Trade finance and international currency: a monetary search approach

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Trade Finance and International Currency: a Monetary Search Approach

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Abstract
The determinants of international currency received a lot of attention since the great recession. Classic literature focused on economy size and openness, but that couldn’t explain why RMB remains largely national, while China is already leading in international trade. In this paper, I verified the importance of financial development for currency internationalization using SWIFT trade finance data. Then I built a two-country monetary search model, where trade takes time, and lack of commitment makes exporter and importer rely on bank-intermediated finance. The agent’s currency choice is related with terms of trade, monetary policy, and financial efficiency. Optimal monetary policy differs according to currency regime. Related topic such as size effect and global imbalance is also discussed.

*I’m indebted to Wing Woo and Athanasios Geromichalos for continuous guidance and support. Data help from SWIFT Institute is greatly appreciated. I also thank Paul Bergin, Katheryn Russ, Colin Carter, Seungduck Lee, Mingzhi Xu, Tian Xia, and participants at UC Davis Macro/International Brownbag for helpful comments and suggestions. All errors are mine. Author’s email: taoliu@ucdavis.edu
1 Introduction

The international monetary system came under serious doubt after the global financial crisis in 2008. Many alternatives have been proposed to replace the exorbitant privilege of US dollar, such as SDR (Zhou, 2009), a multipolar system (Eichengreen, 2011), and a single world currency (Mundell, 2012). Academic research on the determinants of international currency traditionally emphasized economy size and openness, but historical experience shows another picture. US GDP surpassed Great Britain in 1870s, and US share of world export surged to 22.1% in 1913, but the international role of US dollar was essentially zero, while Sterling still invoiced over 60% of world trade by early twentieth century (Broz, 1997). The establishment of FED is believed to speed up the rise of USD with its favorable policy to develop financial market and provide trade credit (Eichengreen and Flandreau, 2012). In this paper I show that a currency would never become international without a sound financial market, which could explain why RMB remains largely national, while China is already leading in international trade.

To that end, I first verify the importance of financial development for a currency to be widely used in cross-border trade, using trade finance data from SWIFT (Society for Worldwide Interbank Financial Telecommunication). A two-country monetary search model is then built to explain this finding. In model, goods are delivered one period after contract, and the lack of commitment calls for bank to provide liquidity to exporter with the fund from investor, who would later get payoff from importer. Banking sector operates at a fixed cost, and exporter receives liquidity at discount, so he would choose the currency with a higher level of profit. Consequently, a currency is never used in international trade if the issuing country doesn’t have a liquid and efficient financial market.

Three currency regimes emerge from the model’s equilibrium: single international currency (SIC), producer currency settlement (PCS), and local currency settlement (LCS). The welfare function consists of gains from domestic trade, international trade, and seigniorage revenue if there’s foreign demand of currency, as in SIC and PCS. For a central bank trying to maximize total welfare, Friedman rule is not necessarily optimal given the trade-off between gains from trade and seigniorage revenue. In addition, the relationship between optimal inflation level and economy size is hump-shaped for the issuing country of international currency: for a relatively large or small economy, seigniorage revenue is dwarfed by gains from domestic or international trade, so lower inflation is better choice.

Moreover, this model provides an intuitive explanation for global imbalance character-
ized by the persistent current account deficit of US. Consider a symmetric model with two countries identical in every aspect except that country 1 currency is the single international currency. Country 1 resident would hold more of his home currency since it’s used in both domestic and international trade. In contrast, country 2 resident would hold less of country 1 currency since he could use it only in cross-border transaction. As a result, the over-consumption tendency of country 1 leads to its trade deficit. This model therefore implies that global imbalance is partially accredited to the status of USD as the dominant international currency.

For definition, a currency becomes international when it’s used by foreigners in locations outside of the issuing country. As store of value, it could be central bank’s foreign reserve or private agent’s investment instrument. As medium of exchange, it could smooth government’s foreign exchange intervention or settle international trade. As unit of account, it denominates financial transaction or becomes the anchor of other currencies. Table 1 summarized all these functions. This paper focused on the international currency used as medium of exchange1.

Table 1: International function of money

<table>
<thead>
<tr>
<th></th>
<th>For government</th>
<th>For private use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Store of value</td>
<td>international reserve</td>
<td>investment instrument</td>
</tr>
<tr>
<td>Medium of exchange</td>
<td>FX intervention</td>
<td>transaction of good/asset</td>
</tr>
<tr>
<td>Unit of account</td>
<td>currency anchor</td>
<td>asset denomination</td>
</tr>
</tbody>
</table>

1In practice, there’s a lot of difference among pricing, invoicing, and settlement currency, although theoretical model usually takes them as equivalent. Pricing or invoicing currency might be considered as unit of account, while settlement currency is naturally classified as medium of exchange. Friberg and Wilander (2008) conducted a questionnaire study on the currency choice of Swedish exporter in 2006, and most firms reported to use the same currency in over 90% of their revenue. Of course, the discrepancy could be large, especially for developing countries. Reiss (2015) found that, for Brazil real, its use as invoicing currency is more than settlement currency, whereas Yu (2013) suggested that RMB was used more as settlement currency than invoicing currency.
Academic research on international currency spans economics and politics. Interested reader could refer to Bénassy-Quéré (2015) for a systematic review. For economic theories that regarded international currency as the outcome of decentralized choice by private agent, they could be loosely classified into trade models, invoicing currency models, and search models.

Trade models mostly used 3-country or N-country general equilibrium to explain the phenomenon of international vehicle currency (IVC) (Rey, 2001; Devereux and Shi, 2013). In models of this fashion, producer and consumer hold only home currency, and foreign exchange transaction is undertaken by financial intermediary or trading post with an increasing-return-to-scale technology that lowers transaction cost with a large trade volume. Under the assumption of cash in advance and PCS, agent’s currency choice is exogenously given. The existence of a general equilibrium with IVC is crucially dependent on economic openness. Therefore, the currency issued by a country intensively engaged in international trade would emerge as IVC. This thick market externality also makes the status of IVC a natural monopoly. The advantage of such model comes from its nature of general equilibrium: the welfare gain of using IVC could be analyzed, and discussion on optimal monetary policy is feasible. Lack of micro-foundation, however, constitutes an obvious drawback: agent’s currency choice is exogenously given, so it’s impossible to explain the rise and fall of different currency regimes.

Invoicing currency models endogenized currency choice by allowing exporters to set price before exchange rate shock is realized. For PCP, there’s uncertainty in foreign demand and production cost, while LCP makes future price unpredictable. So exporters choose invoicing currency mainly to mitigate exchange rate risk. Bacchetta and Van Wincoop (2005) showed that producer’s currency choice is affected by competition in foreign markets: higher level of exporter’s market share and differentiation tends to promote PCP. Goldberg and Tille (2008, 2013) continued this approach to include vehicle currency, and the determinants of invoicing currency include exporter’s motive to limit output volatility, hedge macroeconomic volatility, and reduce transaction cost. For all its success, invoicing currency model is not explicit about the underlying process of currency circulation, and its nature of partial equilibrium also limits welfare analysis.

Search theory focused on the rise of fiat money as medium of exchange. Earlier studies in this field employed two-country two-currency model, but suffered from the indivisibility of output and money (Matsuyama et al., 1993) or inability to reach equilibrium (Trejos and Wright, 1996). With the breakthrough in Lagos and Wright (2005), search theory is now widely applied to topics in international macroeconomics such as home bias puzzle.
(Geromichalos and Simonovska, 2014) and UIP puzzle (Jung and Lee, 2015). This paper is closest in methodology to Lester et al. (2012) and Zhang (2014) that used information theory to discuss the determinants of international currency and its implication for monetary policy.

This paper also follows a long tradition of explicitly modeling bank and credit since Diamond and Dybvig (1983). One difficulty in this field is the conflict between money and credit, as pointed out by Berentsen et al. (2007). There must be an absence of record keeping for money to be essential, but credit requires record keeping in case of default. The inherent tension between money and credit is not present in this model thanks to the institutional setup of trade finance: importer has no incentive to default since that would deny him the ownership of goods. Money could coexist with credit without record keeping, and credit improves welfare by facilitating trade.

The rest of this paper proceeds as follows. Part 2 presents an empirical analysis of international currency use in cross-border trade. Part 3 describes model environment and defines monetary equilibrium. Part 4 undergoes discussion on related topic with numerical example. Part 5 concludes.

2 An empirical analysis with SWIFT dataset

This part documents the practice of trade finance and takes advantage of SWIFT dataset to emphasize the importance of financial development for currency internationalization.

2.1 International trade finance

The timing of payment and delivery is always a big issue for international trade. Without mutual trust or history record, the direct trade between buyer and seller would bring in a lot of uncertainty: buyers don’t know whether they could get goods after payment, and sellers are not guaranteed that they would get paid after delivery. According to the timing

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2 This class of model has a large concern on asset, usually supplied in the manner of Lucas tree. The asset plays as both store of value in its claim to future output, and medium of exchange in acting as collateral to facilitate trade. My model is focused on fiat money as medium of exchange so asset pricing only has minor, if not trivial, effect on equilibrium condition. Moreover, buyer is assumed to get goods one period after contract, so he would never give asset to seller as payment without further guarantee.

3 This conflict is solved in Berentsen et al. (2007) by assuming banks are able to record financial transaction but not goods trade, so fiat money still circulates as medium of exchange but credit becomes feasible. Bignon et al. (2013) continued this approach to discuss the implication for currency union and financial integration. However, search model in this fashion still forces foreign consumption to be settled by foreign currency or credit, so it’s improper for the discussion of international currency.
of payment and delivery, trade finance could be classified into Cash-in-advance (payment before delivery), Open account (payment after delivery), and Bank trade finance. If buyer and seller trust each other, cash-in-advance or open account would be a better choice with relatively lower transaction cost. If sellers don’t trust buyer but believe the credit of buyer’s bank, bank-intermediated finance could help facilitate international trade. Committee on the Global Financial System (2014) estimated that bank trade finance directly supports about one-third of global trade. One mainstream instrument covering half of bank trade finance is letter of credit (LC). Figure 1 illustrates its mechanism, and detailed procedures are relegated to appendix.

