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Based on experimental and anecdotal evidence, an anchoring-adjusted option pricing model is developed in which the volatility of the underlying stock return is used as a starting point that gets adjusted upwards to form expectations about call option volatility. I show that the anchoring price lies within the bounds implied by risk-averse expected utility maximization when there are proportional transaction costs. The anchoring model provides a unified explanation for key option pricing puzzles. Two predictions of the anchoring model are empirically tested and found to be strongly supported with nearly 26 years of options data.

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Anchoring Heuristic in Option Pricing

A call option is widely considered to be a stock surrogate by market professionals[2]as their payoffs are closely related by construction, and move in sync perhaps more than any other pair of assets in the market. This built-in similarity between the payoffs of the two instruments has given rise to a popular trading strategy known as the "stock replacement strategy", which includes replacing the underlying stock in a portfolio with a corresponding call option.[3]

Where would you start if you need to form a risk judgment about a given call option? One expects the risk of a call option to be related to the risk of the underlying stock. In fact, a call option creates a leveraged position in the underlying stock. Hence, a natural starting point is the risk of the underlying stock, which needs to be scaled-up. Defining $\sigma(R_s)$ as the standard deviation of stock returns, and $\sigma(R_c)$ as the standard deviation of call returns, one expects the following to hold:

$$\sigma(R_c) = \sigma(R_s)(1 + A) \tag{0.1}$$

where $(1 + A)$ is the scaling-up factor.

The Black-Scholes model implies a particular value for A, which is equal to $\Omega - 1$ where Ω is the call price elasticity with respect to the underlying stock price. Any other value of A creates a risk-less arbitrage opportunity under Black-Scholes assumptions. For example, if $A < \Omega - 1$, the call option is overpriced in comparison with the cost of replicating portfolio if the risk premium on the underlying stock is positive. One may buy the replicating portfolio and write the call option to make riskless arbitrage profits. Hence, the only plausible value is the correct one.

[2] A few examples of experienced professionals stating this are:
http://web.streetauthority.com/templates/terms/optionsequity.asp
http://www.etf.com/sections/features-and-news/nations-1
http://finance.yahoo.com/news/stock-replacement-strategy-reduce-risk-142949569.html
http://www.optionsuniversity.com/blog/options-universitys-options-101-part-89/

[3] Jim Cramer, the host of popular US finance television program "Mad Money" (CNBC) has contributed to making this strategy widely known among general public.

If one allows for a little 'sand in the gears' in the form of transaction costs while keeping the other assumptions of the Black-Scholes model the same, a whole range of values of A different from the Black-Scholes implied value become plausible. That is, it becomes possible to support incorrect beliefs in equilibrium because such beliefs cannot be arbitraged away.

With transaction costs, the belief about A can be on either side of the correct value in equilibrium; however, there are strong cognitive and psychological reasons to expect that it falls consistently on one side. Due to their strongly similar payoffs, underlying stock volatility is a natural starting point, which is adjusted upwards to form call volatility judgments. Beginning with the early experiments in Tversky and Kahneman (1974), over 40 years of research has demonstrated that starting from (often self-generated) initial values, adjustments tend to be insufficient. This is known as the anchoring bias (see Furnham and Boo (2011) for a literature review). From self-generated anchors, adjustments are insufficient because people tend to stop adjusting once a plausible value is reached (see Epley and Gilovich (2006) (2001) and references therein). Hence, assessments remain biased towards the starting value known as the anchor.

A few examples illustrate the anchoring and adjustment approach quite well. Studies have shown that when asked, most respondents do not know the year George Washington became the first president of America. However, they know that it had to be after 1776 as the declaration of independence was signed in 1776. So, while guessing their answer, they tend to start with 1776 to which they add a few years and then stop once a plausible value has been reached. Such a cognitive process implies that their answers remain biased towards the self-generated anchor as people typically stop adjusting at the edge of plausible values on the side of the anchor.

Another example of this thinking process is provided by the freezing point of Vodka. Most respondents know that Vodka freezes at a temperature below the freezing point of water, so they start from 32 degree Fahrenheit (0 Celsius) and adjust downwards. However, such adjustments are typically insufficient (Epley and Gilovich (2006)).

What is the fair price of a 3-bedroom house in the Devon neighborhood of Chicago? If you know the sale price of a 4-bedroom house in the same neighborhood but in a slightly better location, then you would likely start with that price and adjust downwards for size, location, and other differences. The above examples illustrate that the defining feature of a self-generated anchor is its strong relevance to the problem at hand. Epley and Gilovich (2006) and others show that in tasks that do not involve such self-generated anchors, people tend to choose around the middle value within the set of values deemed plausible by them. Self-generated anchors change this as the process of adjustment stops once a value deemed plausible has been reached. Hence, location of the anchor relative to the set of plausible values becomes an important factor.

The presence of proportional transaction costs opens up an interval of plausible values for A with the correct value near the middle. So, without anchoring, one may expect the perceived value to equal the correct value (A^{BS}) on average. That is, $E[A] \approx A^{BS}$. However, with stock volatility as a self-generated starting point (anchor) due to the similarity between the two instruments, one expects the perceived value to fall firmly on the side of the anchor near the edge of plausible values. It means that the values of A between 0 and the correct value (A^{BS}) are of particular interest. In other words, anchoring implies a value of A such that $0 \leq A < A^{BS}$.

In this article, I investigate the implications of such an anchoring bias for option pricing. Surprisingly, I find that anchoring provides a plausible unified explanation for key option pricing puzzles. In addition, I test two predictions of the anchoring model regarding leverage adjusted option returns and find strong empirical support with nearly 26 years of options data. The puzzles addressed are:

1) The existence of the implied volatility skew in index options, and average implied volatility of at-the-money options being larger than realized volatility. (Rubinstein (1994))

2) Superior historical performance of covered-call writing. (Whaley (2002))

3) Worse-than-expected performance of zero-beta straddles. (Coval and Shumway (2001))

4) Average call returns appear low given their systematic risk. (Coval and Shumway (2001))

5) Average put returns are far more negative than expected. (Bondarenko (2014))

6) Leverage adjusted call and put index returns exhibit patterns inconsistent with the model. (Constantinides, Jackwerth, and Savov (2013))

As the volatility of call returns and the underlying stock returns are related in accordance with equation (0.1), their returns must also be related as follows:[4]

$$E[R_c] = E[R_s] + A \cdot (E[R_s - R_F]) \tag{0.2}$$

where R_F is the risk-free return.

It is possible to use equation (0.2) to test for the anchoring bias in a laboratory experiment. By creating the ideal conditions required for the Black-Scholes (binomial version) model to hold in the laboratory, one can see whether the Black-Scholes prediction holds or not. As the anchoring bias implies a lower value of A, the presence of anchoring would show up as insufficient addition to the stock return to arrive at call return. Furthermore, if the underlying cause of anchoring is the similarity between the payoffs, reducing the similarity (for example, by increasing the strike price) should weaken the pull of the anchor, thereby increasing the distance between the stock return and the call return.

Siddiqi (2012) (by building on the earlier work in Siddiqi (2011) and Rockenbach (2004)) tests the above predictions in a series of laboratory experiments and finds that indeed call options appear to be influenced by such analogical anchoring. The key findings in Siddiqi (2012) are:

1) The call average return is so much less than the Black Scholes prediction that the hypothesis that a call option is priced by equating its return to the return available on the underlying stock outperforms the Black-Scholes hypothesis by a large margin. Note, that there is not even a single observation in Siddiqi (2012) where a participant has priced a call option by equating its return to the return from the underlying stock. The call expected return is almost always larger than the stock expected return; however, it always remains far below the Black-Scholes prediction. That is, it does not deviate from the stock return by the required amount suggesting that analogical anchoring might be taking place.

2) If the similarity between the payoffs of call and its underlying stock is reduced, let's say by increasing the strike price, then the statistical performance of the hypothesis, $E[R_c] = E[R_s]$, weakens. That is, with weakly similar payoffs, participant price a call option in such a way that a large distance is allowed between $E[R_c]$ and $E[R_s]$. With anti-similar payoffs (such that of a put option

[4] The derivation is discussed in section 2.

and the underlying stock), the hypothesis, $E[R_{option}] = E[R_s]$ performs poorly. This suggests that analogical anchoring gets weaker as the analogy between the corresponding call and stock payoffs gets weaker, with the effect disappearing when the payoff similarity disappears.

The experimental evidence in Siddiqi (2012) suggests that anchoring bias directly influences the price of a call option, and much more strongly so at lower strikes due to the call and stock payoffs being closer to each other. There is no evidence of anchoring directly influencing the price of a put option as put and stock payoffs are anti-similar. Hence, the perception of similarity between a call option and its underlying stock appears to be the driving mechanism here. Note, that similar types of situational similarities have been used in the psychology and cognitive science literature to test for the influence of self-generated anchors on judgment (see Epley and Gilovich (2006) (2001) and references therein).

Given the above experimental evidence and widespread market belief that a call option is an equity surrogate, the next step is to take the anchoring idea to its logical conclusion by building an option pricing model and then testing its empirical predictions with field data. This is the contribution of this article. Surprisingly, I find that anchoring adjusted option pricing provides a unified explanation for 6 option pricing puzzles mentioned earlier. Furthermore, I test the predictions of the anchoring model and find strong support with nearly 26 years of options data.

