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# Over-the-Counter Markets, Intermediation, and Monetary Policy\*

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## Abstract

During the Great Recession, the Federal Reserve implemented two monetary policies: cutting interest rates and quantitative easing (QE). I develop a model to examine these two policies in a frictional financial environment. In this model, agents sell assets to acquire money when a consumption opportunity arises, which can only be done through over-the-counter (OTC) markets. In equilibrium, when the interest rate is low (not necessarily zero), households who trade in OTC markets achieve their optimal consumption. When the interest rate is high, QE will raise asset prices and lower households' consumption. The asset price increase indicates a higher liquidity premium, which reflects inefficiency in money reallocation.

**JEL Classification:** E44, E52, E58, G12

**Keywords:** OTC markets, Middlemen, Monetary Policy, QE, Asset Pricing

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# 1 Introduction

OTC markets are large asset markets in which agents search, perform bilateral trade, and bargain. Many assets are traded in OTC markets.<sup>1</sup> Therefore, it is of interest to study how these markets work and how policy affects these markets. Because of search and matching frictions in OTC markets, some OTC trades occur with the help of intermediaries, such as financial brokers and dealers. Due to limited commitment or lack of record keeping among these agents, many OTC markets require assets to facilitate transactions.

OTC markets play an important role in reallocating money.<sup>2</sup> In OTC markets, agents transfer money to those who need it most, increasing liquidity and welfare in the economy. However, OTC markets have suffered due to a scarcity of assets (Caballero 2006; Caballero and Krishnamurthy 2006), that has worsened since the Great Recession (Gorton and Ordonez 2013, 2014). During the crisis, many private assets, such as mortgage-backed securities, have not been accepted in OTC markets. OTC markets are inefficient at reallocating money, and as a result, market liquidity is limited. To increase market liquidity, the Federal Reserve employs two monetary policies: cutting nominal interest rates and quantitative easing (QE). QE is a new policy. To accomplish QE, the Federal Reserve simultaneously increases the level of money in the economy and decreases the level of assets. The traditional wisdom of monetary economics states that this is an effective monetary policy because increasing the level of money raises real money balances.<sup>3</sup> However, I want to reinvestigate this idea through a model using relatively firm microfoundations for money, liquidity, and assets.

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<sup>1</sup>“Many assets, such as mortgage-backed securities, corporate bonds, government bonds, US federal funds, emerging-market debt, bank loans, swaps and many other derivatives, private equity and real estate are traded in ... (OTC) markets. Traders in these markets search for counterparties, incurring opportunity or other costs. When counterparties meet, their bilateral relationship is strategic, prices are set through a *bargaining* process that reflects each investor’s alternatives to immediate trade.” (Duffie et al. 2007, emphasis added)

<sup>2</sup>Reallocation of money in OTC markets has not been well studied. Many OTC papers (Duffie et al. 2005, 2007, Lagos and Rocheteau 2009, Chiu and Koppel 2011, and Lagos et al. 2011) trade assets for goods, instead of money in their models.

<sup>3</sup>The traditional IS-LM model and other relatively modern market segmentation models state that the level of money affects allocations. See Bernanke et al. (1999) for traditional models. Examples of market segmentation models are Alvarez et al. (2001, 2009, 2014) and Chiu (2014). These papers study CIA (cash-in-advance) models. See Kahn (2006) for a broader review.

I study the effects of interest rates and QE in a general equilibrium model with OTC markets and middlemen. The interest rate policy is to alter the growth rate of the money supply, which is equivalent to the inflation rate and, hence, the nominal interest rate on illiquid assets determined by the Fisher equation. The other policy is QE, which involves changing the level of money and assets in circulation. The main contribution of this paper is the new findings regarding QE. The factor that affects allocations is the asset levels in circulation, rather than the level of money.<sup>4</sup> The main point is that, with flexible prices, changing the level of money has no effect on consumption and welfare. This point reflects the classical neutrality of money. However, the level of assets in circulation is significant to allocations. Assume that assets are not sufficiently abundant to satisfy the agents.<sup>5</sup> Given that assets are the medium of exchange in OTC transactions, a decrease in assets will make OTC markets less efficient at reallocating money—thus actually decreasing consumption, liquidity, and welfare. This finding is new. If the Federal Reserve wants to increase liquidity in the economy, it should sell assets, instead of buying them.

Another contribution of the study is the new findings regarding monetary policies on asset pricing. In frictionless models, assets are priced fundamentally, which means asset prices reflect only the discounted stream of dividends. However, in OTC markets, assets are not only stores of value but also a medium of exchange. When the asset supply is not sufficiently large, assets will be priced above the fundamental price and carry a liquidity premium. This paper studies the effects of these two monetary policies on the liquidity premium. The effects of inflation (equivalent to the nominal interest rate) are as follows: Inflation will increase the demand for assets in OTC markets and lead to a higher premium. But asset prices do not always increase with inflation. When inflation is extremely high, money suppliers on OTC markets carry such little cash that only a few assets are sufficient to acquire all the money in

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<sup>4</sup>The finding that the money level has no effect holds in Williamson (2012, 2014a, 2014b) under different assumptions. In Williamson's papers, agents use banks rather than OTC markets to trade. This finding is also established in Rocheteau et al. (2014), which similarly does not look at the case of OTC markets.

<sup>5</sup>See Caballero (2006), Caballero and Krishnamurthy (2006), and Gorton and Ordonez (2013, 2014) for the scarcity of safe assets.

OTC markets; then assets are priced fundamentally again. The effect of QE increases the scarcity of assets in OTC markets and leads to higher asset prices. However, this higher price signifies a tighter liquidity constraint.

This model uses three fundamental factors: OTC markets, middlemen, and monetary policies. The OTC markets section follows Duffie et al. (2005, 2007) and captures two important and realistic characteristics: (a) search and bargaining, and (b) the use of inter-dealer markets to facilitate trade. Middlemen are useful for two reasons. First, they increase the trade probability and the market volume (Rubinstein and Wolinsky 1987; Wright and Wong 2014); second, they make money allocation more efficient. My paper is also related to a strand of monetary literature that supports the agents' ability to rebalance their money holdings when a consumption or investment opportunity arises (Berentsen et al. 2007; Berentsen and Waller, 2011; Boel and Camera 2006; Kocherlakota 2003; Li and Li 2013).<sup>6</sup> Considerable literature exists relating to monetary policy and search; therefore, I put three surveys here: Williamson and Wright (2010), Nosal and Rocheteau (2011), and Lagos et al. (2014).

Three recent papers are highly related to my work: Mattesini and Nosal (2013), Geromichalos and Herrenbrueck (2012), and Lagos and Zhang (2013). Similar to the method described in this paper, they embed an OTC financial market in a monetary model. Mattesini and Nosal (2013) state that entrepreneurs have heterogeneous money demand and trade in OTC markets via dealers only, whereas Geromichalos and Herrenbrueck (2012) portray a scenario in which buyers and sellers trade without dealers in the OTC financial market. Lagos and Zhang (2013) state that money is used to buy assets on the OTC market. The authors of all of these papers specify that assets carry a liquidity premium, and they also discuss the effects of changing inflation. However, none of these papers focuses on the effects of QE.

This paper is organized as follows: Section 2 presents the environment and the

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<sup>6</sup>Some of these papers use assets as collateral, following Kiyotaki-Moore (1997, 2005), whereas others use assets as media of exchange, following Kiyotaki-Wright (1989, 1993). Ferraris and Watanabe (2008, 2011) discuss the relationship between these two styles. Under certain assumptions, He et al. (2014) show that they are mathematically equivalent. In my paper, it does not matter whether assets are used as collateral or as a medium of exchange. I use assets as a medium of exchange to simplify my model.

characteristics of markets. Section 3 derives the value functions of agents and solves the general equilibrium model. The relationship between policies and liquidity is analyzed in Section 4. Section 5 summarizes the findings of this research and offers recommendations for how the Federal Reserve should set monetary policy.

## 2 The Environment

Time is discrete and continues forever. Each time period contains three submarkets: an OTC financial market, a decentralized goods market (DM), and a frictionless centralized market (CM). There are two types of agents: a unit measure of households  $H$  and a measure  $\zeta$  of middlemen  $M$ .  $H$  trades with another  $H$  in OTC markets or via the third party  $M$ .