Figure 1: Mechanism of LC

The timing mismatch between shipment and payment is easily solved by LC: exporter gets timely fund once he shows shipment document and importer is charged only after the delivery of goods. Holding LC is not attractive for banks given its average maturity of 2-3 months, but investors would be interested in this short-term asset with its payment guaranteed by bank credit. So banks would package LC as trade acceptance and sell it to investors. In the end, banks facilitate international trade by playing the role of intermediation among exporter, importer, and investor.

2.2 Empirical analysis of international currency use in trade

With better data availability in recent years, cross-country analysis of trade invoicing currency becomes feasible, and the leading research includes Kamps (2006), Goldberg and...
Tille (2008), Ito and Chinn (2013), and Gopinath (2015). They collect data mainly from the survey of central bank and statistical institute. The importance of market share, product differentiation, and financial development is generally verified. One fallacy of this approach, however, comes from the measurement inconsistency among central banks. Also, its sample size is quite limited, covering only 35 countries and regions.

SWIFT trade finance dataset provides an innovative insight on the currency use of cross-border transaction. It involves 227 countries and regions with detailed information on trade settlement currency. Table 2 is a summary of currency use for several countries and regions. Not surprisingly, US export and import is predominantly settled by USD. With the exception of Germany, most countries use USD as vehicle currency to settle their international trade. China made progress in setting its import with home currency, but not so for export⁴.

Table 2: Currency use of international trade

<table>
<thead>
<tr>
<th></th>
<th>Export</th>
<th></th>
<th>Import</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PCS</td>
<td>LCS</td>
<td>VCS</td>
<td>PCS</td>
</tr>
<tr>
<td>US</td>
<td>98.89%</td>
<td>2.64%</td>
<td>0.37%</td>
<td>1.49%</td>
</tr>
<tr>
<td>UK</td>
<td>2.59%</td>
<td>5.21%</td>
<td>92.20%</td>
<td>7.61%</td>
</tr>
<tr>
<td>Germany</td>
<td>47.51%</td>
<td>5.26%</td>
<td>47.23%</td>
<td>2.74%</td>
</tr>
<tr>
<td>France</td>
<td>32.34%</td>
<td>2.09%</td>
<td>65.57%</td>
<td>3.91%</td>
</tr>
<tr>
<td>Japan</td>
<td>33.71%</td>
<td>6.43%</td>
<td>59.87%</td>
<td>9.70%</td>
</tr>
<tr>
<td>Canada</td>
<td>4.25%</td>
<td>8.95%</td>
<td>86.79%</td>
<td>16.19%</td>
</tr>
<tr>
<td>Australia</td>
<td>1.14%</td>
<td>9.35%</td>
<td>89.51%</td>
<td>8.12%</td>
</tr>
<tr>
<td>China</td>
<td>0.69%</td>
<td>7.10%</td>
<td>92.22%</td>
<td>10.81%</td>
</tr>
<tr>
<td>OPEC</td>
<td>6.35%</td>
<td>9.43%</td>
<td>84.23%</td>
<td>25.27%</td>
</tr>
<tr>
<td>OECD without US</td>
<td>17.67%</td>
<td>7.26%</td>
<td>75.07%</td>
<td>10.20%</td>
</tr>
<tr>
<td>Eurozone</td>
<td>37.90%</td>
<td>4.03%</td>
<td>58.07%</td>
<td>3.91%</td>
</tr>
<tr>
<td>World</td>
<td>11.98%</td>
<td>13.37%</td>
<td>74.65%</td>
<td>13.37%</td>
</tr>
</tbody>
</table>

Notes: Statistics calculated from sample average between 2010 October and 2014 August, using MT400&700 message in SWIFT dataset. PCS for producer currency settlement; LCS for local currency settlement; VCS for vehicle currency settlement, mainly USD in this dataset. Intra Euro-zone trade excluded. Transaction among mainland China, Hong Kong, Macao, and Taiwan regarded as international.

⁴There should be some caution with the interpretation of table 2. Although letter of credit is estimated to directly support one sixth of total merchandise trade, its coverage is unbalanced across regions. Less than 10% of US export is linked with bank trade finance, whereas Asian countries heavily relies on it. This is also true for mainland China: around 30% of its import is financed by letter of credit, but that share is less than 10% for export. For comparison, data from PBOC and China’s custom showed that 20.94% of China’s merchandise trade was settled by RMB in 2015 November.
Table 3 provides panel regression analysis for the determinants of trade settlement currency. Two indicators are used as proxy for financial development: private credit over GDP, and Chinn-Ito index that measures capital account openness. It’s obvious from regression outcome that financial market development matters for currency use in cross-border trade: private credit over GDP is statistically significant for both export and import, with expected sign. This means, all else equal, a sound financial market promotes wider use of its currency in international trade.\(^5\) The following part would build a two-country monetary search model to explain this finding.

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Export</th>
<th>Non-OECD</th>
<th>Total</th>
<th>Export</th>
<th>Non-OECD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity share</td>
<td>-1.788</td>
<td>1.224</td>
<td>-1.087</td>
<td>0.967</td>
<td>-2.045</td>
<td>1.490</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.000</td>
<td>-0.006</td>
<td>0.003</td>
<td>0.002</td>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>Inflation volatility</td>
<td>0.001</td>
<td>0.002</td>
<td>-0.0002</td>
<td>0.001</td>
<td>-0.002</td>
<td>0.011</td>
</tr>
<tr>
<td>Exchange rate (per USD)</td>
<td>-0.037***</td>
<td>-0.077***</td>
<td>-0.020***</td>
<td>-0.021**</td>
<td>-0.039***</td>
<td>-0.011</td>
</tr>
<tr>
<td>Exchange rate volatility</td>
<td>-0.154</td>
<td>0.112</td>
<td>-0.652</td>
<td>-0.146</td>
<td>-0.176</td>
<td>0.404</td>
</tr>
<tr>
<td>Private credit / GDP</td>
<td>0.128***</td>
<td>-0.035</td>
<td>0.099***</td>
<td>0.087***</td>
<td>0.816*</td>
<td>0.100**</td>
</tr>
<tr>
<td>Chinn-Ito index</td>
<td>0.051</td>
<td>0.213</td>
<td>0.012</td>
<td>0.151***</td>
<td>0.069</td>
<td>0.156**</td>
</tr>
<tr>
<td>Product differentiation</td>
<td>-0.012</td>
<td>-0.062</td>
<td>-0.019</td>
<td>-0.030</td>
<td>-0.047</td>
<td>-0.102</td>
</tr>
<tr>
<td>Real GDP</td>
<td>0.061***</td>
<td>0.004</td>
<td>0.036***</td>
<td>0.017</td>
<td>0.002</td>
<td>0.014</td>
</tr>
<tr>
<td>Constant</td>
<td>-2.051***</td>
<td>0.151</td>
<td>-1.323***</td>
<td>-0.948***</td>
<td>-0.347</td>
<td>-1.038</td>
</tr>
<tr>
<td>N</td>
<td>319</td>
<td>76</td>
<td>243</td>
<td>334</td>
<td>79</td>
<td>255</td>
</tr>
<tr>
<td>N (uncensored)</td>
<td>131</td>
<td>66</td>
<td>65</td>
<td>124</td>
<td>64</td>
<td>60</td>
</tr>
</tbody>
</table>

Note: US not included in sample. Data frequency is annual. Dependent variable is the share of a country’s export/import settled in home currency. Econometric method is panel Tobit, since SWIFT dataset is restricted for confidentiality purpose: if the monthly transaction number for a country pair is less than or equal to 4, it’s recorded as 0. Detailed description of independent variable in appendix.

3 The Model

3.1 Environment

Time is discrete and infinite. There are two countries in the world, 1 and 2, each populated with a unit measure of buyer, seller, and investor, who live forever with a discount factor of $\beta \in (0, 1)$. Their identity is fixed over time and their respective population is $\sigma$, $\sigma$, \(^5\)It must be cautioned here that regression significance implies correlation rather than causality. So it’s safer to conclude that financial market development is a necessary but not sufficient condition for currency internationalization.
and \((1 - 2\sigma)\). In addition, each country has a perfect competitive banking sector. Each period is divided into three rounds of centralized market (CM), decentralized market (DM), and financial market (FM). There is divisible and storable fiat money circulating in each country, and its total supply evolves according to \(\hat{M}_i = (1 + \mu_i)M_i\), where \(M_i\) is the stock of country i’s fiat money in current period, and variable with a hat is the level in next period. The growth rate of money supply, \(\mu_i\), is under the control of central bank.

Here I start with brief introduction on model, and a formal description would follow. In DM, sellers are specialized in the production of a perishable differentiated good \(q\) but unable to consume it, while buyers are able to consume but couldn’t produce. Due to imperfect credit and lack of record, a medium of exchange is necessary. Moreover, \(q\) is delivered only at the beginning of next period. Assume domestic agents know each other very well so they agree on the use of open account for settlement, whereas agents from different countries don’t trust each other, so settlement is facilitated by bank-intermediated finance. For international trade, buyers ask bank to issue LC, and sellers get immediate liquidity from bank after showing required document of shipment. In FM, only investors could purchase trade acceptance, which is a one-period nominal bond issued by bank, with a total payoff equal to buyer’s future payment. At the beginning of next period, buyers make payment to get \(q\), and investors receive payoff for their holding of trade acceptance.

In the following CM, buyer, seller, and investor engage in the production of a perishable numéraire good \(X\) and adjust their holdings of fiat money. The timing of model is depicted in figure 2.

