Stochastic input-output modeling

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1. Introduction

Data in the input-output analysis is based on empirical investigations. In such investigations various kinds of errors are inevitable. In addition, some data is not even estimated by means of a sample at all, but just set up on the basis of the expert's opinion. The assumed numbers can be considered as the most probable values only. In reality, other values can occur. Sometimes there is a choice among several acceptable values of at least some parameters. This happens very often in the case of input coefficients in input-output models, when different technologies are considered. Unlike any other economic model, the statistical properties of IO model received only small attention, whereas the more practical applications and theoretical extensions have received great emphasis. One of the main reasons for that was Leontief's early aversion toward the application of probabilistic methods in IO. The second reason could be the fact that the stochastic analysis of errors is difficult to undertake because of the lack of hard data of a probabilistic nature to validate or refute empirically any conclusions drawn from such a study. Therefore the results of traditional input-output studies were almost always expressed as point estimates. Any assessment of the reliability of these estimates is left to the reader.

New econometric methodologies allow an analysis of econometric models with variable parameters, heteroscedastic individuals in panel data or some assumption about heteroscedastic error terms in regression analysis. This progress opened also new perspectives in econometric analysis with respect to input-output theory.

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Stochastic input-output models can be built and estimated mainly on the basis of panel data. By means of econometric methods the established models can be thoroughly investigated.

The most important early investigations on this subject were performed by Miernyk et al. [31'] and Gerking [16]. Miernyk was the first economist who compiled an IO table on the basis of sample information. Gerking performed an explicite stochastic IO analysis, upon the IO table by Miernyk et al. [31']

Early methods of a stochastic approach to IO models were based on time series data. In cases of structural changes in the economy under study the results of such an analysis were not acceptable. An adventage of the stochastic approach was the possibility to test model coefficients by means of methods developed by mathematical statistics. The stochastic approach to input-output models was the of McCamley et al. [29] considerable debate (see e.g. Miernyk [30]).

Difficulties with the collection of relevant data, combined with the complex interrelationships involved, has resulted in few applied input-output studies within a stochastic framework (e.g. Briggs, [2]; Cmiel & Gurgul, [9]; Gurgul, [22]; Simonovits, [43]; West, [49]). More details will be given in the next chapters.

The remainder of the paper is organized as follows. In section 2 early contributions to stochastic models are briefly overviewed. In section 3 main results on input-output analysis in 70's and 80's are presented. In section 4 recent studies on stochastic methods for dynamic and regional integrated models are disscussed. Section 5 concludes the paper.

2. Early contributions to stochastic IO Analysis

Input-output analyses were undertaken in the late 1940s and early 1950s. IO economists investigated the properties of "a Leontief inverse". The most known contribution is that of Sherman and Morrison's [44]. They checked the impact on an inverse matrix given a change in one element of a Leontief matrix. Given a limited number of known changes in the original Leontief matrix, the method was applied to get a new inverse without ever having to invert the new table. In those times the method was very important, because the researcher could not be supported by computer techniques (most calculations were performed by hand). In the following years this method served not only as a method of analysis, but also as a guideline in the conceptual development of stochastic input-output tables and models.

In the literature it is pointed out that the most important direct early contribution to input-output error analysis is in Evans [13]. This study aims at analytical and theoretical questions concerning the contribution of matrix errors to errors in the column vector of activity level estimates. In this study the problem of cumulative and noncumulative errors in the interindustry framework is discussed. Evans was one of the first economists who demonstrated that the error in using a structural matrix for time *t* in conjunction with a forecasted final demand vector depends on the degree to which the forecasted vector differs from a scalar multiple of the initial final demand vector. This finding was very important with respect to future research on IO matrix actualization methods e.g. RAS. Evans applied a simulation in order to analyze stochastic error. His investigations were based on a symmetric, multiplicative error structure. From his simulations it results that errors in structural matrices are not only noncumulative, but partly they cancel out.

Applying the mathematical formulation of Dewyer and Waugh [12] Christ [6] proved the impact of small absolute errors of the direct coefficients on the Leontief inverse. Christ found out that errors were selfcompensating and that the "*errors in the inverse... are probably not as much as an order of magnitude larger than their parent errors in the input-output matrix*" (Christ, [6], pp. 157-158). Although Leontief was in general no enthusiast for probabilistic methods in IO, a partial justification for this stochastic research direction can be found in his contribution: *Mathematical statistics will, however, become very useful, nay indispensable... after all the principal parts of the analytical structure have been erected and one can turn to a more precise fitting and mutual adjustment of its originally roughhewn components*. (Leontief, [13], p. 13).

