Scheduling preferences, parking competition, and bottleneck congestion: A model of trip timing and parking location choices by heterogeneous commuters

Takayama, Yuki and Kuwahara, Masao

19 January 2016

Scheduling preferences, parking competition, and bottleneck congestion: A model of trip timing and parking location choices by heterogeneous commuters*

Yuki Takayama† and Masao Kuwahara‡

January 19, 2016

Abstract

This study examines the effects of location-dependent parking fees and time-varying congestion tolls on the behavior of heterogeneous commuters and their commuting costs. To this end, we develop a model of trip timing and parking location choices by heterogeneous commuters and characterize its equilibrium. By comparing the equilibrium with and without pricing policies, we obtain the following results: (1) without pricing policies, interactions among heterogeneous commuters yield an inefficient distribution of trip timing and parking locations; (2) imposing a parking fee and expanding parking capacity may concentrate the temporal distribution of traffic demand, thereby exacerbating traffic congestion and total commuting cost; (3) the social optimum is achieved by combining a parking fee with a congestion toll; and (4) the revenue obtained from pricing of parking and roads exactly equals the costs for optimal parking and bottleneck capacity; that is, the self-financing principle holds in the model.

JEL classification: D62; H23; R41; R48

Keywords: scheduling preference; parking competition; bottleneck congestion; parking fee

*We are grateful to Daisuke Fukuda for his helpful comments. This research was supported by JSPS Grant-in-Aid for Young Scientists (B) 15K18136. Any remaining errors are ours.

†Institute of Science and Engineering, Kanazawa University, Kakuma-machi, Kanazawa 920-1192, Japan. Phone/Fax: +81-76-234-4915, E-mail: ytakayama@se.kanazawa-u.ac.jp

‡Graduate School of Information Sciences, Tohoku University.

1 Introduction

Traffic demand is highly concentrated within a specific time period, a phenomenon that can be observed in most cities. This concentration is mainly due to the fact that firms in central business districts (CBDs) generally start work at the same time; therefore, numerous commuters want to arrive at the same location simultaneously. This concentration of traffic demand is a major cause of traffic congestion.

Although the temporal distribution of traffic demand tends to be clustered, it is not fully concentrated into a specific time. This is due to the existence of commuter incentives, which cause traffic demand to become decentralized. Avoidance of traffic congestion is one principal reason for commuters shifting their trip timing from the peak period. Parking congestion also leads to temporal traffic dispersion, as pointed out by previous studies (e.g., Arnott et al., 1991; Qian et al., 2012). In particular, commuters who leave home later need to park farther from their workplace and then walk a longer distance when parking spaces are not sufficiently numerous, an outcome that influences commuters' preferences for their trip timing.[1] Empirical evidence (e.g., Beesley, 1965; Quarmby, 1967; Domencich and McFadden, 1975) indicates that the cost per unit walking time is between one to three times higher than that per unit travel time. This shows that a commuter's scheduling preference generated from parking competition is not negligible. Many other commuter incentives exist that serve to shift trip timing from the peak, e.g., to arrive at work earlier than their supervisor, to work efficiently, and so on.

These facts suggest that commuters' scheduling preferences arise from various factors, and therefore, differ significantly among commuters. Furthermore, heterogeneity of commuters in their value of travel time, walking time, and schedule delay may strongly impact the distribution of trip timing. In fact, if we consider interactions between the two groups of commuters, namely Commuter Group A, who place a higher value on shorter travel times, and Commuter Group B, who place a higher value on shorter walking time, the following circular causation creates strong incentives for commuters to be "early birds" (i.e., to arrive at work earlier than other commuters): an early departure by Commuter Group A due to their high cost per unit travel time leads to their occupation of convenient parking spaces, thereby increasing the early-bird incentive for commuters in Group B; conversely, an earlier departure by commuters in Group B exacerbates traffic congestion at earlier time periods, which further increases the early-bird incentive for commuters in Group A. Therefore, it is important to explicitly consider commuter heterogeneity and various factors yielding scheduling preferences (e.g., parking competition, queuing congestion) when analyzing the distribution of trip timing and the efficacy of measures intended to alleviate traffic congestion and parking competition.

A considerable number of studies have attempted to model commuters' choice of trip timing and the formation of traffic congestion. The bottleneck model (Vickrey, 1969; Hendrickson and Kocur, 1981; Arnott et al., 1990) is the most successful model. This equilibrium model provides a simple framework and thus has inspired numerous extensions and modifications. However, as will be discussed in Section 1.1, no studies have so far considered heterogeneity of commuters and multiple factors in generating commuters' scheduling preferences.

This study develops a model of trip timing and parking location choices by heterogeneous commuters that describes traffic congestion and parking competition among commuters. We

[1] If parking spaces face a very high demand, commuters cannot immediately find a vacant parking space and therefore cruise for parking. This also impacts commuters' scheduling preferences (see, e.g., Anderson and de Palma, 2004).

then characterize both equilibrium and socially optimal distributions of trip timing and parking locations. It is shown that an inefficient distribution can arise as an equilibrium, such as when commuters with high cost per unit travel time choose to travel in a highly congested period.

After characterizing the equilibrium and social optimum, this study investigates the effects of location-dependent parking fees and time-varying congestion tolls on the distribution of trip timing and parking locations. This analysis shows that the social optimum is achieved by imposing both of these two pricing policies. Furthermore, the revenue from parking fees and congestion tolls exactly equals the costs of the optimal parking and bottleneck capacity, respectively, i.e., the *self-financing principle* (Mohring and Harwitz, 1962) holds in our model. In contrast, an expansion of parking capacity and the introduction of parking fees may lead to temporal concentration of traffic demand, thereby exacerbating traffic congestion and total commuting cost.

It is noteworthy that not only parking fees but also congestion tolls can be charged at parking gates in our framework. Therefore, the effects of the parking fee and/or the congestion toll presented in this study can be interpreted as applying to location- and/or time-dependent parking prices. As noted by Shoup (2005) and Fosgerau and de Palma (2013), such parking prices are easily imposed as the technology needed to charge for parking is much simpler than that needed to charge for driving in congested traffic.

1.1 Related literature

Since the seminal work of Vickrey (1969), numerous studies have developed models of commuters' choice of trip timing. In the last few decades, many studies have been devoted to integrating parking competition into the standard bottleneck model. Arnott et al. (1991) was the first successful attempt to explore how parking competition among commuters affects their trip timing in addition to investigating the efficiency of various road toll and parking fee policies. Their model has been extended by Zhang et al. (2008, 2011), Qian et al. (2011), Yang et al. (2013), Fosgerau and de Palma (2013), and Liu et al. (2014b), all of which examine the robustness of their results and propose new schemes to improve traffic congestion.[2] Recently, a few studies have developed bottleneck models that incorporate various factors generating the scheduling preferences of commuters. Peer and Verhoef (2013) and Takayama (2015) described commuters' divergence in terms of long-run decisions regarding routine arrival times at work and short-run decisions regarding day-specific trip timings. Fosgerau and Lindsey (2013), Gubins and Verhoef (2014), and Fosgerau and Small (2014) considered the utility of spending time at home,[3] which would reduce commuters' early-bird incentives. However, these studies assumed that commuters are homogeneous.[4]

Bottleneck models with heterogeneous commuters have also been developed, e.g., by Newell (1987), Arnott et al. (1994), Lindsey (2004), Ramadurai et al. (2010), van den Berg and Verhoef (2011), and Doan et al. (2011). These studies demonstrated the properties of equilibrium, social optimum, and optimal pricing strategies. However, they assumed that scheduling preferences are affected only by traffic congestion.

This study extends the model of Arnott et al. (1991) to incorporate commuter heterogeneity.

[2] For a comprehensive review of the literature on parking congestion, see Inci (2015).

[3] Vickrey (1973) and Tseng and Verhoef (2008) also consider a time-varying utility of time spent at home.

[4] Liu et al. (2014a) extended the studies by Yang et al. (2013) and Liu et al. (2014b) to consider commuter heterogeneity. However, in their framework, parking competition does not create commuters' early-bird incentives, that is, commuters' scheduling preferences arise only from traffic congestion.

We then systematically analyze the developed model by utilizing the properties of the complementarity problems that define the equilibrium of our model. This analysis shows that at the equilibrium, workers sort temporally and spatially on the basis of their value of time, and that the effects of parking fees and congestion tolls on the commuting cost of each commuter type differs significantly. Thus, each pricing policy leads to an essentially different distribution of trip timing and parking locations.

This study proceeds as follows. Section 2 formulates our model of trip timing and parking location choice by heterogeneous commuters. In Sections 3 and 4, we characterize the equilibrium and social optimum, respectively. Section 5 analyzes the effects of pricing policies, namely parking fees and congestion tolls. To demonstrate concretely the properties of our model and the effects of pricing policies, we analyze the model under a simple setting in Section 6. Section 7 concludes.

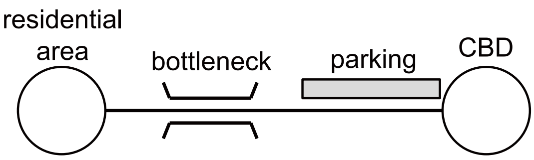


Figure 1: Urban spatial structure

2 The model

2.1 Basic assumptions

We consider a city consisting of a CBD and residential area connected by a single road (Figure 1). The road has a single bottleneck with capacity μ. If arrival rates at the bottleneck exceed its capacity, a queue develops. To model queuing congestion, we employ first-in-first-out (FIFO) and a point queue in which vehicles have no physical length as in standard bottleneck models (e.g., Vickrey, 1969; Hendrickson and Kocur, 1981; Arnott et al., 1990, 1993).

The parking spaces are located between the bottleneck and CBD. The number of parking spaces per unit of distance (i.e., density of parking spaces) is assumed to be constant, d, as in Arnott et al. (1991). Hence, in our model, commuters need to walk from their parking location to the CBD. A parking location is indexed by the distance l from the CBD, which equals the walking distance. Walking time from the parking location l to the CBD is taken to be wl, where w describes the walking time per unit of distance.

There are I types of commuters, each of whom must travel from the residential area to the CBD and who have the same work start time t^*. The number of commuters of type $i \in \mathcal{I} \equiv \{1, 2, \cdots, I\}$, whom we call "commuters i," is fixed and denoted by N_i. The commuting cost $c_i(t, l)$ of commuter i who departs the bottleneck at time t (travels at t), parks at location l, and arrives at the CBD at time $a(t, l) = t + wl$ is expressed as the sum of travel time cost $\alpha_i\{q(t) + c^f\}$, schedule delay cost $s_i(t^* - a(t, l))$, and walking time cost $\lambda_i wl$:

$$c_i(t, l) = \alpha_i \left\{q(t) + c^f\right\} + s_i(t^* - a(t, l)) + \lambda_i wl, \tag{1}$$

where α_i is the value of travel time of commuter i, $q(t)$ denotes queuing time of a commuter traveling at time t, c^f is fixed free flow travel time, and λ_i is the value of walking time of commuter i. Note that the value of c^f does not affect the results of interest since c^f is independent of both trip timing and parking location. Therefore, we set $c^f = 0$. We assume that arriving late is prohibitively costly,[5] i.e., all commuters must be at work by time t^*, and the schedule delay cost function $s_i(t^* - a(t, l))$ is linear: $s_i(t^* - a(t, l)) = \beta_i\{t^* - a(t, l)\}$, where β_i are the early delay cost per unit of time for commuters i.

This study imposes the following assumptions for ensuring the existence of an equilibrium and eliminating unrealistic situations.

Assumption 1. *Parameters α_i, β_i, λ_i, w, μ, and d satisfy the following conditions:*

[5]Prohibiting late arrivals does not affect the fundamental properties of our model. This assumption is made to simplify the analysis and to present clearly the essential properties of our model. The same modeling strategy has been used in many bottleneck models (e.g., Arnott and Kraus, 1993, 1995; Kraus and Yoshida, 2002; Kraus, 2003, 2012; Gubins and Verhoef, 2014).

(a) $\beta_i < \alpha_i$ *for all* $i \in \mathcal{I}$;

(b) $\beta_i < \lambda_i$ *for all* $i \in \mathcal{I}$;

(c) $\beta_i > \{\lambda_i - \beta_i\} w\mu/d$ *for all* $i \in \mathcal{I}$;

(d) $w/d < 1/\mu$.

Condition (a) ensures that an equilibrium in our model satisfies the FIFO property (i.e., vehicles must leave the bottleneck in the same order as their arrival at the bottleneck), which is proved in the literature. This condition requires that the value α_i of travel time is higher than the early delay cost β_i per unit of time for all commuters $i \in \mathcal{I}$, and implies that commuters prefer to wait at the office rather than in the queue. If this condition is violated, there is no equilibrium satisfying the FIFO property.

Condition (b) implies that the early delay cost β_i per unit of time is less than the value λ_i of walking time for all commuters $i \in \mathcal{I}$. This ensures that commuters choose a parking location as close as possible to the CBD and go directly to their office.[6] Note that Beesley (1965), Quarmby (1967), and Domencich and McFadden (1975) all provide supporting empirical evidence.

Under Assumption 1 (b), parking spaces are occupied in strict order from the CBD (i.e., $l = 0$). This indicates that the parking location $l(t)$ of a commuter who travel at time t is given by

$$l(t) = \frac{N(t)}{d}, \tag{2}$$

where $N(t)$ denotes the cumulative number of commuters who have departed the bottleneck by time t, which is expressed using the number $\hat{n}_i(t)$ of commuters i traveling at time t as

$$N(t) = \sum_{i \in \mathcal{I}} \int_{-\infty}^{t} \hat{n}_i(s)\, \mathrm{d}s. \tag{3}$$

Therefore, we can rewrite the commuting cost of commuter i as a function of departure time t at the bottleneck, which we call trip timing t

$$\begin{aligned} \hat{c}_i(t) &= \alpha_i q(t) + \beta_i(t^* - \hat{a}(t)) + \lambda_i \frac{wN(t)}{d} \\ &= \alpha_i q(t) + \beta_i(t^* - t) + (\lambda_i - \beta_i)\frac{wN(t)}{d}, \end{aligned} \tag{4}$$

where $\hat{a}(t) = a(t, l(t))$.

