

# Money Inventories in Search Equilibrium

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## Money Inventories in Search Equilibrium<sup>\*</sup>

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#### Abstract

The paper relaxes the one unit storage capacity imposed in the basic search-theoretic model of fiat money with indivisible real commodities and indivisible money. Agents can accumulate as much money as they want. It characterizes the stationary distributions of money and shows that for reasonable parameter values (e.g. production cost, discounting, degree of specialization) a monetary equilibrium exists. There are multiple stationary distributions of a given amount of money, which differ in their levels of economic activity and welfare. The model reveals two essential features of money. First, the marginal expected utility of money decreases. Second, there exists an endogenous upper bound on the money holdings: agents willingly produce and sell for money up to this bound and refuse to do so if their money holdings exceed this bound.

Keywords: Money, Search Equilibrium, Decentralized Trade

JEL: E00, D83, E52

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## 1 Introduction

The paper extends the basic search-theoretic model of fiat money with indivisible money and indivisible real commodities by considering a model in which agents can accumulate as much money as they want. There are two reasons for doing so. First, in the basic model developed by Kiyotaki and Wright (1991, 1993), agents have a one unit storage capacity, which does not allow them to hold more than one unit of money. Although this limitation seems to be unrealistic, the basic model is very useful to illustrate certain essential features of money and certain aspects of the exchange process without having to determine the distribution of money holdings. By removing this limitation, however, additional properties of money can be derived. It is shown that the marginal value of money decreases; the more money an agent owns, the smaller is the additional "insurance" against a lack of cash that additional money provides. Moreover, there exists an endogenous upper bound on the money holdings. Agents with money holdings below this bound willingly produce and sell for money while agents with money holdings above this bound cease production and selling. Second, relaxing the one unit storage technology allows one to study distributional issues. Without this constraint, there are multiple stationary distributions of a given quantity of money that differ in their levels of economic activity and welfare and, consequently, a redistribution of money affects real economic variables.

Following the seminal work of Kiyotaki and Wright (1991,1993), search models of fiat money have undergone rapid development. A number of papers incorporated bilateral bargaining into their models to derive relative prices endogenously. Articles by Berentsen, Molico, and Wright (1998), Trejos and Wright (1995), and Shi (1995) are the main examples. Other articles have addressed the limitation on the distribution of money imposed by restricting the inventories of individual agents. Two types of related research have been undertaken to remove these restrictions. On the one hand, Molico (1998) and Molico and Musalem (1996) remove simultaneously all restrictions on the accumulation of goods and money. Their models have all the "desirable features" but they are difficult to analyze analytically. The authors, therefore, use numerical methods to derive the bargaining solutions and to characterize the stationary distributions of goods and money.

On the other hand, other authors have studied less modified versions of the basic model. Articles by Corbae and Camera (1997), Green and Zhou (1998a), Li (1994), Shi (1997), and Zhou (1998) are the main examples. Shi considers a model with divisible money and divisible commodities in which the traders bargain bilaterally about how much to trade. Each household consists of many members who pool their money holdings each period, which eliminates aggregate uncertainty for households because the distribution of money is degenerate across households. In the symmetric equilibrium all households hold the same amount of money and, consequently, all commodities are exchanged at the same price.

Zhou and Green and Zhou investigate a model where agents can hold any arbitrary amount of divisible money and bargain bilaterally about the amount of money that is exchanged for one unit of an indivisible commodity. Their focus is on the existence of a monetary equilibrium when all commodities are traded at the same price. In Green and Zhou's model the agents obtain their production good at no cost. In Zhou's model production is costly, which gives rise to an endogenous bound on money holdings. In both models, money holdings are private information and when a buyer meets a seller, the seller makes a take-it-or-leave-it-offer to the buyer (seller-posting-price protocol). Green and Zhou and Zhou conjecture a pattern of exchange in which all trades take place at one price and then provide conditions under which such an equilibrium exists.

Corbae and Camera study a model with indivisible money where agents can accumulate

money up to an exogenous bound. When a buyer and a seller meet, the buyer makes a take-it-or-leave-it-offer about the quantity of the divisible consumption commodity to be exchanged for one unit of money. When the buyers make their offers they know the money holdings of the sellers. Similar to the articles of Green and Zhou and Zhou, Corbae and Camera conjecture a uniform pattern of exchange, and then provide conditions under which such an equilibrium exists.

