased from Ψ to X_1^*. Due to the work of the law of transfer equality, this decrease of X_1 causes a decrease of Y_1, which changes from Ψ to X_1^*. Even though the hoarded money does not disappear, revenue is decreased instead. Accordingly, the budget is lowered from 2Ψ to $\Psi+X_1^*$. Based on the lowered budget line, the realized expenditure of the second basic-term is $X_2=\Psi$ when $X_1=X_1^*$ is satisfied.

This situation can be illustrated as follows.



Point C represents (X_1*, Ψ), which is realized by hoarding H_1*. In this graph, marginal utilities are equalized in Point C. Moreover, this point is derived as fulfilling the law of transfer equality. Since it satisfies conditions of individual optimality, it is an authentic individual optimal solution.[13]

As shown in the preceding graph, the utility of Point C is less than that of Point A, which refers to the non-hoarding state and the social optimal solution. The agent subjectively intends to change their expenditure plan from A to B by hoarding money, but they objectively change it from A to C because they maximize the utility while lowering the budget line. Although the agent hoards money to increase their utility, the hoarding actually decreases it against their intention. Thus, we can see that hoarding which is executed due to an individual's rational judgment actually causes an economic loss.

It is not generally guaranteed that marginal utilities are equalized at Point C, which is located directly below Point B. If marginal utilities are not

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[13] Also note that Point C satisfies the whole budget constraint. This constraint is derived from the individual constraint and the law of transfer equality. Therefore, satisfying these two conditions is a sufficient condition for satisfying the whole constraint. Accordingly, Point C, which fulfills the two conditions, automatically satisfies the whole constraint.

equalized at Point C, the individual optimal solution lies elsewhere. But even if this were the case, the solution is not Point A, the non-hoarding state.

An individual optimal solution must satisfy the following three conditions, the law of equi-marginal utilities, the individual budget constraint, and the law of transfer equality. The first condition requires that $X_1<X_2$ in the case of the future preference type. Further, the second condition requires that $X_1+X_2=\Psi+Y_1$, and the third condition requires that $X_1=Y_1$. Synthesizing these conditions, the individual optimal solution has to satisfy $X_1<\Psi$ and $X_2=\Psi$. This solution is obviously not the non-hoarding state, and its utility is less than the non-hoarding state due to the monotonicity of the nominal utility function. Therefore, an individual optimal solution always gives a lower utility than the social optimal solution in the case where the agent is the future preference type. Thus, we can see that hoarding always brings an economic loss in this case.

This hoarding loss is directly connected with a decrease in nominal expenditure. Nevertheless, the decrease of nominal expenditure corresponds to that of real consumption under the fixed price. Hence, this is also a realization of under-consumption.

7. Concluding Comments

The hoarding loss occurs based on a qualitative difference between the budget constraint of the whole society and that of the individual agent. We will show the two constraints again.

[Whole Budget Constraint] $0\leq X_1\leq\Psi$, $0\leq X_2\leq\Psi$.

[Individual Budget Constraint] $0\leq X_1\leq\Psi$, $0\leq X_2$, $X_1+X_2\leq\Psi+Y_1$.

Graphically, whereas the whole budget constraint is a square, the individual budget constraint is a trapezoid. This difference depends on whether the law of transfer equality is incorporated in the constraint beforehand or not.

In principle, the individual optimal solution must satisfy the law of equi-marginal utilities as traditional economics has taught us. However, this is only an optimal condition in the case where decision-making on expenditure does not vary the budget.

But in the whole society, expended money refluxes as revenue. As a result, expenditure varies the budget. Since this whole expenditure reflux exists, equalizing marginal utilities is not an objective optimal condition. The law of transfer equality is a quantitative expression of this whole expenditure reflux. The whole budget constraint is indeed an objective constraint for expenditure because it incorporates this law. Hence, the non-hoarding state, which is a social optimal solution derived under the whole budget constraint, is an objectively correct optimal solution regardless of whether its marginal utilities are equal or not.

If the agent nevertheless hoards money to equalize marginal utilities, the budget of the entire relevant term is forcibly lowered in order to satisfy the law of transfer equality. This is a reflection of the whole expenditure reflux working in the opposite direction. Since the agent makes a decision with ignoring the reflux, utility of the agent is decreased contrary to their intention.

The cause of the hoarding loss is decision-making without recognizing the law of transfer equality, which is an objective truth of a monetary economy. Why does the agent disregard the truth?

For an individual agent, expenditure refers to the money transferred from the agent to others, and revenue refers to the money transferred from others to the agent. Therefore, expenditure and revenue are surely separate events for an individual agent. Their decision-making under the idea that expenditure does not vary the budget is not based on an erroneous factual judgment. In this sense, the individual agent executes a rational judgment.

But in the whole society, expenditure and revenue are the same events namely money transfer. These two truths are not a contradiction because, even though expenditure of an individual agent does not vary their own revenue, it varies their others' revenue. However, a selfish individual agent ignores this relational truth because their interest is limited to their private range and they form an expenditure plan without considering its impact on their others' revenue. As a result, individual constraint detaches from the whole budget constraint, and the agent falls into an irrational situation despite their rational judgment.

This paper proved this irrationality of rationality under an assumption that the number of economic agents is limited to one and that of basic-terms is limited to two. But in the intended paper, we will assume that the numbers of agents and basic-terms are generalized. Then, we will prove that hoarding behavior by individual rational judgment brings a Pareto inefficient situation.

This proof will assume that agents similarly prefer future expenditure as in this paper. However, the author already knows that hoarding loss can occur even if agents prefer past expenditure or time neutrality. Since this hoarding loss depends on a distribution, it cannot be explained using the representative agent model like this paper. The author also intends to publish a paper, which discusses this distributive hoarding loss in the near future.

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