Do differences in international labor mobility lead to differences in the fiscal multiplier? A theoretical approach

Pfammatter, Andrea Corina

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Andrea Corina Pfammatter
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Abstract: A real business cycle economy with endogenous labor supply and heterogeneous households is modeled. I allow for different degrees of labor migration to assess potential differences in the effects of changes in government consumption on aggregate economic activity. I argue that a relatively elastic labor migration with respect to economic activity may have a positive effect on the effectiveness of fiscal policy because labor migration may influence labor market adjustments after a positive government consumption shock. The findings suggest that there is a positive relationship between labor migration elasticity and the size of the fiscal multiplier. However, whether the relationship is economically meaningful is uncertain and requires further research.

Keywords: Fiscal multiplier, fiscal policy, RBC model, international labor migration.

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1 Introduction

During the last financial crisis many governments all around the world had to respond with expansionary fiscal policy in order to stimulate their recessive economies. When it came to decide on how big the respective stimulus packages should be, there was no broad agreement on the size of the effect of an increase in government consumption on economic activity, which is captured by the so-called fiscal multiplier. A recent contribution to this debate is the article by Ilzetzki et al. (2013). They show that the effect of government spending shocks depends on key country characteristics as for instance the degree of development, the exchange rate regime, openness to trade and public indebtedness. Brinca et al. (2015) contribute to the recent literature on the effectiveness of the fiscal multiplier by suggesting another key country characteristic, which is wealth inequality among the population of an economy. They explain the strong correlation between wealth inequality and the size of the fiscal multiplier by modeling an overlapping generations model with heterogeneous agents and calibrate it to match key characteristics of a number of OECD countries. Their findings are consistent with the strong correlation between wealth inequality and fiscal multipliers found in the data. Within the framework of this thesis, I focus on another potential determinant of the size of the fiscal multiplier. I investigate whether dynamics in international labor migration potentially influence the impact of government spending shocks on output.

Over the past couple of years, immigration became an important political issue among the developed OECD countries. That is because net migration rates to these countries have been rising substantially during the past decades, (Nickell, 2009). In 2014, the majority of the international migration flows into OECD countries were work related. Thus, conditions in the host labor markets were determinant for international migration (Arslan et al., 2014). Therefore, it is not surprising that not only employment rates were negatively affected by the global slowdown but also net migration rates decreased on average across OECD countries (OECD, 2014a). Along with the relationship between economic activity in the host country and net migration flows as well as the renewed interest in fiscal policy, I would like to find out whether differences in labor migration dynamics theoretically lead to economically meaningful differences in the size of the fiscal multiplier.
This Master thesis studies the effect of disturbances in fiscal policy on the economy within a basic neoclassical framework allowing for the possibility of immigration and emigration in response to economic activity. I consider a real business cycle economy that is opened to a certain extend, where migration of labor alters the population size and constellation, the labor force as well as the aggregate capital stock. Standard real business cycle theory generally predicts a positive effect of an increase in government consumption on economic output. However, the size of the fiscal multiplier, crucially depends on how households behave after such a change in fiscal consumption. First of all, it depends on how the increase in government consumption is financed. Within the alternative of altering taxation schemes to finance changes in government consumption, there are two different options. An increase in government consumption can be financed by higher lump-sum or distortionary taxes. In the case of lump-sum taxes, the multiplier is positive however smaller than one, since households adjust their consumption downwards due to the negative wealth effect and increase the supply of labor in order to be able to cover the higher tax expenses\(^1\). However, when changes in government consumption are financed by a higher distortionary tax, the fiscal multiplier is certainly below unity but could even be below zero, depending on how much private consumption as well as investment are affected. Households decrease their labor supply due to the fact that a current income tax lowers the return on labor. Also, private consumption as well as investment crowd out. Thus, if private consumption and investment are relative strongly affected by the increase in government consumption, the multiplier becomes negative.

When it comes to investigating the effects of public spending on economic activity Brinca (2006) approaches the question in an empirical way and uses a VAR (Vector Autoregressive model approach) in a country-specific framework for Sweden. His findings suggest that the growth rate of public investment indirectly affects GDP through the growth rate of private investment. In other words, growth in public investment stimulates aggregate output through private investments. Ramey (2011) summarizes the effects of fiscal policy on macroeconomic aggregates, neoclassical models among others hinge fundamentally on the effect that a change in government consumption has on the labor supply of employees.

\(^1\)The timing of tax collection does not matter since Ricardian equivalence holds. The Ricardian theorem proposes that deficit finance and current taxation are equal since individuals take into account the future taxes they have to pay (Blanchard and Fischer, 1989).
households and how this change in labor supply translates to changes in economic activity. Thus, the labor supply adjustments within the economy together with labor force adjustments across economies induced by higher government consumption and how the resulting changes in the labor market are transmitted to affect the economy’s output leads to the formulation of my research question. How do differences in labor mobility lead to differences in the size of the fiscal multiplier?

By investigating the effects of a change in government consumption on economic activity allowing for labor mobility, this thesis contributes to the theoretical field of research on the effects of fiscal policy. The goal of my thesis is to find out if a higher elasticity of labor mobility has a positive effect on the size of the fiscal multiplier. I find support for this hypothesis even though it is uncertain whether the differences are economically meaningful and therefore further research is needed in order to find out if the findings could have future policy implications.

The remainder of the thesis is organized as follows. After briefly discussing empirical evidence that motivates the form how labor migration modeled, I turn to the main contribution of my thesis: A simple dynamic general equilibrium model that incorporates international labor migration is developed in Section 2, followed by the calibration of the economy in Section 3. In Section 4, I analyze the results of impulse response functions of an increase in government consumption and calculate the fiscal multipliers. Section 5 summarizes the findings and concludes.

1.1 Motivation and empirical evidence

The economic consequences of migration have been intensively studied throughout the past. The impact of migration on the economy can be summarized into three areas. Migration is found to affect labor markets, the public purse and economic growth. According to a publication on the impact of migration on the economy by OECD (2014b), migrants accounted for nearly 50% of the increase in the workforce in the U.S. and 70% in Europe over the past ten years. When it comes to the impact on the public purse, work-related migrants contribute more in taxes and social contributions than they actually receive in benefits. Also, labor migrants relative to migrants that move due to other purposes have the
most positive impact on the public purse. The migration’s impact on economic growth is two-fold. On one hand, migration boosts the working-age population. On the other hand, migrants are endowed with skills that contribute to the human capital of an economy (OECD, 2014b). The fact that migration boosts working-age population and that migrants contribute to the human capital of an economy are the driving sources of my thesis.

