# New GMM Estimators for Dynamic Panel Data Models

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# New GMM Estimators for Dynamic Panel Data Models

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**ABSTRACT**: In dynamic panel data (DPD) models, the generalized method of moments (GMM) estimation gives efficient estimators. However, this efficiency is affected by the choice of the initial weighting matrix. In practice, the inverse of the moment matrix of the instruments has been used as an initial weighting matrix which led to a loss of efficiency. Therefore, we will present new GMM estimators based on optimal or suboptimal weighting matrices in GMM estimation. Monte Carlo study indicates that the potential efficiency gain by using these matrices. Moreover, the bias and efficiency of the new GMM estimators are more reliable than any other conventional GMM estimators.

**KEYWORDS**: Dynamic panel data, Generalized method of moments, Monte Carlo simulation, Optimal and suboptimal weighting matrices.

## I. INTRODUCTION

The econometrics literatures focus on three types of GMM estimators when studying the DPD models. The First is first-difference GMM (DIF) estimator which presented by Arellano and Bond [4], and the second is level GMM (LEV) estimator which presented by Arellano and Bover [5], while the third is system GMM (SYS) estimator which presented by Blundell and Bond [6]. Since the SYS estimator combines moment conditions of DIF and LEV estimators, and it is generally known that using many instruments can improve the efficiency of various GMM estimators (Arellano and Bover [5]; Ahn and Schmidt [2]; Blundell and Bond [6]). Therefore, the SYS estimator is more efficient than DIF and LEV estimators. Despite the substantial efficiency gain, using many instruments has two important drawbacks: increased bias and unreliable inference (Newey and Smith [10]; Hayakawa [8]). Moreover, the SYS estimator does not always work well; Bun and Kiviet [7] showed that the bias of SYS estimator becomes large when the autoregressive parameter is close to unity and/or when the ratio of the variance of the individual effect to that of the error term departs from unity.

In general, an asymptotically efficient estimator can be obtained through the two-step procedure in the standard GMM estimation. In the first step, an initial positive semidefinite weighting matrix is used to obtain consistent estimates of the parameters. Given these consistent estimates, a weighting matrix can be constructed and used for asymptotically efficient two-step estimates. Arellano and Bond [4] showed that the two-step estimated standard errors have a small sample downward bias in DPD setting, and one-step estimates with robust standard errors are often preferred. Although an efficient weighting matrix for DIF estimator under the assumption that the errors are homoskedastic and are not serially correlated is easily derived, this is not the case for the LEV and SYS estimators.

In this paper, we will present new LEV and SYS estimators based on optimal or suboptimal weighting matrices, without increase of the moment conditions of these estimators. The new GMM estimators are more efficiency than the conventional GMM estimators.

This paper is organized as follows. Section II provides the model and reviews the conventional DIF, LEV, and SYS estimators. Section III presents the new GMM estimators. While section IV contains the Monte Carlo simulation study. Finally, section V offers the concluding remarks.

## II. RELATED WORK

Consider a simple DPD process of the form

$$y_{it} = \phi y_{i,t-1} + \mu_i + \varepsilon_{it}, \quad i = 1, \dots, N;\ t = 1, \dots, T. \tag{1}$$

Under the following assumptions:

(i) $\varepsilon_{it}$ are i.i.d across time and individuals and independent of $\mu_i$ and $y_{i1}$ with $E(\varepsilon_{it}) = 0, Var(\varepsilon_{it}) = \sigma_\varepsilon^2$.

(ii) $\mu_i$ are i.i.d across individuals with $E(\mu_i) = 0, Var(\mu_i) = \sigma_\mu^2$.

(iii) The initial observations satisfy $y_{i1} = \frac{\mu_i}{1-\phi} + w_{i1}$ for $i = 1, \dots, N$, where $w_{i1} = \sum_{j=0}^{\infty} \phi^j \varepsilon_{i,1-j}$ and independent of $\mu_i$.

Assumptions (i) and (ii) are the same as in Blundell and Bond [6], while assumption (iii) has been developed by Alvarez and Arellano [3].

Stacking equation (1) over time, we obtain

$$y_i = \phi y_{i,-1} + u_i, \tag{2}$$

where $y_i = (y_{i3}, \dots, y_{iT})', y_{i,-1} = (y_{i2}, \dots, y_{i,T-1})', u_i = (u_{i3}, \dots, u_{iT})'$, with $u_{it} = \mu_i + \varepsilon_{it}$.

Given these assumptions, we get three types of GMM estimators. These include DIF, LEV, and SYS estimators. In general, the GMM procedure used the suggested weighting matrix to get the one-step estimation, and then used the residuals from one-step estimation as a weighting matrix to get the two-step estimation.

In model (2), the individual effect ($\mu_i$) caused a severe correlation between the lagged endogenous variable ($y_{i,-1}$) and the error term ($u_i$). So, to eliminate this effect, Arellano and Bond [4] have used the first differences as:

$$\Delta y_i = \phi \Delta y_{i,-1} + \Delta u_i, \tag{3}$$

where $\Delta y_i = (y_{i3} - y_{i2}, \dots, y_{iT} - y_{i,T-1})'$, $\Delta y_{i,-1} = (y_{i2} - y_{i1}, \dots, y_{i,T-1} - y_{i,T-2})'$ and $\Delta u_i = (u_{i3} - u_{i2}, \dots, u_{iT} - u_{i,T-1})'$, and then they showed that

$$E(H_i^{D\prime} \Delta u_i) = 0, \tag{4}$$

where

$$H_i^D = \begin{pmatrix} y_{i1} & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & y_{i1} & y_{i2} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & y_{i1} & \cdots & y_{i,T-2} \end{pmatrix}. \tag{5}$$

Using (4) as the orthogonal conditions in the GMM, Arellano and Bond [4] constructed the one-step first-difference GMM (DIF1) estimator for $\phi$, which is given by

$$\hat{\phi}^D = (\Delta y_{-1}' H^D W^D H^{D\prime} \Delta y_{-1})^{-1} \Delta y_{-1}' H^D W^D H^{D\prime} \Delta y, \tag{6}$$

where $\Delta y_{-1} = (\Delta y_{1,-1}', \dots, \Delta y_{1,-N}')', \Delta y = (\Delta y_1', \dots, \Delta y_N')', H^D = (H_1^{D\prime}, \dots, H_N^{D\prime})'$, and

$$W^D = \left( \frac{1}{N} \sum_{i=1}^{N} H_i^{D\prime} D H_i^D \right)^{-1}, \tag{7}$$

where $D = FF'$, and $F$ is a $(T-2)\times(T-1)$ first-difference operator matrix

$$F = \begin{pmatrix} -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{pmatrix}. \quad (8)$$

To get the two-step first-difference GMM (DIF2) estimator, the moment conditions are weighted by

$$W_{(2)}^{D} = \left(\frac{1}{N}\sum_{i=1}^{N} H_i^{D\prime}\Delta\hat{u}_i\Delta\hat{u}_i' H_i^{D}\right)^{-1}, \quad (9)$$

where $\Delta\hat{u}_{\mathrm{i}}$ are the fitted residuals from DIF1estimator.

Blundell and Bond [6] showed that when $\phi$ is close to unity and/or $\sigma_\mu^2/\sigma_\varepsilon^2$ increases the instruments matrix (5) becomes invalid. This means that the first-difference GMM estimator has weak instruments problem.