Anchoring and adjustment bias, as modeled here, is a new deviation from the Black-Scholes framework different from the typical deviations that have been considered in the literature so far. There are several fruitful directions of research that have followed from relaxing the Black-Scholes assumptions; however, no existing reduced form option pricing model convincingly explains the puzzles mentioned earlier (see the discussion in Bates (2008)). The two most frequently questioned assumptions in the Black-Scholes model are: 1) The use of geometric Brownian motion as a description for the underlying stock price dynamics. 2) Assuming that there are no transaction costs.

Dropping the assumption of geometric Brownian motion to processes incorporating stochastic jumps in stock prices, stochastic volatility, and jumps in volatility has been the most active area of research. Initially, it was assumed that the diffusive risk premium is the only priced risk factor (Merton (1976), and Hull and White (1987)), however, more recent models now include risk factors due to stochastic volatility, stochastic jumps in stock prices, and in some cases also stochastic jumps in volatility. Based on the assumption that all risks are correctly priced, this approach empirically searches for various risk factors that could potentially matter (see Constantinides, Jackwerth, and Savov (2013), and Broadie, Chernov, & Johannes (2009) among others). These models typically attribute the divergences between objective and risk neutral probability measures to the free "risk premium" parameters within an affine model.[5] Bates (2008) reviews the empirical evidence on stock index option pricing and concludes that options do not price risks in a way which is consistent with existing option pricing models. Many of these models are also critically discussed in Hull (2011), Jackwerth (2004), McDonald (2006), and Singleton (2006).

Another area of research deviates from the Black-Scholes no-arbitrage approach by allowing for transaction costs. Leland (1985) considers a class of imperfectly replicating strategies in the presence of proportional transaction costs and derives bounds in which an option price must lie when there are proportional transaction costs. Hodges and Neuberger (1989) and Davis et al (1993) explicitly derive and numerically compute the bounds under the assumption that utility is exponential with a given risk aversion coefficient. Their bounds are comparable to Leland bounds. Constantinides and Perrakis (2002) show that expected utility maximization in the presence of proportional transaction costs implies that European option prices must lie within certain bounds. Constantinides and Perrakis bounds are generally tighter than the Leland bounds, and are considered the tightest bounds in the literature. I show that the anchoring price always lies within these bounds.

Yet another line of research provides evidence that options might be mispriced. Jackwerth (2000) shows that the pricing kernel recovered from option prices is not everywhere decreasing as predicted by theory, and concludes that option mispricing seems the most likely explanation. Others researchers (see Rosenberg and Engle (2002) among others) also find that empirical pricing kernel is oddly shaped. Constantinides, Jackwerth, and Perrakis (2009) find that S&P 500 index options are possibly overpriced relative to the underlying index quite frequently. Earlier, Shefrin and Statman (1994) put forward a structured behavioral framework for capital asset pricing theory that allows for systematic treatment of various biases.

Hirshleifer (2001) considers anchoring to be an *"important part of psychology based dynamic asset pricing theory in its infancy"* (p. 1535). Shiller (1999) argues that anchoring appears to be an important concept for financial markets. This argument has been supported quite strongly by recent empirical

[5] Bates (2003) writes, "To blithely attribute divergences between objective and risk-neutral probability measures to the free "risk premium" parameters in an affine model is to abdicate one's responsibilities as a financial economist" (page 400).

research on financial markets: 1) Anchoring has been found to matter in the bank loan market as the current spread paid by a firm seems to be anchored to the credit spread the firm had paid earlier (see Douglas, Engelberg, Parsons, and Van Wesep (2015)). 2) Baker, Pan, and Wurgler (2012) provide evidence that peak prices of target firms become anchors in mergers and acquisitions. 3) The role of anchoring bias has been found to be important in equity markets in how analysts forecast firms' earnings (see Cen, Hillary, and Wei (2013)). 4) Campbell and Sharpe (2009) find that expert consensus forecasts of monthly economic releases are anchored towards the value of previous months' releases. 5) Johnson and Schnytzer (2009) show that investors in a particular financial market (horse-race betting) are prone to the anchoring bias.

Given the key role that anchoring appears to play in financial decision making, it seems only natural that anchoring matters for option pricing too, especially given the fact that an option derives its existence from the underlying stock. After all, a call option is equivalent to a leveraged position in the underlying stock. Hence, a clear starting point exists. In fact, given the similarity of their payoffs, one can argue that it would be rather odd if the return on the underlying stock is ignored while forming return expectations about a call option. The Black-Scholes model by-passed the stock return only by assuming costless perfect replication. Clearly, transaction costs are a reality, and they create sufficient room for the anchoring and adjustment heuristic. This article shows that with anchoring, the resulting option pricing formula is almost as simple as the Black-Scholes formula. It seems that anchoring provides the minimum deviation from the Black-Scholes framework that is needed to capture the key empirical properties mentioned earlier. To my knowledge, the anchoring approach developed here is the simplest reduced-form framework that captures the key empirical features of option returns and prices. Furthermore, I show that the anchoring price always lies in a tight region between the Black-Scholes price and the upper bound implied by risk-averse expected utility maximization (within Constantinides and Perrakis (2002) bounds). Hence, transaction costs create sufficient room for anchoring and adjustment heuristic to matter.

There is strong evidence from cognitive science and psychology literature that similarity judgments are a key part of our thinking process. When faced with a new situation, people instinctively search their memories for something similar they have seen before, and mentally co-categorize the new situation with the similar situations encountered earlier to draw relevant lessons. This way of thinking, termed analogy making, is considered the core of cognition and the fuel and fire of thinking by prominent cognitive scientists and psychologists (see Hofstadter and Sander

(2013)). Hofstadter and Sander (2013) write, *"[…] at every moment of our lives, our concepts are selectively triggered by analogies that our brain makes without letup, in an effort to make sense of the new and unknown in terms of the old and known." (Hofstadter and Sander (2013), Prologue page1).*

Mullainathan et al (2008) argue that advertisers are well-aware of this propensity of similarity based judgments and routinely attempt to implant favorable anchors in our brains. That is why we get campaigns like, "we put silk in our shampoo". Obviously, putting silk in the shampoo does not do anything for hair; however, it does implant a favorable anchor in our brains. Imagine walking down the aisle of a supermarket, and remembering the "silk ad" upon seeing the shampoo. Mullainathan et al (2008) argue that, as silky is (presumably) a good quality in hair, the perception of the shampoo improves due to our propensity for similarity based judgments. If superficial anchors like this influence decision making by tapping into our innate ability of making judgments based on similarity, then genuine anchors (such as stock volatility for forming judgments about call volatility) should matter even more.

This article is organized as follows. Section 1 illustrates the anchoring and adjustment approach with a numerical example. Section 2 derives the option pricing formulas when there is anchoring. Section 3 shows that anchoring provides an explanation for the implied volatility skew. Section 4 shows that the anchoring model is consistent with the recent findings about leverage adjusted returns. Section 5 tests two predictions of the anchoring model with nearly 26 years of data and finds strong support. Section 6 shows that the anchoring model explains the superior performance of covered call writing as well as the inferior performance of zero-beta straddles. Section 7 concludes.

1. Anchoring Heuristic in Option Pricing: A Numerical Example

Imagine that there are 4 types of assets in the market with payoffs shown in Table 1. The assets are a risk-free bond, a stock, a call option on the stock with a strike of 100, and a put option on the stock also with a strike of 100. There are two states of nature labeled Green and Blue that have equal probability of occurrence. The risk-free asset pays 100 in each state, the stock price is 200 in the Green state, and 50 in the Blue state. It follows that the Green and Blue state payoffs from the call option are 100 and 0 respectively. The corresponding put option payoffs are 0 and 50 respectively.

Table 1				
	Bond	Stock	Call	Put
Green State	100	200	100	0
Blue State	100	50	0	50

What are the equilibrium prices of these assets? Imagine that the market is described by a representative agent who faces the following decision problem:

$$\max u(C_0) + \beta E[u(C_1)]$$

$$\text{subject to } C_0 = e_0 - S \cdot n_s - C \cdot n_c - P \cdot n_p - P_F \cdot n_F$$

$$\tilde{C}_1 = e_1 + \tilde{X}_s \cdot n_s + \tilde{X}_c \cdot n_c + \tilde{X}_p \cdot n_p + X_F . n_F$$

where C_0 and C_1 are current and next period consumption, e_0 and $e_1$are endowments, $S, C, P, and\ P_F$ denote the prices of stock, call option, put option, and the risk-free asset, and $\tilde{X}_s, \tilde{X}_c, \tilde{X}_p$ and X_F are their corresponding payoffs. The number of units of each asset type is denoted by n_s, n_c, n_p, and n_F with the first letter of the asset type in the subscript (letter F is used for the risk-free asset).

The first order conditions are:

$$1 = E[SDF] \cdot E[R_s] + \rho_s \cdot \sigma[SDF] \cdot \sigma(R_s)$$

$$1 = E[SDF] \cdot E[R_c] + \rho_c \cdot \sigma[SDF] \cdot \sigma(R_c)$$

$$1 = E[SDF] \cdot E\left[R_p\right] + \rho_p \cdot \sigma[SDF] \cdot \sigma\left(R_p\right)$$

$$1 = E[SDF] \cdot R_F$$

where $E[\cdot]$ and $\sigma[\cdot]$ are the expectation and standard deviation operators respectively, SDF is the stochastic discount factor or the inter-temporal marginal rate of substitution of the representative investor, and ρ denotes the correlation of the asset with the SDF.