Figure 2: Model timing
Now I will begin to formalize the setup of physical environment. For tractability, assume
the instantaneous utility function for buyer, seller, and investor in two countries is the
following

\[
U_B = u(q) + U(X) - H
\]
\[
U_S = -c(q) + U(X) - H
\]
\[
U_I = U(X) - H
\]

where \( q, X, \) and \( H \) capture the amount of specialized good, numéraire good, and working
hour. While every agent could produce numéraire good with a linear technology of \( X = H \),
only sellers could produce differentiated good at the cost of \( c(q) \). It’s further assumed that
the optimal consumption in CM is \( X^* \), such that \( U'(X^*) = 1 \). The conventional assumption
on function form also holds, so \( u(0) = c(0) = 0, u'(0) = +\infty, c'(0) = 0, u'' > 0, u'' < 0, c' > 0, c'' > 0 \). For notations below, \( i, j = \{1, 2\}, i \neq j \). The real value of country i’s fiat money
in terms of numéraire good is \( \phi_i \). This model is focused on stationary monetary equilibrium
where the aggregate real balance is constant, therefore \( 1 + \mu_i = \phi_i \hat{\phi}_i \). Central banks adjust
home currency supply through lump-sum transfer to domestic agent when CM opens.

There is separate DM in each country. Buyers could go abroad with a probability of
\((1 - \alpha)\) while sellers stay at home. Buyer and seller meet pairwise and at random, with a
matching function of \( N_i = B_i S_i \), where \( N_i \) is the number of successful matching in country
i, with \( B_i \) and \( S_i \) for the number of buyer and seller in country i’s DM. With this matching
function, the number of meeting between country i buyer and country j seller (\( n_{ij} \)), as well
as the probability for country i buyer to meet country j seller (\( p_{ij} \)) could be determined. DM
in this model functions as international trade market. FM is segmented by capital control.
The banking sector in country i could issue LC denominated in its home currency. The total
cost (\( F_i \)) is assumed to be fixed to reflect economy of scale. \( F_i \) is also a proxy for financial
development. FM in this model represents financial market for short-term investment.

In contrast, CM is open to buyer, seller, and investor from both countries. This Wal-
rasian market allows agents to adjust their holding of home and foreign currency, so it’s
similar to a frictionless foreign exchange market\(^6\). Additionally, since central bank’s lump-
sum transfer is only for domestic agents, extracting seigniorage revenue through inflation is
possible only when a certain currency is demanded by foreigners.

\( ^6 \)This is certainly not without loss of generality, as discussed in Geromichalos and Jung (2015)
The currency regime is endogenized by seller’s binary choice of settlement currency. If financial frictions make international trade unprofitable, international currency would never emerge. Otherwise, sellers would choose whichever currency that brings a higher level of profit.

3.2 Optimal choice and equilibrium

3.2.1 CM Value function

Agent’s CM value function differs according to his type. For buyer, he would want to hold money at the end of CM to enjoy differentiated good in next period, therefore the CM maximization problem for buyer in country i is

\[
W^B_i(\phi_i m^i_i, \phi_j m^j_i) = \max_{\hat{m}_i^i, \hat{m}_j^i} U(X) - H + \beta \mathbb{E}[V^B_i(\hat{\phi}_i \hat{m}_i^i, \hat{\phi}_j \hat{m}_j^i)]
\]

s.t. \( \phi_i \hat{m}_i^i + \phi_j \hat{m}_j^i + X \leq H + \phi_i m^i_i + \phi_j m^j_i + T_i \)

where \( m^j_i \) is country i buyer’s holding of country j currency; \( V^B_i \) is country i buyer’s value function for DM trade; \( T_i \) is the lump-sum transfer from country i central bank. This CM value function could be simplified as

\[
W^B_i(\phi_i m^i_i, \phi_j m^j_i) = W^B_i(0, 0) + \phi_i m^i_i + \phi_j m^j_i
\]

With the observation that buyer’s value function is linear in his holding of money, further simplify this into

\[
W^B_i(\phi_i m^i_i, \phi_j m^j_i) = W^B_i(0, 0) + \phi_i m^i_i + \phi_j m^j_i
\]

For sellers, they don’t have any incentive to hold money in CM since the liquidity he would get from bank is irrelevant with his holding of money. So CM value function for seller is constant with respect to his own money holding.

\footnote{Here I assume away the possibility that sellers accept both currencies at the same time, for two reasons. First, that doesn’t happen very often in reality, given that LC is mostly issued in a single currency. Second, this assumption makes model tractable in the case of indeterminacy. In my model, sellers would choose home currency if both currencies bring the same level of positive profit. In Zhang (2014), accepting home currency doesn’t incur additional information cost for seller, so accepting both currencies is possible. In this model, accepting home currency is also costly for international trade, so sellers would choose a single currency for settlement.}
With similar notations, the CM value function for country i investor is

\[ W^I_i(z_i) = \max_{\hat{z}_i, \hat{a}_i, H, X} U(X) - H + \beta \mathbb{E}[V^I_i(\hat{z}_i, \hat{a}_i)] \]

s.t. \( \phi_i \hat{z}_i + X \leq H + \phi_i z_i + T_i \)

where \( V^I_i(\hat{m}_i, \hat{a}_i) \) is the value function for investor in financial market, related with his holding of home currency (\( \hat{z}_i \)) and trade acceptance (\( \hat{a}_i \)) for next period. Similarly, this value function could be simplified into

\[ W^I_i(z_i) = W^I_i(0) + \phi_i z_i \]

### 3.2.2 Terms of trade in DM

Buyer and seller make a proportional bargaining in DM to determine terms of trade. Buyer’s utility maximization problem is

\[
\max_{q, d(m)} \{ u(q) - \phi d(m) \}
\]

s.t. \( \frac{u(q) - \phi d(m)}{\phi d(m) - \frac{c(q)}{\beta}} = \frac{\theta}{1 - \theta} \)

\[ d(m) \leq m \]

where \( q \) is the amount of differentiated good sellers would produce; \( d(m) \) is the amount of fiat money buyers would pay to sellers; \( \theta \) is buyer’s bargaining power. Since buyers make payment only when \( q \) is delivered at the beginning of next period, seller’s surplus is adjusted by discount factor. The solution is

\[
d(m) = \begin{cases} 
  m^* & \text{if } \phi m > \frac{c(q^*)}{\beta} \\
  m & \text{if } \phi m \leq \frac{c(q^*)}{\beta}
\end{cases}
\]

where \( q^* \) is the level of consumption that would maximize total surplus such that \( \beta u'(q^*) = c'(q^*) \); \( m^* \) is buyer’s payment when total surplus is maximized, so \( \phi m^* = (1 - \theta)u(q^*) + \theta \frac{c(q^*)}{\beta} \).

It will become clear in equilibrium that buyer’s holding of fiat money would never exceed \( m^* \), because excessive money doesn’t increase his gains from trade, but would incur a loss from inflation. Therefore, buyer’s payment to seller is \( \phi m = (1 - \theta)u(q) + \theta \frac{c(q)}{\beta} \), with \( q \leq q^* \), \( \beta u'(q^*) = c'(q^*) \).
3.2.3 Investor decision in FM

Country i investor’s profit maximization problem in FM is

$$\max_{a_i} \{ z_i + (y_i - p_i)a_i \}$$

s.t. \( p_i a_i \leq z_i \)

where \( p_i \) and \( y_i \) are nominal price and payoff of trade acceptance issued by country i’s banking sector. Notice that investor is risk-neutral since his CM value function is linear in \( z \), so he would only want to maximize his expected level of wealth. Individual investor’s demand for trade acceptance is

$$a_i = \begin{cases} 
0 & \text{if } y_i < p_i \\
\frac{z_i}{p_i} & \text{if } y_i \geq p_i 
\end{cases}$$

This result is intuitive: if the payoff is lower than cost, investor’s demand would be zero. For country i investor, the total demand for trade acceptance is \( D = (1 - 2\sigma)a_i \). The total payoff of trade acceptance in country i should be equal to buyer’s total payment for international trade settled in country i currency, so the total supply of trade acceptance is

$$S = \frac{s_i n_{i,m_i}^j + (1-s_{i,j})n_{i,m_i}^j}{y_i}$$

At equilibrium, if trade acceptance is attractive to investor, its payoff must be no less than price, so \( \frac{\tilde{y}_i}{p_i} = \frac{s_i n_{i,m_i}^j + (1-s_{i,j})n_{i,m_i}^j}{(1-2\sigma)z_i} \geq 1 \).

3.2.4 Financial constraint and seller’s decision

More importantly, the addition of bank and investor imposed financial constraint for monetary equilibrium that allows for international trade. The immediate liquidity provided by bank must be able to cover seller’s DM cost. Given a perfect competitive banking sector, zero profit condition holds, so this immediate liquidity is equal to the proceeds from selling trade acceptance, net of banking sector’s fixed cost. Consequently, country i seller’s gain from international trade settled in home currency is

$$\pi_i \equiv \left[ 1 - \frac{F_i}{(1-2\sigma)\phi_i z_i} \right] \left( \frac{1}{1 + R_i} \right) \phi_i m_i^j - c(q_i^j)$$

where \( R_i \equiv \frac{\phi_i}{\beta \phi_i} - 1 \) is the nominal interest rate of country i, and \( q_i^j \) is country j buyer’s purchase of differentiated good settled in country i currency.

From this result, seller’s revenue in DM trade is affected by three factors. First, terms
of trade from proportional bargaining, $\phi_i m^i_j$. Second, discount factor of $(1 + R_i)^{-1}$. Finally, financial friction influenced by the fixed cost of banking sector ($F_i$) and financial market liquidity of $(1 - 2\sigma)\phi_i z_i$. Intuitively, fixed cost is negatively correlated with seller’s revenue, while an increase of financial market liquidity could help improve seller’s profit from DM trade.

