Polish stock market and some foreign markets – dependence analysis by regime-switching copulas

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1. Introduction

The dependencies among international stock markets have been researched in many papers, especially after so-called "Black Monday" (October 1987). This issue is an important point for investors because, in recent years, global markets have become more and more integrated. This is a result of a tendency towards liberalization as well as deregulation in money and capital markets of both developed and developing countries. Moreover, these changes significantly reduced opportunities for international diversification.

The tendency of financial returns to exhibit asymmetric dependence is widely observed in financial markets. It means that in times of crisis, returns (hectic phase of stock market) tend to be more dependent than in the times of stability (quiet phase of stock market). This observation is significant in the respect to the risk of an international portfolio. Because of the increased dependence in the bad times, the investors might lose advantages of diversification when such benefits are the most valuable. Thus, international portfolios in reality may be more risky than the investors think. The occurrence of such asymmetric interdependence increases the cost of diversification with foreign stocks.

These contributions indicated that co-movements between stock markets have increased the probability for the national markets to be influenced by changes

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in the foreign ones. While extensive research examining the international stock market linkages in the USA, Europe, Japan and even the Pacific-Basin stock markets exist, there is no research on the linkages between the main world stock markets and the emerging markets in Central and Eastern Europe, in the context of their growing economic importance.

The goal in this article is to study the evidence of co-movements among the four stock markets, three in Europe, namely the German (Frankfurt Stock Exchange), Austrian (Vienna Stock Exchange) and Polish (Warsaw Stock Exchange); represented by main indices: DAX, ATX and WIG20 respectively and the well-known American index DJIA .

The remainder of the paper is organized in the following way. In section 2 we present the literature overview concerning the dependence concepts, including regime-switching models and copulas and discuss the recent contributions to the subject. In section 3 we overview the models and methodology applied. In the fourth section the dataset and results of computations are presented. Section 5 concludes the paper.

2. Literature overview

A low dependency between two markets implies a good opportunity for an investor to diversify his investments risk. Thus, based on the Gaussian assumption, an investor can significantly reduce his risk by balancing his portfolio with stocks from a foreign stock market. However, it has been observed that market crashes and financial crises often occur in different countries approximately during the same time period, even if the dependency measured by correlation is very low between these markets. Researchers have raised the question of a different dependence structure between markets with the same (pairwise) correlations. These dependence structures could increase, or decrease the diversification benefit compared to the normal distribution assumption. Therefore, the question of dependency among stock markets in time of globalization is a very important topic. The level of dependence between the stock markets can be measured through such variables as stock return, trading volume and volatility. Because of the drawbacks in traditional dependency, measures like correlation, in the recent finance literature, copulas are applied. However, the simplest methodology in investigations of interdependencies was based on causality notion and VAR model. For example, Eun and Shim [17] investigated the relationships among nine major stock markets including Australia, Canada, France, Germany, Hong Kong, Japan, Switzerland, the UK and the US by means of the Vector Autoregressive (VAR) Model. They reported that news in the US market has the most impact on the other markets. Lin *et al.* [32] studied the interdependence between the returns and volatility of Japan and the US market indices using data of high frequency (daytime and overnight returns). They drew conclusion that daytime returns in the US or Japan market were linked with the overnight returns in the other.

Kim and Rogers [28] applied the GARCH model to study the dynamic interdependence between the stock markets of Korea, Japan, and the US. They made a conclusion that the impact of Japanese and the US stock markets has increased since the Korean market became more open for foreign investors. Booth *et al.* [10] applied the EGARCH model in order to find strong interdependence among the Scandinavian stock markets i.e. Danish, Finnish, Norwegian and Swedish. They found that the significant dependence began with the so-called Thailand currency crisis, but it was not noticed after the Hong Kong crisis. Ng [37] found significant causality of the US and Japan stock market to six Asian markets, including : Hong Kong, Korea, Malaysia, Singapore, Taiwan and Thailand. Lee (comp. [47]) on the basis of wavelets technique, which he developed himself, applied to three developed markets US, Germany and Japan and two emerging markets Egypt and Turkey; found that changes in these developed markets have influenced the emerging markets. Antoniou *et al.* (comp. [47]) by application of the VAR-EGARCH model examined the interdependence among three EU markets namely Germany, France and the UK. The results confirmed the cointegration among the stock markets of those countries.

