Foreign exchange rates with the Taylor rule and VECMs

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Abstract

In this project, we challenge the conventional wisdom on exchange rate predictability with the Taylor rule (Molodtsova & Papell, 2009; Rossi, 2013) by employing the vector error correction model (VECM) when the cointegration (CI) rank of our multivariate model is greater than one and less than full. Even though our approach is quite bounded to the finding of a suitable CI rank, our predictions are quite good when compared to a driftless random walk as a benchmark in the long run, whilst the performance in the short run is not. Notwithstanding we claim that we could also obtain better results had we been able to perform a static forecast for three months ahead rather than one (the latter is the only case admitted by the gretl software).

1. Introduction

Several empirics regarding exchange rate projections have been done by researchers and practitioners under which the outcomes could be broadly worthwhile for central bankers, policymakers, or private businesses. Various predictors, models, data specifications, and evaluation methods have been employed. Accordingly, such the variety to a certain extent brings about contradictory and divergent results among the relevant studies. And, the typical toughest benchmark is a random walk without drift. Lately, Rossi (2013) has kindly reviewed and somewhat summarized the recent literature exploring exchange rate determination. She claims that more than a few works of research properly demonstrate the validity of the Taylor rule fundamentals and net foreign assets rather than other predictors. In the midst of econometric models, the empirics stemming from an error correction model (ECM) seem to be preferable to others. These positive pieces of evidence overall then inspire us to investigate exchange rates by making use of the Taylor rule together with an error correction concept since it conceptually well reflects how the Taylor rule mechanizes. However, although she empirically indicates that ECMs are, in some measure, more favored than VECMs, we are still encouraged to reexamine should there exist more than one long-run or CI relationships in the valid foreign-exchange predictor namely the Taylor rule. If so, VECMs could be effective. Supportively for VECM modeling, foreign-exchange VECMs are workable for MacDonald and Taylor (1993) with a monetary model and Clarida and Taylor (1997) with forward-rate analysis. Furthermore, as Molodtsova and Papell (2009) have also successfully employed the Taylor rule fundamental but with linear regressions, we exploit their symmetric and homogenous Taylor rule model and their dataset in exploring our VECM constructions. The contents after this will be theoretical framework and background, data description, empirical methodology, and empirical results.

2. Theoretical framework

2.1 Theoretical Background

The aim of this work is to estimate an exchange rate forecasting model. More specifically, focusing on the relation that should link the exchange rates with related fundamentals we aim to estimate a fundamental specification which proved relatively successful, namely the Taylor rule (Rossi, 2013).

Since the result by Meese and Rogoff, who found that a random walk process is generally able to produce better forecasts than fundamental models (Meese & Rogoff, 1983), many empirical works have tried to revert this conclusion, with mixed results. Rossi (2013) proposes a comprehensive review of the state of the literature, by assessing not only the performance of different specifications, but also how that the choice of different estimation features, have non-trivial consequences on the result of an empirical work.

At the core of the idea of employing fundamental models to forecast Exchange rates (in magnitude, but also directionally) is the fact that at least in the long period, the operators who concretely move the rates will have to reconcile with equilibrium values suggested by the theory. We can observe therefore many different specification forms, which embeds different theoretical perspectives; in particular, the exchange rate (or a related transformation) is often derived as a function of other variables thought to be relevant in explaining its behaviour that will be used as predictors in an econometric setting. Between the most used models, we can list UIP and PPP models, money and output differential models, productivity differentials models (à la Balassa-Samuelson), models based on CA related balances or imbalances, and the one of our interest, the Taylor Rule. In the following section, we will describe how the Taylor Rule could be linked with the exchange rate, in particular by looking at how our main benchmark paper, Molodtsova and Papell (2009).

Taylor rules as exchange rates fundamentals

The Taylor rule, which in its most basic form relates interest rate with inflation and output differentials (Taylor, 1999), is used as a description of monetary policy by central banks. In his seminal paper of 1993, Taylor formalized the idea that central banks could use the interest rate as an instrument to reach its own target of inflation and to move the output if it were to distance itself to much from its natural level (Taylor, 1993). It is of uttermost importance in macroeconomics, as it is often applied as a system closure in new Keynesian models, in alternative with other monetary relations such as money demands. Of course, this relation could also be considered in open economy settings, where it requires considering relative rates.