As shown later, inflation has negative impact on these factors at the same time. For terms of trade, higher inflation would reduce buyer’s trade volume and real balance holding; for discount factor, it erodes the value of future payment; for financial friction, it tends to depress investor’s confidence, thus lowering their purchase of trade acceptance. In short, higher inflation would get amplified by financial market and hugely deteriorate exporter’s welfare.

Similarly, country $i$ seller’s profit from international trade settled in foreign currency is

$$\pi^*_i \equiv \left[1 - \frac{F_j}{(1 - 2\sigma)\phi_j z_j}\right] \left(\frac{1}{1 + R_j}\right) \phi_j m^j_i - c(q^j_i)$$

With these in mind, country $i$ sellers choose settlement currency.\(^8\)

\[
\begin{align*}
    s_i = 1 & \quad \text{if} \ \max\{\pi_i, \pi^*_i\} < 0 \\
    s_i = 1 & \quad \text{if} \ \max\{\pi_i, \pi^*_i\} \geq 0, \ \pi_i \geq \pi^*_i \quad (1) \\
    s_i = 0 & \quad \text{if} \ \max\{\pi_i, \pi^*_i\} \geq 0, \ \pi_i < \pi^*_i
\end{align*}
\]

Finally, currency regime comes from seller’s decision. If $\{s_1, s_2\} = \{1, 0\}$ or $\{0, 1\}$, there’s a single international currency (SIC); if $\{s_1, s_2\} = \{1, 1\}$, both currencies become international, and seller would use home currency for trade settlement, which is producer currency settlement (PCS); if $\{s_1, s_2\} = \{0, 0\}$, there are two international currencies, and international trade is settled by importer’s home currency, which is local currency settlement (LCS). Currency regime is summarized in table 4.

\(^8\)Here I didn’t consider the asymmetric case when international trade is profitable for country $i$ seller but not for country $j$ seller, just for the sake of simplicity. It’s quite easy to include that case and related discussion would be straightforward.
Table 4: Currency regime

<table>
<thead>
<tr>
<th>Regime</th>
<th>Seller’s choice</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIC</td>
<td>{s_1, s_2} = {1, 0}</td>
<td>Country 1 currency is international</td>
</tr>
<tr>
<td></td>
<td>{s_2, s_1} = {1, 0}</td>
<td>Country 2 currency is international</td>
</tr>
<tr>
<td>PCS</td>
<td>{s_1, s_2} = {1, 1}</td>
<td>Two international currencies</td>
</tr>
<tr>
<td></td>
<td>Trade settled in seller’s home currency</td>
<td></td>
</tr>
<tr>
<td>LCS</td>
<td>{s_1, s_2} = {0, 0}</td>
<td>Two international currencies</td>
</tr>
<tr>
<td></td>
<td>Trade settled in buyer’s home currency</td>
<td></td>
</tr>
</tbody>
</table>

3.2.5 Optimal choice for buyer and investor

For buyer and investor, the optimal holding of real balance is available after combining CM and DM value function. For country i buyer, his DM value function is

\[
V_i^B = (p_{ii} + (1 - s_j)p_{ij})(u(q_{ii}^i) - \phi_i m_{ii}^i) + p_{ij} s_j (u(q_{ij}^j) - \phi_j m_{ij}^j) + W_i^B
\]

where \((p_{ii} + (1 - s_j)p_{ij})(u(q_{ii}^i) - \phi_i m_{ii}^i)\) is country i buyer’s expected surplus for DM trade settled in country i currency, while \(p_{ij} s_j (u(q_{ij}^j) - \phi_j m_{ij}^j)\) is his expected surplus for trade settled in country j currency.

Substitute this into the expression of buyer’s CM value function, then his maximization problem becomes

\[
\max_{\hat{m}_i, \hat{m}_j} \left\{ (\beta \hat{\phi}_i - \phi_i) \hat{m}_i^i + \beta (p_{ii} + (1 - s_j)p_{ij}) \theta \left[ u(\hat{q}_i^i) - \frac{c(\hat{q}_i^i)}{\beta} \right] \right. \\
+ (\beta \hat{\phi}_j - \phi_j) \hat{m}_j^j + \beta s_j p_{ij} \theta \left[ u(\hat{q}_j^j) - \frac{c(\hat{q}_j^j)}{\beta} \right] \right\}
\]

Several conventional observation in monetary search model would also apply here. For example, the solution for maximization problem requires \(\beta \hat{\phi}_i - \phi_i < 0\) and \(m < m^*\). The
first order condition for home currency is

\[ R_i = (p_{ii} + (1 - s_j)p_{ij}) \left[ \frac{\theta(u'(q^i_j) - c'(q^i_j)/\beta)}{(1 - \theta)u'(q^i_j) + \theta c'(q^i_j)/\beta} \right] \]  

(2)

This first order condition means buyer’s marginal cost of holding money \((R_i)\) must be equal to the expected marginal benefit. Notice that buyer’s demand for home currency is positive since his meeting with domestic sellers would always use home currency as medium of exchange. This is not true for foreign currency, which depends on foreign seller’s decision.

\[
\begin{align*}
q^i_j &= 0 \quad \text{if } s_j = 0 \\
R_j &= p_{ij} \left[ \frac{\theta(u'(q^i_j) - c'(q^i_j)/\beta)}{(1 - \theta)u'(q^i_j) + \theta c'(q^i_j)/\beta} \right] \quad \text{if } s_j = 1
\end{align*}
\]  

(3)

For simplicity, define \(L(q) = \frac{\theta(u'(q) - c'(q)/\beta)}{(1 - \theta)u'(q) + \theta c'(q)/\beta}\) as liquidity premium. Apply the same procedure to investor’s maximization problem, and combine the first order condition with equilibrium level of \(y_i/p_i\), I could get investor’s optimal holding of home currency.

\[
\begin{align*}
z_i &= 0 \quad \text{if } \{s_i, s_j\} = \{0, 1\} \\
R_i &= \frac{\sigma \phi_i m^i_i + (1 - s_j)\phi_i \phi_j m^i_j}{(1 - 2\sigma)\phi_i z_i} - 1, \text{ otherwise}
\end{align*}
\]  

(4)

This result is also intuitive: if home currency never became international, investor wouldn’t hold any of that; otherwise, investor’s marginal cost of holding home currency should be equal to the rate of return from trade acceptance.

Lastly, money market should clear after agents make choice. Consider the case when international trade is profitable. \(\forall i, j = \{1, 2\}, i \neq j\)

\[
\begin{align*}
\sigma \phi_i m^i_i &= \phi_i M_i \quad \text{if } \{s_i, s_j\} = \{0, 1\} \\
\sigma \phi_i m^i_j + (1 - 2\sigma)\phi_i z_i + F_i &= \phi_i M_i \quad \text{if } \{s_i, s_j\} = \{0, 0\} \\
\sigma \phi_i m^i_i + \sigma \phi_i m^j_i + (1 - 2\sigma)\phi_i z_i + F_i &= \phi_i M_i \quad \text{if } \{s_i, s_j\} = \{1, 0\}, \{1, 1\}
\end{align*}
\]  

(5)

For the first case, country \(i\) currency remains national, so its demand comes from only domestic buyer. For the second case of LCS, its demand comes from home buyer, home investor, and banking sector. For the last case, home buyer, home investor, foreign buyer,
and banking sector would all demand for country i currency.

### 3.2.6 Monetary equilibrium of international trade

With agent’s optimal choice, now it’s possible to define a stationary monetary equilibrium. My main concern is the emergence of international currency, so I would focus on the equilibrium that allows for international trade.

**Definition 1** A stationary monetary equilibrium that allows for international trade is a list of time-invariant values including trade volume \( \{q_{ij}\}_{i,j=1}^2 \), investor’s holding of real balance \( \{\phi_i z_i\}_{i=1}^2 \), and seller’s choice of settlement currency \( \{s_i\}_{i=1}^2 \) such that, given other agent’s behavior,

1. Seller’s choice of \( \{s_i\}_{i=1}^2 \) solves (1);
2. Buyer’s choice of \( \{q_{ij}\}_{i,j=1}^2 \) solves (2)(3);
3. Investor’s choice of \( \{\phi_i z_i\}_{i=1}^2 \) solves (4);
4. Money market clears so that (5) holds.

### 3.3 Hegemony and incumbency advantage

Now consider the case of hegemony when country 1 currency becomes international while country 2 currency remains national \( (s_1 = 1, s_2 = 0) \). Country 1 would be referred to as center country while country 2 as peripheral country. Intuitively, country 1 buyer would never hold foreign currency since his home currency is universally acknowledged and appreciated. In contrast, country 2 buyer would hold home currency for domestic trade and foreign currency for international trade. Moreover, the single international currency makes financial market active only in country 1. Consistent with definition 1, equilibrium condition is explicitly shown in appendix. There is incumbency advantage of country 1 in this international monetary system. Due to the economy of scale in banking sector, country 2 currency would never become international without collective action, government promotion, or a sudden shock that drains financial market liquidity in country 1. This observation is reflected in proposition 1.
**Proposition 1** If country 1 currency is the only international currency, an individual seller would never use country 2 currency for international trade.

**Proof** In this case, country 2 seller couldn’t ask country 1 buyer to pay country 2 currency since neither buyer or investor in country 1 holds foreign currency. For country 1 seller, if he accepted country 2 currency for trade settlement, his profit is

\[
\pi_1^* \leq \epsilon (1 - \theta) \left[ u(q_2^1) - c(q_2^1)/\beta \right] - F_2,
\]

where \( \epsilon \) is the number of seller trying to accept country 2 currency. If \( \epsilon \) is not sufficiently large relative to \( F_2 \), seller’s profit would be negative thanks to the fixed cost in the banking sector of country 2.