A crucial feature of the model developed below is the possibility of labor immigration as well as labor emigration in response to economic opportunities (Barro and Sala-i Martin, 2004). More precisely, I assume that labor migration depends on the performance of economic output. Thus, migrant labor arrives during an economic expansion, when economic output lies above its long-run trend and leaves during a recession, when output lies below its long-run trend. The evidence on economic determinants of international migration flows is extensive. Bergheim (2008) motivates why population growth should be treated as endogenous variable in economic models. He argues that net migration is becoming a more important source of variation in population growth. He finds that the decision to migrate is on one hand driven by country-specific migration policies and on the other hand by the relative economic attractiveness of countries. In line with his arguments, Ortega and Peri (2013), Mayda (2010) and Grogger and Hanson (2011) among others find that international migration flows are highly responsive to income at destination. It is generally perceived that the relative income between source and destination country matters. Thus, if the income differential between source and destination country increases, migration is found to flow from the source country towards the economically more prosperous destination country.

Along with the support of empirical evidence on the relationship between international migration flows and economic activity, I continue with the main contribution of my thesis. In the following Section, a real business cycle economy extended with labor mobility that depends on economic performance, is constructed.
2 A real business cycle model

Within the neoclassical framework, Kydland and Prescott (1982) introduced the real business cycle theory that explains business cycle fluctuations of a micro-founded economy as the efficient answer to real exogenous shocks. Business cycles are caused by optimal responses of rational agents to real shocks, which can be fluctuations in productivity growth or fluctuations in government consumption or consumers’ preferences (Romer, 2012). In the model economy constructed here, a positive government consumption shock financed by an increase in lump-sum taxes is analyzed by allowing for labor migration to flow into and out of the economy.

The economy that is constructed here is drawn on the paper by Canova and Ravn (2000) who analyze the macroeconomic effects of German unification. I simplify their model in some sense and at the same time extend it with features in order to be able to analyze the effects of increased government spending on output in the presence of labor migration. I also follow Galí et al. (2007) who extend a basic neoclassical framework with rule-of-thumb households in order to investigate the effects of government expenditures on private consumption. Even though Galí et al. (2007) do not model international labor mobility, their model is a useful guideline for the setup of the model constructed here. This is because they model two heterogeneous types of households as well as I intend to do and also because they simulate a positive government consumption shock in presence of these two types of households to investigate the impact on macroeconomic variables.

Even though the model is highly stylized, it contains crucial features necessary to analyze and understand the issue of interest. The model developed here is a basic one-sector model with four different types of actors that focuses on one country which allows for labor migration. It features two types of optimizing households, a representative firm that employs all labor and uses total capital stock to produce and a government that runs a period-by-period balanced budget and finances its consumption by lump-sum taxes and distortionary income taxes.
2.1 Demographics

It is assumed that at time 0, native as well as migrant households populated the economy. They arrived all at once in a manner of a land rush, where I normalize the initial size of the total population to one (Barro and Sala-i Martin, 2004). I assume that the two types of households differ in their degree of international labor mobility. Native households represent the immobile type of households while migrant households are assumed to be able to freely move between the economy and the rest of the world. The stock of migrant households is assumed to be influenced by economic output which implies that population growth is not held at a constant exogenous rate but adopts an endogenous manner throughout time. As motivated in Section 1.1, I assume that migrant labor arrives during an economic expansion when it is needed and promptly returns to the country of origin when the economy is finding back to its long-run steady-state level. However, this logic only works in the absence of labor mobility restrictions (Mandelman and Zlate, 2012). Therefore, I assume that there is no border enforcement that would destabilize the connection between higher labor demand during expansion and increase in the stock of migrant households. Equation (1) captures the effect of a deviation of actual output from its steady-state value on the stock of migrant households, where parameter $\theta$ represents the elasticity of migration to and from the economy with respect to output. Thus, if $\theta < 1$ the deviation of output from its steady-state level increases the migrant population to a lower extent. However, if $\theta > 1$ the opposite interpretation holds; the migrant population is relatively more responsive than the actual deviation of the output.

$$x_t = \left(\frac{Y_t}{Y}\right)^\theta$$

While the stock of native households $N^n$ is constant over time, the stock of migrant households $N^m_t$ is a function of $x_t$ and its initial size $N^m$.

$$N^n_t = N^n$$

$$N^m_t = x_t N^m$$

Thus, a sudden increase in economic activity leads to an unexpected expansion of the stock of migrant households within the same period, whereas the native stock remains unchanged. When output is eventually converging back to its steady-state level, the stock of migrant households also converges back to its
steady state. Since the initial population is normalized to 1, the following relation between the two types of households can be derived as

\[ N_m = (1 - N^n) \]  
\[ N_t^m = x_t(1 - N^n) \]  

In steady state, \( x_t = 1 \) which implies that whenever the economy is in a steady state, the size of the population equals its initial size 1, since both stocks of households equal their steady-state sizes. In this context, labor migrants can be seen as a certain type of seasonal workers who on one hand immigrate when there is need for them in the economy, and on the other hand emigrate one-by-one when the economy is finding back to its balanced growth path. Having specified the size of each type of household in (4) and (5), the size of the total population at time \( t \) can be written as

\[ N_t = N^n + N_t^m \]
\[ = N^n + x_t(1 - N^n) \]  

This leads to the expression for the change of the total population \( g_t^p \)

\[ g_t^p = \frac{N_{t+1}}{N_t} = \frac{N^n + x_{t+1}(1 - N^n)}{N^n + x_t(1 - N^n)} \]  

The share of each household in the total population can be written as

\[ \gamma_t \equiv \frac{N^n}{N_t} = \frac{N^n}{N^n + x_t(1 - N^n)} \]
\[ (1 - \gamma_t) \equiv \frac{N_t^m}{N_t} = \frac{N_t^m}{N^n + x_t(1 - N^n)} \]  

2.2 The households

There are many infinitely lived households in the economy who are either migrant or native households. Each household consists of one member which is considered not to change over time. It is assumed that migrant and native households only differ with respect to the ability of labor mobility. Apart from the fact that native households are immobile and migrant households are not, both are equal in every sense. Both types are endowed with the same skill level, have equal returns on labor as well as on capital. Furthermore, I assume that
newcomers directly acquire the political, legal and work rights of the economy, which rules any emerging market inefficiencies out.

Klein and Ventura (2009) and Mandelman and Zlate (2012) investigate dynamic effects of labor movements in a two-location growth model, where physical capital moves freely across the two economies. Their models imply that migrant households are able to move their capital between economies and also that they are able to actually accumulate capital. I adopt their assumption and allow newcomers to move their capital stock with them. Thus, if relatively more migrant households are attracted to the economy, there will be more human as well as more physical capital available for the firms to use for production. The opposite holds, if relatively less migrant households are located in the economy.

Also, I assume that both household types follow a Cobb-Douglas utility function that is monotonically transformed. As a result, utility over consumption and leisure is additively separable and is consistent with balanced growth in the RBC framework. Since all households within a type are identical, I consider a representative native household as well as a representative migrant household, for whom I solve separate utility maximization problems.

