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# The Intensity of Competition in the Hotelling Model: A New Generalization and Applications\*

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## Abstract

I develop a simple Hotelling model which relates the distribution of consumer preferences to the intensity of competition. I impose two properties, mean preserving spread (MPS) and monotone likelihood ratio property (MLRP), on distribution functions. These properties provide a way to represent the intensity of competition in the Hotelling model. Market competition is less intense as the distribution is dispersed in that the MPS raises firms' equilibrium prices. This approach can describe how the intensity of competition influences the effects of firm's various strategies, which has been largely neglected in most papers. Non-uniform distributions can reverse some well-known results derived under the uniform distribution dramatically. They also allow us to discover new results that the uniform distribution could not demonstrate. As examples, I study three issues such as incentives for innovation, preference based price discrimination, and incentives for information sharing.

JEL Classification: D43, D82, L11, L13

Keywords: Hotelling model, intensity of competition, mean-preserving spread (contraction), monotone likelihood ratio property, innovation, preference-based price discrimination, information sharing

## 1. Introduction

The Hotelling model has been extensively used by economists for analyzing various issues in oligopoly markets. A typical assumption in this model is that

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consumer preferences are uniformly distributed. There may be some reasons for this simple assumption. One obvious reason is that it ensures simple closed-form solutions. In addition, a general distribution may not provide further interesting results in many papers.

However, the shape of the distribution of consumer preferences is important in defining market competition. Let us consider two extreme cases. If consumers are indifferent between two products, we have a degenerate distribution which is localized at the middle point in the Hotelling model. In this case, products are homogenous and the market is perfectly competitive. Conversely, if consumers have a significantly strong preference to one good over the other, we can have a two-point distribution in that consumers are located on two end points. Then competition disappears and firms are able to set monopoly prices for their loyal consumers. Therefore, one can hypothesize that any intermediate level of distribution of consumer preferences may represent an intermediate degree of competition between monopoly and perfect competition.

The basic claim of this paper is that the distribution of consumer preferences corresponds to the intensity of market competition. In order to show this, I analyze how a mean-preserving spread (MPS) of distributions changes equilibrium prices. A mean-preserving spread implies that a greater proportion of consumers has a higher relative preference for one good over the other. I find that the MPS raises firms' equilibrium prices even when two firms are asymmetric. Market competition is less intense as the distribution is dispersed. Hence the mean-preserving transformation stands for a change in the intensity of competition in the Hotelling model. This is a new perspective, and a new generalization, on the distribution in the Hotelling model.

One may argue that transportation costs or the size of product differentiation plays the same role for indexing the degree of competition in the Hotelling model.<sup>1</sup> However there is a huge difference between two approaches. A more efficient firm's relative advantage over its rival is changed by the mean-preserving transformation, while it is not by transportation costs.<sup>2</sup> Namely, my approach can demonstrate how the intensity of competition affects competing firms' relative position in the market competition. This feature is very important to analyze the effects of firms' various strategies. When a firm plays a particular strategy to achieve the relative advantage over its rival, the effect of the firm's strategy depends on the intensity of market competition. However, this aspect has been surprisingly neglected in most papers.<sup>3</sup>

As applications and examples, I will show how non-uniform distributions can

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<sup>1</sup>When transportation costs are zero, the model exhibits perfect competition between homogeneous goods. On the contrary, when it is significantly high, two firms become local monopolists.

<sup>2</sup>We will see that the reason is that the equilibrium competitive front is changed by the mean-preserving transformation of consumers' distribution. But it is invariant to transportation costs.

<sup>3</sup>In fact, we may be able to generalize the function of transportation costs, and show the similar analysis. At least technically, this way can be equivalent to considering non-uniform distributions of consumer preferences. But this approach is less intuitive in economic meanings.

reverse some well-known results derived from the uniform distribution. Some acclaimed results in the literature will be changed dramatically by considering the intensity of market competition. Moreover, non-uniform distributions allow us to discover new results that the uniform distribution could not obtain.

First, I study how the intensity of competition influences incentives to innovate. I find that firms have more incentives for R&D as a market becomes more competitive. In other words, more aggressive competition in product market induces more R&D competition. The reason is that the size of relative advantage obtained by innovation is greater as competition increases. But firms' R&D incentives do not depend on the size of transportation costs in this model.

Second, I examine preference-based price discrimination under non-uniform distributions of consumer preferences. The prevailing literature on this issue shares one important result that price discrimination based on consumer preferences is not a profitable strategy. However, a noteworthy result here is that firms can have higher profits from price discrimination in an intensely competitive market such as an inverse U-shaped distribution. In this case, price discrimination based on consumer preferences softens market competition by letting firms compete on less competitive fronts.

Last, I investigate how the intensity of competition affects incentives for information sharing. Most papers in the literature are based on the model with a linear demand, whereby equilibrium profit functions are always convex. In this setting, for example, Gal-Or (1986) shows that firms prefer concealing private information about costs in Bertrand competition, while information sharing in Cournot competition. However, once again, I obtain the opposite result. When market competition is sufficiently mild such as a U-shaped distribution, the equilibrium profit function can be concave, and so information transmission would be preferred in price competition.

There are a few papers which take into account non-uniform distributions or analyze the transformation of consumers' distribution in the Hotelling model. Under non-uniform distributions of consumer preferences, Shilony (1981) and Neven (1986) examine firms' choices of their location, and Bester (1992) shows the existence and uniqueness of the equilibrium in price competition.<sup>4</sup> But these papers do not study the effects of transformation of consumers' distribution. Bloch and Manceau (1999) analyzes the effect of persuasive advertising in the Hotelling model. In their paper, only one firm advertises, and so the distribution shifts towards the advertised product in the way of first-order stochastic dominance (FOSD). But the paper is based on a restricted class of distributions, and the effect of FOSD transformation on equilibrium prices is ambiguous. The paper does not display the systematic variation in equilibrium prices.

In a different vein, Johnson and Myatt show that the dispersion of consumers' valuations leads to rotations of a demand curve. A remarkable result is that a monopolist prefers low dispersion when serving a mass market, while high dispersion when pursuing a small niche market. I believe that this paper is

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<sup>4</sup>Shilony (1981) and Neven (1986) show the similar result that firms may tend to locate inside the market as the distribution becomes more concentrated.

complementary to their paper in understanding the role of consumer preferences more fully. They find that the dispersion of consumers' absolute valuations induces demand rotations, and study some implications in the monopolist's strategies. In contrast, my paper finds that the dispersion of consumers' relative preferences between competing firms results in softening market competition, and studies how the intensity of competition influences the effects of firms' strategies.