To provide an intuition for the third condition (c), we consider the commuters' choice of trip timing t under the commuting cost (4). The time derivative $\mathrm{d}\hat{c}_i(t)/\mathrm{d}t$ of the commuting cost $\hat{c}_i(t)$ is given by

$$\frac{\mathrm{d}\hat{c}_i(t)}{\mathrm{d}t} = \alpha_i \frac{\mathrm{d}q(t)}{\mathrm{d}t} - \beta_i + (\lambda_i - \beta_i)\frac{w \sum_{i \in \mathcal{I}} \hat{n}_i(t)}{d}. \tag{5}$$

This indicates that if $-\beta_i + (\lambda_i - \beta_i) w \sum_{i \in \mathcal{I}} \hat{n}_i(t)/d$ is positive, commuters i can reduce their commuting cost by choosing an earlier t when $\mathrm{d}q(t)/\mathrm{d}t = 0$ (e.g., before the queue develops). This implies that a high walking cost λ_i provides an incentive for commuters i to choose an early as possible

[6]In the case where $\beta_i > \lambda_i$, commuters choose to walk around their office until their work start time t^*.

t so as to get a convenient parking location. To avoid strong parking competition, we introduce condition (c), which ensures $-\beta_i + (\lambda_i - \beta_i)w \sum_{i\in\mathcal{I}} \hat{n}_i(t)/d < 0$ for all t.[7]

Finally, condition (d) excludes the situation where the parking capacity d is unrealistically small. In our model, the maximum walking time is $w \sum_{i\in\mathcal{I}} N_i/d$, and the minimum rush hour length in which commuters pass the bottleneck is $\sum_{i\in\mathcal{I}} N_i/\mu$. Thus, the length of walking time can exceed that of the rush hour. Condition (d) ensures that such a situation does not exist. [8]

2.2 Equilibrium conditions

Commuters i minimize their commuting cost $\hat{c}_i(t)$ in (4) by choosing a trip timing t. An equilibrium is defined as a state that satisfies the following three conditions:

$$\begin{cases} \hat{c}_i(t) = c_i^* & \text{if} \quad \hat{n}_i^*(t) > 0 \\ \hat{c}_i(t) \geq c_i^* & \text{if} \quad \hat{n}_i^*(t) = 0 \end{cases} \quad \forall i \in \mathcal{I}, \tag{6a}$$

$$\begin{cases} \sum_{i\in\mathcal{I}} \hat{n}_i^*(t) = \mu & \text{if} \quad q^*(t) > 0, \\ \sum_{i\in\mathcal{I}} \hat{n}_i^*(t) \leq \mu & \text{if} \quad q^*(t) = 0, \end{cases} \tag{6b}$$

$$\int \hat{n}_i^*(t)\,\mathrm{d}t = N_i \quad \forall i \in \mathcal{I}, \tag{6c}$$

where the asterisks denote equilibrium values.

Condition (6a) represents the no-arbitrage condition for departure time choice. This condition means that at the equilibrium, commuters are unable to reduce commuting costs by unilaterally changing trip timing t. Condition (6b) is the capacity constraint of the bottleneck, which requires that the total departure rate $\sum_{i\in\mathcal{I}} \hat{n}_i^*(t)$ at the bottleneck equals the capacity μ if there is a queue; otherwise, the total departure rate is (weakly) lower than μ. Condition (6c) is flow conservation for commuting demand. These conditions give the equilibrium values of $\hat{n}_i^*(t)$, $q^*(t)$, and c_i^*.

[7]Note that $\sum_{i\in\mathcal{I}} \hat{n}_i(t) \leq \mu$ for all t because of the capacity constraint of the bottleneck.

[8]If condition (d) is not satisfied, all commuters can arrive at the CBD at the same time.

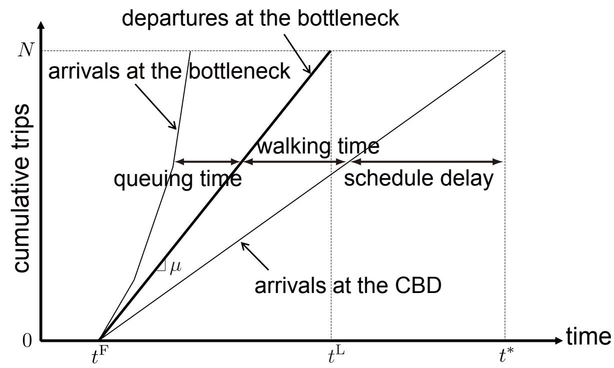


Figure 2: An example of cumulative arrival and departure curves at the equilibrium

3 Equilibrium

In this section, we characterize the equilibrium of our model. From the equilibrium conditions (6), we can show that $\hat{n}_i^*(t)$ has the following property:

Lemma 1. *At the equilibrium, the number $\hat{n}_i^*(t)$ of commuters i who travel at time t is*

$$\sum_{i \in \mathcal{I}} \hat{n}_i^*(t) = \begin{cases} \mu & \text{if} \quad t \in [t^{\mathrm{F}}, t^{\mathrm{L}}], \\ 0 & \text{otherwise}, \end{cases} \tag{7}$$

where t^{F} and t^{L} are the first and last trip timing of commuters, respectively, which are expressed as

$$t^{\mathrm{F}} = t^* - \left(1 + \frac{w\mu}{d}\right) \frac{\sum_{i \in \mathcal{I}} N_i}{\mu}, \qquad t^{\mathrm{L}} = t^{\mathrm{F}} + \frac{\sum_{i \in \mathcal{I}} N_i}{\mu}. \tag{8}$$

Proof. See Appendix A. □

Let $\hat{\mathcal{S}}_i = \{t \in \mathbb{R} \mid \hat{c}_i(t) = c_i^*\}$. Then, from Lemma 1, we can say that the rush hour $\bigcup_{i \in \mathcal{I}} \hat{\mathcal{S}}_i$ equals $[t^{\mathrm{F}}, t^{\mathrm{L}}]$.

Lemma 1 indicates that at the equilibrium, the rush hour must be a single time interval and $N^*(t)$ is given by

$$N^*(t) = \mu(t - t^{\mathrm{F}}) \tag{9}$$

as illustrated in Figure 2. Thus, the commuting cost (4) of commuters i can be rewritten as

$$\hat{c}_i(t) = \alpha_i q(t) - f_i(t - t^{\mathrm{F}}) + \beta_i(t^* - t^{\mathrm{F}}), \tag{10}$$

where f_i is defined as

$$f_i = \beta_i - (\lambda_i - \beta_i)\frac{w\mu}{d} > 0. \tag{11}$$

This leads to the following proposition that characterizes the equilibrium number $\hat{\boldsymbol{n}}^*(t) = [\hat{n}_i^*(t)]$ of commuters i traveling at time t, which we call the equilibrium distribution of trip timing.

Proposition 1. *The equilibrium distribution $\hat{\boldsymbol{n}}^*(t)$ of trip timing coincides with the solution of the following linear programming problem:*

$$\min_{\hat{\boldsymbol{n}}(t)} \sum_{i \in \mathcal{I}} \int \frac{1}{\alpha_i} \left\{ -f_i(\tau - t^{\mathrm{F}}) + \beta_i(t^* - t^{\mathrm{F}}) \right\} \hat{n}_i(\tau) \, \mathrm{d}\tau \tag{12a}$$

$$s.t. \quad \sum_{i \in \mathcal{I}} \hat{n}_i(t) \leq \mu \quad \forall t \in \mathbb{R}, \quad \int \hat{n}_i(t) \, \mathrm{d}t = N_i \quad \forall i \in \mathcal{I}, \quad \hat{n}_i(t) \geq 0 \quad \forall i \in \mathcal{I}, \ \forall t \in \mathbb{R}. \tag{12b}$$

Proof. See Appendix B □

Let us define the (travel) *time-based cost* as the cost converted into equivalent travel time. Then, since the time-based cost of a commuter i is given by the cost divided by α_i, we can say that $\{-f_i(t - t^{\mathrm{F}}) + \beta_i(t^* - t^{\mathrm{F}})\}/\alpha_i$ represents the sum of the *time-based* schedule delay cost and the *time-based* walking time cost of a commuter i who travels at time t. Therefore, Proposition 1 indicates that at the equilibrium, the sum of the total *time-based* schedule delay cost and the total *time-based* walking time cost is minimized, but the sum of total schedule delay cost and walking time cost is not necessarily minimized.[9]

The condition (6a) shows that the commuting cost $\hat{c}_i(t)$ of a commuter i is minimized at $t \in \hat{\mathcal{S}}_i$, and thus, we have

$$\begin{cases} \hat{c}_i(t_i) \leq \hat{c}_i(t_j) \\ \hat{c}_j(t_j) \leq \hat{c}_j(t_i) \end{cases} \qquad \forall t_i \in \hat{\mathcal{S}}_i, \ \forall t_j \in \hat{\mathcal{S}}_j, \ \forall i, j \in \mathcal{I}. \tag{13}$$

This leads to the following condition:

$$\frac{\hat{c}_i(t_i)}{\alpha_i} + \frac{\hat{c}_j(t_j)}{\alpha_j} \leq \frac{\hat{c}_i(t_j)}{\alpha_i} + \frac{\hat{c}_j(t_i)}{\alpha_j} \quad \forall t_i \in \hat{\mathcal{S}}_i, \ \forall t_j \in \hat{\mathcal{S}}_j, \ \forall i, j \in \mathcal{I}. \tag{14}$$

Substituting (10) into this, we have the condition that characterizes the distribution of trip timing:

$$\left(\frac{f_i}{\alpha_i} - \frac{f_j}{\alpha_i} \right) (t_i - t_j) \geq 0 \quad \forall t_i \in \hat{\mathcal{S}}_i, \ \forall t_j \in \hat{\mathcal{S}}_j, \ \forall i, j \in \mathcal{I}. \tag{15}$$

This condition indicates that *commuters depart the bottleneck (arrive the CBD) in order of increasing f_i/α_i*. This implies that $-f_i/\alpha_i$ represents the incentive for a commuter i to be an "early bird" (i.e., to arrive at work earlier than other commuters) and that competition for parking creates commuters' early-bird incentives because $\mathrm{d}(-f_i/\alpha_i)/\mathrm{d}d > 0$. Furthermore, when $f_i/\alpha_i < f_j/\alpha_j$, the parking location of a commuter i must be (weakly) closer to the CBD than that of a commuter j. Therefore, commuters with high λ_i do not necessarily park close to the CBD.

We next investigate the properties of the equilibrium commuting cost $[c_i^*]$. For this, we suppose in this section that $f_{i-1}/\alpha_{i-1} \leq f_i/\alpha_i$ for all $i \in \mathcal{I} \backslash \{1\}$ without loss of generality, and consider $t_{i-1,i}$ such that $t_{i-1,i} \in \hat{\mathcal{S}}_{i-1} \cap \hat{\mathcal{S}}_i$.[10] Then, the difference in the time-based equilibrium commuting cost

[9]It is noteworthy that the sum of the total schedule delay cost and total walking time cost is minimized at the equilibrium if we consider homogeneous commuters.

[10]It follows from Lemma 1 and (15) that for any $i \in \mathcal{I} \backslash \{1\}$, there exists a value of $t_{i-1,i}$ that satisfies $t_{i-1,i} \in \hat{\mathcal{S}}_{i-1} \cap \hat{\mathcal{S}}_i$.

between commuters $i-1$ and i is given by

$$\frac{c_i^*}{\alpha_i} - \frac{c_{i-1}^*}{\alpha_{i-1}} = \left(\frac{\beta_i}{\alpha_i} - \frac{\beta_{i-1}}{\alpha_{i-1}}\right)(t^* - t^{\mathrm{F}}) - \left(\frac{f_i}{\alpha_i} - \frac{f_{i-1}}{\alpha_{i-1}}\right)(t_{i-1,i} - t^{\mathrm{F}}) \quad \forall i \in \mathcal{I}\backslash\{1\}, \tag{16a}$$

$$t_{i-1,i} = t^{\mathrm{F}} + \frac{\sum_{k=1}^{i-1} N_k}{\mu}. \tag{16b}$$

This shows that the values of f_i/α_i, β_i/α_i, and N_i/μ determine the sign of the difference $c_i^*/\alpha_i - c_{i-1}^*/\alpha_{i-1}$. For example, if $\beta_{i-1}/\alpha_{i-1} \geq \beta_i/\alpha_i$ for all $i \in \mathcal{I}\backslash\{1\}$, then the time-based equilibrium commuting cost of a commuter $i-1$ is higher than that of a commuter i, i.e., a commuter traveling earlier incurs a higher time-based commuting cost. It is noteworthy that this property cannot be observed in the standard bottleneck model. In fact, when we consider the case without parking congestion (i.e., $d \to \infty$), (16a) is rewritten as

$$\lim_{d\to\infty}\left\{\frac{c_i^*}{\alpha_i} - \frac{c_{i-1}^*}{\alpha_{i-1}}\right\} = \left(\frac{\beta_i}{\alpha_i} - \frac{\beta_{i-1}}{\alpha_{i-1}}\right)\frac{\sum_{k=i}^{I} N_k}{\mu}. \tag{17}$$

Because $\lim_{d\to\infty} f_i = \beta_i$, this shows that in the standard bottleneck model, a commuter traveling later incurs a higher time-based commuting cost.