One can simplify the first order conditions further by realizing that $\rho_c = \rho_s$, $\rho_p = -\rho_s$, and $\sigma(X_s) = \sigma(X_c) + \sigma(X_p)$. The last condition captures the fact that stock payoff volatility must either show up in call payoff volatility or the corresponding put payoff volatility by construction. One can see this in Table 1 as well where the stock payoff volatility is 75, the call payoff volatility is 50, and the put payoff volatility is 25. It follows that $\sigma(R_p) = \frac{S}{P} \cdot \sigma(R_s) - \frac{C}{p} \cdot \sigma(R_c)$. It is easy to see that with payoffs in Table 1, $\rho_s = -1.$ The first order conditions can be written as:

$$1 = E[SDF] \cdot \frac{125}{S} - \sigma[SDF] \cdot \frac{75}{S}$$

$$1 = E[SDF] \cdot \frac{50}{C} - \sigma[SDF] \cdot \frac{50}{C}$$

$$1 = E[SDF] \cdot \frac{25}{P} + \sigma[SDF] \cdot \left\{\frac{S}{P} \cdot \frac{75}{S} - \frac{C}{P} \cdot \frac{50}{C}\right\}$$

$$1 = E[SDF] \cdot \frac{100}{P_F}$$

Assume that the utility function is $lnC, \beta = 1, e_0 = e_1 = 500,$ and the representative agent must hold one unit of each asset to clear the market. The above first order conditions can be used to infer the following equilibrium prices: $P_F = 46.51163, S = 53.77907, C = 20.34884, and\ P = 13.0814$. It is easy to verify that both the put-call parity as well as the binomial model is satisfied. Replicating the call option requires a long position in $2/3$ of the stock and a short position in $1/3$ of the risk-free bond, so the replication cost is 20.34884, which is equal to the price of the call option. Replicating the put option requires a long position in $2/3$ of the risk-free asset and a short position in $1/3$ of the stock. The replication cost is 13.0814, which is equal to the price of the put option.

One can also verify that equations (0.1) and (0.2) hold with the correct value of A. The correct value of $A = \Omega - 1$. Here, $\Omega = \frac{S}{C} \cdot x$ where x is the number of shares of the stock in the replicating portfolio that mimics the call option. So, in this case, $\Omega = 1.761905.$ As $\sigma(R_s)$ is 1.394595 and $\sigma(R_c)$ is 2.457143, clearly $\sigma(R_c) = \sigma(R_s)(1 + A)$ with the correct value of A. Similarly, it is straightforward to verify that (0.2) also holds with $A = 0.761905$.

Next, I introduce the anchoring bias in the picture. The representative investor uses the volatility of the underlying stock as a starting point, which is scaled-up to form the volatility judgment about the call option with the scaling-up factor allowed to be different from the correct value. The first order conditions can be written as:

$$1 = E[SDF] \cdot \frac{125}{S} - \sigma[SDF] \cdot \frac{75}{S}$$

$$1 = E[SDF] \cdot \frac{50}{C} - \sigma[SDF] \cdot \frac{75}{S} \cdot (1 + A)$$

$$1 = E[SDF] \cdot \frac{25}{P} + \sigma[SDF] \cdot \left\{\frac{S}{P} \cdot \frac{75}{S} - \frac{C}{P} \cdot \frac{75}{S} \cdot (1 + A)\right\}$$

$$1 = E[SDF] \cdot \frac{100}{P_F}$$

The only thing different now is the risk perception of the call option. Instead of $\frac{50}{C}$, the volatility is estimated as $\frac{75}{S} \cdot (1 + A)$. If A takes the correct value, we are back to the prices calculated earlier. However, if there is anchoring bias, the results are different. Table 2 shows the equilibrium prices for $A = 0$ and $A = 0.5$ as well as for $A = 0.761905$, which is the correct value. Put-call parity continues to hold; however, both the call option and the put option are overpriced compared to what it costs to replicate them. With $A = 0$, both options are overpriced by an amount equal to 1.15741. A riskless arbitrage opportunity is created unless there is a little 'sand in the gears' in the form of transaction costs. Allowing for proportional transaction costs of θ in all 4 asset types (buyer pays $(1 + \theta)$ times price and seller receives $(1 - \theta)$ times price), the value that precludes arbitrage in both options is around 1.8%. With $A = 0.5$, the options are overpriced by 0.38466 and the value of θ that precludes arbitrage in both options is now much lower at 0.6%. The point is that transaction costs are a reality and even if rest of the assumptions in the Black-Scholes (binomial) model hold, the presence of transaction costs alone can support incorrect beliefs in equilibrium arising due to the anchoring bias.

Table 2			
	A=0	**A=0.5**	**A=0.761907 (Correct value)**
S	53.53009	53.69633	53.77907
P_F	46.26629	46.44007	46.51163
C	21.41203	20.70219	20.34884
P	14.17824	13.44593	13.0814
Put-Call Parity	Holds	Holds	Holds
Amount by which Call and Put are Overpriced	1.15741	0.38466	0
Perceived $\sigma(R_c)$: $\sigma(R_c) = \sigma(R_s)(1 + A)$	1.401081	2.095115	2.457143
Actual $\sigma(R_c)$: $\sigma(R_c) = \sigma(X_c)/C$	2.335136	2.415203	2.457143
Perceived $E[R_c]$: $E[R_c] = E[R_s] + A \cdot (E[R_s - R_F])$	2.335136	2.415203	2.457143
Actual $E[R_c]$: $E(R_c) = E(X_c)/C$	2.335136	2.415203	2.457143

Note that I have only considered one trading period. Increasing the frequency of trading increases the total transaction cost of replicating an option. It is well-known that in the continuous limit, the total transaction costs of perfect replication are unbounded. Other bounds have been proposed in the literature such as the Leland bounds (Leland (1985)), and the Constantinides and Perrakis bounds (Constantinides and Perrakis (2002)) by assuming imperfect replication and risk-averse expected utility maximization. I show, in section 2, that the anchoring price always lies within these proposed bounds, so it is difficult to see how arbitrage can work to mitigate the anchoring bias.

Table 2 shows that with $A = 0.5$, the perceived risk of the call option is $\hat{\sigma}(R_c) = 2.095115$ $\left(\sigma(R_s)(1 + A)\right)$, whereas the realized $\sigma(R_c)$ is 2.415203. With $A = 0$, the perceived and actual values are 1.401081 and 2.335136. The anchoring bias causes the risk of the call option to be underestimated. Due to the relationship between the volatilities of call, put, and the underlying stock, the put volatility is overestimated.

It is straightforward to verify that equation (0.2) continues to hold with the anchoring bias as well. Assuming that an SDF exists, one can use equation (0.2) directly to price the call option. If $A = 0$, the expected return of the call option (from (0.2)) is 2.335136. Dividing the expected payoff with the expected return results in 21.41203 which is the price of the call option calculated earlier. Similarly, one can verify that the same process yields the correct call price for $A = 0.5$ and $A = 0.761905$. In other words, once A is specified, the call expected return can be calculated directly from (0.2). The expected return can then be used to calculate the correct price of the call option. The corresponding price of the put option follows from put-call parity.

2. Anchoring Heuristic in Option Pricing

It is straightforward to realize that the anchoring approach does not depend on a particular distribution of the underlying stock returns. No matter which distribution one chooses to work with, the idea remains equally applicable. The Black-Scholes model assumes geometric Brownian motion for the underlying stock price dynamics. In this article, since the anchoring approach is illustrated in comparison with the Black-Scholes model, therefore I choose to build the anchoring model with the same set of assumptions. That is, I use the geometric Brownian motion in the continuous limit.

2.1 The anchoring adjusted model

Consider an exchange economy with a representative agent who seeks to maximize utility from consumption over two points, t and $t + 1$. At time t, the agent chooses to split his endowment between investments across N assets and current consumption. At time $t + 1$, he consumes all his wealth.

The decision problem facing the representative agent is:

$$\max u(C_t) + \beta E[u(C_{t+1})]$$

$$\text{subject to } C_t = e_t - \sum_{i=1}^{N} P_i \cdot n_i$$

$$\tilde{C}_{t+1} = e_{t+1} + \sum_{i=1}^{N} \tilde{X}_i \cdot n_i$$

where C_t and C_{t+1} are current and next period consumption, e_t and e_{t+1} are endowments, P_i is the price of asset i, n_i is the number of units of asset i, and $\tilde{X}_i$ is the associated random payoff at $t+1$.

In equilibrium, every asset must satisfy the following:

$$1 = E[SDF] \cdot E[R_i] + \rho_i \cdot \sigma[SDF] \cdot \sigma[R_i]$$

where SDF is the stochastic discount factor, ρ_i is the correlation of asset i's returns with the SDF, and $E[\cdot]$ and $\sigma[\cdot]$ are the expectation and standard deviation operators respectively.