The rest of the article is organized as follows. In Section 2, I formally define MPS and MLRP in the Hotelling model and characterize the equilibrium. In Section 3, I introduce three examples to show the importance of considering non-uniform distributions of consumer preferences. Finally, concluding remarks follow in Section 4.

## 2. Basic Model

Consider a simple market with two competing firms. Each firm produces goods  $A$  and  $B$  with a constant marginal cost of  $c_A$  and  $c_B$  per unit respectively. Each consumer is indexed by  $\theta \in [\underline{\theta}, \bar{\theta}]$ , where  $\underline{\theta} = -\bar{\theta} < 0$ .  $\theta$  represents a consumer's relative preference for the product  $B$  over  $A$ . Consumers are distributed by cumulative distribution function  $F$  over  $\theta$ . The density function  $f$  is symmetric at zero. To ensure nice demand curves, I assume that a hazard rate  $\frac{f(\theta)}{1-F(\theta)}$  is strictly increasing in  $\theta$ . By this assumption, the second order condition is automatically satisfied by the first order condition.<sup>5</sup>

*Assumption 1.* Monotone hazard rate (MHR)

$$\frac{d}{d\theta} \left( \frac{1-F(\theta)}{f(\theta)} \right) \leq 0 \text{ and } \frac{d}{d\theta} \left( \frac{F(\theta)}{f(\theta)} \right) \geq 0.$$

The symmetry of  $f$  ensures the second inequality. Now I consider a sequence of distribution functions,  $(F_0, F_1, \dots, F_k, \dots)$ . They are ordered by mean-preserving spreads as follows.

*Assumption 2.* Mean-preserving Spread (MPS). For  $\theta \in [\underline{\theta}, \bar{\theta}]$  and  $\int_{\underline{\theta}}^{\bar{\theta}} \theta f_k(\theta) d\theta =$

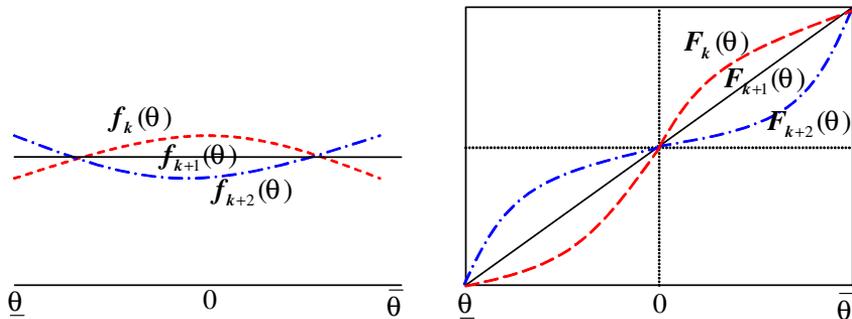
$$\int_{\underline{\theta}}^{\bar{\theta}} \theta f_{k+1}(\theta) d\theta,$$

$$\int_{\underline{\theta}}^{\theta} F_k(x) dx \leq \int_{\underline{\theta}}^{\theta} F_{k+1}(x) dx$$

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<sup>5</sup> Assumption 1 ensures upward reaction functions as well, i.e.,  $\frac{\partial^2 \pi_i}{\partial p_j \partial p_i} \geq 0$ . That is to say, two goods are strategic complements.

Assumption 2 means that a high order distribution is a mean-preserving spread of a low order distribution. The economic interpretation of a mean-preserving spread (contraction) is that the density of consumers with strong preference to one good over the other increases (decreases) in  $k$ . To put it differently, the proportion of loyal consumers grows. Figure 1 represents some possible density functions and corresponding distributions.



[Figure 1]

Since the mean is preserved, all possible distribution functions cross at  $\theta = 0$ . They can cross more than once. But I consider the case in which MPS moves density from the center toward the both tails smoothly, as presented in Figure 1. Formally, to establish the smooth change of MPS, I further assume a monotone likelihood ratio property on the sequence of distributions as follows.

*Assumption 3.* Monotone likelihood ratio property (MLRP)

$$\frac{f_{k+1}(\theta_0)}{f_k(\theta_0)} \leq \frac{f_{k+1}(\theta_1)}{f_k(\theta_1)} \text{ for } \theta_0 \leq \theta_1 \in [0, \bar{\theta}] \text{ and}$$

$$\frac{f_{k+1}(\theta_0)}{f_k(\theta_0)} \geq \frac{f_{k+1}(\theta_1)}{f_k(\theta_1)} \text{ for } \theta_0 \leq \theta_1 \in [\underline{\theta}, 0].$$

Then MPS results in the single crossing at 0 between distribution functions for the interval  $\theta \in (\underline{\theta}, \bar{\theta})$ . The literal meaning of this property is that the likelihood of getting  $|\theta|$  in  $F_{k+1}$  relative to the likelihood of getting  $|\theta|$  in  $F_k$  increases in  $|\theta|$ . When a MPS occurs, we have a stochastically larger density of consumer with higher relative preferences. We will see that this property ensures monotonic changes in firms' reactions functions and equilibrium prices.<sup>6</sup> The following lemma will be often used in future analyses.

<sup>6</sup>MLRP is widely used in the literature of contract theory and auction theory. Its important role is ensuring the optimal compensation scheme to be monotonically increasing and allowing comparison between bidding prices across different bidders or expected revenues across various types of auction.

