Law, Coercion, And Socioeconomic Equilibrium

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2015
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Published in: Review of Economic and Business Studies, 2015, 8(2), 39-50

Abstract: This paper investigates the economic conditions under which the performance of a Judiciary does not impede non-coercive fair socioeconomic allocations under “Strotz-myopia” regarding the law variable, i.e. under a static view of it in an otherwise dynamic context. The law, here, is the positive factor by which consumption volume is multiplied as a result of law introduction in an otherwise fully private social economy. Lexicographic preferences regarding the law is the keyword in establishing non-coercive equilibria either in the static context of a stone-age economy or in the dynamic context of a jungle economy, given in the latter the presence of farsightedness. Nevertheless, such equilibria are found here to exist even under myopia and regardless the presence of lexicographic preferences. We first detect them within a fully private social economy, and we next qualify them by introducing the Judiciary as state officials. The optimality regarding state finances imposes additional restrictions in establishing myopic non-coercive equilibria. In any case, an equilibrium will be stable if it is not influenced by the homotheticity or not of the preferences, i.e. by income distribution considerations. So, any suboptimal behaviour of the Judiciary should be attributed exclusively to the suboptimality of state finances: Macroeconomics does affect law administration.

Keywords: Myopic law preferences; Non-coercive allocations, Homotheticity, Judiciary

JEL classification: K00, O40, D60

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1. INTRODUCTION

The purpose of this paper is to investigate the economic conditions under which the performance of a Judiciary does not impede non-coercive fair socioeconomic allocations under “Strotz-myopia” regarding the law variable, i.e. under a static view of it in an otherwise dynamic context. More specifically, it puts forward the hypothesis that within the context of mainstream economics, preferences need not be lexicographic to have a non-coercive equilibrium even under myopia. But, let us take things one at a time. If one judges from works like those of Sarat and Kearns (1996) and Millhiser (2015) among others, but also from common experience, there are serious grounds of doubting whether the judicial system can be compatible with fair socioeconomic allocations, i.e. with envy-free, equitable, and Pareto efficient allocations. For example, it has become common knowledge through the centuries that there has never been a Judiciary that it didn’t finally succumb to unequal treatment. “Legum servi sumus ut liberi esse possimus”, (i.e. we are slaves of the law so that we can be free) Cicero (106-43 BC),¹ would caution against distrusting Justice.
But, the problem is not with the Justice; it is with the Judiciary. The truth of the matter is that the Judiciary may very well become the source of unfair socioeconomic allocations, *ceteris paribus*. And, once the unfairness is imposed by the coercive power invested with the Judiciary, the subsequent economy falls into the category of what Piccione and Rubinstein (2007) model as *jungle economy*. The Judiciary may be held responsible for fostering a jungle economy and hence, its value should be evaluated within this type of social economy. And, according to Houba, Lutten, and Weikard (2013, 2014), in a farsighted rather than myopic jungle, the equilibrium coincides with lexicographic welfare maximization for which initial wealth is irrelevant; otherwise we have jungle or the same, coercive equilibria.

That is, the cause of fair division can be salvaged only under lexicographic preferences. Under the mentality that what matters primarily is to have law and thereby the people administering it regardless of individual preferences over the misallocation prompted by the Judiciary; which is what, of course, the above quotation from Cicero really signifies. If all are farsighted, they do acknowledge the value of Justice, they tolerate “mishaps” as a necessary evil when administering it in practice, and what would be characterized as misallocation in the absence of this acknowledgement and toleration, becomes now a fair division (see e.g. Whalley and Zhang 2011). Put differently, in a decentralized environment encouraging the formation of rational expectations, the Judiciary is expected to live up to its reputation. A myopic perception of things, a perception based exclusively on short-term self-interest impeding the formation of such long-term expectations, would lead to coercive and hence, unstable equilibria, nurturing socioeconomic unrest.

