An Algorithm for Solving Simple Sticky Information New Keynesian DSGE Model

Chattopadhyay, Siddhartha and Agrawal, Manasi

IIT Kharagpur, IITKharagpur

19 April 2015

Trade and Development Review
Vol. 8, Issue 2, 2015, 120 - 137.
http://www.tdrju.net

An Algorithm for Solving Simple Sticky Information New Keynesian Model[♦]

Siddhartha Chattopadhyay*
Manasi Agrawal[♣]

This paper describes a new algorithm for solving a simple Sticky information New Keynesian model using the methodology of Wang and Wen (2006) for demand and supply shock. Impulse responses for demand and supply shock have been generated and analyzed intuitively. The strength of our algorithm lies in its analytical exposition, which allows to uncover better intuition from the model.

JEL Classification: C22, C61, C63, E63, E52

Keywords: New-Keynesian Model, Algorithm

1. **Introduction**

Sticky price New Keynesian DSGE model is the work horse of modern monetary policy analysis. However, the model suffers from several criticisms as identified by Mankiw and Reis (2002). First, the model fails to produce hump in inflation rate and output to monetary policy shock as observed in the data. Second, the model does not have any endogenous persistence. It simply borrows the persistence of demand and supply shock. Third, the model does not follow the Natural Rate Hypothesis (McCallum, 1998). Fourth, credible disinflation causes booms in the model rather than recessions (Ball, 1994). Mankiw and Reis (2002,

♦ The authors would like to thank seminar participants of Department of HSS, IIT Kharagpur. Research assistance of Vipasha Jain is appreciated. Errors belong to the authors.
* Department of Humanities and Social Sciences, Indian Institute of Technology Kharagpur. Email: chattopadhyay.siddhartha@gmail.com
♣ Department of Humanities and Social Sciences, Indian Institute of Technology Kharagpur. Email: manc.agrawal@gmail.com

2006) has developed a sticky information New Keynesian model that survives all the criticisms mentioned above.[1] This paper extends the original model by introducing a supply shock and proposes a new algorithm for solving model using the methodology of Wang and Wen (2006).

Mankiw and Reis (2002, 2006) developed the sticky information model by assuming, information is costly to acquire and process. As a result, information diffuses slowly through population. Such slow diffusion of information causes information asymmetry among economic agents and induces economic fluctuations in the short run. Mankiw and Reis (2002) derives a backward looking sticky information Phillips curve, which represents the supply side of the economy. The demand side of the economy is represented by a log linearized Quantity Theory of Money. Their paper shows that even such a simple model of sticky information performs better than a sticky price model to match *stylized facts*. An algorithm for solving the model under demand shock is also given in the paper.

Wang and Wen (2006) has devised an ingenious methodology that can solve a wide range of sticky information model very easily. The algorithm first transforms the model in forecast error form and then solves the model by the method of undetermined coefficient. Expressing the model in forecast error forms effectively reduces the numbers of parameters to deal with while solving the model. This greatly reduces the possibility of incurring human errors. Moreover, we show that better intuition can be uncovered from the model when the model is solved using the methodology of Wang and Wen (2006).[2]

We have extended the derivation of sticky information Phillips curve of Mankiw and Reis (2002) by introducing a supply shock in this paper.[3] We have also developed a new algorithm to solve the model based on the methodology of Wang and Wen (2006). We show that, the simple model has its own peculiarities though it survives the criticism of sticky price model mentioned above. We show that, the model produces counter intuitive results to permanent supply shock. A permanent supply shock to the model causes permanent reduction to output

[1]See Woodford (2003) for the development and analysis of sticky price New Keynesian model.
[2]Mankiw and Reis (2006) also appreciated the methodology of Wang and Wen (2006) and one of the future research agenda was to solve the pervasive sticky information model using the methodology of Wang and Wen (2006) and compare its efficiency with their own algorithm. Also see, Verona and Wolter (2013) to solve pervasive stickiness model in Dynare.
[3] See appendix for derivation.

without affecting long-run inflation rate. Mankiw and Reis (2002) fails to identify such peculiarities as they have analyzed the model only under demand shock. We also show that, the model produces hump in output and inflation rate when magnitude of shock and/or persistence is high relative to the endogenous persistence of the model.

Rest of the paper proceeds as follows. Section 2 briefly describes the model. Section 3 gives the algorithm to solve the model. Section 4 analyzes the impulse response separately for demand and supply shock and section 5 concludes.

2. The Model

We briefly describe the model of Mankiw and Reis (2002) in this section. The demand side of the model is represented by the log linearized Quantity Theory of Money. The demand curve is given in equation (1).

$$m_t = p_t + y_t \tag{1}$$

where, m_t, p_t and y_t are respectively the nominal money supply, price level and output at time t. The supply side of the model is represented by a backward looking sticky information Phillips curve. The supply curve is given in equation (2). The derivation of the curve is given in the appendix. We have assumed that, $(1-\theta)$ is the fraction of firm having completely updated information, $\theta \in (0,1)$ and $\Delta y_t = (y_t - y_{t-1})$ is the growth rate of output. π_t is inflation rate at time t and E is the expectation operator. Moreover, $\alpha \in (0,1)$ is a measure of degree of nominal rigidity or strategic complementarity (Mankiw and Reis,2002).[4]

$$y_t = \theta y_{t-1} + \left(\frac{1-\theta}{\alpha}\right)\sum_{j=1}^{\infty}\theta^j[(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t) - E_{t-j}(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t)]$$

$$-\left(\frac{\gamma}{\alpha}\right)(1-\theta L)e_t \tag{2}$$

We have also assumed that growth rate of nominal money supply follows the $AR(1)$ process given in equation (3). Supply shock also follows an AR (1) process