*Lemma 1.* MLRP implies

$$\begin{array}{ll}
 \text{For } \theta \in [\underline{\theta}, 0] & \text{For } \theta \in [0, \bar{\theta}] \\
 \frac{1 - 2F_k(\theta)}{f_k(\theta)} \geq \frac{1 - 2F_{k+1}(\theta)}{f_{k+1}(\theta)} & \frac{1 - 2F_k(\theta)}{f_k(\theta)} \leq \frac{1 - 2F_{k+1}(\theta)}{f_{k+1}(\theta)} \quad (1) \\
 \frac{F_k(\theta)}{f_k(\theta)} < \frac{F_{k+1}(\theta)}{f_{k+1}(\theta)} & \frac{1 - F_k(\theta)}{f_k(\theta)} < \frac{1 - F_{k+1}(\theta)}{f_{k+1}(\theta)} \quad (2) \\
 F_k(\theta) \leq F_{k+1}(\theta) & F_k(\theta) \geq F_{k+1}(\theta) \quad (3)
 \end{array}$$

Proofs are provided in the Appendix. The results are fairly well-known stochastic orders. The inequalities in (2) are often called reverse hazard rate and hazard rate dominance respectively. The inequalities in (3) show a first-order stochastic dominance (FOSD) relationship. The MLRP implies (reverse) hazard rate dominance, which implies the FOSD. Note that a high order distribution dominates a low order distribution in the sense of the FOSD for positive  $\theta$ , whereas the opposite relationship holds for negative  $\theta$ .

From now on, in order to reduce repetition in notations, I use  $F$  and  $f$  without subscript as a representative of distribution and density function respectively. In this simple model, consumers  $\theta < \tilde{\theta} = p_B - p_A$  choose to buy good  $A$ , whereas consumers  $\theta \geq \tilde{\theta}$  choose to buy good  $B$ .<sup>7</sup> The profit function of each firm is given by

$$\pi_A = (p_A - c_A)F(\tilde{\theta}) \text{ and } \pi_B = (p_B - c_B)(1 - F(\tilde{\theta})).$$

Each firm's reaction function is

$$p_A(p_B) = c_A + \frac{F(\tilde{\theta})}{f(\tilde{\theta})} \text{ and } p_B(p_A) = c_B + \frac{1 - F(\tilde{\theta})}{f(\tilde{\theta})}.$$

The solution for two reaction functions must satisfy

$$\theta^* = \Delta + \frac{1 - 2F(\theta^*)}{f(\theta^*)}, \quad (4)$$

where  $\Delta = c_B - c_A$ .  $\theta^*$  indicates the location of marginal consumers at the equilibrium. With this, we can describe firms' equilibrium prices and profits.

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<sup>7</sup>I assume throughout that the value of the goods is sufficiently high so that each consumer buys one unit of either of two goods.

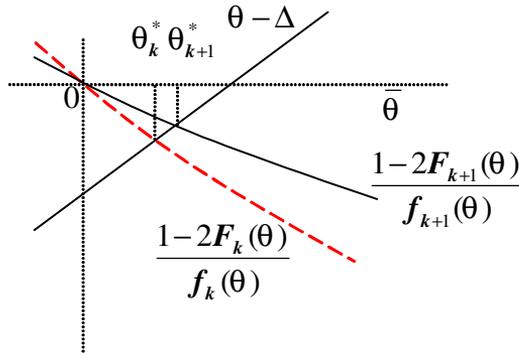
*Proposition 1.* Equilibrium prices and corresponding profits are

$$p_A = c_A + \frac{F(\theta^*)}{f(\theta^*)} \text{ and } p_B = c_B + \frac{(1 - F(\theta^*))}{f(\theta^*)}, \text{ and}$$

$$\pi_A = \frac{F(\theta^*)^2}{f(\theta^*)} \text{ and } \pi_B = \frac{(1 - F(\theta^*))^2}{f(\theta^*)}.$$

Equilibrium prices are represented by the ratio of each firm's market share and the density of marginal consumers at the equilibrium. The symmetric outcome for  $c_A = c_B = c$  is simplified by  $p_i^* = c + \frac{1}{2f(0)}$  and  $\pi_i^* = \frac{1}{4f(0)}$ . This representation of equilibrium prices and profits shows noteworthy information. They depend crucially on the density of marginal consumers at the equilibrium. Note that an increase in the proportion of marginal consumers brings more aggressive competition. The intuition to understand this result is simple. When a firm charges a slightly lower price than its rival, it can attract more consumers. Thus firms' incentives to reduce prices are greater as the density of marginal consumers is greater.

However, if we use the uniform distribution,  $f(\theta) = \frac{1}{2\bar{\theta}}$ , the symmetric equilibrium prices are  $p_A = c + \bar{\theta}$  and  $p_B = c + \bar{\theta}$ . These prices do not exhibit the degree of market competition caused by the shape of distribution. Moreover, firms' price mark-up is always  $\bar{\theta}$ , which is the maximum size of relative preference. But firms can charge higher or lower mark-up than  $\bar{\theta}$  depending on the density of marginal consumers if the distribution of consumer preferences is not uniform.



[Figure 2]

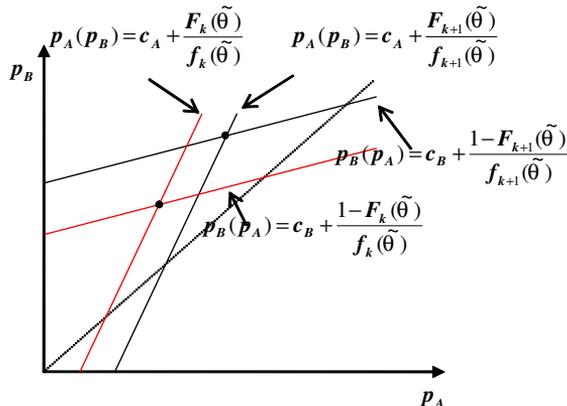
Without loss of generality, I assume that firm  $A$  is more efficient than firm  $B$  by  $\Delta > 0$ . Then Figure 2 shows that the location of marginal consumers  $\theta^*$  is greater than 0. This implies that the more efficient firm sets a lower price.<sup>8</sup>

<sup>8</sup>However, its mark-up is higher, i.e.,  $\frac{F(\theta^*)}{f(\theta^*)} > \frac{1 - F(\theta^*)}{f(\theta^*)}$ .

