Multiproduct Pricing Made Simple

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Abstract

We study pricing by multiproduct firms in the context of unregulated monopoly, regulated monopoly and Cournot oligopoly. Using the concept of consumer surplus as a function of quantities (rather than prices), we present simple formulas for optimal prices and show that Cournot equilibrium exists and corresponds to a Ramsey optimum. We then present a tractable class of demand systems that involve a generalized form of homothetic preferences. As well as standard homothetic preferences, this class includes linear and logit demand. Within the class, profit-maximizing quantities are proportional to efficient quantities. We discuss cost-passthrough, including cases where optimal prices do not depend on other products' costs. Finally, we discuss optimal monopoly regulation when the firm has private information about its vector of marginal costs, and show that if the probability distribution over costs satisfies an independence property, then optimal regulation leaves relative price decisions to the firm.

Keywords: Multiproduct pricing, homothetic preferences, Cournot oligopoly, monopoly regulation, Ramsey pricing, cost passthrough, multidimensional screening.

JEL Classification: D42, D43, D82, L12, L13, L51.

1 Introduction

The theory of multiproduct pricing is a large and diverse subject. Unlike the single-product case, a multiproduct firm must decide about the structure of its relative prices as well as its overall price level. Classical questions include the characterization of optimal pricing by a multiproduct monopolist seeking to maximize profit—or, as with Ramsey pricing, the most efficient way to generate a specified level of profit—when its choice for one price must

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take into account its impact on demand for other products. Additional complexities arise in oligopoly, when a multiproduct firm needs to choose prices to reflect both intra-firm substitution (or complementarity) features and inter-firm interactions. Optimal regulation of a multiproduct firm with private information about its costs, say, must take into account not just its likely average cost across all products but its pattern of relative costs.

In this paper we show how these issues can be illuminated by studying consumer preferences in terms of consumer surplus considered as a function of quantities (rather than the more familiar function of prices).[1] In section 2 we show how profit-maximizing and other Ramsey prices, as well as prices in symmetric Cournot equilibrium, can be expressed as a markup over marginal costs proportional to the derivative of this surplus function. In particular, a product's optimal price is below marginal cost when consumer surplus decreases with the supply of this product. We also show how a Cournot equilibrium corresponds to an appropriate Ramsey optimum, and *vice versa*, which enables us to construct and demonstrate existence of Cournot equilibrium in many cases.

A well-known feature of Ramsey pricing is that when required departures of optimal quantities from efficient quantities are small, then optimal quantities are approximately proportional to the efficient quantities. Thus, a reasonable rule of thumb is often to scale down quantities equiproportionately relative to efficient quantities, rather than to increase prices equiproportionately above marginal cost. For larger departures of prices from costs, though, optimal quantities are generally not proportional to efficient quantities. In section 3, we specialise the demand system so that consumer surplus is a *homothetic* function of quantities, which implies that relative quantities (or relative price-cost markups) do not depend on the weight placed on profit in the Ramsey objective. As shown in section 3.2, this is quite a flexible class of demand systems (much broader than the class where consumer surplus is homothetic in prices), and as well as the obvious case of gross utility being homothetic in quantities it includes linear and logit demands.

In section 3.3, we show that this property, together with assuming constant returns to scale, simplifies the analysis by allowing a multiproduct problem to be decomposed into two steps: first calculate the efficient quantities which correspond to marginal cost pricing, and second solve for the scale factor by which to reduce the efficient quantities. This simplifies

[1]Regarding consumer surplus as a function of quantities is apparently uncommon in the literature. However, in the single-product context Bulow and Klemperer (2012) show this to be a valuable perspective. (They regard consumer surplus as the area between the demand curve and the marginal revenue curve, which is the same thing.)

comparative static analysis, such as how monopoly (and oligopoly) prices vary with cost parameters. In some leading examples there is zero "cross-cost" passthrough—e.g., the most profitable price of each product depends only on its own cost—and more generally there are simple formulas for the size and sign of cross-cost effects.

The fact that a profit-maximizing firm has efficient incentives with respect to the pattern, though of course not the level, of quantities has implications, moreover, for regulation of multiproduct monopoly (section 3.4). It suggests that, in our class of demand systems, it might be optimal for regulation to allow the monopolist considerable discretion over the pattern of relative quantities (or prices). If the probability distribution over costs is such that relative costs and average costs are stochastically independent, this intuition is precisely correct and it is optimal for the choice of relative quantities to be delegated to the firm.

Related literature: Baumol and Bradford (1970), and the many references therein, discuss the economic principles of Ramsey pricing. They suggest (p. 271) that it is plausible that "the damage to welfare resulting from departures from marginal cost pricing will be minimized if the *relative* quantities of the various products sold are kept unchanged from their marginal cost pricing proportions." One aim of our paper is to make this intuition precise in a broad class of demand systems.

Gorman (1961) described a class of utility functions such that income expansion paths (or Engel curves) for quantities demanded were linear. This resembles our class of utility functions, for which Ramsey quantities are equiproportional; that is, where the quantity vector which maximizes consumer utility subject to a profit constraint expand linearly as the profit requirement is relaxed. Gorman's preference family was such that the consumer's expenditure function took the form, $e(p,u) = a(p) + ub(p)$, where a and b are homogeneous degree 1, while Proposition 2 below shows that our family has gross utility of the form $u(x) = h(x) + g(q(x))$ where h and q are homogeneous degree 1.

Cournot oligopoly is studied in a rich literature on single-product firms—see Vives (1999, chapter 4) for an overview of existence, uniqueness and comparative statics of Cournot equilibria. Sometimes a Cournot oligopoly operates as if it maximizes an objective. Bergstrom and Varian (1985) observe that a symmetric oligopoly maximizes a Ramsey objective, while Slade (1994) and Monderer and Shapley (1996) note that oligopolists sometimes maximize a more abstract "potential function". This is useful as it converts the fixed-point problem of calculating equilibrium quantities into a simpler optimization problem. In Proposition 1 we extend this analysis to cover multiproduct cases and, like Bergstrom and Varian (1985), show that oligopolists maximize an appropriate Ramsey objective.

Weyl and Fabinger (2013) discuss the passthrough of costs to prices and its various applications in settings of monopoly and imperfect competition with single-product firms. Within the marketing literature on retailing, a major theme is the extent to which wholesale cost shocks (such as temporary promotions) are passed through into retail prices. Besanko *et al.* (2005) empirically examine the patterns of cost passthrough in a large supermarket chain. They find that own-cost proportional passthrough is more than 60% for most product categories (and sometimes more than 100%), while cross-cost passthrough can take either sign. Moorthy (2005) analyzes a theoretical model where two retailers compete to supply two products to consumers, and as well as cost passthrough within a retailer he discusses how cost shock to the rival affects firm's prices. The sign of most of the passthrough effects depends in an opaque way on the features of various matrices. In section 3.3, our demand system yields some relatively simple multiproduct passthrough relationships.

The optimal regulation of multi-product monopoly is analyzed by Laffont and Tirole (1993, chapter 3). In their main model, cost outturns are observable but the regulator cannot observe cost-reducing effort or the firm's underlying cost type. If the cost function is separable between quantities on the one hand and the firm's effort and type on the other, then the "incentive-pricing dichotomy" holds—pricing should not be used to provide effort incentives. If there is a social cost of public funds, Ramsey pricing is therefore optimal, as characterized by "super-elasticity" formulas for markups. The analysis of regulation in section 3.4 below does not consider effort incentives, but is for the situation studied by Baron and Myerson (1982) where the regulator cannot observe the firm's costs. We extend this model to cover multiproduct situations where a vector of marginal costs is unobserved by the regulator. Building on the approach in Armstrong (1996) and Armstrong and Vickers (2001), we describe a tractable class of situations in which it is optimal to control only the firm's average output, leaving it free to choose relative outputs to reflect its relative costs.

2 A general analysis of multiproduct pricing

Suppose there are $n \geq 2$ products, where the quantity of product i is denoted x_i and the vector of quantities is $x = (x_1, ..., x_n)$. Consumers have quasi-linear utility, which implies that demand can be considered to be generated by a single representative consumer with gross utility function $u(x)$ defined on a (full-dimensional) convex region $R \subset \mathbb{R}^n_+$ which includes zero, where $u(0) = 0$ and u is increasing and concave. (We might have $R = \mathbb{R}^n_+$, so that utility is defined for all non-negative quantity bundles.) We suppose that u is twice continuously differentiable in the interior of R, although marginal utility might be unbounded as some quantities tend to zero.

Faced with price vector $p = (p_1, ..., p_n)$, the consumer chooses quantities $x \in R$ to maximize $u(x) - p \cdot x$. (Here, $a \cdot b \equiv \sum_{i=1}^n a_i b_i$ denotes the dot product of two vectors a and b.) The price vector which induces interior quantity vector $x \in R$ to be demanded, i.e., the inverse demand function $p(x)$, is

$$p(x) \equiv \nabla u(x) \ ,$$

where we use the gradient notation $\nabla f(x) \equiv (\partial f(x)/\partial x_1, ..., \partial f(x)/\partial x_n)$ for the vector of partial derivatives of a function f. To ensure we can invert $p(x)$ to obtain the demand function $x(p)$, assume that the matrix of second derivatives of u, which we write as $Dp(x)$, is non-singular (and hence negative-definite). The revenue generated from quantity vector x is

$$r(x) = x \cdot \nabla u(x) \ ,$$

while the surplus retained by the consumer from x is

$$s(x) \equiv u(x) - r(x) = \left. \frac{d}{dk} k u(x/k) \right|_{k=1} \ . \tag{1}$$

One of this paper's aims is to show the usefulness of the function $s(x)$ for analyzing multiproduct pricing.