Arellano and Bover [5] suggested a new method to eliminate the individual effect from instrumental variables. They considered the level model (2) and then showed that the instrumental variables matrix

$$H_i^{L} = \begin{pmatrix} \Delta y_{i2} & 0 & \cdots & 0 \\ 0 & \Delta y_{i3} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \Delta y_{i,T-1} \end{pmatrix}, \quad (10)$$

which is not contains individual effect and satisfied the orthogonal conditions

$$E(H_i^{L\prime} u_i) = 0. \quad (11)$$

Using (11), Arellano and Bover's [5] one-step level GMM (LEV1) estimator is calculated as:

$$\hat{\phi}^{L} = (y_{-1}' H^{L}\, W^{L}\; H^{L\prime} y_{-1})^{-1}\, y_{-1}' H^{L}\, W^{L}\, H^{L\prime} y, \quad (12)$$

where $y_{-1} = \left(y_{1,-1}', \dots, y_{1,-N}'\right)'$, $y = (y_1', \dots, y_N')'$, $H^{L} = \left(H_1^{L\prime}, \dots, H_N^{L\prime}\right)'$, and

$$W^{L} = \left(\frac{1}{N}\sum_{i=1}^{N} H_i^{L\prime} H_i^{L}\right)^{-1}. \quad (13)$$

To get the two-step level GMM (LEV2) estimator, similarly as in DIF2 estimator, the moment conditions are weighted by

$$W_{(2)}^{L} = \left(\frac{1}{N}\sum_{i=1}^{N} H_i^{L\prime}\hat{u}_i\hat{u}_i' H_i^{L}\right)^{-1}, \quad (14)$$

where $\hat{u}_{\mathrm{i}}$ are the fitted residuals from LEV1estimator.

Blundell and Bond [6] proposed a system GMM estimator in which the moment conditions of the first-difference GMM and level GMM are used jointly to avoid weak instruments and improved the efficiency of the estimator. The moment conditions used in constructing the system GMM estimator are given by

$$E(H_i^{S\prime} u_i^{S}) = 0, \quad (15)$$

where, $u_i^S = (\Delta u_i',\ u_i')'$ and $H_i^S$ is a $2(T - 2) \times (T - 2)\,(T + 1)/2$ block diagonal matrix given by

$$H_i^{S} = \begin{pmatrix} H_i^{D} & 0 \\ 0 & H_i^{L} \end{pmatrix}. \quad (16)$$

Using (15), the one-step system GMM (SYS1) estimator is calculated as:

$$\hat{\phi}^{S} = (y_{-1}^{S\prime} H^{S}\, W_G^{S}\, H^{S\prime} y_{-1}^{S})^{-1} y_{-1}^{S\prime} H^{S}\, W_G^{S}\, H^{S\prime} y^{S}, \quad (17)$$

where $y_{-1}^{S} = \left[(\Delta y_{1,-1}', y_{1,-1}'), \dots, (\Delta y_{N,-1}', y_{N,-1}')\right]'$, $y^{s} = [(\Delta y_1', y_1'), \dots, (\Delta y_N', y_N')]'$, $H^{S} = \left(H_1^{S\prime}, \dots, H_N^{S\prime}\right)'$, and

$$W_G^S = \left(\frac{1}{N}\sum_{i=1}^{N} H_i^{S\prime} G\, H_i^S\right)^{-1}, \tag{18}$$

where
$$G = \begin{pmatrix} D & 0 \\ 0 & I_{T-2} \end{pmatrix}. \tag{19}$$

To get the two-step system GMM (SYS2) estimator, the moment conditions are weighted by

$$W_{G(2)}^S = \left(\frac{1}{N}\sum_{i=1}^{N} H_i^{S\prime}\, \hat{u}_i^S \hat{u}_i^{S\prime}\, H_i^S\right)^{-1}, \tag{20}$$

where $\hat{u}_i^S$ are the fitted residuals from SYS1estimator.

## III. NEW LEV AND SYS GMM ESTIMATORS

In this section, we present the new GMM estimators, depending on the optimal weighting matrix for LEV estimator, and suboptimal weighting matrices for SYS estimator, through the use of these matrices as new weighting matrices in GMM estimation, and then we get new GMM estimators. The new GMM estimators are more efficiency than the conventional GMM (LEV and SYS) estimators.

In level GMM estimation, Youssef et al. [12] showed that $W^L$ is an optimal weighting matrix only in the case of $\sigma_\mu^2 = 0$, i.e. no individual effects case, and they presented an optimal weighting matrix for LEV estimator, in general case, as:

$$W^{OL} = \left(\frac{1}{N}\sum_{i=1}^{N} H_i^{L\prime} J_{T-2}\, H_i^L\right)^{-1}, \tag{21}$$

where
$$J_{T-2} = \begin{pmatrix} 1+\rho & \rho & \rho & \cdots & \rho \\ \rho & 1+\rho & \rho & \cdots & \rho \\ \rho & \rho & 1+\rho & \cdots & \rho \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \rho & \cdots & 1+\rho \end{pmatrix}; \rho = \sigma_\mu^2/\sigma_\varepsilon^2. \tag{22}$$

Note that the use of the weighting matrix $W^{OL}$ can be described as inducing cross-sectional heterogeneity through $\rho$, and also can be explained as partially adopting a procedure of generalized least squares to the level estimation. So using $W^{OL}$, instead of $W^L$, certainly improve the efficiency of level GMM estimator. So, we will present an alternative LEV estimator depending on the optimal weighting matrix, $W^{OL}$, as given in (21). The optimal one-step weighted LEV (WLEV1) estimator is given by

$$\hat{\phi}^{WL} = (y_{-1}' H^L\, W^{OL}\, H^{L\prime} y_{-1})^{-1}\, y_{-1}' H^L W^{OL}\, H^{L\prime} y. \tag{23}$$

To obtain the two-step weighted LEV (WLEV2) estimator, we will suggest the following weighting to the moment conditions:

$$W_{(2)}^{OL} = \left(\frac{1}{N}\sum_{i=1}^{N} H_i^{L\prime} J_{T-2}\, \hat{u}_i \hat{u}_i' J_{T-2}\, H_i^L\right)^{-1}, \tag{24}$$

where $\hat{u}_i$ are residuals from WLEV1 estimator. Note that, we use $J_{T-2}$ in (24) to improve the efficiency of WLEV2, as will be shown in our simulation results below.

In system GMM estimation, Windmeijer [11] showed that the optimal weighting matrix for SYS estimator has only been obtained in case of $\sigma_\mu^2 = 0$, and this matrix is given by:

$$W_c^S = \left(\frac{1}{N}\sum_{i=1}^{N} H_i^{S\prime} G_c\, H_i^S\right)^{-1}, \tag{25}$$

where
$$G_c = \begin{pmatrix} D & C \\ C' & I_{T-2} \end{pmatrix}, \tag{26}$$

and $C$ is a $(T-2)$ square matrix:

$$C = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \ddots & \vdots \\ 0 & -1 & 1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -1 & 1 \end{pmatrix}. \tag{27}$$

Youssef et al. [12] presented the following suboptimal weighting matrices:

$$W_{cj}^S = \left(\frac{1}{N}\sum_{i=1}^N H_i^{S\prime} G_{cj}\, H_i^S\right)^{-1}, \qquad \text{with } \ G_{cj} = \begin{pmatrix} D & C \\ C' & J_{T-2} \end{pmatrix}, \tag{28}$$

$$W_j^S = \left(\frac{1}{N}\sum_{i=1}^N H_i^{S\prime} G_j\, H_i^S\right)^{-1}, \qquad \text{with } \ G_j = \begin{pmatrix} D & 0 \\ 0 & J_{T-2} \end{pmatrix}. \tag{29}$$

So, we will present two alternatives for SYS estimators as:

(a) One-step and two-step weighted SYS (WCJSYS1 and WCJSYS2) estimators which depending on $W_{cj}^S$ instead of $W_G^S$ matrix.
(b) One-step and two-step weighted SYS (WJSYS1 and WJSYS2) estimators which depending on $W_j^S$ instead of $W_G^S$ matrix.