Sharkasi et al. [47] studied, by means of wavelet analysis, the evidence of global co-movements among seven stock markets, three in Europe (namely Irish, UK, and Portuguese), two in the Americas (namely US, and Brazilian) and two in Asia (namely Japanese and Hong Kong).

The papers by Ammermann and Patterson [2], Lim et al. [31], Lim and Hinich [30], Bessler et al. [6] or Bonilla et al. [9] focus on a different pattern of stock price development. They found out long random walk sub-periods which, by alternating with short ones, exhibited significant linear and/or nonlinear correlations. The contributors assumed that these serial dependencies have an episodic character. These serial dependencies were also responsible for the low performance of the forecasting models. The contribution by Nivet [38] dealt with the random walk hypothesis for Warsaw Stock Exchange. Worthington and Higgs [49] dealt with the efficiency on the Hungarian, the Polish, Czech and Russian stock markets. The authors claimed that only the Hungarian stock market followed the random walk. Gilmore and McManus [18] noticed the significant autocorrelations in some of the Central and Eastern European stock markets. Todea et al. [48] used the Hinich–Patterson windowed-test procedure in order to investigate the temporal persistence of linear and, especially, nonlinear interdependencies among six Central and Eastern European stock markets .

Asymmetry of dependence, was analysed by Longin and Solnik [33]. Analyzing the correlations between stock markets over a period of three decades, by means of the constant conditional correlation (CCC) model introduced by Bollerslev [7], the authors discovered that those correlations were not stable over the time period. Moreover, they tended to increase during more volatile periods and depended on some economic variables such as interest rates, buybacks or dividend yields. The authors, in order to document that extreme correlation, defined it as the correlation that existed between returns that were above a certain threshold. The thresholds were different for positive and negative returns. This method, based on extreme value theory, was showed in Ang and Chen [5]. The authors defined a test for asymmetric correlation that was based on comparison of empirical and model-based conditional correlations. They made conclusions that regime-switching models, from the models which were taken into account, were most suitable for modeling of asymmetry. Ang and Bekaert [3] and [4] estimated a Gaussian Markov switching model for international returns. They established two regimes: a bull regime with positive mean, low volatilities and low correlations; and a bear regime with negative returns, high volatilities and correlation.

Regime switching models were introduced in econometrics by Hamilton [21]. Nowadays, they are widely applied in finance. The contribution of Guidolin and Timmermann [19], [20] concerning with interest rate, was based on methodology of regime-switching models. The aforementioned authors used also a regime-switching model for international financial returns. In contribution of Pelletier [44] the regime switching modeling was applied to correlation. The marginals were modeled with the GARCH. The author assumed normal distribution. The model by Pelletier was the "intermediate" model between the constant conditional correlation (CCC) of Bollerslev [8] and the dynamic conditional correlation (DCC) model of Engle [15].

Patton [41] found a significant asymmetry in the dependence of financial returns not only in the marginal distributions, but also in the dependence structure. In his opinion, knowledge of asymmetric dependence allows to achieve significant advantages by a certain kind of investors which have no constraints imposed on the short-sales. Patton [42] and [43] was the first to introduce a theory for the applications of conditional copulas and time-varying models of bivariate dependence of coefficients in order to model foreign exchange rates. Jondeau and Rockinger [26] applied the skewed-t GARCH models for returns with univariate time-varying skewness. Finally, in order to measure the dependence between pairs of countries, they used a time-varying or a switching Gaussian, or Student t copula. The above mentioned authors and Hu [24], suggested to replace the unconditional margins of a copula with conditional margins coming from univariate GARCH models. This led to a special case of the so-called copula based multivariate dynamic (CMD) model. Klein at al. [29] performed an extensive simulation study, and came to conclusions that CMD models were flexible tools for investigating different times series with the GARCH structure for the squared residuals. They pointed out that the copula (mis-) specification should play a key role before the adaption of a CMD model.

Very recently, researchers have also started to combine copulas and regime switching models in bivariate financial data. Rodriguez (2007) and Okimoto (2008) estimated regime-switching copulas for pairs of international stock indices. Okimoto (2008) focused on the US-UK pair, whereas Rodriguez [46] worked with pairs of Latin American and Asian countries. They both applied methodology developed by Ramchand and Susmel [45] who imposed a structure where variances, means and correlations switched together. The authors used the two-variable system. The only exception was contribution by Garcia and Tsafak [17] who estimated a regime-switching model in a four-variable system of domestic and foreign stocks and bonds. They applied a mixture of bivariate copulas to model the dependence between all possible pairs of variables.