We consider the effects of the expansion of the parking capacity d on the equilibrium commuting cost. It follows from (16b) that at the equilibrium, the travel time cost $\alpha_i q(t)$, the schedule delay cost $\beta_i(t^* - \hat{a}(t))$, and the walking time cost $\lambda_i wl(t)$ of a commuter i are expressed as

$$\alpha_i q(t) = \alpha_i \sum_{k=1}^{i}\left(\frac{f_k}{\alpha_k} - \frac{f_i}{\alpha_i}\right)\frac{N_k}{\mu} + \beta_i\left(1 + \frac{w\mu}{d}\right)\frac{\sum_{k\in\mathcal{I}} N_k}{\mu}, \tag{18a}$$

$$\beta_i(t^* - \hat{a}(t)) = \beta_i\left(1 + \frac{w\mu}{d}\right)\left(t^* - t - \frac{w}{d}\sum_{k\in\mathcal{I}} N_k\right), \tag{18b}$$

$$\lambda_i wl(t) = \lambda_i \frac{w\mu}{d}\left\{\left(1 + \frac{w\mu}{d}\right)\frac{\sum_{k\in\mathcal{I}} N_k}{\mu} - (t^* - t)\right\}. \tag{18c}$$

Differentiating these costs with respect to d, we have

$$\frac{\mathrm{d}\{\alpha_i q(t)\}}{\mathrm{d}d} = \frac{w\mu}{d^2}\left\{\alpha_i \sum_{k=1}^{i}\left(\frac{\lambda_k - \beta_k}{\alpha_k} - \frac{\lambda_i - \beta_i}{\alpha_i}\right)\frac{N_k}{\mu} - \beta_i \frac{\sum_{k\in\mathcal{I}} N_k}{\mu}\right\}, \tag{19a}$$

$$\frac{\mathrm{d}\{\beta_i(t^* - \hat{a}(t))\}}{\mathrm{d}d} = \beta_i \frac{w\mu}{d^2}\left(t + \frac{w}{d}\sum_{k\in\mathcal{I}} N_k - t^{\mathrm{F}}\right) > 0, \tag{19b}$$

$$\frac{\mathrm{d}\{\lambda_i wl(t)\}}{\mathrm{d}d} = -\lambda_i \frac{w\mu}{d^2}\left(t + \frac{w}{d}\sum_{k\in\mathcal{I}} N_k - t^{\mathrm{F}}\right) < 0. \tag{19c}$$

(19b) and (19c) shows that expanding parking capacity reduces walking time cost but increases schedule delay cost. Furthermore, (19a) indicates that this expansion can increase the travel time cost. This is because the early-bird incentive $-f_i/\alpha_i$ for commuters i decreases with increases in d. Hence, expanded parking capacity may lead to a temporal concentration of traffic demand, thereby exacerbating the bottleneck congestion. These results indicate that expanding parking capacity may also increase the commuting cost c_i^* and the total commuting cost TC^*, which are given by

$$c_i^* = \beta_i \left(1 + \frac{w\mu}{d}\right) \frac{\sum_{k\in\mathcal{I}} N_k}{\mu} - \alpha_i \sum_{k=1}^{i} \left(\frac{f_i}{\alpha_i} - \frac{f_k}{\alpha_k}\right) \frac{N_k}{\mu}, \tag{20a}$$

$$TC^* = \sum_{i\in\mathcal{I}} \beta_i N_i \left(1 + \frac{w\mu}{d}\right) \frac{\sum_{k\in\mathcal{I}} N_k}{\mu} - \sum_{i\in\mathcal{I}} \alpha_i N_i \sum_{k=1}^{i} \left(\frac{f_i}{\alpha_i} - \frac{f_k}{\alpha_k}\right) \frac{N_k}{\mu}. \tag{20b}$$

In fact, we can easily see that such a situation actually exists by differentiating TC^* with respect to d.

The results obtained above can be summarized as follows:

Proposition 2. *The equilibrium distribution of trip timing has the following properties:*

(a) Commuters with smaller f_i/α_i travel earlier and park closer to the CBD.

(b) Suppose without loss of generality that $f_{i-1}/\alpha_{i-1} \leq f_i/\alpha_i$ for all $i \in \mathcal{I}\backslash\{1\}$. Then, the difference in the equilibrium time-based commuting cost between commuters $i-1$ and i is expressed as

$$\frac{c_i^*}{\alpha_i} - \frac{c_{i-1}^*}{\alpha_{i-1}} = \left(\frac{\beta_i}{\alpha_i} - \frac{\beta_{i-1}}{\alpha_{i-1}}\right)(t^* - t^{\mathrm{F}}) - \left(\frac{f_i}{\alpha_i} - \frac{f_{i-1}}{\alpha_{i-1}}\right)(t_{i-1,i} - t^{\mathrm{F}}) \qquad \forall i \in \mathcal{I}\backslash\{1\}. \tag{21}$$

(c) Expansion of the parking capacity d decreases the walking time cost but increases the schedule delay cost of all commuters. Furthermore, this expansion can increase the total travel time cost (i.e., bottleneck congestion) and total commuting cost.

4 Social optimum

This section characterizes the optimal distribution of trip timing and parking locations. We define the social optimum as a state wherein the total commuting cost is minimized. The total commuting cost $TC(\boldsymbol{n}(t,l))$ is represented as a function of the number $\boldsymbol{n}(t,l) = [n_i(t,l)]$ of commuters who travel at time t and park at location l, which we call distribution of trip timing and parking locations:

$$TC(\boldsymbol{n}(t,l)) = \sum_{i \in \mathcal{I}} \iint c_i(\tau,\ell)\, n_i(\tau,\ell)\, \mathrm{d}\tau\, \mathrm{d}\ell. \tag{22}$$

Therefore, the optimal distribution of trip timing and parking locations coincides with a solution to the following problem:

$$\min_{\boldsymbol{n}(t,l)} TC(\boldsymbol{n}(t,l)) \tag{23a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{I}} \int n_i(t,l)\, \mathrm{d}l \leq \mu \quad \forall t \in \mathbb{R}, \quad \sum_{i \in \mathcal{I}} \int n_i(t,l)\, \mathrm{d}t \leq d \quad \forall l \in \mathbb{R}_+, \tag{23b}$$

$$\iint n_i(t,l)\, \mathrm{d}t\, \mathrm{d}l = N_i \quad \forall i \in \mathcal{I}, \quad n_i(t,l) \geq 0 \quad \forall i \in \mathcal{I},\ \forall t \in \mathbb{R}, \forall l \in \mathbb{R}_+. \tag{23c}$$

Because the queue at the bottleneck is completely eliminated at the social optimum as proved in the studies involving standard bottleneck models, the total commuting cost can be rewritten as

$$TC(\boldsymbol{n}(t,l)) = \iint \Big\{\lambda_i w \ell + \beta_i \{t^* - a(\tau,\ell)\}\Big\} n_i(\tau,\ell)\, \mathrm{d}\tau\, \mathrm{d}\ell \tag{24}$$

The Karush–Kuhn–Tucker (KKT) conditions of this problem are given by

$$\begin{cases} c_i^o(t,l) = c_i^o & \text{if} \quad n_i^o(t,l) > 0 \\ c_i^o(t,l) \geq c_i^o & \text{if} \quad n_i^o(t,l) = 0 \end{cases} \quad \forall i \in \mathcal{I},\ \forall t \in \mathbb{R},\ \forall l \in \mathbb{R}_+, \tag{25a}$$

$$\begin{cases} \sum_{i \in \mathcal{I}} \int n_i^o(t,l)\, \mathrm{d}l = \mu & \text{if} \quad p^q(t) > 0 \\ \sum_{i \in \mathcal{I}} \int n_i^o(t,l)\, \mathrm{d}l \leq \mu & \text{if} \quad p^q(t) = 0 \end{cases} \quad \forall t \in \mathbb{R}, \tag{25b}$$

$$\begin{cases} \sum_{i \in \mathcal{I}} \int n_i^o(t,l)\, \mathrm{d}t = d & \text{if} \quad p^p(l) > 0 \\ \sum_{i \in \mathcal{I}} \int n_i^o(t,l)\, \mathrm{d}t \leq d & \text{if} \quad p^p(l) = 0 \end{cases} \quad \forall l \in \mathbb{R}_+, \tag{25c}$$

$$\iint n_i^o(t,l)\, \mathrm{d}t\, \mathrm{d}l = N_i \quad \forall i \in \mathcal{I}. \tag{25d}$$

where $p^q(t)$, $p^p(l)$, and c_i^o are the Lagrange multipliers for the first, second, and third constraints of (23), respectively, and $c_i^o(t,l)$ is defined as

$$c_i^o(t,l) = \lambda_i w l + \beta_i \{t^* - a(t,l)\} + p^q(t) + p^p(l). \tag{26}$$

Let $\mathcal{S}_i^o = \{(t,l) \in \mathbb{R} \times \mathbb{R}_+ \mid c_i^o(t,l) = c_i^o\}$. Then, the condition (25a) leads to the following condition: for any $(t_i, l_i), (\tilde{t}_i, \tilde{l}_i) \in \mathcal{S}_i^o$ such that $t_i < \tilde{t}_i$,

$$c_i^o(t_i, l_i) + c_i^o(\tilde{t}_i, \tilde{l}_i) \leq c_i^o(t_i, \tilde{l}_i) + c_i^o(\tilde{t}_i, l_i) \quad \forall (t_i, l_i), (\tilde{t}_i, \tilde{l}_i) \in \mathcal{S}_i^o,\ \forall i \in \mathcal{I}. \tag{27}$$

Substituting (26) into this condition, we obtain

$$\begin{cases} c_i^o(t_i, l_i) + c_i^o(\tilde{t}_i, \tilde{l}_i) = c_i^o(t_i, \tilde{l}_i) + c_i^o(\tilde{t}_i, l_i) & \text{if} \quad a(\tilde{t}_i, l_i) \leq t^* \\ c_i^o(t_i, l_i) + c_i^o(\tilde{t}_i, \tilde{l}_i) < c_i^o(t_i, \tilde{l}_i) + c_i^o(\tilde{t}_i, l_i) & \text{if} \quad a(\tilde{t}_i, l_i) > t^* \end{cases} \quad \forall (t_i, l_i), (\tilde{t}_i, \tilde{l}_i) \in \mathcal{S}_i^o, \ \forall i \in \mathcal{I}. \tag{28}$$

The condition (28) characterizes the distribution $n_i^o(t, l)$ of trip timing and parking locations for commuters i. The first condition shows the existence of a situation wherein the total commuting cost does not change even if commuters i change their trip timing and/or parking location, i.e., the optimal distribution is not uniquely determined. The second condition indicates that a commuter i who travels later tends to park closer to the CBD as a late arrival incurs prohibitive cost. This property is quite different from that at the equilibrium.

Condition (25a) also gives us the condition that the commuting costs of commuters i and j satisfy

$$\begin{cases} c_i^o(t_i, l_i) + c_j^o(t_j, l_j) \leq c_i^o(t_j, l_i) + c_j^o(t_i, l_j) \\ c_i^o(t_i, l_i) + c_j^o(t_j, l_j) \leq c_i^o(t_i, l_j) + c_j^o(t_j, l_i) \end{cases} \quad \forall (t_i, t_j) \in \mathcal{S}_i^o, \forall (t_j, l_j) \in \mathcal{S}_j^o, \forall i, j \in \mathcal{I}. \tag{29}$$

Substituting (26) into this condition, we have the following lemma.

Lemma 2. *For any $i, j \in \mathcal{I}$, $(t_i, l_i) \in \mathcal{S}_i^o$, and $(t_j, l_j) \in \mathcal{S}_j^o$, the following conditions are satisfied.*

(a) The trip timing t_i and t_j satisfy

$$\begin{cases} (\beta_i - \beta_j)(t_i - t_j) \geq 0 & \textit{if} \quad a(t_i, l_j) \leq t^*, \ a(t_j, l_i) \leq t^*, \\ t_i < t_j & \textit{if} \quad a(t_i, l_j) \leq t^*, \ a(t_j, l_i) > t^*, \\ t_i > t_j & \textit{if} \quad a(t_i, l_j) > t^*, \ a(t_j, l_i) \leq t^*. \end{cases} \tag{30}$$

(b) If $\lambda_i - \beta_i \geq \lambda_j - \beta_j$, then $l_i \leq l_j$.

Proof. See Appendix C. □

This lemma shows the properties of the optimal distribution $\boldsymbol{n}^o(t, l)$ of trip timing and parking locations. The condition (30) suggests that during time interval $[t^{\mathrm{F}}, t^* - wl_{\max}^{\mathrm{C}}]$, commuters travel in order of increasing β_i, where $l_{\max}^{\mathrm{C}}$ is the closest parking location of commuters with the greatest β_i. In the time interval $[t^* - wl_{\max}^{\mathrm{C}}, t^*]$, commuters who park $l < l_{\max}^{\mathrm{C}}$ travel. This result sharply contrasts to the equilibrium case, in which commuters park in order of increasing distance from the CBD (i.e., park outwards).

Lemma 2 (b) shows that a commuter with larger $\lambda_i - \beta_i$ parks closer to the CBD at the social optimum. This implies that a commuter's parking location is affected not only by the value of walking time λ_i but also by the value of schedule delay β_i. Therefore, commuters with high value of walking time may not park close to the CBD if their value of schedule delay is high.

The following proposition summarizes these results.

Proposition 3. *The optimal distribution of trip timing and parking locations has the following properties:*

(a) For any $i \in \mathcal{I}$ and $(t_i, l_i), (\tilde{t}_i, \tilde{l}_i) \in \mathcal{S}_i^o$ such that $t_i \leq \tilde{t}_i$,

$$\begin{cases} c_i^o(t_i, l_i) + c_i^o(\tilde{t}_i, \tilde{l}_i) = c_i^o(t_i, \tilde{l}_i) + c_i^o(\tilde{t}_i, l_i) & \text{if} \quad a(\tilde{t}_i, l_i) \leq t^*, \\ c_i^o(t_i, l_i) + c_i^o(\tilde{t}_i, \tilde{l}_i) < c_i^o(t_i, \tilde{l}_i) + c_i^o(\tilde{t}_i, l_i) & \text{if} \quad a(\tilde{t}_i, l_i) > t^*. \end{cases} \tag{31}$$

(b) For any $i, j \in \mathcal{I}$, $(t_i, l_i) \in \mathcal{S}_i^o$, and $(t_j, l_j) \in \mathcal{S}_j^o$ such that $t_i \leq t_j$,

$$\left\{\beta_i \leq \beta_j \quad \text{and} \quad a(t_j, l_i) \leq t^*\right\} \quad \text{or} \quad t_j + l_i > t^*, \tag{32a}$$

$$\left\{(\lambda_i - \beta_i) - (\lambda_j - \beta_j)\right\}(l_i - l_j) \leq 0. \tag{32b}$$

This proposition shows that the optimal distribution of trip timing and parking locations has quite different properties from the equilibrium one. Therefore, the planner needs to implement policies to achieve the social optimum. In the next section, we will propose some possible policies and examine their effects.