Li studies the accumulation of commodity inventories in search equilibrium. In his model no agent can accumulate money and money and real commodities must be exchanged one for one. He finds that search efforts and inventory accumulation are too low relative to the social optimum and shows that an inflation tax can improve aggregate welfare.

In the first part of the paper the equilibrium behavior of the agents and the stationary distributions of money holdings are derived. Then, it is shown that for reasonable parameter values (e.g. production cost, discounting, degree of specialization) a monetary equilibrium exists. Finally, the existence of multiple stationary distributions of a given quantity of money is discussed.

### 2 The model

The economy consists of J > 2 distinct nonstorable commodities. The commodities are indivisible and come in units of size one. They are produced and consumed by a large number of infinitely lived agents who differ in their tastes for and in their ability to produce these commodities. Each agent has one favorite commodity, which is called his consumption good. Consuming one unit yields utility U > 0. Consuming one of the other commodities yields zero utility. No agent can produce his own consumption commodity; nevertheless, each agent has the ability to produce one of the other J commodities at cost C, U > C > 0. Particularly, an agent of type s produces commodity s and consumes commodity  $s + 1 \pmod{J}$ . Accordingly, the number of types is  $J^{1}$  In addition to the consumption goods, there is also an object called flat money. Fiat money comes in indivisible units of size one, is storable, and cannot be consumed by any agent. Agents can accumulate and hold as much money as they want.

All agents have to trade to get their consumption good. For this purpose they search for trading partners. Time is discrete and in each period each agent meets one other agent. The order of events in a match is as follows: 1) the traders decide whether to trade and what to trade; 2) the production takes place and the objects change hand; 3) the traders separate and the traded commodities are consumed; 4) when the agents trade real commodities for money they always exchange one unit of a real commodity for one unit of money.<sup>2</sup>

The economy is populated by a continuum of agents with mass 1, the measure of agents of each type is equal, and all agents in each period are randomly matched into pairs with equal probability. This symmetry implies that the agents meet other agents of a particular type with equal probability  $q = J^{-1}$ . Thus, q is the probability of meeting a producer of one's consumption commodity and of meeting an agent eager to get one's product. Denote by  $m_i(t)$  the measure of agents with money holdings i, at time t. In the steady state equilibrium,  $m_i(t) = m_i$ . Denote further by m a set of measures  $m_i$ ,  $i = 0, ..., \infty$ , satisfying  $\sum_{i=0}^{\infty} m_i = 1$ , by  $p_b$  the probability that an agent can buy his consumption commodity, and by  $p_s$  the probability that he can sell his product before he is matched. He can buy his consumption commodity if his partner can produce this good and if the partner is willing to do so for money. He can sell for money if his trading partner is a consumer of his production commodity and has money. Denote the expected utility (value function) of an agent with money holdings i by  $V^i$ . Then, if  $\delta$  is the discount factor, the value functions satisfy

$$V^{0} = p_{s} \max \left\{ \delta V^{1} - C, \delta V^{0} \right\} + (1 - p_{s}) \delta V^{0}$$

$$V^{i} = p_{b} \max \left\{ U + \delta V^{i-1}, \delta V^{i} \right\} + p_{s} \max \left\{ \delta V^{i+1} - C, \delta V^{i} \right\} + (1 - p_{s} - p_{b}) \delta V^{i}, i > 0.$$
(1)

For example, with probability  $p_s$  an agent with no money meets an individual willing to buy his product. The agent proposes (accepts) to sell his product if  $\delta V^1 - C \ge \delta V^0$  and refuses to do so if  $\delta V^1 - C < \delta V^0$ . With probability  $(1 - p_s)$  no trade takes place.

**Definition 1** A monetary equilibrium is a list  $\langle V, m \rangle$  such that:

i) V satisfies (1) taking m as given,
ii) m is stationary taking V as given, and
iii) V > 0.