[4] Also see, Ball and Romer (1990) and Cooper and John (1988).

given in equation (3).

$$\Delta m_t = \rho_{\Delta m} \Delta m_{t-1} + \varepsilon_t^{\Delta m}, \rho_{\Delta m} \in [0,1] \quad (3)$$

$$e_t = \rho_e e_{t-1} + \varepsilon_t^e, \rho_e \in [0,1] \quad (4)$$

where, $\varepsilon_t^{\Delta m}$ and ε_t^e are white noise process with mean zero and finite variance $\sigma_{\Delta m}$ and $\sigma_{\in^e}$. Note, m can also be interpreted as nominal GDP or exogenous shifters of demand curve. The supply shock can be interpreted as technology shock, mark-up shock, oil price shock etc. The value of $\gamma \in (0,1)$ changes with the type and characteristics of the shock. L is the lag operator such that, $L^j(x_t) = x_{t-j}$, where, $j \in (-\infty, \infty)$.

3. The Algorithm

This section describes the algorithm to solve the model using the methodology of Wang and Wen (2006). The model is solved using the method of undetermined coefficients after writing the model is forecast error form. This allows us to deal with smaller number of parameters while solving the model and greatly reduces the possibility of incurring human error. We first solve the supply equation. The next subsection describes how to solve the demand equation.

3.1 The Supply Curve

Note, we have already written the sticky information supply curve in forecast error form (equation, (2)) so that we can apply the methodology of Wang and Wen (2006). The Wang and Wen (2006) solves the model based on the method of undetermined coefficients. To solve the model we assume that output and inflation follows an $MA(\infty)$ process as given respectively in equation (5) and (6) below.

$$y_t = \sum_{j=0}^{\infty} a_{yj}^{\Delta m} \varepsilon_{t-j}^{\Delta m} + \sum_{j=0}^{\infty} a_{yj}^{e} \varepsilon_{t-j}^{e} \quad (5)$$

and,

$$\pi_t = \sum_{j=0}^{\infty} a_{\pi j}^{\Delta m} \varepsilon_{t-j}^{\Delta m} + \sum_{j=0}^{\infty} a_{\pi j}^{e} \varepsilon_{t-j}^{e} \tag{6}$$

with,

$$\sum_{j=0}^{\infty} \left(a_{yj}^{\Delta m}\right)^2 < \infty, \sum_{j=0}^{\infty} \left(a_{yj}^{e}\right)^2 < \infty$$

and,

$$\sum_{j=0}^{\infty} \left(a_{\pi j}^{\Delta m}\right)^2 < \infty, \sum_{j=0}^{\infty} \left(a_{\pi j}^{e}\right)^2 < \infty ,$$

We then rewrite equation (2) as,

$$(1-\theta L)y_t = \left(\frac{1-\theta}{\alpha}\right)[s_{1\pi} + \alpha(s_{1y} - s_{2y}) + \gamma(s_{1e} - s_{2e})] - \frac{\gamma}{\alpha}(1-\theta L)e_t \tag{7}$$

where,

$$s_{1\pi} = \sum_{j=1}^{\infty} \theta^j [\pi_t - E_{t-j}(\pi_t)]$$

$$s_{1y} = \sum_{j=1}^{\infty} \theta^j [y_t - E_{t-j}(y_t)]$$

$$s_{2y} = \sum_{j=1}^{\infty} \theta^j [y_{t-1} - E_{t-j}(y_{t-1})],$$

$$s_{1e} = \sum_{j=1}^{\infty} \theta^j [e_t - E_{t-j}(e_t)],$$

$$s_{2e} = \sum_{j=1}^{\infty} \theta^j [e_{t-1} - E_{t-j}(e_{t-1})],$$

Note, using equation (6) we have,

$$\pi_t - E_{t-1}(\pi_t) = a_{y0}^{\Delta m} \varepsilon_t^{\Delta m} + a_{y0}^{e} \varepsilon_t^{e}$$

$$\pi_t - E_{t-2}(\pi_t) = (a_{y0}^{\Delta m} + a_{y1}^{\Delta m} L)\varepsilon_t^{\Delta m} + (a_{y0}^{e} + a_{y1}^{e} L)\varepsilon_t^{e}$$

$$\pi_t - E_{t-3}(\pi_t) = (a_{y0}^{\Delta m} + a_{y1}^{\Delta m} L + a_{y2}^{\Delta m} L^2)\varepsilon_t^{\Delta m} + (a_{y0}^{e} + a_{y1}^{e} L + a_{y2}^{e} L^2)\varepsilon_t^{e}$$

$$\ldots \qquad \ldots \qquad \ldots$$

This implies,

$$\pi_t - E_{t-j}(\pi_t) = \sum_{k=0}^{j-1} \left[a_{\pi k}^{\Delta m} L^k (\varepsilon_t^{\Delta m})\right] + \sum_{k=0}^{j-1} \left[a_{\pi k}^{e} L^k (\varepsilon_t^{e})\right], \text{ for } j = 1,2,3,... \quad (8)$$

