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Abstract

We study a model of multilateral bargaining in which a buyer attempts to assemble objects owned by multiple sellers. Players can (non-cooperatively) opt out of the bargaining whenever they want. The presence of this option results in an equilibrium in which the buyer implements the project immediately and grabs the entire surplus. It also mitigates the inefficiency associated with nontransparent bargaining protocol. These results are in stark contrast to those obtained in Roy Chowdhury and Sengupta (2012).

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1 Introduction

Roy Chowdhury and Sengupta (2012) study a model of multilateral bargaining, in which a buyer bargains with multiple sellers to obtain their objects, all of which are required to implement his project. The paper searches for an asymptotically efficient equilibrium in which the buyer implements the project and his payoff is bounded away from zero. Asymptotic efficiency refers to the condition $\delta \to 1$ (where $\delta \in (0, 1)$ is the common discount factor), since under this condition the efficiency loss due to delays is arbitrarily close to zero. The paper finds that when

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the buyer's outside option is zero, his payoff approaches zero as $\delta \to 1$ (see lemma RS.1 in section 2.1). Note that only the buyer can **opt out** i.e. leave the game whenever he wants, even if he has no outside option. The sellers do not have any outside option, and also, they cannot opt out. Also, the authors show that if the bargaining protocol is nontransparent[1] (and the buyer has positive outside option), the surplus is not realized.

We show that if all the players are allowed to opt out, even when they do not have any outside options, there exists an equilibrium in which the project is implemented in the first period and the buyer grabs entire surplus (see proposition 1). Also, when the protocol is nontransparent, players' ability to opt out gives rise to an equilibrium in which the surplus is realized immediately (see proposition 2). Hence, we endogenize a player's decision whether or not to continue with the bargaining, whereas Roy Chowdhury and Sengupta (2012) exogenously force all the sellers to keep bargaining till either they sign agreements with the buyer or the buyer opts out. By endogenizing such decisions, the model not only becomes richer, it is more realistic. In most of the real life situations, people can leave the negotiation table anytime they want.

The intuition behind the differences in the results of the two papers is following. In Roy Chowdhury and Sengupta (2012), the sellers cannot walk away from bargaining. This forced coordination among sellers and the perfect complementarity of the objects enables them to extract large sums from the buyer,[2] who cannot credibly threat to opt out since he gets a small but positive payoff by continuing the bargaining. However, in our model, all players are free to choose their path. So even if one of the players in future periods chooses to opt out, the players may as well opt out in the current period. The buyer foresees this in the first period and exploits the situation to the hilt by paying the responders zero.

Ponsatí and Sákovics (1998) show a result similar to ours, but in the context of bilateral bargaining (see lemma PS.1 in section 2.1).

In next section we describe our model, which is a minor modification of Roy Chowdhury and Sengupta (2012)'s model. We reproduce some key results from their paper to facilitate the comparison of results of the two papers. In section 3, we present our results. Finally, we conclude the paper in section 4.

[1] In the model, the buyer bargains with all the sellers simultaneously. While in the baseline model, a seller views offers made to all the sellers (by the buyer), in the model with nontransparent protocol, a seller is blind to the offers made to other sellers.

[2] Note that not all the sellers get large payoffs. Some sellers, particularly those who sell earlier, obtain relatively smaller shares than those who sell later. See Roy Chowdhury and Sengupta (2012, Remark 1).

2 Model

A buyer intends to assemble $n \geq 2$ objects, which are owned by n different sellers. Valuation of each object is normalized to zero. From the buyer's perspective, the objects are perfect complements. Hence, the buyer's utility from assembling m objects is captured by $u(m)$, where