According to the first part of Definition 1, the monetary equilibrium is a Nash equilibrium for a given set of measures m. The second part requires that the economy is in a steady state given the selling and buying activities induced by (1). The third part requires that money has value.<sup>3</sup>

**Lemma 1** In a monetary equilibrium the value functions  $satisfy^4$ 

a)  $0 \le V^i < \frac{p_b U}{1-\delta}, i \ge 0,$ b)  $V^i > V^{i-1}, i \ge 1,$ c)  $U + \delta V^{i-1} > \delta V^i, i \ge 1$ d)  $V^i - V^{i-1} > V^{i+1} - V^i, i \ge 1.$ 

According to a), in a monetary equilibrium the expected lifetime utility is never smaller than the expected utility in the nonmonetary equilibrium and it is strictly smaller than

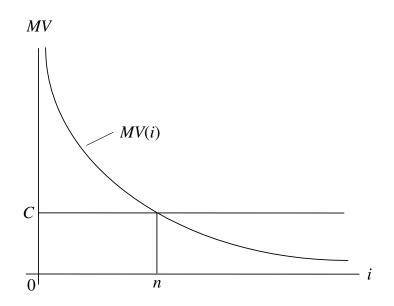


Figure 1: Decreasing marginal value of money.

the expected utility of an agent who has money without bounds. According to b), he expected utility of an agent increases with his money holdings. c) implies that an agent is always willing to buy his consumption commodity and d) that the marginal value of money decreases. The decreasing marginal value of money is due to the fact that the more money an agent owns, the longer is the expected length of time between the date he acquires additional money and the date he spends it. When agents discount future utilities the increasing expected length of time results in a decreasing marginal value of money.

**Proposition 1** In a monetary equilibrium there exists a bound n such that  $\delta V^{i+1} - C < \delta V^i$ ,  $i \ge n$ , and  $\delta V^{i+1} - C \ge \delta V^i$ , i < n.

Proof: Denote by V the expected utility of an agent that never reaches state i = 0, and note that  $\lim_{i\to\infty} V^i = V$ ,  $\lim_{i\to\infty} V^{i+1} = V$ , and  $\lim_{i\to\infty} (V^{i+1} - V^i) = 0$ . According to Lemma 1,  $V^i - V^{i-1} > V^{i+1} - V^i$ ,  $i \ge 1$ . This implies that there exists a n such that  $\delta V^{i+1} - C < \delta V^i$ ,  $i \ge n$ , and  $\delta V^{i+1} - C \ge \delta V^i$ ,  $n > i \ge 0$ . In a monetary equilibrium agents are willing to produce and sell for money if i < nand refuse to do so if  $i \ge n$ . This behavior is summarized in Figure 2 where the horizontal axis displays the money holdings i and the vertical axis the marginal value of money  $MV = \delta(V^{i+1} - \delta V^i)$ . Note also that Proposition 1 determines the probabilities of selling and buying. They are  $p_s = q(1 - m_0)$  and  $p_b = q(1 - m_n)$ , respectively.

Corollary 1 In a monetary equilibrium, the measures m satisfy

$$m_i = m_0^{\left(\frac{n-i}{n}\right)} m_n^{\left(\frac{i}{n}\right)} > 0, \ i = 0, ..., n, \ and \ m_i = 0, \ i > n.$$
 (2)

$$\sum_{i=0}^{n} m_0^{\left(\frac{n-i}{n}\right)} m_n^{\left(\frac{i}{n}\right)} = 1.$$
(3)

If 
$$m_0 \stackrel{\leq}{\equiv} m_n$$
, then  $m_i \stackrel{\leq}{\equiv} m_{i+1}, i = 0, ..., n - 1.$  (4)

Proof: Consider, first, the second part of (2). Assume that, to the contrary of Corollary 1,  $m_i > 0$  for some i > n. According to Proposition 1, agents with money holdings i > n do not produce and sell. However, they are willing to spend money. Hence,  $m_i > 0$ , i > n, is not stationary. Consider, next, the first part of (2). In a steady state, the measure of agents who leave state i equals the measure of agents that enter this state. All steady state conditions are summarized in the following n + 1 equations:

$$n: p_b m_n = p_s m_{n-1} \\ n-1: (p_b + p_s) m_{n-1} = p_b m_n + p_s m_{n-2} \\ \cdots \\ n-i: (p_b + p_s) m_{n-i} = p_b m_{n-i+1} + p_s m_{n-i-1} \\ \cdots \\ 1: (p_b + p_s) m_1 = p_b m_2 + p_s m_0 \\ 0: p_s m_0 = p_b m_1$$

$$(5)$$

One of the equations is redundant and the remaining equations simplify to

$$p_b m_i = p_s m_{i-1}, i = 1, ..., n, \text{ and}$$
(6)

$$m_i = \frac{m_{i-1}^2}{m_{i-2}}, i = 2, .., n$$
 (7)