In a bilateral setting therefore, this rule could be linked to the real exchange rate, once realizing that the Uncovered Interest Parity condition relates the bilateral exchange rates with the interest rates of the two countries. By differencing the Taylor Rules for the two countries then, and rearranging terms, it is possible to model the expected depreciation as a function of the inflation, the output gaps and the interest rates of the two countries, obtaining therefore a testable econometric specification (Rossi, 2013).

Focusing on this strand of literature, our paper takes as an ideal benchmark the work by Molodtsova and Papell (2009). Following the intuition presented before, the authors proceed to estimate a model of exchange rate forecast for the bilateral exchange rate of the USD with various other currencies, in a sample that spans from 1971 to 2006.

The authors take a bit further the basic specification discussed before by testing multiple forms of the econometric specification. This is done in order to capture some theoretical instances, like the fact of central banks acting smoothly on the interest rate, and that the rule may depend also on real interest rate as a condition to keep the nominal exchange rate around the Purchasing Power Parity (PPP).

In particular, the authors consider various divide between models; since moreover these divides are non-exclusive, a given Taylor rule could become any combination between them, which raises greatly their number of estimates. Since our model is inspired by their specification, we will look more in detail to the derivation of the Taylor rule and its variants, in the following subsection.

Benchmarking the various models (estimated with a linear rolling window) against a random walk, and assessing their model performance through a Clark and West procedure, they obtain evidence for short run out of sample predictability of the exchange rate. Their best model reject the null of no predictability at the 5% level for 10 out of 12 countries. Moreover, they compare their Taylor specification against other, such as an interest rate differential, a PPP fundamental model, and three different monetary models, for all of which the predictability is weaker. The caveat here is that not all the combinations of Taylor Rule different models perform at the same level, and that comparing nested models is not a straightforward feature. The authors nonetheless are keen to evidence these limitations, by employing appropriate statistical tests and by explaining clearly the difference between predictability and forecasting.

Chinn (2008) nonetheless challenges the conclusion of the authors for in-sample estimation, and Rogoff and Stavrakeva (2008) do this for the out of sample, which points that the Taylor rule estimates are not robust. Rossi (2013) evidences by how for other references the evidence seems still positive, and her personal analysis displays positive evidence for Taylor rule predictability in the short run. Finally, the new trend in this literature, following the financial crisis, and considering that the Taylor rule might have changed during a period of such distress, is to include indicators of financial stress, or measures of liquidity in the estimation equation, which seems to adapt the methodology to samples of more recent period.

Given this review, we conclude that the Taylor rule specification is still a fertile ground for research, and worth to be testing on it an additional methodology, in our case the VECM. In the next section, we will briefly derive theoretically our basic specification, taken from Molodtsova and Papell (2009).

From the Taylor rule to the exchange rate

In their work, Molodstova and Papell (2009) present many different versions of the Taylor rule. Starting from different theoretical assumptions about how the rule is formed or behave, they propose four different divides to classify their rules.

The first divide is between symmetric and asymmetric models, where the asymmetric version includes the real exchange rate. Since their "Home" country is United States, the coefficient for the real exchange rate is set equal to zero, which causes the asymmetry in this version.

The second divide is between a smoothing and non-smoothing version; it is accepted that Central Banks does not react on the interest rate immediately and completely, but that their response is sluggish. This could be represented in a smoothing model by allowing for a lagged interest rate variable in the equation, where the related coefficient represents the degree at which the rule is adjusted.

The third divide is between allowing for homogeneous or heterogeneous coefficients that in practice means to permit different response coefficients by the central banks; in this case the variables of the home and foreign countries are not included as the same variable already differenced, but separated.

The fourth and last divide, is to include or not a constant in the specification, which is included if the two central banks have different real interest rate or different inflation targets.

Each divide may or may not be present in the final specification, which means that there are various combinations of Taylor rules tested by the authors.