We next discuss some features of $s(x)$. First, the right-hand side of expression (1) shows that $s(\cdot)$ is related to the elasticity of scale of $u(\cdot)$ evaluated at x, and a more concave u allows the consumer to retain more surplus.[2] Second, the derivative of s can be expressed as

$$\nabla s(x) = \left. \frac{d}{dk} p(x/k) \right|_{k=1} \ , \tag{2}$$

[2] If all quantities x are increased by 1 per cent, then u increases by $(1 - s(x)/u(x))$ per cent.

so that an equiproportionate contraction in quantities x moves the price vector in the direction $\nabla s(x)$, i.e., normal to the surface $s(x) = constant$. To see (2), note that

$$\frac{\partial}{\partial x_i} s(x) = \frac{\partial}{\partial x_i}[u(x) - r(x)] = -\sum_j x_j \frac{\partial}{\partial x_i} p_j(x) = -\sum_j x_j \frac{\partial}{\partial x_j} p_i(x) = \left. \frac{d}{dk} p_i(x/k) \right|_{k=1} ,$$

where the third equality follows from the symmetry of cross-derivatives of $p(x)$. Unlike consumer surplus expressed as a function of prices—which is necessarily a decreasing function of prices—here $s(x)$ can increase or decrease with x_i.[3] From (2), a sufficient condition for s to increase with x_i is that p_i decrease with all x_j, which is the case if products are gross substitutes (see Vives (1999, section 6.1)). Note, though, that the above expressions imply that for $x \neq 0$ we have

$$\left. \frac{d}{dk} s(kx) \right|_{k=1} = -x \cdot Dp(x) \cdot x > 0 , \tag{3}$$

where the inequality follows from the matrix $Dp(x)$ being negative-definite. Thus consumer surplus increases as all quantities are increased equiproportionately.

The Ramsey monopoly problem: Now suppose that these products are supplied by a monopolist with differentiable cost function $c(x)$. To sidestep issues of fixed costs and the potential undesirability of producing at all, both with monopoly and in the later analysis of Cournot oligopoly, we suppose that

$$c(0) = 0 \text{ and } c(x) \text{ is convex.} \tag{4}$$

Consider the Ramsey problem of choosing quantities to maximize a weighted sum of profit and consumer surplus. If $\alpha \leq 1$ is the relative weight on consumer surplus, the Ramsey objective is

$$[r(x) - c(x)] + \alpha s(x) = u(x) - c(x) - (1 - \alpha)s(x) . \tag{5}$$

This includes as polar cases profit maximization ($\alpha = 0$) and total surplus maximization ($\alpha = 1$). Standard comparative statics shows that optimal consumer surplus, $s(x)$, in this Ramsey problem weakly increases with α, while optimal profit $[r(x) - c(x)]$ weakly decreases with α. Total surplus is maximized at quantities x^w which involve prices equal to marginal costs, so that $p(x^w) = \nabla c(x^w)$, and assumption (4) ensures that the firm breaks

[3] Likewise, while consumer surplus is necessarily convex as a function of prices, even in the single-product case it is ambiguous whether $s(x)$ is convex or concave (or neither) as a function of quantities.

even with marginal-cost pricing. More generally, the Ramsey problem with weight α has first-order condition for optimal quantities given by

$$p(x) = \nabla c(x) + (1 - \alpha)\nabla s(x) \ . \tag{6}$$

Thus, when $\alpha < 1$ the optimal departure of price from marginal cost is proportional to $\nabla s(x)$. In particular, the Ramsey price for product i is above its marginal cost if surplus $s(x)$ increases with x_i at optimal quantities, while using the product as a "loss leader" is optimal if $s(x)$ decreases with x_i. As we will see later, there are also natural cases where s depends only on the quantities of a subset of products, in which case (6) indicates that the remaining products should be priced at marginal cost. Thus, the function s succinctly determines when it is optimal to set a price above, below, or equal to marginal cost.[4]

When α is close to 1 then choosing $x = \alpha x^w$ approximately solves the Ramsey problem (5) when the cost function $c(x)$ is homogeneous degree 1. To see this, note that when $\alpha \approx 1$ expression (2) implies

$$p(\alpha x) - p(x) \approx (1 - \alpha)\nabla s(x) \ , \tag{7}$$

in which case

$$\begin{aligned} p(\alpha x^w) &\approx p(x^w) + (1-\alpha)\nabla s(x^w) \\ &= \nabla c(x^w) + (1-\alpha)\nabla s(x^w) \\ &= \nabla c(\alpha x^w) + (1-\alpha)\nabla s(x^w) \\ &\approx \nabla c(\alpha x^w) + (1-\alpha)\nabla s(\alpha x^w) \ , \end{aligned}$$

so that $x = \alpha x^w$ approximately satisfies condition (6). (Here, the first strict equality follows from the efficiency of quantities x^w while the second follows from the homogeneity of $c(\cdot)$.) In sum, in the Ramsey problem with constant returns to scale and $\alpha \approx 1$, the efficient quantities should be scaled back equiproportionately by the factor α.

Without making further assumptions, there is little reason to expect that this insight for $\alpha \approx 1$ extends to the situation where a monopolist maximizes profit ($\alpha = 0$), and in general profit-maximizing quantities are not proportional to efficient quantities. To

[4] Expression (6) is an alternative—and arguably more transparent—formulation of the insight in Baumol and Bradford (1970, section VIII) that the gap between price and marginal cost should be proportional to the gap between marginal revenue and marginal cost, where "marginal revenue" takes into account how increasing the supply of one product affects prices for other products.

illustrate, the bold curve on Figure 1 depicts Ramsey quantity vectors as the weight on consumer surplus varies from $\alpha = 0$ to $\alpha = 1$.[5] As shown, these Ramsey quantities are the vectors—the "contract curve" between consumers and the firm—where iso-profit contours (which are curves centred on profit-maximizing quantities) are tangent to iso-welfare contours (centred on efficient quantities). As discussed above, when $\alpha \approx 1$ optimal quantities are approximately proportional to efficient quantities, and so when $\alpha = 1$ the bold line is tangent to the dashed ray from the origin.

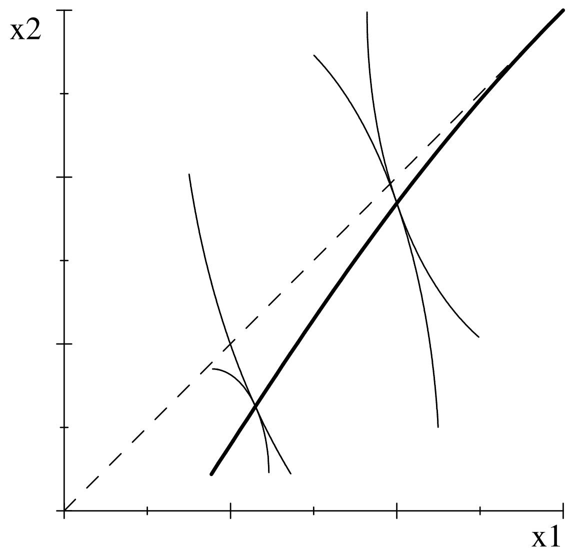

Figure 1: Ramsey quantities as α varies from 0 to 1

Cournot oligopoly: A natural development of our framework is to the Cournot oligopoly setting with m symmetric firms that each supply the full range of products and have the cost function $c(\cdot)$ satisfying (4). Our main result in this context is that equilibrium in this m-player game is closely related to an appropriate Ramsey optimum. Assumption (4) implies that the least-cost way for the industry to supply total quantity vector x is to split this quantity equally between the m firms so that total cost is $mc(\frac{1}{m}x)$. In this case the Ramsey objective (5) becomes

$$u(x) - mc(\tfrac{1}{m}x) - (1-\alpha)s(x) \ , \tag{8}$$

so that the corresponding first-order condition for the optimal vector of total quantity x is

$$p(x) = \nabla c(\tfrac{1}{m}x) + (1-\alpha)\nabla s(x) \ . \tag{9}$$

[5]The figure depicts for $x_1, x_2 \geq \frac{2}{5}$ the example where $u(x) = x_1 + x_2 - \frac{1}{3}(x_1)^3 - \frac{2}{3}(x_2)^{\frac{3}{2}}$ and $c(x) = 0$.

Consider a candidate symmetric equilibrium in which each firm supplies quantity vector $\frac{1}{m}x$ (so that x is total supply). Then a firm must maximize its profit

$$\pi(y) = y \cdot p(\tfrac{m-1}{m}x + y) - c(y)$$

by choosing $y = \frac{1}{m}x$, which from expression (2) has the first-order condition

$$p(x) = \nabla c(\tfrac{1}{m}x) + \tfrac{1}{m}\nabla s(x)\ . \tag{10}$$

Comparing with (10) with (9) reveals that a symmetric Cournot equilibrium, if it exists, has the same first-order conditions as the Ramsey problem (8) when the weight on consumers is $\alpha = \frac{m-1}{m}$.