In addition to the above, we will propose other alternatives SYS (WCSYS1 and WCSYS2) estimators by using $W_c^S$, which given in (25), instead of $W_G^S$ matrix to study the performance of these estimators, especially when $\sigma_\mu^2 > 0$.

In practice, the variance ratio, $\rho$, is unknown. So we will use the suggested estimates by Jung and Kwon [9] for $\sigma_\varepsilon^2$ and $\sigma_\mu^2$:

$$\hat{\sigma}_\varepsilon^2 = \frac{\sum_{i=1}^N \Delta\hat{u}_i'\, \Delta\hat{u}_i}{2N(T-2)}, \tag{30}$$

where $\Delta\hat{u}_i$ are the residuals from DIF1 estimator which given in (6). While $\hat{\sigma}_\mu^2$ is given by

$$\hat{\sigma}_\mu^2 = \frac{\sum_{i=1}^N [\tilde{u}_i'\, \tilde{u}_i - (\Delta\tilde{u}_i'\, \Delta\tilde{u}_i/2)]}{N(T-2)}, \tag{31}$$

where $\tilde{u}_i$ and $\Delta\tilde{u}_i$ are residuals from first-difference and level equations in SYS1 estimator, which given in (17), respectively. Abonazel [1] studied the performance of $\hat{\rho} = \hat{\sigma}_\mu^2/\hat{\sigma}_\varepsilon^2$ and showed that in cases of $\rho < 5$ the bias of $\hat{\rho}$, Bias $(\hat{\rho})$, close to zero, while in the case of increasing $\rho$ (specifically when $\rho \geq 5$) the Bias $(\hat{\rho})$ increases significantly, especially when $\phi$ increases and is close to one.

## IV. Monte Carlo Simulation Results

In this section, we illustrate the small and moderate samples performance of different GMM estimation procedures that are considered according to their weighting matrices. Monte Carlo experiments were carried out based on the following data generating process:

$$y_{it} = \phi y_{i,t-1} + \mu_i + \varepsilon_{it}, \tag{32}$$

where $\mu_i \sim N(0, \sigma_\mu^2)$ is independent across $i$, $\varepsilon_{it} \sim N(0,1)$ is independent across $i$ and $t$, $\mu_i$ and $\varepsilon_{it}$ such that they are independent of each other. We generate the initial conditions $y_{i1}$ as

$$y_{i1} = \frac{\mu_i}{1-\phi} + w_{i1}, \tag{33}$$

where $w_{i1} \sim N(0, \sigma_{w1}^2)$, independent of both $\mu_i$ and $\varepsilon_{it}$ with variance $\sigma_{w1}^2$ that chosen to satisfy covariance stationarity. Since, $\rho$ is characterized by $\sigma_\mu^2/\sigma_\varepsilon^2$, so we choose $\sigma_\mu^2$ = 0, 0.5, 1, and 25. Throughout the experiments, $N$ = 50, 100, and nine parameter settings (i.e., $\phi$ = 0.2, 0.5, 0.9 and $T$ = 5, 10, 15) are simulated. For all experiments we ran 1000 replications and all the results for all separate experiments are obtained by precisely the same series of random numbers.

To compare the small and moderate samples performance, the six different GMM estimation procedures are considered according to their weighting matrix. Specifically, LEV1(2), WLEV1(2), SYS1(2), WCSYS1(2), WCJSYS1(2), and WJSYS1(2). Moreover, we calculate the bias and root mean squared error (RMSE) for each GMM estimator. The bias and RMSE for a Monte Carlo experiment are calculated by

$$\text{Bias} = \frac{1}{1000}\sum_{l=1}^{1000}(\hat{\phi} - \phi);\ \text{RMSE} = \sqrt{\frac{1}{1000}\sum_{l=1}^{1000}(\hat{\phi} - \phi)^2}, \tag{34}$$

where $\phi$ is the true value for $y_{i,t-1}$ parameter in (32), and $\hat{\phi}$ is the estimated value for $\phi$.

The results are given in Tables 1 to 6. Specifically, Tables 1 and 2 present the bias and RMSE of conventional and weighted level GMM estimators for the small, $N$ = 50, and moderate, $N$ = 100, samples, respectively. While Tables 3 to 6 present the bias and RMSE of conventional and weighted system GMM estimators, since Tables 3 and 4 dedicated for $N$ = 50, while Tables 5 and 6 dedicated for $N$ = 100.

From Tables 1 and 2, We can note that in case of $\rho = 0$, the bias and RMSE values for conventional level GMM (LEV1, LEV2) estimators equivalent to the bias and RMSE values for weighted level GMM estimators (WLEV1, WLEV2), the reason that when $\rho = 0$ lead to $\hat{J}_{T-2} \cong I_{T-2}$. Unless $\rho = 0$, WLEV2 estimator is smaller in bias and RMSE than other level GMM estimators, which indicates that the use of $W^{OL}$ as a weighting matrix for level GMM estimator lead to improve the efficiency for this estimator. Moreover, the bias and RMSE for LEV1, LEV2, WLEV1, and WLEV2 estimators in Table 2 are smaller than the bias and MSE in Table 1 because the sample size was increased from 50 to 100.

From Tables 3 to 6, as in results level GMM estimation, we can note that in case of $\rho = 0$, the bias and RMSE values for SYS1 and SYS2 equivalent to the bias and RMSE values for WJSYS1 and WJSYS2. Moreover, WCSYS2 estimator is smaller in bias and RMSE (when $\rho = 0$ only) than other system GMM estimators. But when $0 < \rho \leq 1$, we find that SYS2 and WJSYS2 are smaller in bias and RMSE than other system GMM estimators. Moreover, when $\rho \geq 5$, WCJSYS2 and WJSYS2 estimators are the smallest in bias and RMSE even in the case of increasing $\phi$ and is close to one. Moreover, the bias and RMSE for all system GMM estimators in Tables 5 and 6 are smaller than the bias and MSE in Tables 3 and 4 because the sample size was increased from 50 to 100.

## V. CONCLUSION

We can summarize the main conclusions in the following points:

1. The bias and RMSE of all GMM estimators are increased with increasing by $\rho$. While the bias and RMSE of weighted GMM estimators show a much slower increase whenever $\rho$ increased. Consequently, we conclude that the weighted GMM estimators are more efficiency than the conventional GMM estimators especially when $\rho \geq 5$.

2. In case of $\rho = 0$, the bias and RMSE values for the conventional level GMM (LEV1, LEV2) estimators equivalent to the bias and RMSE values for the weighted level GMM (WLEV1, WLEV2) estimators. Therefore, not any advantage of use the suggested weighting matrices in this case. While in system GMM estimation, when $\rho = 0$, the WCSYS2 estimator performs very well compared with other system GMM estimators.

3. In general, the WLEV2 and WJSYS2 estimators perform very well when compared with other level and system GMM estimators, respectively, in terms of bias and RMSE for all values of $\phi$, $T$, and $\rho$. Theoretically, since system GMM estimation use many instruments about level GMM estimation, thus WJSYS2 estimator is more efficient than the WLEV2, which was confirmed by our simulation study. Consequently, we conclude that the WJSYS2 estimation will provide useful parameter estimates for the practitioner.