There are two types of goods,  $x$  and  $q$ , both of which are perishable:  $x$  is a CM good and is produced with labor linearly, and  $q$  is produced and traded on the DM, where the cost is  $c(q)$  with  $c'(q) > 0$ , and  $c''(q) > 0$ .<sup>7</sup>  $H$ 's preferences are  $U(x) - L + u(q) - c(\tilde{q})$ , where  $\tilde{q}$  is the quantity of DM goods that  $H$  produces for others. This quasi-linear preference setup is important for tractability.  $H$  could buy  $x$  with credit, labor, money, or assets. There are no restrictions on how to pay for  $x$ , whereas  $H$  can only use money to buy  $q$ . The variable  $q^*$  is the optimal  $q$  consumption that satisfies  $u'(q^*) = c'(q^*)$ .  $M$  represents brokers who trade on behalf of  $H$  and receive service fees in terms of  $x$ ;  $M$  does not work or hold assets and consumes  $x$  with linear preferences.

There are two kinds of assets: money  $m$  and real assets  $a$ . The quantity of money  $M_t$  grows at rate  $\mu$ , and  $M_{t+1} = (1 + \mu)M_t$ . Money is injected or withdrawn by lump-sum transfers. In the model, I use  $m_t$  for  $H$ 's money holding.  $\phi$  represents the amount of  $x$  that one unit of money buys, and  $z_t = \phi m_t$  denotes real money balances of the date  $t$  (measured in terms of CM goods). In a steady state,  $\mu$  is also the inflation

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<sup>7</sup>I interpret  $q$  as consumption goods (Lagos and Wright 2005; Rocheteau and Wright 2005); however,  $q$  can also be interpreted as inputs of productions (Silveira and Wright 2010; Mattesini and Nosal 2013), in which case  $u(q)$  stands for the profit function of firms, not the utility function of households. Then agents need to reallocate money when an investment opportunity, rather than a consumption opportunity, arises.

rate,  $\phi_{t+1}/\phi_t = 1 + \mu$ . Given the inflation rate  $\mu$ , the nominal interest rate  $i$  on illiquid assets is determined by the Fisher equation:  $1 + i = (1 + \mu)/\beta$ , where  $\beta$  is the discount factor. Therefore, changing the money growth rate, or the inflation rate, is equivalent to changing the nominal interest rate on illiquid assets. In this paper,  $i$  is a policy variable.

Another policy variable is the amount of assets in circulation,  $A$ . The total supply of assets,  $\bar{A}$ , is fixed. And  $\bar{A} = A + A_g$ , where  $A_g$  is in the hands of the government.  $A_g$  is not used in market transactions. When the government performs QE, it decreases the level of assets,  $A$ , and increases the level of money,  $M_t$ . In my model,  $M_t$  has no effect on allocations due to flexible prices.

Agent H requires money to purchase the DM good,  $q$ ; it accumulates money and asset balances in the CM. Then H brings assets and money to the OTC market. Upon entering the OTC market, H receives a preference shock and determines whether it has a consumption opportunity in the next DM. H becomes a buyer B or a nonbuyer N. Agents B and N have heterogeneous money demand. B receives consumption opportunities and would like to have greater money holdings and fewer real assets, whereas N would like to hold more assets and less money. B and N search randomly on the OTC market to rebalance their money and assets holdings. B could meet N to trade bilaterally, or trade with a third party M, or with no one at all if no meeting occurs. Similarly, N could trade with B, trade with M, or make no trade. After trade occurs in the OTC market, B enters the DM to purchase  $q$  with  $m$ . All households then reenter the CM to consume, rebalance  $m$  and  $a$ , and so on. More formally, in period  $t-1$ , H exits the CM with  $m$  and  $a$ . At the beginning of period  $t$ , H enters the OTC market, and the following sequence of events occurs:

1. Each H receives a preference shock and determines whether it will act as B or N.
2. B and N search to reallocate  $m$  and  $a$ . B may meet N, M, or neither. N may meet B, M, or neither. The OTC market ends.
3. B brings  $m$  and enters the DM to buy; N enters the DM to sell and produce.
4. B and N go to the CM in period  $t$ . They consume, work, clear their debt, and

move to the next OTC.

Figure 1 shows the timeline.

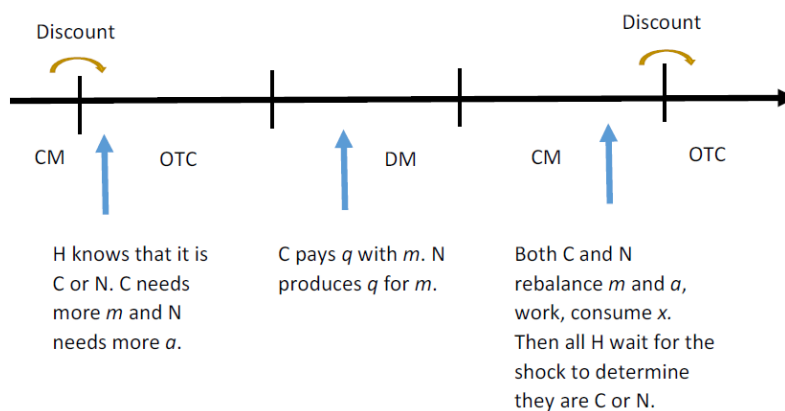


Figure 1: Timeline

Among the three submarkets, CM is frictionless, whereas the DM and the OTC market are frictional. The DM has search, recognizability, and commitment frictions. The OTC market has search and record-keeping frictions. Let me describe the DM frictions first. Following Lagos and Wright (2005), the DM is a search and matching goods market. Each household produces heterogeneous DM goods  $q$ . Buyers search for their personal  $q$ . If B finds a  $q$  that fulfills its needs, B and N will trade. However, B cannot use credit in this trade due to the lack of commitment and therefore, it must use some assets to pay for  $q$ . Here, only one asset, money, can be recognized. The reason is that claims to assets can be counterfeited with zero cost, whereas money cannot be counterfeited; that is the recognizability friction.<sup>8</sup> Generally speaking, DM frictions generate money demand, and H needs to pay with money when acting as B.

OTC markets are search and matching financial markets. In the OTC market, H does not have record-keeping technology. When N trades with counterparts, it does not give credit because it cannot identify who received its money. In other words, if B or M wants to acquire money from N, it must pay with assets. Assets serve as the

<sup>8</sup>Recognizability is a traditional assumption. An alternative assumption is that agents know assets well but treat them asymmetrically. See Zhu and Wallace (2007), Nosal and Rocheteau (2013), and Hu and Rocheteau (2013, 2014).



medium of exchange on OTC markets, which is the key assumption of this paper.<sup>9</sup>

To summarize, H requires money because it may want to buy the DM goods,  $q$ ; however, H does not want to hold large sums of money due to inflation. Then, OTC markets provide a chance to reallocate money between B and N, thereby increasing liquidity and welfare. Assets are used as a medium of exchange in the OTC markets. I use the following Figure 2 to explain OTC markets in greater detail.

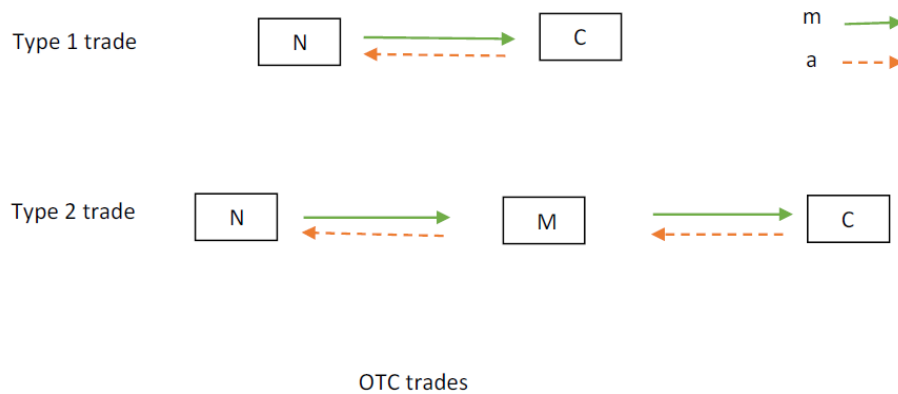


Figure 2: The OTC market

Figure 2 shows how OTC trades function. The solid arrows represent the flow of money, while the dashed arrows represent the flow of assets. There are two types of trade depicted in Figure 2. In type 1, N trades with B, whereas in type 2, N and B trade with M. To clarify, N and B do not trade using the same middleman. B and N meet different Ms, then M makes trades in the inter-dealer market on behalf of its clients. Case 3 exists in the OTC market, which is the “no trade” scenario in which B or N does not meet a counterparty successfully.

The inter-dealer market is a key feature of the OTC market. Figure 3 illustrates how this market works. As in Figure 3, the solid arrows represent the flow of money, whereas the dashed arrows represent the flow of assets.

<sup>9</sup>This assumption seems realistic given that many OTC markets, such as repos and swaps, require assets.