Model by Chollete et al. [13] can be seen as an extension of the Pelletier [44] model to the non-Gaussian case. The authors relaxed the Gaussian assumption, as it was well known that returns were not Gaussian, while retaining the intuitively appealing features of a regime switching structure for dependence. Instead of relying on the Gaussian assumption, they used canonical vines which were flexible multivariate copulas. They also tried to separate the asymmetry in the marginals from the one in the dependence. This could not have been done in a Gaussian switching model. Instead, they relied on copulas and used the flexibility which they had provided in modeling the marginals separately from the dependence structure. Therefore, the authors allowed the marginal distributions to be different from the normal by using the skewed *t* GARCH model of Hansen [23].

Chollete et al. [13] applied their model to a multivariate context. Thus, they made a step towards making this approach feasible for realistic applications. Moreover, they used the canonical vine copula, a new type of copula that was defined in respect to financial variables by Aas et al. (comp. Chollete [13]) and which allowed very general types of dependence.

Finally, the authors estimated the Value at Risk (VaR) and Expected Shortfall (ES) of an equally weighted portfolio for all models and compared them with the Gaussian Model. They found out that the VaR and ES of the canonical vine models were substantially higher than the Student *t* or Gaussian copula models, which implied that the inappropriate usage of the latter models could lead to the underestimation of the risk of a portfolio.

In order to highlight the observed asymmetric dependence in international financial returns, we estimated a bivariate copula based on a regime-switching model. We applied this model to returns from the main indices of the US, German, Austrian and Polish stock markets. The choice of copula, as we already mentioned, is important for the risk management, because it modifies the Value at Risk (VaR) and Expected Shortfall of international portfolio returns. Therefore, we will check dynamics of the interdependence between the mentioned indices. The main goal is to document changes in the dependency and the asymmetry in both quiet and hectic (bull or bear) phase in the stock markets.

3. Model and estimation

3.1. Models for marginal distribution

We use the GARCH(1,1) specification with skewed version of Student- t conditional distribution (Hansen) adopted to each of the stock market log-returns (demeaned and without autocorrelation). This is done to present the volatility clustering, fat tails and skewness in the series. Formally, we apply

$$\begin{aligned} y_{it} &= \sqrt{h_{it}}\,\varepsilon_{it} \\ h_{it} &= \omega_{it} + \alpha_i h_{it-1} + \beta_i \varepsilon_{it-1}^2 \\ \varepsilon_{it} &\sim St(v_i, \lambda_i) \end{aligned}$$

The density of skewed – t distribution is

$$g(z \mid v, \lambda) = \begin{cases} bc\left(1 + \dfrac{1}{v-2}\left(\dfrac{bz+a}{1-\lambda}\right)^2\right)^{\frac{-(v+1)}{2}} & dla\, z < -\dfrac{a}{b} \\ bc\left(1 + \dfrac{1}{v-2}\left(\dfrac{bz+a}{1+\lambda}\right)^2\right)^{\frac{-(v+1)}{2}} & dla\, z \geq -\dfrac{a}{b} \end{cases}$$

where $a = 4\lambda c\left(\dfrac{v-2}{v-1}\right)$, $b = \sqrt{1 + 3\lambda^2 - a^2}$, $c = \dfrac{\Gamma\left(\dfrac{v+1}{2}\right)}{\sqrt{\Pi(v-2)}\,\Gamma\left(\dfrac{v}{2}\right)}$.

The parameters $2 < v < \infty$ and $-1 < \lambda < 1$ control the kurtosis and skewness of distribution, respectively.

3.2. Copula based regime switching model

Our methodology is based on Hamilton [22], Aas et al. [1], Chen [12], Embrechts et al. [14], Jondeau et al. [27], McNeil et al. [36] and is similar to Chollette et al. [13]. We discuss two-state, first-order Markov switching process for s_t. Let $y_t = (y_{1t}, y_{1t})$ be a vector of filtered pairs of stock market returns and $Y_t = (y_t, y_{t-1}, y_{t-2}, \ldots)$ vector containing all observations obtained through date t. We define the density of returns at date t governed by regime j as

$$f\left(y_t \mid y_{t-1}, s_t = j\right) = c^{(j)}\left(F_1\left(y_{1t};\delta_1\right), F_2\left(y_{2t};\delta_2\right)\right) \cdot f_1\left(y_{1t};\delta_1\right) \cdot f_2\left(y_{2t};\delta_2\right)$$