Among assets, if there is a stock, a call and a put option on that stock with the same strike price, and a risk-free asset, then the following must hold in equilibrium:

$$1 = E[SDF] \cdot E[R_s] + \rho_s \cdot \sigma[SDF] \cdot \sigma(R_s)$$

$$1 = E[SDF] \cdot E[R_c] + \rho_c \cdot \sigma[SDF] \cdot \sigma(R_c)$$

$$1 = E[SDF] \cdot E\left[R_p\right] + \rho_p \cdot \sigma[SDF] \cdot \sigma\left(R_p\right)$$

$$1 = E[SDF] \cdot R_F$$

The above equations can be simplified further by realizing that $\rho_c = \rho_s$, $\rho_p = -\rho_s$, and $\sigma\left(R_p\right) = a \cdot \sigma(R_s) - b \cdot \sigma(R_c)$, where $a = \frac{S}{P}$ and $b = \frac{C}{P}$. Also, there exists an A such that $\sigma(R_c) = \sigma(R_s)(1 + A)$. The following simplified equations follow:

$$1 = E[SDF] \cdot E[R_s] + \rho_s \cdot \sigma[SDF] \cdot \sigma(R_s)$$

$$1 = E[SDF] \cdot E[R_c] + \rho_s \cdot \sigma[SDF] \cdot \sigma(R_s)(1 + A)$$

$$1 = E[SDF] \cdot E\left[R_p\right] - \rho_s \cdot \sigma[SDF] \cdot \sigma(R_s)\left(a - b(1 + A)\right)$$

$$1 = E[SDF] \cdot R_F$$

It follows that,

$$E[R_c] = E[R_s] + A \cdot \delta \tag{2.1}$$

$$E[R_p] = R_F - \delta[a - b(1 + A)] \tag{2.2}$$

where R_F is the risk-free rate, and $\delta = E[R_s] - R_F$.

If an SDF exists, then one can use equations (2.1) and (2.2) to price corresponding call and put option without knowing what the SDF is. Equations (2.1) and (2.2) specify the discount rates at which call and put payoffs are discounted to recover their prices. More simply, one can use equation (2.1) to price a call option, and then use put-call parity to price the corresponding put option.

Under the assumption of geometric Brownian motion, if the call volatility is correctly perceived (no anchoring bias), then A is equal to $\Omega - 1$ where Ω is the call price elasticity with respect to the underlying stock price. If one allows for a little 'sand in the gears' in the form of proportional transaction costs, then there always exist a threshold value of A less than $\Omega - 1$ above which arbitrage profits cannot be made, so incorrect beliefs can be supported in equilibrium. Denoting the threshold value by $\bar{A}$, it follows that anchoring bias implies a plausible value of A in the following range: $\bar{A} \leq A < \Omega - 1$.

In the next subsection, I derive the closed form expressions for option prices in the continuous limit of the geometric Brownian motion. We will see that these expressions are almost as simple as the Black-Scholes formulas.

2.2. Anchoring adjusted option pricing: The continuous limit

The continuous time version of (2.1) is:

$$\frac{1}{dt}\frac{E[dC]}{C} = \frac{1}{dt}\frac{E[dS]}{S} + A_K \cdot \delta \tag{2.3}$$

Where $C, and\ S,$ denote the call price, and the stock price respectively. $A_K \geq 0$.

If the risk free rate is r and the risk premium on the underlying stock is δ (assumed to be positive), then, $\frac{1}{dt}\frac{E[dS]}{S} = \mu = r + \delta$. So, (2.3) may be written as:

$$\frac{1}{dt}\frac{E[dC]}{C} = (r + \delta + A_K \cdot \delta) \tag{2.4}$$

The underlying stock price follows geometric Brownian motion:

$$dS = \mu S dt + \sigma S dZ$$

where dZ is the standard Brownian process.

From Ito's lemma:

$$E[dC] = \left(\mu S \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{\sigma^2 S^2}{2}\frac{\partial^2 C}{\partial S^2}\right) dt \tag{2.5}$$

Substituting (2.5) in (2.4) leads to:

$$(r + \delta + A_K \cdot \delta)C = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}(r + \delta)S + \frac{\partial^2 C}{\partial S^2}\frac{\sigma^2 S^2}{2} \tag{2.6}$$

(2.6) describes the partial differential equation (PDE) that must be satisfied if anchoring determines call option prices.

To appreciate the difference between the anchoring PDE and the Black-Scholes PDE, consider the expected return under the Black-Scholes approach, which is given below:

$$\frac{1}{dt}\frac{E[dC]}{C} = \mu + (\Omega - 1)\delta \tag{2.7}$$

Substituting (2.5) in (2.7) and realizing that $\Omega = \frac{S}{C}\frac{\partial C}{\partial S}$ leads to the following:

$$rC = \frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{\partial^2 C}{\partial S^2}\frac{\sigma^2 S^2}{2} \tag{2.8}$$

(2.8) is the Black-Scholes PDE.

In the Black-Scholes world, the correct adjustment to stock return to arrive at call return is $(\Omega - 1)\delta$. By substituting $A = (\Omega - 1)$ in (2.6), it is easy to verify that the Black-Scholes PDE in (2.8) follows. That is, with correct adjustment (2.6) and (2.8) are equal to each other. Clearly, with insufficient adjustment, that is, with the anchoring bias $(A < (\Omega - 1))$, (2.6) and (2.8) are different from each other.

The proportional transaction costs are allowed. Such transaction costs capture brokerage fee, bid-ask spread, transaction taxes, market impact costs etc. Following the standard practice in the literature, I allow for proportional transaction costs in the underlying stock. We assume that the buyer of the stock pays$(1 + \theta)S$, and the seller receives $(1 - \theta)S$, where S is the stock price, and θ is a constant less than one. There are no transaction costs in the bond or the option market. [6]

Constantinides and Perrakis (2002) derive a tight upper bound (CP upper bound) on a call option's price in the presence of proportional transaction costs. In particular, under mild conditions, they show that risk-averse expected utility maximization implies the following upper bound[7]. It is the call price at which the expected return from the call option is equal to the expected return from the underlying stock net of round-trip transaction cost:

$$\bar{C} = \frac{(1 + \theta)S \cdot E[C]}{(1 - \theta)E[S]}$$

It is easy to see that the anchoring price is always less than the CP upper bound. The anchoring prone investor expects a return from a call option which is at least as large as the expected return from the underlying stock. That is, with anchoring, $\frac{E[C]}{C} \geq \frac{E[S]}{S} > \frac{(1-\theta)E[S]}{(1+\theta)S}$. It follows that the maximum price under anchoring is: $\bar{C}_A < \bar{C} = \frac{(1+\theta)S \cdot E[C]}{(1-\theta)E[S]}$.

Re-writing the anchoring PDE with the boundary condition, we get:

$$(r + \delta + A_K \cdot \delta)C = \frac{\partial C}{\partial t} + \frac{\partial C}{\partial S}(r + \delta)S + \frac{\partial^2 C}{\partial S^2}\frac{\sigma^2 S^2}{2} \tag{2.9}$$

where $0 \leq A_K < (\Omega - 1)$, and $C_T = max\{S - K, 0\}$

Note, that the presence of the anchoring bias, $A_K < (\Omega - 1)$, guarantees that the CP lower bound is also satisfied. The CP lower bound is weak and lies substantially below the Black-Scholes

[6] Adding transaction costs in bond and option markets makes the results even stronger.
[7] See Proposition 1 in Constantinides and Perrakis (2002).

price. As the anchoring price is larger than the Black-Scholes price, it follows that it must be larger than the CP lower bound.

One way to interpret the anchoring approach is to think of it as a mechanism that substantially tightens the Constantinides and Perrakis (2002) option pricing bounds. The anchoring price always lies in the narrow region between the Black-Scholes price and the CP upper bound.

There is a closed form solution to the anchoring PDE. Proposition 1 puts forward the resulting European option pricing formulas.

Proposition 1 ***The formula for the price of a European call is obtained by solving the anchoring PDE. The formula is*** $\boldsymbol{C = e^{-A_K \cdot \delta(T-t)}\left\{SN\left(d_1^A\right) - Ke^{-(r+\delta)(T-t)}N\left(d_2^A\right)\right\}}$ ***where*** $\boldsymbol{d_1^A = \frac{ln(S/K)+(r+\delta+\frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}}$ ***and*** $\boldsymbol{d_2^A = \frac{ln\left(\frac{S}{K}\right)+\left(r+\delta-\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}}$

Proof.

See Appendix A. ■

Corollary 1.1 ***There is a threshold value of*** $\boldsymbol{A_K}$ ***below which the anchoring price stays larger than the Black-Scholes price.***

Proof.

See Appendix B.

■

Corollary 1.2 ***The formula for the anchoring adjusted price of a European put option is***

$$\boldsymbol{Ke^{-r(T-t)}\left\{1 - e^{-\delta(T-t)}N(d_2)e^{-A_K(T-t)}\right\} - S\left(1 - e^{-A_K \cdot \delta(T-t)}N(d_1)\right)}$$

Proof.

Follows from put-call parity.

■

As proposition 1 shows, the anchoring formula differs from the corresponding Black-Scholes formulas due to the appearance of δ, and A_K. Note, as expected, if the marginal investor is risk neutral, then the two formulas are equal to each other.