Moreover  $\theta^*$  is increasing in  $\Delta$ . As the cost difference between two firms is larger, the location of marginal consumers will be closer to  $\bar{\theta}$ . Then we can easily show how each firm's mark-up changes with  $\Delta$ . As  $\Delta$  increases, firm  $A$ 's mark-up rises, while firm  $B$ 's mark-up decreases.

$$\frac{\partial}{\partial \Delta} \left( \frac{F(\theta^*)}{f(\theta^*)} \right) > 0 \text{ and } \frac{\partial}{\partial \Delta} \left( \frac{1 - F(\theta^*)}{f(\theta^*)} \right) < 0.$$

Now, a more interesting question is how the equilibrium changes with a mean-preserving transformation of the distribution of consumer preferences. For the symmetric firms, that is  $c_A = c_B = c$ , it is easy to show that the equilibrium prices increase by the MPS from  $F_k(\theta)$  to  $F_{k+1}(\theta)$ . Let  $\theta_k^*$  represent the location of marginal consumers at the equilibrium when firms face the consumer distribution  $F_k(\theta)$ . By symmetry,  $\theta_k^* = \theta_{k+1}^* = 0$ . Firms' corresponding equilibrium prices are  $c + \frac{1}{f_k(0)}$  and  $c + \frac{1}{f_{k+1}(0)}$ . Because  $f_{k+1}(0) \leq f_k(0)$ , equilibrium prices are greater in more dispersed distributions.



[Figure 3]

This result is robust for asymmetric firms. The analysis for asymmetric firms is rather complicated, but more intuitive. Both firms' reaction curves shift outward under the MLRP as represented in Figure 3. Thus, equilibrium prices of both firms are always greater in higher order distributions. A rigorous proof is presented in the Appendix. This result corroborates my basic claim that the distribution of consumer preferences corresponds to the intensity of competition. More dispersed (concentrated) distributions lead to less (more) intense market competition.<sup>9</sup>

<sup>9</sup> Perloff and Salop (1985) and Bester (1992) show that the dispersion of consumers' distribution increases firms' equilibrium prices. However, in their models, the dispersion is enforced by scaling up consumers' distribution; they multiply each consumer's location  $\theta$  by some factor. This is equivalent to increasing transportation costs.

*Proposition 2.* Mean-preserving spread raises both firms' equilibrium prices.

$$\begin{aligned} c_A + \frac{F_{k+1}(\theta_{k+1}^*)}{f_{k+1}(\theta_{k+1}^*)} &> c_A + \frac{F_k(\theta_k^*)}{f_k(\theta_k^*)} \text{ and} \\ c_B + \frac{1 - F_{k+1}(\theta_{k+1}^*)}{f_{k+1}(\theta_{k+1}^*)} &> c_B + \frac{1 - F_k(\theta_k^*)}{f_k(\theta_k^*)}. \end{aligned}$$

One may argue that our result could be captured by transportation costs or the size of product differentiation in the Hotelling model even if consumer preferences are uniformly distributed. In fact, in this model,  $\bar{\theta}$  plays an identical role as transportation costs.<sup>10</sup> Under the uniform distribution, equilibrium prices are  $p_A = \bar{\theta} + \frac{(2c_A+c_B)}{3}$  and  $p_B = \bar{\theta} + \frac{(2c_B+c_A)}{3}$ , thereby increasing in  $\bar{\theta}$ .

However, there is an important difference between two approaches in analyzing market competition. The degree of competition is unvarying over the entire locations under the uniform distribution. Thus the equilibrium location of marginal consumers is invariable with  $\bar{\theta}$ . On the contrary, the mean-preserving transformations change the equilibrium location of marginal consumers. This seemingly small difference yields significant implications about firms' relative (dis)advantage.

A MPS shifts the location of marginal consumers  $\theta^*$  to the right;  $\theta_k^*$  increases as  $k$  increases, i.e.,  $\theta_k^* < \theta_{k+1}^*$ . The equilibrium condition (4) and inequality (1) lead to this result as we see in Figure 2. This implies that the difference between two firms' equilibrium prices is getting smaller. In other words, firm  $A$ 's relative advantage over firm  $B$  decreases as market becomes less competitive.

$$\begin{aligned} \theta_{k+1}^* - \theta_k^* &= \left( \frac{1 - 2F_{k+1}(\theta_{k+1}^*)}{f_{k+1}(\theta_{k+1}^*)} \right) - \left( \frac{1 - 2F_k(\theta_k^*)}{f_k(\theta_k^*)} \right) > 0 \\ \iff &\left[ \frac{F_k(\theta_k^*)}{f_k(\theta_k^*)} - \frac{1 - F_k(\theta_k^*)}{f_k(\theta_k^*)} \right] > \left[ \frac{F_{k+1}(\theta_{k+1}^*)}{f_{k+1}(\theta_{k+1}^*)} - \frac{1 - F_{k+1}(\theta_{k+1}^*)}{f_{k+1}(\theta_{k+1}^*)} \right] \end{aligned}$$

The driving force behind this result is again the assumption of MLRP. The marginal change in equilibrium prices by the MPS is greater for firm  $B$  than firm  $A$ . The intuition is as follows. Firm  $A$  has more incentives to cut its price relative to firm  $B$  when it faces  $F_{k+1}(\theta)$  rather than  $F_k(\theta)$  because it can attract a stochastically larger proportion of consumers.

<sup>10</sup>In a typical Hotelling model, the size of product differentiation is normalized as a unit, and transportation costs are measured by a parameter. On the other hand, our model can be thought of as normalizing transportation costs as a unit, but denotes the size of product differentiation by  $\bar{\theta}$ . Anyway, both ways are equivalent.

*Proposition 3.* As the distribution is transformed by the MPS (As  $k$  increases),  
(1)  $\theta_k^* < \theta_{k+1}^*$ . The location of marginal consumers is closer to the right side.  
(2)  $[p_A(\theta_k^*) - p_B(\theta_k^*)] > [p_A(\theta_{k+1}^*) - p_B(\theta_{k+1}^*)]$ . The difference between two firms' equilibrium prices is getting smaller.

This striking result shows that a firm's relative advantage over the rival depends on the intensity of competition. This implies that the intensity of competition influences the extent of effects of firms' business strategies. It is quite obvious that when firms develop strategies to have competitive advantages over ones' rival, they assess how the current competition acts upon the consequence of a strategy. However, this aspect has been largely left behind in most papers.

In the next section, I provide some applications to show that the consequence of firms' business strategies depends crucially on the intensity of competition captured by non-uniform distributions of consumer preferences. Non-uniform distributions may reveal new results that the uniform distribution could not obtain. In addition, for some issues, we will see that some well-known results obtained from the uniform distribution may collapse under non-uniform distributions.