where F_i oraz f_i ($i = 1,2$) are respectively marginal cumultative distribution functions and probability density functions of stock returns y_i with parameters $\delta_i = \left(\omega_i, \alpha_i, \beta_i, v_i, \lambda_i\right)$. The copula density $c^{(1)}$ is choosen in a way which allows to model asymptotic dependence in the tails. We use mixtures of Archimedean copulas (and its survival versions) along with two parameter copulas BB1, BB4 and BB7. Since one-parameter Archimedean copulas are well known we give solely definitions of used copulas BB1, BB4 and BB7 (see [25]). First of them is defined as follows:

$$C\left(u, v; \theta, \delta\right) = \left\{1 + \left[\left(u^{-\theta} - 1\right)^{\delta} + \left(v^{-\theta} - 1\right)^{\delta}\right]^{1/\delta}\right\}^{-1/\theta}$$

for $\theta > 0$, $\delta \geq 1$.

When $\delta = 1$ copula BB1 becomes Clayton copula and Gumbel copula is obtained as $\theta \to 0$. The tail dependence coefficients based on this copula are $\lambda_U = 2 - 2^{1/\delta}$ and $\lambda_L = 2^{-1/(\delta\theta)}$ Kendall's tau is equal to $1 - 2/\left(\delta\left(\theta + 2\right)\right)$.

BB4 copula belongs to Archimax class of copulas, combined of the extreme value and Archimedean classes (see [11]):

$$C\left(u, v; \theta, \delta\right) = \left\{u^{-\theta} + v^{-\theta} - 1 - \left[\left(u^{-\theta} - 1\right)^{-\delta} + \left(v^{-\theta} - 1\right)^{-\delta}\right]^{-1/\delta}\right\}^{-1/\theta}$$

for $\theta > 0$, $\delta \geq 0$.

When $\delta \to 0$ Clayton copula is obtained and as $\theta \to 0$ Galambos copula is obtained (non-Archimedean, extreme value copula). The upper and lower tail dependence coefficients are equal to $\lambda_U = 2^{-1/\delta}$ and $\lambda_L = \left(2 - 2^{-1/\delta}\right)^{-1/\theta}$, respectively. We did not find formula for Kendall's tau coefficient, so we performed the integration in the equation $\tau = 4E(C(U, V)) - 1$ using a double quadrature.

The last of two-parameter copulas used is BB7 (also called Joe-Clayton) copula:

$$C\left(u, v; \theta, \delta\right) = 1 - \left(1 - \left[\left(1 - \bar{u}^{\theta}\right)^{-\delta} + \left(1 - \bar{v}^{\theta}\right)^{-\delta} - 1\right]^{-1/\delta}\right)^{1/\theta}$$

where $\bar{u} = 1 - u$, $\bar{v} = 1 - v$ and $\theta \geq 1$, $\delta > 0$.

This copula exhibits the asymptotic dependence in the tail with $\lambda_L = 2^{-1/\delta}$ and $\lambda_U = 2 - 2^{-1/\theta}$. For $\theta = 1$ the copula reduces to the Clayton copula, while the $\delta \to 0$ Joe copula is obtained. As above Kendall's tau is computed using numerical integration.

The density $c^{(2)}$ refers to Gaussian copula. The $p_{ij} = P[s_t = j \mid s_{t-1} = i]$ is the probability that state i will be followed by state j. This probabilities are collected in the transition matrix

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix}.$$

3.3. Estimation

The estimation of a model is performed in two steps. The log likelihood function for set of data $\boldsymbol{Y} = (Y_1, Y_2, \ldots, Y_T)$

$$L\left(\boldsymbol{Y}; \delta, \theta\right) = \sum_{t=1}^{T} \log f\left(y_t \mid Y_{t-1}; \delta, \theta\right)$$

can be decomposed in two parts containing marginal densities L_m and the copula L_c. We have

$$L\left(\boldsymbol{Y}; \delta, \theta\right) = L_m\left(\boldsymbol{Y}; \delta\right) + L_c\left(\boldsymbol{Y}; \delta, \theta\right)$$

with

$$L_m(\boldsymbol{Y};\, \delta) = \sum_{t=1}^{T} \left[\log f_1\left(y_{1t} \mid Y_{1t-1};\, \delta_1 \right) + \log f_2\left(y_{2t} \mid Y_{2t-1};\, \delta_2 \right) \right]$$

$$L_c(\boldsymbol{Y};\, \delta,\, \theta) = \sum_{t=1}^{T} \left[\log \boldsymbol{c}\left(F_1\left(y_{1t} \mid Y_{1t-1};\, \delta_1 \right), F_2\left(y_{2t} \mid Y_{2t-1};\, \delta_2 \right);\, \theta \right) \right]$$