5 Effects of parking fees and congestion tolls

In the previous section, we show that the equilibrium distribution of trip timing and parking locations does not coincide with the optimal one. This is because at the equilibrium, commuters will park outwards and queuing congestion occurs. This implies that the introduction of parking fees and congestion tolls may improve the commuting cost of commuters. Therefore, this section examines the effectiveness of such policies.

5.1 Parking fees

We first consider the situation in which the planner imposes a location-dependent parking fee $p^p(l)$ to equalize demand for parking at location l with supply d (i.e., a competitive parking fee). We assume that under this policy, commuters can choose their parking location before their trip. The commuting cost of a commuter i who travels at time t and parks at location l is then expressed as

$$\begin{aligned} c_i^p(t,l) &= p^p(l) + c_i(t,l) \\ &= p^p(l) + \alpha_i q(t) + \lambda_i w l + \beta_i \{t^* - a(t,l)\}., \end{aligned} \tag{33}$$

Since the parking fee $p^p(l)$ equalizes demand and supply for parking at location l, the equilibrium conditions are given by

$$\begin{cases} c_i^p(t,l) = c_i^{p*} & \text{if} \quad n_i^p(t,l) > 0 \\ c_i^p(t,l) \geq c_i^{p*} & \text{if} \quad n_i^p(t,l) = 0 \end{cases} \quad \forall i \in \mathcal{I},\ \forall t \in \mathbb{R},\ \forall l \in \mathbb{R}_+, \tag{34a}$$

$$\begin{cases} \sum_{i \in \mathcal{I}} \int n_i^p(t,l)\, \mathrm{d}l = \mu & \text{if} \quad q(t) > 0 \\ \sum_{i \in \mathcal{I}} \int n_i^p(t,l)\, \mathrm{d}l \leq \mu & \text{if} \quad q(t) = 0 \end{cases} \quad \forall t \in \mathbb{R}, \tag{34b}$$

$$\begin{cases} \sum_{i \in \mathcal{I}} \int n_i^p(t,l)\, \mathrm{d}t = d & \text{if} \quad p^p(l) > 0 \\ \sum_{i \in \mathcal{I}} \int n_i^p(t,l)\, \mathrm{d}t \leq d & \text{if} \quad p^p(l) = 0 \end{cases} \quad \forall l \in \mathbb{R}_+, \tag{34c}$$

$$\iint n_i^p(t,l)\, \mathrm{d}t\, \mathrm{d}l = N_i \quad \forall i \in \mathcal{I}. \tag{34d}$$

Condition (34a) is the no-arbitrage condition for commuters' choice of trip timing and parking location. (34b) and (34c) represent the capacity constraints of the bottleneck and parking, respectively. Note that (34c) can be interpreted as the market clearing condition. In fact, this condition suggests that if demand $\sum_{i \in \mathcal{I}} \int n_i(t,l)\mathrm{d}t$ for parking at location l equals the supply d, then $p^p(l) \geq 0$. Condition (34d) is conservation law of the population.

Let $\mathcal{S}_i^p = \{(t,l) \in \mathbb{R} \times \mathbb{R}_+ \mid c_i^p(t,l) = c_i^{p*}\}$. Then, (34a) gives us the following conditions on $\boldsymbol{n}^p(t,l)$: for any $i \in \mathcal{I}$, $(t_i, l_i), (\tilde{t}_i, \tilde{l}_i) \in \mathcal{S}_i^p$ such that $t_i < \tilde{t}_i$,

$$c_i^p(t_i, l_i) + c_i^p(\tilde{t}_i, \tilde{l}_i) \leq c_i^p(t_i, \tilde{l}_i) + c_i^p(\tilde{t}_i, l_i). \tag{35}$$

Substituting (33) into (35), we obtain

$$\begin{cases} c_i^p(t_i,l_i)+c_i^p(\tilde{t}_i,\tilde{l}_i)=c_i^p(t_i,\tilde{l}_i)+c_i^p(\tilde{t}_i,l_i) & \text{if} \quad a(\tilde{t}_i,l_i)\le t^* \\ c_i^p(t_i,l_i)+c_i^p(\tilde{t}_i,\tilde{l}_i)<c_i^p(t_i,\tilde{l}_i)+c_i^p(\tilde{t}_i,l_i) & \text{if} \quad a(\tilde{t}_i,l_i)> t^* \end{cases} \quad \forall(t_i,l_i),(\tilde{t}_i,\tilde{l}_i)\in\mathcal{S}_i^p,\ \forall i\in\mathcal{I}. \tag{36}$$

These conditions coincide with (28), which implies that the properties of the equilibrium distribution $n_i^p(t,l)$ of trip timing and parking locations of commuters i are the same as those at the social optimum.

Condition (34a) also gives the following conditions that should be satisfied at the equilibrium with the existence of parking fees:

$$\begin{cases} \dfrac{c_i^p(t_i,l_i)}{\alpha_i}+\dfrac{c_j^p(t_j,l_j)}{\alpha_j}\le\dfrac{c_i^p(t_j,l_i)}{\alpha_i}+\dfrac{c_j^p(t_i,l_j)}{\alpha_j} \\ c_i^p(t_i,l_i)+c_j^p(t_j,l_j)\le c_i^p(t_i,l_j)+c_j^p(t_j,l_i) \end{cases} \quad \forall(t_i,l_i)\in\mathcal{S}_i^p,\ \forall(t_j,l_j)\in\mathcal{S}_j^p,\ \forall i,j\in\mathcal{I}, \tag{37}$$

Substituting (33) into (37) yields the following lemma.

Lemma 3. *For any $i,j\in\mathcal{I}$, $(t_i,l_i)\in\mathcal{S}_i^p$, and $(t_j,l_j)\in\mathcal{S}_j^p$, the following conditions hold at the equilibrium with the existence of parking fees:*

(a) The trip timings t_i and t_j satisfy

$$\begin{cases} \left(\dfrac{\beta_i}{\alpha_i}-\dfrac{\beta_j}{\alpha_j}\right)(t_i-t_j)\ge 0 & \text{if} \quad a(t_i,l_j)\le t^*,\ a(t_j,l_i)\le t^*, \\ t_i<t_j & \text{if} \quad a(t_i,l_j)\le t^*,\ a(t_j,l_i)> t^*, \\ t_i>t_j & \text{if} \quad a(t_i,l_j)> t^*,\ a(t_j,l_i)\le t^*. \end{cases} \tag{38}$$

(b) If $\lambda_i-\beta_i\ge\lambda_j-\beta_j$, then $l_i\le l_j$.

Proof. Similar to the proof of Lemma 2. □

This lemma characterizes the equilibrium distribution of trip timing and parking locations with the existence of parking fees. Lemma 3 (a) shows that commuters with smaller β_i/α_i travel earlier during the time interval $[t^{\mathrm{F}},t^*-wl_{\max}^{\mathrm{C}}]$ and that commuters parking at $l\le l_{\max}^{\mathrm{C}}$ travel during $[t^*-wl_{\max}^{\mathrm{C}},t^*]$, where $l_{\max}^{\mathrm{C}}$ is the closest parking location for commuters with the greatest β_i/α_i. Therefore, in general, the optimal distribution of trip timing cannot be achieved only by imposing a parking fee. In contrast, Lemma 3 (b) coincides with Lemma 2 (b), which implies that the existence of parking fees leads to the optimal distribution of parking locations.[11]

In order to investigate the properties of the equilibrium commuting cost c_i^{p*}, we consider the equilibrium trip cost τ_i^{p*} and equilibrium parking cost ρ_i^{p*}, which are defined as

$$c_i^{p*}=\tau_i^{p*}+\rho_i^{p*}, \tag{39a}$$

$$\tau_i^{p*}=\alpha_i q(t_i)+\beta_i(t^*-t_i), \tag{39b}$$

$$\rho_i^{p*}=p^p(l_i)+(\lambda_i-\beta_i)wl_i, \tag{39c}$$

[11] This property has been shown in Arnott et al. (1991), which considers homogeneous commuters.

where $(t_i, l_i) \in \mathcal{S}_i^p$. By using t_{ij} such that $(t_{ij}, l_i) \in \mathcal{S}_i^p$ and $(t_{ij}, l_j) \in \mathcal{S}_j^p$ and l_{ij} such that $(t_i, l_{ij}) \in \mathcal{S}_i^p$ and $(t_j, l_{ij}) \in \mathcal{S}_j^p$, we have

$$\frac{\tau_i^{p*}}{\alpha_i} - \frac{\tau_j^{p*}}{\alpha_j} = \left(\frac{\beta_i}{\alpha_i} - \frac{\beta_j}{\alpha_j}\right)(t^* - t_{ij}). \tag{40a}$$

$$\rho_i^{p*} - \rho_j^{p*} = \left\{(\lambda_i - \beta_i) - (\lambda_j - \beta_j)\right\} w l_{ij}. \tag{40b}$$

(40a) shows that if $\beta_i/\alpha_i > \beta_j/\alpha_j$, then $\tau_i^{p*}/\alpha_i \geq \tau_j^{p*}/\alpha_j$. That is, a commuter traveling later incurs a higher time-based equilibrium trip cost. (40b) indicates that a commuter who parks closer to the CBD incurs a higher equilibrium parking cost.

The results obtained above are summarized as follows.

Proposition 4. *The equilibrium distribution $\boldsymbol{n}^p(t, l) = [n_i^p(t, l)]$ of trip timing and parking locations, the equilibrium trip cost $[\tau_i^{p*}]$, and the equilibrium parking cost $[\rho_i^{p*}]$ with the existence of parking fees satisfy the following conditions:*

(a) For any $i \in \mathcal{I}$, $(t_i, l_i), (\tilde{t}_i, \tilde{l}_i) \in \mathcal{S}_i^p$ such that $t_i < \tilde{t}_i$,

$$\begin{cases} c_i^p(t_i, l_i) + c_i^p(\tilde{t}_i, \tilde{l}_i) = c_i^p(t_i, \tilde{l}_i) + c_i^p(\tilde{t}_i, l_i) & \text{if} \quad a(\tilde{t}_i, l_i) \leq t^*, \\ c_i^p(t_i, l_i) + c_i^p(\tilde{t}_i, \tilde{l}_i) < c_i^p(t_i, \tilde{l}_i) + c_i^p(\tilde{t}_i, l_i) & \text{if} \quad a(\tilde{t}_i, l_i) > t^*. \end{cases} \tag{41}$$

(b) For any $i, j \in \mathcal{I}$, $(t_i, l_i) \in \mathcal{S}_i^p$, and $(t_j, l_j) \in \mathcal{S}_j^p$ such that $t_i \leq t_j$,

$$\left\{\frac{\beta_i}{\alpha_i} \leq \frac{\beta_j}{\alpha_j} \quad \text{and} \quad a(t_j, l_i) \leq t^*\right\} \quad \text{or} \quad t_j + l_i > t^*, \tag{42a}$$

$$\left\{(\lambda_i - \beta_i) - (\lambda_j - \beta_j)\right\}(l_i - l_j) \leq 0. \tag{42b}$$

(c) A commuter traveling later incurs a higher equilibrium time-based trip cost.

(d) A commuter parking closer to the CBD incurs a higher equilibrium parking cost.

This proposition suggests that the optimal distribution of parking locations is achieved by introducing parking fees. However, we should note that because $\beta_i/\alpha_i > f_i/\alpha_i$ for all $i \in \mathcal{I}$, this policy reduces the early-bird incentives for all commuters. This may lead to a temporal concentration of traffic demand, thereby exacerbating the bottleneck congestion. In Section 6, we will observe that such a situation actually exists. This fact suggests that parking policies may in fact exacerbate traffic congestion and should be implemented along with a measure aimed to alleviate peak congestion.

5.2 Congestion tolls

We next examine the effects of a time-varying congestion toll $p^q(t)$. This toll $p^q(t)$ completely eliminates the bottleneck congestion[12] but does not affect commuters' parking location preferences. Thus, under this policy, commuters park outwards. It is noteworthy that since we consider

[12]Note that the tradable bottleneck permits proposed by Akamatsu (2007) and Wada and Akamatsu (2013) have the same effect as the congestion toll.

heterogeneous commuters, the congestion toll $p^q(t)$ does not equal the travel time cost $\alpha_i q(t)$ at the equilibrium without pricing policies and is set so that the travel demand $\sum_{i\in\mathcal{I}} \hat{n}_i(t)$ equals the supply (capacity) μ. Therefore, the equilibrium is defined as a state that satisfies the following conditions:

$$\begin{cases} \hat{c}_i^q(t) = c_i^{q*} & \text{if} \quad \hat{n}_i^q(t) > 0 \\ \hat{c}_i^q(t) \geq c_i^{q*} & \text{if} \quad \hat{n}_i^q(t) = 0 \end{cases} \quad \forall i \in \mathcal{I},\ \forall t \in \mathbb{R}, \tag{43a}$$

$$\begin{cases} \sum_{i\in\mathcal{I}} \hat{n}_i^q(t) = \mu & \text{if} \quad p^q(t) > 0 \\ \sum_{i\in\mathcal{I}} \hat{n}_i^q(t) \leq \mu & \text{if} \quad p^q(t) = 0 \end{cases} \quad \forall t \in \mathbb{R}, \tag{43b}$$

$$\int \hat{n}_i^q(t)\, \mathrm{d}t = N_i \quad \forall i \in \mathcal{I}, \tag{43c}$$

where $\hat{c}_i^q(t)$ is the commuting cost of a commuter i traveling at time t, which is given by

$$\hat{c}_i^q(t) = p^q(t) + \lambda_i \frac{w\mu}{d}(t - t^{\mathrm{F}}) + \beta_i\{t^* - \hat{a}(t)\}. \tag{44}$$

c_i^{q*} denotes the equilibrium commuting cost under the congestion toll.