It is interesting to analyze put option returns under anchoring. Proposition 2 shows that put option returns are more negative under anchoring when compared with put option returns in the Black Scholes model.

Proposition 2 ***Expected put option returns (for options held to expiry) under anchoring are more negative than expected put option returns in the Black Scholes model as long as the underlying stock has a positive risk premium.***

Proof.

See Appendix D.

■

Proposition 2 is quite intriguing given the puzzling nature of empirical put option returns when compared with the predictions of popular option pricing models. Chambers et al (2014) analyze nearly 25 years of put option data and conclude that average put returns are, in general, significantly more negative than the predictions of Black Scholes, Heston stochastic volatility, and Bates SVJ model. See also Bondarenko (2014). Hence, anchoring offers a potential explanation.

Clearly, expected call return under anchoring is a lot less than what is expected under the Black Scholes model due to the anchoring bias. Empirical call returns are found to be a lot smaller given the predictions of the Black Scholes model (see Coval and Shumway (2001)). Hence, anchoring appears to be consistent with the empirical findings regarding both call and put option returns.

3. The Implied Volatility Skew with Anchoring

If anchoring determines option prices (formulas in proposition 1), and the Black Scholes model is used to infer implied volatility, the skew is observed. For illustrative purposes, the following parameter values are used: $S = 100, T - t = 0.25\ year, \sigma = 15\%, r = 0\%, and\ \delta = 4\%.$

An anchoring prone investor uses the expected return of the underlying stock as a starting point that gets adjusted upwards to arrive at the expected return of a call option. Anchoring bias implies that the adjustment is not sufficient to reach the Black-Scholes price.

As long as the adjustment made is smaller than the adjustment required to reach the Black-Scholes price, the implied volatility skew is observed. To reach the Black-Scholes price, it must be: $ln\left(\frac{SN(d_1^A) - Ke^{-(r+\delta)(T-t)}N(d_2^A)}{SN(d_1) - Ke^{-r(T-t)}N(d_2)}\right) \cdot \frac{1}{(T-t)} = \bar{\bar{A}}_K \cdot \delta$. For the purpose of this illustration, assume that the actual adjustment is only a quarter of that.

Table 3 shows the Black-Scholes price, the anchoring price, and the resulting implied volatility. The skew is seen. Table 1 also shows the CP upper bound and Leland prices for various trading intervals by assuming that $\theta = 0.01$. The anchoring price lies within a tight region between the Black-Scholes price and the CP upper bound. Furthermore, implied volatility is always larger than actual volatility. Consistent with empirical findings, it is straightforward to see that regressing actual volatility on implied volatility leads to implied volatility being a biased predictor of actual volatility with the degree of bias rising in the level of implied volatility.

The observed implied volatility skew also has a term-structure. Specifically, the skew tends to be steeper at shorter maturities. Figure 1 plots the implied volatility skews both at a longer time to maturity of 1 year and at a considerably shorter maturity of only one month. As can be seen, the skew is steeper at shorter maturity (the other parameters are $S = 100,\ \sigma = 20\%, r = 2\%, \delta = 3\%, and\ A_K = 0.25\bar{\bar{A}}_K$).

Table 3						
K/S	Black-Scholes	Anchoring	Implied Volatility	CP Upper Bound	Leland Price (Trading Interval 1/250 years)	Leland Price (Trading Interval 1/52 years)
0.95	6.07	6.61	18.32%	6.93	8.36	7.59
1.0	2.99	3.37	16.88%	3.57	5.42	4.50
1.05	1.19	1.40	16.21%	1.50	3.28	2.38

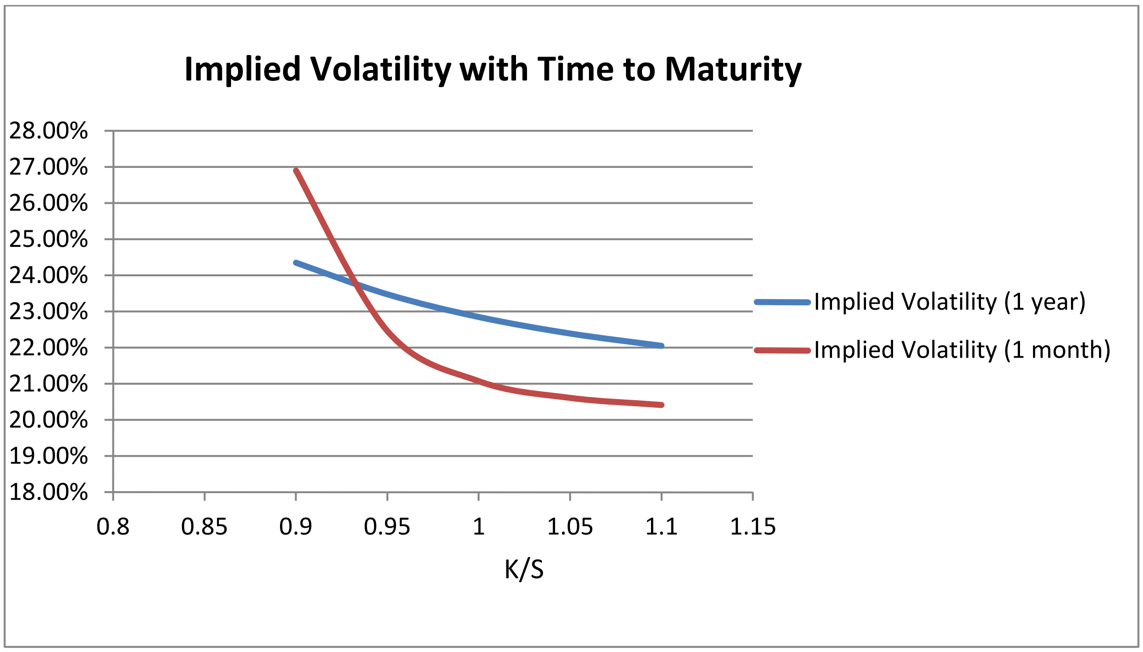

Figure 1

Hence, anchoring provides a relatively straightforward explanation for the implied volatility puzzle. In fact, with anchoring, the skew arises even within the simplest framework of geometric Brownian motion. Next section shows that anchoring is consistent with the recent empirical findings in Constantinides et al (2013) regarding leverage adjusted index option returns.

4. Leverage Adjusted Option Returns with Anchoring

Leverage adjustment dilutes the beta risk of an option by combining it with a risk free asset. Leverage adjustment combines each option with a risk-free asset in such a manner that the overall beta risk becomes equal to the beta risk of the underlying stock. The weight of the option in the portfolio is equal to its inverse price elasticity w.r.t the underlying stock's price:

$$\beta_{portfolio} = \Omega^{-1} \times \beta_{call} + (1 - \Omega^{-1}) \times \beta_{riskfree}$$

where $\Omega = \frac{\partial Call}{\partial Stock} \times \frac{Stock}{Call}$ (i.e price elasticity of call w.r.t the underlying stock)

$$\beta_{call} = \Omega \times \beta_{stock}$$

$$\beta_{riskfree} = 0$$

$$=> \beta_{portfolio} = \beta_{stock}$$

When applied to index options, such leverage adjustment, which is aimed at achieving a market beta of one, reduces the variance and skewness and renders the returns close to normal enabling statistical inference.

Constantinides, Jackwerth and Savov (2013) uncover a number of interesting empirical facts regarding leverage adjusted index option returns. Table 4 presents the summary statistics of the leverage adjusted returns (Table 3 from Constantinides et al (2013)). As can be seen, four features stand out in the data: 1) Leverage adjusted call returns are lower than the average index return. 2) Leverage adjusted call returns fall with the ratio of strike to spot. 3) Leverage adjusted put returns are typically higher than the index average return. 4) Leverage adjusted put returns also fall with the ratio of strike to spot.

The above features are sharply inconsistent with the Black-Scholes/Capital Asset Pricing Model prediction that all leverage adjusted returns must be equal to the index average return, and should not vary with the ratio of strike to spot. Using their dataset, Constantinides et al (2013) reject the Capital Asset Pricing Model. In this section, I show that the anchoring adjusted option pricing model, developed in this article, provides a unified explanation for the above findings. Furthermore, in section 5, we test two predictions of the anchoring model with nearly 26 years of leverage adjusted index returns and find strong empirical support.

Table 4 (Table 3 from Constantinides et al (2013))

Average percentage monthly returns of the leverage adjusted portfolios from April 1986 to January 2012. For comparison, average monthly return on S&P 500 index is 0.86% in the same period.

	Call						Put					
K/S	90	95%	100%	105%	110%	Hi-Lo	90	95%	100%	105%	110%	Hi-Lo
Average monthly returns												
30 days	0.49	0.42	0.21	0.03	-0.02	-0.51	2.18	1.66	1.07	0.80	0.75	-1.43
(s.e)	*0.24*	*0.24*	*0.24*	*0.23*	*0.22*	*0.17*	*0.36*	*0.32*	*0.29*	*0.27*	*0.26*	*0.20*
90 days	0.51	0.44	0.37	0.31	0.21	-0.30	1.15	1.10	0.91	0.81	0.74	-0.40
(s.e)	*0.24*	*0.24*	*0.24*	*0.24*	*0.24*	*0.11*	*0.33*	*0.31*	*0.29*	*0.27*	*0.27*	*0.14*
90-30	0.03	0.02	0.16	0.28	0.23		-1.04	-0.55	-0.16	0.00	-0.01	
(s.e)	*0.02*	*0.02*	*0.03*	*0.06*	*0.11*		*0.11*	*0.07*	*0.03*	*0.02*	*0.02*	

Section 4.1 considers leverage adjusted call returns under anchoring and shows that anchoring provides an explanation for the empirical findings. Section 4.2 does the same with leverage adjusted put returns.