The following result establishes the existence and symmetry of Cournot equilibrium:

Proposition 1 *Suppose there are m Cournot competitors, each of which supplies all n products and has the same cost function $c(x)$ satisfying (4). Then there exists a symmetric Cournot equilibrium in which quantities maximize the Ramsey objective (8) with $\alpha = \frac{m-1}{m}$. There are no asymmetric equilibria. If in addition $r(x)$ is concave there is only one symmetric equilibrium.*

Proof. We first rule out asymmetric equilibria. Consider a (possibly asymmetric) candidate equilibrium in which firm j supplies quantity vector x^j, where $x = \Sigma_j x^j$ is the total supply. At an asymmetric equilibrium, at least one firm must have $x^j \neq \frac{1}{m}x$. In this equilibrium firm j must maximize its profit

$$\pi_j(y) = y \cdot p(\Sigma_{i \neq j} x^i + y) - c(y)$$

by choosing $y = x^j$. In particular, it cannot be profitable to deviate from supplying $y = x^j$ to supplying $y = x^j + \varepsilon(x - mx^j)$, where ε is a scalar. Evaluating the derivative of $\pi_j(x^j + \varepsilon(x - mx^j))$ with respect to ε at $\varepsilon = 0$ therefore yields

$$\begin{aligned} 0 &= [p(x) - \nabla c(x^j) + x^j \cdot Dp(x)] \cdot (x - mx^j) \\ &= [p(x) - \nabla c(x^j) - \tfrac{1}{m}(x - mx^j - x) \cdot Dp(x)] \cdot (x - mx^j) \\ &\geq [p(x) - \nabla c(x^j) + \tfrac{1}{m}x \cdot Dp(x)] \cdot (x - mx^j)\ , \end{aligned} \tag{11}$$

where the inequality (11) follows from the negative-definiteness of the matrix $Dp(x)$. This inequality is strict if $x^j \neq \frac{1}{m}x$, which is the case for some firm in an asymmetric equilibrium, and so summing (11) across the m firms we obtain

$$0 > -m\sum_j(\tfrac{1}{m}x - x^j)\cdot\nabla c(x^j) \geq m\sum_j[c(x^j) - c(\tfrac{1}{m}x)] \geq 0 \ , \tag{12}$$

which is a contradiction. To see the second inequality in (12), note that the convexity of $c(\cdot)$ implies $c(\frac{1}{m}x) - c(x^j) \geq (\frac{1}{m}x - x^j)\cdot\nabla c(x^j)$, while the third inequality in (12) follows directly from the convexity of $c(\cdot)$. We deduce that x cannot be equilibrium supply unless it is symmetrically shared between firms.

Turning to equilibrium existence, note that the Ramsey objective (8) can be written $(1-\alpha)r(x)+\alpha u(x)-mc(\frac{1}{m}x)$. Suppose that quantity vector x solves this Ramsey problem when $\alpha = \frac{m-1}{m}$. Since c is convex, it follows that choosing $x^j = \frac{1}{m}x$ for each j maximizes the function

$$(1-\alpha)r(\Sigma_j x^j) + \alpha u(\Sigma_j x^j) - \Sigma_j c(x^j) \ .$$

In particular, choosing $y = \frac{1}{m}x$ maximizes the function

$$\rho(y) \equiv \tfrac{1}{m}r(\tfrac{m-1}{m}x + y) + \tfrac{m-1}{m}u(\tfrac{m-1}{m}x + y) - c(y) \ .$$

Now consider Cournot competition and a firm's best response when its rivals each supply quantity vector $\frac{1}{m}x$. This firm chooses its quantity vector y to maximize its profit

$$\begin{aligned}
\pi(y) &\equiv y\cdot p(\tfrac{m-1}{m}x + y) - c(y) \\
&= \rho(y) - \tfrac{m-1}{m}\left\{u(\tfrac{m-1}{m}x + y) + (\tfrac{1}{m}x - y)\cdot p(\tfrac{m-1}{m}x + y)\right\} \\
&\leq \rho(y) - \tfrac{m-1}{m}u(x) \ ,
\end{aligned}$$

where the inequality follows from the concavity of u. Since $\pi(\frac{1}{m}x) = \rho(\frac{1}{m}x) - \frac{m-1}{m}u(x)$, we therefore have

$$\pi(\tfrac{1}{m}x) - \pi(y) \geq \rho(\tfrac{1}{m}x) - \rho(y) \geq 0$$

where the final inequality follows since $y = \frac{1}{m}x$ maximizes $\rho(y)$. We deduce it is a Cournot equilibrium for each firm to supply $\frac{1}{m}x$.

Finally, consider the uniqueness of equilibrium. We have already shown there are no asymmetric equilibria, while expression (10) shows that any symmetric equilibrium satisfies the first-order conditions for maximizing the Ramsey objective (8) with $\alpha = \frac{m-1}{m}$. This Ramsey objective can be written as $\frac{1}{m}r(x)+\frac{m-1}{m}u(x)-mc(\frac{1}{m}x)$, and given that u is strictly concave and c is convex, this is strictly concave if $r(x)$ is concave. In this case, there is a unique quantity vector x which satisfies the first-order condition (10), and hence a unique symmetric equilibrium. ■

Thus, with symmetric convex cost functions there are no asymmetric Cournot equilibria, and there exists a symmetric Cournot equilibrium in which total quantities maximize the Ramsey objective (8) with $\alpha = \frac{m-1}{m}$. If revenue r is concave, there is a unique Cournot equilibrium, which coincides with the (unique) optimum for the Ramsey objective (8) with $\alpha = \frac{m-1}{m}$. In this sense, the Cournot problem and the (appropriately weighted) Ramsey problem are the same. This generalizes the second remark in Bergstrom and Varian (1985)—that a symmetric single-product Cournot oligopoly can be considered to maximize a Ramsey objective—to the multiproduct context.

When $c(x)$ is homogeneous degree 1, the number of suppliers has no impact on industry costs and the Ramsey objectives (5) and (8) coincide. In this case, since consumer surplus in the Ramsey problem (5) increases, and profit decreases, with α, we deduce that as the number of competitors increases, a symmetric Cournot equilibrium delivers more surplus to consumers and involves lower industry profit, and with many firms the equilibrium quantities are approximately $\frac{m-1}{m}x^w$ (where x^w is the efficient quantity vector). One can also study how equilibrium prices depend on marginal costs by studying the simple Ramsey problem, as we do below in section 3.3.

The analysis in Proposition 1 assumes firms are symmetric. Among other issues, this assumption means one cannot study the impact of firm-specific cost shocks, for instance, but only industry-wide cost shocks. When Cournot equilibria exist in asymmetric settings it is straightforward to obtain first-order conditions for equilibrium prices. For example, suppose that each firm has constant marginal costs, and firm j has the marginal cost vector $c^j = (c_1^j, ..., c_n^j)$. Then if all firms supply all products in equilibrium, an argument similar to (10) shows that equilibrium prices satisfy

$$p(x) = \tfrac{1}{m}\Sigma_j c^j + \tfrac{1}{m}\nabla s(x) \tag{13}$$

where $\frac{1}{m}\Sigma_j c^j$ is the industry *average* vector of marginal costs.[6] Thus, the Cournot equilibrium here corresponds to a the Ramsey optimum with weight on consumers $\alpha = \frac{m-1}{m}$ and

[6] This generalizes the first remark in Bergstrom and Varian (1985)—that equilibrium industry output depends only on the average marginal cost in the industry, not its distribution—to multiple products. One can show that this Cournot equilibrium exists if (i) inverse demands $p_i(x)$ are each weakly concave and (ii) that the cost vectors c^j are "close enough" that each firm supplies all products in equilibrium.

a hypothetical monopolist with cost function $c(x) = \frac{1}{m}x \cdot [\Sigma_j c^j]$. Another way to allow for firm asymmetries is discussed in the following Bertrand model.

Bertrand oligopoly: Although it is not the focus of this paper, consider briefly one way to model Bertrand competition in this framework. Bliss (1988) and Armstrong and Vickers (2001, section 2) suggested a model where consumers buy all products from one firm or another, so there is one-stop shopping, and firms therefore compete in terms of the surplus they offer their customers. Each consumer has the same gross utility, $u(x)$, when they purchase quantity vector x from a firm, and this utility function is the same at all firms. Firms compete by offering linear prices, so that a consumer obtains surplus $s(x)$ when they buy quantity x from a firm via linear prices, while firm i, say, obtains profit $r(x) - c_i(x)$ from each customer where $c_i(x)$ is this firm's constant-returns-to-scale cost function. (Unlike the previous Cournot model, here it is straightforward to allow firms to have different cost functions.) Consumers differ in their brand preferences for the various firms, say due to the distances they must travel to reach them, and the number of customers a firm attracts increases with the surplus s it offers and decreases with the surplus its rivals offer.

In this framework, each firm's strategy can be broken down into two steps: (i) choose the most profitable way to deliver a given surplus to a customer, and (ii) choose how much surplus to offer its customers. Step (i) is just the Ramsey problem as discussed above, and a firm's optimal prices take the form (6) where α now will reflect the firm's competitive constraints in step (ii) rather than concern for consumer welfare. In equilibrium there is intra-firm efficiency, but with cost differences across firms there will not in general be industry-wide efficiency (i.e., industry profits are not maximized subject to an overall consumer surplus constraint).

The general analysis in this section has introduced the consumer surplus function $s(x)$ and shown its usefulness in analyzing the Ramsey monopoly problem and, by extension, the symmetric Cournot oligopoly problem. In the rest of the paper we develop the analysis of monopoly and oligopoly by supposing that s is homothetic in x. This specification includes a number of familiar multiproduct demand systems, and has notably convenient properties. In particular, the feature of equiproportionate quantity reduction that appeared locally (for $\alpha \approx 1$ in the Ramsey problem or large m in the Cournot equilibrium) in the analysis above, holds globally.

3 A family of demand systems

3.1 Homothetic consumer surplus

The family of demand systems on which we now focus is characterized by the property that consumer surplus $s(x)$ is a homothetic function of quantities x. We first describe which demand systems have this property:

Proposition 2 *Consumer surplus $s(x)$ is homothetic in x if and only if utility $u(x)$ can be written in the form*

$$u(x) = h(x) + g(q(x)) \tag{14}$$

where $h(\cdot)$ and $q(\cdot)$ are homogeneous degree 1 functions and $g(\cdot)$ is concave with $g(0) = 0$.

Proof. First, note that we must have $g(0) = 0$ and g concave in q given that $u(0) = 0$ and $u(\cdot)$ is concave in x. (Since u is concave, when u takes the form (14) for given x the function $k \to kh(x) + g(kq(x))$ is too, so that $g(\cdot)$ is concave.)