Solving (7) recursively yields the first part of (2). Combine (2) with  $\sum_{i=0}^{n} m_i = 1$  to get (3). To see that (4) holds, divide  $m_i$  by  $m_{i+1}$  to get  $\frac{m_0}{m_n} = \left(\frac{m_i}{m_{i+1}}\right)^n$ . To see that  $m_i > 0$ , i = 0, ..., n, assume to the contrary that for some  $i \leq n$ ,  $m_i = 0$ . Then by (2), either  $m_0 = 0$  or  $m_n = 0$ . If  $m_0 = 0$ ,  $m_i = 0$ , i = 0, ..., n - 1, and  $m_n = 1$ , which implies that no agent is willing to sell for money and  $V^i = 0$  for all i contradicting Definition 1. If  $m_n = 0$ ,  $m_i = 0, i = 1, ..., n$ , and  $m_0 = 1$ , which implies that no agent has money and  $V^i = 0$  for all i contradicting Definition 1.

## **3** Existence

For any set of measures m a monetary equilibrium exists if m satisfies (2) and (3) and if no agent has an incentive to either increase his money holdings above n or to refuse to sell for money when i < n. Thus, a stationary distribution is a fixed point of the mapping  $f: D \to D$  where D contains all m. Because (2) and (3) are necessary conditions it is sufficient to study the set  $D_1 = \{m: m \text{ satisfies } (2) \text{ and } (3)\}$  and the mapping  $f: D_1 \to$  $D_1$ . To proceed denote by  $\overline{C}$  the value of C that solves  $C\delta^{-1} = V^n(m) - V^{n-1}(m)$  and by  $\widetilde{C}$  the value of C that solves  $C\delta^{-1} = V^{n+1}(m) - V^n(m)$ .

**Proposition 2** For any  $m \in D_1$  there exist critical values  $\overline{C}$  and  $\widetilde{C}$  constructed in the proof, with  $\overline{C} > \widetilde{C} > 0$ , such that the following is true: if  $\overline{C} \ge C > \widetilde{C}$ , m is stationary, and if  $C > \overline{C}$  or if  $\widetilde{C} \ge C$ , m is not stationary.

Proof: By construction of  $\overline{C}$  and  $\widetilde{C}$ , if  $\overline{C} \ge C > \widetilde{C}$ , agents with money holdings n-1 are willing to produce for money whereas agents with money holdings n are not willing to do so. Then, the concavity property d) of Lemma 1 implies that all agents with money holdings i < n are willing to produce for money and all agents with money holdings  $i \ge n$  are not willing to do so. Finally, m satisfies (2) and (3). This establishes that m is stationary if  $\overline{C} \geq C > \widetilde{C}$ . If  $C > \overline{C}$ , agents with money holding n - 1 are not willing to produce for money and if  $\widetilde{C} \geq C$ , they increase their money holdings above n and m is not stationary. To derive  $\overline{C}$  and  $\widetilde{C}$ , rewrite (1) to get

$$V^{0} = p_{s} \left( \delta V^{1} - C \right) + (1 - p_{s}) \delta V^{0}$$

$$V^{i} = p_{b} \left( U + \delta V^{i-1} \right) + p_{s} \left( \delta V^{i+1} - C \right) + (1 - p_{b} - p_{s}) \delta V^{i}, i = 0, ..., n - 1 \qquad (8)$$

$$V^{n} = p_{b} \left( U + \delta V^{n-1} \right) + (1 - p_{b}) \delta V^{n}.$$

(8) defines a second-order linear nonhomogeneous difference equation with constant coefficients and constant term and two initial conditions. The second equation is the difference equation and the first and the third equations are the two initial conditions. The solution is

$$V^{i} = \phi_{1}\lambda_{1}^{i} + \phi_{2}\lambda_{2}^{i} + \mu, i = 1, .., n.$$
(9)

where  $0 < \lambda_1 < 1$  and  $\lambda_2 > 1$  are the two distinct real roots

$$\lambda_{i} = \frac{1 - \delta \left(1 - p_{b} - p_{s}\right) \mp \sqrt{\left(1 - \delta \left(1 - p_{b} - p_{s}\right)\right)^{2} - 4\delta^{2} p_{b} p_{s}}}{2\delta p_{s}}, \ i = 1, 2.$$
(10)

