Alternative GMM Estimators for First-order Autoregressive Panel Model: An Improving Efficiency Approach

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Abstract

This paper considers first-order autoregressive panel model which is a simple model for dynamic panel data (DPD) models. The generalized method of moments (GMM) gives efficient estimators for these models. This efficiency is affected by the choice of the weighting matrix which has been used in GMM estimation. The non-optimal weighting matrices have been used in the conventional GMM estimators. This led to a loss of efficiency. Therefore, we present new GMM estimators based on optimal or suboptimal weighting matrices. Monte Carlo study indicates that the bias and efficiency of the new estimators are more reliable than the conventional estimators.

Keywords: Dynamic panel data; Generalized method of moments; Kantorovich inequality upper bound; Monte Carlo simulation; Optimal and suboptimal weighting matrices.

1. Introduction

In econometrics literature, the panel data refers to the pooling of observations on a cross-section of households, countries, firms, etc. over several time periods. Panel data is now widely used to estimate dynamic econometric models.[1] Its advantage over cross-section data in this context is obvious: we cannot estimate dynamic models from observations at a single point in time, and it is rare for single cross section surveys to provide sufficient information about earlier time periods for dynamic relationships to be investigated. Its advantages over aggregate time series data include the possibility that underlying microeconomic dynamics may be obscured by aggregation biases, and the scope that panel data offers to investigate heterogeneity in adjustment dynamics between different types of individuals, household, or firms.

The DPD models offer great flexibility to empirical researchers. Many economic phenomena are dynamic in nature. DPD models allow researchers to control for unobserved heterogeneity in adjustment dynamics between different individual units and thereby provide improved insights in such models.

[1] See, e.g., Bond (2002), Baltagi (2013), and Hsiao (2014).

When dynamic models are estimated using panel data, the usual least squares methods lead to inconsistent estimates of the parameters of the models when the time dimension (T) is short regardless of the cross sectional dimension (N). This inconsistency stems from the fact that the disturbance terms are correlated with the lagged endogenous variable. Moreover, under large N fixed T asymptotic, Nickell (1981) showed that the standard maximum likelihood estimator suffers from an incidental parameter problem leading to inconsistency.[2] This has led to an interest in likelihood-based methods that correct for this problem. Some of these methods are based on modifications of the profile likelihood, such as Lancaster (2002) and Dhaene and Jochmans (2012). Other methods start from the likelihood function of the first differences, such as Hsiao *et al.* (2002), Binder *et al.* (2005) and Hayakawa and Pesaran (2014).

The literature has focused on GMM[3] estimators applied to first differences, such as Anderson and Hsiao (1982), Holtz-Eakin *et al.* (1988), and Arellano and Bond (1991). However, the standard GMM estimator obtained after first differencing has been found to suffer from substantial finite sample bias, especially when the instruments are weak and the number of moments is large relative to the cross section sample size.[4]

This low precision of GMM is also evident in more general contexts. To improve the small sample properties of GMM estimators, a number of alternative estimators have been suggested, including, level and system GMM estimators which presented by Arellano and Bover (1995). These estimators are based on use many instrumental variables to improve the efficiency of GMM estimator.[5] However, these estimators still biased and need to further improvement. Hayakawa (2007) examined the asymptotic bias of GMM estimators and proposed new estimators with less bias. Recently, Han and Phillips (2010) developed efficient GMM estimators based on alternative moment conditions arising from the model in first differences.

The main objective of this paper is improving the efficiency of GMM estimators. To achieve this objective, we proposed new approach to improve the efficiency of GMM estimators. Our approach based on finding and using the optimal weighting matrices to obtain more efficient estimators.

This paper is organized as follows. Section 2 provides the model and reviews the conventional level and system GMM estimators. Section 3 presents the new GMM estimators. In Section 4, we consider the efficiency gain for the new estimators against using the identity matrix as an initial weight matrix. While in Section 5, Monte Carlo simulation comparison the performance of various level and system GMM estimators will be introduced. Finally, Section 6 offers the concluding remarks.

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[2] This problem appears when the number of parameters increasing with the sample size. For more details, see Nickell (1981).

[3] Blundell *et al.* (2001), Roodman (2009), and Bun and Sarafidis (2014) provide excellent summaries of the GMM methodology in DPD models.

[4] See Alonso-Borrego and Arellano (1999) and Han *et al.* (2014).

[5] See, e. g., Ahn and Schmidt (1995) and Roodman (2009).

2. The model and GMM estimators

The first-order autoregressive panel model is a simple case of DPD models, and it is defined as the following:

$$y_{it} = \rho y_{i,t-1} + \gamma_i + \varepsilon_{it}; \quad |\rho| < 1; \; i = 1, \dots, N; \; t = 2, \dots, T. \tag{1}$$

Under the following assumptions:

(i) γ_i are i.i.d across individuals with $E(\gamma_i) = 0, Var(\gamma_i) = \sigma_\gamma^2$.
(ii) ε_{it} are i.i.d across time and individuals and independent of γ_i and y_{i1} with $E(\varepsilon_{it}) = 0, Var(\varepsilon_{it}) = \sigma_\varepsilon^2$.
(iii) The initial observations satisfy $y_{i1} = \frac{\gamma_i}{1-\rho} + w_{i1}$ for $i = 1, \dots, N$, where $w_{i1} = \sum_{j=0}^{\infty} \rho^j \varepsilon_{i,1-j}$ and independent of γ_i.

Assumptions (i) and (ii) are the same as in Blundell and Bond (1998), while assumption (iii) has been developed by Alvarez and Arellano (2003).

Stacking equation (1) over time, we obtain

$$y_i = \rho y_{i,-1} + u_i; \quad u_{it} = \gamma_i + \varepsilon_{it}. \tag{2}$$

where $y_i = (y_{i3}, \dots, y_{iT})', y_{i,-1} = (y_{i2}, \dots, y_{i,T-1})', u_i = (u_{i3}, \dots, u_{iT})'$.

Given these assumptions, we get three types of GMM estimators. These include first-difference GMM (DIF) estimator, level GMM (LEV) estimator, and system GMM (SYS) estimator. Blundell and Bond (1998) showed that when ρ is close to unity and/or $\sigma_\gamma^2/\sigma_\varepsilon^2$ increases the DIF estimator has weak instruments problem, i.e., the instruments matrix which been used in the estimation become invalid. As well as, Abonazel (2014) proved that the conventional weighting matrix which has been used in DIF estimator, when the errors that are homoscedastic and that are not serially correlated, is the optimal weighting matrix for this estimator, but this is not the case for LEV and SYS estimators. So, we will not adjust DIF estimator by a new weighting matrix because it will not improve the efficiency of the estimation. Therefore, we focus on the LEV and SYS estimators to improve the efficiency of these estimators. In the following, we briefly review the conventional LEV and SYS estimators with the initials weighting matrices.

2.1. Level GMM estimator

In model (2), the individual effect (γ_i) caused a severe correlation between the lagged endogenous variable ($y_{i,-1}$) and the error term (u_i). To eliminate this effect, Arellano and Bover (1995) suggested a method to eliminate the individual effect from instrumental variables. They considered the level model (2) and then showed that the instrumental variables matrix $L_i = daig(\Delta y_{i2}, \Delta y_{i3}, \dots, \Delta y_{i,T-1})$ which is not contains individual effect and satisfied the orthogonal conditions $E(L_i' u_i) = 0$.

Using these conditions, Arellano and Bover's (1995) one-step LEV estimator is calculated as:

$$\hat{\rho}^l = (y'_{-1} L\, Z^l L' y_{-1})^{-1}\, y'_{-1} L\, Z^l L' y, \tag{3}$$

where $y_{-1} = \left(y'_{1,-1}, \dots, y'_{N,-1}\right)', y = (y'_1, \dots, y'_N)', L = (L'_1, \dots, L'_N)'$, and

$$Z^l = \left(\frac{1}{N}\sum_{i=1}^{N} L'_i L_i\right)^{-1}. \tag{4}$$

2.2. System GMM estimator

Arellano and Bover (1995) proposed a system GMM estimator in which the moment conditions of the first-difference GMM and level GMM are used jointly to avoid weak instruments and improved the efficiency of the estimator. The moment conditions used in constructing the SYS estimator are given by

$$E(S'_i u^s_i) = 0, \tag{5}$$

where, $u^s_i = (\Delta u'_i,\ u'_i)'$ and S_i is a $\{2(T - 2) \times [(T - 2)\,(T + 1)/2]\}$ block diagonal matrix given by

$$S_i = \begin{pmatrix} D_i & 0 \\ 0 & L_i \end{pmatrix};\ \text{with } D_i = \begin{pmatrix} y_{i1} & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & y_{i1} & y_{i2} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & y_{i1} & \cdots & y_{i,T-2} \end{pmatrix}. \tag{6}$$

Using (5), the one-step SYS estimator is calculated as:

$$\hat{\rho}^s = (y^{s\prime}_{-1} S\, Z^s\, S' y^s_{-1})^{-1} y^{s\prime}_{-1} S\, Z^s\, S' y^s, \tag{7}$$

where $y^s_{-1} = \left[(\Delta y'_{1,-1}, y'_{1,-1}), \dots, (\Delta y'_{N,-1}, y'_{N,-1})\right]',\ y^s = [(\Delta y'_1, y'_1), \dots, (\Delta y'_N, y'_N)]'$,

$S = (S'_1, \dots, S'_N)'$, and

$$Z^s = \left(\frac{1}{N}\sum_{i=1}^{N} S'_i G\, S_i\right)^{-1},\ \text{with}\quad G = \begin{pmatrix} FF' & 0 \\ 0 & I_{T-2} \end{pmatrix}, \tag{8}$$

where F is a $(T - 2)\times(T - 1)$ first-difference operator matrix:

$$F = \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{pmatrix}. \tag{9}$$

Even though, the SYS estimator is more efficient than LEV estimator. However, the SYS estimator does not always work well; Bun and Kiviet (2006) showed that the bias of SYS estimator becomes large when the autoregressive parameter is close to unity and/or when the ratio of the variance of the individual effect to that of the error term departs from unity.