$$u(m) = \begin{cases} 1 & if \;\; m = n \\ 0 & if \;\; m < n \end{cases}$$

Players do not have outside options, but they can opt out in any period. Active players are those who are yet to leave the bargaining process. Bargaining takes place over an infinite period horizon. Each period has three stages. Consider odd periods. In stage 1, the buyer makes offers to all the active sellers. Hence, he announces an offer vector, each element of which is an offer to one of the active sellers. In stage 2, the sellers simultaneously respond to the offers made in stage 1. Note that a seller can view not only the offer made to him but the whole vector. Sellers, who accept the offers, are paid immediately by the buyer and they leave the game forever. In stage 3, if the buyer is yet to acquire all the objects, all the active players non-cooperatively decide whether or not to opt out. If any of them decides to opt out, the game ends with all the active players getting zero.[3] Note that in stage 3, the active players can make decisions simultaneously or in any given sequence, our results hold in every case. Similar bargaining occurs in even periods, with roles reversed. In stage 1, all the active sellers simultaneously demand prices from the buyer. In stage 2, the buyer responds. If he accepts any demand, then he pays the corresponding seller immediately, who in turn leaves the game forever. The process in stage 3 remains the same. Hence, the game ends when either the buyer obtains all the objects or any of the active players opts out, whichever occurs first.

All players are risk neutral, and discount at a common rate $\delta \in (0, 1)$ per period. It is a complete information game, hence, we use subgame perfect nash equilibrium (SPE) to analyze the same. Henceforth, "equilibrium"implies SPE.

[3]The buyer may get a negative net payoff if he has paid a positive sum to a seller in an earlier period. However, it is easy to see that this cannot be an equilibrium outcome.

2.1 Previous Results

In this subsection, we present some of the results proved in the literature, one of them is used to prove our result while others are reproduced for the purpose of comparison.

There are two differences between our model and Roy Chowdhury and Sengupta (2012)'s model. In their model, (1) the buyer has an outside option $C \in [0, 0.5)$, and (2) he can opt out in stage 3, but not the sellers. Following are some of the key results from the paper.

Lemma RS. 1 *For $C = 0$ and $\delta \in (0,1)$, in any equilibrium the buyer's payoff is at most $\frac{1-\delta}{1+\delta}$.*[4]

Lemma RS. 2 *Consider a $C \in (0, 0.5)$. Let δ_0 solves $\frac{\delta^3}{1+\delta} = C$. For $\delta \in (\delta_0, 1)$, there exists an equilibrium in which buyer's payoff is $\frac{\delta}{1+\delta}$.*[5]

It is clear from above results that a positive outside option improves buyer's bargaining power and he can possibly grab almost half of the surplus (as $\delta \to 1$), whereas in the absence of any outside option the buyer's payoff is negligible (as $\delta \to 1$).

Roy Chowdhury and Sengupta (2012) analyze a variant of the baseline model in which a seller cannot view the prices the buyer has offered to other sellers. This non-transparency leads to the following result.

Lemma RS. 3 *For $C = 0$ and $\delta \in (0,1)$, the payoff to buyer is $\max\{\frac{1-(n-1)\delta}{1+\delta}, 0\}$. For a $C \in (0, 0.5)$, there exist a $\delta_1 \in (0,1)$ such that if $\delta \in (\delta_1, 1)$, then the game has a unique outcome in which the bargaining ends with the buyer taking his outside in period 1.*[6]

Hence, in the model with non-transparency, if the buyer's outside option is zero, his payoff is also zero (provided $n \geq 3$ and $\delta \geq \frac{1}{n-1}$). And, when he has an outside option, the unique outcome is inefficient in which the surplus is not realized.

Now consider our model as described in section 2. Observe that when only one seller is left in the game, the continuation game is a Rubinstein (1982) game (with

[4] See Roy Chowdhury and Sengupta (2012, Proposition 1).
[5] See Roy Chowdhury and Sengupta (2012, Proposition 2).
[6] See Roy Chowdhury and Sengupta (2012, Proposition 3 and 4).

unit surplus[7]) in which both the players decide in stage 3 of every period whether or not to opt out. This (sub)game has been studied by Ponsatí and Sákovics (1998), and they show the following result.

Lemma PS. 1 *For $\delta \in (0,1)$, there exists an equilibrium in which the bargaining ends in period 1, with the proposer and responder obtaining 1 and 0 respectively.*[8]

3 Results

Consider our model. Following proposition captures one of the main result of this paper, which we prove using lemma PS.1.