But, what exactly “myopia” means within the context of the mainstream, non-jungle view of an intertemporal social economy? As the term suggests, it refers to disregard of the future as follows. To preserve the dynamic character of decisionmaking and keep at the same time the analysis simple, a two-period horizon is assumed in this paper. Within this time framework, myopia should mean decisionmaking about consumption today and tomorrow, disregarding the fact that the consumption planned for tomorrow need not be surrounded by the same legal environment which is preferred for consumption today. The preferences tomorrow for tomorrow’s legal environment may be different from the current preferences for tomorrow’s environment. That is, in a two-period setting, we have to have Strotz’s (1956) sense of myopia whereby future expectations do exist but shape current behavior neglecting the fact that preferences in the future may change. Therefore, the law, as it will be defined immediately, should be entering a time-strongly additive utility function in a weakly separable fashion across periods when myopia is postulated in Strotz’s sense. This, under the presumption that the presence of law corroborates output growth as North (1991), acting thereby multiplicatively on consumption.

Now, this paper argues that within the context of mainstream economics, preferences need not be lexicographic to have a non-coercive equilibrium even under myopia. To obtain such a result suffices law to be entering the utility function in a weakly separable mode regardless the homotheticity of the function. McCoubrey and White (1996) have shown that no universally acceptable definition of law can be produced, but by the term “law” is meant below the positive factor by which consumption volume is multiplied as a result of law introduction in an otherwise fully private social economy. A factor shaped by such diverse institutions as industry regulation within period, social security rules across periods, or theft and robbery laws as handled by the Judiciary as state officials and hence, depending on whether state finances can ensure a sound Judiciary (see e.g. Chemin 2007). So, if sub-
optimal state behavior weakens Judiciary performance after certain equilibrium is thought to have been reached, the solution will be another equilibrium with a different Judiciary, all else being the same including income distribution. Equilibrium is unstable if it depends on socioeconomic stratification.

The next section offers a formal support of our thesis, followed by a section concluding this article with a discussion in connection with the economics of judicial decisionmaking. From the viewpoint of the source of myopia, to dissociate lexicographic preferences from non-coercion is important because such preferences are necessary and sufficient to prompt "generalized myopia", i.e. myopia independent of information representation (Feldman 1992). This perception of the nexus between preferences and myopia derives from Tintner (1941) and Alchian (1950), and there is no a priori reason to presume here that the representation of information does not influence behavior. Anyway, judging from Miceli (2014, Abstract), the approach herein is novel in that it falls neither in the category of the "economic analysis of law"—which concerns the use of economic theory for describing the incentive effects of legal rules (positive analysis) and for prescribing better rules (normative analysis)—and not in the category of "law and economics"—which concerns the relationship between law and markets as alternative institutions for organizing economic activity.” Moreover, our approach is also an intertemporal one close to the mentality whereby sustained growth dominates in importance the matter of static efficiency (see e.g. Cooter and Edlin 2011).

Finally, according to Epstein (2013, xiii): “In the study of judicial behavior, ‘economics’ has multiple meanings. Many scholars view it through a theoretical lens, arguing that economic studies operate under the assumption that the judge is a ‘rational maximizer’... Others focus on whether the research employs the tools of econometrics. A third group might claim that work exploring economics as a substantive matter – say, a paper on the effect of the economy on judicial decisions – qualifies as an economic study of judging.” This paper falls in the realm of the third group. For us, here, the law is put in the service of market exchange across time periods with an eye to investigating whether “Strotz-myopia” over the law variable, a static view of it in dynamic mainstream microeconomics suffices to salvage the case for non-coercive equilibria and thereby the case for fair division under myopia. This is the reason the discussion is made in connection with jungle-dynamic rather than stone-age static equilibria. Myopic non-coercive equilibria are impossible in a jungle economy even under lexicographic preferences, and it is remarkable that such are the preferences fostering stone-age equilibria, too (see e.g. Houba and Weikard 2009).

2. THE FORMAL ARGUMENT

In a few words, this paper starts by pointing out that Justice may not perform its duties satisfactorily because of a number of reasons. It next tries to see if this hampers economic efficiency by focusing on the economic dimension of these reasons. The standard conclusion is that efficiency will not be hampered if people are farsighted and realize that the Judiciary is indispensable to a civil society. Below, we find out that efficiency can still be the case even if people are myopic; that is people need not trust Justice and be lawyers to have a Justice system operating efficiently given sound public finance conditions.

To support our thesis, an economy without a Judiciary is examined first, and the results are next qualified by introducing the Judiciary as state officials. Either case is evaluated under a homothetic utility specification and under an example of non-homothetic utility, both with a
two-period horizon. Intertemporal homotheticity means that rich and poor decisionmakers are equally averse to proportional fluctuations in consumption, and respond alike to the challenges by the legal system. An equilibrium will be unstable if it depends on income distribution and this is the reason the possibility of equilibrium under conditions of non-homotheticity is examined as well.