A very important contributor to stochastic IO analysis was Quandt [36, 37]. In his study from 1958 for analytical and theoretical reasons Quandt explicitly rejected the notion of so called "true" coefficients. In his opinion the errors in coefficients can be treated in a stochastic framework because they show random properties. Quandt wrote "*they would exhibit variations even if they were obtained by taking exhaustive samples every time*" ([36], p. 156). Quandt in his contributions tried to answer the following important research questions:

1. Can analysts theoretically attach confidence limits to the solution?
2. Is it possible to calculate the moments of the distribution of the solution?
3. How are the moments of he distribution of the input coefficients related to the moments of the distributions of the elements of the inverse or of the solution?
4. What, if anything, can be said about the distribution of the input coefficients?
5. What is the meaning of the solution of a probabilistic Leontief system?

In order to answer these questions Quandt performed a simulation of a two-sector region by simplified error structure assumptions. He assumed symmetrical, additive, independent error structures with zero means. For the two-sector region model Quandt derived confidence intervals around gross output solutions.

Quandt's paper from 1959 was based on the assumption of an additive error structure for a three-sector model. From simulation results Quandt drew conclusions that errors in gross output have a tendency to be skewed. He suggested a lognormal distribution as an adequate description of the solution. According to Quandt a lognormal distribution is valid.

In 1960s, Theil [46] tried to establish the relationship of Leontief matrix coefficient errors to errors in the output vectors. In his contribution, the derived inverse matrix is equal to the "true" matrix plus a matrix of error terms. He expressed additive errors in the original Leontief matrix as additive errors in the Leontief inverse. These type of investigations were continued by Park [34] who distinguishes errors in type I and type II output multipliers from additive errors in the augmented input coefficient matrix. Park foud out that errors in the output vectors and multipliers can be expressed as linear combinations of errors in the various components of the original model.

3. The most important contributions to the stochastic approach in 70's and 80's.

Considerable progress in the stochastic analysis of input-output models could be observed in 70's and 80'. McCamley et al. [29] tried in the early 1970s to extend probabilistic analysis to input-output multipliers. The authors used the well known Rao's [38] variance approximation method, in order to derive a formula for the variance of sectoral income multipliers. They assumed row coefficient independence. By this simplification they obtained an analytical approximation based on the distribution of total outputs, the values of the Leontief inverse, and the covariance among elements in the same column of the direct coefficient table. The authors did not analyze theoretically error distributions. Their approach was demonstrated empirically solely.

The most important motivation for studies of the probabilistic nature of input-output models is based on the question of accuracy of IO modeling. This research stream is present in contributions concerning regional modeling by Morrison and Smith [33], McMenamin and Haring [29'], and Malizia and Bond [28]. Based upon contributions by Czamanski and Malizia [7] and Schaffer and Chu [41], Round [39] confronted the accuracy of nonsurvey techniques and survey-based models and did not find significant discrepancies. He paid for the first time much attention to measures of the distance between two interindustry matrices.

McMenamin and Haring [29'] studied in their contribution the accuracy of technical coefficients, total final demand, value added, total intermediate input and output, imports and exports, and multiplier estimates. But they did not focus directly on the accuracy of the total requirement coefficients (Jensen [25]).

In the 1970s researchers started to develop regionalization and recombination methods in an IO context. The important papers by Gerking and Miernyk (Gerking, [16], [17], [18]; Gerking and Pleeter, [19]; Miernyk, [30], [31]) are motivated (especially those by Gerking) by early work of Briggs [2]. Gerking focused on statistical estimation methods for direct coefficients. Miernyk contributed to the methods based only on survey data and professional judgment. Gerking's investigations by his methods were based on comparisons of traditionally calculated and statistically estimated direct coefficient tables. These contributions prompted hectic discussion (e.g., Brown and Giarratani, [4]; Hanseman and Gustafson, [23]; Hanseman, [22']). There followed from this debate increased interest and research activity in stochastic input-output analysis.