Starting from the basic Taylor rule:

$$i_t^* = \pi_t + \phi(\pi_t - \pi^*) + \gamma y_t + r^*$$

where we can see that the optimal nominal interest rate depends on inflation π_t, its distance from the inflation target, the output gap y_t, which is the difference of the actual output with its natural level, and the real interest rate. The authors simplify the notation by defining

$$\mu = r^* - \phi\pi^*$$
$$\lambda = 1 + \phi$$

such that

$$i_t^* = \mu + \lambda\pi_t + \gamma y_t$$

Considering also a variable for the real exchange rate for the Central Bank to make the PPP hold, and a possible smoothing such that the interest rate will follow such a path to readjust as

$$i_t = (1-\rho)i_t^* + \rho i_{t-1} + v_t$$

the whole expression for the extended Taylor rule would become

$$i_t = (1-\rho)(\mu + \lambda\pi_t + \gamma y_t + \delta q_t) + \rho i_{t-1} + v_t$$

By considering the Taylor rules for two different countries, as home and foreign, and by subtracting one from another, and considering UIP, the authors obtain their most general version of their model, which can be modified accordingly for each of the specific divides discussed before. (Notice that in this case we are looking at time $t+1$, that s represents the nominal exchange rate, and that the tilde is used to denote variables of the foreign country):

$$E_t s_{t+1} - s_t = \mu + \delta q_t + \tilde{\lambda}\tilde{\pi}_t + \tilde{\gamma}\tilde{y}_t + \lambda\pi_t + \gamma y_t + \rho i_t - \tilde{\rho}\tilde{\imath}_i$$

As proposed by evidence, ideally an increase in inflation should make the forecasted exchange rate to appreciate. This also happen if the Fed were to raise the interest rate with respect to the foreign country.

In our case, the VECM representation that we will apply includes straightforwardly the symmetric homogenous version with no constant, which is a quite basic version, since it does not allow for asymmetry and heterogeneous coefficients. From one side, in the case of the VECM methodology that we will apply, the number of variables and coefficients greatly affects the resulting estimates and coefficient matrices that one need to "keep track" of, greatly augmenting the involvedness of the method so this version is welcomed. It is therefore implied that, in this analysis, we will try to see if the multivariate methodology is a viable option for the case of Taylor rules, as we will see in a later section, and because of it, this work should be looked at as a starting point for more complete studies. From the other, the Molodsova and Papell's (2009) Taylor Rule that we are going to include as one of the relevant CI relationships in our VECM specification, is just one of them. This is evidenced just to explain that the VECM structure could be already capable to include some of the authors concerns both because of the structure and following the the imposition of restrictions. For instance, the smoothing part could be already treated in a VECM, because the VECM specification is already designed to keep track of the lagged components. For instance for a general VECM $(p-1)$ we would have:

$$\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Psi_i \, \Delta y_{t-i} + \varepsilon_i.$$

Thus, as we can see in the VECM formulation smoothing would not be a concern if it were to be influent in the interest rate part of the system. We are keen however to repeat that the "best" specification of the authors is the symmetric Taylor rule with heterogeneous coefficients, smoothing, with a constant version. Before discussing the VECM methodology in detail, in the next brief subsection we will look at previous applications of it in the exchange rate literature.

VECM and exchange rates

Our original contribution is to consider the Molodstova and Papell (2009) setting in a multivariate setting, in particular within a vector error correction model specification (VECM). Whilst in Rossi (2013) such models are dismissed with respect to single equation ECM (which are simply their univariate version), there could be at least one reason to consider this methodology in the case of Taylor rules specification. Leaving the technical details for the estimation section, where we will discuss the specifics, it suffices here to note that Taylor rules are themselves a sort of error correction model; however, failing to consider that these relationships may include additional long run trends might limit the analysis a bit.

Many works have already applied this methodology to exchange rates forecasts, but their evidence about VECM efficacy with respect to exchange rate predictability has been quite mixed (Rossi, 2013). MacDonald and Taylor (1994) apply it to a monetary model, beating a random walk, while Clarida and Taylor (1997) look at the link between spot and forward rates to obtain also better forecasts than alternative methods. Rapach and Wohar (2002) test a much longer sample for a monetary models, finding evidence of long run monetary models for half of the countries they inspect. Applying the VECM to these countries to perform a forecast yields for them more mixed results. The most negative article is from Diebold et al. (1994), that shows how by applying ECM structures to the exchange spot rate obtains worst results than a Martingale model; given these result they rejects evidence of cointegration for the rate by applying superior tests. It should be noted that none of these examples are applied to a fundamental structure based on a Taylor rule. The most similar structure that we were able to find (Asari et al., 2011), while not using a proper Taylor rule, looks at interest rate and inflation as fundamentals for the Malaysian Ringgit, finding evidence of cointegration and applicability of the methodology in explaining the exchange rate volatility.