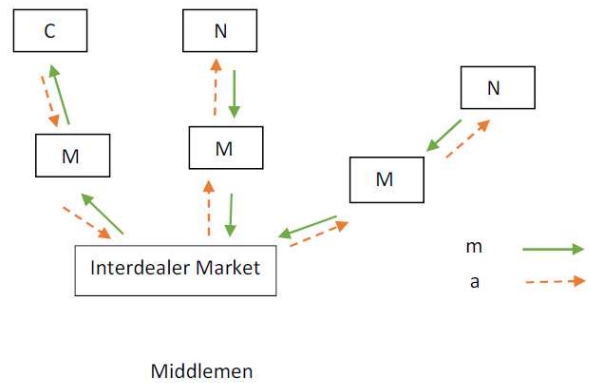


Figure 3: Inter-dealer market

The inter-dealer market is competitive and Walrasian. In this market,  $M$  trades on behalf of  $B$  or  $N$ , collecting fees for its services. However, these fees will not make the reallocation of money less efficient because  $M$  can give credit to  $B$  on fees. Let  $d$  denote the service fee debt. In contrast to  $H$ ,  $M$  has record-keeping technology and enforces the repayment on the next  $CM$ .  $M$  does not have money or asset holdings:  $M$  is a broker. When  $M$  meets  $N$  to trade,  $M$  requires assets. Then  $M$  acquires assets from other  $M$ s, who collect assets from  $B$ . In the inter-dealer market,  $B$ 's asset holdings are used to get  $N$ 's money, which is similar to the OTC trades without middlemen.

Another important characteristic of the inter-dealer market is that  $B$  is able to use money from more than one  $N$ . The amount of money  $B$  receives depends on the  $B$ - $N$  ratio. All terms of trades in OTC markets are determined by bilateral bargaining. This framework allows me to study the effects of changing  $i$  and  $A$  on liquidity, welfare, and asset pricing.

### 3 Analysis

#### 3.1 Value functions

I begin by describing value functions in the CM, considering that H enters the CM with  $m$ ,  $a$ , and debt  $d$ ; then Bellman's equation is the following:

$$V^c(m, a, d) = \max_{x, L, \hat{m}, \hat{a}} \{U(x) - L + \beta V^o(\hat{m}, \hat{a}, 0)\} \quad (1)$$

$$st \quad x = L + (\psi + \rho)a + \phi m - \psi \hat{a} - \phi \hat{m} - T - d, \quad (2)$$

where  $V^c$  and  $V^o$  are the value functions for the CM and the OTC market.  $\phi$  and  $\psi$  are prices of money and assets respectively in terms of  $x$ .  $\rho$  is the dividend of one unit of asset.  $T$  is the lump-sum taxes or transfers;  $T = T_r - \phi(M_{t+1} - M_t)$ ; that is,  $T$  contains money transfers  $\phi(M_{t+1} - M_t)$  and real taxes  $T_r$ . H determines future asset and money holdings,  $\hat{a}$  and  $\hat{m}$ ; and H clears the debt,  $d$ , every period. Substituting the budget equation into Bellman's equation, three choice variables remain. The FOC are found below:

$$\begin{aligned} x : \quad & U'(x) = 1; \\ \hat{m} : \quad & \beta V_1^o(\hat{m}, \hat{a}, 0) = \phi; \\ \hat{a} : \quad & \beta V_2^o(\hat{m}, \hat{a}, 0) = \psi. \end{aligned} \quad (3)$$

The above result is standard in models that build on Lagos and Wright (2005), in which all Hs have the same  $x$  consumption; their money and asset holdings follow Lemma 1.<sup>10</sup>

**Lemma 1.**  *$\hat{m}, \hat{a}$  are history-independent.*

*Proof.* See FOC (3). □

After leaving the CM, H enters the next market, the OTC market. At the beginning

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<sup>10</sup>It may be interesting to study the case where  $q$  is used to produce  $x$ ; this generates an inflation-employment trade-off for reasons similar to but different from Rocheteau et al. (2008), Dong (2011), Berentson et al. (2011), and Aruoba et al. (2011); I leave this idea for further research.

of the OTC market, H becomes B or N; the value function of the OTC market is

$$V^o(\hat{m}, \hat{a}, 0) = \gamma V_b^o(\hat{m}, \hat{a}, 0) + (1 - \gamma) V_n^o(\hat{m}, \hat{a}, 0), \quad (4)$$

where  $V_b^o$  and  $V_n^o$  are the value functions for B and N, and  $\gamma$  and  $1 - \gamma$  are the probabilities of turning into B and N respectively.

In the OTC market, B (who needs additional money) may be matched with N, M, or nobody; therefore, two types of trades are possible. B is represented as type 1 or type 2 in these trades and as type 3 when there are no possible trades. Let  $\alpha_b^1$ ,  $\alpha_b^2$ , and  $\alpha_b^3$  be the probabilities of the three types of B, whereas  $\alpha_n^1$ ,  $\alpha_n^2$ , and  $\alpha_n^3$  are the probabilities for N. To simplify, assume one middleman is guaranteed to meet with at least one buyer or one seller. This gives us the following results:  $\alpha_b^2 = \xi\gamma$ ,  $\alpha_n^2 = \xi(1 - \gamma)$ ;  $\alpha_b^1 = (1 - \xi)m_1(\gamma, 1 - \gamma)/\gamma$ ,  $\alpha_n^1 = (1 - \xi)m_1(\gamma, 1 - \gamma)/(1 - \gamma)$ ; and  $\alpha_b^3 = 1 - \alpha_b^1 - \alpha_b^2$ ,  $\alpha_n^3 = 1 - \alpha_n^1 - \alpha_n^2$ , where  $m_1(\gamma, 1 - \gamma)$  is the OTC markets' matching function.

The following is B's OTC value function:

$$V_b^o(\hat{m}, \hat{a}) = \alpha_b^1 V_b^d(\hat{m} + m_1, \hat{a} - a_1, 0) + \alpha_b^2 V_b^d(\hat{m} + m_2, \hat{a} - a_2, d) + (1 - \alpha_b^1 - \alpha_b^2) V_b^d(\hat{m}, \hat{a}, 0), \quad (5)$$

where  $m_1$  and  $m_2$  are additional units of money that B obtains in type 1 and type 2 meetings, and  $a_1$  and  $a_2$  are assets that B sells in type 1 and type 2 meetings.  $d$  represents the service fees that B must pay at the next CM.  $V_b^d$  is B's value function in the next DM.

Similarly, N's OTC value function is,

$$V_n^o(\hat{m}, \hat{a}, 0) = \alpha_n^1 V_n^d(\hat{m} - m_1, \hat{a} + a_1, 0) + \alpha_n^2 V_n^d(\hat{m} - m_2, \hat{a} + a_2, 0) + (1 - \alpha_n^1 - \alpha_n^2) V_n^d(\hat{m}, \hat{a}, 0) \quad (6)$$

N has no debt.

Finally, I consider the value functions in the DM. B is a potential buyer, whereas N is a potential seller. Let  $\alpha_b$  denote the probability that B makes a trade, and  $\alpha_n$  denote

the probability that N makes a trade;  $m_2(\gamma, 1 - \gamma)$  denotes the DM matching function. Then  $\alpha_b = m_2(\gamma, 1 - \gamma)/\gamma$ , and  $\alpha_n = m_2(\gamma, 1 - \gamma)/(1 - \gamma)$ . The DM value functions for B and N are the following:

$$V_b^d(m, a, d) = V^c(m, a, d) + \alpha_b[u(q) - \phi m] \quad (7)$$

$$V_n^d(m, a, 0) = V^c(m, a, 0) + \alpha_n[-c(\tilde{q}) + \phi \tilde{m}], \quad (8)$$

where  $\tilde{q}$  is the DM goods that N produces for B, and  $\tilde{m}$  is the payment that N receives.  $\tilde{q}$  and  $\tilde{m}$  are not N's choice variables.

When B and N match successfully in the DM market, B consumes DM consumption and pays with money, whereas N produces with cost and receives payment. The amount of DM goods that N produces depends only on B's money holding. Having established the agent's value functions, I will examine the DM and OTC market bargaining processes.

### 3.2 DM bargaining

The DM opens after the OTC market; therefore, the terms of trade in this market will serve as inputs to determine the terms of trade in the OTC market. Hence, I study the DM first and then examine the OTC processes via backward induction.

In the DM, money is used to buy  $q$ . In a bilateral meeting, the terms of trade  $(q, v(q))$  are determined by bargaining. The Kalai-Smorodinsky bargaining process is applied here:

$$\max_{q, v(q)} u(q) - c(q) \quad (9)$$

$$st \quad v(q) = \theta u(q) + (1 - \theta)c(q) \leq \phi m, \quad (10)$$

where  $\theta$  is B's bargaining power, and  $v(q)$  is the function of payment.

The solution to the bargaining problem is described in Lemma 2.