2.1 The Private Sector

One well-known utility specification that might be used in connection with intertemporal homotheticity derives from what Neary (2004) calls “the Dixit-Stiglitz Lite”. Let current and future consumption be \( c_1 \) and \( c_2 \), respectively, so that lifetime consumption in the absence of law is:

\[
\sum_{t=1}^{2} c_t = c_1 + \delta c_2,
\]

where \( \delta \) is a discount factor. The law, as defined earlier, is designated by variable \( L \), and it is assumed to be multiplying the volume of consumption by contributing to output growth. It is assumed to be produced based on statutes, decrees, regulations, and precedents by the legislature-cum-Judiciary, \( \mathcal{L} \), according for simplicity to the production function:

\[
\mathcal{L} = aL^2,
\]

where \( a \) is some positive coefficient. A myopic treatment of it wants it to be invariant over time and hence, it is taken to be the numeraire good so that lifetime budget, \( H \), is:

\[
(aL)^2 + p_1 c_1 + \delta p_2 c_2 = H,
\]

where \( p_1 \) and \( p_2 \) are the prices in periods 1 and 2, respectively. This is the income constraint under which the homothetic Cobb Douglas/Constant Elasticity of Substitution utility:

\[
u = L^{1-n} (c_1^e + c_2^e)^{n/e},
\]

is maximized, where \( n \) is the share parameter and \( e \in (0,1) \) is the substitution parameter excluding the case \( e = 1 \) of perfect substitutability and the case \( e = 0 \) of independent goods. It is clear that \( L \) is separable in (2). The optimal demands then will be:

\[
c_1^* = \frac{\psi}{[H - (aL^*)^2](\delta p_2)^{1/(1-e)}},
\]

\[
c_2^* = \frac{\psi}{[H - (aL^*)^2]p_1^{1/(1-e)}},
\]

and

\[
\mathcal{L}^* = (aL^*)^2 = H - \left[ \frac{p_1^{2-e}/(1-e) + \delta^{1/(1-e)}p_2^{2-e}/(1-e)}{(\delta p_1 p_2)^{1/(1-e)}} \right] \psi,
\]

where \( \psi \equiv p_1^{e/(1-e)} + (\delta p_2)^{e/(1-e)} \) while the fraction gives the elasticity of substitution whose negative is the price elasticity of demand, \( \theta \).

These optima are certainly non-coercive, and in order to arrive at non-coercive equilibria, the supply-side of the economy has to be examined too, given \( \mathcal{L} \) at \( \mathcal{L}^* \). Assuming imperfect competition in each period to utilize \( \theta \), profit maximization occurs when:

\[
p_t \left( 1 + \frac{1}{\theta} \right) = k_i,
\]
where $k_i$ is the constant marginal cost in period $i = 1,2$. Hence,

$$p_i^* = \frac{k_i}{e}; \quad (5)$$

prices depend inversely on the substitution parameter. The fixed factor of production $L^*$ does not enter in this condition, and any positive profits could be considered to be rents to law abiding on the part of firms: $\Pi_i^* = p_i^* c_i - k_i c_i - \epsilon L^* > 0$, where $\Pi^*$ is the optimal form profit. From this last relationship and (5), one obtains that:

$$c_i^* = \frac{e(\epsilon L^* + \Pi_i^*)}{k_i(1 - e)}, \quad (6)$$

which $c_i^*$'s have at equilibrium to be equal with the $c_i^*$'s from (3). These equalities characterize the non-coercive equilibria under the presumed myopia type and homotheticity.\

Nevertheless, one the one hand the “Lite” has been criticized by many (among which Hicks 1965), and on the other hand the issue of the stability of equilibrium has to be addressed by relaxing homotheticity. Accordingly, we continue by capitalizing upon the notion of myopic separability advanced by Kannai, Selden and Wei (2014), who note that myopia does not necessarily presuppose homotheticity or logarithmic period utility. Let utility be given for example by the simple non-homothetic function:

$$u = L(c_1 + \sqrt{c_2}). \quad (7)$$

The optimum quantities under again (1) will now be:

$$c_1^* = \frac{4\delta^2 p_2 (H + p_1^2) - (1 + 4\delta)p_1^2}{8\delta^2 p_1 p_2}, \quad (8i)$$

