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Abstract

We evaluate the predictive performance of a variety of value-at-risk (VaR) models for a portfolio consisting of five assets. Traditional VaR models such as historical simulation with bootstrap and filtered historical simulation methods are considered. We suggest a new method for estimating Value at Risk: the filtered historical simulation GJR-GARCH method based on bootstrapping the standardized GJR-GARCH residuals. The predictive performance is evaluated in terms of three criteria, the test of unconditional coverage, independence and conditional coverage and the quadratic loss function suggested. The results show that classical methods are inefficient under moderate departures from normality and that the new method produces the most accurate forecasts of extreme losses.

Keywords: Value at Risk, bootstrap, GARCH,

JEL codes: C15, C58, G17

1. Introduction

The last years and after the failure of certain big financial institutions at the beginning of the 90s, the Value at Risk (VaR) concept has become the major tool in market risk management worldwide. VaR is defined as the maximum potential loss in value of an asset or a portfolio with a given confidence level over a certain horizon. The popularity of VaR is mostly related to the simplicity of its calculation and interpretation. It summarizes the market risk exposure of all financial instruments in a portfolio into a single number.

The purpose of this paper is to analyze the capacity of the bootstrap to ameliorate the pertinence of VaR methods. We will consider five different VaR estimation methods: historical simulation with bootstrap (BHS), historical simulation with circular block bootstrap (CBBHS) and the filtered historical simulation method (FHS) suggested by Barone-Adesi et al. (1999). The FHS which takes into account parameter estimation error in GARCH models suggested by Christoffersen and Gonçalves (2005) is also examined. Finally, the GARCH model suggested by Glosten, Jagannathan and Runkle (1993) (GJR-GARCH after their surnames) is implemented.

These methods are backtested for their out-of-sample predictive ability by using the tests of unconditional coverage, independence and conditional coverage suggested by Christoffersen (1998) and the quadratic loss function suggested by Lopez (1999).

This paper is organized as follows: various VaR methods are discussed in section 2. Section 3 provides a description of the GARCH models. Section 4 presents techniques of backtesting: test of unconditional coverage, independence and conditional coverage of Christoffersen (1998) and the quadratic loss function suggested by Lopez (1999). Empirical results and predictive performance evaluation of several methods are presented in section 5 using real data. Finally, the last Section concludes this paper.

2. VaR methods

VaR has become a golden standard for measuring and assessing risk. Different methods of VaR are nowadays developed in banks. VaR is defined as the potential loss that is expected to occur over a predetermined period and at a given confidence level α under hypothesis that during a period of times the composition of the portfolio remains unchanged.

We will briefly present four VaR calculation methods and we will focus on the strengths and weaknesses of each method. Let $R_t = \log\left(\frac{p_t}{p_{t-1}}\right)$ be the returns at time t where p_t is the price of portfolio at time t. Then the VaR with confidence level α is given by

$$VaR_t(\alpha) = F^{-1}(1-\alpha), \tag{1}$$

where F^{-1} is the inverse of the cumulative distribution function of the returns R_t.

2.1 Bootstrap historical simulation method

This method is undoubtedly the simplest method in its comprehension and its implementation. It assumes that the distribution of returns will remain the same in the past and in the future and hence historical returns will be used in the forecast of VaR.

Bootstrap historical simulation method (BHS) generates pseudo returns by sampling with replacement from the set of original returns. Then every bootstrap sample allows acquiring an estimate of VaR using (1). In the end, the average of all the VaR estimates gives us the bootstrap estimate of VaR.

2.2 Circular block bootstrap historical simulation method

In the presence of dependence the BHS could give erroneous results. Therefore, alternative approaches have been developed to deal with this problem. Hall (1985) proposed to resample the data using data blocks. The block method can account for a possible phenomenon of memory (that is to say that the value of the series at time t depends on past values present in the studied series). We will use the circular block bootstrap Politis and Romano (1992) which makes better use of the information we have about dependence. This approach assumes the data live on a circle so that $y_{T+1} = y_1, y_{T+2} = y_2$, etc. Thus, all data points get sampled with equal probability.