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**Table 1:** Bias and RMSE for conventional and weighted level GMM estimators when $N = 50$

| GMM Estimator | | $\phi = 0.2$ | | | $\phi = 0.5$ | | | $\phi = 0.9$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ |
| $\rho = 0$ | | | | | | | | | | |
| Bias | LEV1 | -0.0007 | 0.0005 | 0.0026 | 0.0008 | 0.0022 | 0.0008 | -0.0081 | -0.0054 | -0.0004 |
| | LEV2 | 0.0049 | 0.0083 | 0.0096 | 0.0039 | 0.0068 | 0.0047 | -0.0084 | -0.0035 | 0.0006 |
| | WLEV1 | -0.0010 | 0.0005 | 0.0025 | 0.0006 | 0.0021 | 0.0007 | -0.0087 | -0.0064 | -0.0013 |
| | WLEV2 | 0.0032 | 0.0070 | 0.0086 | 0.0018 | 0.0041 | 0.0023 | -0.0138 | -0.0184 | -0.0171 |
| RMSE | LEV1 | 0.1186 | 0.0759 | 0.0622 | 0.1367 | 0.0841 | 0.0656 | 0.1006 | 0.0694 | 0.0584 |
| | LEV2 | 0.1230 | 0.0830 | 0.0678 | 0.1428 | 0.0910 | 0.0720 | 0.1043 | 0.0736 | 0.0633 |
| | WLEV1 | 0.1189 | 0.0760 | 0.0622 | 0.1369 | 0.0841 | 0.0656 | 0.1010 | 0.0699 | 0.0587 |
| | WLEV2 | 0.1240 | 0.0833 | 0.0678 | 0.1450 | 0.0911 | 0.0722 | 0.1084 | 0.0833 | 0.0717 |
| $\rho = 0.5$ | | | | | | | | | | |
| Bias | LEV1 | 0.0339 | 0.0276 | 0.0291 | 0.0346 | 0.0421 | 0.0482 | 0.0445 | 0.0598 | 0.0640 |
| | LEV2 | 0.0298 | 0.0269 | 0.0294 | 0.0295 | 0.0379 | 0.0427 | 0.0421 | 0.0582 | 0.0637 |
| | WLEV1 | 0.0255 | 0.0167 | 0.0177 | 0.0246 | 0.0269 | 0.0307 | 0.0414 | 0.0549 | 0.0584 |
| | WLEV2 | 0.0110 | -0.0028 | 0.0008 | 0.0068 | -0.0056 | -0.0043 | 0.0377 | 0.0409 | 0.0339 |
| RMSE | LEV1 | 0.1404 | 0.0835 | 0.0673 | 0.1530 | 0.0974 | 0.0826 | 0.1158 | 0.0761 | 0.0733 |
| | LEV2 | 0.1473 | 0.0887 | 0.0715 | 0.1608 | 0.1030 | 0.0859 | 0.1237 | 0.0781 | 0.0754 |
| | WLEV1 | 0.1403 | 0.0806 | 0.0620 | 0.1556 | 0.0927 | 0.0741 | 0.1224 | 0.0760 | 0.0706 |
| | WLEV2 | 0.1455 | 0.0823 | 0.0613 | 0.1631 | 0.0940 | 0.0726 | 0.1263 | 0.0780 | 0.0686 |
| $\rho = 1$ | | | | | | | | | | |
| Bias | LEV1 | 0.0572 | 0.0538 | 0.0585 | 0.0824 | 0.0885 | 0.0696 | 0.0754 | 0.0789 | 0.0572 |
| | LEV2 | 0.0463 | 0.0432 | 0.0415 | 0.0644 | 0.0718 | 0.0690 | 0.0744 | 0.0782 | 0.0463 |
| | WLEV1 | 0.0316 | 0.0259 | 0.0388 | 0.0489 | 0.0488 | 0.0679 | 0.0722 | 0.0752 | 0.0316 |
| | WLEV2 | 0.0033 | -0.0008 | 0.0140 | -0.0006 | -0.0025 | 0.0659 | 0.0634 | 0.0587 | 0.0033 |
| RMSE | LEV1 | 0.1029 | 0.0852 | 0.1760 | 0.1243 | 0.1139 | 0.1193 | 0.0844 | 0.0842 | 0.1029 |
| | LEV2 | 0.1032 | 0.0831 | 0.1853 | 0.1197 | 0.1063 | 0.1231 | 0.0851 | 0.0843 | 0.1032 |
| | WLEV1 | 0.0895 | 0.0688 | 0.1793 | 0.1061 | 0.0872 | 0.1203 | 0.0836 | 0.0822 | 0.0895 |
| | WLEV2 | 0.0858 | 0.0667 | 0.1848 | 0.0968 | 0.0742 | 0.1241 | 0.0822 | 0.0761 | 0.0858 |
| $\rho = 25$ | | | | | | | | | | |
| Bias | LEV1 | 0.4281 | 0.4798 | 0.4933 | 0.3976 | 0.4090 | 0.4136 | 0.0985 | 0.0985 | 0.0986 |
| | LEV2 | 0.3240 | 0.3589 | 0.3879 | 0.3746 | 0.3791 | 0.3899 | 0.0985 | 0.0987 | 0.0985 |
| | WLEV1 | 0.2914 | 0.2075 | 0.1623 | 0.3701 | 0.3378 | 0.3093 | 0.0984 | 0.0984 | 0.0985 |
| | WLEV2 | 0.2381 | 0.0781 | 0.0294 | 0.3527 | 0.2340 | 0.1197 | 0.0982 | 0.0982 | 0.0974 |
| RMSE | LEV1 | 0.5754 | 0.4966 | 0.5036 | 0.4312 | 0.4140 | 0.4162 | 0.1006 | 0.0990 | 0.0988 |
| | LEV2 | 0.5799 | 0.3990 | 0.4135 | 0.4321 | 0.3909 | 0.3958 | 0.1010 | 0.0992 | 0.0988 |
| | WLEV1 | 0.5677 | 0.2553 | 0.1941 | 0.4270 | 0.3567 | 0.3231 | 0.1006 | 0.0989 | 0.0988 |
| | WLEV2 | 0.5365 | 0.1400 | 0.0829 | 0.4223 | 0.2721 | 0.1587 | 0.1009 | 0.0988 | 0.0978 |

