Anchoring and Adjustment Heuristic: A Unified Explanation for Equity Puzzles

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Abstract

I model a scenario in which investors do not know the payoff distributions of relatively newer firms and use the payoff distribution of similar well-established firms as starting points. The starting distributions are then adjusted for size, volatility, and other differences. Anchoring bias implies that such adjustments typically fall short. I show that incorporating such anchoring and adjustment heuristic into the standard consumption-based capital asset pricing model provides a unified explanation for 9 asset pricing puzzles including the equity premium puzzle. The anchoring approach achieves these explanations while maintaining the tractable framework of a representative agent with time-additive and isoelastic preferences in a complete market.

Keywords: *The Equity Premium Puzzle, Anchoring Bias, The Risk-Free Rate Puzzle, Countercyclical Equity Premium, Stock Price Volatility, Knightian Uncertainty*

JEL Classification: G02, G11, G12, D80, D81

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Anchoring and Adjustment Heuristic: A Unified Explanation for Equity Puzzles

Any financial asset is just a particular label assigned to some future payoff stream. Valuing an asset requires forming a judgment about this stream. Needless to say, this is a complex task involving a high degree of uncertainty, and especially more so for relatively newer firms. Imagine a well-established firm which has been around for decades. It has lived through a series of good and bad times. Based on its past history, one can hope to form reasonably accurate judgments about such a firm's payoffs, at least in the immediate future. One cannot make such a claim for a relatively new firm, not just because it has not been around long enough to generate a sufficiently rich dataset, but also due to its growing nature, which makes past performance a particularly poor guide to the future.

Well-established firms, known as blue-chips in market parlance, are small in number and typically constitute less than 4% of the firms whose scrips are traded in major stock exchanges of the world. They typically generate large payoffs due to their large asset bases, have high market prices, and are often household names. They get a hugely disproportionate amount of media attention and analysts coverage. A study suggests that roughly 83% of equity analysts cover blue chips only, leaving only 17% analysts for the remaining 96%.[2] So, firms which are intrinsically harder to value, that is, firms which are not considered blue-chip or well-established, are made even harder to value by this lack of analyst coverage.

How does an average investor estimate the future payoff stream of a typical firm? Outside a small number of well-stablished firms, it is difficult to support the claim that an average investor just knows what the future payoff distribution of every firm is. However, this is exactly what standard macro-finance models such as the consumption-based capital asset pricing model (CCAPM) assume. In standard CCAPM (Rubinstein (1976), Lucas (1978), Grossman and Shiller (1981) (1978), Hansen and Singleton (1983) among others), the representative agent is assumed to be omniscient. That is, the agent is assumed to know the relevant structural parameters of all payoff distributions accurately. The standard model does not acknowledge that some payoff distributions are easier to learn than others. Clearly, omniscience is a convenient assumption. I argue, in this article, that this convenience

[2] http://punchinvest.com/wp-content/uploads/2011/11/The-Blind-Spot-Impact-of-Analyst-Coverage1.pdf

comes at a high cost, and is the main reason behind the equity premium puzzle of Mehra and Prescott (1985)). I show that appropriately relaxing this assumption provides a plausible unified explanation for 9 asset pricing puzzles including the equity premium puzzle.

If asked, most economists would probably agree that the payoff distribution of a typical firm is not just unknown but perhaps unknowable. The real challenge is to model decision-making in such a setting. Economic literature has responded to this challenge in two ways: 1) Preferences: By imposing more complex preference structures that distinguish ambiguity from risk such as $\alpha - MaxMin$ Expected Utility Theory $(\alpha - MEU)$ of Ghirardato et al (2004) among others. If the agent is unsure about payoff volatility, then under $\alpha - MEU$, a weighted average of the worst possible and the best possible outcome is taken, with the weights reflecting the ambiguity attitude of the agent. 2) Beliefs: By allowing a Bayesian learning process in which the representative agent continuous to update beliefs in the presence of structural instability. The implications of such a learning process for asset prices are studied (see Weitzman (2007), Bossaerts (2003) (1995) and references therein).

In contrast to the above approaches, an alternate view has taken shape in psychology and cognitive science literature. This view relies on the anchoring and adjustment heuristic, first pointed out in the early experiments of Tversky and Kahneman (1974). A large body of research over the years shows that in order to estimate what they do not know, people have a tendency to start from a closely related answer (known as the self-generated anchor) which they know to be wrong and then make adjustments to it to form judgments. However, adjustments typically fall short. This observation is known as the anchoring bias (see Furnham and Boo (2011) for a literature review). Adjustments are typically insufficient because people tend to stop adjusting once a plausible value is reached (see Epley and Gilovich (2006)). Hence, assessments remain biased towards the starting value known as the anchor.

Epley and Gilovich (2001) write, "*People may spontaneously anchor on information that readily comes to mind and adjust their response in a direction that seems appropriate, using what Tversky and Kahneman (1974) called the anchoring and adjustment heuristic. Although this heuristic is often helpful, the adjustments tend to be insufficient, leaving people's final estimates biased towards the initial anchor value*." (Epley and Gilovich (2001) page. 1)

A few examples illustrate this approach well. When did George Washington became the first president of America? When asked this question, most people do not know the right answer, however, respondents typically reason as follows. The declaration of independence was signed in 1776 so it might have taken a few years after that to elect the first president. That is, they start from 1776 and add a few years to it. However, they stop adding once a plausible year is reached, which is typically less than the right answer. Hence, a typical answer is biased towards the starting value or the anchor of 1776. Another example is the freezing temperature of Vodka. Most respondents know that Vodka freezes at a temperature below the freezing point of water, so they start from 32 degree Fahrenheit (0 Celsius) and adjust downwards. However, adjustments are typically insufficient. To take a third example, what is a fair price for a 3-bedroom house in the Devon neighborhood of Chicago? If you know the price paid for a 4-bedroom house in the same neighborhood but in a slightly better location, you would likely start with that price and adjust for location, size and other differences. Anchoring bias refers to the fact that such adjustments tend to be insufficient.

A large literature in psychology and cognitive science argues that the reasoning illustrated in the above examples is universal, especially for self-generated anchors (see Epley and Gilovich (2006)(2001) for a detailed discussion and references therein). Self-generated anchors are closely related starting points for the situations at hand just like 1776 is relevant to the presidency of George Washington, freezing point of water is a relevant reference point for the feezing point of Vodka, or the price paid for a 4-bedroom house is relevant for the price of a 3-bedroom house in the same neighborhood.

In the preference based approach to ambiguity, such as $\alpha - MEU$, if the agent is unsure about the payoff volatility, effectively a weighted average of extreme outcomes is taken with weights reflecting the agent's ambiguity attitude. In contrast, under the judgment based approach of anchoring and adjustment, the location of the self-generated anchor is the key determining factor. If the anchor lies to the right of the set of plausible values, then the chosen value is close to the right edge of plausible values, consistent with the notion of adjustment from the anchor till a plausible value is reached. If the anchor lies to the left of the set of plausible values, then the chosen value is close to the left edge. This leads to a bias in judgment because instead of picking somewhere in the middle (the average value), values are chosen near the edges of plausible values. In this article, I show that the anchoring and adjustment based judgment approach provides a straightforward resolution of the equity premium and other puzzles even when the anchoring bias is quite small.