The properties of the interindustry framework were subject of intensive research started by Sebald's [42] study of the sensivity of large-scale input-output models to parametric uncertainties. Sebald was the first to investigate worst case percentage tolerances on solution elements, given uncertainty characteristics on model parameters. According to his findings negative tolerances were smaller on average than positive tolerances even though the perturbations were of the same magnitude. He established that tolerance amplification (greater uncertainty on the solution than on the model parameters) is predominant.

Sebald was the originator of the "Most Important Parameter" (MIP) concept. "Given a model solution and a known or assumed uncertainty on each parameter", the MIP problem seeks to "identify the model parameters whose uncertainty is responsible for a significant uncertainty in any element of the solution" (Sebald, [42], pp. 3-4). Due to Sebald a parameter is inverse important if "reasonable variations in its value can affect the solution in some significant way" (Sebald, [42], p. 23). Sebald's results are very important with respect to modern sensitivity analysis directly upon the inverse coefficients.

Bullard and Sebald [5] assumed uncertainty in the 1967 Bureau of Economic Analysis (BEA) accounts at various levels of aggregation. The authors imposed distribution of system parameters that would simulate the activities involved in compiling the table. They used lognormal, normal and "folded normal" distributions. These distributions were based on the characteristics of the mean and the level of certainty assigned to various parameters. Because the error structure captured most closely resembled a multiplicative structure, a mixture of error symmetry was derived. Changing levels of aggregation didnot produce "effectively change in the simulation output uncertainties" (Bullard and Sebald, [5], p. 37).

In line with the above investigations are those by West [47, 48]. He developed a method to rank the technical coefficients according to their importance. Economists received a method of allocating scarce resources for generating or updating input-output data (West recommended to focus on most important coefficients).

He established that multiplicative error terms, and that the absolute error in the *j*th column multiplier is a function not only of the size of the *j*th output multiplier, but also of the magnitude of the output multiplier corresponding to the row sector in which the original error was generated. Further, the error over all output multipliers from an error in one coefficient a_{ij} is the error in that coefficient weighted by the *i*th output multiplier and the *j*th row total of the inverse.

Studies by Simonovits [43] and Goicoechea and Hansen [21] are particularly significant. Taking into account contributions by Quandt and Evans, Simonovits analytically compared the expected value of the Leontief matrix with the expected value of its inverse. Simonovits [43] found that if all the elements of A are independent then $E[(I-A)^{-1}] \geq [I-E(A)]^{-1}$ and if all the coefficients are symmetrically distributed and the row and column sums deterministic, then at least one element of the inverse is underestimated and at least one overestimated. This inequality means that the expected value of the Leontief inverse is underestimated by what Simonovits understood the "practical estimator." These conclusions support Evans' assertion of compensating error effects.

The significant contribution of Goicoechea and Hansen has received much less attention than might have been expected. In this contribution technology coefficients and demand variables of the input-output framework are treated as random variables. The authors transformed the equation into a probabilistic inequality, asserting "that the number of times (expressed as a percentage) inter-industry use and final consumption are less than or equal to the output of sector *i* is $1-\alpha_i$" (Goicoeachea and Hansen, [21], p. 286). Here α_i is an error probability. The coefficients for each industry may have any (known) probability density function. To ensure nonnegativity conditions, there are arbitrarily chosen exponential density functions. All inequalities are transformed into a set of deterministic equivalents.This procedure allows the building of a system of nonlinear equations. In this framework the source and nature of the error structure receive little attention. However this approach enables statements about system structure, explicitly related to industry specific conditions, which are not possible under any of the earlier formulations.

In the 1980s interest in the stochastic approach to regional nonsurvey input-output models can be found in the work of Stevens and Trainer [45], Park et al. [35], Ganseman [14], Wibe [50], Garhart [15], and then Giarratani [20]. Stevens and Trainer performed a series of simulations based on multiplicative error structures and directed toward the relative contribution of errors in technical coefficients and regional purchase coefficients to impacts on multipliers. Stevens and Trainer based their experiments on theoretical matrices. Park et al. used an empirical table. The mentioned analysts established that errors in regional purchase coefficients (RPCs) were of greater importance than those in technical coefficients in affecting regional output multipliers. Ganseman applied stochastic simulation methods to input-output models. Wibe examined the distribution of coefficients of particular input-output models. In his study Garhart [15] investigated the relative contribution to multiplier error of RPCs, but under conditions of either purely additive error or a combination of additive and multiplicative error. Garhart came to the conclusion that analysts should interpret the results of Stevens and Trainer and Park et al. with caution. It results from his investigations that at least equal attention should be paid to technical coefficients as to regional purchase coefficients. Giarratani investigated structure of errors in IO context.