 $\phi_1 = \frac{(1-\lambda_2)(p_b\lambda_2^n U - p_s\lambda_1 C)}{(1-\delta)\left(\lambda_2^{n+1} - \lambda_1^{n+1}\right)} \text{ and } \phi_2 = \frac{(1-\lambda_1)(p_s\lambda_2 C - p_b\lambda_1^n U)}{(1-\delta)\left(\lambda_2^{n+1} - \lambda_1^{n+1}\right)} \text{ are the coefficients, and } \mu = \frac{p_b U - p_s C}{1-\delta}$ is the particular integral. Use (9) to get<sup>5</sup>

$$\bar{C} = \frac{\lambda_1^n \lambda_2^n (\lambda_2 - \lambda_1) U}{\left(\lambda_2^{n+1} - \lambda_1^{n+1}\right) - \left(\lambda_2^n - \lambda_1^n\right)}$$
(11)

To derive  $\tilde{C}$  use  $V^{n+1} = p_b (U + \delta V^n) + (1 - p_b) \delta V^{n+1}$  and (8) to get

$$V^{n+1} - V^n = \left(\frac{\delta p_b}{1 - \delta + \delta p_b}\right) \left(V^n - V^{n-1}\right).$$
(12)

Then, use (12) and (9) to get

$$\tilde{C} = \frac{\lambda_1^n \lambda_2^n \left(\lambda_2 - \lambda_1\right) \delta p_b U}{\left(1 - \delta + \delta p_b\right) \left(\lambda_2^{n+1} - \lambda_1^{n+1}\right) - \delta p_b \left(\lambda_2^n - \lambda_1^n\right)} > 0$$
(13)

(12) implies that  $\overline{C} > \widetilde{C}$ . To see that  $\widetilde{C} > 0$ , note that the numerator and the denominator of (13) are positive because  $\lambda_2^{n+1} - \lambda_1^{n+1} > \lambda_2^n - \lambda_1^n > 0$ .

Proposition 3 For any bound n, a monetary equilibrium exists if

$$C < U \left(\frac{\delta q}{1 - \delta + \delta q}\right)^n.$$
(14)

Proof: First, note that for any  $m \in D_1$  the quantity of money is  $M = \sum_{i=0}^n im_i$  where  $M \in [0, n]$ . Then, define the sets  $D_2 = \{\langle M, n \rangle : M \in (0, n), n \in \{1, ..., \infty\}\}$  and  $D_3 = \{\langle m_0, n \rangle : m_0 \in (0, 1), n \in \{1, ..., \infty\}\}$  and note that there exists a one-to-one correspondence between  $D_1, D_2$ , and  $D_3$ . To see this note, first, that - given n - (3) defines a strictly decreasing function  $m_n = m_n (m_0, n)$  and, therefore,  $m = \{m_0, .., m_i (m_0, n), .., m_n (m_0, n), 0, ..\}$ , which establishes the existence of a one-to-one correspondence between the sets  $D_1$  and  $D_3$ . Next, note that - given n - (4) implies that M is a strictly decreasing function of  $m_0$ , which establishes the existence of a one-to-one correspondence between the sets  $D_2$  and  $D_3$ .

Next, (14) is derived. If  $M \to 0$ ,  $m_0 \to 1$ ,  $m_n \to 0$ ,  $p_b \to q$ ,  $p_s \to 0$ ,  $\lambda_2 \to \infty$ , and by l'Hopital's rule  $\lambda_1 \to \frac{\delta q}{1-\delta+\delta q}$ . Accordingly,  $\lim_{M\to 0} \bar{C} = U\left(\frac{\delta q}{1-\delta+\delta q}\right)^n$ . If  $M \to n$ ,  $m_0 \to 0$ ,  $m_n \to 1$ ,  $p_b \to 0$ ,  $p_s \to q$ ,  $\lambda_1 \to 0$ , and  $\lambda_2 \to \frac{1-\delta+\delta q}{\delta q}$ . Accordingly,  $\bar{C} \to 0$ . Finally, note that  $\bar{C}(M)$  is continuous in (0,n). This and the two limit points imply that if  $C < U\left(\frac{\delta q}{1-\delta+\delta q}\right)^n$ , there exists a M such that  $\bar{C}(M) = C$ . Denote this value by  $\bar{M}$  (see Figure 2 ). By construction,  $\langle \bar{M}, n \rangle$  satisfies  $\bar{C} \ge C > \tilde{C}$ . This establishes the existence of a stationary distribution m and, accordingly, the existence of a monetary equilibrium if  $C < U\left(\frac{\delta q}{1-\delta+\delta q}\right)^n$ .