**Lemma 2.** Define the amount of money that allows a buyer to purchase  $q^*$  ( $q^*$  st,  $u'(q^*) =$

$c'(q^*))$  as  $m^*$ :

$$m^* = \frac{\theta u(q^*) + (1 - \theta)c(q^*)}{\phi}.$$

Then, the solution is as follows:

$$v(q) = \begin{cases} m^* & \text{if } m \geq m^*, \\ m & \text{if } m < m^*. \end{cases} \quad (11)$$

$$q = \begin{cases} q^* & \text{if } m \geq m^*, \\ v^{-1}(\phi m) & \text{if } m < m^*. \end{cases} \quad (12)$$

*Proof.* See Lagos and Wright (2005). □

Let  $\lambda$  denote the Lagrangian multiplier of Equation (10); then  $\lambda = \frac{u'(q)}{v'(q)} - 1$  is also the liquidity premium for money. If Equation (10) is not binding,  $\lambda$  will be 0, and B will acquire optimal DM consumption  $q^*$ . Otherwise,  $\lambda$  is a positive number, and the buyer cannot receive optimal DM consumption due to a money shortage. Let  $\lambda_1, \lambda_2$ , and  $\lambda_3$  denote the liquidity premium for type 1, 2, and 3 meetings. Then  $\lambda_3 \geq \lambda_1, \lambda_2$ , because B has no additional money to use in type 3.

### 3.3 OTC bargaining

Now I study the terms of trade in OTC markets. H becomes B or N. B requires more money while N desires more assets. There are two types of meeting: the type 1 meeting in which B and N trade, and the type 2 meeting in which B and N trade via M. There are three types of bargaining in the OTC market. B bargains with N in the type 1 meeting; B bargains with M in the type 2 meeting; and N also bargains with M in the type 2 meeting as well.

Before considering the bargaining process, I define the scarcity of assets as seen in Lemma 3:

**Lemma 3.**  $\exists \tilde{A}$ , if  $A \geq \tilde{A}$ ,  $\psi = \psi^* = \frac{\rho}{r}$  ( $\psi^*$  is the fundamental price). Otherwise,  $\psi > \psi^*$  for some  $i$ . The scarcity of assets means  $A < \tilde{A}$ .

*Proof.* See the appendix. □

Given the scarcity of assets, I consider the type 1 bargaining problem. Let  $\theta_1$  denote N's bargaining power in type 1 meetings and  $T_{BN}$  denote the total benefit of B and N's trading:

$$\max_{a_1, m_1} T_{BN} = [V_b^d(\hat{m} + m_1, \hat{a} - a_1, 0) - V_b^d(\hat{m}, \hat{a}, 0)] + [V_n^d(\hat{m} - m_1, \hat{a} + a_1, 0) - V_n^d(\hat{m}, \hat{a}, 0)] \quad (13)$$

$$st \quad a_1 \leq \hat{a}; \quad m_1 \leq \hat{m}; \quad (14)$$

$$\theta_1 T_{BN} = (\psi + \rho)a_1. \quad (15)$$

To receive money from N, B must hold assets. As a result, B will suffer from the feasibility constraint (14).

One particular property of this bargaining process is worth emphasizing: the level of assets is relevant when transferring money. Equation (15) is the *key* equation to understand the role of assets. Assets are the medium of exchange for money reallocation. How much money is transferred depends on  $A$ , the level of assets in circulation. If Equation (15) is not binding (B has sufficient assets), then B will achieve  $m^*$ , or B will borrow all available money. Otherwise, B cannot acquire sufficient money due to the shortage of assets.

Now I consider the type 2 meetings, where B and N trade via M. M trades on behalf of its client in the inter-dealer market. Let  $i_2$  denote the interest rate in the inter-dealer market. The inter-dealer market money supply is  $\zeta(1 - \gamma)\hat{m}$ , which is N's total money holdings in the market. N needs to acquire assets when it trades. The level of assets is defined as  $\zeta\gamma\hat{a}$ , which is B's total asset holdings in the market. If the level of assets is insufficient, this shortage of assets will lead to inefficient money reallocation.

Consider the bargaining process between M and N. Let  $\theta_2^N$  denote N's bargaining power in type 2 meetings. N wants to buy assets to receive a return. The total money supply in type 2  $M_N$  should satisfy the following equation:

$$(1 + \theta_2^N i_2) \phi M_N \leq \zeta \gamma (\psi + \rho) a_1 \quad (16)$$

Then B will receive  $\frac{M_N}{\zeta \gamma}$ , which is determined by the market equilibrium. Let  $T_{BM}$  denote the total benefit of B and M's trading and  $\theta_2^M$  denote M's bargaining power in a type 2 meeting between B and M. I represent B and M's bargaining as follows:

$$\max_{a_2, m_2} T_{BM} = [V_b^d(\hat{m} + m_2, \hat{a} - a_1, d) - V_b^d(\hat{m}, \hat{a}, 0)] + (\psi + \rho) a_1 - (1 + i_2) \phi m_2 \quad (17)$$

$$st \quad a_1 \leq \hat{a}; \quad m_2 = \frac{M_N}{\zeta \gamma}. \quad (18)$$

The results of OTC bargaining are summarized in Lemma 4.

**Lemma 4.** *Given  $A$  and  $\phi$ , there exists  $i_j^*$  ( $j = 1, 2$ ), satisfying:*

$$\hat{m} + m_j \begin{cases} = m^* & \text{if } i \leq i_j^* \\ < m^* & \text{if } i > i_j^* \end{cases}$$

And  $\frac{\partial i_j^*}{\partial A} > 0$  when there is a scarcity of assets.

*Proof.* See the appendix. □

B can achieve  $m^*$  in type 1 and type 2 meetings when nominal interest rates are low enough. However, most papers follow the example of Lagos and Wright (2005), which claims that this result can be obtained only under the Friedman rule:  $i=0$ . The special mechanism at work in this model is the uncertainty of OTC markets. H may enter a no-meeting case in the OTC markets; as a result, H wants to retain more money on the CM to create a precautionary savings pool in case the worst case scenario is realized. If H knows that it has definite access to a banker (Berentson et al., 2007), H will decrease its money holdings and fail to achieve  $m^*$ .

When the nominal interest rate is low, B holds a relatively large amount of money



in its portfolio, and requires only slightly more money to obtain optimal consumption. Although the level of assets in circulation is limited, it is already sufficient to ensure that B acquires  $m^*$ . When the nominal interest rate is high, B cannot obtain  $m^*$  due to the shortage of assets or the shortage of the money supply. Higher  $A$  in circulation improves the allocation.

### 3.4 Government budget

Now I consider the government's budget constraint. The government collects real tax and inflation tax to finance its expenditures, and its budget constraint is represented as follows:

$$G + \psi \hat{A}_g = T_r + (\psi + \rho)A_g + \phi[M_{t+1} - M_t] \quad (19)$$

where  $G$  is government spending,  $T_r$  is the tax in term of  $x$ , and  $\hat{A}_g$  represents the government asset holding for the next period. The government holds real assets; it uses dividends from assets to pay government bills, or it collects less tax. The government balances its budget every time period.

The two policy variables are the nominal interest rate,  $i$ , and the level of assets in circulation,  $A$ . The government controls these two variables by changing  $M_{t+1}$  and  $A_g$ . It injects money into the economy by initiating a lump-sum transfer to control the money growth rate  $\mu$ , which is equivalent to  $i$ . Meanwhile, the government could also use money to buy assets (QE) or sell assets for money; these actions cause the levels of money and assets in circulation to fluctuate.

### 3.5 Equilibrium

In this section, I describe the equilibrium, focusing on steady state equilibria, where all Hs have the same money and asset holdings,<sup>11</sup> and the real variables will be constant over time. A steady state equilibrium is defined in Definition 1.

**Definition 1.** A steady state equilibrium is a state in which, given  $i$ ,  $A$ , a list of  $m$ ,  $a$ ,  $d$ ,

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<sup>11</sup>Following Lagos and Wright (2005), households have the same money and asset holdings.

and prices satisfy the following:

- (1)  $\hat{a}$  and  $\hat{m}$  solve H's problem;
- (2) the government balances its budget;
- (3)  $m_1, a_1, m_2, a_2$ , and  $d$  are bargaining solutions;
- (4) prices  $(\phi, \psi, i_2)$  clear markets.

Then Lemma 5 describes the existence and uniqueness of a steady state equilibrium:

**Lemma 5.** *In this model, a unique steady state equilibrium exists.*

*Proof.* See the appendix. □

## 4 Policy

### 4.1 Liquidity and welfare

I examine the effects of monetary policies in this chapter, focusing on consumption, liquidity, and welfare. First, consider the effects of the nominal interest rate,  $i$ , which is equivalent to the money growth rate,  $\mu$ . The effect of  $i$  on real money balances  $z_t$  is described in Proposition 1.