2.2.1 The representative native household

At date $t$, the representative native household maximizes the following expected utility

$$\max_{\{c^n_t, h^n_t, k^n_{t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t \{\log(c^n_t) + A \log(1 - h^n_t)\}$$

subject to the inter-temporal budget constraint and the capital evolution equation

$$c^n_t + i^n_t = (1 - \tau) [w_t h^n_t + R_t k^n_t] - t^n_t$$
$$k^n_{t+1} = (1 - \delta) k^n_t + i^n_t$$

where all lower case letters denote per-capita variables. $c^n$ denotes consumption and $h^n$ is hours worked. Since the time endowment is normalized to one, such that $1 - h^n = l^n$, where $l^n$ denotes leisure. Then, $k^n$ refers to the native household’s capital stock, $\tau$ is a uniform tax rate on labor and capital income,
while $t_t$ denotes lump-sum taxes. Also, $\beta$ is the subjective discount factor, and $A$ is the preference parameter for leisure which measures the intensity of the households’ preferences for leisure relative to consumption. Hence, if $A$ is high, households are willing to supply a relatively small amount of labor to obtain consumption goods and instead prefer to consume relatively more leisure (Fehr, 1999).

The native household’s Euler equation and the optimal labor supply condition are \(^2\)

\[
\frac{1}{c^n_t} = \beta E_t \left\{ [1 + (1 - \tau)R_{t+1} - \delta] \frac{1}{c^n_{t+1}} \right\} \tag{12}
\]

\[
h^n_t = 1 - \frac{A c^n_t}{(1 - \tau)w_t} \tag{13}
\]

The Euler equation captures the idea that the optimizing household is indifferent between consuming in time period $t$ and $t + 1$. Thus, the left-hand side of Equation (12) quantifies the utility loss of saving one unit of consumption in $t$ while the right-hand side amounts the utility gain in $t + 1$ of saving that particular unit of consumption in time period $t$. The optimal labor supply condition in (13) captures the relationship between consumption and labor supply for every given real wage rate in time period $t$. Thus, the Euler equation is the household’s inter-temporal optimality condition, while the labor supply decision is referred to as intra-temporal optimality condition.

\subsection*{2.2.2 The representative migrant household}

The identical utility maximization problem can be solved for the representative migrant household. At date $t$, the representative migrant household maximizes the following expected utility

\[
\max_{\{c^m_t, h^m_t, k^m_{t+1}\}} E_t \sum_{t=0}^\infty \beta^t \{ \log(c^m_t) + A \log(1 - h^m_t) \} \tag{14}
\]

subject to

\[
c^m_t + i^m_t = (1 - \tau) [w_t h^m_t + R_t k^m_t] - t^m_t \]

\[
k^m_{t+1} = (1 - \delta) k^m_t + i^m_t \tag{15}
\]

\(^2\)See Appendix A.1 for the derivation of the first order conditions and calculations.
Analogue to the native household, the solution to the migrant household’s problem can be summarized in the Euler equation and the optimal labor supply conditions.

\[
\frac{1}{c^m_t} = \beta E_t \left\{ \left[ 1 + (1 - \tau) R_t + 1 - \delta \right] \frac{1}{c^m_{t+1}} \right\} \tag{16}
\]

\[
h^m_t = 1 - \frac{Ac^m_t}{(1 - \tau)w_t} \tag{17}
\]

### 2.2.3 Aggregation

In order to get aggregate variables, per native household variables and per migrant household variables, respectively, have to be converted into the same unit. This happens by using the definition of \( \gamma_t \) and \( (1 - \gamma_t) \). Multiplying the per respective household variables by their relative share in the population yields to aggregate migrant household variables and aggregate native household variables. In a next step, the respective aggregate household variables are multiplied by the size of total population \( N_t \) to get absolute aggregates for native households and migrant households. An addition of those yields to aggregate variables.

\[
C_t = N_t \gamma_t c^n_t + N_t (1 - \gamma_t) c^m_t \tag{18}
\]

\[
H_t = N_t \gamma_t h^n_t + N_t (1 - \gamma_t) h^m_t \tag{19}
\]

\[
K_t = N_t \gamma_t k^n_t + N_t (1 - \gamma_t) k^m_t \tag{20}
\]

Within this framework, labor migration does not affect household-specific per-capita variables, since neither the variables \( x, \gamma \) nor \( N \) show up in the optimality conditions of each household type. Thus, whenever the economy is hit by a positive government consumption shock, household-specific per-capita variables are equally affected by it. Differences in the responses to the shock between the two household types will only be observable in aggregate variables, i.e., either in the aggregates of each household type or in aggregates of the total population as defined in (18), (19) and (20).

---

3See Appendix A.1 for the derivation of the first order conditions and calculations for the native household. The same calculations hold for the migrant household.
Hence, the economy perceives an influx of additional migrant households as an increase of the stock of identical migrant households who consume, work and invest the same amount as the households already located in the economy. Therefore, it is implicitly assumed that newcomers are endowed with the same capital stock as the other households in the economy. In the case of emigration, households bring their capital stock, which results in a reduction of the aggregate (migrant) capital stock. However, the migrant per-capita capital stock remains unchanged.

2.3 The representative firm

There is one representative firm that uses two input factors for production $Y_t$; aggregate labor expressed in working hours ($H_t$) and aggregate capital stock ($K_t$). The production function is Cobb-Douglas with constant returns to scale to both input factors.

$$ Y_t = F(K_t, H_t) = K_t^\alpha H_t^{1-\alpha} \quad (21) $$

Labor input is a CES-aggregate of native worker hours ($H^n_t$) and migrant worker hours ($H^m_t$) with an elasticity of substitution of $1/\rho$. Furthermore, it is assumed that the labor market is always cleared, i.e., labor demand equals labor supply which leads to zero unemployment in the economy.

$$ H_t = F(H^n_t, H^m_t) = \left[(H^n_t)^{1-\rho} + (H^m_t)^{1-\rho}\right]^{\frac{1}{1-\rho}} \quad (22) $$

The elasticity of substitution between immigrant and native labor has been subject in the empirical field of research. For instance, Ottaviano and Peri (2012) find a small but significant degree of imperfect substitutability between natives and immigrant workers, i.e., $\rho > 0$. However, Borjas et al. (2008) show that their finding of imperfect substitution is fragile due to heterogeneity in labor market attachment among workers. However, controlling for this heterogeneity, the evidence for immigrant-native complementarity vanishes. Therefore, it cannot be rejected that native and migrant workers with equal skill endowment are perfect substitutes. Relying on (Borjas et al., 2008), I set $\rho$ equal to zero and assume perfect substitutability between migrant and native workers. This simplifies Equation (22) to the following expression