3. New GMM estimators

We present the new GMM estimators; depending on the optimal weighting matrix for LEV estimator and suboptimal weighting matrix for SYS estimator through the use of these matrices as new weighting matrices in GMM estimation, and then we get new GMM estimators. The new GMM estimators are more efficient than the conventional GMM (LEV and SYS) estimators.

In level GMM estimation, Youssef *et al.* (2014a) showed that Z^l is an optimal weighting matrix only in the case of $\sigma_\gamma^2 = 0$, i.e., no individual effects case, and they presented an optimal weighting matrix for LEV estimator, in the general case, as:

$$Z^{ol} = \left(\frac{1}{N}\sum_{i=1}^{N} L_i' A_{T-2} L_i\right)^{-1}; \text{ with } A_{T-2} = I_{T-2} + r l_{T-2} l_{T-2}' \quad (10)$$

where $r = \sigma_\gamma^2/\sigma_\varepsilon^2$ and l_{T-2} is a $(T-2) \times 1$ vector of ones.

Note that the use of the weighting matrix Z^{ol} can be described as inducing cross-sectional heterogeneity through r, and also can be explained as partially adopting a procedure of generalized least squares to the level estimation. So, using Z^{ol}, instead of Z^l, certainly improve the efficiency of LEV estimator. So, we present the optimal LEV estimator depending on the optimal weighting matrix, Z^{ol}, as given in (10). The optimal level GMM (OLEV) estimator is given by

$$\hat{\rho}^{ol} = (y_{-1}' L\, Z^{ol} L' y_{-1})^{-1}\, y_{-1}' L\, Z^{ol} L' y. \quad (11)$$

In system GMM estimation, we use A_{T-2} again in the weighting matrix to improve the efficiency for SYS estimator as follows:

$$Z^{os} = \left(\frac{1}{N}\sum_{i=1}^{N} S_i' G^o S_i\right)^{-1}, \text{ with } G^o = \begin{pmatrix} FF' & 0 \\ 0 & A_{T-2} \end{pmatrix}. \quad (12)$$

So, we present the suboptimal system GMM (SSYS) estimator[6] depending on the suboptimal weighting matrix (Z^{os}):

$$\hat{\rho}^{os} = (y_{-1}' S\, Z^{os} S' y_{-1})^{-1}\, y_{-1}' S\, Z^{os} S' y. \quad (13)$$

In general, an asymptotically efficient estimator can be obtained through the two-step procedure in the standard GMM estimation. In the first step, an initial positive semidefinite weighting matrix is used to obtain consistent estimates of the parameters. Given these, a weighting matrix can be constructed and used for asymptotically efficient two-step estimates. It is well known[7] that the two-step estimated standard errors have a small-sample downward bias in dynamic panel data setting, and one-step estimates with robust standard errors are often preferred. We briefly summarize the two-step procedure as: at first, we use the suggested weighting matrix to get the one-step estimation, and then used the residuals from

[6] Kiviet (2007) proposed a similar estimator using an optimal weighting matrix based on the particular values of ρ, σ_γ^2, and σ_ε^2.

[7] See, e.g., Arellano and Bond (1991) and Windmeijer (2005).

one-step estimation as a weighting matrix to get the two-step estimation. For example, we can get the two-step estimation of $\hat{\rho}^{os}$ by using the following weighting matrix:

$$Z_{(2)}^{os} = \left(\frac{1}{N} \sum_{i=1}^{N} S_i' \hat{u}_i^S \hat{u}_i^{S\prime} S_i \right)^{-1}, \tag{14}$$

where $\hat{u}_i^S$ are the fitted residuals from $\hat{\rho}^{os}$.

To achieve more efficiency for all GMM estimators, we suggest using the three-step estimation that can obtain by replacing the residuals from the two-step estimation into the weighting matrix. Applying this approach to the SSYS estimator, we can get the three-step estimation of $\hat{\rho}^{os}$ by using the following weighting matrix:

$$Z_{(3)}^{os} = \left(\frac{1}{N} \sum_{i=1}^{N} S_i' \hat{u}_i^{+S} \hat{u}_i^{+S\prime} S_i \right)^{-1}, \tag{15}$$

where $\hat{u}_i^{+S}$ are the fitted residuals from the two-step estimation of $\hat{\rho}^{os}$.

In practice, the variance ratio, r, is unknown. So, we use the suggested estimates by Jung and Kwon (2007) for σ_ε^2 and σ_γ^2 as:[8]

$$\hat{\sigma}_\varepsilon^2 = \frac{\sum_{i=1}^{N} \Delta\hat{u}_i' \, \Delta\hat{u}_i}{2N(T-2)}, \quad \hat{\sigma}_\gamma^2 = \frac{\sum_{i=1}^{N} [\tilde{u}_i' \, \tilde{u}_i - (\Delta\tilde{u}_i' \, \Delta\tilde{u}_i/2)]}{N(T-2)}, \tag{16}$$

where $\Delta\hat{u}_i$ are the residuals from one-step DIF estimator, while $\tilde{u}_i$ and $\Delta\tilde{u}_i$ are residuals from first-difference and level equations in one-step SYS estimator, respectively.

Note that if $r = 0$, we get $A_{T-2} = I_{T-2}$ then $Z^{ol} = Z^{l}$ and $Z^{os} = Z^{s}$. Therefrom the proposed GMM (OLEV and SSYS) estimators are equivalent to the conventional GMM (LEV and SYS) estimators. This means that the advantages from OLEV and SSYS estimators are increasing when r is increasing.

4. Efficiency gains

Generally, using the moment conditions, the GMM estimator[9] $\hat{\rho}$ for ρ_0 minimizes

$$\left[\frac{1}{N} \sum_{i=1}^{N} f_i(\rho) \right]' W_N \left[\frac{1}{N} \sum_{i=1}^{N} f_i(\rho) \right], \tag{17}$$

with respect to ρ, where W_N is a positive semidefinite weighting matrix which satisfies $plim_{N\to\infty} W_N = W$, with W a positive definite matrix. Regularity conditions are in place such that $plim_{N\to\infty} N^{-1} \sum_{i=1}^{N} f_i(\rho) = E\{f(\rho)\}$ and

[8] See Abonazel (2014) and Youssef *et al.* (2014b).

[9] See Hansen (1982) and Ogaki (1993). In this section, the same notation as in Liu and Neudecker (1997) is used.

$$\frac{1}{\sqrt{N}}\sum_{i=1}^{N} f_i(\rho_0) \rightarrow N(0,\Omega). \tag{18}$$

Let $F(\rho) = E(\partial f_i(\rho)/\partial \rho)$ and $F_0 \equiv F(\rho_0)$, then $\sqrt{N}(\hat{\rho} - \rho_0)$ has a limiting normal distribution, $\sqrt{N}(\hat{\rho} - \rho_0) \rightarrow N(0, V_W)$, where

$$V_W = (F_0' W F_0)^{-1} F_0' W \Omega W F_0 (F_0' W F_0)^{-1}. \tag{19}$$

It is clear from the expression of the asymptotic variance matrix V_W, in (19), that the efficiency of the GMM estimator is affected by the choice of the weighting matrix W_N.An optimal choice is a weighting matrix for which $W = \Omega^{-1}$. The asymptotic variance covariance matrix is then given by $(F_0' \Omega^{-1} F_0)^{-1}$. For any other W, the GMM estimator is less efficient as:

$$(F_0' \Omega^{-1} F_0)^{-1} \leq (F_0' W F_0)^{-1} F_0' W \Omega W F_0 (F_0' W F_0)^{-1}. \tag{20}$$

To assess the potential loss in efficiency from using this initial weighting matrix, the following expression for the upper bound of the efficiency loss has been derived by Liu and Neudecker (1997) on the basis of the Kantorovich inequality (KI):

$$(F_0' W F_0)^{-1} F_0' W \Omega W F_0 (F_0' W F_0)^{-1} \leq \frac{\left(\lambda_1 + \lambda_p\right)^2}{4\lambda_1 \lambda_p} (F_0' \Omega^{-1} F_0)^{-1}, \tag{21}$$

and the upper bound of KI is UBKI $= \frac{\left(\lambda_1 + \lambda_p\right)^2}{4\lambda_1 \lambda_p}$, where $\lambda_i > 0$; $i = 1, \dots, p$ are the eigenvalues of the $p \times p$ matrix ΩW. By increasing UBKI, more loss of efficiency will be occurred.