To show sufficiency, note that (14) implies that inverse demand is

$$p(x) = \nabla h(x) + g'(q(x))\nabla q(x) \ . \tag{15}$$

Revenue is therefore

$$r(x) = x \cdot p(x) = h(x) + g'(q(x))q(x) \ , \tag{16}$$

where we used the fact that $x \cdot \nabla h(x) \equiv h(x)$ for a homogenous degree 1 function. Consumer surplus $s(x)$ is then

$$s(x) = g(q(x)) - g'(q(x))q(x) \ , \tag{17}$$

which is homothetic since $s(x)$ is an increasing function of the homogenous function $q(x)$. (Since g is concave, $g(q) - g'(q)q$ is an increasing function.)

To show necessity, suppose that consumer surplus $s(x)$ is homothetic, so that $s(x) \equiv G(q(x))$ for some increasing function G and some function $q(x)$ which is homogeneous degree 1. We can write G as $G(q) \equiv g(q) - qg'(q)$ for another function $g(\cdot)$.[7] Then

$$s(\tilde{x}/k) = g(q(\tilde{x})/k) - \frac{q(\tilde{x})}{k} g'(q(\tilde{x})/k) = \frac{d}{dk} kg(q(\tilde{x})/k) \ . \tag{18}$$

[7] Given any function $G(\cdot)$, one can generate the corresponding $g(\cdot)$ using the procedure

$$g(q) = -q \int^q [G(\tilde{q})/\tilde{q}^2] d\tilde{q} \ .$$

This function $g(q)$ is concave given that $G(q)$ is increasing.

Note that (1) can be generalized slightly so that $s(\tilde{x}/k) = \frac{d}{dk} k u(\tilde{x}/k)$, and so (18) can be integrated to yield

$$ku(\tilde{x}/k) = h(\tilde{x}) + kg(q(\tilde{x})/k)$$

for some constant of integration $h(\tilde{x})$. Writing $x = \tilde{x}/k$ this becomes

$$u(x) = \frac{h(kx)}{k} + g(q(x)) \ .$$

Since this holds for all k we deduce that $u(x) = h(x) + g(q(x))$, where $h(x)$ is homogeneous degree 1. ■

This result implies that the set of demand systems in which consumer surplus is homothetic in quantities is broader than that where consumer surplus is homothetic in *prices*. Expressed as a function of prices, consumer surplus is the convex function $v(p) = \max_{x \geq 0}\{u(x) - p \cdot x\}$. Duality implies that $u(x)$ can be recovered from $v(p)$ using the procedure $u(x) = \min_{p \geq 0}\{v(p) + p \cdot x\}$, and if $v(p)$ is homothetic in p then $u(x) = \min_{p \geq 0}\{v(p) + p \cdot x\}$ is homothetic in x. Thus, the utility functions such that consumer surplus is homothetic in prices are simply the homothetic utility functions, i.e., where $h \equiv 0$ in (14), which is a subset of the family of preferences we study. In section 3.2 we discuss familiar instances of the family (14) which do not have homothetic $u(\cdot)$.

For the remainder of the paper we assume that utility $u(x)$, as well as being increasing and concave, can be written in the form (14). Some immediate observations on this preference specification are:

- For a specific utility function $u(x)$ it may not be obvious *a priori* whether it accords with the form (14). However, Proposition 2 implies that this is the case whenever consumer surplus, $s(x) \equiv u(x) - x \cdot \nabla u(x)$, is homothetic, which in practice is easy to check.

- Expression (15) implies that an equiproportionate change in quantities moves the price vector along a straight line in the direction $\nabla q(x)$. In geometric terms, then, quantity vectors on the ray joining x to the origin correspond to price vectors which lie on the straight line starting at $p(x)$ pointing in the direction $\nabla q(x)$.

- If u satisfies (14), then the modified environment in which a subset of these products are removed also satisfies (14). That is, if a subset of products have quantities x_i set equal to zero, the utility function u defined on the remaining products continues to satisfy (14).

- Since g is concave, the function $g(q) - g'(q)q$ in (17) is an increasing function, so surplus s increases with x_i—and the Ramsey price for product i is above marginal cost in (6)—if and only if $q(\cdot)$ increases with x_i.
- When utility takes the form (14), the revenue function (16) takes a similar form, with the same h and q (but with $qg'(q)$ replacing $g(q)$). For this reason, a multiproduct monopolist's problem of maximizing profit—discussed below in section 3.3—is closely connected to the consumer's problem of maximizing surplus, where prices in the consumer's problem correspond to marginal costs in the firm's problem.
- There are three degrees of freedom when choosing a demand system within the class—$q(x)$, $h(x)$ and $g(q)$—and expression (14) provides a useful toolkit for constructing tractable multiproduct demand systems with particular desired properties. For this purpose it is useful to know conditions for the resulting utility function u to be concave. Sufficient conditions to ensure that u in (14) is concave are that h and g are concave <u>and</u> either: (i) q is concave and g is increasing; (ii) q is convex and g is decreasing, or (iii) q is linear in x (which allows g to be non-monotonic).

For the remainder of this subsection we discuss in more detail the implications of this utility specification for the corresponding demand system, denoted $x(p)$. Given prices p, the consumer with utility (14) can maximize her surplus with a simple two-step procedure. We can write quantities x in the form

$$x \equiv q(x) \times \frac{x}{q(x)} \ . \tag{19}$$

Here, $x/q(x)$ is homogeneous degree zero and depends only on the *ray* from the origin on which x lies, while $q(x)$ is homogeneous degree 1 and measures how far along that ray x lies, and so the decomposition (19) represents a generalized form of "polar coordinates" for the quantity vector x. (The coordinate $x/q(x)$ lies on the $(n-1)$-dimensional surface $q \equiv 1$.) Henceforth we refer to $q(x)$ as "composite" quantity and $x/q(x)$ as the "relative" quantities.

We know already that (maximized) consumer surplus, $s(x)$, depends on x only via composite quantity $q(x)$. More generally, consumer surplus with arbitrary quantities x

and prices p can be written in terms of the coordinates in (19) as

$$h(x) + g(q(x)) - p \cdot x = g(q(x)) - q(x)\frac{p \cdot x - h(x)}{q(x)} \ . \tag{20}$$

(Since the function $p \cdot x - h(x)$ is homogeneous degree 1, $(p \cdot x - h(x))/q(x)$ depends only on the relative quantities $x/q(x)$.) Since consumer surplus in (20) is decreasing in $(p \cdot x - h(x))/q(x)$, the consumer should choose relative quantities to minimize this term, regardless of her choice for composite quantity. Therefore, write

$$\phi(p) \equiv \min_{x \geq 0} : \frac{p \cdot x - h(x)}{q(x)} \ , \tag{21}$$

which is an increasing and concave function of p. The envelope theorem implies that its derivative is the optimal choice of relative quantities, so if we write $x^*(p) \equiv \nabla\phi(p)$ the consumer facing prices p chooses relative quantities $x^*(p)$.[8]

Given the relative quantities, $x^*(p)$, the optimal choice of composite quantity, say Q, is easily derived. Consumer surplus in (20) with the optimal relative quantities is the concave function $g(Q) - Q\phi(p)$. Write $\hat{Q}(\phi)$ for the composite quantity which maximizes $g(Q) - Q\phi$, which is necessarily weakly downward-sloping, so that $\hat{Q}(\phi(p))$ is the demand for composite quantity given the price vector p. Price vectors with the same $\phi(p)$ induce the consumer to choose the same composite quantity Q, and so $\phi(p)$ is the "composite" price which corresponds to composite quantity $q(x)$. Since the consumer chooses relative quantities $x^*(p)$ and composite quantity $\hat{Q}(\phi(p))$, from (19) the vector of quantities demanded at prices p is

$$x(p) = \hat{Q}(\phi(p)) \times x^*(p) \ . \tag{22}$$

Here, the function $g(\cdot)$ determines the shape the composite demand function $\hat{Q}(\cdot)$, while the functions $h(\cdot)$ and $q(\cdot)$ combine to determine the form of composite price function $\phi(\cdot)$.

Expression (22) implies that cross or own-price demand effects are

$$\frac{\partial x_i}{\partial p_j} = \hat{Q}(\phi)\phi_{ij} + \hat{Q}'(\phi)\phi_i\phi_j \ . \tag{23}$$

(Here, recall that $x^*(p) = \nabla\phi(p)$, while subscripts to ϕ denote its partial derivatives.) This is akin to the Slutsky Equation from classical demand theory. The first term in (23) represents the substitution effect while staying on the same composite quantity (or

[8] There is a unique vector of relative quantities which solves (21) provided that h and q are quasi-concave with one or both of them strictly so.

consumer surplus) contour, and the second term represents the impact of a price rise on the composite quantity demanded. This second term is negative, while the first term is negative if $j = i$ and could be positive or negative when $j \neq i$. For instance, if utility is homothetic then $\phi(p)$ is positive and homogenous degree 1, and expression (23) has the sign of $\frac{\phi\phi_{ij}}{\phi_i\phi_j} - \frac{-\phi\hat{Q}'(\phi)}{\hat{Q}(\phi)}$. Here the first term is the elasticity of substitution of demand and the second term is the elasticity of composite demand, and the relative sizes of these two elasticities determines the sign of cross-price effects.

Since inverse demand $p(x)$ in (15) induces demand x, it follows that

$$p = \nabla h(x) + g'(Q)\nabla q(x) \Rightarrow x(p) = Q\frac{x}{q(x)} \ . \tag{24}$$

In particular, for fixed Q any price vector of the form $p = \nabla h(x) + g'(Q)\nabla q(x)$ induces the same composite demand Q, and hence the same consumer surplus $s = g(Q) - Qg'(Q)$ and composite price $\phi(p) = g'(Q)$. Conversely, since demand $x(p)$ induces inverse demand p, substituting (22) into the expression for inverse demand (15), and recalling that for positive demand we have $g'(\hat{Q}(\phi)) \equiv \phi$, reveals that

$$p \equiv \nabla h(x^*(p)) + \phi(p)\nabla q(x^*(p)) \ . \tag{25}$$

(Alternatively, this expression is the first-order condition for problem (21).) Expression (25) is the analogue for prices of the change of coordinates for quantities in (19), and decomposes the price vector p into composite price, $\phi(p)$, and "relative prices" which in this context we define to be $x^*(p)$, i.e., the relative quantities which are optimal with prices p. From (24), prices which induce relative quantities x^* lie on the straight line $\phi :\mapsto \nabla h(x^*) + \phi\nabla q(x^*)$, which is not necessarily a ray from the origin, while the coordinate $\phi(p)$ determines how far along such a line the price vector lies.