Contributions from the second half of the 80's by West [49] and Lahiri and Satchell [26] concern directly stochastic input-output models. These works are in a certain sense a continuation of the approach by Simonovits [43]. Lahiri and Satchel examined again the relationship between the expected value of the Leontief inverse and its true value when the direct coefficient table is a stochastic input-output matrix. A misinterpretation of Simonovits theoretical formulation led to some imprecisions. The authors used e.g. the terms *true* and *expected* interchangeably.

Jackson [24] applied in a simulation-based analysis full probability distributions for input-output coefficients. He claims that an alternative to considering the error problem as one of a lack of statistical coherence among true value, observed value, and estimator is to apply the full distribution of direct establishment level coefficients for each *i-j* combination. Although there is a true (if unknown) coefficient for any time period past, in the absence of other information, the best option for future intervals is to approximate these industry aggregates on the basis of the characteristics of the underlying distributions. Further, if we expect systematic variation within the populations of industry aggregates, then asymmetric distributions may be the norm.

Lahiri and Satchel [26] generalized some early results and provided some new ones. They demonstrated at least one case in which Simonovits' claim of underestimation (which is overestimation in Lahiri's framework) holds when the assumption of independence of error terms is relaxed (i.e., biproportionally stochastic, multiplicative, error terms). In addition, the combination of over– and underestimation (compensating effects) given row constraints is shown to hold true when considering either flows or technical coefficient tables. This result were not established by Simonovits. The mentioned contributions are important. However, the probability density function of the inverse was still unknown. In order to calculate the confidence intervals for the solution, we must know more about the error structure, especially the probability density function of direct coefficient errors, ε_{ij}.

West [49] directly investigated the problem of approximating multiplier density functions and moments. Assuming coefficient independence and small, normally distributed error terms, West derived a theoretical expression for the probability density of the deviations from an observed multiplier, given an estimate of the standard error of the input coefficient. West demonstrated that the expected value of this error term is positive and the multiplier distribution is positively skewed. In addition, his results provide the first derivation of theoretical multiplier confidence intervals.

The last two papers represent in the best way the state of stochastic input-output research in 1980s. Lahiri and Satchell offer the broadest generalizations concerning over– and underestimation. West's contribution is most important in terms of its practical utility and summarizes more than 30 years of work in stochastic input-output analysis.

There remain, of course, a great number of studies to be mentioned. With the exception of conclusive, but restrictive, results concerning over– and underestimation (restrictive in respect to the restrictive row constraints), coefficient interdependence has not received adequate attention. The nonlinearity of matrix inversion is a source of immense complexity when coefficient interdependence is allowed. Quandt [36] offered the strongest justification for the independence assumption. He claimed that coefficient column sums of (less than) unity do not necessarily imply interdependence. The assumption of interindustry (or cross-industry) independence is however somewhat arbitrary. Quandt himself was unwilling to defne this assumption strongly. This is supported by a qualifying footnote in the first of his two papers (Quandt, [36], p. 157).

From our point of view the question of relations in input-output models is very important. Usually a change in the total system exceeds the sum of the changes in the direct coefficients. Therefore, any method that examines the importance of single coefficients will capture only limited effects on the system.

Next there arises the question of temporal relationships. Assume, for example, that coefficient $a_{kl}(t)$ is ranked as the most important coefficient at time t. Following an updating procedure based on the initial coefficient rankings, $a_{kl}(t+1)$ may rank considerably lower. In the extreme case, assume that on the basis of coefficient rankings we scrutinize the top 5 percent of the coefficients from time t. After updating, and using the same algorithm, the now top ranked coefficient was not among the original 5 percent. Here arise the questions: How is inverse importance for each time interval related? Can changes in inverse importance be modeled?

The next question concerns the relation between the results of Lahiri and Satchell and West and the aggregation error. Especially interesting is the relation of the aggregation error and other errors assumed e.g. in sensitivity analyses. Moreover, one can easily notice that the distinction between error analysis in a deterministic model and error analysis in a stochastic model is not very clear. Other questions which arise in this context are: What will the implications be if the error structure is not symmetrical? How could the Leontief inverse affect this asymmetry?