Notice the difference between this incumbency advantage and the size effect emphasized by classical literature. Previous studies often argued that the size effect of large economy would help lower the transaction cost of its currency in foreign exchange market, therefore justifying its status of international currency. But proposition 1 shows that economy size alone is not enough. Financial development proves indispensable.

This situation of hysteresis leaves room for policy intervention. Government could promote the internationalization of its currency by decreasing \( F \) through financial reform or deregulation. Another possibility is for central bank to absorb financial friction by becoming market maker. In history, FED took advantage of both options after 1913, and the rise-up of US dollar was largely attributed to that, as vividly described in Eichengreen (2011).

### 3.4 Monetary policy and international trade

With definition 1, the equilibrium condition for SIC, PCS, and LCS could be outlined, and comparative statics on monetary policy become possible. On the part of domestic trade, monetary policy has uniform effect on agent’s welfare level: higher inflation tends to reduce their gains from trade. On the part of investor, it’s also easy to show that higher inflation level erodes confidence and drives down financial market liquidity. The effect of monetary policy on international trade, however, differs according to currency regimes, as shown in proposition 2.

**Proposition 2** Under some general assumptions, higher inflation of international currency would hurt whoever used it for trade settlement.
(i) For SIC, higher inflation of international currency would hurt importer and exporter from both countries, i.e., $\frac{\partial q_1}{\partial R_1} < 0$, $\frac{\partial q_2}{\partial R_1} < 0$, $\frac{\partial \pi_1}{\partial R_1} < 0$, $\frac{\partial \pi^*_2}{\partial R_1} < 0$.

(ii) For PCS, higher inflation of international currency would hurt home exporter and foreign importer, i.e., $\frac{\partial \pi_1}{\partial R_1} < 0$, $\frac{\partial q_2}{\partial R_1} < 0$.

(iii) For LCS, higher inflation of international currency would hurt home importer and foreign exporter, i.e., $\frac{\partial q_1}{\partial R_1} < 0$, $\frac{\partial \pi^*_2}{\partial R_1} < 0$.

Proof in appendix

One interesting observation from proposition 2 is the relationship between nominal exchange rate and net export. This model is quite silent on exchange rate partly because, as shown in proposition 1, it’s the incumbency advantage and financial development that determines the emergence of international currency. A discussion on monetary policy and international trade, however, necessitates the inclusion of exchange rate. In particular, the possibility of ‘beggar thy neighbor’ through nominal depreciation would influence the conduct of monetary policy. Now assume Law of One Price (LOP) for numéraire good holds in this model, and nominal exchange rate is $e_{i/j} \equiv \frac{\phi_j}{\phi_i}$, where $e_{i/j}$ is the nominal exchange rate of country i currency per country j currency. Given that $\phi_i = (1 + \mu_i)\hat{\phi}_i$ in stationary monetary equilibrium, a higher inflation level of home currency would lead to nominal depreciation, whose effect on international trade differs according to currency regime.

For SIC, the result is unclear and contingent on parameter value. For PCS, higher inflation and home currency depreciation would hurt home exporter and foreign importer, thus lowering home export and net export, given that home import is insulated from this shock. For LCS, home currency depreciation would hurt home importer and foreign exporter, thus lowering home import and increasing home net export. Therefore, in this model, ‘beggar thy neighbor’ through nominal depreciation is possible in LCS, impossible in PCS, and uncertain in hegemony. These observations are summarized in table 5. Of course, the conduct of monetary policy is over-simplified in model. In reality, a sterilized FX intervention could depreciate home currency and stabilize money supply at the same time. A more elaborate model is required for in-depth discussion.
Table 5: Monetary policy and international trade

<table>
<thead>
<tr>
<th></th>
<th>Hegemony</th>
<th>PCS</th>
<th>LCS</th>
</tr>
</thead>
<tbody>
<tr>
<td>home importer</td>
<td>$\frac{\partial q^1}{\partial R^1} &lt; 0$</td>
<td>$\frac{\partial q^1}{\partial R^1} = 0$</td>
<td>$\frac{\partial q^1}{\partial R^1} &lt; 0$</td>
</tr>
<tr>
<td>home exporter</td>
<td>$\frac{\partial \pi^1}{\partial R^1} &lt; 0$</td>
<td>$\frac{\partial \pi^1}{\partial R^1} &lt; 0$</td>
<td>$\frac{\partial \pi^1}{\partial R^1} = 0$</td>
</tr>
<tr>
<td>foreign importer</td>
<td>$\frac{\partial q^2}{\partial R^1} &lt; 0$</td>
<td>$\frac{\partial q^2}{\partial R^1} &lt; 0$</td>
<td>$\frac{\partial q^2}{\partial R^1} = 0$</td>
</tr>
<tr>
<td>foreign exporter</td>
<td>$\frac{\partial \pi^2}{\partial R^1} &lt; 0$</td>
<td>$\frac{\partial \pi^2}{\partial R^1} = 0$</td>
<td>$\frac{\partial \pi^2}{\partial R^1} &lt; 0$</td>
</tr>
</tbody>
</table>

| home net export     | ?        | $\frac{\partial NX^1}{\partial R^1} < 0$ | $\frac{\partial NX^1}{\partial R^1} > 0$ |
| foreign net export  | ?        | $\frac{\partial NX^2}{\partial R^1} > 0$ | $\frac{\partial NX^2}{\partial R^1} < 0$ |

Notes: Country 1 is regarded as home country. In the first column, country 1 currency emerged as the single international currency. For the second column, international trade is settled by seller’s home currency. For the last column, international trade is settled by buyer’s home currency.

3.5 Welfare analysis and optimal monetary policy

A prominent advantage of monetary search model is the tractability of agent’s asset holding and welfare level, which is important for the conduct of optimal monetary policy if central bank is assumed to maximize the social welfare of its own country. In this model, social welfare consists of seigniorage revenue, gains from trade, and a loss from banking sector’s fixed cost if this country issues international currency. For simplicity, additional welfare gain from consuming numéraire good is omitted.

If country 1 issues the only international currency, for example, social welfare level at
the end of each period is the following.

\[
W_1 = \mu_1 \sigma_1 m_1^2 + n_{11} [\beta u(q_1^1) - c(q_1^1)] + n_{12} \theta [\beta u(q_1^1) - c(q_1^1)] + n_{21} \left\{ \frac{F_1}{(1 - 2\sigma)\phi_1 z_1} \right\} \left( \frac{1}{1 + R_1} \right) \phi_1 m_1^2 - c(q_1^2) \}

\]

\[
W_2 = -\mu_1 \sigma_1 m_2^2 + n_{22} [\beta u(q_2^2) - c(q_2^2)] + n_{21} \theta [\beta u(q_2^1) - c(q_2^1)] + n_{12} \left\{ \frac{F_1}{(1 - 2\sigma)\phi_1 z_1} \right\} \left( \frac{1}{1 + R_1} \right) \phi_1 m_1^1 - c(q_1^1) \}

With similar procedure, the welfare level for PCS and LCS is shown in appendix.

From previous assumptions and proposition 2, each country’s gain from international trade is decreasing in the nominal interest rate of international currency. For seigniorage revenue, recall that central bank’s lump-sum transfer is only applied to domestic agent, and other agents need to purchase that currency in CM. Therefore, seigniorage revenue is possible only when there’s foreign demand for that country’s currency. Without loss of generality, assume seigniorage revenue is increasing in the growth rate of money supply, which gives incentive to deviate from Friedman rule.

Seigniorage revenue would cancel out in the summation of each country’s welfare, so Friedman rule is optimal for a social planner trying to maximize total welfare. In addition, it’s inefficient to issue two international currencies since that would incur fixed cost of banking sector in both countries. Social planner would let a country with lower \( F \) issue a single international currency. For each country aimed at maximizing its own welfare, Friedman rule is not optimal if there’s foreign demand of its currency, which includes the case of PCS and hegemony when a country issues the only international currency. These observations are summarized in proposition 3.

**Proposition 3** In a stationary monetary equilibrium with international trade, Friedman rule of zero nominal interest rate is not always optimal.

1. For social planner trying to maximize total welfare, Friedman rule is optimal, and countries with higher degree of financial development would issue international currency.
2. For central bank trying to maximize the social welfare of its own country, Friedman rule is not optimal when there’s is foreign demand of its currency.

Figure 3 shows the second case of central bank facing trade off between seigniorage revenue and gains from trade. It plots the relationship between nominal interest rate and the welfare level of a country that issues the only international currency. For illustration, the function form is borrowed from Lagos and Wright (2005) with \( u(q) = \ln(q + b) - \ln(q) \), \( c(q) = q \), \( b = 0.0001 \). Additionally, \( \alpha = 0.5 \), \( \beta = 0.966 \), \( \sigma = 0.3 \), \( \theta = 0.5 \), \( F = 0.01 \). Friedman rule is clearly not optimal, since the welfare level is maximized around 16% of nominal interest rate. Also, the status of international currency would be lost if nominal interest rate is raised above 25%, putting a limit central bank’s conduct of monetary policy, which has been intensively discussed in Zhang (2014).

Figure 3: Optimal monetary policy

One interpretation of proposition 3 is to regard international currency as public good, in the spirit of Kindleberger (1986). Center country makes investment in banking sector and financial institution to facilitate trade. Peripheral country takes advantage of international currency as well as the system of payment and settlement. Seigniorage revenue conveys negative externality since center country tends to inflate and overproduce international
currency, and that would hurt the rest of world. For a social planner, that externality is internalized and canceled out, retaining Friedman rule as the optimal monetary policy.

4 Discussion of related topic

4.1 Size effect

Classical and recent literature uniformly favored large economy as provider of international currency due to size effect. For example, Devereux and Shi (2013) built a DSGE model for quantitative analysis, and concluded that large country is in a good position to provide international currency, since large trade volume would reduce transaction cost in FX market. In what follows I would use a numerical example to re-evaluate this issue.