Circular block Bootstrap historical simulation method (CBBHS) generates pseudo returns by applying circular block bootstrap to original returns. Then every bootstrap sample allows acquiring an estimate of VaR using (1). In the end, the average of all the VaR estimates gives us the bootstrap estimate of VaR.

2.3 Filtered Historical Simulation

To overcome the problem arising with the assumption of constant volatility, Barone-Adesi et al. (1999) introduced the filtered historical simulation method (FHS). This method is a Monte Carlo based approach that combines volatility models and the BHS method. Therefore, it can accommodate the volatility clustering, asymmetry and heavy tails of the empirical distribution of returns. According to the GARCH (1, 1) model, returns are generated as

$$R_t = \sigma_t z_t, \tag{2}$$

where $Z_t \sim N(0,1)$ and

$$\hat{\sigma}_{t+1}^2 = \hat{\omega} + \hat{\alpha} R_t^2 + \hat{\beta} \hat{\sigma}_t^2, \tag{3}$$

where R_t denotes the daily return, z_t are i.i.d standard normal random variables, σ_{t+1}^2 denotes the conditional variance and the parameters ω, α *and* β satisfy $\omega > 0, \alpha \geq 0$ and $\beta \geq 0$ to ensure the positivity of the conditional variance.

Step 1. GARCH model is fitted to the return data via quasi-maximum likelihood. Once the parameters $(\hat{\omega}, \hat{\beta}, \hat{\alpha})$ are calculated, the conditional variance is calculated using (3), with $\hat{\sigma}_1^2 = \frac{\hat{\omega}}{1-\hat{\alpha}-\hat{\beta}}$. Then, the standardized returns, $\hat{Z}_t = \frac{R_t}{\hat{\sigma}_t}$, are computed.

Step 2. A sample of T standardized bootstrapped returns $\hat{Z}_t^*$ $(t = 1, \dots, T)$ is generated.

Step 3. The FHS VaR is estimated via

$$VaR_{T+1}^{*i} = \hat{\sigma}_{T+1} \hat{Z}_{SHF,T}^*,$$

where $\hat{Z}_{SHF,T}^*$ is the α-th quantile of the empirical distribution of $\hat{Z}_t^*$.

Step 4. By repeating step 2 and 3 N times we acquire a series of FHS VaR_{T+1}^{*i}, $i = 1, \dots, N$. The estimator of VaR is

$$\widehat{VaR} = \frac{1}{N} \sum_{i=1}^{N} VaR_{T+1}^{*i}.$$

However, estimation of the parameters of GARCH model introduces a source of error due to the normality assumption of the innovations. Pascual et al. (2006) used bootstrap which allows incorporating the uncertainty of the estimated parameters without distributional assumptions on the sequence of innovations. Christoffersen and Gonçalves (2005) implemented this method to obtain confidence intervals for the VaR and expected shortfall. Furthermore, Hartz et al. (2006) used the bootstrap procedure proposed by Pascual et al. (2006) as a bias-correction method, for improving the VaR forecasting ability of the normal-GARCH model. Estimation of the FHS VaR as suggested by Christoffersen and Gonçalves (2005) takes place as follows.

Step 1. GARCH model is fitted to the return data by maximum likelihood. Once the parameters $(\hat{\omega}, \hat{\beta}, \hat{\alpha})$ are calculated, the conditional variance, using (3), is calculated.

Step 2. The standardized returns are calculated: $\hat{Z}_t = \frac{R_t}{\hat{\sigma}_t}$.