That is, the equity premium puzzle is resolved even if the agent goes almost all of the way and falls only slightly short while making adjustments.

In this article, I show that the anchoring and adjustment heuristic provides a unified explanation for the following puzzles:

1) The high equity premium puzzle (Mehra and Prescott (1985), Mehra (2003)).
2) The low risk-free rate puzzle (Weil (1989)).
3) The excess volatility puzzle (Barsky and Delong (1993), Shiller (1981), and LeRoy and Porter (1981)).
4) Countercyclical equity Sharpe ratio and countercyclical equity premium (Harvey (1989), Li (2001)).
5) The small size anomaly (Banz (1981) among others).
6) The value premium (Fama and French (1998)).
7) The momentum effect (Jegadeesh and Titman (1993)).
8) Positive excess return associated with stock-splits (Ikenberry (1996).
9) Negative excess return from reverse stock-splits (Kim et al (2008)).

Given the wide-spread used of anchors in decision making, a natural starting point for estimating the unknown payoff distribution of a typical firm is the known payoff distribution of a well-established firm in the same sector. Plausibly, one can estimate the payoff distribution of a typical firm by starting from the distribution of a similar well-established firm and then making adjustments for size, volatility and other differences. However, such cognitive adjustments expose one to the anchoring bias.

Imagine one is interested in Cisco system's stock in February 1990. Cisco in 2015 is a network technology giant and considered a blue-chip stock with over 30 years of history behind it. However, back in 1990, its stock was launched at a price of 6 cents (on split-adjusted basis). Not much was known about Cisco in 1990, then only 6 years old, in the relevant market segment largely dominated by IBM. How would one go about forming a judgment about Cisco's stock in 1990? Where-else would one start if not by looking at the performance of the established market leader at that time, which was IBM, and attempt to make appropriate adjustments for much smaller size, volatility, and growing nature of the new firm? That is, it makes a lot of sense to start from the payoff distribution of IBM, and then apply a series of cognitive operations to it with the aim of adjusting for size, volatility, and other differences.

Of course, with time, the business model of Cisco was better understood; however, the firm also grew and now is counted among large-cap blue chip stocks. Other start-ups and relative new comers now occupy the same spot that Cisco had in 1990. And, arguably, just like for Cisco in 1990, for these newer small companies, one may start from Cisco's payoff distribution and attempt to make appropriate adjustments to form relevant judgments. The point is that a given firm may go through several classifications over its lifetime. A small-cap stock of yesterday, if it does not go bust, may be a large-cap stock of today, with newer small cap stocks taking its place. The identities of firms within the categories of large-cap and small-cap change, but the percentages of stocks in each category remain more or less the same.[3] So, the impact of the anchoring bias may never disappear, as there will always be small-cap stocks that are valued by making adjustments to large-cap stocks. Learning may alleviate the bias in the stock of a particular small company if it does not go bust, but the time it takes to do that, may mean a classification change to large-cap well-established stock, with some other small-cap taking its place.

Not only the anchoring and adjustment view is psychologically accurate but it also is the case that financial analysts and investors alike are, quite plausibly, strongly prone to it. Quite sensibly, a financial analyst aims to (is trained to) place his analysis of a particular firm in the context of the industry in which the firm operates. The industrial landscape is shaped by well-established firms in that particular sector. As every firm is viewed from the lens of the industrial landscape shaped by well-established firms, the conditions are already ripe for anchoring to take place. Cen et al (2013) find that earnings per share forecasts of a given firm made by professional analysts are strongly influenced by the average earnings per share in that particular sector. That is, analysts seem to be anchored to the "industry norm".

In this article, I consider anchoring at the level of estimating payoff distributions. Hirshleifer (2001) considers anchoring to be an *"important part of psychology based dynamic asset pricing theory in its infancy"* (p. 1535). Shiller (1999) argues that anchoring appears to be an important concept for financial markets. This argument has been supported quite strongly by recent empirical research on

[3] http://www.investopedia.com/articles/analyst/010502.asp

financial markets: 1) Anchoring has been found to matter in the bank loan market as the current spread paid by a firm seems to be anchored to the credit spread the firm had paid earlier (see Douglas, Engelberg, Parsons, and Van Wesep (2015)). 2) Baker, Pan, and Wurgler (2012) provide evidence that peak prices of target firms become anchors in mergers and acquisitions. 3) Campbell and Sharpe (2009) find that expert consensus forecasts of monthly economic releases are anchored towards the value of previous months' releases. 4) Johnson and Schnytzer (2009) show that investors in a particular financial market (horse-race betting) are prone to the anchoring bias.

Intriguingly, recent research on anchoring shows that it is relevant for valuing financial assets: 1) Option Pricing: Siddiqi (2015a) shows that adjusting the Black-Scholes model for anchoring provides a unified explanation for a number of option pricing puzzles. 2) Asset pricing: Siddiqi (2015b) shows that adjusting CAPM for anchoring provides a plausible unified explanation for the size, value, and momentum effects. The ability to address a diverse range of puzzles is a testament to the power of this approach.

Mullainathan et al (2008) argue that advertisers frequently make use of the fact that people are anchoring-prone. That is why we get campaigns like, "we put silk in our shampoo". With a campaign like this, advertisers are attempting to implant a 'quality' anchor in the minds of consumers so that their shampoo gets anchored with "silk", which is beneficial as "being silky" is presumably a good quality in hair. Whereas advertisers maybe attempting to implant 'superficial anchors' through media campaigns, arguably, anchoring is an even more powerful force when anchors are self-generated and salient to the problem at hand. Ariely (2008) argues that we tend to compare things that are similar or related in some way. Some cognitive scientists consider thinking by analogy and comparison as the fuel and fire of thinking (see Hofstadter and Sander (2013)).

In section 1, I briefly discuss the nature of the equity premium puzzle and two broad categories of proposed explanations. Section 2 provides a numerical comparison of the standard CCAPM with the anchoring adjusted CCAPM. The purpose is to build intuition. Section 3 develops the anchoring adjusted CCAPM. Section 4 shows that anchoring adjusted CCAPM provides a plausible unified explanation for 9 asset pricing puzzles including the equity premium puzzle. Section 5 concludes.

1. Consumption-Based Asset Pricing and the Equity Premium Puzzle

In the simplest version of consumption based asset pricing model[4](CCAPM), the price of an asset is equal to the present value of expected payoffs plus an adjustment term for risk. The adjustment term for risk is usually (almost always) negative and depends both on the quantity of risk as well as the price of risk. It is equal to the covariance of the asset's payoffs with the stochastic discount factor (SDF) or equivalently with the inter-temporal marginal rate of substitution (IMRS) of the representative investor. Intuitively, the risk term depends on how badly an asset is expected to perform in bad times. An asset that performs worse in bad times is riskier (with a negative risk adjustment term of a larger magnitude) and gets a lower price when compared with an asset that performs better.