4.1 Leverage adjusted call returns with anchoring

Applying leverage adjustment to a call option means creating a portfolio consisting of the call option and a risk-free asset in such a manner that the weight on the option is Ω_K^{-1}. It follows that the leverage adjusted call option return is:

$$\Omega_K^{-1} \cdot \frac{1}{dt} \cdot E\frac{[dC]}{C} + (1 - \Omega_K^{-1})r \tag{4.1}$$

Substituting from (2.4) and realizing that anchoring implies that $A_K = m \cdot (\Omega_K - 1)$ where $0 \leq m < 1$, (4.1) can be written as:

$$\delta\{m \cdot (1 - \Omega_K^{-1}) + \Omega_K^{-1}\} + r \tag{4.2}$$

From (4.2) one can see that as the ratio of strike to spot rises, leverage adjusted call return must fall. This is because Ω_K rises with the ratio of strike to spot (Ω_K^{-1}falls).

Note that call price elasticity w.r.t the underlying stock price under the anchoring model is:

$$\Omega_K = \frac{S}{\left(SN(d_1^A) - Ke^{-(r+\delta)(T-t)}N(d_2^A)\right)} \cdot N(d_1^A) \quad (4.3)$$

Substituting (4.3) in (4.2) and simplifying leads to:

$$R_{LC} = \mu - \delta \cdot \frac{K}{S} \cdot e^{-(r+\delta)(T-t)} \cdot \frac{N(d_2^A)}{N(d_1^A)} \cdot (1-m) \quad (4.4)$$

R_{LC} denotes the expected leverage adjusted call return with anchoring. Note if $m = 1$, then the leverage call return is equal to the CAPM/Black-Scholes prediction, which is $R_{LC} = \mu$. With anchoring, that is, with $0 \leq m < 1$, the leverage adjusted call return must be less than the average index return as long as the risk premium is positive. Hence, the anchoring model is consistent with the empirical findings that leverage adjusted call returns fall in the ratio of strike to spot and are smaller than average index returns.

Figure 2 is a representative graph of leverage adjusted call returns with anchoring ($r = 2\%, \delta = 5\%, \sigma = 20\%$). Apart from the empirical features mentioned above, one can also see that as expiry increases, returns rise sharply in out-of-the-money range. This feature can also be seen in Table 4.

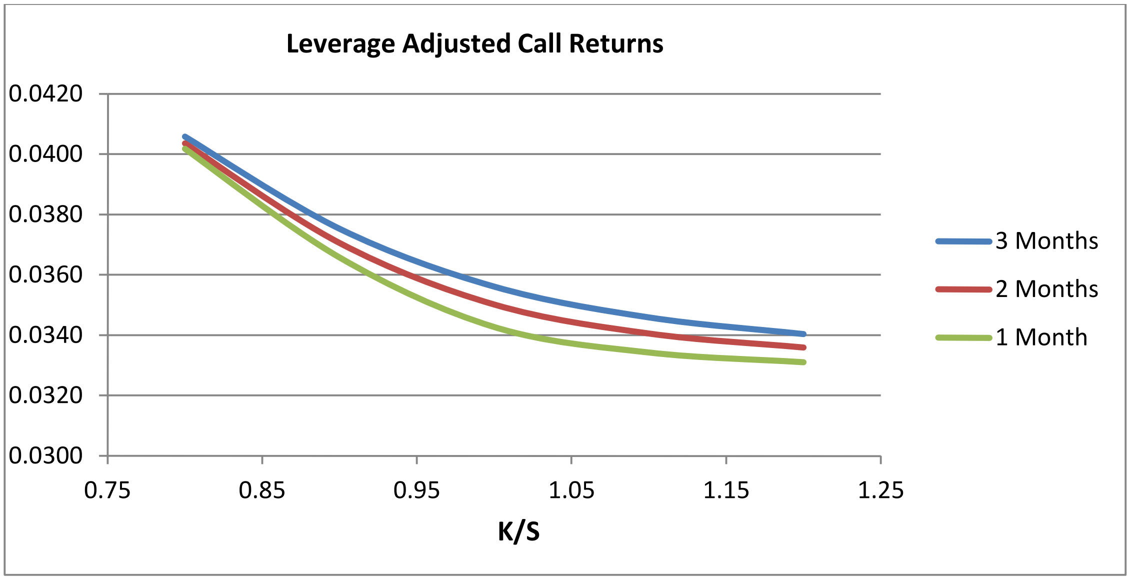

Figure 2

4.2 Leverage adjusted put returns with anchoring

Using the same logic as in the previous section, the leverage adjusted put option return with anchoring can be shown to be as follows:

$$R_{LP} = \mu + \delta \cdot \frac{K}{S} \cdot e^{-(r+\delta)(T-t)} \cdot \frac{e^{-A_K(T-t)} \cdot N(d_2^A)}{\left(1-e^{-A_K(T-t)}N(d_1^A)\right)} \cdot (1-m) \tag{4.5}$$

As can be seen from the above equation, the CAPM/Black-Scholes prediction of $R_{LP} = \mu$ is a special case with $m = 1$. That it, the CAPM/Black-Scholes prediction follows if there is no anchoring bias. With the anchoring bias, that is, with $0 \leq m < 1$, leverage adjusted put return must be larger than the underlying return if the underlying risk premium is positive. It is also straightforward to verify that anchoring implies that R_{LP} falls as the ratio of strike to spot increases.

Figure 3 is a representative plot of the leverage adjusted put returns for 1, 2, and 3 months to expiry ($r = 2\%, \delta = 5\%, \sigma = 20\%$). One can also see that returns are falling substantially at lower strikes as expiry increases. This feature can also be seen in the data presented in Table 4.

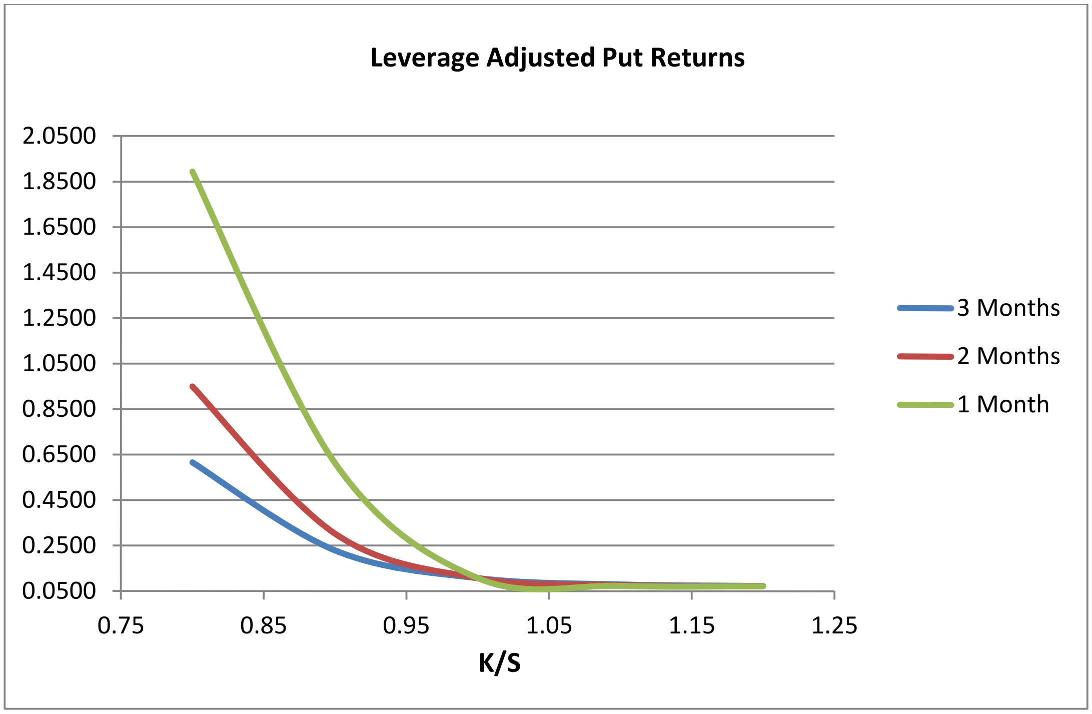

Figure 3

5. Empirical Predictions of the Anchoring Model

By considering Figure 3 and Figure 2, the following two predictions of the anchoring model follow immediately[8]:

Prediction 1. *At low strikes* $(K < S)$, *the difference between leverage adjusted put and call returns must fall as the ratio of strike to spot increases at all levels of expiry.*

Figure 3 shows a very sharp dip in leverage adjusted put returns at low strikes. The dip is so sharp that it should dominate the difference between put and call returns in the low strike range. At higher strikes, the decline in put and call returns is of the same order of magnitude.

Prediction 2. *The difference between leverage adjusted put and call returns must fall as expiry increases at least at low strikes.*

[8] Technical proofs of these predictions are available from the author upon request.