Condition (43a) is the no-arbitrage condition for commuters' trip timing choices. (43b) denotes the bottleneck capacity constraints, which ensure that the bottleneck congestion is completely eliminated at the equilibrium. Condition (43c) provides the flow conservation for commuting demand.

We can easily show that equilibrium conditions (43) are equivalent to the following optimization problem:

Proposition 5. *The equilibrium distribution $\hat{\boldsymbol{n}}^q(t) = [\hat{n}_i^q(t)]$ of trip timing with the existence of congestion tolls coincides with the solution of the following linear programming problem:*

$$\min_{\hat{\boldsymbol{n}}(t)} \sum_{i\in\mathcal{I}} \int \left\{ \lambda_i \frac{w\mu}{d}(\tau - t^{\mathrm{F}}) + \beta_i\{t^* - \hat{a}(\tau)\} \right\} \hat{n}_i(\tau)\, \mathrm{d}\tau \tag{45a}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{I}} \hat{n}_i(t) \leq \mu \quad \forall t \in \mathbb{R}, \qquad \int \hat{n}_i(t)\, \mathrm{d}t = N_i \quad \forall i \in \mathcal{I}, \quad \hat{n}_i(t) \geq 0 \quad \forall i \in \mathcal{I},\ \forall t \in \mathbb{R}. \tag{45b}$$

Proof. Similar to the proof of Proposition 1. □

This proposition suggests that the sum of total schedule delay cost and total walking time cost, which equals the total commuting cost minus total toll revenue, is minimized at the equilibrium. Hence, the congestion toll achieves the social optimum under condition (2) (i.e., parking spaces are occupied in strict order from the CBD). Furthermore, by comparing Propositions 1 and 5, we can see that commuters with large α_i (rich commuters) will gain a great deal from the congestion toll, but commuters with small α_i (poor commuters) may lose.

Let $\hat{\mathcal{S}}_i^q = \{t \in \mathbb{R} \mid \hat{c}_i^q(t) = c_i^{q*}\}$. Then, as in Section 3, condition (43a) gives the following condition that is satisfied at the equilibrium with the existence of congestion tolls:

$$\hat{c}_i^q(t_i) + \hat{c}_j^q(t_j) \leq \hat{c}_i^q(t_j) + \hat{c}_j^q(t_i) \quad \forall t_i \in \hat{\mathcal{S}}_i^q,\ \forall t_j \in \hat{\mathcal{S}}_j^q,\ \forall i, j \in \mathcal{I}. \tag{46}$$

Substituting (44) into (46), we have

$$\left(f_i - f_j\right)(t_i - t_j) \geq 0 \quad \forall t_i \in \hat{\mathcal{S}}_i^q,\ \forall t_j \in \hat{\mathcal{S}}_j^q,\ \forall i, j \in \mathcal{I}. \tag{47}$$

This condition shows that commuters travel in order of increasing f_i and that the value of the travel time α_i does not affect the equilibrium distribution $\hat{\boldsymbol{n}}^q(t)$ of trip timing. Furthermore, since commuters park outwards in this case, the congestion toll cannot significantly improve the distribution of parking locations.

We suppose in this section that $f_{i-1} \leq f_i$ for all $i \in \mathcal{I}\backslash\{1\}$ without loss of generality. Then, by using $t_{i-1,i}$ such that $t_{i-1,i} \in \hat{\mathcal{S}}_{i-1}^q \cap \hat{\mathcal{S}}_i^q$, we can easily show that the equilibrium commuting cost c_i^{q*} satisfies

$$c_i^{q*} - c_{i-1}^{q*} = (\beta_i - \beta_{i-1})(t^* - t^{\mathrm{F}}) - (f_i - f_{i-1})(t_{i-1,i} - t^{\mathrm{F}}), \tag{48a}$$

$$t_{i-1,i} = t^{\mathrm{F}} + \frac{\sum_{k=1}^{i-1} N_k}{\mu}, \tag{48b}$$

$$t^{\mathrm{F}} = t^* - \left(1 + \frac{w\mu}{d}\right)\frac{\sum_{k\in\mathcal{I}} N_k}{\mu}. \tag{48c}$$

This implies that similar to the equilibrium with no pricing policies, the values of f_i, β_i, and $N1/\mu$ determine the sign of the difference $c_i^{q*} - c_{i-1}^{q*}$. Hence, if $\beta_{i-1} \geq \beta_i$ for all $i \in \mathcal{I}\backslash\{1\}$, then the equilibrium commuting cost of a commuter $i-1$ is higher than that of a commuter i, i.e., a commuter who travels earlier incurs a higher commuting cost.

The equilibrium commuting cost c_i^{q*} of a commuter i is obtained from (48) as

$$c_i^{q*} = \beta_i\left(1 + \frac{w\mu}{d}\right)\frac{\sum_{k\in\mathcal{I}} N_k}{\mu} - \sum_{k=1}^{i}(f_i - f_k)\frac{N_k}{\mu}. \tag{49}$$

Therefore, if $f_{i-1}/\alpha_{i-1} \geq f_i/\alpha_i$ (i.e., $\alpha_{i-1} \leq \alpha_i$) for all $i \in \mathcal{I}\backslash\{1\}$, we have

$$c_i^{q*} - c_i^* = \alpha_i \sum_{k=i}^{I}\left(\frac{f_i}{\alpha_i} - \frac{f_k}{\alpha_k}\right)\frac{N_k}{\mu} - \sum_{k=1}^{i}(f_i - f_k)\frac{N_k}{\mu}. \tag{50}$$

This indicates that a value of $i^q \in \mathcal{I}\backslash\{1\}$ exists such that $c_k^{q*} - c_k^* \geq 0$ for all $k \in \{1, \cdots, i^q - 1\}$ and $c_k^{q*} - c_k^* \leq 0$ for all $k \in \{i^q, \cdots, I\}$, which implies that rich commuters gain but poor commuters lose from the imposition of a congestion toll.

The total commuting cost TC^q and the total toll revenue P^q are obtained as

$$TC^q = \sum_{i\in\mathcal{I}}\left\{\beta_i N_i\left(1 + \frac{w\mu}{d}\right)\frac{\sum_{k\in\mathcal{I}} N_k}{\mu} - N_i\sum_{k=1}^{i}(f_i - f_k)\frac{N_k}{\mu}\right\}, \tag{51a}$$

$$P^q = \sum_{i\in\mathcal{I}} N_i\left\{\sum_{k=1}^{i} f_k\frac{N_k}{\mu} - f_i\frac{N_i}{2\mu}\right\}. \tag{51b}$$

This shows that the expansion of the parking capacity d may increase the total commuting cost TC^q but must decrease $TC^q - P^q$. That is, the expansion of the parking capacity reduces the total commuting cost if the planner redistributes the toll revenue.

We summarize the results obtained above in the following proposition.

Proposition 6. *The equilibrium distribution of trip timing with the existence of congestion tolls has the following properties:*

(a) Commuters with smaller f_i travel earlier and park closer to the CBD.

(b) Suppose without loss of generality that $f_{i-1} \leq f_i$ for all $i \in \mathcal{I}\backslash\{1\}$. Then, the difference in the equilibrium time-based commuting cost between commuters $i-1$ and i is expressed as

$$c_i^{q*} - c_{i-1}^{q*} = (\beta_i - \beta_{i-1})(t^* - t^{\mathrm{F}}) - (f_i - f_{i-1})(t_{i-1,i} - t^{\mathrm{F}}) \qquad \forall i \in \mathcal{I}\backslash\{1\}. \tag{52}$$

(c) If the planner redistributes the toll revenue, the expansion of the parking capacity d must decrease the total commuting cost; otherwise, the expansion may increase the total commuting cost.

5.3 Combination of parking fees and congestion tolls

We finally consider the effects of introducing parking fees and congestion tolls. In this case, the distribution $\boldsymbol{n}^{pq}(t,l) = [n_i^{pq}(t,l)]$ of trip timing and parking locations, the congestion toll $p^q(t)$, the parking fee $p^p(l)$, and the commuting cost c_i^{pq} at the equilibrium are obtained from the following conditions:

$$\begin{cases} c_i^{pq}(t,l) = c_i^{pq*} & \text{if} \quad n_i^{pq}(t,l) > 0 \\ c_i^{pq}(t,l) \geq c_i^{pq*} & \text{if} \quad n_i^{pq}(t,l) = 0 \end{cases} \quad \forall i \in \mathcal{I},\ \forall t \in \mathbb{R},\ \forall l \in \mathbb{R}_+, \tag{53a}$$

$$\begin{cases} \sum_{i\in\mathcal{I}} \int n_i^{pq}(t,l)\, \mathrm{d}l = \mu & \text{if} \quad p^q(t) > 0 \\ \sum_{i\in\mathcal{I}} \int n_i^{pq}(t,l)\, \mathrm{d}l \leq \mu & \text{if} \quad p^q(t) = 0 \end{cases} \quad \forall t \in \mathbb{R}, \tag{53b}$$

$$\begin{cases} \sum_{i\in\mathcal{I}} \int n_i^{pq}(t,l)\, \mathrm{d}t = d & \text{if} \quad p^p(l) > 0 \\ \sum_{i\in\mathcal{I}} \int n_i^{pq}(t,l)\, \mathrm{d}t \leq d & \text{if} \quad p^p(l) = 0 \end{cases} \quad \forall l \in \mathbb{R}_+, \tag{53c}$$

$$\iint n_i^{pq}(t,l)\, \mathrm{d}t\, \mathrm{d}l = N_i \quad \forall i \in \mathcal{I}, \tag{53d}$$

where $c_i^{pq}(t,l)$ is the commuting cost of a commuter i who travels at time t and parks at location l

$$c_i^{pq}(t,l) = p^q(t) + p^p(l) + \lambda_i w l + \beta_i\{t^* - a(t,l)\}. \tag{54}$$

Because conditions (53) coincide with (25), the optimal distribution of trip timing and parking locations is achieved by introducing parking fees and congestion tolls.

Let $\mathcal{S}_i^{pq} = \{(t,l) \in \mathbb{R}\times\mathbb{R}_+ \mid c_i^{pq}(t,l) = c_i^{pq*}\}$. We then examine the properties of the equilibrium trip cost τ_i^{pq*} and equilibrium parking cost ρ_i^{pq*}, which are defined as

$$c_i^{pq*} = \tau_i^{pq*} + \rho_i^{pq*}, \tag{55a}$$

$$\tau_i^{pq*} = p^q(t_i) + \beta_i(t^* - t_i), \tag{55b}$$

$$\rho_i^{pq*} = p^p(l_i) + (\lambda_i - \beta_i) w l_i, \tag{55c}$$

where $(t_i, l_i) \in \mathcal{S}_i^{pq}$. By using t_{ij} such that $(t_{ij}, l_i) \in \mathcal{S}_i^{pq}$ and $(t_{ij}, l_j) \in \mathcal{S}_j^{pq}$ and l_{ij} such that $(t_i, l_{ij}) \in \mathcal{S}_i^{pq}$

and $(t_j, l_{ij}) \in \mathcal{S}_j^{pq}$, we have $\tau_i^{pq*} - \tau_j^{pq*}$ and $\rho_i^{pq*} - \rho_j^{pq*}$ as follows

$$\tau_i^{pq*} - \tau_j^{pq*} = \left(\beta_i - \beta_j\right)(t^* - t_{ij}). \tag{56a}$$

$$\rho_i^{pq*} - \rho_j^{pq*} = \left\{(\lambda_i - \beta_i) - (\lambda_j - \beta_j)\right\} w l_{ij}. \tag{56b}$$

Condition (56a) shows that a commuter traveling later incurs a higher time-based equilibrium trip cost. Condition (56b) indicates that a commuter who parks closer to the CBD incurs a higher equilibrium parking cost.

These results establish the following proposition.

Proposition 7. *The equilibrium distribution of trip timing and parking locations, the equilibrium trip cost* $[\tau_i^{pq*}]$*, and the equilibrium parking cost* $[\rho_i^{pq*}]$ *with the existence of parking fees and congestion tolls have the following properties.*

(a) The equilibrium distribution of trip timing and parking locations coincides with the optimal one.

(b) A commuter traveling later incurs a higher equilibrium trip cost.

(c) A commuter parking closer to the CBD incurs a higher equilibrium parking cost.

The effects of the parking fee and congestion toll on the distribution of trip timing and parking locations are summarized in Table 1. The results obtained in this section demonstrate that introducing pricing policies significantly changes the distribution of trip timing and parking locations, thereby leading to complex changes in commuters' commuting costs. As an example, we consider a case where for all $i \in \mathcal{I}\backslash\{1\}$,

$$f_{i-1} > f_i, \quad \beta_{i-1} > \beta_i, \quad \lambda_{i-1} - \beta_{i-1} > \lambda_i - \beta_i, \quad \alpha_{i-1} > \alpha_i, \tag{57a}$$

$$\frac{f_{i-1}}{\alpha_{i-1}} < \frac{f_i}{\alpha_i}, \quad \frac{\beta_{i-1}}{\alpha_{i-1}} < \frac{\beta_i}{\alpha_i}, \quad \frac{\lambda_{i-1} - \beta_{i-1}}{\alpha_{i-1}} < \frac{\lambda_i - \beta_i}{\alpha_i}. \tag{57b}$$

That is, commuters with smaller (larger) i are rich (poor). In this case, Propositions 2, 4, 6, and 7 give us the difference in the commuting costs between commuters $i-1$ and i for all $i \in \mathcal{I}\backslash\{1\}$ as

$$\frac{c_i^*}{\alpha_i} > \frac{c_{i-1}^*}{\alpha_{i-1}}, \tag{58a}$$

$$\frac{\tau_i^{p*}}{\alpha_i} > \frac{\tau_{i-1}^{p*}}{\alpha_{i-1}}, \qquad \rho_i^{p*} < \rho_{i-1}^{p*}, \tag{58b}$$

$$c_i^{q*} < c_{i-1}^{q*}, \tag{58c}$$

$$\tau_i^{pq*} < \tau_{i-1}^{pq*}, \qquad \rho_i^{p*} < \rho_{i-1}^{p*}. \tag{58d}$$

This shows that the effect of each pricing policy on each type of commuter can differ quite radically. Hence, the planner should understand these mechanisms to ensure that any implemented road/parking policy will be effective.