In standard CCAPM (Rubinstein (1976), Lucas (1978), Grossman and Shiller (1981), Hansen and Singleton (1983) among others), the price of risk is typically the coefficient of relative risk aversion, and the quantity of risk depends on the covariance of payoffs with consumption growth. Mehra and Prescott (1985) show that in order to justify the historically observed high equity premium, an implausibly large price of risk is needed. This is because the quantity of risk seems small as historical consumption growth has little volatility. Mehra and Prescott (1985) show that the historical data implies a risk aversion coefficient of around 30 whereas a value of around 1 to 3 seems reasonable. A related anomaly is the low risk-free rate puzzle put forward in Weil (1989): As consumption tends to grow with time, the desire to smooth consumption should increase the demand for borrowing causing the risk-free rate to rise; however, inconsistent with this prediction, the historically observed risk-free rate is too low. Apart from high equity premium and low risk-free rate, the strongly countercyclical nature of the equity premium is also a related puzzle along with high stock price volatility given the considerably smaller volatility in fundamentals.

In standard CCAPM, the first order condition yields[5]:

$$P_i = \frac{E[X_i]}{R_F} + Cov(q_i, X_i) \tag{1.1}$$

[4] The model is discussed in section 3

[5] The derivation is discussed in section 3.

where P_i is the current price of asset i, $E[X_i]$ is the expected next period payoff, R_F is the (gross) risk-free rate return between now and the next period, q is the SDF or equivalently IMRS of the representative investor evaluated at optimal allocation, and $Cov(q, X_i)$ is the covariance of payoffs with the SDF.

In (1.1), q is an indicator of bad times, so the risk-adjustment term is equal to the judgment of the representative investor regarding the covariance of an asset's future payoffs with the state of the economy. CCAPM requires that such risk judgments are correctly formed for every asset in the economy. This is a strong assumption especially given the fact that such covariances are not only just difficult to estimate but are also unknowable in many cases. Firms differ in terms of history, data availability, and how much media and analyst attention they get. Some stocks have been around for decades and belong to well-known and well-established companies while others are relative new comers. In standard CCAPM, no allowance has been made for the fact that some firms have lived through a series of good and bad times, so forming risk judgments about them is easier when compared with firms that have just started operating. In other words, standard CCAPM views every firm with the same lens of omniscience. Differences of information availability across firms are simply brushed away by assuming that correct risk judgments are formed. Of course, omniscience is a convenient assumption. However, this convenience comes at a great cost. I argue, in this article, that the inability of standard CCAPM to explain the equity premium and related puzzles is the price paid for assuming omniscience.

It is not just that information availability differs between well-established and newer firms, the usefulness of information as a window to the future also differs. Current financials provide more of a window to the future for a well-established firm when compared with a newer firm. Some market professionals advocate an 80-20 rule for well-established vs. newer firm, which says that for evaluating a well-established firm, place an 80% weight on company financials and 20% weight on your judgment, whereas for a newer firm, place only a 20% weight on current financials and an 80% weight on judgment.[6] So, it is not just that less is known about the future potential performance of a newer stock, it also is true that much less is actually knowable, and meticulously studying current company financials does not solve the problem as the financials are practically out-of-date the

[6] http://www.investopedia.com/articles/stocks/07/micro_cap.asp

moment they are filed. Hence, Knightian uncertainty (unknowable distribution) is a plausible description especially for relatively newer stocks.

Given the fact that not only much less information is available about a newer firm but also that this information is of little use as a window to the future, where would one start if one needs to form risk judgments? Plausibly, one starts by considering the payoff distribution of a similar well-established firm in the same sector. That is, one starts by considering a similar firm for which information is plentiful, and then one makes adjustments for the smaller size, greater payoff volatility, and other differences. To my knowledge, anchoring adjusted CCAPM is the smallest deviation from the standard CCAPM that provides a unified explanation for high equity premium and related asset pricing puzzle. In this sense, it provides the simplest explanation.

Since the seminal work of Mehra and Prescott (1985), a large number of explanations for the equity premium puzzle (and the related risk-free rate puzzle) have been put forward. Mehra (2003) provides a survey of prominent explanations, and argues that they fail to resolve the puzzle across crucial dimensions. Some well-known explanations can be classified into two broad categories: 1) Approaches that increase the price of risk without requiring a high risk aversion coefficient. 2) Approaches that increase the quantity of risk.

Some examples of the first category of explanations include the habit-formation model of Constantinides (1990), and the "prospect theory" and "mental accounting" based explanation of Benartzi and Thaler (1995) (see Grant and Quiggin (2004) for a discussion of some prominent explanations).

The second category prominently includes the small probability disaster state approach initially proposed in Reitz (1988) and extended in Barro (2006), Gabaix (2012) and others. The approach essentially argues that the *ex-ante* perceived quantity of risk is larger than what is measured in a finite sample because there is a very small probability of a very large disaster. The *ex-ante* probability of disaster is larger than the *ex post* frequency of disasters observed in a finite sample so *ex ante* quantity of risk is larger than the *ex post* quantity of risk. However, this approach does not jibe well with the well-documented human tendency of ignoring very small probabilities of very large losses in decision making (see Dhami and al-Nowaihi (2010) for a discussion on this human tendency). Noting the human tendency of ignoring very small probabilities, Kahneman and Tversky

(1979, p.282) write, "*The simplification of prospects can lead the individual to discard events of extremely low probability and to treat events of extremely high probability as if they were certain*".

Kunreuther et al (1978) provide detailed evidence that people buy inadequate insurance against low probability events such as earthquakes, floods, hurricanes etc in areas that are prone to such natural disasters even when actuarially fair insurance is available. The tendency to ignore very small probability events of large losses is also reflected in what is known as the Becker paradox. Becker (1968) famously proposed that the best way to deter crime is to impose the severest possible penalty with the smallest probability. In fact, under the expected utility theory, if infinitely severe punishment is imposed, although with a very small probability, then the crime would be deterred completely. However, empirical evidence does not support the Becker proposition. This is known as the Becker paradox. Bar-Ilan (2000) use empirical evidence to show that a version of Becker paradox is illustrated by the traffic accidents caused by people running red traffic lights. Here the punishment can be thought of as self-imposed and may result in death or serious injury due to accidents. Similar examples are people not using seat belts in countries where they are not mandatory, and driving while talking on cell phones etc. For a detailed discussion of these and many other examples of people ignoring very low probability events involving large losses, see Dhami and al-Nowaihi (2010).

Effectively, the anchoring adjusted CCAPM increases the *ex-ante* perceived quantity of risk without requiring the small-probability- large-disaster-states. Hence, the anchoring approach does not suffer from the problems of the disaster state approach, and achieves high *ex ante* perceived quantity of risk needed to resolve the puzzles.