Similarly using equation (5) we have,

$$y_t - E_{t-j}(y_t) = \sum_{k=0}^{j-1} \left[a_{yk}^{\Delta m} L^k (\varepsilon_t^{\Delta m})\right] + \sum_{k=0}^{j-1} \left[a_{yk}^{e} L^k (\varepsilon_t^{e})\right], \text{ for } j = 1,2,3,... \quad (9)$$

Also note that equation (5) gives,

$$y_{t-1} - E_{t-1}(y_{t-1}) = 0$$

$$y_{t-1} - E_{t-2}(y_{t-1}) = a_{y0}^{\Delta m} L\varepsilon_t^{\Delta m} + a_{y0}^{e} L\varepsilon_t^{e}$$

$$y_{t-1} - E_{t-3}(y_{t-1}) = \left(a_{y0}^{\Delta m} L + a_{y1}^{\Delta m} L^2\right)\varepsilon_t^{\Delta m} + \left(a_{y0}^{e} L + a_{y1}^{e} L^2\right)\varepsilon_t^{e}$$

This implies,

$$y_{t-1} - E_{t-1}(y_{t-1}) = 0,$$

$$y_{t-1} - E_{t-j}(y_{t-1}) = \sum_{k=0}^{j-1} \left[a_{yk}^{\Delta m} L^{k+1} \left(\varepsilon_t^{\Delta m}\right)\right] + \sum_{k=0}^{j-1} \left[a_{yk}^{e} L^{k+1} (\varepsilon_t^{e})\right], \text{ for } j = 2,3,... \quad (10)$$

$$\varepsilon_t - E_{t-j}(e_t) = \sum_{k=0}^{j-1} \left[\rho_e^k L^k (\varepsilon_t^{e})\right], \text{ for } j = 0,1,2,3,... \quad (11)$$

$$e_{t-1} - E_{t-1}(e_{t-1}) = 0,$$

$$e_{t-1} - E_{t-j}(e_{t-1}) = \sum_{k=0}^{j-1} \left[\rho_e^k L^{k+1} (\varepsilon_t^{e})\right], \text{ for } j = 2,3,4,... \quad (12)$$

Now, using equation (8) we can calculate,

$$s_{1\pi} = \sum_{j=1}^{\infty} \theta^j \left[\pi_t - E_{t-j}(\pi_t)\right]$$

$$= \frac{\theta}{1-\theta}\left[a_{\pi 0}^{\Delta m} + \theta a_{\pi 1}^{\Delta m} L + \theta^2 a_{\pi 2}^{\Delta m} L^2 + ...\right]\varepsilon_t^{\Delta m} + \frac{\theta}{1-\theta}\left[a_{\pi 0}^{e} + \theta a_{\pi 1}^{e} L + \theta^2 a_{\pi 2}^{e} L^2 + ...\right]\varepsilon_t^{e} \quad (13)$$

Similarly, by using equation (9) we can calculate,

$$s_{1y} = \sum_{j=1}^{\infty} \theta^j \left[y_t - E_{t-j}(y_t) \right]$$

$$= \frac{\theta}{1-\theta}\left[a_{y0}^{\Delta m} + \theta a_{y1}^{\Delta m} L + \theta^2 a_{y2}^{\Delta m} L^2 + \ldots\right]\varepsilon_t^{\Delta m} + \frac{\theta}{1-\theta}\left[a_{y0}^{e} + \theta a_{y1}^{e} L + \theta^2 a_{y2}^{e} L^2 + \ldots\right]\varepsilon_t^{e} \quad (14)$$

And, by using equation (10) we can calculate,

$$s_{2y} = \sum_{j=1}^{\infty} \theta^j \left[y_{t-1} - E_{t-j}(y_{t-1}) \right]$$

$$= \frac{\theta}{1-\theta}\left[\theta a_{y0}^{\Delta m} L + \theta^2 a_{y1}^{\Delta m} L^2 + \theta^3 a_{y2}^{\Delta m} L^3 + \ldots\right]\varepsilon_t^{\Delta m} + \frac{\theta}{1-\theta}\left[\theta a_{y0}^{e} L + \theta^2 a_{y1}^{e} L^2 + \theta^3 a_{y2}^{e} L^3 + \ldots\right]\varepsilon_t^{e} \quad (15)$$

Similarly, by using equation (11) we have,

$$s_{1e} = \sum_{j=1}^{\infty} \theta^j \left[e_t - E_{t-j}(e_t) \right]$$

$$= \frac{\theta}{1-\theta}\left[1 + \theta \rho_e L + \theta^2 \rho_e^2 L^2 + \ldots\right]\varepsilon_t^{e} \quad (16)$$

and by using equation (12) we can calculate,

$$s_{2e} = \sum_{j=1}^{\infty} \theta^j \left[e_{t-1} - E_{t-j}(e_{t-1}) \right]$$

$$= \frac{\theta}{1-\theta}\left[\theta L + \theta^2 \rho_e L^2 + \theta^3 \rho_e^2 L^3 + \ldots\right]\varepsilon_t^{e} \quad (17)$$