Figure 3 shows that put returns fall drastically with expiry at low strikes. They rise marginally at higher strikes with expiry. Figure 2 shows that call returns rise with expiry throughout and relatively more so at higher strikes. It follows that the difference between put and call returns should fall with expiry at least at low strikes if not throughout.

Next, I use the dataset developed in Constantinides et al (2013) to test these predictions. Constantinides et al (2013) use Black-Scholes elasticities evaluated at implied volatility for constructing leverage adjusted returns. As the anchoring model elasticities are very close to Black-Scholes elasticities evaluated at implied volatility, the dataset can be used to test the prediction of the anchoring model. The dataset used in this paper is available at http://www.wiwi.uni-konstanz.de/fileadmin/wiwi/jackwerth/Working_Paper/Version325_Return_Data.txt

The construction of this dataset is described in detail in Constantinides et al (2013). It is almost 26 years of monthly data on leverage adjusted S&P-500 index option returns ranging from April 1986 to January 2012.

5.1. Empirical findings regarding prediction 1

Wilcoxon signed rank test is used as it allows for a direct observation by observation comparison of two time series. The following procedure is adopted:

1) The dataset has the following ratios of strikes to spot: 0.9, 0.95, 1.0, 1.05, and 1.10. For each value of strike to spot, the difference between leverage adjusted put and call returns is calculated.
2) Pair-wise comparisons are made between time series of 0.9 and 0.95, 0.95 and 1.0, 1.0 and 1.05, and 1.05 and 1.10. Such comparisons are made for each level of maturity: 30 days, 60 days, or 90 days.
3) The first time series in each pair is dubbed series1, and the second time series in each pair is dubbed series 2. That is, for the pair, 0.9 and 0.95, 0.9 is Series 1, and 0.95 is Series 2.
4) For each pair, if the prediction is true, then Series 1>Series 2. This forms the alternative hypothesis in the Wilcoxon signed rank test, which is tested against the null hypothesis: Series 1 = Series 2

Table 5			
Put minus Call Return (Monthly) Leverage Adjusted (April 1986 to January 2012) Series 1 Strike (%spot)	Put minus Call Return (Monthly) Leverage Adjusted (April 1986 to January 2012) Series 2 Strike (%spot)	Maturity (days)	Wilcoxon Signed Rank Test Null Hypothesis: Series 1=Series 2 Alternate Hypothesis: Series 1>Series 2 P-Value
0.9	0.95	30	5.62883E-14
0.95	1	30	2.33147E-14
1	1.05	30	0.095264801
1.05	1.1	30	0.378791967
0.9	0.95	60	2.23715E-06
0.95	1	60	2.08904E-11
1	1.05	60	1.31059E-09
1.05	1.1	60	0.978440796
0.9	0.95	90	0.002029759
0.95	1	90	2.84604E-08
1	1.05	90	0.10253709
1.05	1.1	90	0.696743837

Table 5 shows the results. As can be seen from the table, when call is in-the-money, the difference between leverage adjusted put and call returns falls with strike to spot at all levels of expiry (Series 1 is greater than Series 2). Hence, null hypothesis is rejected, in accordance with prediction of the anchoring model. As expected, the p-values are quite large for out-of-the-money call range, so null cannot be rejected for out-of-the-money call range.

5.2 Empirical findings regarding prediction 2

To test prediction 2, the procedure adopted is very similar to the one used for prediction 1:

1) For each level of strike to spot, the following pair-wise comparisons are made: 30 days vs 60 days, 60 days vs 90 days, 30 days vs 90 days.

2) The first time series in each pair is dubbed Series 1, and the second time series is labeled Series 2. If prediction 2 is true, then Series 1 > Series 2. This forms the alternate hypothesis against the null: Series 1 = Series 2.

3) Wilcoxon signed rank test is conducted for each pair.

Table 6 shows the results. As can be seen, the null is rejected in favor of the alternate hypothesis throughout. Hence, both the predictions of the anchoring model are strongly supported in the data.

Table 6			
Put minus Call Ret. (Monthly) Leverage Adjusted (April 1986 to January 2012) Series 1 Maturity (Days)	**Put minus Call Ret. (Monthly) Leverage Adjusted (April 1986 to January 2012) Series 2 Maturity (Days)**	**Strike (% spot)**	**Wilcoxon Signed Rank Test Null Hypothesis: Series1=Series2 Alternate: Series1>Series2 P-Value**
30	60	0.9	0.0000
60	90	0.9	0.0000
30	90	0.9	0.0000
30	60	1.1	0.0125
60	90	1.1	0.0033
30	90	1.1	0.0020

6. The Profitability of Covered Call Writing with Anchoring

The profitability of covered call writing is quite puzzling in the Black Scholes framework. Whaley (2002) shows that BXM (a Buy Write Monthly Index tracking a Covered Call on S&P 500) has significantly lower volatility when compared with the index, however, it offers nearly the same return as the index. In the Black Scholes framework, the covered call strategy is expected to have lower risk as well as lower return when compared with buying the index only. See Black (1975). In fact, in an efficient market, the risk adjusted return from covered call writing should be no different than the risk adjusted return from just holding the index.

The covered call strategy (S denotes stock, C denotes call) is given by:

$$V = S - C$$

With anchoring, this is equal to:

$$V = S - e^{-A_K \cdot \delta(T-t)}\left\{SN(d_1^A) - Ke^{-(r+\delta)(T-t)}N(d_2^A)\right\}$$

$$=> V = \left(1 - e^{-A_K \cdot \delta(T-t)}N(d_1^A)\right)S + e^{-A_K \cdot \delta(T-t)}N(d_2^A)Ke^{-(r+\delta)(T-t)} \qquad (6.1)$$

The corresponding value under the Black Scholes assumptions is:

$$V = \left(1 - N(d_1)\right)S + N(d_2)Ke^{-r(T-t)} \tag{6.2}$$

A comparison of 6.1 and 6.2 shows that covered call strategy is expected to perform much better with anchoring when compared with its expected performance in the Black-Scholes world. With anchoring, covered call strategy creates a portfolio which is equivalent to having a portfolio with a weight of $1 - e^{-A_K \cdot \delta(T-t)} N(d_1^A)$ on the stock and a weight of $e^{-A_K \cdot \delta(T-t)} N(d_2^A)$ on a hypothetical risk free asset with a return of $r + \delta + A_K \cdot \delta$. The stock has a return of $r + \delta$ plus dividend yield. This implies that, with anchoring, the return from covered call strategy is expected to be comparable to the return from holding the underlying stock only. The presence of a hypothetical risk free asset in 6.1 implies that the standard deviation of covered call returns is lower than the standard deviation from just holding the underlying stock. Hence, the superior historical performance of covered call strategy is consistent with anchoring.

6.1 The Zero-Beta Straddle Performance with Anchoring

Another empirical puzzle in the Black-Scholes/CAPM framework is that zero beta straddles lose money. Goltz and Lai (2009), Coval and Shumway (2001) and others find that zero beta straddles earn negative returns on average. This is in sharp contrast with the Black-Scholes/CAPM prediction which says that the zero-beta straddles should earn the risk free rate. A zero-beta straddle is constructed by taking a long position in corresponding call and put options with weights chosen so as to make the portfolio beta equal to zero:

$$\theta \cdot \beta_{Call} + (1 - \theta) \cdot \beta_{Put} = 0$$

$$=> \theta = \frac{-\beta_{Put}}{\beta_{Call} - \beta_{Put}}$$

Where $\beta_{Call} = N(d_1) \cdot \frac{Stock}{Call} \cdot \beta_{Stock}$ and $\beta_{Put} = (N(d_1) - 1) \cdot \frac{Stock}{Put} \cdot \beta_{stock}$

It is straightforward to show that with anchoring, where call and put prices are determined in accordance with proposition 1, the zero-beta straddle earns a significantly smaller return than the risk free rate (with returns being negative for a wide range of realistic parameter values). See

Appendix C for proof. Intuitively, with anchoring, both call and put options are more expensive when compared with Black-Scholes prices. Hence, the returns are smaller, and are typically negative.

Anchoring provides a unified explanation for key option pricing puzzles even in the simplest setting of geometric Brownian motion. Furthermore, two novel predictions of the anchoring model are strongly supported in the data.

7. Conclusions

Intriguing option pricing puzzles include: 1) the implied volatility skew, 2) superior historical performance of covered call writing, 3) worse-than-expected performance of zero beta straddles, and 4) the puzzling findings regarding leverage adjusted index option returns. Furthermore, it is well known that average put returns are far more negative than what the theory predicts, and average call returns are smaller than what one would expect given their systematic risk.

If the volatility of the underlying stock returns is used as a starting point which gets adjusted upwards to arrive at call option volatility, then the anchoring bias implies that such adjustments are insufficient. There is considerable field and experimental evidence of the role of anchoring in option pricing. In this article, an anchoring-adjusted option pricing model is put forward. The model provides a unified explanation for the puzzles mentioned above. Furthermore, the anchoring price lies within the bounds implied by risk-averse expected utility maximization. Two novel predictions of the model are empirically tested and found to be strongly supported in the data.