The set $Y_{it} = (y_{i1}, y_{i2}, ..., y_{it})$ contains returns of variable number $\boldsymbol{i}$ up to time $\boldsymbol{t}$, $\delta = (\delta_1,\, \delta_2)$ collects parameters of pairs of marginals, while $\theta = (\theta_1,\, \theta_2,\, \boldsymbol{P})$ collects copulas parameters along with transition probabilities. Using the maximum likelihood estimation we get

$$\hat{\delta}_i = \arg\max_{\delta_i} \sum_{t=1}^{T} \left[\log f_i\left(y_{it} \mid y_i^{t-1};\, \delta_i \right) \right]$$

and

$$\hat{\theta} = \arg\max_{\theta} L_c\left(\boldsymbol{Y};\, \hat{\delta},\, \theta \right).$$

To estimate parameters of the regime switching model we use the Hamilton filter:

$$\hat{\xi}_{t|t} = \frac{\hat{\xi}_{t|t} \odot \eta_t}{\mathbf{1}^T \left(\hat{\xi}_{t|t-1} \odot \eta_t \right)}$$

$$\hat{\xi}_{t+1|t} = \boldsymbol{P}^T \hat{\xi}_{t|t}$$

$$\eta_t = \begin{bmatrix} \boldsymbol{c}^{(1)}\left(F_1\left(y_{1t};\, \delta_1 \right),\, F_2\left(y_{2t};\, \delta_2 \right);\, \theta_1 \right) \\ \boldsymbol{c}^{(2)}\left(F_1\left(y_{1t};\, \delta_1 \right),\, F_2\left(y_{2t};\, \delta_2 \right);\, \theta_2 \right) \end{bmatrix}$$

where $\hat{\xi}_{t|t}$ and $\hat{\xi}_{t+1|t}$ are vectors that collect conditional probabilities $P[s_t = j|Y_t;\, \theta]$ and $P[s_{t+1} = j|Y_t;\, \theta]$ ($\odot$ means element-by-element multiplication, $\mathbf{1}$ is vector of 1s). The log likelihood function for observed data is defined as

$$L_c(\boldsymbol{Y};\, \delta,\, \theta) = \sum_{t=1}^{T} \log\left(\mathbf{1}^T \left(\hat{\xi}_{t|t-1} \odot \eta_t\right)\right).$$

The initial value $\hat{\xi}_{1|0}$ is specified as the limiting probabilities of the Markov process:

$$\hat{\xi}_{1|0} = \begin{bmatrix} \dfrac{1-p_{22}}{2-p_{11}-p_{22}} \\ \dfrac{1-p_{11}}{2-p_{11}-p_{22}} \end{bmatrix}.$$

4. Dataset and results

The dataset used in the research contains main indices of four countries: Germany (DAX), Poland (WIG20), US (DJIA) and Austria (ATX). The DJIA is the most important index of the largest stock market in the world. We assume that it has the greatest impact on the other stock markets. The German stock market index DAX from Frankfurt Stock Exchange is the index of the largest of the European Union's economy. It is widely assumed that the Austrian stock market index represented on Vienna Stock Market by main index ATX is strongly associated with German Stock Market Index. Moreover, Vienna Stock Exchange is the local rival in the Central and Eastern Europe of the Warsaw Stock Exchange. The Warsaw Stock Exchange represents an emerging market. After computing weekly log-returns, we get a sample of 804 observations over the period from 1995-01-13 to 2010-06-04. In the table 1 we present some main descriptive statistics.