Step 3. Obtain $\{R_t^*, t = 1, \dots, T\}$ using the recursive formulas

$$\hat{\sigma}_{t+1}^{2*} = \hat{\omega} + \hat{\alpha} R_t^{2*} + \hat{\beta} \hat{\sigma}_t^{2*}$$

$$R_t^* = \sigma_t^* Z_t^*, \ t = 1, \dots, T, \tag{4}$$

where $\hat{\sigma}_1^{2\,*} = \hat{\sigma}_t^2 = \frac{\widehat{\omega}}{1-\hat{\alpha}-\widehat{\beta}}$ and Z_t^* is a random draws with replacement from the original standardized returns series

Step 4. Re-estimate the GARCH (1,1) for the replicate sample by maximum likelihood, resulting in a new set of parameters $\left(\widehat{\omega}^*, \hat{\beta}^*, \hat{\alpha}^*\right)$

Step 5. A bootstrap prediction of volatility $\hat{\sigma}_{T+1}^{2\,*}$ is obtained by using $\widehat{\sigma_{T+1}^{2*}} = \widehat{\omega^*} + \widehat{\alpha^*} R_T^{2*} + \widehat{\beta^*}\widehat{\sigma_T^{2}}^*$, given the initial values $R_T^* = R_T$ and

$$\hat{\sigma}_T^2 = \frac{\widehat{\omega}^*}{1-\hat{\alpha}^*-\hat{\beta}^*} + \hat{\alpha}^* \sum_{j=0}^{T-2} \hat{\beta}^{*j} \left[R_{T-j-1}^2 - \left(\frac{\widehat{\omega}^*}{1-\hat{\alpha}^*-\widehat{\beta}^*}\right)\right] \tag{5}$$

The equation (5) uses the initial series of returns therefore σ_T^2 is small when returns at the end of period are small and large when these one are large. Therefore, $\hat{\sigma}_{t+1}^{2\,*}$ incorporates the variability due to parameter estimation and take into account the state of process when we make an estimate.

Step 5. Estimation of the FHS VaR takes place as

$$VaR_{T+1}^{*i} = \hat{\sigma}_{T+1} \hat{Z}_{SHF,T}^*,$$

where $\hat{Z}_{SHF,T}^*$ is the α empirical quantile of $\hat{Z}_t^*$.

Step 6. Steps 3, 4 and 5 are repeated N times and a series of VaR, $VaR_{T+1}^i,\ i = 1, \dots, N$ is acquired. The FHS estimator of VaR is given by

$$\widehat{VaR} = \frac{1}{N} \sum_{i=1}^{N} VaR_{T+1}^{*i}.$$

2.3 GJR-GARCH (1, 1) model

GARCH models allow us to express the conditional variance as a linear function of lagged squared returns and lagged conditional variance terms and it can successfully capture several characteristics of financial time series, such as heavy tails and volatility clustering. Bollerslev et al. (1992) showed that the GARCH(1,1) model (given in (2) and (3)) fits adequately to many financial time series.

GARCH model presents some implementation drawbacks, since bad and good news, have the same impact on the conditional variance. Indeed, equity markets often seem to display a strong asymmetry, a negative return boosts volatility by more than a positive return of the same absolute magnitude (Engle and Ng, 1993).

To overcome this limitation, asymmetric GARCH models were introduced. The most popular models proposed to capture the asymmetric effects are the Exponential GARCH (EGARCH) model of Nelson (1991), the Asymmetric GARCH (AGARCH) model of Engle and Ng (1993), and the GJR-GARCH model (Glosten et al., 1993). Engle and Ng (1993) compared asymmetric volatility models and concluded that the GJR-GARCH is the best.

