The role of networks in firms' multi-characteristics competition and market-share inequality

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Abstract

We develop a location analysis spatial model of firms' competition in multi-characteristics space, where consumers' opinions about the firms' products are distributed on multilayered networks. Firms do not compete on price but only on location upon the products' multi-characteristics space, and they aim to attract the maximum number of consumers. Boundedly rational consumers have distinct ideal points/tastes over the possible available firm locations but, crucially, they are affected by the opinions of their neighbors. Our central argument is that the consolidation of a dense underlying consumers' opinion network is the key for the firm to enlarge its market-share. Proposing a dynamic agent-based analysis on firms' location choice we characterize multi-dimensional product differentiation competition as adaptive learning by firms' managers and we argue that such a complex systems approach advances the analysis in alternative ways, beyond game-theoretic calculations.

Keywords: location choice, networks, multi-characteristics space, consumer behavior, decision heuristics, agent-based model, political competition

JEL Classification: C63, C65, L14, R39, D72

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1. Introduction

Product characteristics are typically considered as given when economists study firms' strategies and behavior. But firms in industries with product differentiation actually choose the features of their products based on consumers' preferences. The aim of this paper is to suggest a way to understand the complexity in such markets using complex networks, which mathematically are described by graph objects. We develop a stylized theoretical model to examine the issue of product differentiation in a multi-dimensional space with consumers' choices emerging from consumers' opinion-based multilayered networks. We ask two questions: First, how networks affect market share at the firm level? Second, how these networks affect firms' locations?

Here we propose a multi-dimensional model of firms' locational choice in the product-characteristics space that describes a finite number of firms competing for customers.[1] In this model, consumers have tastes/requirements that place them on their ideal points in the multi-characteristics space. For every firm we model the network of consumers' opinions about its product (i.e. for every firm there is a corresponding opinion network describing consumers' opinion about the firm's product: the nodes represent the consumers and a link between two nodes represents their exchange of opinion about the corresponding firm's product), on which (opinion network) a contagion dynamics can take place. Consumers are represented by a node on each network and can be active only in one of the networks (purchase of one product from one firm) at the end of each time period (at the moment of the purchase). Each consumer has also the option not to purchase, and in that case she will be inactive in all networks. Before the end of each time period, and while the consumers think about the purchase, they are more likely to change opinions and examine different options. They have some form of product characteristics opinion at the beginning, but as they gain experience and acquire more information about the products, their opinion can change, but it stabilizes over time (see Hoeffler and Ariely, 1999; West et al., 1996). This "uncertainty reduction" is captured by dynamics that slow down until the purchase moment when they become completely frozen. These dynamics are implemented in this work with the simulated annealing

[1] Other firm-level models of multidimensional product choice within the agent-based framework are, for example, Page and Tassier (2007) and Babutsidze (2015).

algorithm. An illustration of a possible final configuration of the described system is shown in Figure 1.

Crucially, consumers are affected by the opinion of their peers, and they tend to be active in the network where the majority of their peers are also active.[2] In addition, the probability of changing their choice decreases as the purchase moment approaches, in line with the behavioral literature's argument that agents change and adapt their behavior over time, reflecting both their own learning and changes in their environment (Hoeffler and Ariely, 1999).

Our model would apply also to political competition, as studied by Downs (1957) and many others after him. More specifically, it could be perfectly placed within the scope of agent-based models described in Laver and Sergenti (2011). However, in this work we will stick to the firm affairs' language.

Our aim is to provide insights for the role of networks in (a) the firms' location problem and (b) consumers' choices. We consider consumers' networks as 'social' networks since they are based on the social interaction of opinion exchange, without making any assumption about the existence of friendship relations, in a Facebook like view of a social network. We describe the dynamics of 'social' influence in the consumers' networks and we model the uncertainty reduction preceding the consumers' final choice by implementing the simulated annealing algorithm.[3] At the end of the simulated annealing calculation we have the number of consumers that opt to purchase from each firm. This is affected by the average connectivities of the consumers' opinion networks. Hence, we observe "regions" in the average connectivity space where the active nodes belonging to some opinion networks with high average connectivity percolate the system, while nodes of the remaining opinion networks with lower average connectivities are concentrated in disconnected clusters indicating high market share inequality. Nevertheless, "regions" where the average connectivity of the consumers' opinion networks are comparable manifest a market that sustains low market share inequality.

[2] Two consumers are peers when there is a link between them, i.e. when they exchange opinions.
[3] An early application of the simulated annealing in economics is the work of Goffe et al. (1994) who suggested that simulated annealing could be used to optimize the objective function of various econometric estimators.