We have shown that combining parking fees with congestion tolls achieves the social optimum. However, some commuters may be worse off compared with the no-pricing equilibrium if the planner does not appropriately redistribute the revenue earned from the pricing of road and parking. Therefore, as a benchmark of appropriate redistribution, we propose that the revenue is used for financing the bottleneck and parking capacity as in Mohring and Harwitz (1962), Strotz

Table 1: Equilibrium distribution of trip timing and parking locations with/without policies

policy	the order of trip timing	the order of parking location
no-pricing policies	ascending order of f_i/α_i	ascending order of f_i/α_i
parking fee	ascending order of β_i/α_i *	descending order of $\lambda_i - \beta_i$
congestion toll	ascending order of f_i	ascending order of f_i
parking fee + congestion toll	ascending order of β_i *	descending order of $\lambda_i - \beta_i$

* $t \in [t^{\mathrm{F}}, t^* - wl^{\mathrm{C}}_{\max}]$

(1965), Keeler and Small (1977), Arnott et al. (1993), Arnott and Kraus (1995), Yang and Meng (2002), Akamatsu (2007), and Verhoef (2007).

We consider that the planner chooses a parking and bottleneck capacity, d and μ, so as to minimize the social cost, which is defined as the sum of the total commuting cost $TC(\boldsymbol{n}(t,l))$ defined in (24), parking capacity cost $K^p(d)$, and bottleneck capacity cost $K^q(\mu)$. Therefore, the optimal parking and bottleneck capacity, d^o and μ^o, is obtained from the following problem:

$$\min_{\boldsymbol{n}(t,l),d,\mu} TC(\boldsymbol{n}(t,l)) + K^p(d) + K^q(\mu) \quad \text{s.t.} \quad (23\text{b}), (23\text{c}). \tag{59}$$

The capacity costs $K^p(d)$ and $K^q(\mu)$ are assumed to be homogeneous of degree one in d and μ, respectively.

The KKT conditions of this problem are given by (25) and the following two conditions

$$\begin{cases} \dfrac{\mathrm{d}K^q(\mu)}{\mathrm{d}\mu} - \displaystyle\int p^q(\tau)\mathrm{d}\tau = 0 & \text{if} \quad \mu > 0, \\ \dfrac{\mathrm{d}K^q(\mu)}{\mathrm{d}\mu} - \displaystyle\int p^q(\tau)\mathrm{d}\tau \geq 0 & \text{if} \quad \mu = 0, \end{cases} \tag{60a}$$

$$\begin{cases} \dfrac{\mathrm{d}K^p(d)}{\mathrm{d}d} - \displaystyle\int p^p(\ell)\mathrm{d}\ell = 0 & \text{if} \quad d > 0, \\ \dfrac{\mathrm{d}K^p(d)}{\mathrm{d}d} - \displaystyle\int p^q(\tau)\mathrm{d}\ell \geq 0 & \text{if} \quad d = 0. \end{cases} \tag{60b}$$

Since $K^q(\mu)$ and $K^p(d)$ are homogeneous of degree 1, the conditions (60) yield

$$K^q(\mu^o) = \mu^o \int p^q(t)\mathrm{d}t, \tag{61a}$$

$$K^p(d^o) = d^o \int p^p(l)\mathrm{d}l. \tag{61b}$$

This leads to the following proposition.

Proposition 8. *Suppose that the parking and bottleneck capacity costs, $K^p(d)$ and $K^q(\mu)$, are homogeneous of degree one in d and μ, respectively. Then, parking fee revenues equal the cost $K^p(d^o)$ for the optimal parking capacity d^o, and congestion toll revenues equal the cost $K^q(\mu^o)$ for the optimal bottleneck capacity μ^o.*

This proposition shows that *the self-financing result holds in our model.*

6 A simple example

This section analyzes the model in a simple setting to show concretely properties of equilibrium and effects of pricing policies. We assume that commuters are divided in two groups (i.e., $I = 2$) and that $N_1 = N_2 = N/2$. In addition, the parameters $\alpha_i, \beta_i, \lambda_i$ are assumed to satisfy

$$\frac{f_1}{\alpha_1} < \frac{f_2}{\alpha_2}, \qquad \frac{\beta_1}{\alpha_1} < \frac{\beta_2}{\alpha_2}, \qquad \frac{\lambda_1 - \beta_1}{\alpha_1} < \frac{\lambda_2 - \beta_2}{\alpha_2}, \tag{62a}$$

$$f_1 > f_2, \qquad \beta_1 > \beta_2, \qquad \lambda_1 - \beta_1 > \lambda_2 - \beta_2. \tag{62b}$$

Note that this assumption gives

$$\alpha_1 > \alpha_2, \quad \lambda_1 > \lambda_2, \quad \beta_1 - \beta_2 > f_1 - f_2. \tag{63}$$

Both (62) and (63) imply that commuters 1 and 2 can be considered rich and poor commuters, respectively.

6.1 Equilibrium with no pricing policies

We first consider a situation in which neither parking fees nor congestion tolls are imposed. Let t_i^{F} and t_i^{L} be the first and last trip timing of commuters i, respectively. Then, Proposition 2 and (62) gives t_i^{F} and t_i^{L} as follows:

$$t_1^{\mathrm{F}} = t^{\mathrm{F}} = t^* - \left(1 + \frac{w\mu}{d}\right)\frac{N}{\mu}, \qquad t_1^{\mathrm{L}} = t_2^{\mathrm{F}} = t^* - \frac{w}{d}N - \frac{N}{2\mu}, \qquad t_2^{\mathrm{L}} = t^{\mathrm{L}} = t^* - \frac{w}{d}N. \tag{64}$$

It follows from (6a) and this that the equilibrium commuting cost c_i^* and queuing time $q(t)$ are obtained as

$$c_1^* = f_1\frac{N}{\mu} + \lambda_1\frac{w}{d}N, \qquad c_2^* = \alpha_2\left(\frac{f_1}{\alpha_1} + \frac{f_2}{\alpha_2}\right)\frac{N}{2\mu} + \lambda_2\frac{w}{d}N, \tag{65a}$$

$$q(t) = \begin{cases} \dfrac{f_1}{\alpha_1}(t - t_1^{\mathrm{F}}) & \text{if} \quad t \in [t_1^{\mathrm{F}}, t_1^{\mathrm{L}}], \\ \dfrac{f_2}{\alpha_2}(t - t_2^{\mathrm{F}}) + \dfrac{f_1}{\alpha_1}\dfrac{N}{2\mu} & \text{if} \quad t \in [t_2^{\mathrm{F}}, t_2^{\mathrm{L}}]. \end{cases} \tag{65b}$$

Figure 3 illustrates the queuing time $q(t)$. Thus, total queuing time Q^* and total commuting cost TC^* at the equilibrium are given by

$$Q^* = \left[3\frac{f_1}{\alpha_1} + \frac{f_2}{\alpha_2}\right]\frac{N^2}{8\mu}, \tag{66a}$$

$$TC^* = \left[\left(2 + \frac{\alpha_2}{\alpha_1}\right)f_1 + f_2 + 2\,(\lambda_1 + \lambda_2)\,\frac{w\mu}{d}\right]\frac{N^2}{4\mu}. \tag{66b}$$

Differentiating (65) and (66) with respect to d, we have

$$\frac{\mathrm{d}c_1^*}{\mathrm{d}d} = -\beta_1\frac{w}{d^2}N < 0, \tag{67a}$$

$$\frac{\mathrm{d}c_2^*}{\mathrm{d}d} = \left\{\frac{\alpha_2}{\alpha_1}\lambda_1 - \lambda_2 - \frac{\alpha_2}{\alpha_1}\beta_1 - \beta_2\right\}\frac{w}{2d^2}N, \tag{67b}$$

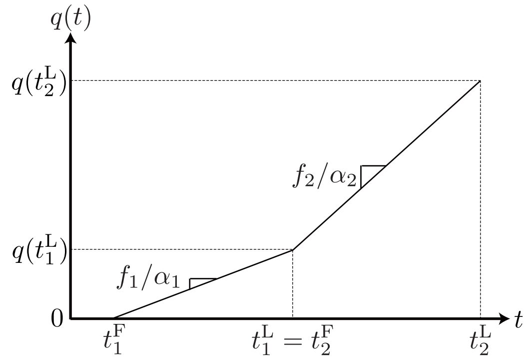


Figure 3: Equilibrium queuing time $q(t)$

$$\frac{\mathrm{d}Q^*}{\mathrm{d}d} = \left\{3\frac{\lambda_1 - \beta_1}{\alpha_1} + \frac{\lambda_2 - \beta_2}{\alpha_2}\right\} \frac{w}{8d^2} N^2 > 0, \tag{67c}$$

$$\frac{\mathrm{d}TC^*}{\mathrm{d}d} = \left\{\frac{\alpha_2}{\alpha_1}\lambda_1 - \lambda_2 - \left(2 + \frac{\alpha_2}{\alpha_1}\right)\beta_1 - \beta_2\right\} \frac{w}{4d^2} N^2. \tag{67d}$$

(67) indicates that expanding parking capacity d exacerbates traffic congestion. Furthermore, if $\lambda_1 > (\lambda_2 + 2\beta_1 + \beta_2)\alpha_1/\alpha_2 + \beta_1$, then expanding parking capacity also increases the total commuting cost.

6.2 Equilibrium with parking fees

We next examine the effects of instituting parking fees. Let l_i^{C} and l_i^{F} be the parking location for commuters i closest to and farthest away from the CBD, respectively. Then, t_i^{F}, t_i^{L}, l_i^{C}, and l_i^{F} are obtained from Proposition 4 and (62):

$$t_1^{\mathrm{F}} = t^{\mathrm{F}} = t^* - \frac{N}{\mu}, \qquad t_2^{\mathrm{F}} = t^* - \left(1 + \frac{w\mu}{d}\right)\frac{N}{2\mu}, \qquad t_2^{\mathrm{L}} = t^* - \frac{w}{2d}N, \qquad t_1^{\mathrm{L}} = t^{\mathrm{L}} = t^*, \tag{68a}$$

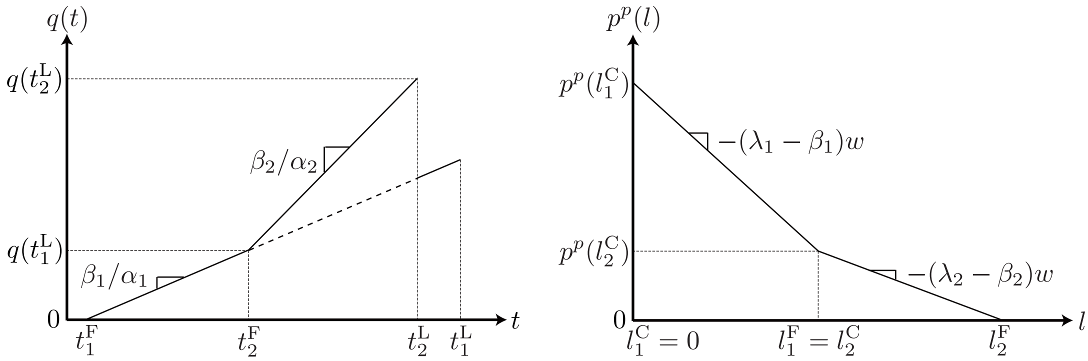
$$l_2^{\mathrm{F}} = \frac{N}{d}, \qquad l_2^{\mathrm{C}} = l_1^{\mathrm{F}} = \frac{N}{2d}, \qquad l_1^{\mathrm{C}} = 0. \tag{68b}$$

Condition (34a) and this give us the equilibrium commuting cost c_i^p, queuing time $q(t)$, and parking fee $p^p(l)$ as follows (Figure 4):

$$c_1^p = (3\beta_1 + \beta_2 - f_1 - f_2)\frac{N}{2\mu}, \qquad c_2^p = \left\{\left(1 - \frac{w\mu}{d}\right)\frac{\alpha_2}{\alpha_1}\beta_1 + \left(3 + \frac{w\mu}{d}\right)\beta_2 - 2f_2\right\}\frac{N}{2\mu}, \tag{69a}$$

$$q(t) = \begin{cases} \dfrac{\beta_1}{\alpha_1}(t - t_1^{\mathrm{F}}) & \text{if} \quad t \in [t_1^{\mathrm{F}}, t_2^{\mathrm{F}}], [t_2^{\mathrm{L}}, t_1^{\mathrm{L}}], \\ \dfrac{\beta_1}{\alpha_1}\dfrac{N}{2\mu} + \dfrac{\beta_2}{\alpha_2}(t - t_2^{\mathrm{F}}) & \text{if} \quad t \in [t_2^{\mathrm{F}}, t_2^{\mathrm{L}}], \end{cases} \tag{69b}$$

$$p^p(l) = \begin{cases} (\lambda_1 - \beta_1)w(l_1^{\mathrm{F}} - l) + (\lambda_2 - \beta_2)w\dfrac{N}{2d} & \text{if} \quad l \in [l_1^{\mathrm{C}}, l_1^{\mathrm{F}}], \\ (\lambda_2 - \beta_2)w(l_2^{\mathrm{F}} - l) & \text{if} \quad l \in [l_2^{\mathrm{C}}, l_2^{\mathrm{F}}], \end{cases} \tag{69c}$$



(a) Queuing time $q(t)$ (b) Parking fee $p^p(t)$

Figure 4: Equilibrium with parking fees

Thus, we can easily see the effects from imposing parking fees upon commuting cost, total queuing time, and total commuting cost as

$$c_1^{p*} - c_1^* = \{3(\lambda_1 - \beta_1) - (\lambda_2 + \beta_2)\} \frac{w}{2d} N, \tag{70a}$$

$$c_2^{p*} - c_2^* = \left\{ \frac{\alpha_2}{\alpha_1}(\lambda_1 - 2\beta_1) + (\lambda_2 - 2\beta_2) \right\} \frac{w}{2d} N, \tag{70b}$$

$$Q^p - Q^* = \left[3\frac{\beta_1 - f_1}{\alpha_1} + \frac{\beta_1}{\alpha_1}\left(2 - \frac{w\mu}{d}\right) + \frac{\beta_2 - f_2}{\alpha_2} \right] \frac{N^2}{8\mu} > 0, \tag{70c}$$

$$TC^p - P^p - TC^* = \left[\left(1 + 2\frac{\alpha_2}{\alpha_1}\right)\lambda_1 - \left(5 + 4\frac{\alpha_2}{\alpha_1}\right)\beta_1 + 3\lambda_2 - 5\beta_2 \right] \frac{w}{d}\frac{N^2}{8}, \tag{70d}$$

where Q^p, TC^p, P^p are the total queuing time, total commuting cost, and total revenue from parking at the equilibrium with parking fees, respectively. This indicates that $Q^p > Q^*$ and that $TC^p - P^p > TC^*$ if the first bracket on the right hand side of (70d) is positive. This result clearly shows that imposing parking fees can increase not only traffic congestion but also total commuting cost.