2. Anchoring in Asset Pricing: A Numerical Example

To fix ideas, consider a simple case of two risky assets (L and S) and one risk-free asset (F). There is one time-period marked by two points in time, t and $t + 1$. There are two states of nature at $t + 1$ called the Green and Blue states. The chance of each is 50%. The current time is t. One risky asset (L) belongs to a well-established firm with large payoffs. The second risky asset is a newer asset (S) with much smaller payoffs. The payoffs from L, S and F are shown in Table 1.

Table 1 Payoffs from well-established, relatively new, and risk-free asset			
Asset Type	**Price**	**Green State**	**Blue State**
L	P_L	200	100
S	P_S	25 (Omniscience) 30 (Anchoring)	5 (Omniscience) 0 (Anchoring)
F	P_F	110	110
Agent faces Knightian uncertainty about S and uses the payoff distribution of L as a starting point for forming judgments about the payoff distribution of S. Anchoring bias implies that he fails to adjust fully and the judgment remains biased towards the starting value.			

L is a well-established asset and investors know the true distribution which is 200 in the Green state and 100 in the Blue state, implying a mean of 150 and a standard deviation of 50. S is a newer asset and investors face Knightian uncertainty about its true payoff distribution. Assume that they use the payoff distribution of L as a starting point to which a series of cognitive operations are applied to generate a plausible distribution for S. The idea is that S and L are similar (in the same sector). Investors understand L, but do not know much about S, so they start from what they know and then attempt to make appropriate adjustments.

Due to smaller asset base, S must have smaller payoffs, so the first cognitive operation is adjusting for size to get to the expected payoff of S. Let's say the expected payoff of L is divided by 10 to get the expected payoff of S. As the expected payoff of L is 150, the expected payoff of S is inferred to be 15. Assume that the expected payoff is correctly estimated to obtain 15.

Estimating the payoff standard deviation is harder. The payoff standard deviation of the well-established asset is 50. Starting from 50, one cannot simply divide by 10 (as we did for expected payoff) because the percentage fluctuation of the smaller asset around the mean is likely to be larger than the corresponding percentage fluctuation of the older well-established asset. If the payoffs of the new asset are 25 and 5 in the Green and Blue states respectively, then the correct value of standard deviation is 10, which is obtained by subtracting 40 from 50 (or equivalently division by 5). For the purpose of illustration, assume that, starting from 50, the agent goes nearly the full way but not quite to 10, he stops adjusting at 15. That is, the standard deviation is inferred to be 15 instead of the correct value of 10. This is the anchoring bias. With such a bias, the payoff distribution is inferred to be 30 and 0 in the Green and Blue states respectively, whereas the corresponding correct values are 25 and 5.

To fully appreciate the implications of this bias, we need to compare the outcomes under omniscience with outcomes under Knightian uncertainty with anchoring. In the standard consumption-based asset pricing approach, one simply assumes omniscience for convenience. This article is aimed at replacing the assumption of omniscience with a much more reasonable assumption of Knightian uncertainty with anchoring. This is the only change and the rest of the framework is left unchanged.

In section 1.1, in the context of our example, we examine the case of omniscience. Continuing with the same example, in section 1.2, we look at the implications of the anchoring bias.

2.1. CCAPM with Omniscience

Given the payoffs of L, F, and S (assuming omniscience) in Table 1, what are the equilibrium prices of these assets? This question is answered next.

Suppose there exists a representative agent with a time separable utility function who maximizes the following:

$$max_{n_L, n_s, n_F} U(c_t) + \beta E_t[c_{t+1}]$$

where n_L, n_s, and n_F are the number of shares of L, S, and F respectively. The current and next period consumption are c_t and c_{t+1} respectively, and β is the time discount factor.

The agent maximizes expected utility of consumption subject to the following constraints:

$$c_t = e_t - n_L P_L - n_s P_s - n_F P_F$$

$$\tilde{c}_{t+1} = e_{t+1} + n_L \tilde{X}_L + n_s \tilde{X}_s + n_F X_F$$

where $\tilde{X}_L$, $\tilde{X}_s$ *and* X_F are payoffs of L, S, and F respectively and are given in Table 1. The agent receives endowments e_t and e_{t+1} at t and $t+1$ respectively. P_L, P_s, and P_F denote prices.

The first order conditions of the maximization problem are:

$$P_L = E_t[SDF_i \cdot X_{Li}]$$

$$P_s = E_t[SDF_i \cdot X_{si}]$$

$$P_F = E_t[SDF_i] \cdot X_F$$

where $SDF_i = \frac{\beta U'(c_{t+1i})}{U'(c_t)}$ evaluated at optimal allocation, and i is the state indicator.

Assume that the representative agent must hold one unit of each asset to clear the market. Assume that utility function is $ln(c)$, $\beta = 1$, and $e_t = e_{t+1} = 500$. It follows that $P_L = 70.20785$, $P_s = 6.83484$, and $P_F = 52.849335$. The SDF is $\{0.44326, 0.517637\}$. That is, $E[SDF] = 0.480449$, and $\sigma(SDF) = 0.037189$. The Sharpe ratio of the large asset is equal to the Sharpe ratio of the small asset at 0.077404. The present value of the Sharpe ratio is 0.037189. In other words, the present value of the Sharpe ratio is equal to the standard deviation of the stochastic discount factor. That is, the following is true:

$$\frac{E_t[R_{it+1}] - R_F}{R_F \cdot \sigma_t(R_{it+1})} = \sigma_t(SDF) \tag{2.1}$$

(2.1) is the capital market line (Sharpe (1964)) equivalent of Hansen-Jagannathan bound (Hansen and Jagannathan (1991)), which corresponds to the mean-variance frontier. In our example, with omniscience, both the L.H.S and the R.H.S in (2.1) are equal to 0.037189.

The equity premium puzzle is a puzzle because empirically L.H.S in (2.1) has been found to be much larger than the R.H.S in (2.1). With historical US data, the equity premium is 6%, the risk free rate is 1%, and the standard deviation of returns is 18%. This implies a present value of equity Sharpe ratio equal to 0.33. The standard deviation of SDF estimated from consumption data is around 0.02. Hence, the L.H.S and the R.H.S are different by more than an order of magnitude.

2.2 CCAPM with Anchoring

I continue with the same example; however, I replace the assumption of omniscience with the assumption that investors face Knightian uncertainty about the payoff distribution of the new asset, and use the payoff distribution of the well-established asset as a starting point to which a series of cognitive operations are applied. Recall that we have assumed that the expected payoff is correctly estimated to be 15; however, there is anchoring bias in the standard deviation, as it is estimated to be 15, whereas the correct value is 10. It follows that the payoff distribution is estimated to be 30 and 0 in the Green and Blue states respectively, whereas, the corresponding correct values are 25 and 5.