From consumers' perspective, we begin with a very basic idea. Consumers' decision on purchases cannot be different from most other decisions people make in their daily lives, in the sense that some process for acquiring information and evaluating it is necessary. To the best of our knowledge, this paper is the first study that incorporates the role of opinion exchange and processing on a multilayered network, in an attempt to shed light on the understanding of how the market-share inequality and competition in a multi-characteristics space are affected by the presence of consumers' interactions. Taking into account these multiple layers is crucial, as is shown by the considerable current interest in multilayered systems (Buldyrev et al., 2010; Vespignani, 2010; Parshani et al., 2011; Baxter et al., 2012; Bashan et al., 2012; Gomez et al., 2013; Radicchi and Arenas, 2013; De Domenico et al., 2013; Radicchi, 2014; Garas, 2016). On the other hand, to make significant advances in understanding consumers' decisions, we must device a method for studying this process. Such method would allow us to observe consumers' behavior from up close, to dig below the surface and watch consumers as they try to exchange information about a myriad of alternative products while refining the overwhelmingly volume of information of their multiple characteristics. Behavioral decision theory guides the process-oriented, complex systems-framework we present in the next sections in an effort to develop a new set of measures for studying market proceedings.

A few papers consider location models where consumers are distributed on a graph. Mavronicolas et al. (2008), Feldmann et al. (2009), Nunez and Scarsini (2015), Fournier and Scarsini (2014) consider Hotelling models on graphs studying some very interesting properties of the economic system at the firm-level. To the best of our knowledge this is the first analysis of product complexity that places the spotlight to the consumers' perspective, and treats complexity using the multiple characteristics of the products over which consumers have tastes. By doing so, we characterize product differentiation competition as adaptive learning by firms' managers in a complex system with limited information feedback. The results reported in the subsequent sections are intended to demonstrate that such a complex systems approach provides a plausible basis for understanding market-share inequality and firms' competition in the product-characteristics space and can advance the analysis of these topics in alternative ways.

The rest of the paper is organized as follows. In Section 2 the model is introduced: the complexity of the market is described analytically by applying graph theory in networks. Section 3 gives the simulation strategy and discusses the simulation results. Section 4 concludes.

2. The market as a complex system

Following the relevant literature (Potts, 2000; Schweitzer et al., 2009; Easley and Kleinberg, 2010; Jackson, 2010), at any point in time t, the market is represented by a graph of a set of consumers and a set of products/firms, G_t, and the relationships between them represented in $l_t(G_t)$

$$E_s^t = \{G_t, l_t(G_t)\} \tag{1}$$

where $G_t = \{g_i : g_i \in \mathbb{R}^k\}_{i=1}^{g}$ is the set of economic agents on the product-characteristics space $\mathbb{R}^k$ and $l_t(G_t) = \{l_{ij}\}_t$ is a $g \times g$ matrix that summarizes the connections between the economic agents, where, if $l_i, l_j \in E_s^t$ denotes the existence of a connection between economic agents i and j, then

$$l_{ij} = \begin{cases} 0 & \Rightarrow g_i g_j \notin E_s^t \\ 1 & \Rightarrow g_i g_j \in E_s^t \end{cases} \tag{2}$$

In this system there are two subsets of economic agents, *consumers*, $C_t = \{c_i : c_i \in \mathbb{R}^k\}_{i=1}^{|C|_t}$, and *firms*, $F_t = \{f_i : f_i \in \mathbb{R}^k\}_{i=1}^{|F|_t}$. We consider $|F|_t$ competing firms for which $|F|_t$ different opinion networks exist. These opinion networks represent the different opinion exchange patterns of consumers for the different $|F|_t$ firms. For example, two consumers may exchange their opinions for firm A but not for firm B, hence there is a link between them apparent in opinion network A but not in network B. Suppose that in every period each consumer i is represented in each network and must make a decision on buying a product from a firm, hence to stay active in one of the $\left|F\right|_t$ opinion networks. For simplicity, let us assume $f_i f_j \notin l_t(G_t), \forall f_i f_j \in G_t$, thus omitting relationships between the different opinion networks.[4] In this market, the earnings of firm j immediately follow from the number of active consumers (nodes) in opinion network j at the end of any given period *t*, since any active node in opinion network j at the end of period *t* represents a purchase from firm j.[5] Furthermore, the number of active nodes in each network is determined by the tastes and the choices of each of the consumers in the market and following Simon (1955) and Tversky and Kahneman (1974) we explain below the process of these choices assuming bounded rationality.