6.3 Equilibrium with congestion tolls

We investigate the properties of the equilibrium with congestion tolls in operation. $t_i^{\mathrm{F}}, t_i^{\mathrm{L}}$ is readily obtained as in Section 6.1

$$t_2^{\mathrm{F}} = t^{\mathrm{F}} = t^* - \left(1 + \frac{w\mu}{d}\right)\frac{N}{\mu}, \qquad t_2^{\mathrm{L}} = t_1^{\mathrm{F}} = t^* - \frac{w}{d}N - \frac{N}{2\mu}, \qquad t_1^{\mathrm{L}} = t^{\mathrm{L}} = t^* - \frac{w}{d}N. \tag{71}$$

The equilibrium commuting cost c_i^q and congestion toll $p^q(t)$ are represented as (Figure 5)

$$c_1^q = (f_1 + f_2)\frac{N}{2\mu} + \lambda_1 \frac{w}{d}N, \quad c_2^q = f_2\frac{N}{\mu} + \lambda_2\frac{w}{d}N, \tag{72a}$$

$$p^q(t) = \begin{cases} f_2(t - t_2^{\mathrm{F}}) & \text{if} \quad t \in [t_2^{\mathrm{F}}, t_2^{\mathrm{L}}], \\ f_2\dfrac{N}{2\mu} + f_1(t - t_1^{\mathrm{F}}) & \text{if} \quad t \in [t_1^{\mathrm{L}}, t_1^{\mathrm{L}}]. \end{cases} \tag{72b}$$

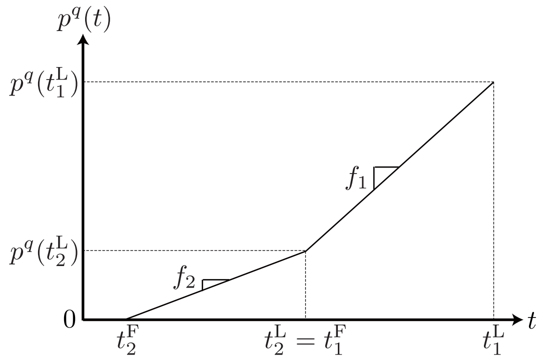


Figure 5: Congestion toll $p^q(t)$ without parking fees

Thus, the difference between c_i^q and c_i^* is given by

$$c_1^q - c_1^* = -(f_1 - f_2)\frac{N}{2\mu} < 0, \tag{73a}$$

$$c_2^q - c_2^* = \left(-\frac{\alpha_2}{\alpha_1}f_1 + f_2\right)\frac{N}{2\mu} > 0. \tag{73b}$$

This shows that the impact of congestion tolling varies among commuters. Furthermore, we can easily obtain the difference between the total commuting costs, TC^q and TC^*:

$$TC^q - TC^* = \left\{2f_2 - \left(1 + \frac{\alpha_2}{\alpha_1}\right)f_1\right\}\frac{N^2}{4\mu}, \tag{74a}$$

$$TC^q - P^q - TC^* = \left\{-2\left(1 + \frac{\alpha_2}{\alpha_1}\right)f_1 - (f_1 - f_2)\right\}\frac{N^2}{8\mu} < 0, \tag{74b}$$

where P^q is the total toll revenue. This indicates that $TC^q > TC^*$ if $2f_2 > (1+\alpha_2/\alpha_1)f_1$ and that $TC^q - P^q < TC^*$, which suggests that the appropriate redistribution of toll revenue is quite important for achieving a Pareto improvement. In this example, if the planner equally redistributes toll revenue, a Pareto improvement is achieved:

$$c_1^q - P^q/N - c_1^* = -f_1\frac{N}{2\mu} - (f_1 - f_2)\frac{N}{8\mu} < 0, \tag{75a}$$

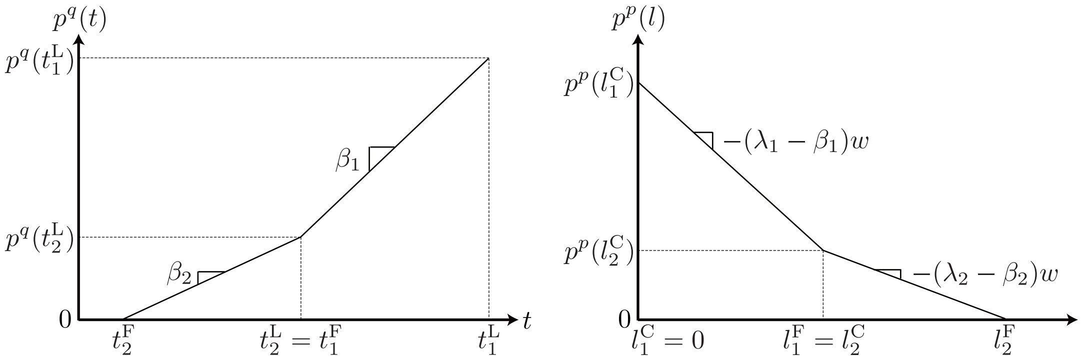
$$c_2^q - P^q/N - c_2^* = \left[-4\frac{\alpha_2}{\alpha_1}f_1 - (f_1 - f_2)\right]\frac{N}{8\mu} < 0. \tag{75b}$$

6.4 Equilibrium with both parking fees and congestion tolls

We finally consider the case wherein both parking fees and congestion tolls are imposed. Propositions 3 and 7, and (62) give t_i^{F}, t_i^{L}, l_i^{C}, and l_i^{F} as follows:

$$t_2^{\mathrm{F}} = t^{\mathrm{F}} = t^* - \frac{N}{\mu}, \qquad t_2^{\mathrm{L}} = t_1^{\mathrm{F}} = t^* - \frac{N}{2\mu}, \qquad t_1^{\mathrm{L}} = t^{\mathrm{L}} = t^*, \tag{76a}$$

$$l_2^{\mathrm{F}} = \frac{N}{d}, \qquad l_2^{\mathrm{C}} = l_1^{\mathrm{F}} = \frac{N}{2d}, \qquad l_1^{\mathrm{C}} = 0. \tag{76b}$$



(a) Congestion toll $p^q(t)$ (b) Parking fee $p^p(t)$

Figure 6: Equilibrium with parking fees and congestion tolls

As illustrated in Figure 6, commuting cost c_i^{pq}, parking fee $p^p(l)$, and congestion toll $p^q(t)$ are given by

$$c_1^{pq} = (2\beta_1 + 2\beta_2 - f_1 - f_2)\,\frac{N}{2\mu}, \qquad c_2^{pq} = (2\beta_2 - f_2)\,\frac{N}{\mu}, \tag{77a}$$

$$p^q(t) = \begin{cases} \beta_2(t - t_2^{\mathrm{F}}) & \text{if} \quad t \in [t_2^{\mathrm{F}}, t_2^{\mathrm{L}}], \\ \beta_2 \dfrac{N}{2\mu} + \beta_1(t - t_1^{\mathrm{F}}) & \text{if} \quad t \in [t_1^{\mathrm{F}}, t_1^{\mathrm{L}}]. \end{cases} \tag{77b}$$

$$p^p(l) = \begin{cases} (\lambda_1 - \beta_1)w(l_1^{\mathrm{F}} - l) + (\lambda_2 - \beta_2)w\dfrac{N}{2d} & \text{if} \quad l \in [l_1^{\mathrm{C}}, l_1^{\mathrm{F}}], \\ (\lambda_2 - \beta_2)w(l_2^{\mathrm{F}} - l) & \text{if} \quad l \in [l_2^{\mathrm{C}}, l_2^{\mathrm{F}}], \end{cases} \tag{77c}$$

Comparing c_i^{pq} and c_i^*, we have

$$c_1^{pq} - c_1^* = \left(2\beta_1 + 2\beta_2 - 3f_1 - f_2 - 2\lambda_1 \frac{w\mu}{d}\right)\frac{N}{2\mu}, \tag{78a}$$

$$c_2^{pq} - c_2^* = \left(4\beta_2 - \frac{\alpha_2}{\alpha_1} f_1 - 3f_2 - 2\lambda_2 \frac{w\mu}{d}\right)\frac{N}{2\mu}. \tag{78b}$$

This shows that commuters generally cannot reduce commuting costs only as a result of imposition of parking fees and congestion tolls. As in Section 6.3, a Pareto improvement is achieved by equally redistributing the revenue P^{pq} obtained from parking fees and congestion tolls:

$$c_1^{pq} - P^q/N - c_1^* = \left[-4(\beta_1 - \beta_2) - 7\frac{w\mu}{d}\beta_1 - 3f_1 - f_2\right]\frac{N}{8\mu} < 0, \tag{79a}$$

$$c_2^{pq} - P^q/N - c_2^* = -\left[4(\beta_1 - \beta_2) - (f_1 - f_2) + 8\beta_2 \frac{w\mu}{d} + 4\frac{\alpha_2}{\alpha_1} f_1\right]\frac{N}{8\mu} < 0. \tag{79b}$$

7 Conclusions

This study developed a theoretical model of trip timing and parking location choices by heterogeneous commuters by extending the framework of Arnott et al. (1991) and examined the effects of parking fees and congestion tolls. We showed that the equilibrium distribution of trip timing and parking locations is quite different from the optimum due to strategic interactions among heterogeneous commuters. Furthermore, we clarified that these strategic interactions limit the efficacy of policies intended to alleviate traffic congestion and parking competition. More specifically, expanding parking capacity and introducing parking fees may lead to temporal concentration of traffic demand, thereby exacerbating traffic congestion and increasing total commuting cost. In contrast, a combination of parking fees and congestion tolls achieves an optimal distribution of trip timing and parking locations.

Our results suggest that the effects of each pricing policy upon each commuter type are significantly different. This implies that the "winner" and "loser" among commuters will vary according to which pricing policy is imposed, and thus, appropriate redistribution of the revenue from such policies is quite important to achieve a Pareto improvement. As a benchmark of appropriate redistribution, we proposed that the revenue is used for financing parking and road capacity. We then demonstrated that the revenue from parking fees and congestion tolls exactly equals the costs of optimal parking and bottleneck capacity, respectively; that is, the *self-financing principle* (Mohring and Harwitz, 1962) holds for the model.

This study introduces a number of assumptions to clearly present how strategic interactions among heterogeneous commuters result in an inefficient outcome and limit the efficacy of pricing policies. For example, in our model, commuters need not search vacant parking lots and cannot use public transport.[13] Since relaxing these assumptions may lead to changes in commuters' choice of trip timing and parking location, it would be valuable for future research to extend our model in these directions.

[13] Many studies have considered cruising to search for a parking lot (e.g., Glazer and Niskanen, 1992; Anderson and de Palma, 2004; Calthrop and Proost, 2006; Arnott and Inci, 2006; Arnott and Rowse, 2009; Arnott and Inci, 2010; Arnott et al., 2015; Inci and Lindsey, 2015; Geroliminis, 2015) and public transport (e.g., Yang et al., 2013; Liu et al., 2014a,b).

A Proof of Lemma 1

Suppose to the contrary that there exist $t_i \in \hat{\mathcal{S}}_i$, $t_j \in \hat{\mathcal{S}}_j$, and $t_k \in (t_i, t_j)$ such that $\sum_{m\in\mathcal{I}} \hat{n}^*_m(t_i) = \sum_{m\in\mathcal{I}} \hat{n}^*_m(t_j) = \mu$ and $\sum_{m\in\mathcal{I}} \hat{n}^*_m(t_k) < \mu$ (i.e., $q(t_k) = 0$). Then, it follows from equilibrium condition (6a) that $\hat{c}_i(t_i) \leq \hat{c}_i(t_k)$. Thus, if $\hat{a}(t_k) \leq t^*$, we have

$$\begin{aligned}
q(t_i) &\leq \frac{\beta_i}{\alpha_i}(t_i - t_k) + \frac{1}{\alpha_i}(\lambda_i - \beta_i)\frac{w}{d}\{N(t_k) - N(t_i)\} \\
&\leq -\frac{1}{\alpha_i}\left\{\beta_i - (\lambda_i - \beta_i)\frac{w\mu}{d}\right\}(t_k - t_i) < 0.
\end{aligned} \tag{80}$$

Similarly, if $\hat{a}(t_k) \geq t^*$, the condition $\hat{c}_j(t_j) \leq \hat{c}_j(t_k)$ gives

$$\begin{aligned}
q(t_j) &\leq \gamma_j(t_k - t_j) + \left(\lambda_j + \gamma_j\right)\frac{w}{d}\left\{N(t_k) - N(t_j)\right\} \\
&\leq \left\{\gamma_j + \left(\lambda_j + \gamma_j\right)\frac{w\mu}{d}\right\}(t_k - t_j) < 0.
\end{aligned} \tag{81}$$

But this contradicts the equilibrium condition (6b).