**Table 2:** Bias and RMSE for conventional and weighted level GMM estimators when $N$ = 100

| GMM Estimator | | $\phi = 0.2$ | | | $\phi = 0.5$ | | | $\phi = 0.9$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ |
| $\rho = 0$ | | | | | | | | | | |
| Bias | **LEV1** | 0.0040 | -0.0007 | 0.0015 | 0.0003 | -0.0005 | 0.0001 | -0.0097 | -0.0019 | -0.0020 |
| | **LEV2** | 0.0059 | 0.0037 | 0.0060 | 0.0010 | 0.0020 | 0.0035 | -0.0091 | -0.0005 | -0.0009 |
| | **WLEV1** | 0.0040 | -0.0007 | 0.0015 | 0.0002 | -0.0005 | 0.0000 | -0.0098 | -0.0021 | -0.0023 |
| | **WLEV2** | 0.0056 | 0.0035 | 0.0058 | 0.0001 | 0.0011 | 0.0026 | -0.0111 | -0.0050 | -0.0069 |
| RMSE | **LEV1** | 0.0915 | 0.0560 | 0.0431 | 0.0982 | 0.0623 | 0.0462 | 0.0781 | 0.0490 | 0.0424 |
| | **LEV2** | 0.0939 | 0.0594 | 0.0466 | 0.1006 | 0.0651 | 0.0491 | 0.0790 | 0.0509 | 0.0443 |
| | **WLEV1** | 0.0916 | 0.0560 | 0.0431 | 0.0984 | 0.0623 | 0.0462 | 0.0782 | 0.0491 | 0.0425 |
| | **WLEV2** | 0.0944 | 0.0594 | 0.0465 | 0.1014 | 0.0655 | 0.0491 | 0.0806 | 0.0532 | 0.0468 |
| $\rho = 0.5$ | | | | | | | | | | |
| Bias | **LEV1** | 0.0090 | 0.0165 | 0.0161 | 0.0201 | 0.0273 | 0.0260 | 0.0288 | 0.0455 | 0.0518 |
| | **LEV2** | 0.0080 | 0.0163 | 0.0155 | 0.0164 | 0.0232 | 0.0218 | 0.0259 | 0.0426 | 0.0489 |
| | **WLEV1** | 0.0049 | 0.0108 | 0.0102 | 0.0148 | 0.0186 | 0.0167 | 0.0258 | 0.0415 | 0.0460 |
| | **WLEV2** | -0.0022 | 0.0004 | 0.0005 | 0.0048 | -0.0001 | -0.0019 | 0.0223 | 0.0305 | 0.0261 |
| RMSE | **LEV1** | 0.0973 | 0.0566 | 0.0473 | 0.1131 | 0.0676 | 0.0543 | 0.0879 | 0.0609 | 0.0606 |
| | **LEV2** | 0.0995 | 0.0585 | 0.0495 | 0.1148 | 0.0693 | 0.0548 | 0.0914 | 0.0610 | 0.0603 |
| | **WLEV1** | 0.0970 | 0.0553 | 0.0455 | 0.1136 | 0.0653 | 0.0506 | 0.0909 | 0.0604 | 0.0576 |
| | **WLEV2** | 0.0989 | 0.0561 | 0.0457 | 0.1147 | 0.0650 | 0.0497 | 0.0926 | 0.0592 | 0.0530 |
| $\rho = 1$ | | | | | | | | | | |
| Bias | **LEV1** | 0.0255 | 0.0271 | 0.0257 | 0.0372 | 0.0474 | 0.0503 | 0.0482 | 0.0630 | 0.0663 |
| | **LEV2** | 0.0180 | 0.0196 | 0.0199 | 0.0271 | 0.0334 | 0.0350 | 0.0452 | 0.0595 | 0.0642 |
| | **WLEV1** | 0.0152 | 0.0137 | 0.0115 | 0.0252 | 0.0270 | 0.0272 | 0.0454 | 0.0591 | 0.0613 |
| | **WLEV2** | 0.0038 | -0.0005 | -0.0012 | 0.0109 | 0.0007 | -0.0003 | 0.0428 | 0.0505 | 0.0452 |
| RMSE | **LEV1** | 0.1081 | 0.0656 | 0.0530 | 0.1220 | 0.0827 | 0.0728 | 0.1299 | 0.0723 | 0.0714 |
| | **LEV2** | 0.1094 | 0.0634 | 0.0523 | 0.1245 | 0.0793 | 0.0658 | 0.1388 | 0.0712 | 0.0704 |
| | **WLEV1** | 0.1070 | 0.0600 | 0.0472 | 0.1216 | 0.0736 | 0.0588 | 0.1401 | 0.0712 | 0.0682 |
| | **WLEV2** | 0.1074 | 0.0587 | 0.0475 | 0.1231 | 0.0692 | 0.0522 | 0.1388 | 0.0682 | 0.0600 |
| $\rho = 25$ | | | | | | | | | | |
| Bias | **LEV1** | 0.3035 | 0.3520 | 0.3661 | 0.3476 | 0.3508 | 0.3585 | 0.0978 | 0.0978 | 0.0981 |
| | **LEV2** | 0.1770 | 0.1826 | 0.1961 | 0.3138 | 0.2867 | 0.2933 | 0.0977 | 0.0975 | 0.0978 |
| | **WLEV1** | 0.1485 | 0.0974 | 0.0717 | 0.3054 | 0.2343 | 0.1965 | 0.0976 | 0.0976 | 0.0979 |
| | **WLEV2** | 0.1115 | 0.0473 | 0.0195 | 0.2929 | 0.1513 | 0.0848 | 0.0975 | 0.0971 | 0.0968 |
| RMSE | **LEV1** | 0.3819 | 0.3719 | 0.3798 | 0.4223 | 0.3588 | 0.3626 | 0.1007 | 0.0982 | 0.0983 |
| | **LEV2** | 0.3418 | 0.2266 | 0.2283 | 0.4329 | 0.3088 | 0.3067 | 0.1010 | 0.0980 | 0.0981 |
| | **WLEV1** | 0.3122 | 0.1409 | 0.0994 | 0.4218 | 0.2608 | 0.2151 | 0.1009 | 0.0981 | 0.0981 |
| | **WLEV2** | 0.2917 | 0.1042 | 0.0614 | 0.4211 | 0.1831 | 0.1092 | 0.1009 | 0.0977 | 0.0972 |