2.2 Model motivation

The reason why we chose a VECM model in order to predict the nominal exchange rate is that we want to exploit the eventual presence of more than one cointegration relationships among the variables we have in the model. Indeed, when a Taylor rule is considered cointegration is in place. Then, given that we cannot be sure that this is the only cointegration relationship that exists among our variables of interest we set up a VECM framework in order to investigate whether there is room to improve the predictive power of Taylor rule based model by adding more common stochastic trends when found. In particular, we focus on those cases where, according to Johansen test, we find the CI matrix having rank greater than one but less than four. This is done because if the rank is zero it means that all the variables are neither stationary nor share any common stochastic trend, hence there is no rationale why our model should perform better than others. In this case we should run a first difference VAR, but this is at odds with the empirical investigation we aim to carry out and furthermore rules out the existence of any Taylor rule relationship in the data. If, instead, the rank is one (and presumably this is the Taylor rule) there is no reason why our model should do better than those presented in the literature since we do not exploit any additional information, and because of this our empirical investigation is of no interest. If the CI matrix has full rank this means all the variables are stationary and a normal VAR can be run, even this case is of no interest since there is not any long run driver in our model which can catch up in order to improve our model predictions. Finally, if our rank is two or three then it must be the case that one CI relationship is the Taylor rule whilst the other may be identified according to the economic theory. In particular we are keen to consider UIP, PPP and Harrod-Balassa-Samuelson effect as candidate for the second CI (or even third) CI vector of interest.

Of course the advantage of exploiting the eventual presence of additional CI relationships in our model comes at some cost. We have to keep the number of parameters to estimate within a reasonable range and this implies not to have too many variables in our model. This is a hard task when an exchange rate model is to be estimated. However, we come up with four variables which are the log of the nominal exchange rate between US and a foreign country, inflation differential, interest rate differential and output gap differential between US and a foreign country respectively. They are the essential ingredients to estimate an exchange rate model with a Taylor rule inside. The fact of employing those variables in our model leads us to impose some quite restrictive assumptions: the Taylor rule must be symmetric and the coefficients homogeneous. Notwithstanding, we believe that the additional information stemming from the CI relationships overcomes the burden of the above limitations.

2.3 Performance evaluation

We use the F-test in order to test the statistical significant difference between the forecast-error second moment (i.e. mean squared error or MSE) of our Taylor rule model and that of a random walk without drift, which is such a typical exchange rate model benchmark. The null and alternative hypotheses are as follows.

$$H_0: E\left[\varepsilon_{t+h|t}^{TR}\right]^2 = E\left[\varepsilon_{t+h|t}^{RW}\right]^2$$
$$H_1: E\left[\varepsilon_{t+h|t}^{TR}\right]^2 \neq E\left[\varepsilon_{t+h|t}^{RW}\right]^2$$

where ε: forecast error ($actual - predicted$, for a model of interest)

$\varepsilon_{t+h|t}^{TR} = \Delta s_{t+h} - \widehat{\Delta s}_{t+h} = s_{t+h} - \hat{s}_{t+h}$ $(\Delta s_{t+h} - \widehat{\Delta s}_{t+h} = s_{t+h} - s_t - (\hat{s}_{t+h} - s_t))$

$\varepsilon_{t+h|t}^{RW} = \Delta s_{t+h} = s_{t+h} - s_t$

$s = \ln(nominal\ exchange\ rate)$

h: forecast horizon

TR: Taylor rule

RW: a random walk without drift

For the alternative hypothesis (H_1), it can be either $E\left[\varepsilon_{t+h|t}^{TR}\right]^2 < E\left[\varepsilon_{t+h|t}^{RW}\right]^2$, i.e. the Taylor rule fundamental outperforms the benchmark; or, $E\left[\varepsilon_{t+h|t}^{TR}\right]^2 > E\left[\varepsilon_{t+h|t}^{RW}\right]^2$, i.e. a random walk without drift is superior.