### 3. Applications

#### 3.1 Incentives for Innovation

I extend the basic model by incorporating the possibility of R&D. In the first stage, two firms invest in innovation. In the second stage, price competition follows. I assume symmetric costs for simplicity, thereby  $c_A = c_B = c$ . Firms conduct process innovations which reduce marginal production costs from  $c$  to  $c - \lambda$ . This R&D competition is a winner-take-all contest. Hence we need to formulate the winner's payoff and loser's payoff, which are denoted by  $\pi^W$  and  $\pi^L$  respectively. Following the previous analysis, each payoff can be represented as

$$\pi^W = \frac{F(\theta^*)^2}{f(\theta^*)} \text{ and } \pi^L = \frac{(1 - F(\theta^*))^2}{f(\theta^*)},$$

$$\text{where } \theta^* = \lambda + \frac{1 - 2F(\theta^*)}{f(\theta^*)}.$$

Let us consider the following simple R&D competition.  $I_A$  and  $I_B$  are each firm's investment levels respectively. Define  $p(I_A, I_B)$  as the probability that firm A wins, while  $1 - p(I_A, I_B)$  as the probability that it loses. Then  $1 - p(I_A, I_B)$  is firm B's winning probability and  $p(I_A, I_B)$  is B's corresponding losing probability. To ensure an interior solution, I assume  $\frac{\partial p(I_A, I_B)}{\partial I_A} > 0$ ,  $\frac{\partial^2 p(I_A, I_B)}{\partial I_A^2} < 0$ ,  $\frac{\partial p(I_A, I_B)}{\partial I_B} < 0$ , and  $\frac{\partial^2 p(I_A, I_B)}{\partial I_B^2} > 0$ . Each firm's expected profit is

$$\begin{aligned}\pi_A &= p(I_A, I_B)\pi^W + (1 - p(I_A, I_B))\pi^L - I_A \text{ and} \\ \pi_B &= (1 - p(I_A, I_B))\pi^W + p(I_A, I_B)\pi^L - I_B.\end{aligned}$$

The first-order conditions are  $\frac{\partial p(I_A, I_B)}{\partial I_A}(\pi^W - \pi^L) - 1 = 0$  and  $-\frac{\partial p(I_A, I_B)}{\partial I_B}(\pi^W - \pi^L) - 1 = 0$ . Accordingly, given that the model is symmetric, we have  $I_A^* = I_B^* = I^*$  satisfying  $\pi^W - \pi^L = \frac{1}{p'(I^*, I^*)}$  at the equilibrium.<sup>11</sup>

This equilibrium condition implies that  $(\pi^W - \pi^L)$  can be thought of as R&D incentives, because the equilibrium level of investment increases in  $(\pi^W - \pi^L)$ . Therefore we need to analyze how R&D incentives are changed by the transformation of consumer distributions. Note that the difference between the winner's and loser's payoffs can be simplified as

$$\pi^W - \pi^L = \frac{2F(\theta^*) - 1}{f(\theta^*)} = \lambda - \theta^*.$$

It can be easily shown that  $(\pi^W - \pi^L)$  is smaller as the order of distribution rises, because we have  $\lambda - \theta_k^* > \lambda - \theta_{k+1}^*$  by Proposition 3. That is to say, the firms' R&D incentives are greater in the more competitive market.<sup>12</sup> The intuition to understand this result comes from Proposition 3. The winner's relative advantage over the loser is getting smaller as market competition is less intense, because the winner has more incentives to cut its price relative to the loser in the less competitive market.

*Proposition 4.* R&D incentives are greater as market competition becomes more intense.

If we adopt the uniform distribution, we are not able to capture the effect of the intensity of competition on innovation incentives. Under the uniform distribution, the winner's and loser's payoffs are  $\pi^W = 2\bar{\theta}(\frac{\lambda}{6\bar{\theta}} + \frac{1}{2})^2$  and  $\pi^L = 2\bar{\theta}(\frac{\lambda}{6\bar{\theta}} - \frac{1}{2})^2$  respectively. Each payoff depends on  $\bar{\theta}$ , the size of product differentiation. However, note  $\pi^W - \pi^L = \frac{2}{3}\lambda$ . Interestingly, R&D incentives do

<sup>11</sup>Consequently, each firm has an equal chance of winning R&D at the symmetric equilibrium, i.e.  $p(I_A, I_B) = \frac{1}{2}$ .

<sup>12</sup>In fact, there are some other ways to study the effect of market competition on R&D incentives. A typical way is considering the number of firms in a market. For example, Loury (1979) and Lee and Wilde (1980) study how the equilibrium level of investment is changed by the number of firms in a market. Several papers compare Bertrand competition to Cournot competition because the Bertrand model yields more competitive market outcome than the Cournot model does. For example, Delbono and Denicolo (1990), Bester and Petrakis (1993), and Bonanno and Haworth (1998) compares incentives for innovation between Cournot and Bertrand competition. In addition to these approaches, I examine this issue through the intensity of competition in the Hotelling model which I have developed in the previous section.

not rely on  $\bar{\theta}$ . Even if we include transportation costs in the Hotelling model, they are incapable of showing the effect of intensity of competition on innovation incentives. In this sense, the shape of consumer distribution may be more appropriate in some cases than transportation costs to demonstrate how the intensity of competition affects firms' strategies.

### 3.2 preference-based Price Discrimination

Recently, preference or behavior-based price discrimination has been widely studied. The Hotelling model has a good nature to analyze this issue because this type of price discrimination is based on brand preference. A large body of literature shares one important result that firms' profits are more likely to decrease by price discrimination. Bester and Petrakis (1996), Chen (1997), Fudenberg and Tirole (2000), and Armstrong (2006) are examples. However, I will show that this result may not hold in a general distribution. In particular, firms can increase their profits from the preference-based price discrimination in intense competition, where consumer preferences follow an inverse U-shaped distribution.

For simplicity, I assume that firms are symmetric, and marginal costs are zero,  $c_A = c_B = 0$ . As a benchmark, I summarize symmetric equilibrium prices and profits without price discrimination as below.

$$p_i^* = \frac{1}{2f(0)} \text{ and } \pi_i^* = \frac{1}{4f(0)} \quad (5)$$

Now, suppose that a firm is able to observe whether a consumer has more preference for its good or its rival's, i.e., whether  $\theta$  is greater or smaller than 0. Both firms offer different prices to different turfs. As in Bester and Petrakis (1996), this scenario can be thought of as targeted coupons offered to the rival's turf. Also, this can be interpreted as the second-period poaching competition as in Fudenberg and Tirole (2000).