**Table 3:** Bias and RMSE for conventional and weighted system GMM estimators when $N = 50$ and $\rho = 0, 0.5$

| GMM Estimator | | $\phi = 0.2$ | | | $\phi = 0.5$ | | | $\phi = 0.9$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ |
| $\rho = 0$ | | | | | | | | | | |
| Bias | SYS1 | -0.0139 | -0.0164 | -0.0145 | -0.0227 | -0.0227 | -0.0235 | -0.0332 | -0.0541 | -0.0558 |
| | SYS2 | -0.0068 | -0.0159 | -0.0199 | -0.0170 | -0.0208 | -0.0373 | -0.0127 | -0.0503 | -0.0695 |
| | WCSYS1 | -0.0086 | -0.0050 | -0.0010 | -0.0172 | -0.0056 | -0.0032 | -0.0107 | -0.0082 | -0.0040 |
| | WCSYS2 | -0.0028 | -0.0052 | -0.0115 | -0.0117 | -0.0048 | -0.0262 | -0.0016 | -0.0071 | -0.0401 |
| | WCJSYS1 | -0.0129 | -0.0074 | -0.0036 | -0.0238 | -0.0103 | -0.0083 | -0.0172 | -0.0180 | -0.0173 |
| | WCJSYS2 | -0.0043 | -0.0075 | -0.0130 | -0.0143 | -0.0092 | -0.0290 | -0.0047 | -0.0163 | -0.0473 |
| | WJSYS1 | -0.0144 | -0.0165 | -0.0146 | -0.0239 | -0.0229 | -0.0236 | -0.0369 | -0.0573 | -0.0580 |
| | WJSYS2 | -0.0070 | -0.0159 | -0.0199 | -0.0175 | -0.0210 | -0.0374 | -0.0146 | -0.0533 | -0.0709 |
| RMSE | SYS1 | 0.0989 | 0.0603 | 0.0476 | 0.1204 | 0.0669 | 0.0514 | 0.1095 | 0.0852 | 0.0732 |
| | SYS2 | 0.1041 | 0.0611 | 0.0591 | 0.1193 | 0.0664 | 0.0677 | 0.0929 | 0.0819 | 0.0905 |
| | WCSYS1 | 0.0905 | 0.0529 | 0.0405 | 0.1045 | 0.0528 | 0.0391 | 0.0744 | 0.0368 | 0.0248 |
| | WCSYS2 | 0.1038 | 0.0548 | 0.0536 | 0.1166 | 0.0542 | 0.0586 | 0.0834 | 0.0379 | 0.0627 |
| | WCJSYS1 | 0.0924 | 0.0538 | 0.0415 | 0.1078 | 0.0551 | 0.0412 | 0.0804 | 0.0431 | 0.0328 |
| | WCJSYS2 | 0.1045 | 0.0557 | 0.0545 | 0.1181 | 0.0562 | 0.0606 | 0.0863 | 0.0434 | 0.0687 |
| | WJSYS1 | 0.0993 | 0.0603 | 0.0476 | 0.1211 | 0.0671 | 0.0515 | 0.1143 | 0.0887 | 0.0754 |
| | WJSYS2 | 0.1042 | 0.0611 | 0.0591 | 0.1196 | 0.0665 | 0.0678 | 0.0954 | 0.0851 | 0.0918 |
| $\rho = 0.5$ | | | | | | | | | | |
| Bias | SYS1 | 0.0103 | -0.0062 | -0.0052 | 0.0042 | -0.0090 | -0.0104 | 0.0233 | 0.0268 | 0.0196 |
| | SYS2 | 0.0179 | -0.0058 | -0.0145 | 0.0117 | -0.0077 | -0.0294 | 0.0115 | 0.0266 | -0.0041 |
| | WCSYS1 | 0.0365 | 0.0655 | 0.0965 | 0.0360 | 0.0795 | 0.1122 | 0.0410 | 0.0632 | 0.0695 |
| | WCSYS2 | 0.0318 | 0.0616 | 0.0460 | 0.0317 | 0.0760 | 0.0411 | 0.0290 | 0.0623 | 0.0319 |
| | WCJSYS1 | 0.0103 | 0.0046 | 0.0069 | 0.0025 | 0.0076 | 0.0060 | 0.0128 | 0.0399 | 0.0417 |
| | WCJSYS2 | 0.0197 | 0.0044 | -0.0073 | 0.0137 | 0.0080 | -0.0197 | 0.0076 | 0.0394 | 0.0124 |
| | WJSYS1 | 0.0019 | -0.0124 | -0.0106 | -0.0116 | -0.0205 | -0.0202 | 0.0017 | 0.0113 | 0.0040 |
| | WJSYS2 | 0.0143 | -0.0116 | -0.0178 | 0.0036 | -0.0185 | -0.0353 | -0.0049 | 0.0112 | -0.0160 |
| RMSE | SYS1 | 0.1198 | 0.0639 | 0.0467 | 0.1377 | 0.0723 | 0.0506 | 0.1043 | 0.0574 | 0.0418 |
| | SYS2 | 0.1205 | 0.0645 | 0.0592 | 0.1342 | 0.0724 | 0.0678 | 0.1112 | 0.0572 | 0.0510 |
| | WCSYS1 | 0.1170 | 0.0928 | 0.1116 | 0.1292 | 0.1039 | 0.1243 | 0.0893 | 0.0683 | 0.0714 |
| | WCSYS2 | 0.1228 | 0.0902 | 0.0764 | 0.1369 | 0.1018 | 0.0749 | 0.1075 | 0.0681 | 0.0520 |
| | WCJSYS1 | 0.1122 | 0.0602 | 0.0453 | 0.1299 | 0.0694 | 0.0480 | 0.1266 | 0.0665 | 0.0555 |
| | WCJSYS2 | 0.1208 | 0.0614 | 0.0569 | 0.1379 | 0.0705 | 0.0629 | 0.1309 | 0.0666 | 0.0527 |
| | WJSYS1 | 0.1214 | 0.0654 | 0.0475 | 0.1449 | 0.0765 | 0.0543 | 0.1348 | 0.0713 | 0.0488 |
| | WJSYS2 | 0.1206 | 0.0659 | 0.0601 | 0.1370 | 0.0761 | 0.0707 | 0.1335 | 0.0710 | 0.0600 |

**Table 4:** Bias and RMSE for conventional and weighted system GMM estimators when $N$ **=** 50 and $\rho$ = 1, 25

| GMM Estimator | | $\phi = 0.2$ | | | $\phi = 0.5$ | | | $\phi = 0.9$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ |
| $\rho = 1$ | | | | | | | | | | |
| Bias | SYS1 | 0.0107 | 0.0056 | 0.0288 | 0.0168 | 0.0111 | 0.0544 | 0.0531 | 0.0466 | 0.0107 |
| | SYS2 | 0.0108 | -0.0074 | 0.0247 | 0.0174 | -0.0124 | 0.0455 | 0.0522 | 0.0263 | 0.0108 |
| | WCSYS1 | 0.1197 | 0.1669 | 0.0652 | 0.1383 | 0.1862 | 0.0650 | 0.0774 | 0.0817 | 0.1197 |
| | WCSYS2 | 0.1136 | 0.0923 | 0.0497 | 0.1335 | 0.0934 | 0.0561 | 0.0762 | 0.0533 | 0.1136 |
| | WCJSYS1 | 0.0113 | 0.0078 | 0.0146 | 0.0201 | 0.0158 | 0.0446 | 0.0643 | 0.0636 | 0.0113 |
| | WCJSYS2 | 0.0115 | -0.0061 | 0.0199 | 0.0205 | -0.0094 | 0.0380 | 0.0633 | 0.0396 | 0.0115 |
| | WJSYS1 | -0.0045 | -0.0081 | 0.0012 | -0.0099 | -0.0127 | 0.0360 | 0.0421 | 0.0345 | -0.0045 |
| | WJSYS2 | -0.0034 | -0.0160 | 0.0095 | -0.0082 | -0.0278 | 0.0298 | 0.0413 | 0.0168 | -0.0034 |
| RMSE | SYS1 | 0.0694 | 0.0509 | 0.1473 | 0.0784 | 0.0529 | 0.1007 | 0.0666 | 0.0557 | 0.0694 |
| | SYS2 | 0.0695 | 0.0587 | 0.1424 | 0.0784 | 0.0645 | 0.1130 | 0.0662 | 0.0527 | 0.0695 |
| | WCSYS1 | 0.1456 | 0.1813 | 0.1452 | 0.1580 | 0.1954 | 0.0959 | 0.0802 | 0.0827 | 0.1456 |
| | WCSYS2 | 0.1404 | 0.1157 | 0.1467 | 0.1544 | 0.1150 | 0.1103 | 0.0795 | 0.0638 | 0.1404 |
| | WCJSYS1 | 0.0665 | 0.0491 | 0.1404 | 0.0810 | 0.0556 | 0.1121 | 0.0875 | 0.0702 | 0.0665 |
| | WCJSYS2 | 0.0671 | 0.0573 | 0.1466 | 0.0815 | 0.0641 | 0.1253 | 0.0871 | 0.0597 | 0.0671 |
| | WJSYS1 | 0.0689 | 0.0512 | 0.1556 | 0.0804 | 0.0552 | 0.1153 | 0.0704 | 0.0552 | 0.0689 |
| | WJSYS2 | 0.0689 | 0.0602 | 0.1465 | 0.0800 | 0.0702 | 0.1270 | 0.0701 | 0.0560 | 0.0689 |
| $\rho = 25$ | | | | | | | | | | |
| Bias | SYS1 | 0.3544 | 0.3410 | 0.3207 | 0.3645 | 0.3439 | 0.3251 | 0.0977 | 0.0972 | 0.0965 |
| | SYS2 | 0.3209 | 0.3380 | 0.2642 | 0.3497 | 0.3431 | 0.2816 | 0.0969 | 0.0971 | 0.0941 |
| | WCSYS1 | 0.5101 | 0.6288 | 0.6855 | 0.4133 | 0.4516 | 0.4644 | 0.0981 | 0.0987 | 0.0991 |
| | WCSYS2 | 0.4660 | 0.6266 | 0.5613 | 0.3911 | 0.4510 | 0.3992 | 0.0973 | 0.0986 | 0.0966 |
| | WCJSYS1 | 0.2334 | 0.1345 | 0.0909 | 0.2496 | 0.2977 | 0.2491 | 0.0941 | 0.0972 | 0.0996 |
| | WCJSYS2 | 0.2205 | 0.1333 | 0.0745 | 0.3186 | 0.2970 | 0.2176 | 0.0938 | 0.0970 | 0.0971 |
| | WJSYS1 | 0.1978 | 0.0820 | 0.0499 | 0.2995 | 0.2278 | 0.1720 | 0.0939 | 0.0966 | 0.0962 |
| | WJSYS2 | 0.1741 | 0.0813 | 0.0396 | 0.2972 | 0.2272 | 0.1499 | 0.0936 | 0.0965 | 0.0938 |
| RMSE | SYS1 | 0.4304 | 0.3635 | 0.3360 | 0.3919 | 0.3522 | 0.3305 | 0.0998 | 0.0976 | 0.0967 |
| | SYS2 | 0.4064 | 0.3612 | 0.2849 | 0.3887 | 0.3517 | 0.2913 | 0.1014 | 0.0975 | 0.0948 |
| | WCSYS1 | 0.5609 | 0.6355 | 0.6877 | 0.4332 | 0.4534 | 0.4650 | 0.0994 | 0.0988 | 0.0991 |
| | WCSYS2 | 0.5349 | 0.6337 | 0.5679 | 0.4215 | 0.4529 | 0.4025 | 0.1011 | 0.0987 | 0.0970 |
| | WCJSYS1 | 0.3936 | 0.1930 | 0.1273 | 3.5190 | 0.3332 | 0.2755 | 0.1023 | 0.1000 | 0.1012 |
| | WCJSYS2 | 0.3727 | 0.1920 | 0.1159 | 0.3972 | 0.3328 | 0.2482 | 0.1026 | 0.1000 | 0.0991 |
| | WJSYS1 | 0.6546 | 0.1377 | 0.0862 | 0.3705 | 0.2702 | 0.2021 | 0.1024 | 0.0972 | 0.0965 |
| | WJSYS2 | 0.3285 | 0.1369 | 0.0826 | 0.3711 | 0.2699 | 0.1846 | 0.1024 | 0.0971 | 0.0946 |