Proposition 1 *For $\delta \in (0,1)$, there exists an equilibrium in which the buyer implements the project in period 1, and obtains the entire surplus.*

Proof. Let the notation $m(t,k)$ represent the number of sellers active in stage k of period t. Consider the following strategy profile, denoted by E.

> **E:** If $m(t,1) = 1$, the outcome of the subgame is dictated by lemma PS.1. If $m(t,1) > 1$, then following strategies are played. If t is odd, buyer offers zero to each seller. A seller accepts ≥ 0. In stage 3, if $m(t,3) = 1$, then only the buyer opts out. Otherwise, all the active players opt out. If t is even, each seller demands 1. Buyer rejects any demand > 0. In stage 3, all the active sellers opt out, but the buyer does not.

It can be verified that E constitutes an equilibrium, and hence proves the result. ■

Note that the proposition is true even when the buyer has a positive (≤ 1) outside option. Observe the contrast between proposition 1 and lemma RS.1 and RS.2. If players have the option to leave the game anytime, even though there are no benefits to leaving, the buyer can grab entire surplus. However, if the sellers are not allowed to opt out, the buyer's payoff is almost zero (as $\delta \to 1$), which improves to (almost) 0.5 in the presence of an outside option for the buyer. The intuition behind this contrast is following. In both the models, the objects are perfect complements for the buyer, hence entire surplus is up for grabs even when

[7]Payments made to previous sellers are sunk from the buyer's perspective. Hence, the entire surplus is available in every subgame.

[8]See Ponsatí and Sákovics (1998, Lemma 1).

most of the sellers have left after selling their objects. In fact, when $n-1$ sellers have left, the last seller obtains almost half of the surplus (as $\delta \to 1$). This creates an incentive for the sellers to wait and be the last one to make deal, provided they are sure that no player is going to destroy the surplus by opting out in the future. Roy Chowdhury and Sengupta (2012) ensure such coordination by disallowing the sellers from opting out. Buyer, although can opt out, does not do so if he gets a positive sum however small, making his opting out incredible. Hence (some) sellers are able to extract large shares of the surplus, leaving the buyer with infinitesimal share. However, in our model, such coordination is not guaranteed. An active player may opt out anytime. If even one player in future periods chooses to opt out, the players may as well opt out in the current period. The buyer anticipates this in period 1 and takes advantage of this situation by offering the sellers zero, which they accept.

Now we analyze a variant of our model — a model with nontransparent protocol. In this model, while a seller views the offer made to him by the buyer, the rest of the offer vector remains hidden from him. It can be verified that E is still an equilibrium. Hence, proposition 1 is true in this case as well. Following proposition formally states the same.

Proposition 2 *For $\delta \in (0, 1)$, there exists an equilibrium in which the buyer implements the project in period 1, and obtains the entire surplus.*

Proof. See equilibrium E in the proof of proposition 1. ■

Note that the proposition 2 holds even if the buyer has a positive (≤ 1) outside option. Observe the difference between proposition 2 and lemma RS.3. When sellers cannot opt out, nontransparency results in inefficiency i.e. the buyer takes his (positive) outside option and the surplus is destroyed. However, when all the players are allowed to opt out, there exists an equilibrium in which the buyer implements the project immediately and obtains the entire surplus. The intuition behind this difference is similar to that discussed in the context of proposition 1. In Roy Chowdhury and Sengupta (2012), due to forced coordination among the sellers and the perfect complementarity, the sellers are in a position to extract most of the surplus from the buyer, making it profitable for the buyer to take his outside option. However, when all the players are allowed to opt out, the coordination is not ensured, resulting in an equilibrium in which buyer exploits the situation to his benefit.

4 Conclusion

Players' abilities to opt out of the bargaining, even when they have no outside options, critically affect the outcome in multilateral bargaining. Ability to opt out may inhibit coordination among sellers, which significantly improves the bargaining power of the central player i.e. the buyer, who in turn may confiscate the entire surplus. The result remains true even when the protocol is nontransparent. Hence, the inefficiency associated with the nontransparent protocol is mitigated.

References

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