**Proposition 1.** *Let  $z_t = \phi m_t$  denote real money balances, then  $\frac{\partial z_t}{\partial i} < 0$ .*

*Proof.* See the appendix. □

Real money balances  $z_t$  decrease with the nominal interest rate,  $i$ , which is the cost of holding money.<sup>12</sup> This is not a surprising result; a higher inflation tax leads to lower money holdings, and therefore, to greater money demand when H turns to B. B with no OTC meetings uses its own money to purchase in next DM. I call these agents type 3 buyers. Other Bs have an OTC trade with N or M. We call the meeting with N the type 1 meeting, and the meeting with M the type 2 meeting. DM consumption is

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<sup>12</sup>The only cost of holding money is the inflation tax here. However, it is also worth mentioning that one disadvantage of money is that it may be more susceptible to loss or theft, as in He et al. (2005, 2008) or Sanches and Williamson (2010). I leave this consideration for future research.

heterogeneous in these three types. Figure 4 shows the relationship between  $i$  and DM consumption in the three types for a given assets level,  $A$ . The solid line represents B's DM consumption in type 2 meetings, the dashed line denotes B's DM consumption in type 1 meetings, and the dotted line denotes B's DM consumption with no OTC trade.

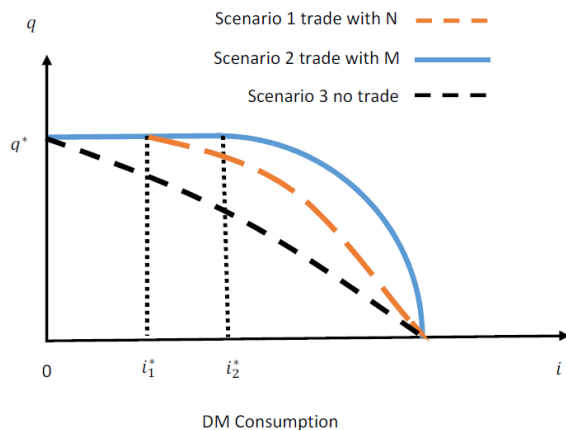


Figure 4: DM consumption in three OTC types

Type 3 B has no additional money and therefore consumes fewest DM goods. Type 1 B can acquire money from one N while type 2 B has access to N's money via the inter-dealer market. Therefore, type 2 could obtain a larger amount of money than the other two types can. When the nominal interest rate is low, B can obtain enough money to buy  $q^*$  in both type 1 and type 2 meetings.

I calculate the weighted average of all three types of DM consumption to derive the overall effects. The weighted average is  $q_w = \alpha_b^1 q_1 + \alpha_b^2 q_2 + \alpha_b^3 q_3$ , which is a straightforward indicator of consumption; and hence, of welfare and liquidity. Higher  $q_w$  translates into higher welfare and liquidity. From this point forward in the analysis, I consider the relationship between  $q_w$  and monetary policies, beginning with the nominal interest rate,  $i$ .

$q_w$  measures welfare and liquidity, due to the quasi-linear utility setup. If policy changes (cutting interest rates and QE) take place, CM consumption  $x$  will be constant, and the total amount of labor will stay the same. These results follow Lagos and Wright (2005). Therefore, the overall CM consumption and labor are independent of policies. We then focus on the DM consumption to measure welfare and liquidity. A

higher  $q_w$  indicates higher liquidity and welfare.

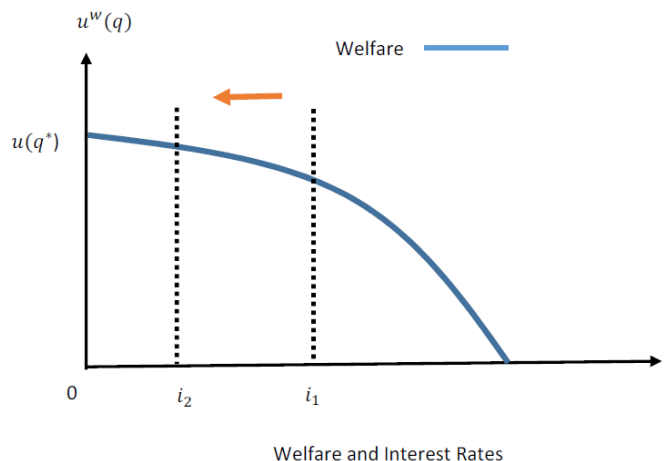


Figure 5: The nominal interest rate and weighted DM consumption

Figure 5 shows the relationship between  $q_w$  and  $i$ . The curve in Figure 5 slopes downward. At a low level of inflation, although B can acquire  $q^*$  in type 1 and 2 meetings, B’s weighted consumption still decreases due to the drop of the type 3 consumption. The effect of cutting the nominal interest rate is very clear: It increases consumption, and hence, liquidity and welfare.

In considering QE, I will focus on decreasing asset levels to align with the Federal Reserve’s actions in the real world. Proposition 2 summarizes the effects of QE.

**Proposition 2.**  $\forall A$ , there exist two cutoffs,  $\tilde{i}_1$  and  $\tilde{i}_2$ , if  $\tilde{i}_1 < i < \tilde{i}_2$ ; QE decreases DM consumption and thereby decreases liquidity and welfare.

*Proof.* See the appendix. □

Figure 6 illustrates the effect of QE, which lowers the level of assets in circulation. The solid line represents B’s weighted DM consumption before QE, whereas the dashed line denotes B’s weighted DM consumption after QE.

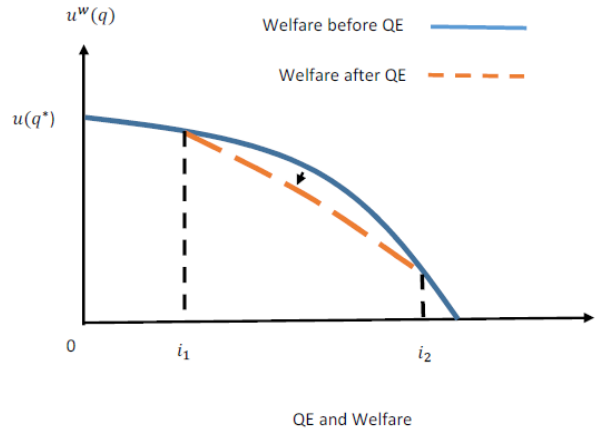


Figure 6: QE and weighted DM consumption

To explain Proposition 2 in more detail, I examine DM consumption in all OTC types. Type 3 B's DM consumption is not affected because it finds no trade in the OTC market. DM consumption in type 1 or type 2 OTC meetings is more complex. Figure 7 shows the effects of QE on type 1 or type 2 DM consumption. The solid line denotes B's type 1 or type 2 DM consumption before QE, whereas the dashed line represents DM consumption after QE.

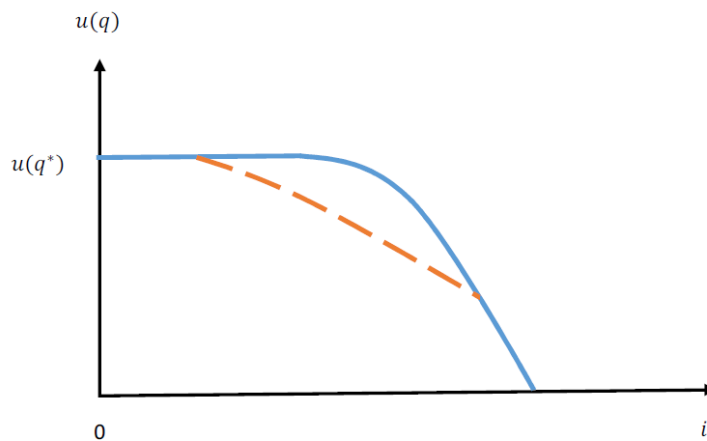


Figure 7: QE and Type 2 or Type 3 DM consumption

The figure depicts three parameter regions. In the low and extremely high interest rate regions, assets are sufficient, and QE has no effect. When the interest rate is low, the households themselves carry relatively large amounts of money; they require little money to obtain  $m^*$ . Therefore, a few assets are enough to facilitate all needed OTC

transactions. In the case of extremely high interest rates, the money supply in the OTC market is limited. A few assets are sufficiently abundant to obtain all the money in OTC trades. Assets are sufficient in the two cases above. When assets are sufficient, QE has no effect because the decreasing level of assets is irrelevant.

When the interest rate is high but not extremely high, there is an adequate money supply on OTC markets. Then the stock of assets is the most important factor in facilitation OTC transactions. It is possible that assets are not enough to ensure that B acquires a satisfying amount of money. And QE makes assets scarcer and money allocation less efficient. These mechanisms, therefore, lower DM consumption and reduce liquidity.