In my model, economy size is approximated by national population, i.e., the total number of buyer, seller, and investor. A change in population would, according to matching function, directly influence the number and probability of the meeting between buyer and seller, thus affecting equilibrium outcome. Figure 4 shows the relationship for center country’s population and its optimal nominal interest rate. Parameter value and function forms are identical to those in figure 3. Center country’s population is ranged from 0.1 to 10, while peripheral country’s population stays at 1. A hump-shape is surprising at first sight, but the composition of center country’s welfare in figure 5 demystifies everything. In essence, size effect alters the degree of trade-off between seigniorage revenue and gains from trade. For a large economy, gains from domestic trade dominate its total welfare, so higher inflation is not a good choice. Similarly, gains from international trade makes up the biggest part of welfare for a small open economy, reducing the attractiveness of reaping seigniorage revenue. It’s therefore reasonable to think of figure 4 as a continuation of proposition 3 in exploration of center country’s optimal monetary policy. Size effect is crucial here not because of its absolute value, but in affecting the desirability of seigniorage revenue: if gains from trade loom larger and larger from size effect, convergence to Friedman rule becomes a better choice. In other words, it is the structure of economy, the share of trade in its total welfare, that determines whether a country is qualified as natural provider of international currency.

To summarize, hegemony is reasonable for a unipolar world dominated by economic superpower, while multiple international currencies make sense in a multipolar world with evenly distributed economy size.
4.2 Global imbalance

The 2008 financial crisis brought into attention the huge current account deficit of United States, known as global imbalance now. Many studies regard it as transitory phenomenon due to the saving glut of Asian countries (Bernanke, 2005) or the monetary policies of United states and exchange rate policy of emerging countries (Obstfeld and Rogoff, 2009). My model of international currency, however, illustrates that global imbalance is a sustainable and structural symptom arising from the arrangement of monetary system. If there’s only a single international currency, the center country would have trade deficit at equilibrium in a perfect symmetric model, whereas multiple international currencies would help reduce this global imbalance.

Now consider a perfect symmetric two country model where both countries are identical in size, openness, and monetary policy, so that $n_{12} = n_{21}$, $p_{12} = p_{21}$, $p_{11} = p_{22}$, $R_1 = R_2 > 0$. If country 1 issues the only international currency, equilibrium condition indicates

$$R_1 = (p_{11} + p_{12})L(q_1^1) = p_{21}L(q_2^1),$$

which naturally leads to the observation that $q_1^1 > q_2^1$, given $p_{11} + p_{12} > p_{21}$ and $L'(q) < 0$. From terms of trade, $\phi_m = (1 - \theta)u(q) + \theta^2\frac{q}{p}$, so $\phi_1m_1^1 > \phi_1m_1^2$. Intuitively, country 1 buyer would hold more real balance of country 1 currency since he expects a higher chance of meeting with home or foreign seller, while country 2 buyer would hold less real balance of international currency since his trade with home sellers is still settled in country 2 currency.
The trade balance for country 1 is therefore

\[ \text{TB}_1 \equiv \text{EX}_1 - \text{IM}_1 = n_{21}q^2_1 - n_{12}q^1_1 < 0. \]

Obviously, even in a perfect symmetric model, the single issuing country of international currency would have persistent trade deficit, which doesn’t hold for other currency regimes. In PCS,

\[
\begin{align*}
R_1 &= p_{11}L(q^1_1) = p_{21}L(q^2_1) \\
R_2 &= p_{12}L(q^1_2) = p_{22}L(q^2_2)
\end{align*}
\]

With \( q^2_1 = q^1_2 \), trade balance of both countries is zero. Similarly for LCS

\[
\begin{align*}
R_1 &= (p_{11} + p_{12})L(q^1_1) \\
R_2 &= (p_{22} + p_{21})L(q^2_2)
\end{align*}
\]

With \( q^1_1 = q^2_2 \), trade balance is also zero for both countries. This finding echoes Liu and Zhou (2015), who built a DSGE model to show the sustainability of US current account deficit resulting from the status of dollar as an international currency\(^9\).

It should be cautioned here this model doesn’t provide any normative analysis on global imbalance, since agents would always benefit from international trade, irrelevant with current account surplus or deficit. So this application only states that a system of multiple international currencies is desirable if global imbalance proves problematic.

The case of asymmetric model is complicated and sensitive to parameter value. For simplicity, the following discussion is limited to the case of hegemony where country 1 issues the only international currency. First consider the effect of monetary policy. Differentiate the trade balance of country 1 with respect to the nominal interest rate of country 1, and the result follows.

\[
\frac{\partial \text{TB}_1}{\partial R_1} = \frac{1}{R_1} \left[ \frac{\text{EX}_1}{\epsilon_L(q^2_1)} - \frac{\text{IM}_1}{\epsilon_L(q^1_1)} \right], \quad \epsilon_L(q) = \frac{\partial L}{\partial q} \frac{q}{L}
\]

From this, the effect of monetary policy on current account is crucially dependent on the

\(^9\)The mechanism of their model is quite different from mine. Like most invoicing currency model, they presumed CIA to introduce fiat money. US dollar is also exogenously assumed to be the only international currency. US trade deficit is determined by foreign demand of dollar. With positive long-run growth of global economy, there would be a structural global imbalance, whose magnitude is affected by the degree of openness, substitution elasticity between home and foreign goods, and the relative size of US economy to the rest of world.
elasticity of liquidity premium and trade volume: if \( \frac{EX_1}{\epsilon_L(q_1^1)} > \frac{IM_1}{\epsilon_L(q_1^1)} \), higher level of interest rate would deteriorate center country’s current account, otherwise inflation would help reduce global imbalance. Next consider country size effect, illustrated in figure 6 and 7 with numerical example. Both figures plot the relationship between the population and current account of the country that issues the only international currency. The function form and parameter value still follows those in figure 3, with the only exception of \( \alpha \) that represents preference shock. The level of nominal interest rate is welfare-maximizing. With a low level of \( \alpha \), as in figure 6, there’s no monotone relationship between country size and trade balance, whereas global imbalance deepened with population when \( \alpha \) is relatively high in figure 7.

![Figure 6: Asymmetric case: \( \alpha = 0.2 \)](image)

![Figure 7: Asymmetric case: \( \alpha = 0.5 \)](image)

### 4.3 International vehicle currency

The model could be easily extended into N-country case to account for the emergence of IVC. Model details and equilibrium conditions are derived in appendix for 3-country model. Here I would show a main finding from model implication: sellers in different countries would choose the same settlement currency for the export to a certain country, as long as they are identical in bargaining power and cost function. In other words, sellers display herding behavior in their choice of settlement currency.

Consider country \( j \) seller’s profit from trade with country \( i \) buyer, settled in country \( k \) currency, and its expression follows.

\[
\pi_{ij}^k = \left( 1 - \frac{F_k}{(1 - 2\sigma)\phi_k z_k} \right) \frac{1}{1 + R_k} \phi_k m_i^j - c_j(q_k^i)
\]
Notice that seller’s feature would influence only terms of trade and cost function: higher level of bargaining power brings in better terms of trade, and efficient production improves profitability. If sellers are identical in these two aspects, their profit would be equal for the export to a certain country, which leads to the rise of a common IVC to reduce financial friction. Therefore, exporter of commodity or homogeneous good would choose the same currency for settlement, which is consistent with empirical literature. Goldberg and Tille (2008) showed that US dollar is the dominant invoicing currency for the international trade of commodity and homogeneous goods\textsuperscript{10}.

5 Concluding remarks

China has been trying to internationalize RMB since great recession, and several measures are taken to accelerate this process, including currency swap agreement, offshore market development, cross-border trade settlement, and capital account liberalization. Among these, trade settlement is a natural starting point given China’s leading role in merchandise trade. In contrast with traditional view of thick market externality and natural monopoly, recent literature highlighted the importance of a deep and liquid financial market. In this paper, I verified this finding with SWIFT trade finance data, and built a two-country monetary search model to discuss the determinants of international currency. This illustrative model also emphasized government’s role in taking initiative to foster market, and explored the conduct of monetary policy in different regimes. For future research, modeling financial market in a more meaningful way is desirable for asset pricing as well as the impact of capital account liberalization.

RMB’s recent success in joining SDR basket reaffirmed China’s grand plan of financial reform and deregulation, although the outcome of such bold action remains uncertain, especially given the recent chaos in stock market. A monetary system with multiple international currencies is beneficial to United States, who has been long accused of exorbitant privilege, as well as peripheral countries often criticized for excessive reserve accumulation. Whether RMB is a qualified candidate in this race to new world, we shall wait and see.

\textsuperscript{10}Their explanation for this phenomenon is different. They argued that commodity price shows excessive volatility, and IVC is used to reduce exchange rate risk.
References


A Appendix

A.1 Data source and description

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<th>Source</th>
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<td>SWIFT dataset</td>
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<td>IMF</td>
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<td>Inflation volatility</td>
<td>coefficient of variation for monthly inflation</td>
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<td>Exchange rate</td>
<td>nominal exchange rate against USD, in log</td>
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<td>Exchange rate volatility</td>
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</tr>
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<td>UN comtrade</td>
</tr>
<tr>
<td>Real GDP</td>
<td>in log</td>
<td>+</td>
<td>World Bank</td>
</tr>
</tbody>
</table>

A.2 Letter of credit step by step

Figure 1 shows the working mechanism of LC. The following step of LC is in order.

Step 1 Exporter and importer determine terms of trade and sign business contract.

Step 2 Importer would go to issuing bank, show the contract, and apply for LC. The issuing
bank usually asks for a certain amount of collateral from importer before LC is issued.

*Letter of credit* is “a commitment by a bank on behalf of the buyer that payment would be made to the beneficiary provided that the terms and conditions stated in LC has been met, consisting of the presentation of specified documents” (US department of commerce). The issuing bank would make payment only a certain days after shipment, and that’s the maturity of LC, usually around 3 months.