Keeping everything else the same as in section 2.1, the prices of the three assets can be calculated and are: $P_L = 70.16025$, $P_s = 6.61245$, and $P_F = 52.930625$. The SDF is $\{0.44083, 0.521545\}$. That is, $E[SDF] = 0.481188$, and $\sigma(SDF) = 0.040358$.

Note that the distribution of SDF has changed: both the mean and standard deviation have gone up. Intuitively, the distribution of SDF depends on the state-wise distribution of aggregate risk in the economy. Overestimating the risk of the small asset, has led to an increase in the level of perceived aggregate risk. This has led to changes in SDF corresponding to lower prices of equities and a higher price of the risk free asset. In other words, higher perceived aggregate risk lowers the risk-free rate and increases equity returns.

The empirical Sharpe ratio of the large asset is now different from the empirical Sharpe ratio of the new asset. The empirical Sharpe ratio of the large asset is 0.083871. The present value is 0.040358. Hence, (2.1) still holds for the large asset (although with a much larger SDF volatility). The empirical Sharpe ratio of the new asset is 0.125806 with a present value of 0.060536, which is larger than the standard deviation of the SDF (0.040358). Hence, (2.1) does not hold for the new asset. It is easy to verify that the following holds for the new asset instead[7]:

$$\frac{E_t[R_{st+1}] - R_F}{R_F \cdot \left\{\sigma_t(R_{st+1}) + \frac{(1-m)\left(\sigma(X_L) - \sigma(X_s)\right)}{P_s}\right\}} = \sigma_t(SDF) \tag{2.2}$$

where the subscript s denotes the new asset, the subscript L denotes the old asset, P is price, and m is the fraction of correct distance the agent goes starting from the standard deviation of the old

[7] This expression is derived in the next section.

asset. That is, $0 \leq m \leq 1$. If $m = 1$, there is no anchoring bias, and (2.2) converges to (2.1). If $0 \leq m < 1$, then there is anchoring bias, and (2.2) and (2.1) are different.

Realizing that omniscience is just a convenient assumption, and anchoring when faced with Knightian uncertainty is a more realistic description, it is hard to see how this bias can be mitigated. Even if investors realize that they are anchoring-prone (as some probably do), how can they guard against it? Mitigating anchoring requires a willingness to pay a higher price. One needs to become less cautious and less prudent and throw caution to the wind. This advice goes against what is culturally known as better judgment, which favors caution and prudence. This makes anchoring a rare bird among cognitive biases. For a typical cognitive bias, such as overconfidence and optimism, one can imagine an institution that effectively ties Ulysses to the mast to save him from the Sirens by calling for caution and prudence. With anchoring, the equivalent of tying Ulysses to the mast does not exist as the opposite of caution and prudence is required to deal with this bias.

With anchoring, as this example illustrates, Hansen Jagannathan bound changes. The present value of the Sharpe ratio is no longer the lower bound for the standard deviation of the stochastic discount factor for anchoring influenced assets. Equity returns rise. The rise is substantially greater for anchoring prone assets. The risk free rate falls due to an increase in perceived aggregate risk. This looks promising regarding the equity premium puzzle. To directly address whether anchoring explains the equity premium puzzle, I consider the general case in the next section. I find that anchoring not only provides a plausible explanation for the equity premium puzzle, but also provides a unified explanation for related asset pricing puzzles.

3. Incorporating Anchoring in Asset Pricing

I consider a simple exchange economy with only two points in time, now and the future. (Mathematically, the first-order conditions from a multi-period version must decompose anyway into an overlapping sequence of first-order conditions from the two-points-in-time model. However, such details are not needed for the main message of this article, so I avoid creating unnecessary clutter of notation which could be distracting.). The purpose is to study whether anchoring can explain the asset pricing puzzles. For this purpose, I use a stark model in which everything except the most basic structure has been set aside.

As in the last section, consider an exchange economy described by a representative agent who maximizes:

$$U(c_t) + \beta E_t[c_{t+1}]$$

subject to:

$$c_t = e_t - \sum_{i=1}^{N} n_i P_i$$

$$\tilde{c}_{t+1} = e_{t+1} + \sum_{i=1}^{N} n_i \tilde{X}_i$$

The total number of asset types is N. The other symbols have the same meanings as in the last section. In other words, the agent receives an endowment from which he purchases financial assets as well as consumes. The assets generate payoffs in the future, which along with any future endowment, are consumed.

For each asset type, the following must be true in equilibrium:

$$P_i = E_t[SDF_h \cdot X_h] \tag{3.1}$$

where $SDF_h = \frac{\beta U'(c_{t+1h})}{U'(c_t)}$ evaluated at the optimal allocation, and h is the state index. In general, SDF_h with anchoring is different from SDF_h with omniscience as the state-wise distribution of perceived aggregate risk is different in the two cases.

For ease of reference, I label a well-established asset that supplies the starting payoff distribution as the "leader" stock. The stocks for which the representative agent perceives Knightian uncertainty are termed the "normal" stocks.

Well-established stocks have larger payoff variances (standard deviations) than their respective follower stocks. This is due to their larger payoff sizes. Of course, one expects the leader stocks to have lower return variances (standard deviations) than their respective follower stocks. This is due to the larger prices of leader stocks. As an illustration of this feature, suppose the possible payoffs of the leader firm stock, in the next period, are 300, 350, and 400 with equal chance of each. The variance of these payoffs can be calculated easily and is equal to 1666.667. In a risk-neutral world, with zero risk-free interest rate, the price must be 350, so corresponding (gross) returns are: 0.857, 1, 1.143. Hence, the return variance is 0.010. Assume that the next period payoffs of the normal firm are 0, 35, and 70. The variance of these payoffs is 816.667. The risk neural price (with zero risk-free rate) is 35 leading to possible returns of 0, 1, and 2. The corresponding return variance is 0.66. As can be seen in this example, the payoff variance of the normal firm stock is smaller than the payoff variance of the leader firm stock, whereas the return variance of the normal firm is much larger.

In sections 3.1, 3.2, and 3.3, the following three cases are described:

1) One leader stock and one normal stock

2) One leader stock and many normal stocks

3) Many leader and many normal stocks

3.1 One Leader and One Normal Stock

This is the simplest case as there are only three assets in the market: two risky assets, and one risk-free asset. Using L for the leader stock, S for the normal stock and F for the risk-free asset, their respective prices in equilibrium (from (3.1)) must be:

$$P_L = \frac{E[X_L]}{R_F} + \rho_L \cdot \sigma(SDF) \cdot \sigma(X_L) \tag{3.2}$$

$$P_s = \frac{E[X_s]}{R_F} + \rho_s \cdot \sigma(SDF) \cdot \sigma^A(X_s) \tag{3.3}$$

$$P_F = E[SDF] \cdot X_F \tag{3.4}$$

where ρ_i is the correlation coefficient of asset i with the SDF. The superscript A indicates that the standard deviation of the normal stock payoffs is anchoring influenced. In particular, the following holds:

$$\sigma^A(X_s) = (1-m)\sigma(X_L) + m\sigma(X_s) \tag{3.5}$$

where m is the fraction of correct distance the representative agent goes while starting from the standard deviation of the leader firm's payoffs. Note that with $m = 1$, there is no anchoring bias.