Figure 8 shows the relationships between this two policies.

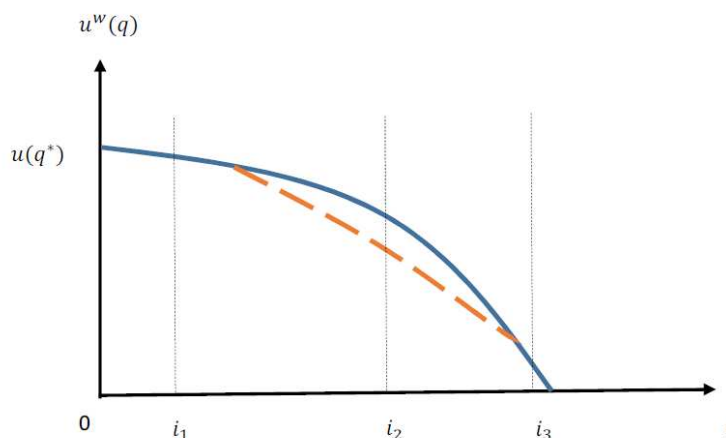


Figure 8: Relationships between interest and QE

Interest rates determine money holding and asset demand endogenously. High inflation leads to a scarcity of assets. If the Federal Reserve employs QE in a particular region, the liquidity of the economy will decrease because of a decrease of  $A$ .

These propositions have new policy implications. First, if the Federal Reserve wants to keep liquidity high, it should continue the low nominal interest rate policy. Second, the Federal Reserve should inject liquid assets into the economy to increase liquidity, rather than injecting money and withdrawing assets.<sup>13</sup>

<sup>13</sup>I assume the total stock of assets is fixed. Some assets are in the government's hands while others are in circulation. However, it may be interesting to relax this assumption and allow the private sector

## 4.2 Asset pricing

I am now prepared to examine how assets are priced and study the effects of QE on asset pricing. Assets are priced as described in Lemma 6:

**Lemma 6.** *Given the scarcity of assets, the asset price is hump-shaped. If the nominal interest rate,  $i$ , is extremely low or very high, assets will be priced fundamentally. Otherwise, asset prices will be higher than the fundamental price, and they will increase with the nominal interest rate.*

*Proof.* See the appendix. □

Figure 9 shows how asset prices change with the nominal interest rate.

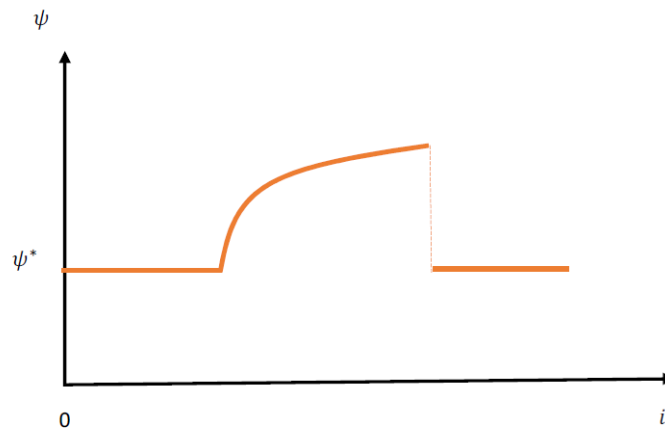


Figure 9: Asset pricing

The effects of QE on asset pricing are summarized in Proposition 3, taking the asset pricing curve into account.

**Proposition 3.** *QE decreases the level of assets in circulation,  $A$ . If the nominal interest rate,  $i$ , is low or very high, assets will be priced fundamentally. Otherwise, the price of assets will increase.*

*Proof.* See the appendix. □

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and the government to produce liquid assets and inject them in the economy, as noted in Gorton and Ordóñez (2013) and Williamson (2014a, 2014b). They suggest that the government should issue more treasury bills, which are considered safe and accepted in transactions. I will leave this asset creation and fiscal policy project for future study.

The effects of QE on asset pricing are shown in Figure 10. The dotted line represents the asset pricing curve before QE, and the solid line denotes the asset pricing curve after QE.

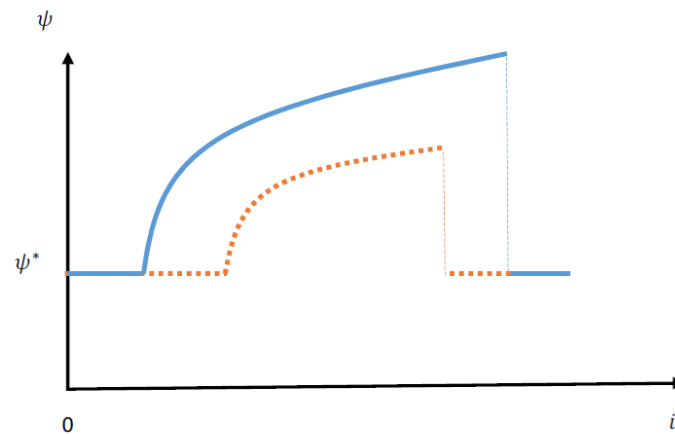


Figure 10: QE and asset pricing

Asset prices are related to the liquidity of the economy. When assets are sufficient to meet needs in OTC markets, assets will be priced fundamentally. This is true during both low and high inflation periods. Otherwise, assets are scarce and carry a liquidity premium, which translates to higher asset prices. When QE decreases  $A$ , assets become scarcer and asset prices rise. These high prices result from an asset shortage in OTC markets, which leads to inefficient money allocation.

## 5 Conclusion

In this paper, I study two monetary policies, cutting interest rates and QE, in a general equilibrium model with frictional OTC markets. OTC markets serve to reallocate money across agents.<sup>14</sup> Through this research, I discover new findings regarding the implementation of QE. Traditional monetary wisdom, such as the IS-LM model

<sup>14</sup>Money is needed here to buy consumption goods (Lagos and Wright 2005; Rocheteau and Wright 2005); however, money demand can also be generated to buy inputs of productions (Silveira and Wright 2010; Mattesini and Nosal 2013), in which case the benefit represents the profit function of firms, not the utility function of households. Then agents need to reallocate money when an investment opportunity, rather than a consumption opportunity, arises.



or the relatively morden market segmentation models, states that raising money levels increases liquidity in the economy, whereas I find that the level of liquid assets is the significant factor affecting allocations, from the perspective of OTC market efficiency. A higher level of assets leads to a more efficient money allocation—and hence increases welfare and liquidity.

I also examine the relationship between cutting interest rates and QE. The nominal interest rate determines the demand for assets. If the nominal interest rate is above a cutoff, OTC markets will suffer from a scarcity of assets. Given the scarcity, QE decreases asset levels furthermore, which leads to assets becoming scarcer and carrying a larger liquidity premium. These higher asset prices reflect lower consumption.

This project provides a new perspective on understanding the effect of QE. One interpretation of this perspective relates to investigating the long-term effects of QE. In the long run, most economists believe that prices adjust freely in markets. This implies that economists need to focus on the level of liquidity, instead of the level of money. When assets are useful (as a medium of exchange or collateral), the government should not collect these assets and bury them in the New York Federal Reserve. My policy recommendation is that the government should channel these useful assets back into the market, rather than injecting money, which only increases price levels.

## Appendix

Proof of Lemma 3. The proof of Lemma 3 depends on Lemma 7 and 8:

**Lemma 7.** *If assets do not carry a liquidity premium, assets will be priced fundamentally.*

*Proof.* Consider the Euler equation for asset holding:

$$(1+r)\psi_{-1} = (\psi + \rho) + \gamma\alpha_b\alpha_b^1(1-\theta_1)\frac{\partial T_{BN}}{\partial a} + \gamma\alpha_b\alpha_b^2(1-\theta_2^M)\frac{\partial T_{BM}}{\partial a}, \quad (20)$$

If assets do not carry a liquidity premium, then  $\frac{\partial T_{BN}}{\partial a} = \frac{\partial T_{BM}}{\partial a} = 0$ . Therefore,  $(1+r)\psi_{-1} = \psi + \rho$ . At a steady state equilibrium,  $\psi_{-1} = \psi$ .  $\psi = \psi^* = \rho/r$ .  $\square$

The amount of money that B needs to achieve  $m^*$  is  $m^d = m^* - z_t/\phi$ , where  $m^d$  is for money demand. The supply of money that each N holds is  $m^s = z_t/\phi$ .