\footnote{Note that the inverse of the elasticity of substitution is the elasticity of complementarity, i.e., $\rho$.}
\[ H_t = F(H_t^n, H_t^m) = H_t^n + H_t^m \]  

Concerning the second input factor capital, both households own the aggregate capital stock and rent it out to the representative firm. This leads to the following maximization problem of the representative firm

\[
\max_{\{H_t, K_t\}} K_t^\alpha H_t^{1-\alpha} - w_t H_t - R_t K_t
\]

where \( w_t \) is the real wage paid to the workers, \( R_t \) is the rental rate of capital, and \( \alpha \) is the capital share of income. The first order conditions of the firm’s maximization problem are summarized below.

\[ K_t : R_t = \alpha \left[ \frac{K_t}{H_t} \right]^{(\alpha-1)} \]

\[ H_t : w_t = (1 - \alpha) \left[ \frac{K_t}{H_t} \right]^\alpha \]

### 2.4 The government

The aim of the government in the economy is to stimulate economic activity by a one time increase in government consumption. The government is forced to balance its budget on a period-by-period basis. I assume that government consumption is financed by distortionary as well as non-distortionary taxation. I follow Baxter and King (1993) and do not explicitly consider the alternative of debt financing here. Since the fiscal experiment that I analyze in the next section is financed by an increase in the lump-sum tax rate, Ricardian equivalence holds. Debt financing, when the tax rate is held constant, has the same implications as financing government consumption by lump-sum taxation. Thus, the public finance rule can be written as

\[
\tau(w_t H_t + R_t K_t) + T_t = G_t
\]

where \( T_t = N_t [\gamma t^n_t + (1 - \gamma) t^m_t] \). Also, taxes are evenly distributed across households, which leads to \( t^n = t^m \) in every time period \( t \). Government consumption, in deviation from its deterministic steady-state value, follows an exogenous stochastic AR(1)-process with persistence parameter \( \rho_G \) and \( \epsilon_t \sim N(0, \sigma^2) \), where \( G \) corresponds to the steady state of government consumption (Dupaigne, 2014).
\[
\log(G_t) = (1 - \rho_G) \log(G) + \rho_G \log(G_{t-1}) + \epsilon_t
\] (28)

2.5 The equilibrium

2.5.1 The households

It is straightforward to transform both Euler equations to get two equilibrium conditions for the economy. Plug in the factor prices \( R_{t+1} \) into both Eulers stated in (16) and (12).

\[
\frac{1}{e_t^{cm}} = \beta \mathbb{E}_t \left\{ \left[ 1 + \alpha (1 - \tau) \left( \frac{K_{t+1}}{H_{t+1}} \right)^{\alpha - 1} - \delta \right] \frac{1}{e_{t+1}^{cm}} \right\} 
\] (29)

\[
\frac{1}{e_t^{cn}} = \beta \mathbb{E}_t \left\{ \left[ 1 + \alpha (1 - \tau) \left( \frac{K_{t+1}}{H_{t+1}} \right)^{\alpha - 1} - \delta \right] \frac{1}{e_{t+1}^{cn}} \right\} 
\] (30)

Both labor supply conditions in (17) and (13) can be rewritten as

\[
h_t^m = 1 - \frac{A e_t^m}{(1 - \alpha)(1 - \tau) \left( \frac{K_t}{H_t} \right)^\alpha} 
\] (32)

\[
h_t^n = 1 - \frac{A e_t^n}{(1 - \alpha)(1 - \tau) \left( \frac{K_t}{H_t} \right)^\alpha} 
\] (33)

2.5.2 The resource constraint

The economy’s resource constraint is obtained by first expressing both households’ budget constraints into aggregate terms. Second, sum them as well as the government budget constraint up and use Equations (18), (19) and (20) to simplify the equations\(^5\). This leads to the following expression for the economy’s resource constraint

\[
C_t + I_t + G_t = Y_t 
\] (34)

Equation (34) shows how total income/output is used in the economy. It is divided between aggregate consumption, government consumption and investment.

\(^5\)See Appendix A.2 for a detailed derivation of both households’ budget constraints in aggregate per-capita terms as well as calculations that yield to the economy’s resource constraint.
Now, given the initial position of the economy, a competitive equilibrium can be defined as a sequence of prices \( \{ R_t, w_t \}_{t=0}^{\infty} \), taxes \( \{ \tau, \tau^m, \tau^n \}_{t=0}^{\infty} \), population share \( \{ \gamma \}_{t=0}^{\infty} \) and quantities \( \{ e^n_t, h^n_t, k^n_t, e^m_t, h^m_t, k^m_t \}_{t=0}^{\infty} \) such that:

1. Given factor prices \( \{ R_t, w_t \}_{t=0}^{\infty} \) and taxes \( \{ \tau, \tau^n \}_{t=0}^{\infty} \), the allocation \( \{ e^n_t, h^n_t, k^n_t \}_{t=0}^{\infty} \) solves the native household’s maximization problem in (10).

2. Given factor prices \( \{ R_t, w_t \}_{t=0}^{\infty} \) and taxes \( \{ \tau, \tau^m \}_{t=0}^{\infty} \), the allocation \( \{ e^m_t, h^m_t, k^m_t \}_{t=0}^{\infty} \) solves the migrant household’s maximization problem in (14).

3. Given factor prices \( \{ R_t, w_t \}_{t=0}^{\infty} \), the allocation \( \{ H_t, K_t \}_{t=0}^{\infty} \) solves the firm’s maximization problem in (24).

4. Taxes \( \{ \tau, T_t \}_{t=0}^{\infty} \) are such that the government’s budget constraint in (27) is satisfied.

5. All markets clear.

6. Total population evolves according to 
\[
g_t^p = \frac{N^n + x_{t+1} (1-N^n)}{N^n + x_t (1-N^n)}
\]

### 3 Calibration and multipliers

In order to solve the nonlinear stochastic general equilibrium model I use the DYNARE implementation for Matlab which takes a first-order linear approximation around the steady state. Following previous research as for instance Galí et al. (2007) or Riguzzi et al. (2014), I choose the variance of \( \epsilon \) in Equation (28) such that the increase in government consumption amounts to 1% of steady-state output.\footnote{\textsuperscript{6}The shock’s standard deviation of 0.05 multiplied by 0.2, which is the fraction of output that is dedicated to government consumption in steady state, then corresponds to a 1% of steady-state output.}

### 3.1 The steady state

The economy is assumed to initially be in steady state, in which all variables (in per capita terms as well as in levels) are constant over time. Since in steady state, \( x = \frac{Y}{Y} = 1 \), there is no migration flowing into or out of the economy which implies that the economy is closed in every respect. This implies that the relative change of the population size defined in (7) simplifies to 
\[
g_t^p = g^p = 1.
\]

Since neither the native population nor the migrant population changes in the

---

\[
\text{14}
\]
steady state, the total population hence does not change neither. This means that the shares, $\gamma$ and $(1 - \gamma)$ are also constant in steady state. Furthermore, since both household types have an identical utility structure and face the same factor prices they do not differ from one and another. All endogenous variables in steady state can be expressed as functions of capital per hours worked $K/H$. Due to the fact that the size of the population equals 1, aggregate variables equal per-capita variables.