Subtracting (14) from (15) we have,

$$\alpha(s_{1y} - s_{2y}) = \alpha \sum_{j=1}^{\infty} \theta^j \{[y_t - E_{t-j}(y_t)] - [y_{t-1} - E_{t-j}(y_{t-1})]\}$$

$$= \frac{\alpha\theta(1-\theta L)}{1-\theta}\left[a_{y0}^{\Delta m} + \theta a_{y1}^{\Delta m} L + \theta^2 a_{y2}^{\Delta m} L^2 + \ldots\right]\varepsilon_t^{\Delta m}$$

$$- \frac{\alpha\theta(1-\theta L)}{1-\theta}\left[a_{y0}^{e} + \theta a_{y1}^{e} L + \theta^2 a_{y2}^{e} L^2 + \ldots\right]\varepsilon_t^{e} \qquad (18)$$

and by subtracting (16) from (17) we have,

$$\gamma(s_{1e} - s_{2e}) = \gamma \sum_{j=1}^{\infty} \theta^j \{(e_t - E_{t-j}(e_t)] - [e_{t-1} - E_{t-j}(e_{t-1})]\}$$

$$= \frac{\gamma\theta(1-\theta L)}{1-\theta}[1 + \theta\rho_e L + \theta^2 \rho_e^2 L^2 + \ldots]\varepsilon_t^e \qquad (19)$$

Now, by substituting equation (13), (18) and (19) to equation (7) and simplifying gives me the following expression of the sticky information Phillips curve,

$$y_t = \theta \sum_{j=0}^{\infty} \theta^j \left[\alpha^{-1}(1-\theta L)^{-1} a_{\pi j}^{\Delta m} + a_{yj}^{\Delta m}\right] L^j \varepsilon_t^{\Delta m}$$

$$+ \theta \sum_{j=0}^{\infty} \theta^j \left[\alpha^{-1}(1-\theta L)^{-1`} a_{\pi j}^{e} + a_{yj}^{e} + \alpha^{-1}\gamma\rho_e^j\right] L^j \varepsilon_t^e - \frac{\gamma}{\alpha}(1-\rho_e L)^{-1}\varepsilon_t^e$$

(20)

We can expand (20) to get,

$$y_t = [\theta\alpha^{-1} a_{\pi 0}^{\Delta m} + a_{y0}^{\Delta m}] \in_t^{\Delta m} + [\theta^2 \alpha^{-1} a_{\pi 0}^{\Delta m} + \alpha^{-1} a_{\pi 1}^{\Delta m} + a_{y1}^{\Delta m}]\varepsilon_{t-1}^{\Delta m}$$

$$+ [\theta^2 \alpha^{-1} a_{\pi 0}^{\Delta m} + \alpha^{-1} a_{\pi 1}^{\Delta m} + \alpha^{-1} a_{\pi 2}^{\Delta m} + a_{y2}^{\Delta m}]\varepsilon_{t-2}^{\Delta m} + \ldots$$

$$+ [\theta(\alpha^{-1} a_{\pi 0}^{e} + a_{y0}^{e} + \alpha^{-1}\gamma) - \frac{\gamma}{\alpha}]\varepsilon_t^e$$

$$+ [\theta^{2e}(\alpha^{-1} a_{\pi 0}^{e} + \alpha^{-1} a_{\pi 1}^{e} + a_{y1}^{e} + \alpha^{-1}\gamma\rho_e) - \frac{\gamma}{\alpha}\rho_e]\varepsilon_{t-1}^e$$

$$+ [\theta^3(\alpha^{-1} a_{\pi 0}^{e} + \alpha^{-1} a_{\pi 1}^{e} + \alpha^{-1} a_{\pi 2}^{e} + a_{y2}^{e} + \alpha^{-1}\gamma\rho_e^2) - \frac{\gamma}{\alpha}\rho_e^2]\varepsilon_{t-2}^e + \ldots$$

(21)

Now, by equating coefficients of (21) and (5) we have,

$$a_{yj}^{\Delta m} = \alpha^{-1}\frac{\theta^{j+1}}{(1-\theta^{j+1})}\sum_{k=0}^{j} a_{\pi k}^{\Delta m}\text{, for } j = 0,1,2,3,... \quad (22)$$

$$a_{yj}^{e} = \alpha^{-1}\frac{\theta^{j+1}}{(1-\theta^{j+1})}\sum_{k=0}^{j} a_{\pi k}^{e} - \frac{\gamma}{\alpha}\gamma\rho_e^j\text{, for } j = 0,1,2,3,... \quad (23)$$

3.2 The Demand Curve

The demand curve given in equation (1) can be written as,

$$\pi_t = \Delta m_t - \Delta y_t \quad (24)$$

Note, by using equation (5) and (3), we can write equation (24) as,[5]

$$\pi_t = [1-a_{y0}^{\Delta m}]\varepsilon_t^{\Delta m} + \left[\rho_{\Delta m} - (a_{y1}^{\Delta m} - a_{y0}^{\Delta m})\right]\varepsilon_{t-1}^{\Delta m} + \left[\rho_{\Delta m}^2 - (a_{y2}^{\Delta m} - a_{y1}^{\Delta m})\right]\varepsilon_{t-2}^{\Delta m} + ...$$
$$+ [-a_{y0}^{e}]\varepsilon_t^{e} + [a_{y1}^{e} - a_{y0}^{e}]\varepsilon_{t-1}^{e} + [a_{y2}^{e} - a_{y1}^{e}]\varepsilon_{t-2}^{e} + ... \quad (25)$$