One feature of (3.5) is the following: the greater the distance between $\sigma(X_L)$ and $\sigma(X_s)$, higher would be the magnitude of error in estimating the standard deviation, given m. Starting from $\sigma(X_L)$, one needs to cover a greater distance if $\sigma(X_s)$ is further away. This is consistent with empirical evidence from psychology literature that the magnitude of error is higher when the judgment task is more difficult (see Kudryavtsev and Cohen (2010)). Reasonably, a firm which is less like the leader firm is more difficult to value than a firm which is more similar to the leader firm. The formulation in (3.5) automatically captures this feature. An example clarifies this point. Suppose the leader stock has a payoff volatility of 100, whereas there are two other stocks with payoff volatilities 50 and 10 respectively. Suppose the anchoring prone marginal investor is able to go 90% of the way in each case, while making adjustments. That is, $m = 0.90$. The adjustment term in the first case is 45, and in the second case is 81. The estimated volatility in the first case is 55, which implies an error of 5. The estimated volatility in the second case is 19, which implies a larger error of 9. That is, greater the distance between a normal firm and the associated leader firm, larger is the magnitude of error, given the intensity of anchoring (as long as m is greater than 0 and less than 1).

Substituting (3.5) in (3.3) and re-arranging leads to:

$$E[R_s] = R_F - \rho_s \cdot \sigma(SDF) \cdot \sigma(R_s) \cdot R_F - \rho_s \cdot \sigma(SDF) \cdot R_F \cdot (1-m) \cdot \frac{\left(\sigma(X_L) - \sigma(X_s)\right)}{P_s} \qquad (3.6)$$

$$=> \frac{E[R_s] - R_F}{R_F \cdot \left\{\sigma(R_s) + \dfrac{(1-m)\left(\sigma(X_L) - \sigma(X_s)\right)}{P_s}\right\}} = -\rho_s \cdot \sigma(SDF) \qquad (3.7)$$

As almost all stocks have expected returns higher than the risk-free rate, one can safely assume that $-1 \leq \rho_s < 0$. So, in the rest of the article, from this point onwards, for simplicity and ease of exposition, I assume that all stock payoff correlations with the SDF are negative. It follows that,

$$\frac{E[R_s] - R_F}{R_F \cdot \left\{\sigma(R_s) + \frac{(1-m)\big(\sigma(X_L) - \sigma(X_s)\big)}{P_s}\right\}} = |\rho_s| \cdot \sigma(SDF) \tag{3.8}$$

So,

$$\frac{E[R_s] - R_F}{R_F \cdot \left\{\sigma(R_s) + \frac{(1-m)\big(\sigma(X_L) - \sigma(X_s)\big)}{P_s}\right\}} \leq \sigma(SDF) \tag{3.9}$$

Hence, the Hansen-Jagannathan bound is no longer valid for the normal stock, and is replaced by (3.9). It is straightforward to check that the Hansen-Jagannathan bound remains valid for the leader stock and is given by[8]:

$$\frac{E[R_L] - R_F}{R_F \cdot \{\sigma(R_L)\}} \leq \sigma(SDF) \tag{3.10}$$

It is easy to see that the aggregate market portfolio satisfies:

$$\frac{E[R_M] - R_F}{R_F \cdot \left\{\sigma(R_M) + \frac{|\rho_s|}{|\rho_M|}\frac{(1-m)\big(\sigma(X_L) - \sigma(X_s)\big) \cdot n'_s}{P_M}\right\}} = |\rho_M| \cdot \sigma(SDF) \tag{3.11}$$

where R_M is the return on the market portfolio, ρ_M is the correlation of the market portfolio's return with the SDF, and n'_s is the number of shares of the normal stock outstanding. It follows that

$$\frac{E[R_M] - R_F}{R_F \cdot \left\{\sigma(R_M) + \frac{|\rho_s|}{|\rho_M|}\frac{(1-m)\big(\sigma(X_L) - \sigma(X_s)\big) \cdot n'_s}{P_M}\right\}} \leq \sigma(SDF) \tag{3.12}$$

[8] However, the volatility of SDF must be large than what consumption data implies.

3.1 One Leader and Many Normal Stocks

It is easy to extend the anchoring approach to a situation in which there is one well-established stock and a large number of normal stocks. Suppose there are k types of normal stocks. By closely following the same steps as in the previous section, I obtain the following lower bound with the aggregate market portfolio:

$$\frac{E[R_M]-R_F}{R_F\cdot\left\{\sigma(R_M)+\sum_{i=1}^{k}\frac{|\rho_{si}|}{|\rho_M|}\frac{(1-m)\big(\sigma(X_L)-\sigma(X_{si})\big)\cdot n'_{si}}{P_M}\right\}} \le \sigma(SDF) \quad (3.13)$$

3.2. Many Leader and Many Normal Stocks

It is natural to expect that every sector has its own leader firm whose stock is used as a starting point to form judgments about other firms in the same sector. I assume that there are Q sectors and every sector has one leader firm. I assume that the number of normal firms in every sector is k. That is, the total number of normal firms in the market is $Q\times k$. As the total number of leader firms is Q. The total number of all firms (both leader and normal) in the market is $Q+(Q\times k)$.

Following a similar set of steps as in the previous two sections, I obtain the following lower bound with the aggregate market portfolio:

$$\frac{E[R_M]-R_F}{R_F\cdot\left\{\sigma(R_M)+\sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{|\rho_{sqi}|}{|\rho_M|}\frac{(1-m)\left(\sigma(X_{Lq})-\sigma(X_{sqi})\right)\cdot n'_{sqi}}{P_M}\right\}} \le \sigma(SDF) \quad (3.14)$$

The expected return on the market portfolio is given by:

$$E[R_M]=R_F+|\rho_M|\cdot\sigma(SDF)\cdot\sigma(R_M)\cdot R_F+\sigma(SDF)\cdot R_F \cdot\sum_{q=1}^{Q}\sum_{i=1}^{k}|\rho_{sqi}|\frac{(1-m)\left(\sigma(X_{Lq})-\sigma(X_{sqi})\right)\cdot n'_{sqi}}{P_M} \quad (3.15)$$

The price of the market portfolio is given by:

$$P_M = \frac{E[X_M]}{R_F} - |\rho_M| \cdot \sigma(SDF) \cdot \sigma(X_M) - \sigma(SDF) \cdot \sum_{q=1}^{Q} \sum_{i=1}^{k} |\rho_{sqi}| (1-m) \left(\sigma(X_{Lq}) - \sigma(X_{sqi})\right) \cdot n'_{sqi} \qquad (3.16)$$

where n'_{sqi} is the number of shares outstanding of the normal stock i belonging to sector q.