On a balanced growth path the return on capital, i.e., the interest rate is assumed to be constant over time.

$$r_t = (1 - \tau)R_{t+1} - \delta$$  \hfill (35)$$

$$r_t = \alpha(1 - \tau)\left[\frac{K_{t+1}}{H_{t+1}}\right]^{\alpha - 1} - \delta$$  \hfill (36)$$

Hence, if the left hand side of (36) is constant, the right hand side must also be constant over all $t$. This is in line with Kaldor’s facts since $\left[\frac{K_{t+1}}{H_{t+1}}\right]^{\alpha - 1} = \frac{Y_{t+1}}{K_{t+1}}$ is the capital-output ratio, which is constant on a balanced growth path.

$$r = \alpha(1 - \tau)\frac{Y}{K} - \delta$$  \hfill (37)$$

Imposing $K_t/H_t = K_{t+1}/H_{t+1} = K/H$ in the Euler equations for both households in (29) and (30), leads to the fact that consumption growth is also constant over all $t$. However, since there is no growth within this framework, this leads to equal consumption levels over all $t$, i.e. that $c^a_t = c^a_{t+1} = c^a$ and $c^m_t = c^m_{t+1} = c^m$. The value for $K/H$ is obtained by solving one of the Eulers with respect to $K/H$.

$$\frac{K}{H} = \left[\frac{\alpha(1 - \tau)}{1/\beta + \delta - 1}\right]^{\frac{1}{\alpha - 1}}$$  \hfill (38)$$

The derivation of the remaining steady stats is straightforward. The steady-state value for the factor prices can be written as

$$R = \alpha \left(\frac{K}{H}\right)^{\alpha - 1} = \left[\frac{1/\beta + \delta - 1}{(1 - \tau)}\right]$$  \hfill (39)$$

$$w = (1 - \alpha) \left(\frac{K}{H}\right)^{\alpha} = (1 - \alpha) \left[\frac{\alpha(1 - \tau)}{1/\beta + \delta - 1}\right]^{\frac{\alpha}{\alpha - 1}}$$  \hfill (40)$$

Rewriting and dividing the resource constraint in (34) by $Y$, consumption-
output ratio can be written as

\[
\frac{C}{Y} = 1 - \omega - \delta \left[ \frac{K}{H} \right]^{1-\alpha} 
\]

\[
= 1 - \omega - \delta \left[ \frac{\alpha(1-\tau)}{1/\beta + \delta - 1} \right] 
\]  

Concerning the optimal labor supply of each household, the following relationship holds \(^7\).

\[
H = \frac{\alpha(1-\tau)}{\alpha(1-\tau) + A \frac{C}{Y}} 
\]  

Due to the fact the variable that captures labor migration does not show up in steady state, a numerical analysis is necessary to shed light on the dynamics of the model when the economy developed above is hit by a shock to government consumption.

### 3.2 Calibration

The calibration of the model is chosen to match U.S. data. I follow the calibration in Galí et al. (2007) where each period is assumed to be a quarter. The values are summarized in Table 1. The preference parameter \(A\) is chosen as in Baxter and King (1993) such that households decide to work 20\% of their time in steady state \(^8\). According to the Migration Policy Institute, the share of foreign born in the U.S. population was 13\% in 2013. To my knowledge the value for the year 2013 is the most recent figure available, which I will therefore use in my calibration. This leads to a steady-state \(\gamma\) of 0.87. When it comes to the autoregressive parameter of government consumption, Galí et al. (2007) choose as benchmark value of 0.9, which matches the half-life of the responses of government consumption. Hence, it reflects the relatively high persistence of the government spending to its own shock.

---

\(^7\)See Appendix A.3 for calculations.

\(^8\)See Appendix A.4 for calculations.
Table 1: Parameter values or ranges

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$A$</td>
<td>3.5</td>
<td>Preference parameter</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.87</td>
<td>initial share of native households</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$1/3$</td>
<td>Total labor share of income</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
<td>Capital depreciation rate</td>
</tr>
<tr>
<td>$\rho_G$</td>
<td>0.9</td>
<td>Persistence parameter</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.2</td>
<td>exogenous income tax rate</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\in [0, 1]$</td>
<td>labor migration elasticity</td>
</tr>
</tbody>
</table>

Due to lack of suitable data, the labor migration elasticity $\theta$ is arbitrarily set and ranges between 0 and 1, where $\theta = 0$ describes a closed economy, in which labor migration is ruled out. When $\theta = 1$, labor migration reacts perfectly elastic to a change in output relative to its steady-state value. It is important to keep in mind when interpreting the results later that $\theta$ is an arbitrary number.

In order to get an idea how much the size of the migrant population within an economy changes in reality, I examine data on the stock of foreign-born population in the U.S. between 1994 and 2013 (OECD, 2015). Between those years, the stock of foreign-born population continuously increased apart from 1999 and 2008 when there was a negative change relative to the previous year. The average annual %-change amounts to approximately 3.6%, i.e., 0.88% in per average quarter terms. Remember though that this value includes all kinds of migration flows that influence the stock of foreign-born in the U.S. and not only work-related migration as it is the case in the model studied here. Thus, 0.88% is used as upper-bound threshold for the simulated relative percentage change of the stock of migrant households. However, even though the simulated percentage change may be below that threshold, this does not directly imply that the results found are economically meaningful, since it is still unclear how much the simulated percentage change of the migrant population must be below that threshold. Thus, any finding on the size of the fiscal multiplier must be treated as suggestive rather than authoritative result.

Table 2 summarizes the steady-state values for the endogenous variables. Following Galí et al. (2007) I set steady-state government consumption to 20% of
steady-state output, which roughly corresponds to the average share of government expenditures in postwar U.S. data. This implies a steady-state income tax level of 0.2 when assuming that the steady-state lump-sum taxes equal zero.