Now, by equating coefficients of (6) and (25) we can get,

$$a_{\pi 0}^{\Delta m} = 1 - a_{y0}^{\Delta m} \quad (26)$$

$$a_{\pi j}^{\Delta m} = \rho_{\Delta m}^{j} - (a_{yj}^{\Delta m} - a_{y(j-1)}^{\Delta m})\text{, for } j = 1,2,3,... \quad (27)$$

and,

$$a_{\pi 0}^{e} = -a_{y0}^{e} \quad (28)$$

$$a_{\pi j}^{e} = \left(a_{yj}^{e} - a_{y(j-1)}^{e}\right)\text{, for } j = 1,2,3,... \quad (29)$$

Note from equation (26) and (27) we can calculate,

$$\sum_{k=0}^{j} a_{\pi k}^{\Delta m} = \frac{1-\rho_{\Delta m}^{j+1}}{1-\rho_{\Delta m}} - a_{yj}^{\Delta m} \quad (30)$$

---

[5] Note, $a_{yj} = 0$ and $a_{\pi j} = 0$ for $j = -1,-2,...$

and similarly, from equation (28) and (29) we can calculate,

$$\sum_{k=0}^{j} a_{\pi k}^{e} = -a_{yj}^{e} \tag{31}$$

Now, substituting equation (30) to (22) yields,

$$\begin{aligned} a_{yj}^{\Delta m} &= \left(\frac{A(j+1)}{1+A(j+1)} \frac{1-\rho_{\Delta m}^{j+1}}{1-\rho_{\Delta m}} \right) \text{ for } \rho_{\Delta m} \in [0,1) \\ &= \left(\frac{A(j+1)}{1+A(j+1)} \right)(j+1) \text{ for } \rho_{\Delta m} = 1 \end{aligned} \tag{32}$$

and by substituting equation (31) to (23) we have,

$$a_{yj}^{e} = -\left(\frac{1}{1+A(j+1)} \right) \frac{\gamma}{\alpha} \rho_{e}^{j} \tag{33}$$

where,

$$A(j+1) = \alpha^{-1} \frac{\theta^{j+1}}{(1-\theta^{j+1})}$$

Note, coefficients of output calculated in equation (32) and (33) enable us to calculate the coefficients of inflation rate from equation (26), (27) and equation (28) and (29) easily.

4. The Impulse Response

The quarterly impulse response of the model is analyzed separately for demand and supply shock in this section. We have first analyzed the impulse response under demand shock. Impulse response under supply shock is analyzed next. To generate impulse response we have used, $\alpha = 0.2$ and $\theta = 0.8$ following Mankiw and Reis (2002, 2006). Note, $\theta = 0.8$ implies that we have assumed 20% firm has completely updated information. We also set $\gamma = 1$ for our analysis.

4.1 Impulse Response under Demand Shock

We assume, $e_t = 0$ to analyze the impulse response under demand shock. Note when $e_t = 0$ we have, $a^e_{yj} = a^e_{\pi j} = 0, \forall j$. Figure 1 and Figure 2 show the impulse response of inflation and output under a 10% positive demand shock with persistence 0.8 and 1 respectively. Figure 1 shows hump shaped response to both output and inflation rate. Wealso see from Figure 1 that though both output and inflation rate rises in short run due a temporary increase in money growth, both converge to their long run level over time.

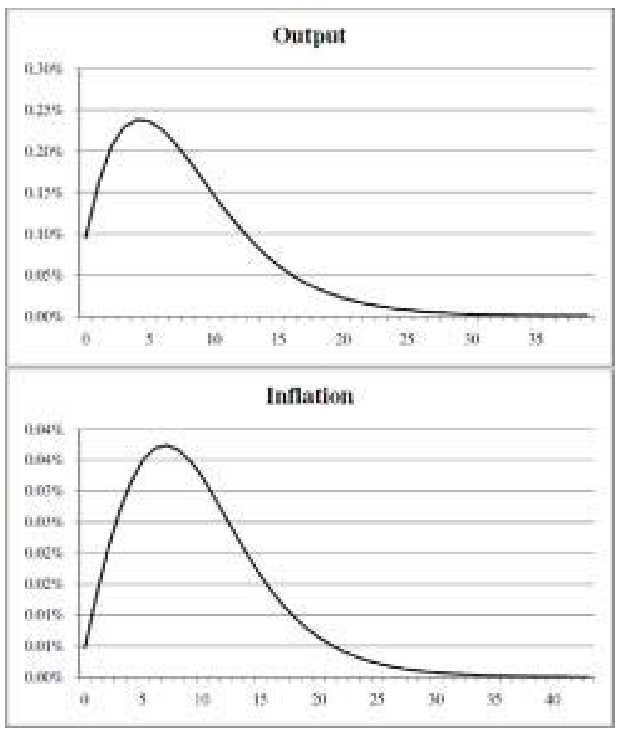

Figure 1: Impulse Response to Temporary Increase in Money Growth

To explain the impulse response portrayed in Figure 1, note when $\rho_{\Delta m} \in (0,1)$, $\lim_{j\to\infty}\left(a^{\Delta m}_{y^j}\right) = \lim_{j\to\infty}\left(\frac{A(j+1)}{1+A(j+1)}\right)\frac{1-\rho^{j+1}_{\Delta m}}{1-\rho_{\Delta m}} \to 0$ .Moreover,