Let  $\bar{p}_i$  denote the price offered to a consumer in its own turf, while  $\hat{p}_i$  represents the poaching price offered to a consumer in its rival's turf. Price competition in firm  $B$ 's turf yields the following profit functions.

$$\hat{\pi}_A = \hat{p}_A(F(\tilde{\theta}) - \frac{1}{2}) \text{ and } \bar{\pi}_B = \bar{p}_B(1 - F(\tilde{\theta})). \quad (6)$$

Equilibrium prices and profits are

$$\begin{aligned}
\widehat{p}_A &= \frac{F(\theta^*) - \frac{1}{2}}{f(\theta^*)} \text{ and } \bar{p}_B = \frac{1 - F(\theta^*)}{f(\theta^*)} \\
\widehat{\pi}_A &= \frac{(F(\theta^*) - \frac{1}{2})^2}{f(\theta^*)} \text{ and } \bar{\pi}_B = \frac{(1 - F(\theta^*))^2}{f(\theta^*)}, \\
\text{where } \theta^* &= \frac{\frac{3}{2} - 2F(\theta^*)}{f(\theta^*)}.
\end{aligned} \tag{7}$$

Similarly, we can find equilibrium prices and profits in competition of firm A's turf, which are simply symmetric. If consumers are uniformly distributed, we can easily find that  $\bar{p}_i = \frac{2\bar{\theta}}{3}$  and  $\widehat{p}_i = \frac{\bar{\theta}}{3}$ . Both prices are lower than the non-discrimination symmetric equilibrium price  $\bar{\theta}$ . Thus, firms are worse-off by price discrimination definitely. To explain this result, Armstrong (2006) writes "discrimination acts to intensify competition ... when firms differ in their view of which markets are strong and which are weak." Similarly, Corts (1996) uses the term "best response asymmetry" and Anderson and Leruth (1993) says "firms compete on more fronts".

However, if  $f(\theta^*)$  is sufficiently small compared to  $f(0)$ , discriminating prices can be greater than non-discriminating price. Then we have a possibility that equilibrium profits are greater with price discrimination. This result stands in sharp contrast to the previous literature. Formally, the equilibrium profits with price discrimination are given by

$$\pi_i^{PD} = \frac{2F(\theta^*)^2 - 3F(\theta^*) + \frac{5}{4}}{f(\theta^*)}. \tag{8}$$

A sufficient condition for  $\pi_i^{PD} > \pi_i^*$  is  $f(0) > 2f(\theta^*)$ . This condition suggests that the total number of marginal consumers do matter for the comparison of equilibrium profits. Without price discrimination, it is  $f(0)$ , whereas it is  $2f(\theta^*)$  with price discrimination.<sup>13</sup> Interestingly, an inverse U-shaped distribution can yield the situation in which the preference-based price discrimination is a profitable strategy.

■ *Example.* Consider the following density function  $f(\theta) = -\frac{1}{\theta^2}|\theta| + \frac{1}{\theta}$ . The corresponding distribution function is  $F(\theta) = \frac{1}{2} + \frac{\theta^2}{2\theta^2} + \frac{1}{\theta}\theta$  for  $\theta < 0$  and  $F(\theta) = \frac{1}{2} - \frac{\theta^2}{2\theta^2} + \frac{1}{\theta}\theta$  for  $\theta > 0$ . Without price discrimination, the symmetric equilibrium prices and profits are  $p_i^* = \frac{\bar{\theta}}{2}$  and  $\pi_i^* = \frac{\bar{\theta}}{4}$ . With price discrimination,

<sup>13</sup>The necessary and sufficient condition is weaker, because the convexity of equilibrium profit functions allows firms to have additional gains through price discrimination. I will discuss this property in the next section.

equation (7) becomes  $\theta^* = \frac{3}{4}\bar{\theta} - \sqrt{\left(\frac{3}{4}\bar{\theta}\right)^2 - \frac{1}{4}}$ . It can be shown that the sufficient condition  $f(0) > 2f(\theta^*)$  holds if  $\bar{\theta} < \sqrt{\frac{3}{8}}$ .

*Proposition 5.* If market competition is very intense, firms' equilibrium profits can be greater with the preference-based price discrimination.

In general, as more consumers have strong preferences, firms may want to price discriminate more. However, paradoxically, preference-based price discrimination is a profitable strategy when consumers have relatively weak preferences to both firms. Competition for the whole market is very intense, but competition in the segmented markets is less intense. Following Anderson and Leruth's words, we can say "firms compete on less competitive fronts" through price discrimination.

### 3.3 Incentive for Information Sharing

Here I investigate how the intensity of competition influences incentives to share private information. There are many papers which address this issue. Vives (1990) and Gal-Or (1985, 1986) show that the incentives for information sharing depend on the nature of competition (Cournot or Bertrand) and the nature of the information structure (demand or costs). In particular, regarding private information about costs, Gal-Or (1986) shows that information sharing is a dominant strategy with Cournot competition and concealing is a dominant strategy with Bertrand competition.

One critical point in the literature is that the analyses are based on a linear demand curve. With a linear demand curve, equilibrium profit functions are convex. This is a crucial driving force behind their results. However, the shape of value functions changes with the intensity of competition, and it can be concave. Thus I will show in the simplest form that their results can be reversed.

Let us consider the following textbook example. Suppose firm  $B$ 's marginal cost is  $c$ , while firm  $A$ 's marginal cost is uncertain. It can be either  $c_H$  or  $c_L$  with equal probability, where  $c_L < c_H$ . Information is asymmetric. Firm  $A$  knows its own marginal cost and firm  $B$ 's. But firm  $B$  knows its cost and only that firm  $A$ 's marginal cost is either  $c_H$  or  $c_L$  with equal probability.<sup>14</sup> All other things are common knowledge. I assume  $(c_H - c) = (c - c_L)$  so that firm  $A$  does not have any *ex ante* cost advantage. I define marginal consumers in each state as  $\tilde{\theta}_H = p_B - p_A(c_H)$  and  $\tilde{\theta}_L = p_B - p_A(c_L)$ .