3.2 Special cases

If $u(\cdot)$ is itself homothetic—for instance, if demand takes the familiar CES form—then (14) is trivially satisfied by setting $h \equiv 0$. In this case, expression (21) implies that $\phi(p)$ is homogeneous degree 1, and $x^*(p) = \nabla\phi(p)$ is homogeneous degree zero. More generally, homothetic demand is an instance of the subclass of (14) where h takes the linear form $h(x) = a \cdot x$, when ϕ in (21) is a function that is homogenous degree 1 in the "adjusted" price vector $(p - a)$.

Linear demand: Another instance of this subclass with linear $h(\cdot)$ is linear demand, where utility $u(x)$ takes the quadratic form

$$u(x) = a \cdot x - \tfrac{1}{2} x \cdot M \cdot x \tag{26}$$

for constant vector $a > 0$ and (symmetric) positive-definite matrix M. Here, inverse demands are $p(x) = a - Mx$, and utility takes the form (14) by writing $h(x) = a \cdot x$, $q(x) = \sqrt{x \cdot M \cdot x}$ and $g(q) = -\frac{1}{2}q^2$. Here, $\nabla q(x) = Mx$ and so expression (24) implies that the set of price vectors which correspond to the same relative quantities takes the form $p = a - tM \cdot x$ for scalar t, which are rays originating from the vector of choke prices a. It may be that $q(x)$ and therefore $s(x)$ decrease with x_i when off-diagonal elements of M are negative (which corresponds to products being complements).

Logit demand: Suppose that consumer demand takes the logit form

$$x_i(p) = \frac{e^{a_i - p_i}}{1 + \sum_j e^{a_j - p_j}} \ , \tag{27}$$

where $a = (a_1, ..., a_n)$ is a constant vector. It follows that inverse demand is

$$p_i(x) = a_i - \log \frac{x_i}{1 - q(x)} \ , \tag{28}$$

where $q(x) \equiv \sum_j x_j$ is total quantity. This inverse demand function (28) integrates to give the utility function

$$u(x) = a \cdot x - \sum_j x_j \log x_j - (1 - q(x)) \log (1 - q(x)) \ .$$

(As with any demand system resulting from discrete choice, the utility function is only defined on the domain $\Sigma_i x_i \leq 1$.[9]) This utility can be written in the required form (14) as

$$u(x) = \underbrace{a \cdot x + \sum_i x_i \log \frac{q(x)}{x_i}}_{h(x)} + \ g(q(x)) \ . \tag{29}$$

Here, $h(x)$ as labelled is homogenous degree 1, as is total output $q(x)$, while $g(q)$ is equal to the entropy function $g(q) = -q \log q - (1 - q) \log(1 - q)$, which is concave in $0 \leq q \leq 1$. Since $g'(q) = \log(1 - q) - \log q$, demand for composite quantity as a function of composite

[9]If one wishes only to consider non-negative prices, from (28) one should further restrict attention to quantity vectors which satisfy $x_i \leq (1 - \Sigma_j x_j) e^{a_i}$ for $i = 1, ..., n$.

price takes the logistic form.[10] With logit, as with homogeneous goods, consumer surplus is a function only of total quantity, and product differentiation is reflected separately in the $h(x)$ term. Since $\nabla q(x) \equiv (1, ..., 1)$, the set of prices which correspond to the same relative quantities takes the form $p + (t, ..., t)$, as can be seen directly from (27). This contrasts with the subclass with linear $h(x)$, where these lines were not parallel but emanated from a point. More generally, with any demand system in the subclass with linear $q(x) = b \cdot x$, the set of prices which correspond to quantity vectors on a given ray from the origin take the form of parallel straight lines.

Systems of strictly complementary products: A common situation is where consumers purchase a single unit of a "base product", and then combine this with variable quantities of one or more complementary products. For instance, a consumer may need to gain entry to a theme park before they can go on the rides (Oi, 1971), or needs to buy a printer along with a suitable quantity of ink in order to print. To illustrate how these situations sometimes fit into our framework, suppose there is a continuum of consumers indexed by scalar θ, where the type-θ consumer has gross utility $U(y) + \theta$ if she consumes quantity y of the combined service. (The following discussion also applies if y is a vector of multiple services.) Adding over the population of consumers implies that aggregate gross utility if x_1 consumers (those with the highest value of θ) each consume quantity y of combined service takes the form $x_1 U(y) + g(x_1)$ for some increasing concave function $g(\cdot)$, where g is determined by the distribution of θ. If x_2 denotes the quantity of combined service across all consumers, so that $x_2 = x_1 y$, it follows that aggregate utility in terms of the quantity vector (x_1, x_2) is

$$u(x_1, x_2) = x_1 U(x_2/x_1) + g(x_1) \ . \tag{30}$$

Clearly, this utility function fits into our family (14), where $h(x) = x_1 U(x_2/x_1)$ and composite quantity is just $q(x) = x_1$. This is another instance of the subclass with linear $q(x)$, but here $s(x)$ is a function only of x_1, the number of active consumers. The set of prices which correspond to the same relative quantities—i.e., the same usage per active consumer—are horizontal lines with p_2 constant.

[10] Anderson *et al.* (1988) have previously noted the connection between logit demand and entropy. The entropy function makes it difficult to obtain closed-form solutions for optimal prices or quantities with logit demand. However, if we modify (29) slightly so that $g(q) \propto q(1-q)$, demand for composite quantity is linear rather than logistic and explicit formulas can be obtained.

3.3 Analysis

In this section we discuss how to maximize welfare and profit, as well as calculate oligopoly outcomes, when the demand system satisfies (14) and the cost function satisfies

$$c(x) \text{ is convex and homogeneous degree 1.} \tag{31}$$

Consider again the problem of maximizing a weighted sum of profit and consumer surplus. If $0 \leq \alpha \leq 1$ is the relative weight on consumer surplus, the Ramsey objective (5) is

$$[r(x) - c(x)] + \alpha s(x) = \alpha g(q(x)) + (1-\alpha) g'(q(x)) q(x) - q(x) \frac{c(x) - h(x)}{q(x)} \ . \tag{32}$$

Here, we used the formulas (16)-(17).

As with the consumer's problem in section 3.1, the Ramsey problem can conveniently be solved by means of the change of variables (19). Expression (32) shows how the Ramsey objective can be written in terms of composite quantity $q(x)$ and the relative quantities $x/q(x)$. Expression (32) is decreasing in the term $(c(x) - h(x))/q(x)$, and so relative quantities should be chosen to minimize this term. As in (21), write

$$\kappa = \min_{x \geq 0} : \frac{c(x) - h(x)}{q(x)} \ , \tag{33}$$

which is solved by choosing relative quantities $x = x^*$, say. (Since the quantity vector which minimizes (33) is indeterminate up to a scaling factor, as in section 3.1 we normalize x^* so that $q(x^*) = 1$.[11]) We deduce that maximizing any Ramsey objective involves choosing the same relative quantities x^*, in contrast to the case depicted in Figure 1. In particular, profit-maximizing quantities ($\alpha = 0$) are proportional to the efficient quantities corresponding to marginal-cost pricing ($\alpha = 1$). That is, the unregulated firm has an incentive to choose its relative quantities in an efficient manner, and the sole inefficiency arises from it supplying too little composite quantity.

Given this choice for relative quantities, the optimal choice for composite quantity Q is easily derived. Expression (32) with relative quantities x^* is the function

$$\alpha g(Q) + (1-\alpha) g'(Q) Q - \kappa Q \ . \tag{34}$$

[11] There is a unique vector of relative quantities which solves (33) provided that $(c-h)$ is quasi-convex and q is quasi-concave with one of them strictly so. To illustrate this analysis, suppose that $q(x_1, x_2) = \sqrt{x_1 x_2}$, $h(x_1, x_2) = 0$ and $c(x_1, x_2) = \sqrt{c_1^2 x_1^2 + c_2^2 x_2^2}$. Then one can check that $x^* = (\frac{c_2}{c_1}, \frac{c_1}{c_2})$ and $\kappa = \sqrt{2 c_1 c_2}$.

(A sufficient condition for (34) to be concave in Q for all α is that "composite revenue" $g'(Q)Q$ be concave.) The vector of quantities which solves the Ramsey problem is then Qx^*, where Q maximizes expression (34). The optimal composite quantity Q increases with α and decreases with κ, and satisfies the Lerner formula

$$\frac{g'(Q) - \kappa}{g'(Q)} = (1 - \alpha)\eta(Q) \ , \tag{35}$$

where

$$\eta(Q) \equiv -\frac{Qg''(Q)}{g'(Q)} \tag{36}$$

is the elasticity of inverse demand for composite quantity.

Since relative quantities are the same in all Ramsey problems, so too are relative price-cost margins. As discussed in section 3.1, this is because an equiproportionate reduction in efficient quantities causes the price vector to move in a straight line away from the vector of marginal costs. In more detail, the optimal quantities for the Ramsey problem are Qx^*, where Q satisfies (35), and in particular let the composite quantity which maximizes total surplus (i.e., when $\alpha = 1$) be denoted Q^w, so that $g'(Q^w) = \kappa$. Then the price-cost margins in the Ramsey problem with composite quantity Q are

$$\begin{aligned} p(Qx^*) - \nabla c(Qx^*) &= p(Qx^*) - \nabla c(Q^w x^*) \\ &= p(Q^w x^*) - \nabla c(Q^w x^*) + [g'(Q) - g'(Q^w)]\nabla q(x^*) \\ &= [g'(Q) - \kappa]\nabla q(x^*) \ . \end{aligned} \tag{37}$$

(Here, the first equality follows from ∇c being homogeneous degree zero, the second follows from (15), and the final equality follows since prices equal marginal costs and $g' = \kappa$ when $Q = Q^w$.) These margins are proportional to $\nabla q(x^*)$, and shrink equiproportionately when Q is larger. Product i is used as a loss leader, in the sense that its price is below marginal cost, in each Ramsey problem when composite quantity q decreases with x_i at x^*.