Step 3 The issuing bank would send LC to advising bank for scrutiny.

Step 4 After checking details, advising bank would notify exporter so that he could prepare shipment.

Step 5 Exporter makes shipment and gets the required document, especially bill of lading (B/L).

*Bill of lading* is a document issued by carrier which details a shipment of merchandise and gives title of that shipment to a specified party, usually its holder.

Step 6 Exporter sends required document to advising bank for payment.

Step 7 After checking the required document, advising bank would notify the issuing bank. In principle, exporter needs to wait until maturity of LC, but he is usually in urgent need of liquidity, so advising bank would make payment to exporter at discount.

Step 8 The principle of “borrow short and lend long” makes advising bank unwilling to hold LC, given its short maturity. Advising bank would sell combine LC and other required documents as trade acceptance and sell it to any interested parties.

The set of documents including LC and B/L is referred to as trade acceptance or banker’s acceptance, whose payment is guaranteed by both issuing bank and advising bank, making it attractive for short-term investment.

Step 9 Upon maturity, anyone holding trade acceptance could go to issuing bank for payment. After checking the required document, issuing bank would notify importer. The importer then makes payment and gets shipment.
A.3 Equilibrium condition of different currency regimes

According to definition 1, the equilibrium condition for country 1 currency to emerge as the single international currency is the following.

For buyer
\[
\begin{align*}
R_1 &= (p_{11} + p_{12})L(q_1^1) = p_{21}L(q_1^2) \quad (1.1) \\
R_2 &= p_{22}L(q_2^2) \quad (1.2)
\end{align*}
\]

For seller
\[
\begin{align*}
\pi_1 &= \left[1 - \frac{F_1}{(1-2\sigma)\phi_1 z_1}\right]\left(\frac{1}{1+R_1}\right)\phi_1 m_1^2 - c(q_1^1) > 0 \quad (1.3) \\
\pi_1^* &= \left[1 - \frac{F_1}{(1-2\sigma)\phi_1 z_1}\right]\left(\frac{1}{1+R_1}\right)\phi_1 m_1^2 - c(q_1^1) > 0 \quad (1.4)
\end{align*}
\]

For investor
\[
R_1 = \frac{n_{1,2}\phi_1 m_1^1 + n_{2,1}\phi_1 m_1^2}{(1-2\sigma)\phi_1 z_1} - 1 \quad (1.5)
\]

For money market
\[
\begin{align*}
\sigma \phi_1 m_1^1 + \sigma \phi_1 m_2^2 + (1-2\sigma)\phi_1 z_1 + F_1 &= \phi_1 M_1 \quad (1.6) \\
\sigma \phi_2 m_2^2 &= \phi_2 M_2 \quad (1.7)
\end{align*}
\]

If equations (1.1)-(1.8) are satisfied at the same time, country 1 currency would emerge as the only international currency. Similarly, the equilibrium condition for PCS is the following.

For buyer
\[
\begin{align*}
R_1 &= p_{11}L(q_1^1) = p_{21}L(q_1^2) \quad (2.1.1) \\
R_2 &= p_{12}L(q_1^2) = p_{22}L(q_2^2) \quad (2.1.2)
\end{align*}
\]

For seller
\[
\begin{align*}
\pi_1 > 0, \quad \pi_1 > \pi_1^* \quad (2.1.3) \\
\pi_2 > 0, \quad \pi_2 > \pi_2^* \quad (2.1.4)
\end{align*}
\]
For investor

\[
\begin{align*}
R_1 &= \frac{n_2 \phi_1 q_1^2}{(1 - 2 \sigma) \phi_2 z_2} - 1 \quad (2.1.5) \\
R_2 &= \frac{n_1 \phi_2 q_2^2}{(1 - 2 \sigma) \phi_1 z_1} - 1 \quad (2.1.6)
\end{align*}
\]

For money market

\[
\begin{align*}
\sigma \phi_1 m_1^1 + \sigma \phi_1 m_1^2 + (1 - 2 \sigma) \phi_1 z_1 + F_1 &= \phi_1 M_1 \quad (2.1.7) \\
\sigma \phi_2 m_2^2 + \sigma \phi_2 m_2^2 + (1 - 2 \sigma) \phi_2 z_2 + F_2 &= \phi_2 M_2 \quad (2.1.8)
\end{align*}
\]

For LCS, equilibrium condition is the following.

For buyer

\[
\begin{align*}
R_1 &= (p_{11} + p_{12}) L(q_1^1) \quad (2.2.1) \\
R_2 &= (p_{21} + p_{22}) L(q_2^2) \quad (2.2.2)
\end{align*}
\]

For seller

\[
\begin{align*}
\pi_1^* > 0 \quad (2.2.3) \\
\pi_2^* > 0 \quad (2.2.4)
\end{align*}
\]

For investor

\[
\begin{align*}
R_1 &= \frac{n_2 \phi_1 q_1^2}{(1 - 2 \sigma) \phi_2 z_2} - 1 \quad (2.2.5) \\
R_2 &= \frac{n_1 \phi_2 q_2^2}{(1 - 2 \sigma) \phi_1 z_1} - 1 \quad (2.2.6)
\end{align*}
\]

For money market

\[
\begin{align*}
\sigma \phi_1 m_1^1 + (1 - 2 \sigma) \phi_1 z_1 + F_1 &= \phi_1 M_1 \quad (2.2.7) \\
\sigma \phi_2 m_2^2 + (1 - 2 \sigma) \phi_2 z_2 + F_2 &= \phi_2 M_2 \quad (2.2.8)
\end{align*}
\]

A.4 Proof of proposition 2

Most of this proof is straightforward except the part of exporter’s gain from international trade, which requires additional assumption on function form.

**Lemma 1** If exporter’s financial loss is more sensitive than DM cost function in response to interest rate shock, i.e., \((\epsilon_f + \epsilon_c) > 0\), exporter’s gain from international trade is decreasing in nominal interest rate, i.e., \(\frac{\partial \pi^*}{\partial R} < 0\).
Proof Let $\beta^E \equiv (1 - \frac{E}{(1-2\sigma)\phi_z})(1 + R)^{-1}$ denote the effective discount factor for seller. Without loss of generality, assume $\beta^E < \beta$ so that financial friction would reduce seller’s gain from trade. Therefore, $(\beta - \beta^E) > 0$ is a measure of seller’s financial loss. Combine the expression of buyer’s payment in DM and seller’s profit in section (3.2.4), I could get

$$\pi = \beta^E (1 - \theta) \left[ u(q) - \frac{c(q)}{\beta} \right] - \frac{1}{\beta} (\beta - \beta^E) c(q).$$

Given that $\frac{\partial \phi_z}{\partial R} < 0$, it’s easy to find that $\frac{\partial \beta^E}{\partial R} < 0$. With the previous condition in proportional bargaining, $q < q^*$, $u'(q^*) = \frac{c'(q^*)}{\beta}$, $u' < 0$, $c' > 0$, the first item is decreased in $R$.

For the second item, differentiate with respect to $R$, I could get $\frac{c_f}{\beta R} (\epsilon_f + \epsilon_c)$, where $f \equiv (\beta - \beta^E)$ captures the degree of seller’s financial loss, $\epsilon_f \equiv \frac{\partial f}{\partial R}$ is the elasticity of financial loss on nominal interest rate. Similarly, $\epsilon_c$ is the elasticity of seller’s DM cost in response to interest rate shock. Obviously, $\epsilon_f > 0$, $\epsilon_c < 0$. A sufficient condition for $\frac{\partial \pi}{\partial R} < 0$ is $\epsilon_f + \epsilon_c > 0$. Therefore, as long as financial loss is more sensitive to the change of nominal interest rate, higher inflation level would decrease exporter’s gain from international trade.

Another helpful observation is the property of buyer’s liquidity premium. Recall its definition.

$$L(q) \equiv \frac{\theta (u'(q) - \frac{c'(q)}{\beta})}{(1 - \theta)u'(q) + \theta \frac{c'(q)}{\beta}}$$

Take differentiation with respect to $q$, I could get the following result.

$$L'(q) = \frac{\theta}{\beta} \left[ (1 - \theta)u'(q) + \frac{\theta}{\beta} c'(q) \right]^{-2} (u''c' - u'c'')$$

With previous assumption on function form, $u' > 0$, $c' > 0$, $u'' < 0$, $c'' > 0$, it’s obvious that $L'(q) < 0$, so buyer’s liquidity premium is decreasing in his trade volume. After these preparations, now it’s easy to prove proposition 2.