3. Data description

Regarding our symmetrical Taylor rule model with homogeneous coefficients across a certain pair of countries (Molodtsova & Papell, 2009; Rossi, 2013), the investigated monthly data taken into account includes logarithmic nominal exchange rates, interest rate differentials, inflation rate differentials, and output gap differentials, from 1971 to 2006 as per Molodtsova and Papell (2009). By considering the US as the domestic country, the corresponding explored foreign countries with the results drawn from our Taylor-rule VECM models are Japan, Canada, Germany, and Portugal (as our study concentrates on the cases with cointegration rank more than one but less than full rank, suitably with how a VECM is advantageous). To be in line with the introduction of EUR currency in January 1999, the latter two European countries are explored only up to such point in time.

Moreover, we have employed the data kindly constructed and provided by Molodtsova and Papell (2009) where the spot exchange rates are originally from the Federal Reserve Bank of Saint Louis database and the others from the IMF's International Financial Statistics (IFS) database. More specifically, interest rates are proxied by money market rates, inflation rates calculated using consumer price index (CPI), and output gaps as deviations of actual outputs from their certain trends in which the proxy of actual ones are seasonally adjusted industrial production index. Pertaining to an output-gap trend, in our paper and consistent with Molodtsova and Papell (2009), it can be either linear, quadratic, or Hodrick-Prescott (HP) filtered. As not provided by Molodtsova and Papell (2009), we computed all linear, quadratic and HP-filtered output-gap trends in Excel suitably, via TREND function, LINEST function up to the power of two (i.e. quadratic polynomial), and HP-filter add-in function kindly built by Annen[1] with the monthly-data smoothing parameter of 14,400, respectively. Furthermore, following Molodtsova and Papell (2009), we computed the trends conditional on the previous 26 months. Hence, the sample estimation starts from March 1973 with the in-sample of March 1973 to February 1982, containing 108 observations, and the out-of-sample afterwards.

4 Empirical methodology

The empirical strategy we adopt is the standard one for VECM models. At first we select the optimal number of lags according to suggestions from the info criteria, then we perform the Johansen test and if the rank is greater than one and less than four we go to estimate our VECM

[1] Please refer to https://ideas.repec.org/c/dge/qmrbcd/165.html for the HP filter Excel add-in by Annen.

model. At this point we perform the usual diagnostic analysis and check for error autocorrelation until lag twelve, if errors are autocorrelated we increase the lag order of our model and redo the CI test, we redo the VECM estimation and so on until we have dealt with autocorrelation of the residuals. Then comes the most difficult part: imposing restrictions. As we know if, in general, our model has rank r we have to impose $r * r$ "just identifying" restrictions in order to have only and only one β CI-vectors matrix and one α loading coefficient matrix such that $\Pi = \alpha_{n*r} * \beta'_{r*n}$, i.e. they give rise to only one long run matrix, Π. We need to impose restrictions in order to identify our CI vectors according to the Taylor rule and UIP, PPP or Harrod-Balassa-Samuelson. In order to do that, we often need to impose an arbitrary number of "over-identifying" restrictions which, in turn, need to pass the usual LR test in order to be accepted. In the next section we explain in details how we identified each CI vector of the models we estimated.

4.1 Japan

With linear output-gap differential

We find a CI rank of 2, and looking at the unrestricted estimates we find that, according to the Heuristic approach proposed by gretl of imposing the four just identifying restrictions, the first CI vector is already specified as a Taylor rule. The estimated coefficients for inflation and output gap differentials are quite big (one is -185.85 and the other is -95.175) and of the right sign. The restricted coefficient for the interest rates differential is 1. The point is that here we would like to have coefficients for inflation and output-gap differentials to be of the magnitude of 1, but if we impose such restrictions, they are not accepted so we have to keep those coefficients which, even if very big, are not theoretically inconsistent with the NEK-Model determinacy conditions. The second CI vector is very similar to the first one. The difference is only that the coefficients for inflation and output-gap differentials now are -8.34 and -5.24 respectively, whilst the restricted coefficient for log of the nominal exchange rate is 1. This seems to suggest that, even if UIP does not hold in the canonical formulation, interest rates differential and log of nominal exchange rate seem to be linked by a stable long run relationship which can be obtained by subtracting the second CI vector from the first:

$$i_t - i_t^* - s_t = 177.51(\pi_t - \pi_t^*) + 89.93(Ygap_t - Ygap_t^*)$$

Then we think of this model specification as satisfactory and we can exploit the additional information provided by this "adjusted" UIP specification. In other words both nominal exchange rate and interest rates seem to have a stable long run relationship with the same fundamentals according to the same sign.