We use UBKI as a measure for the efficiency gains of the new GMM estimators. Since OLEV is the optimal estimator for level GMM estimation, according to Youssef *et al.* (2014a). So, we can conclude that the UBKI value for OLEV is one. This means that the OLEV estimator achieves the maximum degree of the efficiency. But in system GMM estimation, Windmeijer (2000) showed that the optimal weighting matrix for SYS estimator has only been obtained in case of $r = 0$. Therefore, we present SSYS estimator to suit the different cases for r. Also, we measure the efficiency gains from use SSYS especially in cases $r > 0$.

For simplicity, let us assume that $T = 3$ and using the moment conditions in (5), then we get there are two over-identifying moment conditions:

$$E[y_{i1}(\Delta y_{i3} - \rho \Delta y_{i2})] = 0; E[\Delta y_{i2}(y_{i3} - \rho y_{i2})] = 0,$$

and Ω matrix is given by:

$$\Omega = \underset{N\to\infty}{\text{plim}} \frac{1}{N}\sum_{i=1}^{N} S_i^{'} u_i^S u_i^{S'} S_i \ = \sigma_\varepsilon^2 \begin{bmatrix} 2\sigma_y^2 & -(1-\rho)\sigma_y^2 \\ -(1-\rho)\sigma_y^2 & \dfrac{2(\sigma_\gamma^2+\sigma_\varepsilon^2)}{1+\rho} \end{bmatrix},$$

where $\sigma_y^2 = \frac{\sigma_\gamma^2}{(1-\rho)^2} + \frac{\sigma_\varepsilon^2}{1-\rho^2}$. Firstly, in case of the conventional weighting matrix for the SYS estimator (Z^s), we get

$$\Omega W_{SYS} = \begin{bmatrix} \sigma_\varepsilon^2 & -\dfrac{1}{2}\sigma_y^2(1-\rho^2) \\ -\dfrac{1}{2}\sigma_\varepsilon^2(1-\rho) & \sigma_\gamma^2+\sigma_\varepsilon^2 \end{bmatrix}.$$

where $W_{SYS} = \left(\underset{N\to\infty}{\text{plim}} \frac{1}{N}\sum_{i=1}^{N} S_i' G\, S_i\right)^{-1}$, then the upper bound is:

$$\text{UBKI(SYS)} = \frac{\left(\sigma_\gamma^2 + 2\sigma_\varepsilon^2\right)^2}{\sigma_\varepsilon^2\left[4\left(\sigma_\gamma^2+\sigma_\varepsilon^2\right) - \sigma_y^2(1-\rho)^2(1+\rho)\right]}. \tag{22}$$

Secondly, in case of the suboptimal weighting matrix for the SSYS estimator (Z^{os}), we get

$$\Omega W_{SSYS} = \begin{bmatrix} \sigma_\varepsilon^2 & -\dfrac{\sigma_y^2(1-\rho^2)}{2(1+r)} \\ -\dfrac{1}{2}\sigma_\varepsilon^2(1-\rho) & \dfrac{(\sigma_\gamma^2+\sigma_\varepsilon^2)}{(1+r)} \end{bmatrix}.$$

where $W_{SSYS} = \left(\underset{N\to\infty}{\text{plim}} \frac{1}{N}\sum_{i=1}^{N} S_i' G^o S_i\right)^{-1}$, then the upper bound is:

$$\text{UBKI(SSYS)} = \frac{\left[\sigma_\gamma^2 + (2+r)\sigma_\varepsilon^2\right]^2}{(1+r)\sigma_\varepsilon^2\left[4\left(\sigma_\gamma^2+\sigma_\varepsilon^2\right) - \sigma_y^2(1-\rho)^2(1+\rho)\right]}. \tag{23}$$

Since r value must be nonnegative. So, always $UBKI(\text{SSYS}) \leq \text{UBKI(SYS)}$. This means that the SSYS estimator is more efficient than SYS estimator when $r > 0$. But if $r = 0$, the efficiency of SSYS and SYS estimators are equal, i.e., $UBKI(\text{SSYS}) = \text{UBKI(SYS)}$ in this case. So, the efficiency gains from use SSYS estimator is increasing when r is increasing.

Figure 1 presents the efficiency bounds ($UBKI(\text{SYS})$ and $UBKI(\text{SSYS})$) for system GMM estimation for various values of ρ and r and when $T = 3$. Figure 1 shows that the $UBKI(\text{SYS})$ value is increasing when ρ and r are increasing, but it is more influenced by r. In contrast, the $UBKI(\text{SSYS})$ value is not affected much by increase ρ and r. So, we can say, in general, that the SSYS estimator is more efficient than SYS estimator.

To study the effect of time dimension on the efficiency of system GMM estimators, let us assume that $T = 4$, then we calculate the efficiency bounds again for SYS and SSYS estimators. Briefly and without mention to the calculations details, we summarize the values of the efficiency bounds in case of $T = 4$ in Figure 2. Generally, this Figure shows that the values of $UBKI$(SYS) and $UBKI$(SSYS) are increased because of increasing T. Nevertheless, the $UBKI$(SSYS) values still less than the $UBKI$(SYS) values when all values of ρ and r. This means that SSYS estimator is still more efficient than SYS estimator even if the time dimension is increasing.

5. The simulation study

We make Mote Carlo simulation to illustrate the moderate and large samples performance of the conventional (LEV and SYS) and new GMM (OLEV and SSYS) estimators in different situations of $\rho, r,$ and T. We use R language to create our program to set up Monte Carlo simulation and this program is available if requested.

5.1. Design of the simulation

Monte Carlo experiments were carried out based on the following data generating process:

$$y_{it} = \rho y_{i,t-1} + \gamma_i + \varepsilon_{it}, \tag{24}$$

To perform the simulation under the assumptions (i) to (iii), the model in (24) was generated as follows:

1. The individual effects, γ_i, were generated as independent normally distributed random variables, with mean zero and variance σ_γ^2, and is independent across i.

2. The disturbances, ε_{it}, were generated as independent normally distributed random variables, with mean zero and variance σ_ε^2, and is independent across i and t, γ_i and ε_{it} such that they are independent of each other.

3. We generate the initial conditions y_{i1} as $y_{i1} = \frac{\gamma_i}{1-\rho} + w_{i1}$, where $w_{i1} \sim N(0, \sigma_{w1}^2)$, independent of both γ_i and ε_{it} with the variance σ_{w1}^2 chosen to satisfy covariance stationarity.

4. The variance ratio, r, is characterized by $\sigma_\gamma^2/\sigma_\varepsilon^2$, this means that the variance ratio depends on σ_γ^2 and σ_ε^2. To determine the values for r, we must choose different pairs of σ_γ^2 and σ_ε^2 as: $(\sigma_\gamma^2, \sigma_\varepsilon^2) = (0.25, 0.5), (0.25, 0.25), (2.5, 0.5), (2.5, 0.25), or\ (12.5, 0.5)$. These suggested values indicate three levels for r, the first level is when $r < 1$, while the second level is when $r = 1$, and the third level when $r > 1$.

5. The values of N were chosen to be 150, 400, and 600 to represent moderate and large samples for the number of individuals.

6. The values of T were chosen to be 4 and 8 to represent different size for the time dimension.

7. The values of ρ were chosen to be 0.25 and 0.85 to represent different values for the autoregressive parameter.

8. For all experiments we ran 1000 replications and all the results of all separate experiments are obtained by precisely the same series of random numbers.

To compare the moderate and large samples performance, the four different GMM estimation procedures are considered according to their weighting matrix; specifically LEV, OLEV, SYS, and SSYS estimators. To raise the efficiency of the comparison between these estimators, we calculate three (from one-step to three-step) estimates for each estimator, and we refer to these estimates as "step 1", "step 2", and "step 3" in simulation tables. Moreover, we calculate the bias and root mean squared error (RMSE) for each estimator. The bias and RMSE for a Monte Carlo experiment are calculated by

$$\text{Bias} = \frac{1}{1000}\sum_{l=1}^{1000}(\hat{\rho}_l - \rho); \quad \text{RMSE} = \sqrt{\frac{1}{1000}\sum_{l=1}^{1000}(\hat{\rho}_l - \rho)^2},$$

where ρ is the true value for $y_{i,t-1}$ parameter in (24), and $\hat{\rho}_l$ is the estimated value for ρ in the trial number l.

5.2. The simulation results

The results are given in Tables 1 to 5. Specifically, Tables 1 and 2 present the bias and RMSE of conventional and new GMM estimators in cases of $r = 0.5$ and 1, respectively. While in case of $r > 1$ (5, 10, and 25) are presented in Tables 3 to 5, respectively. In our simulation study, the main factors which effect on the bias and RMSE values for all GMM estimators are ρ, T, N, and r. From Tables 1 to 5, we can summarize these effects for all estimators (conventional and new) in following points:

- As ρ increases the bias increases and RMSE increases.
- As T increases the bias increases and RMSE decreases.
- As N increases the bias decreases and RMSE decreases.
- As r increases the bias increases and RMSE increases.