$\lim_{j\to\infty}\left(\frac{A(j+1)}{1+A(j+1)}\right)$ falls and tends 0 but $\lim_{j\to\infty}\left(\frac{1-\rho_{\Delta m}^{j+1}}{1-\rho_{\Delta m}}\right)$ rises and tends to $\frac{1}{1-\rho_{\Delta m}}$.

This trade-off between the first and the second component of $a_{yj}^{\Delta m}$ produces the hump in output. We have checked that there is no hump in output when the persistence of demand shock is smaller relative to the degree of information asymmetry, i.e., the trade-off between two terms of $a_{yj}^{\Delta m}$ is smaller.

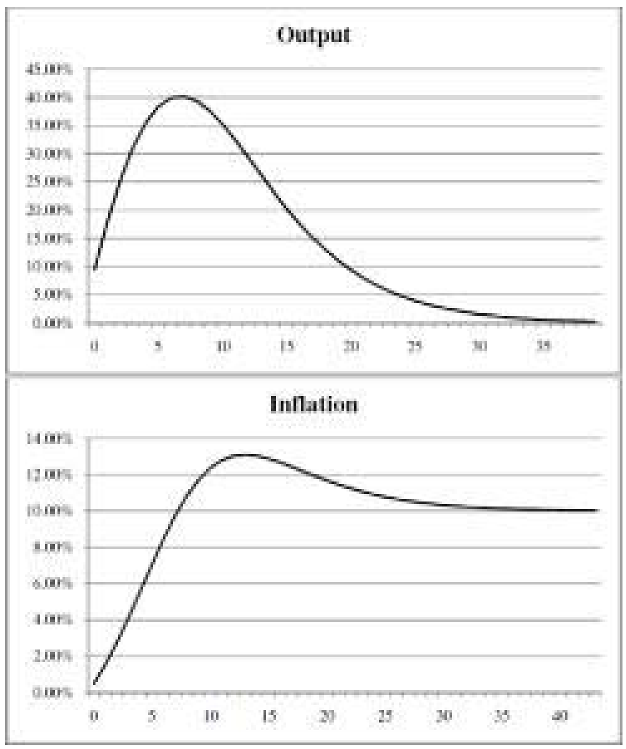

Figure 2: Impulse Response to Permanent Increase in Money Growth

Figure 2 shows that the simple sticky information model follows the Natural Rate Hypothesis (McCallum, 1998). We see from Figure 2 that a permanent rise in money growth only increases inflation rate permanently but not output. To explain note that,

$$\lim_{j\to\infty}\left(a^{\Delta m}_{y^j}\right)=\lim_{j\to\infty}\left(\left(\frac{A_{j+1}}{1+A(j+1)}\right)(j+1)\right)\to 0$$

and,

$$\lim_{j\to\infty}\left(a^{\Delta m}_{\pi j}\right) = \lim_{j\to\infty}\left(\rho^{j}_{\Delta m}-\left(a^{\Delta m}_{yj}-a^{\Delta m}_{y(j-1)}\right)\right)\to 1 .$$

We see hump shaped response in output even under permanent demand. The reason behind the hump in output is again the trade-off between first and second term of $a^{\Delta m}_{yj}$. Note, while the first term of $a^{\Delta m}_{yj}$ falls and tends to zero, the second term rises and goes to infinity. This produces the hump in output as shown in Figure 2.

4.2 Impulse Response under Supply Shock

We assume, $\Delta m_t = 0$ to analyze the impulse response under supply shock. Note, $\Delta m_t = 0$ implies we have, $a^{\Delta m}_{yj} = a^{\Delta m}_{\pi j} = 0\ \forall j$. Figure 3 portrays the impulse response of a 10% contractionary supply shock of persistence 0.8. The supply shock reduces output and increases inflation rate as expected. We see hump shaped response of output and inflation in short run but both go back to their long run level as time progresses. The hump in output obtained under supply shock is also due to the relative strength of the persistence of supply shock and degree of information asymmetry as in under demand shock.

Figure 4 shows the impulse response of a permanent supply shock of same magnitude. The figure shows even if there is a permanent shift in output, inflation comes back to its long run level after initial fluctuations as time progresses. The intuition follows directly from equation (29) and (33). Note we have,

$$\lim_{j\to\infty}\left(a^{e}_{y^j}\right)=\lim_{j\to\infty}-\left(\frac{1}{1+A(j+1)}\right)\frac{\gamma}{\alpha}\rho^{j}_{e}\to -\frac{\gamma}{\alpha}$$

and

$$\lim_{j\to\infty}\left(a^{e}_{\pi j}\right)=\lim_{j\to\infty}\left(\left(a^{e}_{yj}-a^{e}_{y(j-1)}\right)\right)\to 0 .$$

This explains the impulse response portrayed in Figure 4.