Table 2: Steady-state values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k/h$</td>
<td>20.94</td>
<td>Aggregate capital-labor ratio</td>
</tr>
<tr>
<td>$R$</td>
<td>0.04</td>
<td>Real interest rate</td>
</tr>
<tr>
<td>$w$</td>
<td>1.84</td>
<td>Real wage</td>
</tr>
<tr>
<td>$c/y$</td>
<td>0.61</td>
<td>Aggregate consumption-output ratio</td>
</tr>
<tr>
<td>$h$</td>
<td>0.2</td>
<td>Per-capita hours worked</td>
</tr>
<tr>
<td>$k$</td>
<td>4.19</td>
<td>Capital stock</td>
</tr>
<tr>
<td>$y$</td>
<td>0.55</td>
<td>Output level</td>
</tr>
<tr>
<td>$g$</td>
<td>0.11</td>
<td>Government consumption level</td>
</tr>
<tr>
<td>$c$</td>
<td>0.34</td>
<td>Private consumption level</td>
</tr>
</tbody>
</table>

3.3 Fiscal multiplier

The fiscal multiplier quantifies the effect on economic output from a fiscal expansion. Thus, if an increase of 1 unit in government consumption causes an increase of aggregate output by 0.5 units, the resulting multiplier amounts to 0.5. I follow Ilzetzki et al. (2013) for the calculation of two different types of the fiscal multiplier. The impact multiplier describes the immediate impact of the increase in government consumption on aggregate output, whereas the cumulative multiplier depicts the effect of the change in government consumption over a longer forecast horizon $T$.

\[
\text{Impact multiplier} = \frac{\Delta Y_t}{\Delta G_t} \quad (44)
\]

\[
\text{Cumulative multiplier} = \frac{\sum_{t=1}^{T} \Delta Y_t}{\sum_{t=1}^{T} \Delta G_t} \quad (45)
\]

Typically, the cumulative multiplier is larger than the impact multiplier due to the fact that there may be lags in the effects (Spilimbergo et al., 2009; Chinn, 2013). Thus, if for example the maximum effect on output occurs in some periods after the shock actually has hit the economy, the impact multiplier results to be smaller than the cumulative multiplier evaluated at the time period
where output is affected the most relative to all other periods considered in the analysis. Within the framework chosen here, the impact multiplier is always larger than the cumulative multiplier evaluated at any other time period because the effect of an increase in government consumption on output results to be largest at impact. I list plots that show impact as well as cumulative multipliers for different $\theta$ in the Appendix 10.

4 Results

In the present section, I analyze the effects of a 1% increase in government consumption of steady-state output in the economy described above. The focus particularly lies on the influence of the labor mobility parameter $\theta$. It is to find out how sensitive the impact multiplier is with respect to the responsiveness of labor migration to a positive government consumption shock.

The impact multiplier in Equation (44) can be rewritten since the simulated temporary and exogenous shock to government consumption amounts to 1% of steady-state output $9$.

$$\text{Impact multiplier} = \frac{\Delta Y_t}{\Delta G_t} = 100 \cdot \frac{Y_t - Y}{Y}$$

(46)

Within this framework, the impact multiplier can be interpreted as just the relative change of aggregate output times the inverse of the exogenous shock to government consumption. Figure 1 summarizes impact multipliers with different values for the migration elasticity $\theta$, ranging from 0 to 1. The positive relationship between the impact multiplier and the migration elasticity supports the idea that relatively more responsive labor mobility induces a higher effectiveness of fiscal policy. To find a plausible explanation for this positive relationship, an inspection of the impulse response functions of the variables is necessary. Furthermore, it will give instructive information on the different transition mechanisms that lead to that result.

$^9$See Appendix A.5 for calculations.
Figure 1: The relationship between impact multiplier and labor migration elasticity.

Figures 2a and 2b show impulse response functions of aggregate variables to the government consumption shock with $\theta = 0$ and $\theta = 1$, respectively. The impulse responses of the variables seem to look quite similar in both figures. In both cases output and labor supply jump up to a certain level and eventually converge back to their steady state levels. These positive responses of output and labor supply are expected. A change in government consumption financed by an increase in lump-sum taxes has a negative wealth effect on households since households' income reduces by the same amount as the increase in government consumption. This negative wealth effect leads to an increase in labor supply at any given real wage in order to increase labor income that is supposed to cover the increase in their tax burden. Higher labor supply translates into higher labor demand which in turn yields to an increase in output. Due to the negative wealth effect, households not only reduce leisure but also consumption, which is confirmed by the impulse response function of aggregate private consumption. Also aggregate investment is crowded out and decreases at impact before it converges back to its initial steady state.
Even though, the inspection of these figures may have helped to get an instructive idea what happens in the economy after a government consumption shock depending on the degree of the labor migration elasticity, it is still unclear what makes fiscal policy more effective in presence of labor mobility. In a next step, impulse response functions of labor supply, investment and consumption for \( \theta = 0 \) and \( \theta = 1 \) are compared in order to see how these differ depending on the size of \( \theta \).

4.1 Labor supply

Figure 3 shows the difference in the responsiveness of aggregate labor supply depending on the size of \( \theta \). The responsiveness of labor supply with labor migration is higher than without labor migration. Since there is no unemployment in the economy, an increase in total population due to the arrival of new migrant households leads immediately to an increase in the labor force. A difference between the impulse response functions of migrant per-capita labor supply would imply that households would take into account that there are more households supplying labor and would therefore adjust their supply of work hours. Since this is not the case, an adjustment of the intensive margin due to labor migration can be ruled out. No matter if there is labor migration or not, households’ labor supply response to the positive government consumption shock remains the same. This makes sense since each household experiences the same increase in taxes and therefore increases labor supply in the same way. This leads to the conclusion that the difference in the responsiveness of the aggregate labor supply only arises from the increase in the extensive margin, i.e., more households
in the economy means more labor supplied, nevertheless at the same optimal level of hours per person as without labor migration.

Figure 3: Impulse response function of aggregate labor supply depending on \( \theta \).

4.2 Investment

Inspecting the two impulse response functions in Figure 4, leads to the observation that aggregate investment responds relatively less negative in the presence of labor migration than without. The crowding-out effect on investment is mitigated through the positive effect on the migrant capital stock enhanced by the arrival of additional households. See the impulse response functions of the individual households’ capital stocks in Figure 8 in the Appendix. While aggregate migrant capital stock jumps from its steady state at impact and from there decreases due to the increase in the real interest rate, native capital stock remains at its steady-state level at impact and decreases from there. However, both capital stocks recover and eventually find back to their steady states.
4.3 Consumption

Similar to the response of aggregate investments, aggregate consumption instantaneously decreases below its steady-level and converges back to its initial steady state. However, the negative effect on consumption is mitigated by the inflow of migrant households. Aggregate migrant consumption increases at first due to the fact that the share of migrant households in total population increases. However, the positive effect on aggregate migrant consumption is of short duration. Newcomer households perceive the negative wealth effect of the increase in government consumption straight away and decrease their consumption in order to cope with the relatively higher tax burden.
Thus, there is less crowding out of private consumption as well as investment in the presence of labor mobility. Newcomers enhance higher aggregate consumption as well as higher aggregate investment at arrival to the economy which mitigates the negative wealth effect of the increased tax burden. This implies that relatively more labor as well as capital is available for the representative firm to use for production. Therefore, also output happens to be relatively higher than in a closed economy scenario.