$$c_2^* = \frac{p_1^2}{4(\delta p_2)^2}, \quad (8ii)$$

and

$$L^* = \frac{4\delta^2 p_2 (H + p_1^2) - p_1^2}{8\delta^2 p_2}. \quad (8iii)$$

Next, (4) may be rewritten as follows: $p_i = \theta_i k_i / 1 - \theta_i$, where $\theta_i$ is the elasticity of demand in period $i$. This in conjunction with $\Pi_i^* = p_i^* c_i - k_i c_i - \epsilon L^* > 0$ yields that:

$$c_i^* = \frac{(1 - \theta_i)(\epsilon L^* + \Pi_i^*)}{(2\theta_i - 1)k_i}. \quad (9)$$

Monopolistic power implies presumably that $\theta_i > 1/2 \Rightarrow 2\theta_i - 1 > 0$. The non-coercive equilibria are described now by the equalities between $c_i^*$'s from (9) and $c_i^*$'s from (8), given $L$ at $L^*$ rather than at $L^*$. A number of such equilibria may be produced depending on the particular non-homothetic utility function employed each time, and a good many such functions may be specified.
2.2 The Introduction of the State-cum-Judiciary

Recall that the multiplicative factor $L$ is the output of $\mathcal{L}$. Under a balanced-budget and social-welfare minded state whose only responsibility is the promotion and enactment of growth-contributing legislature, and assuming that state expenses are financed wholly through an income tax at a proportional rate $t$, this rate might be viewed as the price of $\mathcal{L}$ and the $p$’s as price ratios relative to $t$. The budget constraint (1) becomes:

$$t(aL)^2 + p_1c_1 + \delta p_2c_2 = t(1 - t)H. \quad (10)$$

In a state like this, non-coercive equilibria such as those described earlier will continue holding.\(^5\) The same holds when in addition to an income tax, a profits tax is levied on the firm given the standard public-finance proposition that corporate taxation does not influence decisionmaking on the part of the firm.

In so far as a sales tax at rate $\tau$ is concerned, it is easily checked that $\tau$ would enter multiplicatively in the denominator of (6) and (9). Under homothetic preferences, a non-coercive equilibrium can be ensured only under a particular non-linear relationship between $t$ and $\tau$ as follows: The budget constraint is now:

$$t(aL)^2 + (1 - \tau)(p_1c_1 + \delta p_2c_2) = t(1 - t)H. \quad (11)$$

Equating the after tax demand and supply $c$’s yields that the equilibrium relationship between $t$ and $\tau$ should be such that the ratio $p_1/\delta p_2$ equals the fraction $k_2[(eL^s)^2 + \Pi_1^2]/k_1[(eL^s)^2 + \Pi_2^2)]^{1-e}$, which, of course, is a quite restrictive condition. And, in so far as our non-homothetic example is concerned, one finds out that equilibrium presupposes that $\tau = 4(1 - \theta_2)[(eL^s)^2 + \Pi_2^2]\delta p_2^2$ and $t = 1 \pm \sqrt{1 - 4\Phi}$, and hence, the even more restrictive condition that, where

$$\Phi = \frac{2(1 - \theta_1)[(eL^s)^2 + \Pi_1^2]p_1 + [(1 + 4\delta)p_1^2 - 4\delta^2 p_2^2p_1^2](2\theta_1 - 1)k_1(1 - \theta_2)(L^s + \Pi_2^2)p_2}{4\delta^2B(2\theta_1 - 1)k_1(1 - \theta_2)[(eL^s)^2 + \Pi_2^2]p_2^2}$$

Moreover, there is no a priori reason to reject one of the solutions for $t$. It appears in general that indirect business taxation makes it very difficult to attain non-coercive equilibria.