The GJR-GARCH(1,1) model allows negative and positive past shocks to have a different effect on future conditional second moments. This model simply augments the standard GARCH (1, 1) formulation with an additional ARCH term conditional on the sign of the past innovation

$$R_t = \sigma_t Z,$$

$$\sigma_{t+1}^2 = \omega + \alpha R_t^2 + \theta I_t R_t^2 + \beta\, \sigma_t^2 = \omega + (\alpha + \theta I_t) R_t^2 + \beta\, \sigma_t^2,$$

where R_t denotes the daily return, Z_t are i.i.d standard random variables, σ_{t+1}^2 denotes the conditional variance, I_t denotes the indicator function, $I_t = 1$ if $R_t < 0$ and $I_t = 0$ otherwise and the parameters ω, α, β and θ satisfy $\omega > 0, \alpha \geq 0, \beta \geq 0$ and $\alpha + \theta > 0$ to ensure the positivity of the conditional variance. Thus, the effect of positive innovation is given by α and the effect of negative return is given by α+θ. Hence this model allows a response of volatility to news with different coefficients for good and bad news.

The distribution assumed for the standardized returns in equation of GJR-GARCH can have an impact on the conditional volatility estimate. Indeed, there is no distribution that consistently produces better results for all returns series. Furthermore, Kuester et al. (2006) have found that the distribution used in the filtering stage influence the FHS VaR obtained. That is why we will apply the method of Christoffersen and Gonçalves (2005) which use Pascual et al. (2006) method that forecast the GARCH density based on a bootstrap procedure to obtain forecast of conditional volatilities which allows it incorporating the parameter uncertainty without relying on any particular assumption about the distribution of standardized returns. We will also replacing the GARCH model by GJR-GARCH. Thus, our method can incorporate parameter uncertainty, accommodate the volatility clustering, asymmetry and heavy tails of the empirical distribution of returns.

3. Evaluating VaR methods

Various tests have been suggested for evaluating VaR model accuracy. In this paper, statistical adequacy will be tested based on Christoffersen (1998) and Lopez (1999) backtesting measures.

3.1 Unconditional coverage

Denoting the daily return of the portfolio at time t as R_t.The ex-ante VaR at time t denoted as VaR_t. Christoffersen (1998) defines the sequence of VaR violations as

$$I_t(\alpha) = \begin{cases} 1 & if \;\; R_t < VaR_t \\ 0 & if \;\; R_t \geq VaR_t \end{cases}.$$

The unconditional coverage test implicitly assumes that the violations are independent and it examines whether the observed violation rate is statistically equal to the expected one. Nevertheless, if the unconditional cover allows making sure that the proportion of the exceptions for given period guarantees the nominal cover, it cannot detect clustered violations. For that purpose, Christoffersen (1998) introduced a second test, which is the test of independence.

3.2 Tests of independence

The second test proposed by Christoffersen (1998) checks whether violations are time dependent. Indeed, this test verifies that VaR violations observed at two different dates must be independently distributed. Thus, this test verifies whether the variable $I_t(\alpha)$ associated with the exception in the date t, is independent from $I_{t-k}(\alpha)$ for any value of k different from 0.

3 .3 Conditional coverage

Finally, the third test of Christoffersen (1998) is the test of conditional coverage which jointly examines the conjecture that the total number of failures is statistically equal to the expected one and the VaR violations are independent. Thus, the conditional coverage test can be decomposed into a test of the unconditional coverage plus a test of independence.

3.4 Loss functions

In most cases, there is more than one model that satisfies Christoffersen's tests and hence, the risk manager cannot select a unique method. To overcome this shortcoming, Lopez (1999) suggested measuring the accuracy of VaR forecasts based on a loss function. We are going to consider the quadratic function which takes into account the scale of the exceptions

$$L_{t+1} = \begin{cases} 1 + (R_{t+1} - VaR_{t+1})^2 & if \;\; R_{t+1} \leq VaR_{t+1} \\ 0 & if \;\; R_{t+1} \geq VaR_{t+1} \end{cases}. \quad (6)$$

This loss function (6) accounts for the magnitude of the tail losses and adds a score of 1 whenever a violation is observed. Under this framework, a method will be preferred if it minimizes the total loss $\sum_{t=1}^{T} L_t$.