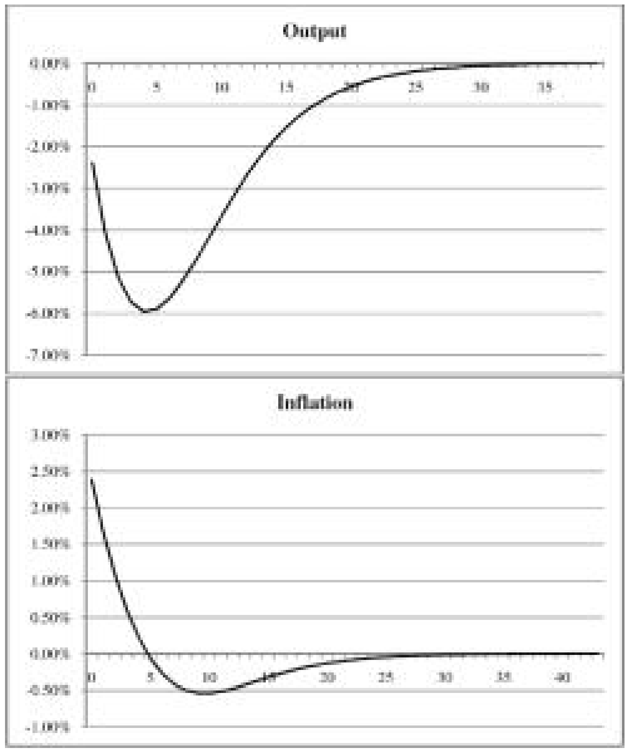

Figure 3: Impulse Response to Temporary Contractionary Supply Shock

Note, the counter intuitive result portrayed in Figure: 4 that, a permanent supply shock causing a permanent departure of output from its long-run value and not to inflation is unidentified in Mankiw and Reis (2002) as they have analyzed the model with demand shock only.

5. Conclusion

This paper extends the derivation of sticky information Phillips curve of Mankiw and Reis (2002) with a supply shock and develops an new algorithm to solve the model using the Methodology of Wang and Wen (2006). The model is solved by the method of undetermined coefficients after transforming into forecast error form. This greatly reduces the possibility of making human error as noted by Wang and Wen (2006). Beside this, the major strength of the algorithm lies in its analytical exposition which allows us to uncover better intuition from the model.

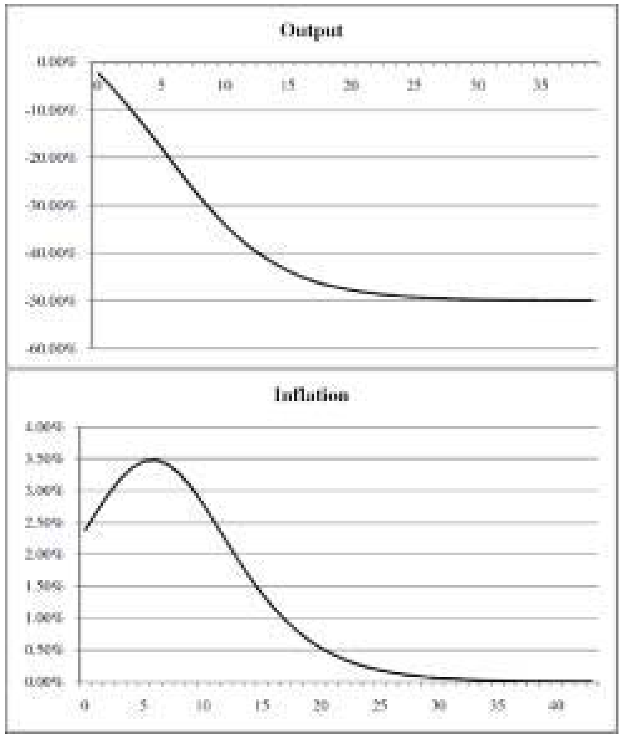

Figure 4: Impulse Response to Permanent Contractionary Supply Shock

For example, the impulse responses of the model shows that, the hump shaped response of inflation and output depends on the magnitude and/or strength of the persistence of shock relative to its endogenous persistence, θ. We see that the model fails to produce hump when persistence of shock is small compared to the endogenous persistence, θ. We also see that the simple model has its own peculiarities too. The model follows the Natural Rate Hypothesis (McCallum, 1998) but produces counter intuitive results to permanent supply shock. A permanent supply shock to the model causes permanent reduction to output without affecting long-run inflation rate. Since Mankiw and Reis (2002) have analyzed the model with only demand shock, they fail to discover the peculiarities of the model associated with supply shock.

Our algorithm can be easily extended to solve a full blown sticky information New Keynesian DSGE model where the demand side is represented by expectational IS equation, supply side is represented by sticky information Phillips curve and nominal interest rate is determined by Taylor rule that follows

Taylor principle. This is known as canonical sticky information model. To solve the model, we assume that inflation and output follows an $MA(\infty)$ process. The canonical model has two roots. One root is the coefficient of inflation rate in Taylor rule, which is greater than one under Taylor principle and other root is $\theta \in (0,1)$, the fraction of firm whose information is not completely updated. The unstable root allows us to solve the expectational IS equation forward. The forward solution of expectational IS solves the coefficient of inflation rate through the method of undetermined coefficient. The stable root on the other hand solves the sticky information Phillips curve backward. The backward solution of the supply curve calculates the coefficients of output through the method of undetermined coefficient.