**Lemma 8.**  $\exists \hat{i}$ , s.t.  $m^d = m^s$  if  $i = \hat{i}$ .

*Proof.* From Proposition 1, we know  $\partial z_t/\partial i < 0$ ; therefore,  $\partial m^s/\partial i < 0$ , and we also have  $\partial m^d/\partial i = -\frac{\partial z_t/\partial i}{\phi} > 0$ . Under the Friedman rule,  $m^d = 0$ ,  $m^s = m^*$ . Due to continuity, we know that a cutoff  $\hat{i}$  exists, st  $m^d = m^s$  if  $i = \hat{i}$ . And we also have  $m^d < m^s$  if  $i < \hat{i}$ ; whereas  $m^d > m^s$  if  $i > \hat{i}$ .  $\square$

Let  $T_{N1}$  and  $T_{N2}$  denote the benefits that N gets in a type 1 or type 2 trade. Then let  $\tilde{A}$  satisfy  $\psi^*\tilde{A} = \max\{T_{N1}(\hat{i}), T_{N2}(\hat{i})\}$ . We need to show that, when  $A \geq \tilde{A}$ , assets will be fundamentally priced under all  $i$ .

Now looking at the case  $i \leq \hat{i}$ , from the proof of Lemma 8, we know  $\phi m^d \leq z_t(\hat{i})$ . That is, money supply in OTC markets is sufficient to satisfy agents' needs. Assets are sufficient to acquire all the money N holds.. Therefore, B achieves  $m^*$ . Assets carry no liquidity premium. We know that assets are priced fundamentally due to Lemma 7. When  $i > \hat{i}$ , then  $\phi m^s \leq z_t(\hat{i})$ ; therefore, assets carry no liquidity premium because there is no remaining money. B obtains all the money available in OTC markets. Then assets are priced fundamentally due to Lemma 7, and Lemma 3 holds. *Q.E.D.*

Proof of Lemma 4. It is easy to show that buyers cannot get  $m^*$  if  $i$  is high (see proof of Lemma 3), so instead, I focus here on the low inflation case. The proof of Lemma 4 depends on Lemma 9:

**Lemma 9.** *Given  $0 < A < \tilde{A}$ ;  $\exists i_1(A), i_2(A)$ , st  $\psi^* A \geq \max\{T_{N1}[i(A)], T_{N2}[i(A)]\}$  if  $i < i_1(A)$  or  $i \geq i_2(A)$ . Then  $\psi = \psi^*$  if  $i < i_1(A)$  or  $i \geq i_2(A)$ .*

*Proof.* See proof of Lemma 8. □

We know that if  $i_1(A) < \hat{i}$ , money supply will be sufficient to achieve  $m^*$ , and if  $i \leq i_1(A)$ , assets will be sufficient to achieve  $m^*$ . Therefore, the feasibility constraints in type 1 and type 2 are not binding if  $i \leq i_1(A)$ ; B achieves  $m^*$ . *Q.E.D.*

Proof of Lemma 5. To prove the existence and uniqueness of equilibrium, I depend on the results of Lemma 1 and Proposition 1. Given real money balances  $z_t$ , the prices  $(\phi, \psi, i_2)$  and allocations  $(m_1, a_1, m_2, a_2, d)$  are determined. Therefore, the existence and uniqueness of  $z_t$  is sufficient to show the existence and uniqueness of equilibrium. Lemma 1 and Proposition 1 show that  $z_t$  is unique. Therefore, I can prove the existence and uniqueness of equilibrium. *Q.E.D.*

Proof of Lemma 6. The proof of Lemma 6 depends on Lemma 7 and Lemma 9. Lemma 7 shows that when assets carry no liquidity premium, assets are priced fundamentally. Lemma 9 shows that in cases of low and extremely high inflation, assets are sufficiently abundant,  $\psi = \psi^*$ . When inflation is neither too low nor too high, increasing the interest rate leads to a higher liquidity premium; see Venkateswaran and Wright (2013) for the proof. This is the Mundell-Tobin effect. At the higher interest cutoff  $i_2(A)$ , where assets are sufficient to capture all the OTC money, asset prices will jump to the fundamental prices. *Q.E.D.*

Proof of Proposition 1. The Euler equation for money holding is needed to prove this proposition:

$$1 + i = 1 + (1 - \gamma)(\alpha_l^1 \Lambda_1 \theta_1 \frac{\partial T_{BN}}{\partial m} + \alpha_l^2 \Lambda_2 \theta_2^N i_2) + \gamma \alpha_b [\alpha_b^1 \lambda_1 + \alpha_b^2 \lambda_2 + \alpha_b^3 \lambda_3], \quad (21)$$

where  $\Lambda_1$  and  $\Lambda_2$  are indicators of whether nonbuyers have sufficient cash in type 1

and type 2 OTC trades. Equation (21) should hold at any  $i$ .

Proposition 1 is demonstrated through proof by contradiction techniques. First, I examine the  $\Lambda_1 = \Lambda_2 = 0$  case, simplifying the Euler equation above as  $1 + i = 1 + \gamma\alpha_b[\alpha_b^1\lambda_1 + \alpha_b^2\lambda_2 + \alpha_b^3\lambda_3]$ . Then I assume that  $\tilde{i}$  exists, satisfying,  $\partial z/\partial i \geq 0$  if  $i=\tilde{i}$ . I look at the interest rate  $i' = \tilde{i} + \delta$ , where  $\delta$  is a small, positive number; due to the principle of continuity,  $z(i') \leq z(\tilde{i})$ . Then the RHS of Equation (21) does not decrease with  $i$ . Because  $z_t$  determines consumption allocations in all three types, a higher  $z_t$  means a lower  $\lambda$ . However, the LHS of Equation (21) strictly increases with  $i$ . The strictly increasing LHS and the nonincreasing RHS cannot coexist, and hence, Equation (21) cannot hold at  $i'$ ; this is a contradiction. Therefore, real money balance  $z_t$  is not strictly decreasing if  $\Lambda_1 = \Lambda_2 = 0$ . Using a similar methodology, we can prove the real money balance is strictly decreasing with  $i$  if  $\Lambda_1 = 1, \Lambda_2 = 0, \Lambda_1 = 0, \Lambda_2 = 1$ , and  $\Lambda_1 = \Lambda_2 = 0$ . *Q.E.D.*

Proof of Proposition 2. The proof of Proposition 2 depends on Lemma 4, Equation 15, and Equation 16. There are three parameter regions in Proposition 2. First, I focus on the low interest rate region. Let  $\tilde{i}_1 = \min\{i_1^*, i_2^*\}$ . Lemma 4 shows that B acquires  $m^*$  in both type 1 and type 2 OTC meetings. Therefore, Equations 15 and 16 are not binding. QE decreases  $A$ , but B still obtains  $m^*$  due to these nonbinding constraints; therefore, QE has no effect. Asset levels are sufficient. Now I consider the extremely high nominal interest rate region. B cannot achieve  $m^*$  there. However, B acquires all available money in both type 1 and type 2 meetings. Then QE has no effect because no money remains. Finally, I consider the region where inflation is high but not extremely high; then Equation 15 or Equation 16 is binding. A lower  $A$  leads to less money being transferred in OTC markets; and lower weighted DM consumption. *Q.E.D.*

Proof of Proposition 3. The proof of Proposition 3 requires the Euler equation for asset holding:

$$(1 + r)\psi_{-1} = (\psi + \rho) + \gamma\alpha_b\alpha_b^1(1 - \theta_1)\frac{\partial T_{BN}}{\partial a} + \gamma\alpha_b\alpha_b^2(1 - \theta_2^M)\frac{\partial T_{BM}}{\partial a},$$

LHS is the cost of holding one extra unit of assets, whereas RHS is the benefit of

holding an extra unit of assets. If the interest is very low, B acquires  $q^*$ . Then  $\frac{\partial T_{BN}}{\partial a} = 0$  and  $\frac{\partial T_{BM}}{\partial a} = 0$ . That is, assets carry no liquidity premium. If the interest is extremely high, the OTC money supply will be limited, and assets will carry no liquidity premium due to the shortage of money. Otherwise, we have  $\frac{\partial T_{BN}}{\partial a} > 0$ ,  $\frac{\partial^2 T_{BN}}{\partial a \partial A} > 0$ ; and  $\frac{\partial T_{BM}}{\partial a} > 0$ ,  $\frac{\partial^2 T_{BM}}{\partial a \partial A} > 0$  from FOC and SOC. In other words, the asset price is above the fundamental price, and prices of assets increase when QE decreases  $A$  in the economy. *Q.E.D.*