4. Empirical Results

The main propose of this study is to empirically assess different VaR estimation methods.

4.1 Data and descriptive statistics

The data set used throughout this paper consists of daily returns of portfolio composed of five shares: AXA, Peugeot, L'Oreal, Air France and Air liquid and it is assumed that the portfolio weights are equal and constant over time. The sample period is from April 24, 2003 to February 27, 2009 with the total of 1500 observations. The sub-period from March 22, 2006 has been reserved for backtesting purposes. We computed the daily log returns which defined by $R_p(t) = \log(p_t/p_{t-1})$, where p_t is the price of portfolio at time t. All computations were implemented with MATLAB 7.1 and R 3.1.

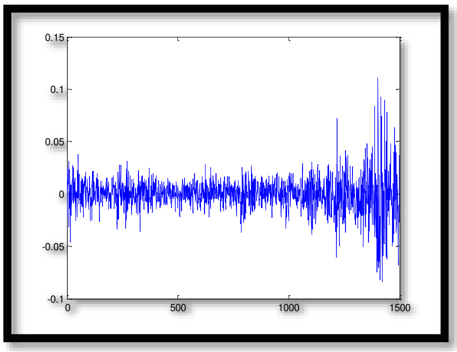


Figure 1. Evolution of daily portfolio returns from April 24, 2003 to February 27, 2009

Mean	Minimum	Maximum	Standard deviation	Skewness	Kurtosis	Jaque-Bera	Q^2-Test
-0.0003	-0.0845	0.111	0.0158	0.0846	10.0107	727.88	45.7726

Table 1. Summary statistics of the returns.

Figure 1 shows that daily returns of portfolio are highly volatile. Thus, we can conclude that the portfolio returns have the features of volatility clustering.

From Table 1 we see that the mean of daily log returns is close to zero. The maximum and minimum statistics are quite large in absolute value, indicating the presence of extreme returns. Positive skewness implies that positive extreme returns are more likely to occur than the normal distribution forecasts. Kurtosis is greater than 3, which means that extreme events occur more frequently in the sample data than can be expected under normal distribution.

Furthermore, the Jarque-Bera normality test provides further evidence of the non normality of the distribution..

Moreover, the Ljung-Box Q^2 test shows a significant autocorrelation, since there is volatility clustering, that is, the returns are not i.i.d and GARCH type models should be considered when estimating VaR. Hence, any VaR method must account for volatility clustering, excess kurtosis and skewness at the same time.

4.2 Out-of-sample evaluation

For evaluating performance of competing VaR methods, we compare VaR in terms of out-of-sample one-step-ahead predictive ability. For each method, a rolling window of 750 observations of daily data are used in estimation, and used to form a VaR forecast for day 751. After this, data from day 2 till 751 is used in estimation to obtain a VaR forecast for day 752. This procedure is repeated until the VaR of all observations will be forecast. The results will be firstly examined graphically and secondly by using the violation rate, the likelihood ratio tests statistics for the unconditional coverage, independence and conditional coverage and the quadratic loss function (6). In all cases, bootstrap was used with 1000 re-samples.

4.2.1 Graphical analysis

Figure 2(a) shows that the VaR estimates computed with the BHS and CBBHS methods are representative of actual portfolio losses. We can see also that this method is not an efficient VaR estimator since it does not react adequately to the changes in the market and occurrence of extreme events that is to say it does not adapt adequately and timely to the changes. Indeed, the subprime and financial crises increase dramatically market volatility. The BHS and CBBHS methods (Figure 2(a)) do not update the VaR number quickly enough when market volatility rises.