In the next section, I use (3.14), (3.15), and (3.16) to show that anchoring provides a plausible unified explanation for 9 asset pricing puzzles including the equity premium puzzle.

4. Anchoring and Asset Pricing Puzzles

The standard consumption-based asset pricing model is a general equilibrium model that assumes a representative agent who is omniscient (accurately knows the payoff distribution of every asset in the market). The empirical record of this model is quite poor and a large number of phenomena exist that are inconsistent with its predictions. I argue, in this article, that the reason for this poor performance is the unreasonable assumption of omniscience. It is likely that investors face Knightian uncertainty, especially for newer stocks. I assume that investors face Knightian uncertainty for less established stocks and use the payoff distribution of similar well-established stocks as starting points that are adjusted for forming judgments about the payoff distributions of newer stocks. Anchoring bias implies that such adjustments tend to be insufficient.

In this section, I show that replacing the assumption of an omniscient representative agent with the assumption that the representative agent is anchoring-prone provides a plausible unified explanation for the following puzzles: 1) High equity premium, 2) Low risk free rate, 3) Countercyclical equity premium, 4) High stock price volatility, 5) Size effect, 6) Value effect

7) Momentum effect, 8) Abnormal positive returns and high volatility after stock-splits, 9) Abnormal negative returns and low volatility after reverse stock-splits.

4.1 The Equity Premium Puzzle

If there is no anchoring bias, then the following must be true (Hansen and Jagannathan (1991)):

$$\frac{E[R_M] - R_F}{R_F \cdot \{\sigma(R_M)\}} \leq \sigma(SDF) \tag{4.1}$$

The equity premium puzzle, first identified in Mehra and Prescott (1985), can be easily seen with the above formulation. The historical average return on US equity market is 7%, the average risk free rate is 1%, and the historical average standard deviation of returns is 18%. With these values, the L.H.S in (4.1) is equal to 0.33. The R.H.S estimated from consumption data is around 0.02. Hence, there is a very wide gap between the L.H.S and the R.H.S. This is the equity premium puzzle.

With anchoring, the corresponding lower bound is given in (3.14). That is, there is an additional term in the denominator. The additional term is $\sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{|\rho_{sqi}|}{|\rho_M|}\frac{(1-m)\left(\sigma(X_{Lq})-\sigma(X_{sqi})\right)\cdot n'_{sqi}}{P_M}$. Anchoring provides a plausible explanation for the equity premium puzzle if for reasonable values for this term, the R.H.S and the L.H.S in (3.14) are equal to each other. I create a higher obstacle for anchoring by assuming that $\sigma(SDF)$ is correctly estimated from consumption data to be 0.02 even though with anchoring the perceived $\sigma(SDF)$ must be higher.

With the above historical values, if the anchoring term is 2.79, then the L.H.S and the R.H.S in (3.14) are equal to each other. In order to make things harder for the anchoring explanation, while choosing values below, I always err on the side of choosing values that make this term smaller.

There are about 5000 listed firms in the US equity market. Assuming that 5% of these are well-established, we get 4750 as the number of firms that are influenced by anchoring. Studies suggest that less than 4% of the firms are considered as blue-chip or well-established. Hence, my estimate of the number of anchoring prone stocks is on the conservative side. I set the anchoring parameter at $m = 0.98$. That is, the anchoring bias is kept quite small at only 2%. Continuing to make things difficult for the anchoring explanation, I underestimate typical $\sigma(X_{Lq}) - \sigma(X_{sqi})$ by assuming that a typical leader firm has a payoff standard deviation only 2 times larger than a typical normal firm. I assume that a typical normal firm has only 1000 shares outstanding, and the value of market portfolio is overestimated to be worth 30000 times the typical standard deviation of a normal stock. Even with such large values chosen to create harder obstacles for the anchoring explanation, the anchoring term is equal to 2.85 if typical $\frac{|\rho_{sqi}|}{|\rho_M|}$ is 0.45.

To try another value, if $\sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{|\rho_{sqi}|}{|\rho_M|}\frac{(1-m)\left(\sigma(X_{Lq})-\sigma(X_{sqi})\right)\cdot n'_{sqi}}{P_M}$ is 1.5, then a reasonable value of 0.035 follows for $\sigma(SDF)$. With $\sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{|\rho_{sqi}|}{|\rho_M|}\frac{(1-m)\left(\sigma(X_{Lq})-\sigma(X_{sqi})\right)\cdot n'_{sqi}}{P_M}=2$, a value of 0.0275 follows for $\sigma(SDF)$. Hence, the equity premium puzzle is surprisingly easy to explain with anchoring.

4.2 The Low Risk Free Rate Puzzle

Weil (1989) is the first to point out that in a standard consumption-based asset pricing model, the risk-free rate is counterfactually high. With anchoring, we do not need to invoke high risk aversion to explain the equity premium puzzle, so the problem of exacerbating the risk-free rate puzzle by using a high risk-aversion parameter is avoided. In fact, the risk-free rate tends to be quite low naturally with anchoring as the perceived aggregate risk is high, which pushes up the price of the risk-free asset lowering the risk-free return. This has also been illustrated with the example in section 1.

To put things into perspective, the risk-free rate is the inverse of $E[SDF]$. It is easy to see that $E[SDF]\approx\sigma[SDF]\cdot f$. Without anchoring, f is the inverse Sharpe-ratio. With anchoring, f is substantially larger and is given by $\frac{\sigma(R_M)+\sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{|\rho_{sqi}|}{|\rho_M|}\frac{(1-m)\left(\sigma(X_{Lq})-\sigma(X_{sqi})\right)\cdot n'_{sqi}}{P_M}}{E[R_M]-R_F}$. Using a value of 1.5 for $\sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{|\rho_{sqi}|}{|\rho_M|}\frac{(1-m)\left(\sigma(X_{Lq})-\sigma(X_{sqi})\right)\cdot n'_{sqi}}{P_M}$ and 0.0355 for $\sigma[SDF]$ as in the last sub-section, the risk-free rate of approximately 1% follows. Hence, the risk-free rate puzzle is also easily resolved with anchoring.

4.3 The Countercyclical Equity Premium/Sharpe Ratio

With anchoring, the return on the market portfolio is given in (3.15). That is, there is an additional term which is equal to $\sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{|\rho_{sqi}|}{|\rho_M|}\frac{(1-m)\left(\sigma(X_{Lq})-\sigma(X_{sqi})\right)\cdot n'_{sqi}}{P_M}$. This additional term is countercyclical as volatilities are higher and market portfolio worth less in recessions. Assuming that standard deviations increase by a factor g, and price falls by a factor h, the anchoring term is higher by a factor of $g \cdot h$ in recessions. Hence, a countercyclical equity premium is consistent with anchoring.

4.4 High Stock Price Volatility

Shiller (1981) and LeRoy and Porter (1981) show that market prices are much more volatile than what can be justified by fundamentals. This feature is also easily seen with anchoring. Anchoring lowers the price of equity. That is, the anchoring price is less than the what can be justified based on fundamentals ex-post. It follows that the equity return volatility must necessarily be larger than what can be justified by fundamentals. This is because $\sigma(R_s) = \frac{\sigma(X_s)}{P_s}$.