Table 1

Descriptive statistics of weekly returns (period from 1995-01-13 to 2010-06-04)

	mean	std.	skewness	kurtosis	minimum	maksimum
DAX	0.0015	0.0337	–0.7435	7.7140	–0.2442	0.1257
WIG20	0.0014	0.0411	–0.1000	4.6137	–0.1664	0.1601
DJIA	0.0012	0.0250	–0.9982	10.7494	–0.2034	0.1005
ATX	0.0009	0.0324	–1.7557	19.9658	–0.3343	0.1797

In all of those cases financial series show non-normality. All returns have negative skewness (relatively low in the case of WIG20) and high kurtosis (very high in the case of returns from the Austrian Traded Index). The hypotheses of normality are rejected with high probabilities by the Jarque-Bera test. We use AR filter to remove autocorrelation from the series (with the exception of the Polish Stock Market Index) and apply the univariate GARCH(1,1) models with skewed-*t* conditional distribution. The results exhibit a lack of normality in raw returns and confirm stylized facts about stock market returns. The skewness parameter was not significantly different from 0 (with *p-value* equal to 0.2956) only in WIG20 series. The table 2 contains the final results (standard errors are given in brackets). In all of the cases unconditional variance ω is not statistically significant.

Table 2

Estimates of marginal models (GARCH(1,1) with skew – *t* distribution)

	DAX	**WIG20**	**DJIA**	**ATX**
ω	0.00007 (0.00004)	0.00011 (0.00009)	0.00008 (0.00006)	0.00007 (0.00005)
α	0.20021 (0.05486)	0.09843 (0.04557)	0.22331 (0.07156)	0.17240 (0.06708)
β	0.75896 (0.06433)	0.83131 (0.07335)	0.65643 (0.12601)	0.76753 (0.09651)
v	10.19309 (4.12384)	7.82445 (2.04678)	7.40179 (2.02597)	6.42837 (1.61125)
λ	–0.30043 (0.06175)	–0.05191 (0.04960)	–0.20783 (0.04924)	–0.22553 (0.04440)

The correctness of specified marginals is tested using GOF tests, including Kolmogorov-Smirnov, χ^2, Kuiper and Berkowitz, for uniformity of the Probability Integral Transform of margins. The computed *p-values* are reported in the table 3.

To be sure that our model should contain the copula with asymmetric dependence in the tails structure, we performed the likelihood ratio test based on the mixture of the Clayton copula and its rotated version (also Gumbel and rotated Gumbel), as described in Manner [34]. The tests are labeled as test 1 and test 2, respectively. The results confirmed our supposition in the most of cases (the table 4 contains *p-values*; small vaules of the probability indicate rejecting the hypothesis of the symmetry in dependence).

Table 3

Testing results for uniformity of PIT (*p-values*)

	DAX	WIG20	DJIA	ATX
K-S	0.60439	0.90537	0.86031	0.96001
χ^2	0.49282	0.71619	0.21163	0.59170
Kuiper	0.56907	0.90435	0.65143	0.82670
Berkowitz	0.9896	0.34661	0.99163	0.58599

Table 4

Results of testing asymmetry in dependence structure (*p-values*)

	DAX-WIG20	DAX-DJIA	DAX-ATX	WIG20-DJIA	WIG20-ATX	DJIA-ATX
test 1	0.12618	0.13558	0.00063	0.03057	0.00001	0.00001
test 2	0.08379	0.32142	0.00043	0.04941	0.00005	0.00001

We model the dependence structure using the regime switching model presented in sections 3.2 and 3.3. The selection of models that fit best is based on AIC and BIC criterions. For all pairs of series model with either BB4 or BB7 copula the first regime is preferred. In the cases of pairs DAX-WIG20 and DAX-DJIA we replaced the Archimedean copula with the Student–*t* copula (which exhibits symmetric tail dependence), but the values of information criterions were not decreased. Table 5 contains results of this estimation.

Parameters $\theta_1^{(1)}$ and $\theta_1^{(2)}$ refer to BB4 copula, while the parameter θ_2 is the correlation coefficient of the Gaussian copula. All parameters are highly significant (exceptions are correlation coefficients for WIG20-ATX with *p-value* 0.0836 and DJIA-ATX with 0.031).

We can observe high persistence of regimes. The first regime is more persistent (except from the pair DAX-WIG20). The interesting characteristics computed from the estimated transition probabilities are expected times of return of the process to regimes and the duration in regimes. The First of them is less for regime of "tail dependence". The ratios of expected times of return (first regime/second regime) range from 0.17 (DJIA-ATX) to 0.65 (WIG20-DJIA) (pair DAX-WIG20 is excluded here). The time of duration in the first regime is much longer than in the second one, and ranges from 1.5 to 5.9.