Quandt's last question concerned the meaning of the solution of a probabilistic Leontief system. For those analysts who have viewed stochastic input-output models with respect to impact of random error structure derived from measurement and sampling, aggregation, or sales/purchases reconciliation methods, the meaning is easily generalized. Given an inability to gather precise data and to define and categorize activities adequately, we will always have a measure of imprecision in an input-output data base. Estimates of the character of this embedded imprecision enable us to make mathematical statements about the precision of the inverse solution. From an alternative perspective, the analyst's primary interest lies in the probability distribution of outcomes from future changes in the structure of final demand. The probability assessment is based on a knowledge and description of the tools developed in stochastic input-output studies. It provides a strong foundation on which can be built a probabilistic perspective.

4. Recent achievements in stochastic IO analysis

Gurgul [22], Ćmiel and Gurgul [9], [10] and [11] demonstrated that the observed instability of input-output and capital matrices in a dynamic Leontief model can be put into the framework of stochastic analysis. The instability of matrices in input-output analysis can be accompanied by stochastic time lags. This fact implies that a general solution to such a stochastic input-output model may be very complex. From the economic point of view the most interesting path of growth is the so called balanced growth path which represents a particular solution to dynamic input-output models. In the mentioned studies Ćmiel and Gurgul defined a stochastic balanced growth path. According to our knowledge their contributions are the first studies concerning stochastic dynamic IO models with time lags.

In 1997 Bródy stated that if flow matrix A is a positive and stochastic matrix and its size grows to infinity, then the matrix behaves like a single dyad, i.e. tends towards the equilibrium point or path very fast, possibly reaching the equilibrium path in one or a few steps of iteration. Bródy derived this conjecture from his empirical observation: "The greater the (random) matrix A is, the more elements (or sectors, or branches) it possesses, the smaller will be its second eigenvalue in relation to the maximal eigenvalue, and the faster will be its convergence to equilibrium". Białas and Gurgul [1'] demonstrated that Bródy's hypothesis is in general not true.

Molnar and Simonovits [32] gave a solution to a similar problem as Bródy formulated. They replaced Bródy's random framework with a deterministic one, and rescaled the matrices to have unit dominant roots. They considered the matrices of non-negative entries and normalized the matrices in such a way that each column sum was equal to 1. Such matrices are called "stochastic matrices" and have a Frobenius eigenvalue equal to 1. A well–known economic example of a stochastic matrix (in the context of input-output theory) is the matrix of flow coefficients, which represents a self-replacing system (static, closed Leontief system). This means that such a system is able to replace itself, but does not yield any surplus.

In their paper [1] Bidard and Schatteman try to justify Bródy's conjectures (Bródy [3]) and discuss some mathematical and economic aspects of the problem.

The stochastic nature of regional purchase coefficients has been justified in the study by Stevens et al. [44'] on econometric modeling of these coefficients. This line of research on uncertainty in regional models was continued in work by Rey et al. [40]. The authors examined the nature of uncertainty in integrated econometric and input-output regional models. They distungished 3 sources of uncertainty: input-output coefficient uncertainty, econometric model parameter uncertainty and econometric disturbance term uncertainty. They were unable by means of simulation experiments to answer the question which source of uncertainty is most important in integrated regional models.

5. Conclusions

Unlike any other economic model the statistical properties of the input-output model have received only token attention, whereas the more practical applications and theoretical extensions have received great emphasis. It is curious that analysts have not focused on the statistical character of such a widely used economic modeling framework. There are however, scattered though the massive input – output literature, attempts to capture the stochastic nature of the input – output modeling framework. In most studies the effects of error structures form the focus. In more recent literature there are attemps to model dynamic input-output models and integrated regional input-output models.

Our lack of knowledge about the coefficients of input-output and capital matrices can be taken into account by randomization of these matrices. The random approach can be also applied to parameters of time lags. Hence, balanced growth factors (and balanced growth paths) become also random. In order to compute the stochastic characteristics of these random values respective formulas can be derived. The next effort should concerned with the determining of distribution and stochastic characteristics of the Frobenius eigenvector in order to establish a stochastic balanced growth path.

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