Similar conclusions are reached when the state is allowed to borrow in which case $\delta = 1/1 + r$, where $r$ is the interest rate on bonds, $B$:

$$B_1 = \delta[L_2 - tH - \tau(p_1c_1 + p_2c_2) + B_2],$$

which given that $L = (eL)^2$ and hence, $B$ should not not change over time, becomes:

$$(1 - \delta)B = \delta[(eL)^2 - tH - \tau(p_1c_1 + p_2c_2)],$$

where the bracketed term on the right is the budget deficit. Solving for $tH$, inserting the result in (10) and manipulating terms gives the budget constraint:

$$t(eL)^2 + [1 + \tau(1 - t)]p_1c_1 + [\delta + \tau(1 - t)]p_2c_2 = (1 - t)(L - rB). \quad (12)$$

The quantities of $c$ in (3) become:
\[ c_1' = \frac{\Psi'}{[(1 - 2t)(\epsilon L^*)^2 - (1 - t)rB][\{\delta + \tau(1 - t)\}p_2]^{1/(1 - e)'}} \]

and

\[ c_2' = \frac{\Psi'}{[(1 - 2t)(\epsilon L^*)^2 - (1 - t)rB][\{\delta + \tau(1 - t)\}p_1]^{1/(1 - e)'}} \]

where \( \Psi' = \{[1 + \tau(1 - t)]p_1]^{e/(1 - e)} + \{[\delta + \tau(1 - t)]p_2]^{e/(1 - e)} \). Equating with the after-sales-tax supplies of \( c \), the relationship between \( t \) and \( \tau \) consistent with non-coercive equilibrium under homothetic preferences becomes:

\[ \frac{[\tau(1 - t)]p_1}{[\delta + \tau(1 - t)]p_2} = \frac{k_2[(\epsilon L^*)^2 + N^*_1]}{(k_1[(\epsilon L^*)^2 + N^*_2])^{1 - e}} \]

which is certainly more complicated than when the left-hand term is only \( p_1/\delta p_2 \).

And, of course, one needs not go on with the tedious algebra surrounding the non-homothetic case to conclude that the condition for the equilibrium relationship between taxes will be even more stringent than without borrowing. More important is the observation that homotheticity, income distribution, is not responsible for the additional restrictions in establishing non-coercive equilibria in the presence of the state. Responsible is the state per se regardless income distribution and the social choice rule sustaining it. To have absence of coercion suffices to have a benevolent state from the viewpoint that it does not consist of a rent-seeking bureaucracy rather than from the Italian public finance perspective that: “If fiscal decisions are made by a ruling class, it is evident that they can only be carried out through coercion” (Domenicantonio 1998, 3).

3. CONCLUDING REMARKS

To sum up, the law was put in the service of market exchange across time periods with an eye to investigating whether “Strotz-myopia” over the law variable, a static view of it in dynamic mainstream microeconomics suffices to salvage the case for non-coercive equilibria and thereby the case for fair division under myopia. This was the reason the discussion was made in connection with jungle-dynamic rather than stone-age static equilibria. Myopic non-coercive equilibria are impossible in a jungle economy even under lexicographic preferences, and it is remarkable that such are the preferences fostering stone-age equilibria, too. Yet, such equilibria do come up in our analysis without lexicographic preferences; and they are stable equilibria, since they are not influenced by the homotheticity or not of the utility function, i.e. by income distribution matters. Also, the additional restrictions in establishing myopic non-coercive equilibria in the presence of the state were found to be owing to the state per se regardless income distribution and the social choice rule sustaining it.

The ethical side of the law, the value called “law”, has prompted many to urge to undermine its economics (see e.g. Jain 2010); from the viewpoint of economics, what they really propose is a lexicographic vision of it: “Without justice, what else is the State but a great band of robbers?”, St Augustine (354-430 AD) would ask. But, it is the economic rather than moral dimension of the law which is of concern to economics. Economics may even prescribe laws that are not acceptable on grounds of morality; a temporary, for instance, measure to make black money official to cope with an urgent government budget distress. For
us, here, the economic aspect of the law which was of concern was its administration by the 
Judiciary given its prudence and the prudence of the law: How can state finances distort 
Judiciary’s prudence and induce subsequently coercion in the presence of myopic law 
preferences on the part of the public? And, to answer this question, one need not necessarily 
preserve any particular preference pattern suffices to obey the axioms of choice. It is also a 
question originating in admitting that public finance decisions and macroeconomics do matter 
in assessing judicial performance.