## References

- [1] F. Alvarez, A. Atkeson, and C. Edmond. Sluggish Responses of Prices and Inflation to Monetary Shocks in an Inventory Model of Money Demand. *Quarterly Journal of Economics*, 124(3):911–967, 2009.
- [2] F. Alvarez, A. Atkeson, and P. Kehoe. Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets. *Journal of Political Economy*, 110(1):73–112, 2014.
- [3] F. Alvarez, R. Lucas, and W. Weber. Interest Rates and Inflation. *American Economic Review*, pages 219–225, 2001.
- [4] B. Aruoba, C. Waller, and R. Wright. Money and Capital. *Journal of Monetary Economics*, 58:98–116, 2011.
- [5] A. Berentsen, G. Camera, and C. Waller. Money, Credit and Banking. *Journal of Economic Theory*, 135:171–195, 2007.
- [6] A. Berentsen, G. Menzio, and R. Wright. Inflation and Unemployment in the Long Run. *American Economic Review*, 101:371–398, 2011.
- [7] B. Bernanke, M. Gertler, and S. Gilchrist. The Financial Accelerator in a Quantitative Business Cycle Framework. *Handbook of macroeconomics*, 1:1341–1393, 1999.
- [8] P. Boel and G. Camera. Efficient Monetary Allocations and the Illiquidity of Bonds. *Journal of Monetary Economics*, 53(7):1693–1715, 2006.
- [9] R. Caballero. On the Macroeconomics of Asset Shortages. Technical report, National Bureau of Economic Research, 2006.
- [10] R. Caballero and A. Krishnamurthy. Bubbles and Capital Flow Volatility: Causes and Risk Management. *Journal of Monetary Economics*, 53(1):35–53, 2006.
- [11] J. Chiu. Endogenously Segmented Asset Market in an Inventory-Theoretic Model of Money Demand. *Macroeconomic Dynamics*, 18(02):438–472, September 2014.
- [12] J. Chiu and T. Koepl. Trading Dynamics with Adverse Selection and Search: Market Freeze, Intervention and Recovery. Technical report, Bank of Canada Working Paper, 2011.
- [13] M. Dong. Inflation and Unemployment in Competitive Search Equilibrium. *Macroeconomic Dynamics*, 15(S2):252–268, 2011.
- [14] D. Duffie, N. Garleanu, and L. H. Pedersen. Over-the-Counter Markets. *Econometrica*, 73:1815–1847, 2005.

- [15] D. Duffie, N. Garleanu, and L. H. Pedersen. Valuation in Over-the-Counter Markets. *Review of Financial Studies*, 20(6):1865–1900, July 2007.
- [16] L. Ferraris and M. Watanabe. Collateral Secured Loans in a Monetary Economy. *Journal of Economic Theory*, 143(1):405–424, November 2008.
- [17] L. Ferraris and M. Watanabe. Collateral Fluctuations in a Monetary Economy. *Journal of Economic Theory*, 146(5):1915–1940, September 2011.
- [18] I. Fisher. *The Theory of Interest*. Kelley, 1965.
- [19] M. Friedman. *The Optimum Quantity of Money*. Transaction Publishers, 2005.
- [20] A. Geromichalos and L. Herrenbrueck. Monetary Policy, Asset Prices, and Liquidity in Over-the-Counter Markets. *Working Papers, University of California, Davis, Department of Economics*, 2012.
- [21] G. Gorton and G. Ordonez. The Supply and Demand for Safe Assets. 2013.
- [22] G. Gorton and G. Ordonez. Collateral Crises. *American Economic Review*, 104(December 2011):343–378, 2014.
- [23] C. He, R. Wright, and Y. Zhu. Housing and Liquidity. In *forthcoming, Review of Economic Dynamics*. Society for Economic Dynamics, 2014.
- [24] P. He, L. Huang, and R. Wright. Money and Banking in Search Equilibrium. *International Economic Review*, 46(2):637–670, 2005.
- [25] P. He, L. Huang, and R. Wright. Money, Banking, and Monetary Policy. *Journal of Monetary Economics*, 55(6):1013–1024, 2008.
- [26] T. Hu and G. Rocheteau. On the Coexistence of Money and Higher-return Assets and its Social Role. *Journal of Economic Theory*, 148(6):2520–2560, 2013.
- [27] T. Hu and G. Rocheteau. Monetary Policy and Asset Prices : A Mechanism Design Approach. *Journal of Money, Banking, and Credit, forthcoming*, 2014.
- [28] E. Kalai. Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons. *Econometrica*, 45:1623–1630, 1977.
- [29] E. Kalai and M. Smorodinsky. Other Solutions to Nash’s Bargaining Problem. *Econometrica*, pages 513–518, 1975.
- [30] A. Khan. The Role of Segmented Markets in Monetary Policy. *Business Review*, (Q4):1–8, 2006.

- [31] N. Kiyotaki and J. Moore. Credit Cycles. *Journal of Political Economy*, 105:211, 1997.
- [32] N. Kiyotaki and J. Moore. Liquidity and Asset Prices. *International Economic Review*, 46(2):317–349, 2005.
- [33] N. Kiyotaki and R. Wright. On Money as a Medium of Exchange. *Journal of Political Economy*, pages 927–954, 1989.
- [34] N. Kiyotaki and R. Wright. A Search-Theoretic Approach to Monetary Economics. *American Economic Review*, 83:63–77, 1993.
- [35] N. Kocherlakota. Societal Benefits of Illiquid Bonds. *Journal of Economic Theory*, 108:179–193, 2003.
- [36] R. Lagos and G. Rocheteau. Liquidity in Asset Markets With Search Frictions. *Econometrica*, 77(2):403–426, 2009.
- [37] R. Lagos, G. Rocheteau, and P. O. Weill. Crises and Liquidity in Over-the-Counter Markets. *Journal of Economic Theory*, 146:2169–2205, 2011.
- [38] R. Lagos, G. Rocheteau, and R. Wright. The Art of Monetary Theory : A New Monetarist Perspective. *forthcoming, Journal of Economic Literature*, 2014.
- [39] R. Lagos and R. Wright. A Unified Framework for Monetary Theory and Policy Analysis. *Journal of Political Economy*, 113:463–484, 2005.
- [40] R. Lagos and S. Zhang. A Model of Monetary Exchange in Over-the-Counter Markets. 2013.
- [41] Y. Li and Y. Li. Liquidity and Asset Prices: a New Monetarist Approach. *Journal of Monetary Economics*, 60(4):426–438, 2013.
- [42] F. Mattesini and E. Nosal. Cash-in-the-Market Pricing in a Model with Money and Over-the-Counter Financial Markets. 2013.
- [43] R. Mundell. Inflation and Real Interest. *Journal of Political Economy*, pages 280–283, 1963.
- [44] E. Nosal and G. Rocheteau. *Money, Payments, and Liquidity*. MIT press, 2011.
- [45] E. Nosal and G. Rocheteau. Pairwise Trade, Asset Prices, and Monetary Policy. *Journal of Economic Dynamics and Control*, 37(1):1–17, 2013.
- [46] G. Rocheteau, P. Rupert, K. Shell, and R. Wright. General Equilibrium with Non-convexities and Money. *Journal of Economic Theory*, 142(1):294–317, 2008.



- [47] G. Rocheteau and R. Wright. Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium. *Econometrica*, 73(1):175–202, 2005.
- [48] G. Rocheteau, R. Wright, and S. Xiao. Open Market Operations. 2014.
- [49] A. Rubinstein and A. Wolinsky. Middlemen. *Quarterly Journal of Economics*, 102:581–594, 1987.
- [50] D. Sanches and S. Williamson. Money and Credit with Limited Commitment and Theft. *Journal of Economic Theory*, 145(4):1525–1549, 2010.
- [51] R. Silveira and R. Wright. Search and the Market for Ideas. *Journal of Economic Theory*, 145(4):1550–1573, 2010.
- [52] J. Tobin. Money and Economic Growth. *Econometrica*, pages 671–684, 1965.
- [53] V Venkateswaran and R Wright. Pledgability and Liquidity: A New Monetarist Model of Financial and Macroeconomic Activity. 2013.
- [54] S. Williamson. Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach. *American Economic Review*, 102(6):2570–2605, 2012.
- [55] S. Williamson. Central Bank Purchases of Private Assets. *Federal Reserve Bank of St. Louis Working Paper Series*, (2014-026), 2014.
- [56] S. Williamson. Scarce Collateral, the Term Premium, and Quantitative Easing. *FRB of St. Louis Working Paper No*, 2014.
- [57] S. Williamson and R. Wright. New Monetarist Economics: Models. In *Handbook of Monetary Economics*, volume 3, pages 25–96. Elsevier B.V., 2010.
- [58] R. Wright and Y. Wong. Buyers, Sellers, and Middlemen: Variations on Search-theoretic Themes. *International Economic Review*, 55(2):375–397, 2014.
- [59] T. Zhu and N. Wallace. Pairwise Trade and Coexistence of Money and Higher Return Assets. *Journal of Economic Theory*, 133(1):524–535, 2007.