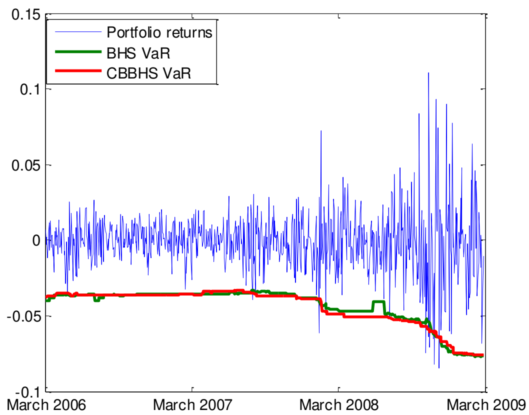
In order to incorporate volatility clustering in bootstrap method we use FHS. We can see from Figure 2(b) that the FHS method provides a major improvement over the BHS method. Indeed it reacts much stronger to volatility changes in returns than the others methods do. Therefore, it seems to describe more efficiently the tails of the empirical distribution.

A comparison between the FHS method proposed by Barone-Adesi et al. (1999) and that proposed by Christoffersen and Gonçalves (2005) is presented in Figure 2(b). The Christoffersen and Gonçalves (2005) FHS VaR estimates are more accurate than the Barone-Adesi et al. (1999) FHS VaR estimates. Thus, we have evidence to say that when the bootstrap is applied to estimate GARCH parameters the estimated VaR will be more precise.

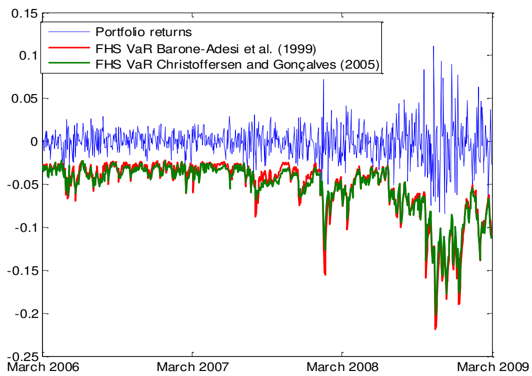
To highlight the pertinence of asymmetric GARCH models, we are going to compare results given by FHS-GJR-GARCH method with those given by FHS-GARCH method offered by Christoffersen and Gonçalves in (2005) (see Figure 2(c)).

Figure 2(c) shows that the FHS-GJR-GARCH VaR estimates are generally higher and considerably more volatile than the FHS-GARCH VaR estimates. Therefore, this method seems to capture the portfolio risk adequately. Indeed, this method can accommodate the volatility clustering, leverage effect, asymmetry and heavy tails of the empirical distribution of portfolio returns. Furthermore, this method allows incorporating parameter uncertainty in GARCH model.

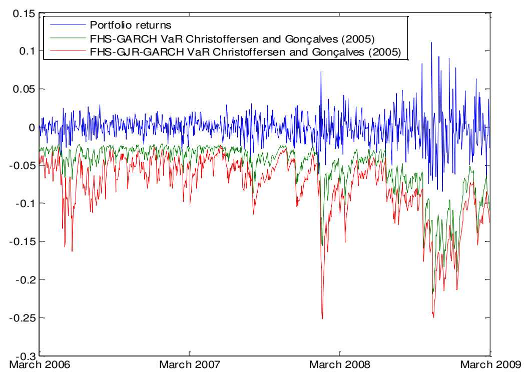
The majority of methods performed poorly at the 99 percent level of confidence. In addition, we observe that conditional VaR forecasts increase with increasing volatility but also decrease with decreasing volatility. Unconditional models on the other hand, produce VaR forecasts that react to changing market conditions slowly. The FHS with GJR-GARCH volatility model produces more accurate forecasts and performs better than all other methods.



(a)



(b)



(c)

Figure 2. Portfolio returns and 99 percent VaR estimates of (a) the BHS method, (b) FHS as suggested by Barone-Adesi et al. (1999) and Christoffersen and Gonçalves (2005) and (c) FHS-GJR-GARCH method with the FHS-GARCH suggested by Christoffersen and Gonçalves (2005).