With anchoring, the price of a normal firm stock is given by:

$$P_s = \frac{E[X_s]}{R_F} - |\rho_s| \cdot \sigma(SDF) \cdot \sigma(X_s) - |\rho_s| \cdot \sigma(SDF) \cdot (1-m) \cdot \left(\sigma(X_L) - \sigma(X_s)\right) \qquad (4.2)$$

That is, price is influenced by an additional term which is equal to $(1-m) \cdot \left(\sigma(X_L) - \sigma(X_s)\right)$. Hence, news unrelated to fundamentals of a given stock, that is, idiosyncratic news only related to the leader stock in the sector also influences the stock price. As most news coverage and analyst attention (83%) is on a small number of well-established firms (about 4% of the total number of publicly listed firms), rest of the market also reacts to this news, which is unrelated to particular fundamentals of anchored firms.

At the aggregate level, the price of the market portfolio is given in (3.16). That is, there is an additional term equal to $\sigma(SDF) \cdot \sum_{q=1}^{Q}\sum_{i=1}^{k}|\rho_{sqi}|(1-m)\left(\sigma(X_{Lq}) - \sigma(X_{sqi})\right) \cdot n'_{sqi}$. As most news is generated about well-established stocks, this additional term becomes an additional source of volatility, and magnifies the impact of news.

4.5 Size, Value, and Momentum Effects

Expected return on a normal firm stock with anchoring is given by:

$$E[R_s] = R_F + |\rho_s| \cdot \sigma(SDF) \cdot R_F \left\{\sigma(R_s) + \frac{(1-m)\big(\sigma(X_L)-\sigma(X_s)\big)}{P_s}\right\} \tag{4.3}$$

Keeping all else the same, smaller size payoffs (of small-cap firm stocks) mean lower price and lower $\sigma(X_s)$. That is, the additional term due to anchoring $\frac{(1-m)\big(\sigma(X_L)-\sigma(X_s)\big)}{P_s}$ rises with smaller size. Hence, anchoring is consistent with the size premium: small-cap stocks tend to outperform large-cap stocks.

Value premium means that growth stocks tend to underperform value stocks. Keeping price the same, a growth stock has lower book value of equity when compared with a value stock. This is typically due to a smaller asset base, which is fast growing due to aggressive investments made by the growth firm. This implies a higher payoff standard deviation for the growth stock in (4.3). The additional term due to anchoring falls as $\sigma(X_s)$ rises. Hence, anchoring is consistent with the value premium as well. Alternatively, keeping book value the same, growth stocks have higher prices and payoff volatilities, which lower the anchoring term.

According to (4.3), in a given cross-section of stocks, keeping everything else the same, low "m" stocks do better than high "m" stocks. But, how can we identify low vs high "m" stocks? Plausibly, we can identify them by looking at their recent performances. Stocks that have received unusually good news recently are "winning stocks", and stocks that have received unusually bad news recently are "losing stocks". Winning stocks are likely to get more strongly anchored to the leader stock as their recent success makes them more like the leader. For losing stocks, their recent bad spell makes them less like the leader. That is, "m" falls for winning stocks and rises for losing stocks. So, winning stocks continue to outperform losing stocks till the effect of differential news on "m" dissipates, and "m" returns to its normal level. This is the momentum effect. This effect is also consistent with the anchoring model.

4.6 Stock-Splits and Reverse Stock-Splits

Stock-splits and reverse stock-splits appear to be merely accounting changes. A stock-split increases the number of shares proportionally. In a 2-for-1 split, a person holding one share now holds two shares. In a 3-for-1 split, a person holding one share ends up with three shares and so on. A reverse stock-split is the exact opposite of a stock-split. Stock-splits and reverse stock- splits appear to be merely changes in denomination, that is, they seem to be accounting changes only with no real impact on returns. With consumption based asset pricing without anchoring, the impact of a stock-split on the equilibrium price of stock i can be seen in the following equation:

$$P_i = \frac{E[X_i]}{R_F} - |\rho_i| \cdot \sigma(SDF) \cdot \sigma(X_i) \tag{4.4}$$

A 2-for-1 split divides the standard deviation as well as the mean of payoffs by 2, so the price gets divided by 2 also. As both the price and the expected payoff are divided by 2, there is no change in expected return. As both the standard deviation of payoffs and the price are divided by 2, there is no change in the standard deviation of returns either. Hence, a stock-split and a reverse stock-split should not change the expected return or the standard deviation of returns, according to the standard CCAPM.

The situation is considerable different with anchoring adjusted CCAPM. The equilibrium price of a 'normal' stock is now given by the following equation:

$$\begin{aligned} P_i = \frac{E[X_i]}{R_F} - |\rho_i| \cdot \sigma(SDF) \cdot \sigma(X_i) - |\rho_i| \cdot \sigma(SDF) \cdot (1-m) \\ \cdot \left(\sigma(X_L) - \sigma(X_i)\right) \end{aligned} \tag{4.5}$$

Due to the presence of an additional term in (4.5) when compared with (4.4), dividing the expected payoff and the standard deviation of $i's$ payoff by 2 lowers the price beyond division by 2. As price gets divided by more than 2, and the expected payoff and the standard deviation of payoff get divided by 2, both the expected return and the standard deviation of returns should rise after a split. The opposite conclusion holds for a reverse stock-split. The expected return as well as the standard deviation of returns should fall after a reverse stock-split.

Empirical evidence strongly supports the above predictions. Using data from 1975 to 1990, Ikenberry et al (1996) shows that stock-splits are associated with 8% positive abnormal returns after one year, and 16% abnormal returns over three years. Ikenberry et al (2003) uses data from 1990 to 1997 and confirms the earlier findings. Gharghori et al (2015) find that option market traders do expects an increase in return volatility after the split. Kim et al (2008) examine the long-run performance of 1600 firms with reverse stock-splits and reports negative abnormal returns. Illahi (2012) finds that return volatility decreases after a reverse stock-split. Hence, anchoring provides a potential explanation for the empirical findings regarding stock-splits and reverse stock-splits.

5. Conclusions

The standard consumption-based asset pricing model is a general equilibrium model which assumes an omniscient representative agent who is able to form correct expectations regarding the future payoff distributions of all available assets. Clearly, this is an unreasonable assumption. It is much more plausible to assume that investors face Knightian uncertainty, especially when it comes to relatively newer firms of smaller size. I assume that, when faced with Knightian uncertainty, investors use the payoff distributions of similar well-established stocks as starting points which are then adjusted to form the required judgments. Anchoring bias implies that such adjustments typically fall short. I propose only one change in the standard model: The replacement of an omniscient representative agent with an anchoring-prone representative agent. I show that this change is sufficient to provide a plausible unified explanation for 9 asset pricing puzzles including the equity premium puzzle.

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