Therefore, the implications of labor mobility, as it is modeled here, leads to a relatively larger impact multiplier because of the mitigating effects of a larger population on the crowding out of private consumption and investment. However, for this to be an economically meaningful result, it is necessary to make sure that the arbitrarily chosen range for $\theta$ produces reasonable changes in the population. If the effect of the government consumption shock on the size of total population is too large, then the mitigating effect on the crowding out of private consumption and investment results to be unreasonably large which in turn leads to an unreasonably large effect on output. The result of a higher effectiveness of fiscal policy in the presence of labor mobility may not be an economically meaningful but rather a random result that crucially depends on the choice of $\theta$. To reduce that risk impulse response functions of the labor
migration variable $x$ is analyzed and the relative percentage change of the stock of migrant households is calculated.

Table 3 summarizes the changes of the stock of migrant households relative to the initial size of 0.13. The impulse responses of $x_t$ suggest that the increase in the size of the migrant population in the shock period lies between 0.04% when $\theta = 0.1$ and 0.5% when $\theta = 1$. This result seems fairly plausible when comparing it to 0.88%. However, even though all values lie below real data, it is unclear how close to reality the simulations are since it is unknown how large in average the fraction of employment-related migrants in the total stock is. Therefore, it is very important that the results are interpreted with caution.

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Stock size at impact</th>
<th>relative %-change</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>13005.76</td>
<td>0.044%</td>
</tr>
<tr>
<td>0.2</td>
<td>13011.66</td>
<td>0.090%</td>
</tr>
<tr>
<td>0.3</td>
<td>13017.70</td>
<td>0.136%</td>
</tr>
<tr>
<td>0.4</td>
<td>13023.90</td>
<td>0.184%</td>
</tr>
<tr>
<td>0.5</td>
<td>13030.25</td>
<td>0.233%</td>
</tr>
<tr>
<td>0.6</td>
<td>13036.77</td>
<td>0.283%</td>
</tr>
<tr>
<td>0.7</td>
<td>13043.46</td>
<td>0.334%</td>
</tr>
<tr>
<td>0.8</td>
<td>13050.32</td>
<td>0.387%</td>
</tr>
<tr>
<td>0.9</td>
<td>13057.37</td>
<td>0.441%</td>
</tr>
<tr>
<td>1</td>
<td>13064.61</td>
<td>0.497%</td>
</tr>
</tbody>
</table>

Notes: The size of the stock is scaled up by 100'000 to facilitate the reading. %-change is relative to the steady-state stock size of 13’000.

The results from the simulation of a 1% shock to government consumption, suggest that a higher labor migration elasticity leads to a higher effectiveness of expansionary fiscal policy. Also, the difference between the size of the multiplier increases as the elasticity increases. Thus, between the arbitrarily chosen range of $[0, 1]$ for $\theta$, the relationship between the impact multiplier and the labor migration elasticity is convex. This makes sense since the more elastic the labor migration with respect to the deviation of economic activity to its steady
state, the more labor as well as the more capital flow into the economy, which in turn translates into relatively more output. The simulated changes in the stock of migrant households result to be in a plausible range, which may indicate an economically meaningful relationship between international labor migration and the effectiveness of fiscal policy. However, for this result to have future policy implications further research is needed in order to figure out how plausible the simulated changes in stock of migrant households are.

5 Conclusion

The aim of this thesis was to find out whether differences in the labor migration elasticity lead to economically meaningful differences in the size of the fiscal multiplier. In order to do so, I constructed a real business cycle model with native and migrant households that only differ in their ability to move across economies. The findings support the hypothesis that a higher labor migration elasticity leads to a larger fiscal multiplier. The reason why the size of the impact multiplier is larger is because the crowding out of aggregate private consumption and investment caused by a 1 % increase in government consumption are mitigated by initial increases of aggregate migrant consumption as well as aggregate migrant investment. However, the question whether the differences in the size of the fiscal multipliers are economically meaningful has not a clear answer. Even though the simulated changes in the migrant population lie in a plausible range, it is not certain if these simulated changes match the changes in the data. The differences in the size of the multipliers are only economically meaningful if they match. Thus, it needs further research to find out if international labor migration is a determinant of the size of fiscal multipliers. The results only suggest the possibility for labor mobility to be a determinant of the effectiveness of fiscal policy.

Having this in mind, it would be really interesting for future research to investigate further about possible implications of international labor mobility on fiscal policy outcomes. A good starting point for future research would be to answer the question if the simulated changes in the stock of migrant households match changes in the stock of work-related migrant households in the data. Due to the fact that the model developed here includes relatively basic features of an econ-
omy, it would be interesting to see how the results would change if more realistic features would be modeled. For instance, Ferriere and Navarro (2014) suggests that the assumption of indivisible labor allows to break the tight link between government consumption, private consumption and hours worked, which implies a milder reaction of aggregate private consumption for a given increase in labor. Within the framework of this thesis, indivisible labor may lead to even milder crowding out of private consumption. Furthermore, choosing a new Keynesian framework that includes wage as well as price rigidities in order to analyze and understand the relationship between labor mobility and fiscal policy outcomes would be interesting as well.
References


Klein, P. and G. Ventura (2009). Productivity differences and the dynamic
effects of labor movements. *Journal of Monetary Economics* 56(8), 1059–
1073.


1274.


A Calculations

A.1 The representative native household’s maximization problem

The natives’ Lagrangian and the first order conditions can be written as

\[
L = E_t \sum_{t=0}^{\infty} \beta^t \{ \log(c^n_t) + A \log(1-h^n_t) \} - \sum_{t=0}^{\infty} \lambda_t \left\{ c^n_t + k^n_{t+1} - (1 - \delta)k^n_t \right\} - (1 - \tau) [w_t h^n_t - R_t k^n_t] + t^n_t \]

\[
c^n_t : \beta^t \frac{1}{c^n_t} = \lambda_t \tag{47}
\]

\[
h^n_t : \beta^t \frac{A}{(1-h^n_t)} = (1 - \tau) w_t \lambda_t \tag{48}
\]

\[
k^n_{t+1} : \lambda_t = E_t \{ \lambda_{t+1} [1 + (1 - \tau)R_{t+1} - \delta] \} \tag{49}
\]

In order to get to the inter-temporal Euler equation use Equation (47) in (49).

\[
\beta^t \frac{1}{c^n_t} = E_t \left\{ \beta^{t+1} \frac{1}{c^n_{t+1}} [1 + (1 - \tau)R_{t+1} - \delta] \right\} \Leftrightarrow \frac{1}{c^n_t} = \beta E_t \left\{ [1 + (1 - \tau)R_{t+1} - \delta] \frac{1}{c^n_{t+1}} \right\}
\]

The intra-temporal labor supply condition of the native household is achieved by dividing (48) by (47).

\[
\frac{A c^n_t}{(1-h^n_t)} = (1 - \tau) w_t
\]
A.2 The resource constraint

Before the individual budget constraints as well as the finance rule of the government can be aggregated, they individual budget constraints have to be transformed such that they are expressed in the same unit. Remember that the individual budget constraints hold for one native household and one migrant household, respectively. Since only one person lives in every household, the budget constraints are expressed in per migrant capita and per native capita, respectively. Thus, multiply the variables by their respective share of total population to express the variables in aggregate per-capita terms. Multiplying them by the size of total population $N_t$ leads to variables in aggregate terms. Note that the budget constraint of the government stated in (27) is already expressed in aggregate terms. Afterwards, all three constraints summed up will eventually lead to the resource constraint of the whole economy.