Table 1 indicates that, in case of r close to zero, the bias and RMSE of OLEV and SSYS estimators are approximately equivalent to the bias and RMSE of LEV and SYS estimators, respectively. Therefore, we don't get a significant advantage from use the suggested weighting matrices (Z^{ol} and Z^{os}) in this case.

Tables 2 to 5 indicate that, in case of $r \geq 1$, the bias and RMSE of all estimators (conventional and new) are increased with increasing by r. While the bias and RMSE of OLEV and SSYS estimators show a much slower increase whenever r increased. Consequently, we conclude that OLEV and SSYS estimators are more efficient than the LEV and SYS estimators especially when $r > 1$.

In general, the all estimators in "step 2" are smaller in bias and RMSE than "step 1". While in "step 3", the bias and RMSE of all estimators are the smallest if $r > 1$ only. Moreover, the bias and RMSE of OLEV (step 3) are approximately equivalent to the bias and RMSE of OLEV (step 2) in all simulation situations.

From Tables 3 to 5, we can note that, in case of $r > 1$, the bias of SSYS (step 3) is the smallest in most situations except the case of ρ and T are small (i.e., ρ = 0.25, T = 4). In this case, the OLEV (step 2 or step 3) estimator is the smallest in bias. However, the RMSE of SSYS (step 2 or step 3) is the smallest in all situations.

Since, the OLEV and SSYS estimators based on the variance ratio estimation, $\hat{r} = \hat{\sigma}_\gamma^2/\hat{\sigma}_\varepsilon^2$. So, we use the Monte Carlo simulation again to illustrate the performance of $\hat{\rho}$ in different cases of sample size (N = 50, 100) and different values of the variance ratio ($r = 0$, 0.5, 1, 5, 10, and 25). We summarize the simulation result in Figure 3. We can note that in cases of $r = 0$, 0.5, and 1 the bias of $\hat{r}$ close to zero, while in the case of increasing r (specifically when $r \geq 5$) the bias of $\hat{r}$, Bias $(\hat{r}) = \hat{r} - r$, increases significantly, especially when ρ increases (close to one). So, the advantage of the OLEV and SSYS estimators decrease as ρ grows to unity because a high ρ leads to an unreliable estimate of r. However, still the OLEV and SSYS estimators are the smallest in bias and RMSE.

6. Conclusion

The LEV and SYS estimators for DPD models are efficient estimators. This efficiency is affected by the choice of the weighting matrix. Our Monte Carlo results indicate that the LEV and SYS estimators are vulnerable to an increase in r. One of the most distinguishing features in these experiments was that biases and RMSE increase with r in most cases. So, we proposed the new weighting matrices which included the r ratio to improve the efficient of LEV and SYS estimators. And then we get the new estimators (OLEV and SSYS).

By using the KI, we show that the OLEV and SSYS estimators are more efficient than LEV and SYS estimators, respectively. Moreover, the potential efficiency gain for OLEV and SSYS estimators becomes large when r increases. This is also confirmed by our simulation study. Since, the bias and RMSE of SSYS are smaller, in most situations, than the bias and RMSE of OLEV especially when $r > 1$. Consequently, we conclude that the SSYS estimator will provide useful parameter estimates for the practitioner.

References

Abonazel, M. R. (2014). Some estimation methods for dynamic panel data models. PhD thesis. Institute of Statistical Studies and Research. Cairo University.

Ahn, S.C., Schmidt, P. (1995). Efficient estimation of models for dynamic panel data. *Journal of Econometrics* 68:5–28.

Alonso-Borrego, C., Arellano, M. (1999). Symmetrically normalized instrumental variable estimation using panel data. *Journal of Business and Economic Statistics* 17:36–49.

Alvarez, J., Arellano, M. (2003). The time series and cross-section asymptotics of dynamic panel data estimators. *Econometrica* 71:1121–1159.

Anderson, T., Hsiao, C. (1982). Formulation and estimation of dynamic models using panel data. *Journal of Econometrics* 18:47–82.

Arellano, M., Bond, S. (1991). Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. *Review of Economic Studies* 58:277–98.

Arellano, M., Bover, O. (1995). Another look at the instrumental variable estimation of error-components models. *Journal of Econometrics* 68:29-51.

Baltagi, B. H. (2013). *Econometric Analysis of Panel Data*. 5th ed. Chichester: John Wiley and Sons.

Binder, M., Hsiao, C., Pesaran, M. H. (2005). Estimation and inference in short panel vector autoregressions with unit roots and cointegration. *Econometric Theory* 21:795–837.

Blundell, R., Bond, S. (1998). Initial conditions and moment restrictions in dynamic panel data models. *Journal of Econometrics* 87:115–143.

Blundell, R., Bond, S., Windmeijer F. (2001). Estimation in dynamic panel data models: improving on the performance of the standard GMM estimator. In: Baltagi, B., Fomby, T., Carter Hill, R., eds.

Nonstationary Panels, Panel Cointegration, and Dynamic Panels. Advances in econometrics. Vol. 15. Emerald Group Publishing Limited.
Bond, S. R. (2002). Dynamic panel data models: A guide to micro data methods and practice. *Portuguese Economic Journal* 1:141–162.
Bun, M., Sarafidis, V. (2014). Dynamic panel data models. In: Baltagi, B. H., ed. *The Oxford Handbook of Panel Data.* Oxford: Oxford University Press.
Bun, M., Kiviet, J. (2006). The effects of dynamic feedbacks on LS and MM estimator accuracy in panel data models. *Journal of Econometrics* 132:409–444.
Dhaene, G., Jochmans, K. (2012). Split-panel jackknife estimation of fixed-effect models, Working Paper. In press, *Review of Economic Studies.* Available online 12 February 2015, http://www.restud.oxfordjournals.org/content/early/2015/03/29/restud.rdv007.abstract.
Han, C., Philips, P. (2010). GMM estimation for dynamic panels with fixed effects and strong instruments at unity. *Econometrics Theory* 26:119–151.
Han, C., Phillips, P., Sul, D. (2014). X-differencing and dynamic panel model estimation. *Econometric Theory* 30:201–251.
Hansen, L.P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica* 50:1029–1054.
Hayakawa, K. (2007). Small sample bias properties of the system GMM estimator in dynamic panel data models. *Economics Letters* 95:32–38.
Hayakawa, K. Pesaran, M.H. (2014). Robust standard errors in transformed likelihood estimation of dynamic panel data models with cross-sectional heteroskedasticity. Working Paper, Hiroshima University. In press, *Journal of Econometrics*. Available online 20 April 2015, http://www.sciencedirect.com/science/article/pii/S0304407615001244.
Holtz-Eakin, D., Newey, W., Rosen, H.S. (1988). Estimating vector autoregressions with panel data. *Econometrica* 56:1371–1395.
Hsiao, C. (2014). *Analysis of Panel Data*. 3rd ed. Cambridge: Cambridge University Press.
Hsiao, C., Pesaran, M.H, Tahmiscioglu, A.K. (2002). Maximum likelihood estimation of fixed effects dynamic panel data models covering short time periods. *Journal of Econometrics* 109:107–150.
Jung, H., Kwon, H. (2007). An alternative system GMM estimation in dynamic panel models, Hi-Stat Discussion Paper No. 217. Hitotsubashi University.
Kiviet, J. F. (2007). On the optimal weighting matrix for the GMM system estimator in dynamic panel data models, Discussion Paper: 2007/08. University of Amsterdam.
Lancaster, T. (2002). Orthogonal parameters and panel data. *Review of Economic Studies* 69:647–666.
Liu, S., Neudecker, H. (1997). Kantorovich inequalities and efficiency comparisons for several classes of estimators in linear models. *Statistica Neerlandica* 51:345–355.
Nickell, S. (1981). Biases in dynamic models with fixed effects. *Econometrica* 49:1417–1426.
Ogaki, M. (1993). Generalized method of moments: econometric applications. In: Maddala, G .S., Rao, C. R., Vinod, H. D, eds. *Handbook of Statistics*. vol.11. Amsterdam: Elsevier Science Publishers.
Roodman, D. (2009). A note on the theme of too many instruments. *Oxford Bulletin of Economics and Statistics* 71:135–158.
Windmeijer, F. (2000). Efficiency comparisons for a system GMM estimator in dynamic panel data models. In: Heijmans, R., Pollock D., Satorra, A., eds. *Innovations in Multivariate Statistical Analysis.* Advanced Studies in Theoretical and Applied Econometrics, vol. 36. Dordrecht: Kluwer Academic Publishers.
Windmeijer, F. (2005). A Finite sample correction for the variance of linear efficient two-step GMM estimators. *Journal of Econometrics* 126:25–51.
Youssef, A., El-sheikh, A., Abonazel, M. (2014a). Improving the efficiency of GMM estimators for dynamic panel models. *Far East Journal of Theoretical Statistics* 47:171–189.
Youssef, A., El-sheikh, A., Abonazel, M. (2014b). New GMM estimators for dynamic panel data models. *International Journal of Innovative Research in Science, Engineering and Technology* 3: 16414–16425.