The challenge for financial economics is to enrich the elegant option pricing framework sufficiently so that it captures key empirical regularities. This article shows that incorporating the anchoring bias in the option pricing framework provides the needed enrichment, while preserving the elegance of the framework. Furthermore, anchoring works regardless of the distributional assumptions that are made about the underlying stock behavior. Hence, it is easy to combine anchoring with jump diffusion and stochastic volatility approaches. This is the subject of future research.

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Appendix A

The anchoring adjusted PDE can be solved by converting to heat equation and exploiting its solution.

Start by making the following transformations in (2.6):

$$\tau = \frac{\sigma^2}{2}(T - t)$$

$$x = ln\frac{S}{K} => S = Ke^x$$

$$C(S,t) = K \cdot\ c(x,\tau) = K \cdot c\left(ln\left(\frac{S}{K}\right), \frac{\sigma^2}{2}(T - t)\right)$$

It follows,

$$\frac{\partial C}{\partial t} = K \cdot \frac{\partial c}{\partial \tau} \cdot \frac{\partial \tau}{\partial t} = K \cdot \frac{\partial c}{\partial \tau} \cdot \left(-\frac{\sigma^2}{2}\right)$$

$$\frac{\partial C}{\partial S} = K \cdot \frac{\partial c}{\partial x} \cdot \frac{\partial x}{\partial S} = K \cdot \frac{\partial c}{\partial x} \cdot \frac{1}{S}$$

$$\frac{\partial^2 C}{\partial S^2} = K \cdot \frac{1}{S^2} \cdot \frac{\partial^2 C}{\partial x^2} - K \cdot \frac{1}{S^2} \frac{\partial C}{\partial x}$$

Plugging the above transformations into (2.6) and writing $\tilde{r} = \frac{2(r+\delta)}{\sigma^2}$, and $\tilde{\epsilon} = \frac{2A\delta}{\sigma^2}$ we get:

$$\frac{\partial c}{\partial \tau} = \frac{\partial^2 c}{\partial x^2} + (\tilde{r} - 1)\frac{\partial c}{\partial x} - (\tilde{r} + \tilde{\epsilon})c \qquad (D1)$$

With the boundary condition/initial condition:

$$C(S,T) = max\{S - K, 0\}\ becomes\ c(x,0) = max\{e^x - 1,0\}$$

To eliminate the last two terms in (D1), an additional transformation is made:

$$c(x,\tau) = e^{\alpha x + \beta\tau} u(x,\tau)$$

It follows,

$$\frac{\partial c}{\partial x} = \alpha e^{\alpha x + \beta\tau} u + e^{\alpha x + \beta\tau} \frac{\partial u}{\partial x}$$

$$\frac{\partial^2 c}{\partial x^2} = \alpha^2 e^{\alpha x + \beta\tau} u + 2\alpha e^{\alpha x + \beta\tau} \frac{\partial u}{\partial x} + e^{\alpha x + \beta\tau} \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial c}{\partial \tau} = \beta e^{\alpha x + \beta\tau} u + e^{\alpha x + \beta\tau} \frac{\partial u}{\partial \tau}$$

Substituting the above transformations in (D1), we get:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} + (\alpha^2 + \alpha(\tilde{r} - 1) - (\tilde{r} + \tilde{\epsilon}) - \beta)u + \big(2\alpha + (\tilde{r} - 1)\big)\frac{\partial u}{\partial x} \qquad \text{(D2)}$$

Choose $\alpha = -\frac{(\tilde{r}-1)}{2}$ and $\beta = -\frac{(\tilde{r}+1)^2}{4} - (\tilde{\epsilon})$. (D2) simplifies to the Heat equation:

$$\frac{\partial u}{\partial \tau} = \frac{\partial^2 u}{\partial x^2} \qquad (D3)$$

With the initial condition:

$$u(x_0,0) = max\{(e^{(1-\alpha)x_0} - e^{-\alpha x_0}), 0\} = max\left\{\left(e^{\left(\frac{\tilde{r}+1}{2}\right)x_0} - e^{\left(\frac{\tilde{r}-1}{2}\right)x_0}\right), 0\right\}$$

The solution to the Heat equation in our case is:

$$u(x,\tau) = \frac{1}{2\sqrt{\pi\tau}} \int_{-\infty}^{\infty} e^{-\frac{(x-x_0)^2}{4\tau}} u(x_0,0)dx_0$$

Change variables: $= \frac{x_0-x}{\sqrt{2\tau}}$, which means: $dz = \frac{dx_0}{\sqrt{2\tau}}$. Also, from the boundary condition, we know that $u > 0 \; iff \; x_0 > 0$. Hence, we can restrict the integration range to $z > \frac{-x}{\sqrt{2\tau}}$

$$u(x,\tau) = \frac{1}{\sqrt{2\pi}} \int_{-\frac{x}{\sqrt{2\pi}}}^{\infty} e^{-\frac{z^2}{2}} \cdot e^{\left(\frac{\tilde{r}+1}{2}\right)(x+z\sqrt{2\tau})} dz - \frac{1}{\sqrt{2\pi}} \int_{-\frac{x}{\sqrt{2\tau}}}^{\infty} e^{-\frac{z^2}{2}} \cdot e^{\left(\frac{\tilde{r}-1}{2}\right)(x+z\sqrt{2\tau})} dz$$

$$=: H_1 - H_2$$

Complete the squares for the exponent in H_1:

$$\frac{\tilde{r}+1}{2}\left(x + z\sqrt{2\tau}\right) - \frac{z^2}{2} = -\frac{1}{2}\left(z - \frac{\sqrt{2\tau}(\tilde{r}+1)}{2}\right)^2 + \frac{\tilde{r}+1}{2}x + \tau\frac{(\tilde{r}+1)^2}{4}$$

$$=: -\frac{1}{2}y^2 + c$$

We can see that $dy = dz$ and c does not depend on z. Hence, we can write:

$$H_1 = \frac{e^c}{\sqrt{2\pi}} \int_{-x/\sqrt{2\pi} - \sqrt{\tau/2}(\tilde{r}+1)}^{\infty} e^{-\frac{y^2}{2}} dy$$

A normally distributed random variable has the following cumulative distribution function:

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\frac{y^2}{2}} dy$$

Hence, $H_1 = e^c N(d_1)$ where $d_1 = {}^{x}\!/\!_{\sqrt{2\pi}} + \sqrt{{}^{\tau}\!/\!_{2}}\,(\tilde{r}+1)$

Similarly, $H_2 = e^f N(d_2)$ where $d_2 = {}^{x}\!/\!_{\sqrt{2\pi}} + \sqrt{{}^{\tau}\!/\!_{2}}\,(\tilde{r}-1)$ and $f = \frac{\tilde{r}-1}{2}x + \tau\frac{(\tilde{r}-1)^2}{4}$

The anchoring adjusted European call pricing formula is obtained by recovering original variables:

$$C = e^{-A\cdot\delta(T-t)}\{SN(d_1^A) - Ke^{-(r+\delta)(T-t)}N(d_2^A)\}$$

Where $\boldsymbol{d_1^A} = \frac{\boldsymbol{ln(S/K)+(r+\delta+\frac{\sigma^2}{2})(T-t)}}{\boldsymbol{\sigma\sqrt{T-t}}}$ and $\boldsymbol{d_2^A} = \frac{\boldsymbol{ln\left(\frac{S}{K}\right)+\left(r+\delta-\frac{\sigma^2}{2}\right)(T-t)}}{\boldsymbol{\sigma\sqrt{T-t}}}$

Appendix B

By definition, at the threshold:

$$e^{-A_K\delta(T-t)}\{SN\left({d_1}^A\right) - Ke^{-(r+\delta)(T-t)}N\left({d_2}^A\right)\} = SN(d_1) - Ke^{-r(T-t)}N(d_2) \qquad \text{(E1)}$$

Solving (E1) for A_K gives the threshold value:

$$ln\left(\frac{SN(d_1^A)-Ke^{-(r+\delta)(T-t)}N(d_2^A)}{SN(d_1)-Ke^{-r(T-t)}N(d_2)}\right)\cdot\frac{1}{(T-t)} = |\overline{A_K}|\cdot\delta \qquad \text{(E2)}$$

Appendix C

Following Coval and Shumway (2001) and some algebraic manipulations, the return from a zero-beta-straddle can be written as:

$$r_{straddle} = \frac{-\Omega_c C + S}{\Omega_c P - \Omega_c C + S}\cdot r_{call} + \frac{\Omega_c P + S}{\Omega_c P - \Omega_c C + S}\cdot r_{put}$$

Where C and P denote call and put prices respectively, r_{call} is expected call return, r_{put} is expected put return, and Ω_c is call price elasticity w.r.t the underlying stock price.

Under anchoring:

$$r_{call} = \mu + A\cdot\delta$$

$$r_{put} = \frac{(\mu + A\cdot\delta)C - \mu S + rPV(K)}{P}$$

Substituting r_{call} and r_{put} in the expression for $r_{straddle}$, and simplifying implies that as long as the risk premium on the underlying is positive, it follows that:

$$r_{straddle} < r$$

Appendix D

Note that for a put option, if the underlying stock has a positive risk premium, then the expected put payoff must be less than its price. That is, expected put return is negative. The proof follows directly from realizing that if the risk premium on the underlying stock is positive, the price of a put option under anchoring is larger than the price of a put option in the Black Scholes model.