As Posner (2005, 1259) notes: “judicial behavior is best understood as a function of the 
incentives and constraints that particular legal systems place on their judges.” And, public 
economics and the macroeconomy do shape the economics of these constraints regardless the 
difference of legal systems across countries. This difference may be influencing the incentives 
but is not important at the level of the finances surrounding constraints. Of course, Siegel 
(1999, 1581) might disagree with this position on the grounds that “economic analysis 
provides an inadequate account of judicial behavior because economic models are 
 incompatible with a jurisprudence that recognizes basic rule-of-law values.” That is, state 
finances should not matter in so far as the “independence” side of the judicial system is 
concerned; but there is the “accountability” side too, the responsiveness of judicial 
decisionmaking to societal needs as framed by the incumbent political regime (see e.g. 
Contini and Mohr 2007). And, the public economy does come to play a significant role in 
practicing law even through this roundabout route. After all: “Even though judges may be 
independent from political control, they may become dependent on other forces, such as 
 senior judges in a judicial hierarchy, with just as much potential to distort individual decision-
making as more conventional political influence” (Garoupa and Ginsburg 2009, 6).

This is even more important when as e.g. Hatlebakk (2012) observes, myopic preferences 
on the part of the public is expected to be the case in low-income economies, where social-
welfare concerns permeate all manifestations of the state. Given a paternalistic social 
objective aiming at maximizing the sum over ex post utilities in such economies, taxation and 
government borrowing become critical to ensuring smooth intertemporal distribution (see e.g. 
Roeder 2009). Of course, myopia in these studies is taken to mean emphasis on the short-run 
by “the poor”, but this emphasis might be used to rationalize the assumption made in this 
paper that people do not care if their law preferences will change in the future. And, it is the 
vast majority of poor people in low-income economies, which might be taken to rationalize 
the robustness of our results to income distribution matters. But, in a developed economy, 
with its middle and high-income classes, myopia should be related only with the low-income 
class. This would be an interesting extension of this paper, which however lies beyond its 
scope.

But, we can still contemplate on this informally by noting that what in essence this paper 
claims is that people need not trust Justice and be lawyers to have a Justice system operating 
efficiently even under adverse economic circumstances if, of course, the subsequent 
macroeconomic policy does not impede the performance of the Judiciary. Within the 
development-stage nexus advanced in the previous paragraph, such a thesis adds to the 
esteeam of the legal system in a less developed country, which is important given the 
allegations about the connection of Justice in these countries with corruption (see e.g. Priks 
2011). In so far as developed countries are concerned, these countries are democracies with 
efficiency-friendly institutions (Pejovich and Colombatto 2008, chapter 5) expected to be
countering the allegations that democracy results in partisan court rulings, which need not be the optimal ones. It is for this perhaps reason that “a considerable number of judges [in developed democracies] think that such tendencies have no significant influence on the management of justice” (Llano 2013, Abstract). From this point of view, our thesis here adds to the esteem of the legal system in a developed-economy democracy as well.

ACKNOWLEDGEMENTS

I am grateful to an anonymous reviewer for useful comments and suggestions. Any remaining errors are my own.

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Footnotes

1 De Legibus (On the Laws), Book 1; see e.g.: http://www.gnomikologikon.gr/latin-quotes.html.

2 Under myopia in a three-period model of past-present-future, the future may be disregarded completely (see e.g., Kurz 1987). This is indeed genuine myopia and contrasts with the change-in-tastes approach adopted herein as a matter of necessity in a two-period setting. In such a setting, the complete disregard of the future would be meaningless. Also, a second type of change-in-tastes myopia is identified by Brown and Lewis (1981) when the discounting of the future diminishes with the passage of time. But, it is a myopia concept clearly for infinitely lived agents.

3 Judging for example from Davis and Trebilcock (2008), this assumption about the positive effect of law on growth may be a “heroic” one.

4 It is interesting to note that the case of within-period perfect competition is equivalent to perfect substitutability across periods: $p_i^* = k_i$ either when $\Pi_i^* = 0$ or when $e = 1$.

5 The difference $[H - (aL)^2]$ in the denominator of (3i) and (3ii) is now: $t[(1 - t)H - (aL)^2]$, while the term $H$ in (3iii) is multiplied by $(1 - t)$ and the term in the denominator of the square root is multiplied by $t^2$. The minus sign accompanying the square root in (3iii) becomes now $(\pm)$ as the equation in $L$ is quadratic and there is no a priori reason to reject any of the two roots. Also, $H$ in the numerator of (8i) and (8iii) is multiplied by $t(1 - t)$, the denominator of (8iii) is multiplied by $t$ too, whereas (8ii) remains unchanged.

6 Book IV of The City of God; see e.g., http://files.libertyfund.org/pll/quotes/200.html