Because  $\overline{C} > \widetilde{C}$ , typically a continuum of stationary distributions exist that differ in their value of M. To see this, consider Figure 2, which shows  $\overline{C}$  and  $\widetilde{C}$  as functions of

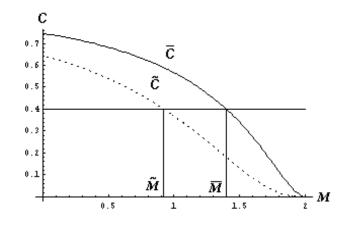


Figure 2:  $\overline{C}$  and  $\widetilde{C}$  as functions of M.

M when n = 2, U = 1,  $\delta = 0.95$ , and  $q = \frac{1}{3}$ . Given these values, any set of measures  $\langle M, n = 2 \rangle$  satisfying  $\overline{C} \geq C > \widetilde{C}$  is stationary. For example, if C = 0.4, all elements in the set  $\{\langle M, n = 2 \rangle : M \in (\widetilde{M}, \overline{M}]\}$  satisfy  $\overline{C} \geq C > \widetilde{C}$ . There are also multiple stationary distributions of a given amount of money, which differ in their levels of welfare and in their velocities of money. To see this, consider the parameters U = 1, C = 0.6,  $\delta = 0.95$ , and  $q = \frac{1}{3}$  and the following two stationary distributions of the same amount of money:  $\langle M = 0.5, n = 1 \rangle$  and  $\langle M = 0.5, n = 2 \rangle$ . To compare welfare, define welfare by  $W = \sum_{i=0}^{n} m_i V^i$ , which measures the ex ante expected utility of all agents (or a single agent) before money is distributed among them, and note that  $W(\langle M = 0.5, n = 1 \rangle) < W(\langle M = 0.5, n = 2 \rangle)$ . The higher welfare in the second equilibrium is due to the higher level of trade, that is, the higher level of the velocity of money.

#### 4 Discussion

The paper relaxes the one unit storage technology imposed in the search theoretic models of fiat money developed by Kiyotaki and Wright (1991, 1993). The following results emerge from the model. First, if the cost of production, the degree of impatience, and the degree of specialization are not too high, a monetary equilibrium exists. Particularly, if the degree of impatience vanishes, for almost all parameter values a monetary equilibrium exists. Second, the paper derives the stationary distribution of money holdings and derives two essential features of money that cannot be derived in the basic model. It is shown that the expected marginal value of money decreases and that in a monetary equilibrium there is an endogenous upper bound on the money holdings: agents willingly produce and sell for money up to this bound and refuse to do so if their money holdings exceed this bound. Third, there are multiple stationary distributions of a given quantity of money. The equilibria differ in their levels of economic activity and welfare and, consequently, a redistribution of money affects real economic variables.

While the paper investigates in detail the exchange process, it does not explore the determination of exchange rates. One way to derive relative prices is to let the traders bargain about the quantity of a real commodity that changes hands for one nondivisible unit of money. This extension would result in price dispersion because the bargaining solutions would depend on the money holdings of the bargainers. An other extension would be to consider nonstationary distributions and the convergence property of the model. See, for example, Green and Zhou (1998b) who consider the convergence property in a related model when there are no constraints on money holdings and traders are assumed to have overtaking-criterion preferences rather than discounting or Berentsen (1998) who studies the convergence property of the model for a given upper bound on money holdings.

#### Endnotes

<sup>1</sup>The complete specialization in production and in consumption, which eliminates barter transactions, simplifies the exposition, yet is by no means necessary. The assumption that real commodities are nonstorable, excludes the possibility that real commodities serve as a medium of exchange. See Kiyotaki and Wright (1989) who analyze commodity money in a related model.

<sup>2</sup>A seller who meets a buyer with several units of money could demand more than one unit of money for his product. However, it is assumed that the agents store their money holdings at home and visit the market with one unit of money only.

<sup>3</sup>This part is necessary because for any set of measures there is always a nonmonetary equilibrium where no agent accepts money. This equilibrium satisfies i) and ii) of Definition 1. However, it does not satisfy iii) because V = 0.

<sup>4</sup>The proof is available by request.

<sup>5</sup>Details of the derivation are available by request.

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