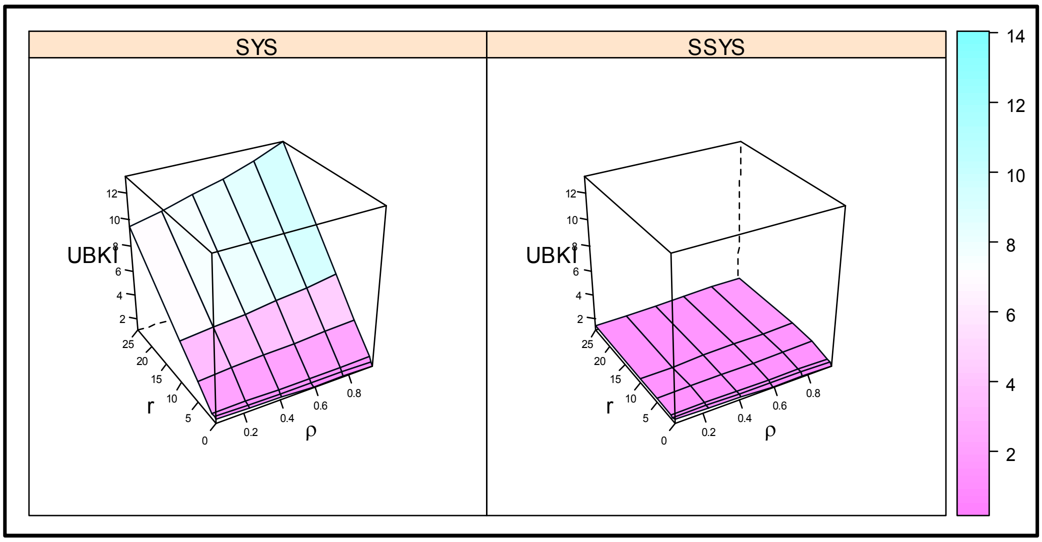

Figure 1: Kantorovich inequality efficiency bounds for SYS and SSYS estimators when $T = 3$

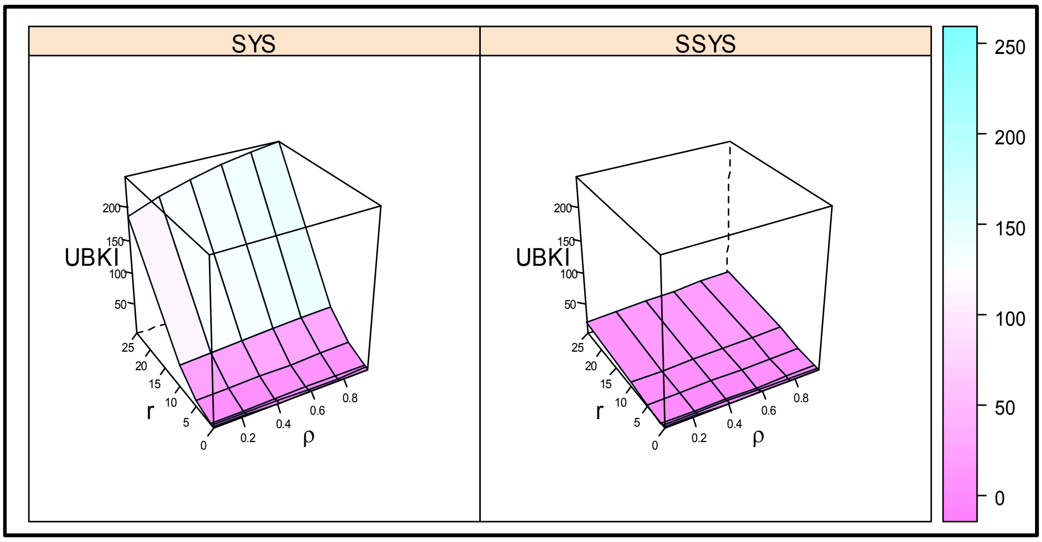

Figure 2: Kantorovich inequality efficiency bounds for SYS and SSYS estimators when $T = 4$

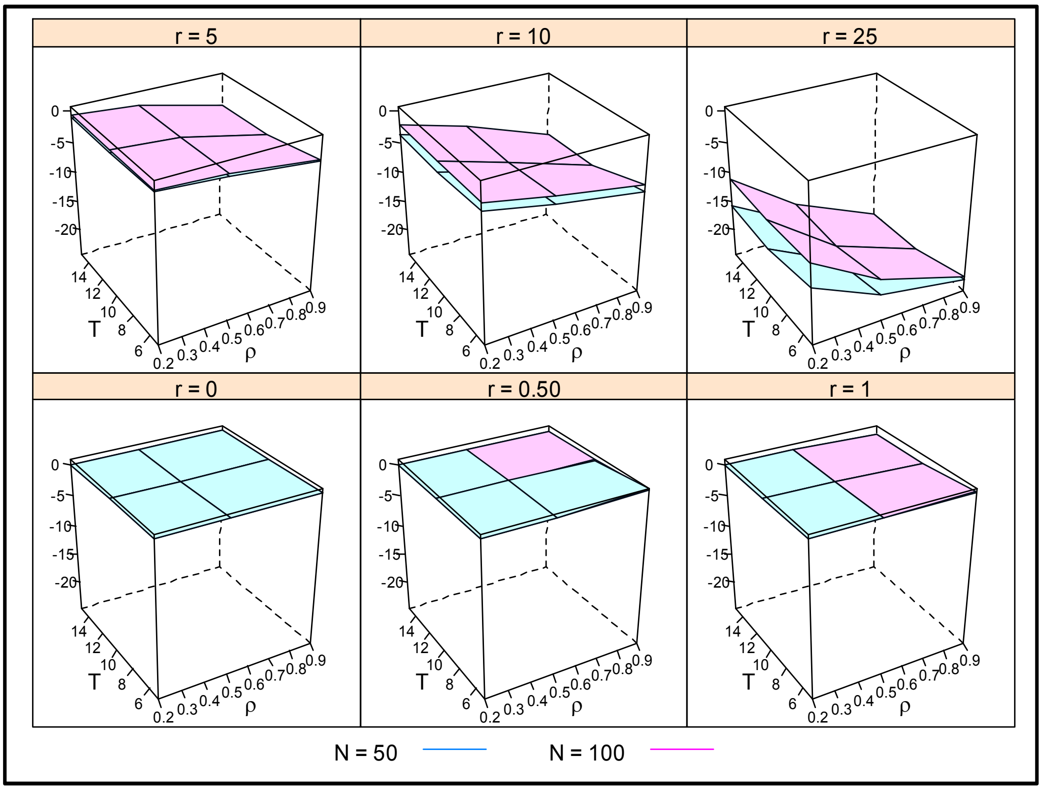

Figure 3: Monte Carlo simulation results of Bias ($\hat{r}$) as a function of ρ and T when N = 50 and 100

Table 1: **Bias** and *RMSE* for conventional and new GMM estimators when $r = 0.25/0.5 = 0.5$