The native household

Remember the native budget constraint per native household from Equation (11)

$$c^n_t + k^n_{t+1} = (1 - \delta)k^n_t + (1 - \tau_t) [w_t h^n_t + R_t k^n_t] - t^n_t$$

Multiply the variables by the share of the native household in total population and the size of total population $N_t$.

$$N_t \gamma_t c^n_t + N_t \gamma_t k^n_{t+1} = (1 - \delta)N_t \gamma_t k^n_t + (1 - \tau_t)N_t \gamma_t [w_t h^n_t + R_t k^n_t] - \gamma_t N_t t^n_t \iff$$

$$C^n_t + K^n_{t+1} = (1 - \delta)K^n_t + (1 - \tau_t) [w_t H^n_t + R_t K^n_t] - T^n_t$$

The migrant household

Remember the migrant budget constraint from Equation (15)

$$c^m_t + k^m_{t+1} = (1 - \delta)k^m_t + (1 - \tau) [w_t h^m_t + R_t k^m_t] - t^m_t$$

Analogue to the derivation of the native household’s budget constraint in aggregate terms, multiply both sides by the share of the migrant household in the
economy \((1 - \gamma_t)\) as well as by \(N_t\).

\[
\begin{align*}
N_t (1 - \gamma_t) c_t^m &+ N_t (1 - \gamma_t) k_{t+1}^m = (1 - \delta) N_t (1 - \gamma_t) k_t^m \\
&+ (1 - \tau) N_t (1 - \gamma_t) [w_t h_t^m + R_t K_t^m] - N_t (1 - \gamma_t) k_t^m \iff \\
C_t^m + K_{t+1}^m &= (1 - \delta) K_t^m + (1 - \tau) [w_t H_t^m + R_t K_t^m] - T_t^m
\end{align*}
\]

**Aggregation**

Remember that the government budget constraint is defined as \(\tau (w_t H_t + R_t K_t) + T_t = G_t\). Then, summing up the three budget constraints yields to the following expression and using the expressions in (18), (19) and (20) for \(C_t\), \(H_t\) and \(K_t\) yields to the following

\[
\begin{align*}
C_t + C_t^m + K_t^m + K_{t+1}^m + G_t &= (1 - \delta) (K_t^m + K_t) + (1 - \tau) \left[ w_t \left( H_t^m + H_t \right) + R_t \left( K_t^m + K_t \right) \right]
\end{align*}
\]

Rearranging and plugging in the firms' first order conditions for \(R_t\) and \(w_t\).

\[
C_t + I_t + G_t = Y_t
\]

where \(I_t = K_{t+1} - (1 - \delta) K_t\) and \(Y_t = K_t^m H_t^{1-\alpha}\).
A.3 The steady state

To get to the steady-state value for the labor supply rewrite the optimal labor supply condition in either (17) or (13), skip the superscripts and plug in the steady-state value for the real wage in (40). Eventually, solve for $h$. Note that in steady state the size of total population equals 1, which leads to the fact the aggregate variables equal per-capita variables.

\[ H = 1 - \frac{AC}{(1 - \tau)w} = 1 - \frac{AC}{(1 - \alpha)(1 - \tau)\frac{Y}{H}} = 1 - \frac{AH\frac{C}{Y}}{(1 - \alpha)(1 - \tau)} \]

\[(1 - \alpha)(1 - \tau)H = (1 - \alpha)(1 - \tau) - Ah\frac{C}{Y} \]

\[ (1 - \alpha)(1 - \tau)H + AH\frac{C}{Y} = (1 - \alpha)(1 - \tau) \]

\[ H \left[ (1 - \alpha)(1 - \tau) + A\frac{C}{Y} \right] = (1 - \alpha)(1 - \tau) \]

\[ H = \frac{(1 - \alpha)(1 - \tau)}{\left[(1 - \alpha)(1 - \tau) + A\frac{C}{Y}\right]} \]

A.4 Calibration

The preference parameter $A$ is calculated from the steady-state labor supply in (43), assuming that $H = 0.2$ and the steady-state government consumption share of income is defined as $\omega$.

\[ H = \frac{(1 - \alpha)(1 - \tau)}{\left[(1 - \alpha)(1 - \tau) + A\frac{C}{Y}\right]} \]

\[ H \left[ (1 - \alpha)(1 - \tau) + A \left(1 - \omega - \delta \left[\frac{K}{H}\right]^{1-\alpha}\right) \right] = (1 - \alpha)(1 - \tau) \]

\[ HA \left(1 - \omega - \delta \left[\frac{K}{H}\right]^{1-\alpha}\right) = (1 - \alpha)(1 - \tau) - H(1 - \alpha)(1 - \tau) \]

\[ A = \frac{(1 - \alpha)(1 - \tau)(1 - H)}{H \left(1 - \omega - \delta \left[\frac{K}{H}\right]^{1-\alpha}\right)} \]

\[ A = 3.5045 \]
A.5 Fiscal multipliers

The impact multiplier can be rewritten as

\[
\frac{\Delta Y_t}{\Delta G_t} = \frac{Y_t - Y_{t-1}}{G_1 - G_{t-1}} = \frac{Y_t - Y}{G_1 - G}
\]

\[
= \frac{Y_t - Y}{(0.2Y + 0.01Y) - 0.2Y} = \frac{Y_t - Y}{0.01Y}
\]

\[
= 100 \cdot \frac{Y_t - Y}{Y}
\]
B Figures

B.1 Impulse response functions

Figure 6: Impulse responses of the population variables, where $\theta = 1$.

Figure 7: Impulse responses of factor prices, where $\theta = 1$. 
Figure 8: Impulse responses of aggregate consumption, labor supply and capital stock for both type of households, where $\theta = 1$.

Figure 9: Impulse responses of per-capita consumption, labor supply and capital stock for both type of households.
B.2 Fiscal multipliers

Figure 10: Impact multiplier and cumulative multiplier for different $\theta$. 

(a) $\theta = 0.1$

(b) $\theta = 0.5$

(c) $\theta = 1$