Then, in a similar fashion we impose some other over-identifying restrictions on the second CI vector. We impose the coefficient for inflations to be 0 and the one for nominal exchange rate to be -1, those restrictions are accepted, so on the second CI vector we have a relationship very similar to the previous one, but now what is only the output-gap differential to be in relation with the "adjusted" UIP (note, we say adjusted because if we add and subtract from the left hand side s_{t-1} we effectively have UIP to be stationary in the long run). Thus, we estimate also this version of the model and do forecast.

With HP filtered output-gap differential

Here the same reasoning applies as before, the only differences are that the estimated coefficients are even bigger and that the output-gap coefficient now has opposite (positive) sign in the fully restricted model.

4.2 Canada

In this case, we find a suitable rank only for quadratic filtered output-gap, this is rank 3. Because of such a high rank we are quite disappointed with the just identifying restriction imposed, by default, by Gretel. Since it follows the heuristic approach we have the first 3 by 3 block of the β matrix as an identity matrix and this is clearly at odds with the Taylor rule identification. It is clear that we cannot have a suitable CI matrix in the case of the unrestricted model, so we directly move to the over-identifying restrictions.

The only way we have been able to obtain theoretically consistent CI vectors, i.e. such that the LR test does not reject our restriction at the 5 percent (although our p-value is pretty small, just 0.05723), is the following one: the first C vector is such that the log of nominal exchange rate has coefficient 1 and the inflation differentials is -2.82 and all the other coefficients are 0; the second CI vector is the Taylor rule, i.e. the coefficient of interest rates differential is 1, those inflation differential is -1, and that of output-gap differentials is -195.63 (which is pretty high but still theoretically consistent with the determinacy of NEK model); finally, the last CI vector is an UIP adjusted for both inflation and output-gap differentials. The economic interpretation we can give to the first CI relationship is that the nominal exchange rate overreacts to inflation differentials movements which may hint a kind of Harrod-Balassa-Samuelson effect, indeed if we assume that the productivity in the traded goods sector is lower in the US than it is in Canada, then the US will have a lower inflation rate than (via non-traded goods inflation) Canada and this suggests a depreciation of the real exchange rate (which is expressed in dollars) in order to keep a stationary relationship.

Finally in the third CI vector we find that, as in the previous case, we have that UIP has a long run relationship with economic fundamentals. Thus, we proceed with forecast.

4.3 Germany

With linear output-gap differential

This case is analogous to that of Japan with linear trended output gap (we have rank 2), so the same conclusions apply also here for the unrestricted model. Anyways, when we go to impose overidentifying restrictions we find from one side that the Taylor rule takes into account only inflation differentials (the coefficient is -10.002 and still the interest rates differential coefficient is restricted to be 1), and from the other we find that UIP holds, or better that the difference between interest rates differential and log of exchange rate is stationary (and still if we add and subtract one lag of nominal exchange rate we have a stationary relationship that is UIP follows one lag of nominal exchange rate). Those are our two CI vectors and given that our restrictions pass the usual LR test we can proceed with forecast.

With HP filtered output-gap differential

This case is analogous to the previous one (i.e. linear output-gap differential).

4.4 Portugal

With linear and HP-filtered output-gap differentials

Also in this case we have rank 2 and we are in the same situation as it was for Japan linear trended output gap for the unrestricted model. When we impose overidentifying restrictions the first CI vector is unchanged (i.e. Taylor rule), and the second is such that the nominal interest rate is in a stationary relationship with the output-gap differential having a non-significant coefficient, this implies that nominal exchange rate is stationary by itself, so here there is no such a big rational to have additional information brought by the additional CI vector, notwithstanding we do forecast also in this case. The case for HP filtered output gap is identical.

With quadratic output-gap differential

Also here we have rank 2, and the case for the unrestricted model is pretty similar to the one of Japan with linear trended output gap. When we go to impose overidentifying restrictions we end up by imposing a coefficient of 1 to inflation differential, but at the end we are in a situation that differs very little from the previous one, the only difference is that now in the second CI vector there is a true CI relationship. Anyways, we proceed with forecast in both cases.