	LEV			OLEV			SYS			SSYS		
	Step 1	Step 2	Step 3	Step 1	Step 2	Step 3	Step 1	Step 2	Step 3	Step 1	Step 2	Step 3
$N = 150$												
$\rho = 0.25, T = 4$	**0.0042**	**0.0036**	**0.0037**	**0.0024**	**-0.0006**	**-0.0007**	**0.0003**	**0.0054**	**0.0065**	**-0.0030**	**0.0045**	**0.0063**
	0.1076	*0.1079*	*0.1081*	*0.1078*	*0.1082*	*0.1082*	*0.0946*	*0.0886*	*0.0898*	*0.0951*	*0.0887*	*0.0898*
$\rho = 0.25, T = 8$	**0.0086**	**0.0081**	**0.0081**	**0.0051**	**-0.0022**	**-0.0023**	**-0.0026**	**0.0005**	**0.0016**	**-0.0048**	**-0.0001**	**0.0014**
	0.0565	*0.0577*	*0.0580*	*0.0560*	*0.0567*	*0.0568*	*0.0482*	*0.0449*	*0.0467*	*0.0486*	*0.0450*	*0.0467*
$\rho = 0.85, T = 4$	**0.0096**	**0.0070**	**0.0069**	**0.0073**	**0.0037**	**0.0040**	**-0.0006**	**-0.0060**	**-0.0144**	**-0.0167**	**-0.0129**	**-0.0182**
	0.1424	*0.1441*	*0.1442*	*0.1442*	*0.1452*	*0.1449*	*0.1165*	*0.1179*	*0.1307*	*0.1402*	*0.1278*	*0.1355*
$\rho = 0.85, T = 8$	**0.0352**	**0.0314**	**0.0310**	**0.0302**	**0.0205**	**0.0202**	**0.0143**	**0.0065**	**0.0022**	**0.0029**	**-0.0003**	**-0.0021**
	0.0660	*0.0665*	*0.0667*	*0.0661*	*0.0659*	*0.0656*	*0.0596*	*0.0547*	*0.0588*	*0.0673*	*0.0593*	*0.0615*
$N = 400$												
$\rho = 0.25, T = 4$	**0.0034**	**0.0032**	**0.0032**	**0.0028**	**0.0016**	**0.0016**	**0.0020**	**0.0034**	**0.0035**	**0.0008**	**0.0032**	**0.0034**
	0.0644	*0.0646*	*0.0646*	*0.0644*	*0.0645*	*0.0645*	*0.0626*	*0.0530*	*0.0531*	*0.0581*	*0.0530*	*0.0531*
$\rho = 0.25, T = 8$	**0.0049**	**0.0043**	**0.0042**	**0.0034**	**0.0001**	**0.0001**	**0.0010**	**0.0022**	**0.0025**	**0.0001**	**0.0021**	**0.0025**
	0.0344	*0.0345*	*0.0345*	*0.0342*	*0.0344*	*0.0344*	*0.0358*	*0.0260*	*0.0262*	*0.0293*	*0.0259*	*0.0262*
$\rho = 0.85, T = 4$	**0.0028**	**0.0022**	**0.0022**	**0.0021**	**0.0006**	**0.0007**	**-0.0020**	**-0.0064**	**-0.0094**	**-0.0082**	**-0.0080**	**-0.0101**
	0.0685	*0.0690*	*0.0691*	*0.0690*	*0.0695*	*0.0695*	*0.0724*	*0.0669*	*0.0713*	*0.0797*	*0.0686*	*0.0717*
$\rho = 0.85, T = 8$	**0.0146**	**0.0118**	**0.0117**	**0.0119**	**0.0065**	**0.0064**	**0.0051**	**-0.0003**	**-0.0020**	**-0.0004**	**-0.0022**	**-0.0028**
	0.0393	*0.0393*	*0.0393*	*0.0394*	*0.0389*	*0.0388*	*0.0410*	*0.0337*	*0.0347*	*0.0407*	*0.0338*	*0.0347*
$N = 600$												
$\rho = 0.25, T = 4$	**0.0018**	**0.0017**	**0.0017**	**0.0013**	**0.0004**	**0.0004**	**-0.0003**	**0.0014**	**0.0015**	**-0.0013**	**0.0013**	**0.0015**
	0.0536	*0.0537*	*0.0537*	*0.0537*	*0.0537*	*0.0537*	*0.0479*	*0.0449*	*0.0450*	*0.0480*	*0.0449*	*0.0450*
$\rho = 0.25, T = 8$	**0.0027**	**0.0022**	**0.0022**	**0.0016**	**-0.0006**	**-0.0006**	**0.0000**	**0.0007**	**0.0009**	**-0.0006**	**0.0006**	**0.0009**
	0.0276	*0.0278*	*0.0278*	*0.0274*	*0.0278*	*0.0278*	*0.0240*	*0.0207*	*0.0208*	*0.0240*	*0.0207*	*0.0208*
$\rho = 0.85, T = 4$	**0.0025**	**0.0020**	**0.0020**	**0.0020**	**0.0008**	**0.0009**	**-0.0004**	**-0.0049**	**-0.0065**	**-0.0046**	**-0.0058**	**-0.0069**
	0.0538	*0.0539*	*0.0539*	*0.0542*	*0.0545*	*0.0544*	*0.0562*	*0.0516*	*0.0535*	*0.0625*	*0.0531*	*0.0543*
$\rho = 0.85, T = 8$	**0.0134**	**0.0115**	**0.0114**	**0.0115**	**0.0075**	**0.0075**	**0.0070**	**0.0007**	**-0.0005**	**0.0033**	**-0.0003**	**-0.0009**
	0.0337	*0.0336*	*0.0336*	*0.0336*	*0.0333*	*0.0333*	*0.0329*	*0.0254*	*0.0263*	*0.0343*	*0.0259*	*0.0265*

RMSE values are written in *italics* under the bias values which are written in **bold**

Table 2: **Bias** and *RMSE* for conventional and new GMM estimators when $r = 0.25/0.25 = 1$

	LEV			OLEV			SYS			SSYS		
	Step 1	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**
$N = 150$												
$\rho = 0.25, T = 4$	**0.0068**	**0.0023**	**0.0020**	**0.0018**	**-0.0041**	**-0.0042**	**0.0043**	**0.0078**	**0.0079**	**-0.0021**	**0.0056**	**0.0073**
	0.1200	*0.1225*	*0.1228*	*0.1212*	*0.1227*	*0.1227*	*0.1023*	*0.0921*	*0.0926*	*0.1033*	*0.0922*	*0.0926*
$\rho = 0.25, T = 8$	**0.0222**	**0.0154**	**0.0147**	**0.0122**	**0.0005**	**0.0004**	**0.0053**	**0.0046**	**0.0048**	**-0.0013**	**0.0025**	**0.0041**
	0.0642	*0.0637*	*0.0638*	*0.0611*	*0.0604*	*0.0604*	*0.0506*	*0.0455*	*0.0472*	*0.0503*	*0.0453*	*0.0472*
$\rho = 0.85, T = 4$	**0.0564**	**0.0542**	**0.0541**	**0.0536**	**0.0518**	**0.0522**	**0.0403**	**0.0342**	**0.0202**	**0.0221**	**0.0239**	**0.0155**
	0.3378	*0.3368*	*0.3371*	*0.3405*	*0.3337*	*0.3336*	*0.1487*	*0.1433*	*0.1654*	*0.1710*	*0.1526*	*0.1677*
$\rho = 0.85, T = 8$	**0.0685**	**0.0637**	**0.0631**	**0.0623**	**0.0549**	**0.0544**	**0.0421**	**0.0336**	**0.0274**	**0.0248**	**0.0212**	**0.0180**
	0.0901	*0.0900*	*0.0902*	*0.0890*	*0.0890*	*0.0886*	*0.0715*	*0.0668*	*0.0704*	*0.0770*	*0.0706*	*0.0733*
$N = 400$												
$\rho = 0.25, T = 4$	**0.0028**	**0.0017**	**0.0016**	**0.0009**	**-0.0008**	**-0.0008**	**0.0020**	**0.0042**	**0.0037**	**-0.0003**	**0.0035**	**0.0036**
	0.0766	*0.0773*	*0.0774*	*0.0769*	*0.0773*	*0.0772*	*0.0656*	*0.0575*	*0.0574*	*0.0649*	*0.0574*	*0.0574*
$\rho = 0.25, T = 8$	**0.0087**	**0.0059**	**0.0058**	**0.0050**	**0.0009**	**0.0008**	**0.0019**	**0.0008**	**0.0008**	**-0.0005**	**0.0005**	**0.0008**
	0.0373	*0.0365*	*0.0365*	*0.0361*	*0.0356*	*0.0356*	*0.0314*	*0.0256*	*0.0258*	*0.0311*	*0.0256*	*0.0258*
$\rho = 0.85, T = 4$	**0.0162**	**0.0140**	**0.0139**	**0.0135**	**0.0116**	**0.0118**	**0.0045**	**0.0020**	**-0.0092**	**-0.0082**	**-0.0053**	**-0.0124**
	0.2163	*0.2218*	*0.2218*	*0.2219*	*0.2222*	*0.2221*	*0.1041*	*0.0922*	*0.1069*	*0.1168*	*0.1027*	*0.1087*
$\rho = 0.85, T = 8$	**0.0381**	**0.0334**	**0.0330**	**0.0331**	**0.0274**	**0.0272**	**0.0250**	**0.0150**	**0.0101**	**0.0142**	**0.0095**	**0.0070**
	0.0582	*0.0575*	*0.0575*	*0.0572*	*0.0559*	*0.0558*	*0.0503*	*0.0424*	*0.0442*	*0.0526*	*0.0436*	*0.0449*
$N = 600$												
$\rho = 0.25, T = 4$	**0.0010**	**0.0002**	**0.0002**	**-0.0003**	**-0.0015**	**-0.0015**	**0.0007**	**0.0028**	**0.0025**	**-0.0007**	**0.0024**	**0.0025**
	0.0594	*0.0599*	*0.0599*	*0.0596*	*0.0598*	*0.0598*	*0.0504*	*0.0447*	*0.0448*	*0.0502*	*0.0447*	*0.0448*
$\rho = 0.25, T = 8$	**0.0067**	**0.0046**	**0.0045**	**0.0042**	**0.0013**	**0.0012**	**0.0024**	**0.0010**	**0.0011**	**0.0008**	**0.0009**	**0.0010**
	0.0301	*0.0297*	*0.0297*	*0.0293*	*0.0293*	*0.0293*	*0.0251*	*0.0203*	*0.0204*	*0.0248*	*0.0202*	*0.0204*
$\rho = 0.85, T = 4$	**0.0066**	**0.0044**	**0.0045**	**0.0047**	**0.0024**	**0.0027**	**0.0005**	**-0.0002**	**-0.0088**	**-0.0092**	**-0.0043**	**-0.0101**
	0.0995	*0.1014*	*0.1014*	*0.1008*	*0.1012*	*0.1012*	*0.0879*	*0.0723*	*0.0811*	*0.0995*	*0.0760*	*0.0825*
$\rho = 0.85, T = 8$	**0.0302**	**0.0262**	**0.0260**	**0.0261**	**0.0217**	**0.0216**	**0.0203**	**0.0110**	**0.0070**	**0.0117**	**0.0074**	**0.0053**
	0.0481	*0.0472*	*0.0472*	*0.0473*	*0.0457*	*0.0456*	*0.0426*	*0.0348*	*0.0357*	*0.0441*	*0.0355*	*0.0360*