**Table 5:** Bias and RMSE for conventional and weighted system GMM estimators when $N = 100$ and $\rho = 0, 0.5$

| GMM Estimator | | $\phi = 0.2$ | | | $\phi = 0.5$ | | | $\phi = 0.9$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ |
| $\rho = 0$ | | | | | | | | | | |
| Bias | SYS1 | -0.0034 | -0.0085 | -0.0072 | -0.0110 | -0.0126 | -0.0127 | -0.0240 | -0.0283 | -0.0343 |
| | SYS2 | -0.0022 | -0.0040 | -0.0072 | -0.0065 | -0.0085 | -0.0125 | -0.0055 | -0.0161 | -0.0335 |
| | WCSYS1 | -0.0025 | -0.0013 | -0.0011 | -0.0072 | -0.0039 | -0.0014 | -0.0065 | -0.0020 | -0.0033 |
| | WCSYS2 | -0.0009 | 0.0003 | -0.0013 | -0.0036 | -0.0032 | -0.0014 | -0.0008 | 0.0000 | -0.0031 |
| | WCJSYS1 | -0.0049 | -0.0024 | -0.0019 | -0.0109 | -0.0058 | -0.0033 | -0.0088 | -0.0052 | -0.0079 |
| | WCJSYS2 | -0.0014 | -0.0003 | -0.0020 | -0.0045 | -0.0043 | -0.0033 | -0.0014 | -0.0019 | -0.0076 |
| | WJSYS1 | -0.0036 | -0.0085 | -0.0072 | -0.0116 | -0.0126 | -0.0127 | -0.0252 | -0.0293 | -0.0349 |
| | WJSYS2 | -0.0023 | -0.0041 | -0.0072 | -0.0067 | -0.0086 | -0.0125 | -0.0059 | -0.0167 | -0.0342 |
| RMSE | SYS1 | 0.0782 | 0.0437 | 0.0327 | 0.0839 | 0.0480 | 0.0337 | 0.0844 | 0.0560 | 0.0494 |
| | SYS2 | 0.0769 | 0.0448 | 0.0331 | 0.0788 | 0.0473 | 0.0339 | 0.0639 | 0.0424 | 0.0486 |
| | WCSYS1 | 0.0712 | 0.0373 | 0.0288 | 0.0709 | 0.0384 | 0.0265 | 0.0547 | 0.0249 | 0.0174 |
| | WCSYS2 | 0.0765 | 0.0428 | 0.0294 | 0.0778 | 0.0437 | 0.0269 | 0.0589 | 0.0277 | 0.0177 |
| | WCJSYS1 | 0.0716 | 0.0377 | 0.0290 | 0.0721 | 0.0393 | 0.0272 | 0.0571 | 0.0264 | 0.0199 |
| | WCJSYS2 | 0.0766 | 0.0430 | 0.0296 | 0.0781 | 0.0442 | 0.0276 | 0.0597 | 0.0286 | 0.0200 |
| | WJSYS1 | 0.0783 | 0.0437 | 0.0327 | 0.0843 | 0.0480 | 0.0337 | 0.0861 | 0.0570 | 0.0502 |
| | WJSYS2 | 0.0770 | 0.0448 | 0.0331 | 0.0789 | 0.0474 | 0.0339 | 0.0644 | 0.0430 | 0.0493 |
| $\rho = 0.5$ | | | | | | | | | | |
| Bias | SYS1 | -0.0005 | -0.0013 | -0.0026 | 0.0037 | -0.0018 | -0.0050 | 0.0153 | 0.0219 | 0.0157 |
| | SYS2 | 0.0038 | 0.0022 | -0.0026 | 0.0044 | 0.0002 | -0.0049 | 0.0040 | 0.0171 | 0.0155 |
| | WCSYS1 | 0.0121 | 0.0365 | 0.0524 | 0.0160 | 0.0457 | 0.0670 | 0.0271 | 0.0457 | 0.0541 |
| | WCSYS2 | 0.0085 | 0.0248 | 0.0513 | 0.0118 | 0.0304 | 0.0658 | 0.0151 | 0.0376 | 0.0536 |
| | WCJSYS1 | 0.0005 | 0.0048 | 0.0035 | -0.0001 | 0.0058 | 0.0057 | 0.0065 | 0.0289 | 0.0302 |
| | WCJSYS2 | 0.0050 | 0.0060 | 0.0035 | 0.0052 | 0.0052 | 0.0056 | 0.0018 | 0.0236 | 0.0299 |
| | WJSYS1 | -0.0043 | -0.0044 | -0.0053 | -0.0048 | -0.0080 | -0.0099 | -0.0014 | 0.0112 | 0.0033 |
| | WJSYS2 | 0.0028 | 0.0004 | -0.0052 | 0.0017 | -0.0035 | -0.0096 | -0.0066 | 0.0079 | 0.0033 |
| RMSE | SYS1 | 0.0868 | 0.0457 | 0.0345 | 0.1053 | 0.0513 | 0.0368 | 0.0851 | 0.0481 | 0.0357 |
| | SYS2 | 0.0850 | 0.0464 | 0.0346 | 0.0963 | 0.0480 | 0.0370 | 0.0944 | 0.0463 | 0.0356 |
| | WCSYS1 | 0.0844 | 0.0598 | 0.0653 | 0.0988 | 0.0672 | 0.0781 | 0.0740 | 0.0520 | 0.0565 |
| | WCSYS2 | 0.0859 | 0.0535 | 0.0644 | 0.0966 | 0.0569 | 0.0770 | 0.0891 | 0.0489 | 0.0561 |
| | WCJSYS1 | 0.0812 | 0.0423 | 0.0318 | 0.0982 | 0.0459 | 0.0340 | 0.1024 | 0.0493 | 0.0428 |
| | WCJSYS2 | 0.0855 | 0.0459 | 0.0322 | 0.0971 | 0.0466 | 0.0343 | 0.1012 | 0.0487 | 0.0426 |
| | WJSYS1 | 0.0873 | 0.0462 | 0.0348 | 0.1092 | 0.0528 | 0.0383 | 0.1113 | 0.0549 | 0.0406 |
| | WJSYS2 | 0.0849 | 0.0465 | 0.0350 | 0.0970 | 0.0486 | 0.0384 | 0.1071 | 0.0526 | 0.0404 |