Then firm  $A$ 's profit function in each state is written as  $\pi_A(c_H) = (p_A(c_H) - c_H)F(\tilde{\theta}_H)$  and  $\pi_A(c_L) = (p_A(c_L) - c_L)F(\tilde{\theta}_L)$ . Firm  $B$  anticipates that firm  $A$ 's price will be  $p_A(c_H)$  or  $p_A(c_L)$ . Firm  $B$  solves  $E\pi_B = \frac{1}{2}(p_B - c)(1 - F(\tilde{\theta}_H)) + \frac{1}{2}(p_B - c)(1 - F(\tilde{\theta}_L))$ . Then the Bayesian Nash equilibrium is characterized by

<sup>14</sup>One justification for this situation is that firm  $A$  may be a new entrant or it may employ a new technology.

$$\theta_H^I = c - c_H + \frac{1 - 2F(\theta_H^I)}{2f(\theta_H^I)} \text{ and } \theta_L^I = c - c_L + \frac{1 - 2F(\theta_L^I)}{2f(\theta_L^I)}, \quad (9)$$

where  $\theta_L^I = -\theta_H^I > 0$  by symmetry. Note that this implies  $f(\theta_L^I) = f(\theta_H^I)$  and  $F(\theta_L^I) = 1 - F(\theta_H^I)$ . Equilibrium prices are  $p_A(c_H) = c_H + \frac{F(\theta_H^*)}{f(\theta_H^*)}$ ,  $p_A(c_L) = c_L + \frac{F(\theta_L^*)}{f(\theta_L^*)}$ , and  $p_B = c + \frac{(1-F(\theta_H^*)) + (1-F(\theta_L^*))}{f(\theta_H^*) + f(\theta_L^*)} = c + \frac{1}{2f(\theta_L^*)}$ . Now we are interested in firm  $A$ 's *ex ante* expected profit. Those are given, respectively, by

$$E\pi_A^I = \frac{1}{2} \frac{F(\theta_H^I)^2}{f(\theta_H^I)} + \frac{1}{2} \frac{F(\theta_L^I)^2}{f(\theta_L^I)}.$$

Now, let us consider the case that firm  $A$  shares the information about its marginal cost. Then firm  $A$ 's *ex ante* expected profit is simply the average of equilibrium profits in each state under complete information.

$$E\pi_A^S = \frac{1}{2} \frac{F(\theta_H^S)^2}{f(\theta_H^S)} + \frac{1}{2} \frac{F(\theta_L^S)^2}{f(\theta_L^S)},$$

where  $\theta_H^S = (c - c_H) + \frac{1 - 2F(\theta_H^S)}{f(\theta_H^S)}$  and  $\theta_L^S = (c - c_L) + \frac{1 - 2F(\theta_L^S)}{f(\theta_L^S)}$  (10)

One can easily find  $\theta_H^I < \theta_H^S < 0 < \theta_L^S < \theta_L^I$  by comparing (9) with (10). More generally,  $(\theta_H^I, \theta_L^I)$  can be thought of as an MPS of  $(\theta_H^S, \theta_L^S)$ . This implies that the shape of equilibrium profit function determines when firm  $A$  has higher profit. If the distribution is uniform,  $E\pi_A^I = \frac{\bar{\theta}}{2} + \frac{(c_H - c_L)^2}{32\bar{\theta}}$  and  $E\pi_A^S = \frac{\bar{\theta}}{2}$ . Since  $E\pi_A^I > E\pi_A^S$ , concealing information is a dominant strategy, which is consistent with Gal-Or (1986).

However, the expected equilibrium profits can be concave in  $\theta_L$ . The first derivative gives us  $\frac{\partial E[\pi_A^I]}{\partial \theta_L} = (2F(\theta_L) - 1) - \frac{[F(\theta_L)^2 + (1-F(\theta_L))^2]f'(\theta_L)}{2f(\theta_L)^2}$ . Under the uniform distribution, the second term disappears because  $f'(\theta_L) = 0$ . Thus, immediately, we can see that the expected equilibrium profit is a convex function. If the distribution is not uniform, the second derivative is given as

$$\frac{\partial^2 E[\pi_A^I]}{\partial \theta_L^2} = 4f(\theta_L) - \left[ \frac{\partial E[\pi_A^I]}{\partial \theta_L} \left( \frac{f'(\theta_L)}{f(\theta_L)} \right) + E[\pi_A^I] \frac{\partial}{\partial \theta_L} \left( \frac{f'(\theta_L)}{f(\theta_L)} \right) \right].$$

In a U-shaped distribution of consumer preferences, we have  $\frac{f'(\theta_L)}{f(\theta_L)} > 0$ , and  $\frac{\partial}{\partial \theta_L} \left( \frac{f'(\theta_L)}{f(\theta_L)} \right)$  can be positive. Hence there is a possibility of having a concave profit function.<sup>15</sup> If this is the case, information sharing is a dominant strategy.

■ *Example.* Consider the following density function  $f(\theta) = \frac{1}{\theta^2} |\theta|$ . The corresponding distribution function is  $F(\theta) = \frac{1}{2} - \frac{\theta^2}{2\theta^2}$  for  $\theta < 0$  and  $F(\theta) = \frac{1}{2} + \frac{\theta^2}{2\theta^2}$  for  $\theta > 0$ . In this case, equation (9) becomes  $\theta_H^I = \frac{2}{3}(c - c_H)$  and  $\theta_L^I = \frac{2}{3}(c - c_L)$ , while equation (10) becomes  $\theta_H^S = \frac{1}{2}(c - c_H)$  and  $\theta_L^S = \frac{1}{2}(c - c_L)$ . It can be easily shown that  $E\pi_A = \frac{1}{2} \frac{F(\theta_H)^2}{f(\theta_H)} + \frac{1}{2} \frac{F(\theta_L)^2}{f(\theta_L)} = \frac{1}{4} \left( \frac{\theta_L^I}{\theta^2} + \frac{\theta^2}{\theta_L} \right)$ . Therefore,  $E\pi_A^I \gtrless E\pi_A^S$  corresponds to  $\bar{\theta}^2 \lesseqgtr \sqrt[4]{\frac{c - c_L}{3}}$ .

*Proposition 6.* The equilibrium profit function can be concave, and information sharing can be a dominant strategy in price competition if market competition is mild.

Information sharing allows firms to compete on farther within fronts in the Hotelling model.<sup>16</sup> Thus, through information transmission, firm *A* can induce competition on less competitive fronts in a U-shaped distribution.