RMSE values are written in *italics* under the bias values which are written in **bold**

Table 3: **Bias** and *RMSE* for conventional and new GMM estimators when $r = 2.5/0.5 = 5$

	LEV			OLEV			SYS			SSYS		
	Step 1	Step 2	Step 3	Step 1	Step 2	Step 3	Step 1	Step 2	Step 3	Step 1	Step 2	Step 3
$N = 150$												
$\rho = 0.25, T = 4$	**0.0226**	**-0.0007**	**-0.0024**	**-0.0055**	**-0.0125**	**-0.0124**	**0.0358**	**0.0394**	**0.0330**	**0.0193**	**0.0279**	**0.0279**
	0.2306	*0.2394*	*0.2394*	*0.2355*	*0.2361*	*0.2361*	*0.1634*	*0.1277*	*0.1245*	*0.1427*	*0.1214*	*0.1212*
$\rho = 0.25, T = 8$	**0.0881**	**0.0361**	**0.0290**	**0.0277**	**0.0097**	**0.0096**	**0.0486**	**0.0240**	**0.0138**	**0.0055**	**0.0052**	**0.0052**
	0.1226	*0.0890*	*0.0854*	*0.0824*	*0.0762*	*0.0761*	*0.0819*	*0.0571*	*0.0529*	*0.0593*	*0.0483*	*0.0495*
$\rho = 0.85, T = 4$	**0.1216**	**0.1204**	**0.1205**	**0.1207**	**0.1195**	**0.1199**	**0.1113**	**0.1037**	**0.0898**	**0.0775**	**0.0809**	**0.0752**
	0.2133	*0.2126*	*0.2129*	*0.2149*	*0.2114*	*0.2117*	*0.1582*	*0.1763*	*0.2128*	*0.1570*	*0.1700*	*0.2043*
$\rho = 0.85, T = 8$	**0.1162**	**0.1121**	**0.1113**	**0.1118**	**0.1068**	**0.1064**	**0.1045**	**0.0959**	**0.0897**	**0.0922**	**0.0852**	**0.0801**
	0.1225	*0.1203*	*0.1200*	*0.1200*	*0.1169*	*0.1167*	*0.1114*	*0.1060*	*0.1047*	*0.1083*	*0.1037*	*0.1029*
$N = 400$												
$\rho = 0.25, T = 4$	**0.0143**	**0.0033**	**0.0029**	**0.0024**	**-0.0009**	**-0.0009**	**0.0165**	**0.0157**	**0.0120**	**0.0092**	**0.0113**	**0.0110**
	0.1179	*0.1190*	*0.1190*	*0.1180*	*0.1181*	*0.1181*	*0.1016*	*0.0716*	*0.0697*	*0.0921*	*0.0702*	*0.0694*
$\rho = 0.25, T = 8$	**0.0373**	**0.0115**	**0.0100**	**0.0100**	**0.0029**	**0.0029**	**0.0194**	**0.0045**	**0.0018**	**0.0004**	**0.0007**	**0.0009**
	0.0634	*0.0488*	*0.0483*	*0.0476*	*0.0461*	*0.0461*	*0.0437*	*0.0279*	*0.0276*	*0.0352*	*0.0271*	*0.0275*
$\rho = 0.85, T = 4$	**0.0631**	**0.0601**	**0.0602**	**0.0606**	**0.0590**	**0.0595**	**0.0586**	**0.0531**	**0.0394**	**0.0382**	**0.0419**	**0.0310**
	0.1764	*0.1799*	*0.1800*	*0.1803*	*0.1801*	*0.1803*	*0.1401*	*0.1319*	*0.1521*	*0.1393*	*0.1333*	*0.1516*
$\rho = 0.85, T = 8$	**0.0841**	**0.0755**	**0.0743**	**0.0751**	**0.0686**	**0.0681**	**0.0751**	**0.0635**	**0.0556**	**0.0580**	**0.0502**	**0.0446**
	0.0908	*0.0849*	*0.0841*	*0.0848*	*0.0796*	*0.0792*	*0.0827*	*0.0740*	*0.0716*	*0.0748*	*0.0686*	*0.0682*
$N = 600$												
$\rho = 0.25, T = 4$	**0.0112**	**0.0030**	**0.0028**	**0.0027**	**0.0003**	**0.0004**	**0.0122**	**0.0098**	**0.0076**	**0.0062**	**0.0071**	**0.0071**
	0.1000	*0.1012*	*0.1011*	*0.1005*	*0.1003*	*0.1003*	*0.0867*	*0.0572*	*0.0552*	*0.0785*	*0.0561*	*0.0550*
$\rho = 0.25, T = 8$	**0.0246**	**0.0070**	**0.0063**	**0.0062**	**0.0018**	**0.0018**	**0.0133**	**0.0027**	**0.0014**	**0.0011**	**0.0010**	**0.0011**
	0.0483	*0.0390*	*0.0387*	*0.0383*	*0.0373*	*0.0373*	*0.0347*	*0.0216*	*0.0214*	*0.0289*	*0.0212*	*0.0214*
$\rho = 0.85, T = 4$	**0.0446**	**0.0402**	**0.0400**	**0.0410**	**0.0389**	**0.0390**	**0.0367**	**0.0253**	**0.0131**	**0.0140**	**0.0172**	**0.0096**
	0.1473	*0.1548*	*0.1546*	*0.1548*	*0.1545*	*0.1544*	*0.1111*	*0.1052*	*0.1235*	*0.1281*	*0.1110*	*0.1253*
$\rho = 0.85, T = 8$	**0.0687**	**0.0584**	**0.0572**	**0.0580**	**0.0517**	**0.0514**	**0.0607**	**0.0489**	**0.0410**	**0.0414**	**0.0347**	**0.0299**
	0.0763	*0.0689*	*0.0681*	*0.0690*	*0.0635*	*0.0631*	*0.0694*	*0.0602*	*0.0578*	*0.0614*	*0.0552*	*0.0549*

RMSE values are written in *italics* under the bias values which are written in **bold**

Table 4: **Bias** and *RMSE* for conventional and new GMM estimators when $r = 2.5/0.25 = 10$