Proof of Proposition 2

On the part of exporters, assume $\epsilon_f + \epsilon_c > 0$ always holds.
For single international currency

\[
\begin{align*}
\frac{\partial q_1}{\partial R_1} &= ((p_{11} + p_{12})L'(q_1^1))^{-1} < 0 \\
\frac{\partial q_2}{\partial R_1} &= (p_{21}L'(q_1^2))^{-1} < 0
\end{align*}
\]

For PCS

\[
\begin{align*}
\frac{\partial q_1}{\partial R_1} &= 0 \\
\frac{\partial q_2}{\partial R_1} &= (p_{21}L'(q_1^2))^{-1} < 0
\end{align*}
\]

For LCS

\[
\begin{align*}
\frac{\partial q_1}{\partial R_1} &= ((p_{11} + p_{12})L'(q_1^1))^{-1} < 0 \\
\frac{\partial q_2}{\partial R_1} &= 0
\end{align*}
\]

A.4.1 Welfare level

For PCS, the welfare level is the following.

\[
W_1 = \mu_1 \sigma_1 m^2_1 - \mu_2 \sigma_2 m^2_2 + n_{11} \left[ \beta u(q_1^1) - c(q_1^1) \right] + n_{12} \left[ \beta u(q_2^1) - c(q_2^1) \right] + n_{21} \left[ \frac{1}{1 - 2 \sigma} \frac{1}{\phi_1 z_1} \right] \left( \frac{1}{1 + R_1} \right) \phi_1 m^2_1 - c(q_1^2) \right] - F_1
\]

\[
W_2 = \mu_2 \sigma_2 m^2_2 - \mu_1 \sigma_1 m^2_1 + n_{22} \left[ \beta u(q_2^2) - c(q_2^2) \right] + n_{21} \left[ \beta u(q_1^2) - c(q_1^2) \right] + n_{12} \left[ \frac{1}{1 - 2 \sigma} \frac{1}{\phi_2 z_2} \right] \left( \frac{1}{1 + R_2} \right) \phi_2 m^2_2 - c(q_2^2) \right] - F_2
\]

For LCS, welfare level is the following.
\[ W_1 = n_{11} \left[ \beta u(q^1_1) - c(q^1_1) \right] + n_{12} \theta \left[ \beta u(q^1_1) - c(q^1_1) \right] \]
\[ + n_{21} \left\{ \left[ 1 - \frac{F_2}{(1 - 2\sigma)\phi_{2z_2}^2} \right] \left( \frac{1}{1 + R_2} \right) \phi_{2m_2}^1 - c(q^2_2) \right\} - F_1 \]

\[ W_2 = n_{22} \left[ \beta u(q^2_2) - c(q^2_2) \right] + n_{21} \theta \left[ \beta u(q^2_2) - c(q^2_2) \right] \]
\[ + n_{12} \left\{ \left[ 1 - \frac{F_1}{(1 - 2\sigma)\phi_{1z_1}^1} \right] \left( \frac{1}{1 + R_1} \right) \phi_{1m_1}^2 - c(q^1_1) \right\} - F_2 \]

**A.5 Three-country model**

The potential payment system in three-country model is quite numerous, and this part is concerned about the rise of international vehicle currency (IVC), which is used to settle trade between non-issuing countries. The assumption in two-country model could be easily applied here, requiring only minor change of notation. \( \forall i, j, k \in 1, 2, 3 \), \( p_{ij} \) is the probability of successful matching between country i buyer and country j seller; \( n_{ij} \) is the corresponding number of meeting; \( q^i_j \) is country i buyer’s holding of country j currency; \( \pi^k_{ij} \) is country j seller’s profit from his trade with country i buyer, settled in country k currency. Most importantly, here I assume sellers in different countries are identical in bargaining power and cost function, so that the consistency from proposition 4 would hold. For simplicity, the following discussion covers only the case of single and double international currency.

**A.5.1 Single dominance**

Now consider a case of hegemony in three-country model, assuming country 1 issues the only international currency. Figure 8 shows the payment system in this case, where all the international trade is settled in country 1 currency. The following equilibrium condition is in order.
For buyer

\[
\begin{align*}
R_1 &= (p_{11} + p_{12} + p_{13})L(q_1^1) = (p_{21} + p_{23})L(q_1^3) = (p_{31} + p_{32})L(q_1^2) \quad (3.1.1) \\
R_2 &= p_{22}L(q_2^2) \\
R_3 &= p_{33}L(q_3^3)
\end{align*}
\]

For seller

\[
\begin{align*}
\pi_{21} &= \pi_{23} = J_1 \phi_1 m_1^2 - c(q_1^2) > 0 \quad (3.1.4) \\
\pi_{31} &= \pi_{32} = J_1 \phi_1 m_1^3 - c(q_1^3) > 0 \quad (3.1.5)
\end{align*}
\]

For investor

\[
[(1 - 2\sigma)\phi_1 z_1](1 + R_1) = (n_{12} + n_{13})\phi_1 m_1^1 + (n_{21} + n_{23})\phi_1 m_1^2 + (n_{31} + n_{32})\phi_1 m_1^3 \quad (3.1.6)
\]

Money market

\[
\begin{align*}
\sigma(\phi_1 m_1^1 + \phi_1 m_1^2 + \phi_1 m_1^3) + (1 - \sigma)\phi_1 z_1 + F_1 &= \phi_1 M_1 \quad (3.1.7) \\
\sigma \phi_2 m_2^2 &= \phi_2 M_2 \quad (3.1.8) \\
\sigma \phi_3 m_3^3 &= \phi_3 M_3 \quad (3.1.9)
\end{align*}
\]

The incumbency advantage from proposition 1 still applies here: as long as individual sellers enjoy positive profit from international trade, they have no incentive to deviate from the existing equilibrium.

A.5.2 Dual dominance: PCS

Now consider the case of double international currencies where the international trade between country 1 and 2 is settled through PCS, while country 3 relies on other country’s currency for settlement. To achieve consistency of decision, country 3 sellers choose country 1 currency to settle trade with country 2, which is the same as country 1 seller’s choice. Apply a similar procedure to other seller’s choice, and the payment pattern is shown in figure 9, with the following equilibrium condition. For buyer

\[
\begin{align*}
R_1 &= p_{11}L(q_1^1) = (p_{21} + p_{23})L(q_1^3) = (p_{31} + p_{32})L(q_1^2) \quad (3.2.1) \\
R_2 &= p_{22}L(q_2^2) = (p_{13} + p_{12})L(q_2^1) \\
R_3 &= p_{33}L(q_3^3)
\end{align*}
\]

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For seller

\[
\begin{align*}
\pi_{12}^2 &= \pi_{13}^2 = J_2\phi_2 m_2^1 - c(q_2^1) > 0 \\
\pi_{21}^1 &= \pi_{23}^1 = J_1\phi_1 m_1^2 - c(q_1^2) > 0 \\
\pi_{31}^1 &= \pi_{32}^1 = J_1\phi_1 m_1^3 - c(q_1^3) > 0 \\
\pi_{21}^1 > \pi_{21}^2 &\Rightarrow J_1\phi_2 m_2^1 - c(q_2^1) > J_2\phi_2 m_2^2 - c(q_2^2) \\
\pi_{12}^2 > \pi_{12}^1 &\Rightarrow J_2\phi_2 m_2^1 - c(q_2^1) > J_1\phi_1 m_1^1 - c(q_1^1)
\end{align*}
\] (3.2.4)

For investor

\[
\begin{align*}
[(1 - 2\sigma)\phi_1 z_1](1 + R_1) &= (n_{21} + n_{23})\phi_1 m_1^2 + (n_{31} + n_{32})\phi_1 m_1^3 \\
[(1 - 2\sigma)\phi_2 z_2](1 + R_2) &= (n_{12} + n_{13})\phi_2 m_2^1
\end{align*}
\] (3.2.9)

Money market

\[
\begin{align*}
\sigma(\phi_1 m_1^2 + \phi_1 m_1^3) + (1 - 2\sigma)\phi_1 z_1 + F_1 &= \phi_1 M_1 \\
\sigma(\phi_2 m_2^2 + \phi_2 m_2^1) + (1 - 2\sigma)\phi_2 z_2 + F_2 &= \phi_2 M_2 \\
\sigma\phi_3 m_3^2 &= \phi_3 M_3
\end{align*}
\] (3.2.11, 3.2.12, 3.2.13)

With double international currencies, the incumbency advantage in proposition 1 is no longer present. The existence of such equilibrium requires not only positive profit for sellers, but also the incentive-compatible condition in (3.2.7) and (3.2.8), otherwise deviation is justified.

A.5.3 Dual dominance: LCS

For another possibility of double international currency, assume the trade between country 1 and 2 to be settled through LCS. Figure 10 shows the payment system, and several equilibrium conditions follow. For buyer

\[
\begin{align*}
R_1 &= (p_{11} + p_{12} + p_{13})L(q_1^1) = (p_{31} + p_{32})L(q_1^3) \\
R_2 &= (p_{22} + p_{21} + p_{23})L(q_2^2) \\
R_3 &= p_{33}L(q_3^3)
\end{align*}
\] (3.3.1, 3.3.2, 3.3.3)
For seller

\[
\begin{align*}
\pi_{12} &= \pi_{13} = J_1 \phi_1 m_1^1 - c(q_1^1) > 0 \hspace{1cm} (3.3.4) \\
\pi_{21} &= \pi_{23} = J_2 \phi_2 m_2^2 - c(q_2^2) > 0 \hspace{1cm} (3.3.5) \\
\pi_{31} &= \pi_{32} = J_1 \phi_1 m_1^3 - c(q_1^3) > 0 \hspace{1cm} (3.3.6)
\end{align*}
\]

For investor

\[
\begin{align*}
[ (1 - 2\sigma) \phi_1 z_1 ] (1 + R_1) &= (n_{12} + n_{13}) \phi_1 m_1^2 + (n_{31} + n_{32}) \phi_1 m_1^3 \hspace{1cm} (3.3.7) \\
[ (1 - 2\sigma) \phi_2 z_2 ] (1 + R_2) &= (n_{21} + n_{23}) \phi_2 m_2^2 \hspace{1cm} (3.3.8)
\end{align*}
\]

Money market

\[
\begin{align*}
\sigma (\phi_1 m_1^1 + \phi_1 m_1^3) + (1 - 2\sigma) \phi_1 z_1 + F_1 &= \phi_1 M_1 \hspace{1cm} (3.3.9) \\
\sigma \phi_2 m_2^2 + (1 - 2\sigma) \phi_2 z_2 + F_2 &= \phi_2 M_2 \hspace{1cm} (3.3.10) \\
\sigma \phi_3 m_3^3 &= \phi_3 M_3 \hspace{1cm} (3.3.13)
\end{align*}
\]

One interesting feature in this system is the pattern of IVC. For PCS in figure 9, the trade between country 2 and 3 is completely settled by country 1 currency, while for LCS in figure 10, there’s no such dominant IVC.

Figure 8: Three-country model: single international currency
Figure 9: Three-country model: dual international currency, PCS

Figure 10: Three-country model: dual international currency, LCS