5. Empirical results

The empirical results of exchange rate (the US as domestic and the others as foreign) projection represented below, including unrestricted and restricted VECMs, respectively, only regards the analytical cases with cointegration rank more than one nonetheless less than full rank in accordance with the VECM exploitation (i.e. the cases with cointegration rank found zero or full are not further investigated). For projection consistency, all the estimated VECMs have also been properly tested to ensure that the forecast errors are serially uncorrelated and normally distributed. In respect of long-run relationship exploration, certain paired countries of the US come up with various cointegration ranks via the Johansen test based on a particular output-gap differential type (computed using either linear, quadratic, or HP-filtered trend) used in an estimation.

5.1 Unrestricted VECMs

Paired country, output-gap trend	Cointegration rank	Forecast horizon									
		1m	3m (1q)	6m (2q)	9m (3q)	1y	2y	3y	4y	5y	> 5y
Japan, linear[a]	2	TR	TR	TR	RW	RW	RW	RW	RW	TR	TR* (~9y)
Japan, HP-filtered[a]	2	TR	TR	TR	TR	TR	TR	TR	TR	TR	TR* (~11y)
Canada, quadratic	3	TR	TR	TR	TR	TR	RW***	TR***	TR***	TR***	TR*** (~6y)
Germany, linear[a]	2	RW***	RW	RW	TR	TR	TR	TR	TR	TR	TR* (~11y)
Germany, HP-filtered[a]	2	RW***	RW	RW	TR	TR	TR	TR	TR	TR	TR* (~11y)
Portugal, linear[a]	2	RW	RW	RW	TR**	TR***	TR***	TR***	TR**	TR	TR (~6y)
Portugal, quadratic[a]	2	RW***	RW	RW	RW	RW	RW	RW	RW	RW	TR** (~7y)
Portugal, HP-filtered[a]	2	RW	RW	RW	TR**	TR***	TR***	TR***	TR	TR	TR (~6y)

Table 1: **Unrestricted** VECM statistical results
m: month(s), q: quarter (s), y: year(s)
TR: Taylor rule better, RW: a random walk without drift better
Statistical significance: *** at 1%, ** at 5%, and * at 10%
a: has a theory consistent specification with just identifying restrictions

From our unrestricted VECM prediction results shown in **Table 1** above as comparing the Taylor rule against a random walk without drift, it can be seen that, overall, the Taylor rule model is able to outperform such a typical benchmark significantly at several long horizons. To be more specific, for the US vs. Japan by both linear and HP-filtered means of the output gap differential, the long-term relation between the exchange rate and the Taylor rule fundamental occurs at a considerably long horizon as being significant at approximately ten years. This statistical significance is also applied to the German cases, both linear and HP-filtered. At a shorter horizon of nearly seven years, the validity of the Taylor rule conception exists significantly as well for the Portuguese instance per quadratic output gap computation. Then again, the Taylor rule fundamental becomes valid significantly at the forecast horizons of three years onwards for the Canadian case with the quadratic output-gap differential. Take the US vs. Portugal in terms of linear and HP-filtered output-gap differentials for consideration, the existence of the Taylor rule is statistically significant at a relatively shorter horizon, three-quarter up to three-year ahead.

5.2 Restricted VECMs

Paired country, output-gap trend	Cointegration rank	Forecast horizon									
		1m	3m (1q)	6m (2q)	9m (3q)	1y	2y	3y	4y	5y	> 5y
Japan, linear	2	TR	TR	TR	TR	TR	TR	TR	TR	TR	TR* (~10y)
Japan, HP-filtered	2	TR	TR	TR	TR	TR	TR	TR	TR	TR	TR* (~10y)
Canada, quadratic	3	RW	TR	TR	TR	TR	RW***	TR***	TR***	TR***	TR** (~6y)
Germany, linear	2	RW***	RW	TR	TR	TR	TR	TR	TR	TR	TR (~6y)
Germany, HP-filtered	2	RW***	RW	TR	TR	TR	TR	TR	TR	TR	TR (~6y)
Portugal, linear	2	RW	RW	RW	TR***	TR***	TR***	TR***	TR**	TR	RW (~6y)
Portugal, quadratic	2	RW***	RW	RW	RW	RW	RW	RW**	RW	RW	RW** (~7y)
Portugal, HP-filtered	2	RW	RW	RW	TR***	TR***	TR***	TR***	TR**	TR	RW (~6y)