	LEV			OLEV			SYS			SSYS		
	Step 1	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**
$N = 150$												
$\rho = 0.25, T = 4$	**0.0862**	**0.0349**	**0.0299**	**0.0306**	**0.0174**	**0.0178**	**0.0950**	**0.0845**	**0.0709**	**0.0499**	**0.0524**	**0.0512**
	0.3183	*0.3325*	*0.3308*	*0.3254*	*0.3257*	*0.3259*	*0.2149*	*0.1721*	*0.1656*	*0.1728*	*0.1517*	*0.1548*
$\rho = 0.25, T = 8$	**0.1568**	**0.0629**	**0.0464**	**0.0457**	**0.0230**	**0.0231**	**0.0957**	**0.0538**	**0.0319**	**0.0096**	**0.0066**	**0.0051**
	0.1884	*0.1167*	*0.1057*	*0.1013*	*0.0902*	*0.0902*	*0.1254*	*0.0863*	*0.0692*	*0.0631*	*0.0506*	*0.0501*
$\rho = 0.85, T = 4$	**0.1365**	**0.1366**	**0.1364**	**0.1363**	**0.1364**	**0.1366**	**0.1310**	**0.1249**	**0.1259**	**0.1082**	**0.1079**	**0.1103**
	0.1678	*0.1707*	*0.1706*	*0.1698*	*0.1708*	*0.1709*	*0.1577*	*0.1694*	*0.2006*	*0.1673*	*0.1729*	*0.2066*
$\rho = 0.85, T = 8$	**0.1373**	**0.1360**	**0.1357**	**0.1359**	**0.1345**	**0.1343**	**0.1304**	**0.1271**	**0.1244**	**0.1243**	**0.1216**	**0.1194**
	0.1404	*0.1396*	*0.1394*	*0.1395*	*0.1386*	*0.1385*	*0.1336*	*0.1318*	*0.1316*	*0.1308*	*0.1295*	*0.1296*
$N = 400$												
$\rho = 0.25, T = 4$	**0.0266**	**0.0028**	**0.0017**	**0.0009**	**-0.0027**	**-0.0027**	**0.0309**	**0.0261**	**0.0197**	**0.0147**	**0.0155**	**0.0159**
	0.1704	*0.1710*	*0.1708*	*0.1704*	*0.1691*	*0.1691*	*0.1380*	*0.0877*	*0.0820*	*0.1088*	*0.0800*	*0.0790*
$\rho = 0.25, T = 8$	**0.0674**	**0.0189**	**0.0160**	**0.0166**	**0.0093**	**0.0093**	**0.0388**	**0.0106**	**0.0038**	**0.0016**	**0.0010**	**0.0010**
	0.0915	*0.0586*	*0.0574*	*0.0569*	*0.0546*	*0.0546*	*0.0599*	*0.0308*	*0.0277*	*0.0367*	*0.0267*	*0.0270*
$\rho = 0.85, T = 4$	**0.1118**	**0.1105**	**0.1103**	**0.1109**	**0.1099**	**0.1100**	**0.1059**	**0.1004**	**0.0959**	**0.0796**	**0.0830**	**0.0865**
	0.1743	*0.1756*	*0.1754*	*0.1757*	*0.1755*	*0.1754*	*0.1545*	*0.1632*	*0.1977*	*0.1615*	*0.1684*	*0.1949*
$\rho = 0.85, T = 8$	**0.1165**	**0.1120**	**0.1115**	**0.1123**	**0.1101**	**0.1099**	**0.1094**	**0.1015**	**0.0962**	**0.0973**	**0.0917**	**0.0872**
	0.1210	*0.1178*	*0.1175*	*0.1182*	*0.1166*	*0.1164*	*0.1142*	*0.1090*	*0.1068*	*0.1088*	*0.1046*	*0.1037*
$N = 600$												
$\rho = 0.25, T = 4$	**0.0200**	**0.0040**	**0.0035**	**0.0029**	**0.0006**	**0.0006**	**0.0228**	**0.0173**	**0.0125**	**0.0130**	**0.0108**	**0.0105**
	0.1341	*0.1330*	*0.1328*	*0.1322*	*0.1317*	*0.1317*	*0.1114*	*0.0636*	*0.0583*	*0.0888*	*0.0591*	*0.0569*
$\rho = 0.25, T = 8$	**0.0500**	**0.0122**	**0.0106**	**0.0108**	**0.0057**	**0.0057**	**0.0291**	**0.0059**	**0.0019**	**0.0014**	**0.0007**	**0.0007**
	0.0722	*0.0475*	*0.0469*	*0.0466*	*0.0453*	*0.0453*	*0.0477*	*0.0226*	*0.0209*	*0.0299*	*0.0205*	*0.0206*
$\rho = 0.85, T = 4$	**0.0902**	**0.0883**	**0.0884**	**0.0882**	**0.0880**	**0.0880**	**0.0931**	**0.0884**	**0.0738**	**0.0715**	**0.0718**	**0.0643**
	0.2068	*0.2155*	*0.2156*	*0.2171*	*0.2156*	*0.2155*	*0.1465*	*0.1407*	*0.1608*	*0.1411*	*0.1395*	*0.1767*
$\rho = 0.85, T = 8$	**0.1080**	**0.1028**	**0.1022**	**0.1026**	**0.1004**	**0.1002**	**0.1008**	**0.0924**	**0.0862**	**0.0871**	**0.0810**	**0.0763**
	0.1131	*0.1097*	*0.1093*	*0.1095*	*0.1078*	*0.1076*	*0.1060*	*0.1006*	*0.0986*	*0.0989*	*0.0954*	*0.0946*

RMSE values are written in *italics* under the bias values which are written in **bold**

Table 5: **Bias** and *RMSE* for conventional and new GMM estimators when $r = 12.5/0.5 = 25$

	LEV			**OLEV**			**SYS**			**SSYS**		
	Step 1	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**	**Step 1**	**Step 2**	**Step 3**
$N = 150$												
$\rho = 0.25, T = 4$	**0.1804**	**0.1040**	**0.0986**	**0.0970**	**0.0857**	**0.0863**	**0.1951**	**0.1815**	**0.1566**	**0.1141**	**0.1136**	**0.1071**
	0.7134	*0.7851*	*0.7888*	*0.7847*	*0.7888*	*0.7891*	*0.3216*	*0.2896*	*0.2784*	*0.2469*	*0.2466*	*0.2470*
$\rho = 0.25, T = 8$	**0.2838**	**0.1321**	**0.0941**	**0.0918**	**0.0603**	**0.0604**	**0.1977**	**0.1480**	**0.1124**	**0.0280**	**0.0227**	**0.0188**
	0.3119	*0.1890*	*0.1560*	*0.1489*	*0.1249*	*0.1249*	*0.2252*	*0.1832*	*0.1554*	*0.0753*	*0.0639*	*0.0623*
$\rho = 0.85, T = 4$	**0.1545**	**0.1526**	**0.1525**	**0.1538**	**0.1527**	**0.1528**	**0.1335**	**0.1313**	**0.1217**	**0.1081**	**0.1111**	**0.1152**
	0.4956	*0.4570*	*0.4570*	*0.4938*	*0.4578*	*0.4578*	*0.1704*	*0.1647*	*0.1851*	*0.1830*	*0.1855*	*0.1884*
$\rho = 0.85, T = 8$	**0.1427**	**0.1416**	**0.1414**	**0.1416**	**0.1404**	**0.1403**	**0.1395**	**0.1373**	**0.1357**	**0.1365**	**0.1345**	**0.1330**
	0.1440	*0.1433*	*0.1432*	*0.1433*	*0.1423*	*0.1423*	*0.1409*	*0.1393*	*0.1388*	*0.1387*	*0.1374*	*0.1370*
$N = 400$												
$\rho = 0.25, T = 4$	**0.0625**	**0.0066**	**0.0038**	**0.0044**	**-0.0009**	**-0.0009**	**0.0731**	**0.0596**	**0.0463**	**0.0339**	**0.0283**	**0.0260**
	0.2738	*0.2859*	*0.2849*	*0.2819*	*0.2824*	*0.2824*	*0.2023*	*0.1432*	*0.1326*	*0.1375*	*0.1108*	*0.1144*
$\rho = 0.25, T = 8$	**0.1467**	**0.0373**	**0.0283**	**0.0293**	**0.0201**	**0.0202**	**0.0943**	**0.0412**	**0.0194**	**0.0048**	**0.0020**	**0.0011**
	0.1734	*0.0867*	*0.0816*	*0.0813*	*0.0771*	*0.0771*	*0.1161*	*0.0650*	*0.0450*	*0.0395*	*0.0291*	*0.0290*
$\rho = 0.85, T = 4$	**0.1259**	**0.1247**	**0.1247**	**0.1251**	**0.1241**	**0.1243**	**0.1235**	**0.1187**	**0.1143**	**0.1003**	**0.1038**	**0.1026**
	0.1596	*0.1627*	*0.1627*	*0.1625*	*0.1624*	*0.1624*	*0.1537*	*0.1543*	*0.1835*	*0.1533*	*0.1610*	*0.1804*
$\rho = 0.85, T = 8$	**0.1282**	**0.1239**	**0.1232**	**0.1240**	**0.1213**	**0.1211**	**0.1239**	**0.1196**	**0.1167**	**0.1141**	**0.1109**	**0.1084**
	0.1306	*0.1274*	*0.1269*	*0.1276*	*0.1253*	*0.1251*	*0.1265*	*0.1234*	*0.1222*	*0.1206*	*0.1188*	*0.1183*
$N = 600$												
$\rho = 0.25, T = 4$	**0.0532**	**0.0144**	**0.0131**	**0.0120**	**0.0100**	**0.0100**	**0.0594**	**0.0421**	**0.0334**	**0.0298**	**0.0225**	**0.0229**
	0.2124	*0.2173*	*0.2169*	*0.2168*	*0.2154*	*0.2154*	*0.1658*	*0.1089*	*0.1010*	*0.1097*	*0.0867*	*0.0909*
$\rho = 0.25, T = 8$	**0.1078**	**0.0234**	**0.0191**	**0.0202**	**0.0140**	**0.0140**	**0.0678**	**0.0216**	**0.0078**	**0.0028**	**0.0007**	**0.0003**
	0.1322	*0.0673*	*0.0653*	*0.0647*	*0.0627*	*0.0627*	*0.0870*	*0.0399*	*0.0277*	*0.0316*	*0.0223*	*0.0225*
$\rho = 0.85, T = 4$	**0.1176**	**0.1164**	**0.1164**	**0.1164**	**0.1161**	**0.1161**	**0.1133**	**0.1097**	**0.0983**	**0.0947**	**0.0956**	**0.0907**
	0.1767	*0.1844*	*0.1845*	*0.1829*	*0.1844*	*0.1844*	*0.1486*	*0.1583*	*0.1813*	*0.1453*	*0.1456*	*0.1626*
$\rho = 0.85, T = 8$	**0.1229**	**0.1174**	**0.1166**	**0.1175**	**0.1144**	**0.1142**	**0.1182**	**0.1139**	**0.1106**	**0.1061**	**0.1033**	**0.1008**
	0.1259	*0.1219*	*0.1213*	*0.1219*	*0.1194*	*0.1192*	*0.1212*	*0.1184*	*0.1169*	*0.1134*	*0.1121*	*0.1116*

RMSE values are written in *italics* under the bias values which are written in **bold**