By virtue of the Ramsey-Cournot result in Proposition 1, these properties extend to symmetric Cournot oligopoly. To summarise:

Proposition 3 *Suppose that utility takes the form (14) and cost takes the form (31). As more weight is placed on consumer surplus in the Ramsey problem, the composite quantity increases, the composite price decreases, each individual quantity increases equiproportionately, and each price-cost margin contracts equiproportionately. The same is true in symmetric Cournot equilibrium as the number of firms increases.*

An important special case involves constant marginal costs, so that $c(x) \equiv c \cdot x$ for a constant vector of marginal costs $c = (c_1, ..., c_n)$. In this case, κ in (33) is simply $\phi(c)$ where the function $\phi(\cdot)$ is defined in (21), while $x^* = x^*(c)$. In this context, consider how optimal prices relate to the firm's costs. This analysis is most transparent using the change of variables for prices (and costs) in (25), so that $\phi(p)$ is the composite price and $x^*(p)$ are relative prices (and similarly for the cost vector). As discussed earlier, in any Ramsey problem it is optimal to choose relative quantities equal to the relative quantities which correspond to efficient marginal-cost pricing. This immediately implies that it is optimal to choose relative prices equal to the firm's relative costs, so that $x^*(p) = x^*(c)$. The optimal markup of composite price over composite cost is then given by (35), and the optimal composite price, $\phi(p)$, decreases with the weight on consumer surplus, α, and increases with composite cost, $\phi(c)$.[12]

What does this mean for price-cost relationships: how does p_i depend on c_j? Expressions (35) and (37) imply that optimal prices satisfy

$$p - c = (1 - \alpha)\eta(Q)g'(Q) \times \nabla q(x^*(c)) \ . \tag{38}$$

Consider first the subclass where h takes the linear form $h(x) = a \cdot x$. Since optimal quantities are $Qx^*(c)$, expression (15) shows that prices satisfy

$$p - a = g'(Q) \times \nabla q(x^*(c)) \ . \tag{39}$$

Putting (38) and (39) together implies that

$$p - c = \frac{(1 - \alpha)\eta(Q)}{1 - (1 - \alpha)\eta(Q)} \times (c - a) \ . \tag{40}$$

In particular, when preferences are homothetic, so that $a = 0$, we obtain the familiar result that proportional price-cost markups are the same across products.

In the iso-elastic case where η is constant, expression (40) implies that the optimal price for product i depends only on c_i, not on any other product's cost, and so there is no "cross-cost" passthrough in prices, even though there may be substantial cross-price effects in the demand system. Moreover, provided that the consumer can obtain positive

[12]Since the profit-maximizing firm's choice of composite quantity falls with $\phi(c)$, and since consumer surplus, s, is an increasing function of composite quantity, we deduce that the firm necessarily offers a lower level of consumer surplus when unit cost c_i increases. Our family of demand systems therefore excludes the possibility explored by Edgeworth (1925) that imposing a linear tax on a product supplied by a multiproduct monopolist could reduce all of its prices.

utility with a subset of products (e.g., if $\rho > 0$ in the CES specification), the optimal price for one product is unaffected if the firm is restricted to offer a subset of products (or even just that product).[13] For instance, if u is homothetic and $g(Q) = \frac{1}{\gamma}Q^{\gamma}$, where $0 < \gamma < 1$, then $\eta \equiv 1 - \gamma$ and (40) implies that the most profitable prices (i.e.,when $\alpha = 0$) are

$$p = \frac{1}{\gamma}c \ .$$

Likewise, with linear demand we have $\eta \equiv -1$ and expression (40) implies that the profit-maximizing prices are[14]

$$p = \tfrac{1}{2}(a + c) \ . \tag{41}$$

More generally within this subclass with linear h, expression (40) and the fact that the most profitable Q decreases with each cost implies that all cross-cost passthrough terms for p_i have the same sign as $(a_i - c_i)\eta'(Q)$.[15]

Alternatively, consider the subclass where composite quantity takes the linear form $q(x) = b \cdot x$. Then (38) implies that

$$p - c = (1 - \alpha)\eta(Q)g'(Q) \times b \ . \tag{42}$$

In the example with complementary products where utility is (30) and $b = (1, 0)$, this expression implies there is marginal-cost pricing for usage ($p_2 = c_2$), and in particular changes in c_1 have no impact on p_2. However, there is cross-cost passthrough in the other direction: since the optimal Q decreases with both costs, it follows that p_1 increases or decreases with c_2 according to whether $\eta(Q)g'(Q)$ decreases or increases with Q. In the logit example utility is (29) and $b = (1, ..., 1)$, so the price-cost margin $p_i - c_i$ is the same for

[13] Shugan and Desiraju (2001) discuss these points in the context of linear demand with two products. In the context of product line pricing, Johnson and Myatt (2015) explore when it is that a firm's optimal price for one product variant can be calculated by supposing that the firm only supplies that variant. (They consider both monopoly and Cournot settings.)

[14] It usually makes sense only to consider non-negative quantities, in which case (41) is only valid if a and c are such that the optimal quantities, $x = \frac{1}{2}M^{-1} \cdot (a - c)$, are positive. In the case of substitutes, where the matrix M necessarily has all non-negative elements, a necessary condition for this is that each $a_i \geq c_i$. (When M has all non-negative elements, the operation $x \mapsto Mx$ takes $\mathbb{R}^n_+$ into itself.) However, with complements, it is possible to have all x_i positive and some $a_i - c_i$ negative. In such cases, (41) indicates that $p_i < c_i$ for those products with $a_i < c_i$.

[15] One can analyze how the optimal *quantity* supplied of one product is affected by cost changes to other products in a similar manner to the consumer demand expression (23). For instance, since profit-maximizing quantities satisfy the first-order condition $\nabla r(x) = c$, where c is the vector of constant marginal costs, there will be a dependence of one product's supply on another product's cost unless $r(x)$ is additively separable in quantities, which is (essentially) only the case if demand for one product does not depend on other prices.

each product i, and the common markup $(1-\alpha)\eta(Q)g'(Q) = (1-\alpha)/(1-Q)$, increases with $Q < 1$. Since the optimal Q decreases with each c_i, it follows that cross-cost passthrough is negative and one product's price decreases with each other product's cost.

As shown in Proposition 1, with m firms each with a cost function satisfying (31) the outcome with Cournot oligopoly coincides with the Ramsey optimum, provided we set the weight on consumer interests in the Ramsey problem equal to $\alpha = \frac{m-1}{m}$. With our demand system (14), then, expression (35) implies that equilibrium composite quantity satisfies

$$\frac{g'(Q) - \kappa}{g'(Q)} = \frac{1}{m}\eta(Q) \ ,$$

and Proposition 3 has the corollary that as the number of competitors increases composite quantity increases, composite price decreases, each individual quantity increases equiproportionately, and each price-cost margin contracts equiproportionately. In addition, when $c(x) = c \cdot x$ and $h(x) = a \cdot x$, expression (40) implies that

$$p - c = \frac{\eta(Q)}{m - \eta(Q)} \times (c - a) \ . \tag{43}$$

As in the Ramsey problem, cross-cost passthrough terms for product i have the same sign as $(a_i - c_i)\eta'(Q)$. (Here, a change in one product's cost is assumed to be industry-wide, not firm-specific.) So if $\eta(Q)$ is constant, then the equilibrium price for one product does not depend on the costs of other products, and nor is one product's price affected when only a subset of products is supplied by the industry.

Firm-specific cost shocks can be analyzed using expression (13). Thus, provided each firm supplies all products in equilibrium, when $h(x) = a \cdot x$ expression (43) continues to apply provided that the vector c is interpreted to be the industry average vector of marginal costs. To illustrate, with linear demand the equilibrium price for product i with m firms is

$$p_i = \tfrac{1}{m+1}\left(a_i + \Sigma_j c_i^j\right) \ ,$$

where c_i^j is firm j's cost for product i. Thus an increase in cost c_i^j will be passed through at rate $\frac{1}{m+1}$ to product i's price and will have no impact on prices for other products.

3.4 Regulation with asymmetric information about costs

When the demand system falls within the family (14), we have seen that the unregulated monopolist will choose an efficient pattern of relative quantities, even though it supplies too little composite quantity. This suggests that, in some circumstances at least, the optimal way to regulate market power when the firm has private information about its costs is to control only its composite quantity, leaving it free to choose relative quantities to reflect its private information.

To explore this issue, we consider optimal regulation of multiproduct monopoly by extending the analysis of the single-product case by Baron and Myerson (1982). Specifically, there is common knowledge about the demand system, which we assume satisfies (14), but the firm has private information about its costs. In particular, suppose the firm has the vector of constant marginal costs, $c = (c_1, ..., c_n)$. Optimal regulation can be analyzed by way of a "direct" scheme whereby the firm reports its cost vector, say $\tilde{c}$, to the regulator, and conditional on this report the firm is instructed to supply a vector of quantities X and receives a net transfer T funded by consumers (in addition to the usual revenue $r(X)$). The revelation principle implies that we can restrict attention to mechanisms in which the firm is given an incentive to report its cost vector truthfully, so that $\tilde{c} = c$. The regulator places weight $0 \leq \beta \leq 1$ on profit relative to consumer surplus, where profit includes the transfer T and consumer surplus includes the deduction for the transfer T.