#### 4. Concluding Remarks

I have set forth the simple Hotelling model which relates the distribution of consumer preferences to the intensity of competition. The imposition of MPS and MLRP on the distributions provides a way of analyzing the change of equilibrium prices. The analyses in this article highlight the importance of taking into account non-uniform distributions in the Hotelling model in the sense that the effects of firms' strategies change with the intensity of competition. Through three examples, I emphasized how non-uniform distributions of consumer preferences can play a significant role by reversing some well-known results or by discovering new results. In this sense, the uniform distribution typically assumed in the Hotelling model may be very restrictive.

There are many possible extensions of my paper. We may be able to study firms' strategies to change consumers' preferences. For example, advertising or

<sup>15</sup>In most models, the shape of equilibrium profit functions is determined by whether marginal competition increases or decreases by a given parameter, which is captured by  $\frac{\partial}{\partial \theta_L} \left( \frac{f'(\theta_L)}{f(\theta_L)} \right)$  in this model. If marginal competition increases, the equilibrium profit function is displayed by a concave curve. My another working paper, Kim and Bang (2007), studies this issue in the dynamic competition model.

<sup>16</sup>Under Cournot competition where reaction functions are downward sloping, information sharing allows firms to compete on farther away fronts in the sense that the low cost type firm produces more and the high cost type firm produces less. I believe that this is why Gal-Or (1986) finds that information sharing is a dominant strategy with Cournot competition.

promotion may play a role of changing the distribution of consumers preferences as in Bloch and Manceau (1999). Although these are important issues in marketing literatures, we do not have a formal economic model. Of all things, I believe that the simple framework developed in this article will be very useful for further researches on various topics.

I conclude the paper by presenting another interesting perspective on the distribution of consumer preferences. In fact, in the Hotelling model, each consumer's valuations for the two products are perfectly negatively correlated. In contrast, they are perfectly positively correlated in the Bertrand model. These two familiar models can be thought of as limiting cases of a general model in which each consumer's valuations are independently distributed. Chen and Riordan (2006) shows this general model with a rectangular area. Nevertheless, this is nothing but the shift of the distribution of consumer preferences in the Hotelling model. At last, the Hotelling model with a general distribution of consumer preferences is the most generalized model.

## Appendix

*Proof of Lemma 1.* For  $\theta \in [0, \bar{\theta}]$ , the definition of MLRP gives us

$$f_{k+1}(\theta_0)f_k(\theta_1) \leq f_{k+1}(\theta_1)f_k(\theta_0). \quad (\text{A1})$$

Integrate both sides over  $\theta_0$  from 0 to  $\theta_1$ . We have  $(F_{k+1}(\theta_1) - \frac{1}{2})f_k(\theta_1) \leq f_{k+1}(\theta_1)(F_k(\theta_1) - \frac{1}{2})$ . Rearrange, and this inequality can be rewritten by

$$\frac{1 - 2F_k(\theta)}{f_k(\theta)} \leq \frac{1 - 2F_{k+1}(\theta)}{f_{k+1}(\theta)}.$$

Similarly, integrate both sides in (A1) over  $\theta_1$  from  $\theta_0$  to  $\bar{\theta}$ , and it turns out that  $f_{k+1}(\theta_0)(1 - F_k(\theta_0)) \leq (1 - F_{k+1}(\theta_0))f_k(\theta_0)$ . This inequality is called hazard rate dominance,

$$\frac{1 - F_k(\theta)}{f_k(\theta)} < \frac{1 - F_{k+1}(\theta)}{f_{k+1}(\theta)}.$$

Let us define the hazard rate of  $F$  by  $\mu_k(\theta) \equiv \frac{f_k(\theta)}{1 - F(\theta)}$ . If we write  $-\mu_k(\theta) = \frac{d}{d\theta} \ln(1 - F(\theta))$ , then the distribution function can be written as  $F_k(\theta) = 1 - \exp(-\int_0^\theta \mu_k(x)dx)$ . It is straightforward to show FOSD as follows.

$$F_{k+1}(\theta) = 1 - \exp(-\int_0^\theta \mu_{k+1}(x)dx) \leq 1 - \exp(-\int_0^\theta \mu_k(x)dx) = F_k(\theta).$$

Likewise, we can easily show the opposite stochastic orders for  $\theta \in [\underline{\theta}, 0]$ .

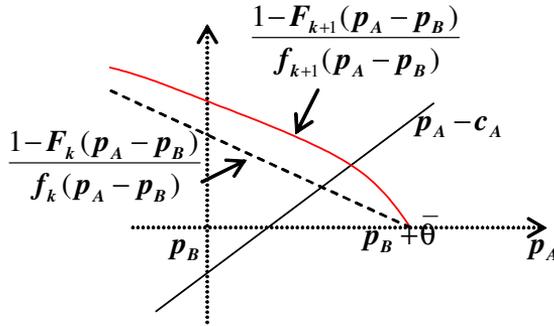
*Proof of Proposition 2.* For the distribution  $F_k(\theta)$ , firms' reaction functions are given by

$$p_A(p_B) = c_A + \frac{F_k(p_B - p_A)}{f_k(p_B - p_A)} \text{ and } p_B(p_A) = c_B + \frac{1 - F_k(p_B - p_A)}{f_k(p_B - p_A)}.$$

First, let's analyze how the MPS affects  $p_A$ , given  $p_B$ . Since the density function is symmetric at 0, firm A's reaction function can be written by

$$p_A - c_A = \frac{1 - F_k(p_A - p_B)}{f_k(p_A - p_B)}.$$

The left-hand side  $p_A - c_A$  is increasing in  $p_A$ , while the right-hand side  $\frac{1 - F_k(p_A - p_B)}{f_k(p_A - p_B)}$  is decreasing in  $p_A$ . These are represented in Figure A1. Note that they must intersect on the range where  $p_A$  is greater than  $p_B$ . By inequality (3) in Lemma 1, we have  $\frac{1 - F_k(\theta)}{f_k(\theta)} < \frac{1 - F_{k+1}(\theta)}{f_{k+1}(\theta)}$ . Therefore,  $p_A$  is always greater in a higher order distribution, given  $p_B$ . This implies that firm A's reaction function shifts outward. Similarly, we can show  $p_B$  is also greater in a higher order distribution, given the level of  $p_A$ .



[Figure A1]

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