Table 5

Estimates of dependence structure

	DAX-WIG20	DAX-DJIA	DAX-ATX	WIG20-DJIA	WIG20-ATX	DJIA-ATX
Copula in first regime	BB4	BB4	BB7	BB4	BB4	BB7
$\theta_1^{(1)}$	0.657 (0.109)	0.661 (0.154)	1.293 (0.956)	0.591 (0.089)	0.823 (0.08)	1.105 (.517)
$\theta_1^{(2)}$	0.510 (0.078)	1.123 (0.12)	1.303 (0.578)	0.360 (0.066)	0.314 (0.122)	0.940 (.372)
θ_2	0.449 (0.027)	0.485 (0.059)	0.232 (0.054)	0.272 (0.05)	0.147 (0.085)	0.109 (.053)
p_{11}	0.998 (0.004)	0.972 (0.011)	0.998 (0.001)	0.996 (0.003)	0.994 (0.005)	0.994 (.002)
p_{22}	0.998 (0.002)	0.922 (0.02)	0.993 (0.005)	0.993 (0.007)	0.991 (0.013)	0.966 (.018)

The correctness of the copula specification is tested using the aforementioned tests applied to $C(U|V)$ and $C(V|U)$ (see [35]). If the copula of U given V (and vice versa) are uniformely distributed, the model is accepted. Table 6 contains results (*p-values*) of testing (first rows accompanied to cell containing pair of indices refer to tests for $C(U|V)$, second for $C(V|U)$).

Table 6

Testing of uniformity (*p-values*)

	K-S	χ^2	Kuiper	Berkowitz
DAX-WIG20	0.5278	0.5849	0.6078	0.9999
	0.6725	0.2224	0.4176	0.2915
DAX-DJIA	0.4574	0.325	0.5212	0.9839
	0.8028	0.485	0.6444	0.9762
DAX-ATX	0.2964	0.2432	0.0960	0.9258
	0.3892	0.4899	0.8448	0.4764

Tabela 6 cont.

	K-S	χ^2	Kuiper	Berkowitz
WIG20-DJIA	0.5743	0.4918	0.711	0.3182
	0.9606	0.5286	0.8824	0.9988
WIG20-ATX	0.8334	0.5181	0.5646	0.7311
	0.484	0.4073	0.2669	0.9609
DJIA-ATX	0.8085	0.1376	0.5499	0.9516
	0.3395	0.5370	0.4248	0.3181

In figure 1 we present smoothed probabilities of being in first regime.