Table 2: **Restricted** VECM statistical results
m: month(s), q: quarter (s), y: year(s)
TR: Taylor rule better, RW: a random walk without drift better
Statistical significance: *** at 1%, ** at 5%, and * at 10%

Subsequent to restricting our initial unrestricted VECMs according to relevant theoretical frameworks, e.g. the Taylor rule, uncovered interest rate parity (UIP), or purchasing power parity (PPP) (refer to the section of **Empirical methodology**) such that the corresponding Chi-squared p-values of all the examined cases are appropriately accepted (i.e. so that a restricted model is statistically workable), **Table 2** demonstrates that the statistically significant outperformance of the Taylor rule still appears rather similarly to the unrestricted results previously described (refer to the restricted VECMs for each of our analytical cases and their Chi-squared p-values represented in **Appendix**). Nevertheless, the restricted empirics are slightly different from the unrestricted estimations such that the Taylor rule fundamental being superior to a random walk without drift in determining exchange rates is not statistically significant for the US v. Germany and even, on the other hand, not discovered for quadratic output-gap case of the US vs. Portugal.

After all, at longer horizons as hypothetically anticipated, the Taylor rule to a certain extent can deal with the so-called Meese and Rogoff puzzle, i.e. the inferiority of economic models towards a random walk.

5.3 Additional prediction clarification

The outperformance over a random walk of our Taylor-rule-fundamental VECMs with a proper cointegration rank possibly could even have been statistically significant at shorter horizons, such as two-quarter, three-quarter, one-year ahead, and so forth, than empirically found above for more than just a few cases (Portugal, linear and Portugal, HP-filtered). It is for the reason that, between one-month and longer-time ahead projections, we have computed them distinctively but from the same estimations using monthly data.

For one-month horizon, our forecasts are simply 'static' (a usual forecast function in statistical packages, including gretl that we use) as always using a previous actual observation to do prediction at each out-of-sample month. The corresponding MSE is then computed from the whole out-of-sample static forecast series.

Regarding our analytical horizons longer than one month, the predictions stem from 'dynamic' forecasting (the other forecast function in statistical packages) as using a previous estimated (instead of actual) observation after the first forecast in the out-of-sample range. For instance, our one-year-ahead estimate is dynamically computed over the first twelve out-of-sample months. This might be alternatively explained that dynamic forecasts assume that there does not exist actual observations after a specified in-sample interval hence using previous estimate(s) rather than the actual and this quite corresponds to a long-horizon anticipation. Thereafter, the resultant MSE of a longer-horizon projection is calculated through the forecast errors of all the previous dynamically computed values. Thus, this MSE perhaps does not fully reflect a longer-horizon (longer than one-month ahead) prediction, which probably to some extent degrades statistical significant outcomes.

However, to have a more appropriate MSE computed from the errors of numerous particularly-longer-horizon estimates not just from the entirely previous dynamically prediction errors of one longer-horizon estimate, it can be done throughout conducting re-estimations for each of several particularly-longer-horizon forecasts over a time series. Unfortunately, countless re-estimations might be somewhat excessively time demanding relative to our time constraint. Accordingly, we have tried our best in providing contributable results by coming up with and implementing such an aforementioned alternative.

6. Conclusion

Our aim at this point was to exploit the presence of possible common trends in order to improve short run predictions of the exchange rate. As expected our models performed quite well in long run predictions. As we can see from the tables, we perform pretty well in dynamic forecasts from nine months ahead onwards. One might think that our performance in the short run is pretty poor, but we should consider the fact that when a dynamic forecast is run, we have very few observations as we consider the time horizon of the predictions in place. When such a small number of observations in considered it is quite difficult to have one model outperforming the other. In order to have a more specific and precise evaluation of our model in the short run, we would need a static forecast for n months ahead; unfortunately this options was allowed only for one month ahead in the used software; of course to overcome this hurdle would be the first natural direction of procedure had we more time to devote to this research. Anyways, what we can infer is that with a static forecast of n months ahead we could obtain better results than a dynamic forecast, as we can see from the long run results of the dynamic forecasting.

Finally, what we can say, in those cases where we find a suitable cointegration rank, than it is an indicator that a simple Taylor rule would not be enough to predict exchange rates, and then the methodology of our work could provide a viable option to enrich the analysis of the kind of Molodtsova and Papell (2009).

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