**Table 6:** Bias and RMSE for conventional and weighted system GMM estimators when $N$ = 100 and $\rho$ = 1, 25

| GMM Estimator | | $\phi = 0.2$ | | | $\phi = 0.5$ | | | $\phi = 0.9$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ | $T = 5$ | $T = 10$ | $T = 15$ |
| | $\rho = 1$ | | | | | | | | | |
| Bias | SYS1 | 0.0101 | 0.0050 | 0.0012 | 0.0201 | 0.0094 | 0.0060 | 0.0392 | 0.0442 | 0.0382 |
| | SYS2 | 0.0076 | 0.0043 | 0.0014 | 0.0175 | 0.0076 | 0.0059 | 0.0254 | 0.0389 | 0.0380 |
| | WCSYS1 | 0.0328 | 0.0666 | 0.0925 | 0.0380 | 0.0861 | 0.1138 | 0.0478 | 0.0640 | 0.0704 |
| | WCSYS2 | 0.0154 | 0.0420 | 0.0908 | 0.0282 | 0.0596 | 0.1119 | 0.0338 | 0.0572 | 0.0699 |
| | WCJSYS1 | 0.0063 | 0.0065 | 0.0027 | 0.0107 | 0.0104 | 0.0071 | 0.0276 | 0.0497 | 0.0505 |
| | WCJSYS2 | 0.0070 | 0.0054 | 0.0029 | 0.0153 | 0.0083 | 0.0070 | 0.0206 | 0.0444 | 0.0502 |
| | WJSYS1 | -0.0003 | -0.0026 | -0.0055 | 0.0029 | -0.0058 | -0.0068 | 0.0227 | 0.0331 | 0.0261 |
| | WJSYS2 | 0.0046 | -0.0003 | -0.0052 | 0.0105 | -0.0025 | -0.0067 | 0.0145 | 0.0287 | 0.0259 |
| RMSE | SYS1 | 0.0941 | 0.0488 | 0.0354 | 0.1082 | 0.0574 | 0.0401 | 0.0882 | 0.0579 | 0.0472 |
| | SYS2 | 0.0886 | 0.0475 | 0.0354 | 0.0996 | 0.0540 | 0.0401 | 0.0934 | 0.0551 | 0.0471 |
| | WCSYS1 | 0.0985 | 0.0892 | 0.1048 | 0.1105 | 0.1048 | 0.1235 | 0.0782 | 0.0677 | 0.0717 |
| | WCSYS2 | 0.0893 | 0.0674 | 0.1032 | 0.1044 | 0.0824 | 0.1217 | 0.0886 | 0.0637 | 0.0712 |
| | WCJSYS1 | 0.0879 | 0.0460 | 0.0330 | 0.1027 | 0.0552 | 0.0381 | 0.1037 | 0.0665 | 0.0577 |
| | WCJSYS2 | 0.0882 | 0.0465 | 0.0331 | 0.1014 | 0.0536 | 0.0383 | 0.1021 | 0.0635 | 0.0574 |
| | WJSYS1 | 0.0949 | 0.0484 | 0.0359 | 0.1123 | 0.0585 | 0.0405 | 0.1104 | 0.0616 | 0.0457 |
| | WJSYS2 | 0.0886 | 0.0470 | 0.0358 | 0.1001 | 0.0545 | 0.0405 | 0.1055 | 0.0593 | 0.0456 |
| | $\rho = 25$ | | | | | | | | | |
| Bias | SYS1 | 0.2554 | 0.2277 | 0.2122 | 0.3041 | 0.2776 | 0.2567 | 0.0970 | 0.0963 | 0.0956 |
| | SYS2 | 0.2126 | 0.2013 | 0.2110 | 0.2838 | 0.2665 | 0.2558 | 0.0952 | 0.0958 | 0.0955 |
| | WCSYS1 | 0.3864 | 0.5284 | 0.5992 | 0.3670 | 0.4136 | 0.4353 | 0.0968 | 0.0979 | 0.0985 |
| | WCSYS2 | 0.3269 | 0.5007 | 0.5979 | 0.3356 | 0.4050 | 0.4344 | 0.0956 | 0.0974 | 0.0983 |
| | WCJSYS1 | 0.1205 | 0.0422 | 0.0250 | 0.2701 | 0.1749 | 0.1180 | 0.0924 | 0.0630 | 0.0987 |
| | WCJSYS2 | 0.1148 | 0.0369 | 0.0247 | 0.2463 | 0.1682 | 0.1176 | 0.0911 | 0.0794 | 0.0986 |
| | WJSYS1 | 0.0934 | 0.0249 | 0.0128 | 0.2170 | 0.1226 | 0.0764 | 0.0924 | 0.0956 | 0.0951 |
| | WJSYS2 | 0.0889 | 0.0216 | 0.0126 | 0.2154 | 0.1174 | 0.0761 | 0.0907 | 0.0951 | 0.0950 |
| RMSE | SYS1 | 0.3209 | 0.2497 | 0.2279 | 0.3426 | 0.2890 | 0.2639 | 0.0998 | 0.0967 | 0.0958 |
| | SYS2 | 0.2919 | 0.2280 | 0.2268 | 0.3359 | 0.2805 | 0.2631 | 0.1007 | 0.0963 | 0.0957 |
| | WCSYS1 | 0.4452 | 0.5390 | 0.6040 | 0.3978 | 0.4178 | 0.4366 | 0.0989 | 0.0981 | 0.0986 |
| | WCSYS2 | 0.4078 | 0.5164 | 0.6028 | 0.3787 | 0.4108 | 0.4357 | 0.1004 | 0.0977 | 0.0984 |
| | WCJSYS1 | 0.3235 | 0.0858 | 0.0546 | 0.5440 | 0.2191 | 0.1473 | 0.0975 | 1.1273 | 0.0998 |
| | WCJSYS2 | 0.2281 | 0.0800 | 0.0545 | 0.3401 | 0.2146 | 0.1469 | 0.0988 | 0.5963 | 0.0997 |
| | WJSYS1 | 0.2005 | 0.0712 | 0.0459 | 0.3051 | 0.1672 | 0.1040 | 0.0983 | 0.0962 | 0.0954 |
| | WJSYS2 | 0.1897 | 0.0661 | 0.0459 | 0.3038 | 0.1631 | 0.1037 | 0.0988 | 0.0957 | 0.0953 |