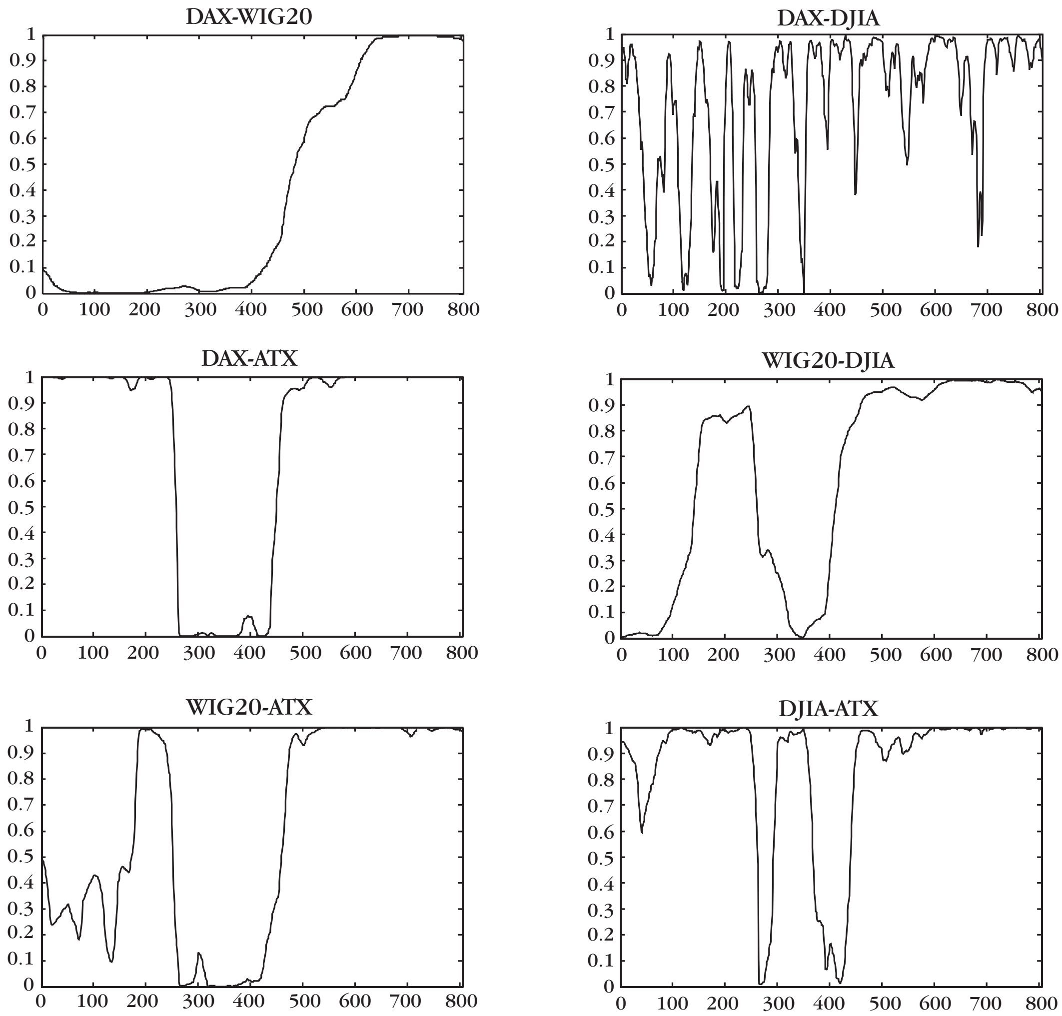

Figure 1. Smoothed probabilities (first regime)

As can be observed in almost all of the cases, starting from the beginning of year 2007 dependence structure is described by copulas from the first regime which exhibits asymmetry in the tail dependence. It could be interpreted as financial contagion effect caused by appearance of crisis in US. All financial markets begin to react on signals from American stock market. In the table 7 dependence measures are presented. We compute them as weighted average of Kendall's tau of choosen copulas and unconditional probabilites of first and second regime.

Table 7

Unconditional dependence parameters

	DAX-WIG20	DAX-DJIA	DAX-ATX	WIG20-DJIA	WIG20-ATX	DJIA-ATX
τ	0.342	0.520	0.384	0.259	0.252	0.304
λ_U	0.114	0.395	0.234	0.088	0.068	0.109
λ_L	0.191	0.413	0.472	0.213	0.284	0.409

The first observation is that the unconditional dependence in the lower tail is more than twice as strong as in the upper tail. It is a well-known fact that correlations calculated with different conditions can exhibit essential differences. It has been found that correlations depending on a large price or trading volume movements are higher than those based on small movements. This observation is known in the literature as the "correlation breakdown". In addition, the investors react stronger to bad news than to good news. Therefore, it is not surprising that the movement of prices in reaction to bad news is more pronounced than the response to good news. Thus, correlation breakdown effect impose higher correlation in the lower tail. The second observation is that the strongest dependence is between the German and the US stock markets, measured either by Kendall's tau or tail dependence coefficients (which are very close in magnitude). This observation is also understandable. Both stock markets belong to the largest stock markets in the world. The third feature of interdependences among listed stock exchanges is a high level of all measures for the pair DAX-ATX. This results come from the strong general dependence of the Austrian economy on German economy. The fourth finding is that probably in the bull phase (upper tail) or in the quiet phase, impulses from DJIA are transmitted to Warsaw and Vienna stock exchanges indirectly via the Frankfurt Stock Exchange, but in the bear phase (lower tail) this impact of NYSE is more direct, as pairwise correlation coefficients between DIJA and the rest of stock indices are in magnitude higher than the correlation coefficients between other indices under study.

5. Concluding remarks

The establishment of the structure of interdependences among financial markets plays a crucial role in making investment decisions. The lack of knowledge about interdependencies among financial markets and their changes over the time may be the source of false investment decisions. The copula based regime switching model is a flexible tool that makes possible to model the changing, over the time period, structure of interdependencies among capital markets. The estimation of the model parameters allows researcher to compute mean time of remaining the financial variable (e.g. equity price or trading volume) in a certain state and time of coming back to the previous state. The computations by means of copula based regime switching models delivered results concerning interdependencies among WIG20, DIJA, DAX and ATX which are in the line with the economic theory and previous findings of other authors.

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