Expression (25) implies we can decompose the cost vector c into composite costs, $\phi(c)$, and relative costs, $x^*(c)$, so that $c = \nabla h(x^*(c)) + \phi(c)\nabla q(x^*(c))$. From this perspective, the firm reports its private cost information in terms of coordinates $(\tilde{\phi}, \tilde{x}^*)$, and conditional on this report the regulator instructs it to supply composite quantity Q and relative quantities x^*.

It is useful to study first the situation where the coordinate $x^*(c)$ can be directly observed by the regulator, before considering the more realistic situation where it is not.[16] If the regulator knows the firm's costs satisfy $x^*(c) = x^*$, say, then (25) implies that the cost vector lies on the straight line $\nabla h(x^*) + \kappa\nabla q(x^*)$, where κ denotes the firm's composite cost, $\phi(c)$. However, the regulator does not know where on this line the cost vector lies, and so needs to solve a one-dimensional screening problem. The following result describes optimal regulation in this situation, which adapts by-now standard arguments in Baron

[16] This approach is similar to that in Armstrong (1996, section 4.4) and Armstrong and Vickers (2001, Proposition 5), where a multidimensional screening problem is solved by supposing that the principal can observe all-but-one dimension of the agent's private information, and then finding conditions which ensure that the incentive scheme offered to these group of agents does not actually depend on the observed parameters.

and Myerson (1982, section 3) to our multiproduct context.[17] (The proof of this result is in the appendix to this paper.)

Lemma 1 *Suppose the regulator knows the firm's cost vector satisfies $x^*(c) = x^*$ and believes the firm's composite cost $\kappa = \phi(c)$ has cumulative distribution function $F(\kappa \mid x^*)$ and associated density function $f(\kappa \mid x^*)$. Provided (44) weakly increases with κ, optimal regulation requires the firm with composite cost κ to supply the composite quantity, $\hat{Q}(\phi(p))$, corresponding to the composite price*

$$\phi(p) = \kappa + (1-\beta)\frac{F(\kappa \mid x^*)}{f(\kappa \mid x^*)} \tag{44}$$

and to supply the efficient relative quantities x^*.

This result shows that relative quantities are not distorted from the efficient relative quantities, x^*, while if $\beta < 1$ expression (44) shows that composite price is above composite cost, and hence that composite quantity is below the efficient level, in order to reduce the rent enjoyed by the firm. Although the result is expressed as the firm being required to offer efficient relative quantities, it is clear that the firm would anyway choose to do this if it had discretion to choose relative quantities.

Consider now the more natural case where the regulator cannot observe $x^*(c)$. In general, with current techniques this seems to be an intractable problem. However, in the special case where the distribution for c is such that $\phi(c)$ and $x^*(c)$ are independent random variables, the previous result provides the solution:

Proposition 4 *Suppose the distribution for c is such that $\phi(c)$ and $x^*(c)$ are stochastically independent, where the firm's composite cost $\kappa = \phi(c)$ has cumulative distribution function $F(\kappa)$ and associated density function $f(\kappa)$. Provided (45) weakly increases with κ, optimal regulation requires the firm with composite cost κ to supply the composite quantity, $\hat{Q}(\phi(p))$, corresponding to the composite price*

$$\phi(p) = \kappa + (1-\beta)\frac{F(\kappa)}{f(\kappa)} \tag{45}$$

and to supply the efficient relative quantities $x^*(c)$.

[17] Baron and Myerson also show how to solve the problem when (44) is not increasing. Sappington (1983) solves a distinct multiproduct regulation problem with scalar private information, where the firm can affect the balance between fixed and variable costs.

Proof. If the two cost coordinates are independent, the regulation of composite quantity in Lemma 1 does not depend on $x^*(c)$. (Not only is the required composite quantity independent of $x^*(c)$ in (44), but so is the associated transfer function $T(\kappa)$ in expression (53) in the appendix.) Consider therefore the regulatory scheme in which, if the firm reports its composite cost is κ and its relative costs are $\tilde{x}^*$, it is instructed to supply the composite quantity corresponding to the composite price (45), it is instructed to supply relative quantities $\tilde{x}^*$, and it receives the transfer $T(\kappa)$ in (53). For any given report κ, then, the firm will choose its report $\tilde{x}^*$ to maximize its profit. As in section 3.3 this is always achieved by minimizing (33), so that the firm reports its true relative costs $x^*(c)$. By construction, this policy therefore implements the optimal policy when the regulator could observe $x^*(c)$ directly, and so the policy must also be optimal when the regulator cannot observe $x^*(c)$ directly. ■

This optimal scheme gives an incentive for the firm to supply higher composite quantity, but does not attempt to influence its choice of relative quantities.[18] Intuitively, when $\phi(c)$ and $x^*(c)$ are stochastically independent the firm is always happy for the regulator to observe its relative cost parameter $x^*(c)$ but not its composite cost $\phi(c)$. That is, for a given composite quantity, the firm and the regulator have aligned preferences with respect to the choice of relative quantities. The regulator gains by delegating to the firm the choice of relative quantities, to enable the firm to make use of its private information about relative costs, and it does this by using a transfer scheme which depends only on the composite quantity supplied. However, if $\phi(c)$ and $x^*(c)$ are correlated, observing the firm's relative costs $x^*(c)$ is informative about the firm's composite cost $\phi(c)$, which gives the firm an incentive to mis-report its relative costs as well as its composite cost.

To illustrate Proposition 4, suppose that there are two products and utility takes the additive form $u(x) = \sqrt{x_1} + \sqrt{x_2}$, which can be written as (14) with $h = 0$, $q(x) = \left(\sqrt{x_1} + \sqrt{x_2}\right)^2$ and $g(Q) = \sqrt{Q}$. In this case (33) implies

$$\kappa = \phi(c) = \frac{1}{\frac{1}{c_1} + \frac{1}{c_2}} \, , \tag{46}$$

while $x^*(c) = ((\frac{c_2}{c_1+c_2})^2, (\frac{c_1}{c_1+c_2})^2)$ depends only on the cost ratio $\frac{c_1}{c_2}$. Thus the method requires that the distribution for (c_1, c_2) be such that $\frac{1}{c_1} + \frac{1}{c_2}$ and $\frac{c_1}{c_2}$ are independent

[18] Note that when $\beta = 1$, so that the regulator cares only about unweighted surplus, expression (44) implies that Q does not depend on x^* regardless of the distribution for c. In this case, it is optimal to set prices equal to marginal costs for all firms, as discussed in Loeb and Magat (1979).

variables. Since the sum and ratio of two i.i.d. exponential variables are independent, the method works if each c_i is an independent draw from a distribution such that $1/c$ is exponentially distributed.[19] Specifically, suppose that each c_i has support $[0, \infty)$ and CDF $\Pr\{c \leq t\} = e^{-\frac{1}{t}}$. Then $1/\kappa$ is the sum of two exponential variables and so κ has CDF $F(\kappa) = (1 + \frac{1}{\kappa})e^{-\frac{1}{\kappa}}$ and corresponding density $f(\kappa) = \frac{1}{\kappa^3}e^{-\frac{1}{\kappa}}$. Expression (45) then implies that optimal composite price for the type-κ firm is $\kappa + (1-\beta)\kappa^2(1+\kappa)$, which increases with κ as required, and (52) then implies that the optimal individual prices are

$$p_i = c_i \left[1 + (1-\beta)\kappa(1+\kappa)\right] \tag{47}$$

where κ is the function of c given by (46).[20]

In expression (47) there is marginal-cost pricing for those firms with $\kappa = 0$, and so $p_i = c_i$ whenever the other product has minimum cost $c_j = 0$. The regulated price for one product is an increasing function of the other product's cost, even though this demand system has no cross-price effects and there is no statistical correlation in costs across products. Note also that all types of firm participate in the mechanism, and unlike in Armstrong (1996, section 3) there is no "exclusion" in the optimal scheme. However, so long as $\beta < 1$, when the firm has high costs the prices in (47) are above the unregulated profit-maximizing prices (which with this demand system are $p_i = 2c_i$), a possibility that was noted in the single-product context by Baron and Myerson (1982, page 292).

Similar analysis could be applied in the alternative situation with price-cap regulation, when transfer payments to the firm are not feasible and instead the regulator specifies the set of quantity (or price) vectors from which the firm is permitted to choose. If, hypothetically, the regulator could observe the firm's relative costs $x^*(c)$, but not its composite cost $\phi(c)$, it could calculate the optimal set of quantity vectors from which the firm can select, which plausibly will all be proportional to the efficient pattern $x^*(c)$.[21] In this case, the regulator can let the iso-x^* firms choose from a set of composite quantities, and let the firm choose its pattern of relative quantities freely. When $\phi(c)$ and $x^*(c)$ are stochastically

[19] Another way to describe this distribution is that c_i comes from an inverse-χ^2 distribution with 2 degrees of freedom.

[20] Another example which works nicely is when $h = 0$ and $q(x) = \sqrt{x_1 x_2}$, so that $\phi(c) = 2\sqrt{c_1 c_2}$. In this case Proposition 4 applies when the distribution for (c_1, c_2) is such that the product $c_1 c_2$ and the ratio c_1/c_2 are stochastically independent, which is the case when each c_i is an independent draw from a log-normal distribution.

[21] It is possible that it is optimal to leave gaps in the set of permitted quantities. Even in the simpler single-product case, Alonso and Matouschek (2008) and Amador and Bagwell (2014) show how the regulator sometimes chooses to leave gaps in the set of permitted prices.

independent, it follows that the permitted set of composite quantities does not depend on $x^*(c)$, and this set of composite quantities then constitutes the optimal price-cap scheme.

4 Conclusions

In this paper we have studied a range of multiproduct pricing questions in terms of consumer surplus considered as a function of quantities. With a monopolist, this is the classical profit-maximization problem, or more generally the Ramsey problem of welfare maximization subject to a profit constraint. With oligopoly, by contrast, the multiproduct pricing question is one of equilibrium among independent firms, not optimization by a single decision-maker. We have shown, however, that in perhaps the most natural multiproduct oligopoly model, with symmetric Cournot firms, there is Ramsey-Cournot equivalence. With suitable welfare weights as between profit and consumer surplus, Ramsey quantities are Cournot equilibrium quantities, and vice versa. Solutions to Ramsey optimization problems are therefore equilibrium solutions too.