Furthermore, commuting cost (4) suggests that $\sum_{m\in\mathcal{I}} \hat{n}^*_m(t) < \mu$ (i.e., $q(t) = 0$) for all $t \in \mathbb{R}_+$ cannot be an equilibrium. Therefore, we obtain Lemma 1. □

B Proof of Proposition 1

The Karush–Kuhn–Tucker (KKT) conditions of problem (12) is expressed as

$$\begin{cases}
q^*(t) + \dfrac{1}{\alpha_i}\hat{c}_i(t) = c^*_i & \text{if} \quad \hat{n}^*_i(t) > 0, \\
q^*(t) + \dfrac{1}{\alpha_i}\hat{c}_i(t) \geq c^*_i & \text{if} \quad \hat{n}^*_i(t) = 0,
\end{cases} \tag{82a}$$

$$\begin{cases}
\sum_{i\in\mathcal{I}} \hat{n}^*_i(t) = \mu & \text{if} \quad q^*(t) > 0, \\
\sum_{i\in\mathcal{I}} \hat{n}^*_i(t) \leq \mu & \text{if} \quad q^*(t) = 0,
\end{cases} \tag{82b}$$

$$\int \hat{n}^*_i(t)\,\mathrm{d}t = N_i \quad \forall i \in \mathcal{I}, \tag{82c}$$

where $q^*(t)$ and c^*_i are the Lagrange multipliers for the first and second constraints, respectively. Because these conditions are equivalent to the equilibrium conditions (6) with (10), the equilibrium distribution $\hat{\boldsymbol{n}}^*(t)$ of trip timing coincides with the KKT points for the problem (12). □

C Proof of Lemma 2

We prove Lemma 2 (a) and (b), in turn.

(a) It follows from the condition (25b) that for any $i, j \in \mathcal{I}$, $(t_i, l_i) \in \mathcal{S}^o_i$, and $(t_j, l_j) \in \mathcal{S}^o_j$,

$$c^o_i(t_i, l_i) + c^o_j(t_j, l_j) \leq c^o_i(t_j, l_i) + c^o_j(t_i, l_j). \tag{83}$$

Substituting (26) into this, we have (30). □

(b) At the social optimum, a commuter's parking location lies in $[0, \sum_{k\in\mathcal{I}} N_k/d]$, i.e., for any $i \in \mathcal{I}$ and $(t_i, l_i) \in \mathcal{S}_i^o$, $l_i \in [0, \sum_{k\in\mathcal{I}} N_k/d]$. It follows from this that for any $i \in \mathcal{I}$, there exists $j \in \mathcal{I}$ and l_{ij} such that $(t_i, l_{ij}) \in \mathcal{S}_i^o$ and $(t_j, l_{ij}) \in \mathcal{S}_j^o$. Furthermore, the condition (25b) gives

$$c_i^o(t_i, l_{ij}) = c_i^o(\tilde{t}_i, l_i) \qquad \forall (t_i, l_{ij}), (\tilde{t}_i, l_i) \in \mathcal{S}_i^o, \tag{84a}$$

$$c_j^o(t_j, l_{ij}) = c_j^o(\tilde{t}_j, l_j) \qquad \forall (t_j, l_{ij}), (\tilde{t}_j, l_j) \in \mathcal{S}_j^o. \tag{84b}$$

From (84), $p^p(l_i) - p^p(l_j)$ such that $l_j \leq l_{ij} \leq l_i$ is given by

$$p^p(l_i) - p^p(l_j) = (\lambda_i - \beta_i)w(l_j - l_i) + \left\{(\lambda_i - \beta_i) - (\lambda_j - \beta_j)\right\} w(l_{ij} - l_j). \tag{85}$$

The condition (25b) implies that $c_i^o(t, l)$ satisfies

$$c_i^o(t_i, l_i) \leq c_i^o(t_i, l_j). \tag{86}$$

Substituting (26) into this, we obtain

$$p^p(l_i) - p^p(l_j) \leq (\lambda_i - \beta_i)w(l_j - l_i). \tag{87}$$

Combining (85) into this yields

$$\left\{(\lambda_i - \beta_i) - (\lambda_j - \beta_j)\right\} (l_{ij} - l_j) \leq 0. \tag{88}$$

This leads to Lemma 2 (b), and we thus complete the proof. □

References

Akamatsu, T. (2007) "Tradable network permits: A new scheme for the most efficient use of network capacity," *mimeograph*.

Anderson, S. P. and de Palma, A. (2004) "The economics of pricing parking," *Journal of Urban Economics*, Vol. 55, No. 1, pp. 1–20.

Arnott, R., de Palma, A., and Lindsey, R. (1990) "Economics of a bottleneck," *Journal of Urban Economics*, Vol. 27, No. 1, pp. 111–130.

Arnott, R., de Palma, A., and Lindsey, R. (1991) "A temporal and spatial equilibrium analysis of commuter parking," *Journal of Public Economics*, Vol. 45, No. 3, pp. 301–335.

Arnott, R., de Palma, A., and Lindsey, R. (1993) "A structural model of peak-period congestion: A traffic bottleneck with elastic demand," *American Economic Review*, Vol. 83, No. 1, pp. 161–179.

Arnott, R., de Palma, A., and Lindsey, R. (1994) "The welfare effects of congestion tolls with heterogeneous commuters," *Journal of Transport Economics and Policy*, Vol. 28, No. 2, pp. 139–161.

Arnott, R., Inci, E., and Rowse, J. (2015) "Downtown curbside parking capacity," *Journal of Urban Economics*, Vol. 86, pp. 83–97.

Arnott, R. and Inci, E. (2006) "An integrated model of downtown parking and traffic congestion," *Journal of Urban Economics*, Vol. 60, No. 3, pp. 418–442.

Arnott, R. and Inci, E. (2010) "The stability of downtown parking and traffic congestion," *Journal of Urban Economics*, Vol. 68, No. 3, pp. 260–276.

Arnott, R. and Kraus, M. (1993) "The Ramsey problem for congestible facilities," *Journal of Public Economics*, Vol. 50, No. 3, pp. 371–396.

Arnott, R. and Kraus, M. (1995) "Financing capacity in the bottleneck model," *Journal of Urban Economics*, Vol. 38, No. 3, pp. 272–290.

Arnott, R. and Rowse, J. (2009) "Downtown parking in auto city," *Regional Science and Urban Economics*, Vol. 39, No. 1, pp. 1–14.

Beesley, M. E. (1965) "The value of time spent in travelling: Some new evidences," *Economica*, Vol. 32, No. 126, pp. 174–185.

van den Berg, V. A. C. and Verhoef, E. T. (2011) "Winning or losing from dynamic bottleneck congestion pricing? The distributional effects of road pricing with heterogeneity in values of time and schedule delay," *Journal of Public Economics*, Vol. 95, No. 7-8, pp. 983–992.

Calthrop, E. and Proost, S. (2006) "Regulating on-street parking," *Regional Science and Urban Economics*, Vol. 36, No. 1, pp. 29–48.

Doan, K., Ukkusuri, S., and Han, L. (2011) "On the existence of pricing strategies in the discrete time heterogeneous single bottleneck model," *Transportation Research Part B: Methodological*, Vol. 45, No. 9, pp. 1483–1500.

Domencich, T. A. and McFadden, D. (1975) *Urban Travel Demand: A Behavioral Analysis*: North-Holland.

Fosgerau, M. and Lindsey, R. (2013) "Trip-timing decisions with traffic incidents," *Regional Science and Urban Economics*, Vol. 43, No. 5, pp. 764–782.

Fosgerau, M. and de Palma, A. (2013) "The dynamics of urban traffic congestion and the price of parking," *Journal of Public Economics*, Vol. 105, pp. 106–115.

Fosgerau, M. and Small, K. A. (2014) "Endogenous scheduling preferences and congestion," *Working Paper, University of California at Irvine*.

Geroliminis, N. (2015) "Cruising-for-parking in congested cities with an MFD representation," *Economics of Transportation*, Vol. 4, No. 3, pp. 156–165.

Glazer, A. and Niskanen, E. (1992) "Parking fees and congestion," *Regional Science and Urban Economics*, Vol. 22, No. 1, pp. 123–132.

Gubins, S. and Verhoef, E. T. (2014) "Dynamic bottleneck congestion and residential land use in the monocentric city," *Journal of Urban Economics*, Vol. 80, pp. 51–61.

Hendrickson, C. and Kocur, G. (1981) "Schedule delay and departure time decisions in a deterministic model," *Transportation Science*, Vol. 15, No. 1, pp. 62–77.

Inci, E. (2015) “A review of the economics of parking,” *Economics of Transportation*, Vol. 4, No. 1-2, pp. 50–63.

Inci, E. and Lindsey, R. (2015) “Garage and curbside parking competition with search congestion,” *Regional Science and Urban Economics*, Vol. 54, pp. 49–59.

Keeler, T. E. and Small, K. A. (1977) “Optimal peak-load pricing, investment and service levels in urban expressways,” *The Journal of Political Economy*, Vol. 85, No. 1, pp. 1–25.

Kraus, M. (2003) “A new look at the two-mode problem,” *Journal of Urban Economics*, Vol. 54, No. 3, pp. 511–530.

Kraus, M. (2012) “Road pricing with optimal mass transit,” *Journal of Urban Economics*, Vol. 72, No. 2-3, pp. 81–86.

Kraus, M. and Yoshida, Y. (2002) “The commuter’s time-of-use decision and optimal pricing and service in urban mass transit,” *Journal of Urban Economics*, Vol. 51, No. 1, pp. 170–195.

Lindsey, R. (2004) “Existence, uniqueness, and trip cost function properties of user equilibrium in the bottleneck model with multiple user classes,” *Transportation Science*, Vol. 38, No. 3, pp. 293–314.

Liu, W., Yang, H., Yin, Y., and Zhang, F. (2014a) “A novel permit scheme for managing parking competition and bottleneck congestion,” *Transportation Research Part C: Emerging Technologies*, Vol. 44, pp. 265–281.

Liu, W., Yang, H., and Yin, Y. (2014b) “Expirable parking reservations for managing morning commute with parking space constraints,” *Transportation Research Part C: Emerging Technologies*, Vol. 44, pp. 185–201.

Mohring, H. and Harwitz, M. (1962) *Highway Benefit: An Analytical Framework*: Northwestern University Press.

Newell, G. F. (1987) “The morning commute for nonidentical travelers,” *Transportation Science*, Vol. 21, No. 2, pp. 74–88.

Peer, S. and Verhoef, E. T. (2013) “Equilibrium at a bottleneck when long-run and short-run scheduling preferences diverge,” *Transportation Research Part B: Methodological*, Vol. 57, pp. 12–27.

Qian, Z. S., Xiao, F. E., and Zhang, H. M. (2011) “The economics of parking provision for the morning commute,” *Transportation Research Part A: Policy and Practice*, Vol. 45, No. 9, pp. 861–879.

Qian, Z. S., Xiao, F. E., and Zhang, H. M. (2012) “Managing morning commute traffic with parking,” *Transportation Research Part B: Methodological*, Vol. 46, No. 7, pp. 894–916.

Quarmby, D. A. (1967) “Choice of travel mode on the journey to work: Some findings,” *Journal of Transport Economics and Policy*, Vol. 1, No. 3, pp. 273–314.

Ramadurai, G., Ukkusuri, S. V., Zhao, J., and Pang, J. S. (2010) “Linear complementarity formulation for single bottleneck model with heterogeneous commuters,” *Transportation Research Part B: Methodological*, Vol. 44, No. 2, pp. 193–214.

Shoup, D. C. (2005) *The High Cost of Free Parking*: Planners Press.

Strotz, R. H. (1965) “Urban transportation parables,” in Margolis, J. ed. *The Public Economy of Urban Communities*: Resources for the Future, pp. 127–169.

Takayama, Y. (2015) “Bottleneck congestion and distribution of work start times: The economics of staggered work hours revisited,” *Transportation Research Part B: Methodological*, Vol. 81, No. 3, pp. 830–847.

Tseng, Y. Y. and Verhoef, E. T. (2008) “Value of time by time of day: A stated-preference study,” *Transportation Research Part B: Methodological*, Vol. 42, pp. 607–618.

Verhoef, E. T. (2007) “Second-best road pricing through highway franchising,” *Journal of Urban Economics*, Vol. 62, No. 2, pp. 337–361.

Vickrey, W. S. (1969) “Congestion theory and transport investment,” *American Economic Review*, Vol. 59, No. 2, pp. 251–260.

Vickrey, W. S. (1973) “Pricing, metering, and efficiently using urban transportation facilities,” *Highway Research Record*, Vol. 476, pp. 36–48.

Wada, K. and Akamatsu, T. (2013) “A hybrid implementation mechanism of tradable network permits system which obviates path enumeration: An auction mechanism with day-to-day capacity control,” *Transportation Research Part E: Logistics and Transportation Review*, Vol. 60, pp. 94–112.

Yang, H., Liu, W., Wang, X., and Zhang, X. (2013) “On the morning commute problem with bottleneck congestion and parking space constraints,” *Transportation Research Part B: Methodological*, Vol. 58, pp. 106–118.

Yang, H. and Meng, Q. (2002) “A note on ”highway pricing and capacity choice in a road network under a build-operate-transfer scheme”,” *Transportation Research Part A: Policy and Practice*, Vol. 36, No. 7, pp. 659–663.

Zhang, X., Huang, H. J., and Zhang, H. M. (2008) “Integrated daily commuting patterns and optimal road tolls and parking fees in a linear city,” *Transportation Research Part B: Methodological*, Vol. 42, No. 1, pp. 38–56.

Zhang, X., Yang, H., and Huang, H. J. (2011) “Improving travel efficiency by parking permits distribution and trading,” *Transportation Research Part B: Methodological*, Vol. 45, No. 7, pp. 1018–1034.