4.2.2 Comparison of the VaR methods

VaR methods are compared with respect to four criteria: violation rate and likelihood ratio tests statistics for the unconditional coverage, independence and conditional coverage (see Table 2).

The violation rate is defined as the number of violations divided by the total number of forecasts. If the VaR method is correctly specified, the failure rate should be equal to pre-specified VaR level. Thus, at 99 percent confidence level, it is only allowed for violation rate to be violating around 1 percent.

VaR methods	VaR violations (violation rate)	LR_{uc}	LR_{ind}	LR_{cc}	Loss function
BHS	11 (1.47 percent)	1.4613	0.3288	1.7901	11.0022
CBBHS	10 (1.335 percent)	0.769	0.271	1.067	10.0018
FHS	7 (0.935 percent)	0.033	0.132	0.184	7.000
FHS GARCH	5 (0.668 percent)	0.947	0.067	1.028	5.000
FHS GJR-GARCH	0 (0.000 percent)	Undefined	Undefined	Undefined	0.0000

Table 2. Violation rates and the likelihood ratio test statistics for the unconditional coverage, independence and conditional coverage. The numbers in bold indicate rejection of the null hypothesis at the 5 percent significance level.

According to this criterion, the best model among these methods is the FHS-GJR-GARCH since it is able to forecast correctly all the losses. The next best model is the FHS-GARCH having 5 violations and the third best model is the FHS which has 7 violations.

At first, we examined whether the observed violation rate was equal to the expected one. We then checked whether the probability of a violation occurring in one day is independent from events occurred the day before. Finally, we applied the conditional coverage test.

Based on the violation rate and the three backtesting measures, the methods that give accurate forecasts are the CBBHS, BHS, the FHS, the FHS and the GJR-GARCH models.

4.2.3 Model selection

According to the backtesting results, there are many methods that perform well. Therefore, we use the quadratic loss function to choose the VaR method which yields the least loss. Table 2 presents the summary results of the quadratic loss function approach.

The results lead to the conclusion that even though CBBHS, BHS or FHS and FHS GARCH provide correct conditional coverage for 99 percent confidence level, risk managers should be very careful when using it. Amongst these methods, the FHS-GJR-GARCH is preferred since it minimizes the value of the quadratic loss function (6). Therefore, the combination of an HS method and an asymmetric volatility model yields the best results.

6. Conclusions

The results of this paper provide an important implication to investors, risk managers and financial institutions. It show that the consideration of the asymmetric effect of the positive and negative returns in conditional variance and the use of bootstrap to reduce parameters estimation errors in GARCH models are necessary for accurate estimation of risk.

We proposed a new method to estimate VaR, the FHS GJR-GARCH which is a bootstrap based approach that combines asymmetric volatility model and the BHS method. In addition, it extends the resampling technique of Pascual et al. (2006) which accounts for parameters estimation error in GARCH models.

The BHS methods do not update VaR quickly when market volatility increases. The FHS GJR-GARCH method is the best performer since it provides more precise VaR estimates and its testing results are more significant. We saw that bootstrap reduces significantly the error of the parameters in the GARCH(1,1) model and improves considerably the performance of VaR methods. In addition, the consideration of the asymmetric effect of the positive and negative returns in conditional variance allows giving more precise results. Consequently, investors are encouraged to use the FHS GJR-GARCH method to estimate VaR.

The methods presented and studied above are well-suited for providing forecasts of portfolio returns. However, the dependence in the comouvements of asset returns in portfolio is important. Thus, to expand on the work done in this paper, FHS GJR-GARCH method needs to be improved in ways that account for correlations. In other words, a multivariate volatility models should be adopted to have a complete picture of the risk. A possible suggestion, for further research could be the dynamic conditional correlation GJR-GARCH (DCC-GJR-GARCH) model which can accommodate the volatility clustering, correlation dynamics, leverage effect and heavy tails of the empirical distribution.

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