The other main aim of the paper has been to show how multiproduct monopoly analysis is made simpler when consumer surplus is a homothetic function of quantities. Whether the firm's objective is profit or a Ramsey combination of profit and consumer surplus, it optimizes by selecting efficient (i.e., welfare-maximizing) quantities scaled back equiproportionately. The resulting optimal markups yield, for example, transparent results on multiproduct cost passthrough, including instances of without cross-cost passthrough. With Ramsey-Cournot equivalence, these results extend to the Cournot oligopoly setting with symmetric firms.

The family of demand systems with consumer surplus a homothetic function of quantities is of course restrictive. But it includes a number of familiar yet diverse special cases, including CES and linear demands and discrete choice models such as logit. Moreover, it shows how those special cases are themselves instances of wider sub-classes of demand systems, involving $h(x) \equiv 0$, linear $h(x)$ and linear $q(x)$ respectively.

Finally, the family of demand systems analyzed in this paper provides a natural basis on which to explore the intuition that regulation of multiproduct monopoly should focus on the general level of prices and not the pattern of relative prices. Indeed we showed how that intuition can sometimes be precisely correct, thereby contributing to the theory of multi-dimensional screening.

Appendix

Proof of Lemma 1: Given x^*, suppose the regulator's additional transfer to the firm when the latter reports its composite cost $\tilde{\kappa}$ is denoted $T(\tilde{\kappa})$ and its required quantity vector is $X(\tilde{\kappa})$. If the firm's true composite cost is $\kappa = \phi(c)$, from (25) its cost vector is $c = \nabla h(x^*) + \kappa \nabla q(x^*)$. The type-$\kappa$ firm's maximum profit from participating in this scheme is therefore

$$\Pi(\kappa) = \max_{\tilde{\kappa}} : T(\tilde{\kappa}) + r(X(\tilde{\kappa})) - X(\tilde{\kappa}) \cdot [\nabla h(x^*) + \kappa \nabla q(x^*)] \ , \tag{48}$$

and it is willing to participate provided that $\Pi(\kappa) \geq 0$. Here, profit in (48) satisfies the envelope condition

$$\Pi'(\kappa) = -X(\kappa) \cdot \nabla q(x^*) \ , \tag{49}$$

where the right-hand side of (49) is evaluated at the firm's true cost κ since we are considering truthful mechanisms.

If the support of the distribution for κ for this iso-x^* group of firms is $[\kappa_{\min}, \kappa_{\max}]$, expected welfare under the scheme is

$$\int_{\kappa_{\min}}^{\kappa_{\max}} (u(X(\kappa)) - r(X(\tilde{\kappa})) - T(X(\kappa)) + \beta \Pi(\kappa)) \, f(\kappa \mid x^*) d\kappa$$

$$= \int_{\kappa_{\min}}^{\kappa_{\max}} (u(X(\kappa)) - X(\kappa) \cdot [\nabla h(x^*) + \kappa \nabla q(x^*)] - (1-\beta)\Pi(\kappa)) \, f(\kappa \mid x^*) d\kappa \ , \tag{50}$$

where the equality follows after substituting for $r + T$ in (48). The regulator wishes to maximize (50) subject to the participation and incentive constraints of the firm. We solve this by solving a "relaxed" problem in which only the participation constraint for the highest-κ firm is considered and where only the local incentive constraints captured by (49) are considered. (We then check *ex post* that the remaining constraints are satisfied.)

Proof. Given (49) and $\Pi(\kappa_{\max}) = 0$, integrating the term $\int \Pi f d\kappa$ by parts implies that (50) can be written as

$$\int_{\kappa_{\min}}^{\kappa_{\max}} \left(u(X(\kappa)) - X(\kappa) \cdot \left[\nabla h(x^*) + \left\{ \kappa + (1-\beta) \frac{F(\kappa \mid x^*)}{f(\kappa \mid x^*)} \right\} \nabla q(x^*) \right] \right) f(\kappa \mid x^*) d\kappa \ . \tag{51}$$

This expression can then be maximized pointwise with respect to the vector $X(\kappa)$. Since consumer demand at price vector p maximizes $u(x) - p \cdot x$, it follows that $X(\kappa)$ is consumer demand corresponding to the price vector

$$p = \nabla h(x^*) + \left\{ \kappa + (1-\beta) \frac{F(\kappa \mid x^*)}{f(\kappa \mid x^*)} \right\} \nabla q(x^*) \ . \tag{52}$$

Expression (24) and its subsequent discussion then implies that the price vector (52) satisfies (44), and the composite quantity Q corresponding to this price vector maximizes $g(Q) - Q\phi(p)$, or in the notation from section 3.1 composite quantity is $\hat{Q}(\phi(p))$. Expression (24) also implies that the relative quantities corresponding to prices (52) are just x^* for each κ.

At this candidate solution with quantities $X(\kappa) = \hat{Q}(\phi(p)) \times x^*$, $\Pi'(\kappa)$ in (49) is equal to $-\hat{Q}(\phi(p)) \leq 0$ and so Π is weakly decreasing in κ and the participation constraint is satisfied for all κ given it is satisfied for $\kappa_{\max}$. Since Π in (48) is necessarily convex in κ, incentive compatibility requires that Π' in (49) be weakly increasing in κ, which is the case provided that (44) is weakly increasing. Standard arguments (see Lemma 1 in Baron and Myerson) show that it is also sufficient for incentive compatibility that Π' in (49) weakly increase with κ. For reference later, it is useful to note that the corresponding incentive payment $T(\kappa)$ can be calculated from (48) and (49) to be

$$T(\kappa) = \int_{\kappa}^{\kappa_{\max}} \hat{Q}\left(\tilde{\kappa} + (1-\beta)\frac{F(\tilde{\kappa} \mid x^*)}{f(\tilde{\kappa} \mid x^*)} \right) d\tilde{\kappa} - (1-\beta)\frac{F(\kappa \mid x^*)}{f(\kappa \mid x^*)} \hat{Q}\left(\kappa + (1-\beta)\frac{F(\kappa \mid x^*)}{f(\kappa \mid x^*)} \right) \tag{53}$$

which depends on x^* only via its impact on F/f. ■

References

Alonso, Ricardo, and Niko Matouschek (2008), "Optimal delegation", *Review of Economic Studies* 75(1), pp. 259–293.

Amador, Manuel, and Kyle Bagwell (2014), "Regulating a monopolist with uncertain costs without transfers", mimeo.

Anderson, Simon, André de Palma and Jacques-François Thisse (1988), "The CES and the logit: two related models of heterogeneity", *Regional Science and Urban Economics* 18(1), pp. 155–164.

Armstrong, Mark (1996), “Multiproduct nonlinear pricing”, *Econometrica* 64(1), pp. 51–75.

Armstrong, Mark, and John Vickers (2001), “Competitive price discrimination”, *Rand Journal of Economics* 32(4), pp. 579-605.

Baron, David, and Roger Myerson (1982), “Regulating a monopolist with unknown costs”, *Econometrica* 50(4), pp. 911–930.

Baumol, William, and David Bradford (1970), “Optimal departures from marginal cost pricing”, *American Economic Review* 60(3), pp. 265–283.

Bergstrom, Theodore, and Hal Varian (1985), “Two remarks on Cournot equilibria”, *Economics Letters* 19(1), pp. 5–8.

Besanko, David, Jean-Pierre Dubé and Sachin Gupta (2005), “Own-brand and cross-brand retail pass-through”, *Marketing Science* 24(1), pp. 123-137.

Bliss, Christopher (1988), “A theory of retail pricing”, *Journal of Industrial Economics* 36(4), pp. 375–391.

Bulow, Jeremy, and Paul Klemperer (2012), “Regulated prices, rent seeking, and consumer surplus”, *Journal of Political Economy* 120(1), pp. 160–186.

Edgeworth, Francis (1925), *Papers Relating to Political Economy*, McMillan and Co.

Gorman, W.M. (1961), “On a class of preference fields”, *Metroeconomica* 13(2), pp. 53-56.

Johnson, Justin, and David Myatt (2015), “The properties of product line prices”, *International Journal of Industrial Organization* 143, pp. 182–188.

Laffont, Jean-Jacques, and Jean Tirole (1993), *A Theory of Incentives in Procurement and Regulation*, MIT Press.

Loeb, Martin, and Wesley Magat (1979), “A decentralized method for utility regulation”, *Journal of Law and Economics* 22(2), pp. 399–404.

Monderer, Dov, and Lloyd Shapley (1996), “Potential games”, *Games and Economic Behavior* 14(1), pp. 124–143.

Moorthy, Sridhar (2005), “A general theory of pass-through in channels with category management and retail competition”, *Marketing Science* 24(1), pp. 110-122.

Oi, Walter (1971), "A Disneyland dilemma: two-part tariffs for a Mickey Mouse monopoly", *Quarterly Journal of Economics* 85(1), pp. 77–96.

Sappington, David (1983), "Optimal regulation of a multiproduct monopoly with unknown technological capabilities", *Bell Journal of Economics* 14(2), pp. 453–463.

Shugan, Steven, and Ramarao Desiraju (2001), "Retail product-line pricing strategy when costs and products change", *Journal of Retailing* 77(1), pp. 17–38.

Slade, Margaret (1994), "What does an oligopoly maximize?", *Journal of Industrial Economics* 42(1), pp. 45-61.

Vives, Xavier (1999), *Oligopoly Pricing*, MIT Press.

Weyl, Glen, and Michal Fabinger (2013), "Pass-through as an economic tool: principles of incidence under imperfect competition", *Journal of Political Economy* 121(3), pp. 528-583.