The canonical model follows the Natural Rate Hypothesis (McCallum, 1998) but produces hump only to inflation rate and not to output. However, the pervasive stickiness model (Mankiw and Reis, 2006) produces hump to both output and inflation rate when the magnitude and/or persistence of the shock is large enough relative to the endogenous persistence as we have seen for the simple model. A detail exposition of the algorithm solving both canonical and pervasive stickiness model under demand shock is given in Chattopadhyay (2011).[6]

References

[1] Ball, L. (1994). Credible disinflation with Staggered Price Setting. *American Economic Revie.*84. 282-89.

[2] Ball, L. and D. Romer (1990). Real Rigidities and Non-Neutrality of Money. *Review of Economic Studies.* 57. 183-203.

[3] Calvo, G. A. (1983). Staggered Prices in a Utility Maximizing Framework, *Journal of Monetary Economics.* 12(3). 983-98.

[4] Chattopadhyay, S. (2011). *Monetary Policy and New Keynesian Macroeconomics.* Unpublished Ph.D. Dissertation, University at Albany, State University of New York. USA.

[6] The analysis of the canonical and pervasive stickiness model under supply shock is in our future research.

[5] Cooper, R. and A. John (1988), Coordinating Coordination Failures in Keynesian Models. *The Quarterly Journal of Economics.* 103. 441-63.

[6] Mankiw, N. G. and R. Reis (2002). Sticky information Versus Sticky Prices: A Proposal to Replace The New Keynesian Phillips Curve. *The Quarterly Journal of Economics.* 117, 1295-1328.

[7] Mankiw, N. G. and R. Reis (2006). Pervasive Stickiness. *American Economic Review.*96(2). 1164-69.

[8] McCallum, B. T. (1998). Stickiness: A Comment, *Carnegi-Rochester Conference Series on Public Policy.* 68. 3657-63.

[9] Verona, F. and M. H. Wolters (2013). Sticky Information Models in Dynare. *Centre Pour La Resherche Economique Et Ses Applications, France. Working Paper No.* 11.

[10] Wang, P. and Y. Wen (2006). Solving Linear Difference Systems with Lagged Expectations by a Method of Undetermined Coefficients. *Federal Reserve Bank of St. Louis, Working Paper No. 2006-003-C.*

[11] Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Economics.* Princeton University Press. Princeton and Oxford.

Appendix:

Sticky Information Phillips Curve

Desired/optimal price level of a generic firm j with completely updated

information

is,

$$p_{t,0}(j) = p_{t,0} = p_t + \alpha y_t + \gamma e_t \tag{34}$$

Therefore,

$$p_{t,0} - p_{t-1,0} = \pi_t + \alpha \Delta y_t + \gamma \Delta e_t \tag{35}$$

Assuming $(1-\theta)$ as the fraction of firms with updated information and Calvo price setting (Calvo, 1983) the price of firms who have updated their information $(t-j)$ period ahead is,

$$p_{t,j} = (1-\theta)\theta^{j} E_{t-j}(p_{t,0}) \tag{36}$$

Therefore, the aggregate price level of the economy

$$p_t = \sum_{j=0}^{\infty} p_{t,j} = (1-\theta)\sum_{j=0}^{\infty} \theta^{j} E_{t-j}(p_{t,0})$$

Simplifying we get,

$$p_t = \left(\frac{1-\theta}{\theta}\right)[\alpha y_t + \gamma e_t] + (1-\theta)\sum_{j=0}^{\infty} \theta^{j} E_{t-j-1}(p_{t,0}) \tag{37}$$

Similarly,

$$p_{t-1} = (1+\theta)\sum_{j=0}^{\infty} \theta^{j} E_{t-j-1}(p_{t-1,0}) \tag{38}$$

Define, $\pi_t = p_t - p_{t-1}$. Now, subtracting equation (37) from (28) and using equation (35) we have,

$$\pi_t = \left(\frac{1-\theta}{\theta}\right)[\alpha y_t + \gamma e_t] + (1-\theta)\sum_{j=0}^{\infty} \theta^{j} E_{t-j-1}(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t) \tag{39}$$

Note, by adding and subtracting $(1-\theta)\sum_{j=1}^{\infty} \theta^{j}(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t)$ to the R.H.S. of equation (39) and using $\sum_{j=1}^{\infty} \theta^{j} = \frac{\theta}{1-\theta}$ and $\left(\frac{\theta}{\alpha}\right)\gamma\Delta e_t + \left(\frac{1-\theta}{\alpha}\right)\gamma e_t = \left(\frac{\gamma}{\alpha}\right)(1-\theta L)e_t$ gives,

$$y_t = \theta_{y-1} + \left(\frac{1-\theta}{\alpha}\right)\sum_{j=1}^{\infty} \theta^{j}[(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t) - E_{t-j}(\pi_t + \alpha\Delta y_t + \gamma\Delta e_t)] - \left(\frac{\gamma}{\alpha}\right)(1-\theta L)e_t \tag{40}$$