2.1 Consumers' behavior

Traditionally the assumed heuristic for consumers is a maximization rule over some utility function defined over the set of products' quantities. However, this rule is inconsistent with extensive evidence presented in behavioral economics and marketing literature. Here we assume that each consumer's preferences can be characterized by an ideal economic position in some k-dimensional product characteristics space, and we look closely on the decisions made by consumers when choosing which firm's product to buy. Having switched from formal analysis to computation, we depart from the classical analytically tractable models first, by assuming as baseline decision rule that consumers are affected by the opinion of their peers and second, by making the appropriate behavioral assumptions following the relevant literature of behavioral economics, discussed below.

Behavioral decision theory is psychological in its orientation, beginning with the view of humans as limited information processors or, perhaps more accurately, as "boundedly rational information processors" (Simon, 1955, 1956, 1957, 1959). Humans have developed a large number of cognitive mechanisms to cope with the overwhelming volume of information in the modern societies. These mechanisms are adopted automatically without any conscious and are cognitive shortcuts for making

[4] We assume that all consumers buy in the equilibrium (covered market is a quite standard assumption in the literature, e.g. Irmen and Thisse, 1998), where price is exogenously given. Firms do not compete on price but only on location.

[5] We assume that any product may be produced at the same constant marginal cost, which is normalized to zero without loss of generality.

certain judgments and inferences with considerably less alternatives than those dictated by rational choice, focusing attention on a small subset of all possible information.

Kahneman and Tversky (1973, 1974, 1984) and Tversky and Kahneman (1973, 1974) have identified three general cognitive *heuristics* that decision makers adopt in the process of information gathering and analysis: (a) *decomposition,* which refers to braking a decision down into its component parts, each of which is presumably easier to evaluate than the entire decision; (b) *editing* or *pruning,* which refers to simplifying a decision by eliminating (ignoring) otherwise relevant aspects of the decision; (c) *decision heuristics,* which are simplifying the choice between alternatives thus providing cognitive efficiency.

These *heuristics* have direct application to consumers' choices. Consumers face a myriad of alternative products and there is compelling evidence which suggest that consumers simplify their decisions with a *consider-then choose* decision process in which they first identify a set of products, *the consideration set*, for further evaluation and then choose from the consideration set. In seminal observational research Payne (1976) identified that consumers use consider-then-choose decision processes. This heuristic is firmly rooted in both the experimental and prescriptive marketing literature (e.g. Bronnenberg and Vanhonacker, 1996; Brown and Wildt, 1992; DeSarbo et al., 1996; Hauser and Wernerfelt, 1990; Jedidi et al., 1996; Mehta et al., 2003; Montgomery and Svenson, 1976; Paulssen and Bagozzi, 2005; Roberts and Lattin, 1991; Shocker et al., 1991; Wu and Rangaswamy, 2003).

In the context of our model, this means that the set of alternatives each consumer, i, considers, B_i^t, is a subset of the overall set of firms/products in the system. So we can define the consideration set for consumer i as

$$B_i^t = \{ f_j \in F_t / \left\| f_j - c_i \right\| \le \varepsilon \} = \{ f_j \in F_t / b_i^t(f_j) = 1 \} \subset F_t \quad (3)$$

where b_i^t is a mapping which assigns elements in the overall set of alternatives F_t to either the consideration set or not, so $b_i^t : F_t \to \{0,1\}$, such that $b_i^t(f_j) = 1$ indicates that the firm/product j is considered by consumer i.

2.2 Consumers' multilayered opinion networks

At the beginning of time period t we consider $|F|_t$ networks, representing the consumers' different opinion exchange patterns for the $|F|_t$ competing firms. During the time period t, each consumer i is represented in $|B|_i^t$ networks and can be active in any of these networks. In particular, $b_i^t(f_j) = 0$ if consumer i is inactive in network j and $b_i^t(f_j) = 1$ if consumer i is active in network j. At the end of time period t, the activity of a consumer in network j corresponds to the consumer's purchase of firm j 's output, hence each consumer can be active only on one network on the purchase moment (i.e., if $b_i^t(f_j) = 1$ then $b_i^t(f_m) = 0$ for $m \neq j$). Nevertheless we leave to the consumer the freedom not to make a purchase (i.e., $b_i^t(f_1) = b_i^t(f_2) = ... = b_i^t(f_{|B|_i^t}) = 0$). Moreover, consumers are influenced by their network peers, hence we assume that, on the purchase time, if the majority of consumer i's peers are active in network j, the consumer will be active in the same network j, provided that she is not already active in another network.

Based on the above discussion, the $|B|_i^t$ mathematical constraints that consumer i needs to satisfy at the end of the period time t are:

$$b_i^t(f_j) = \left[1 - \prod_{\lambda \in Nb_j^t(c_i)} (1 - b_\lambda^t(f_j))\right] \prod_{\substack{k=1 \\ k \neq j}}^{|B|_i^t} (1 - b_i^t(f_k)), \quad \text{for} \quad j = 1, 2 ... |B|_i^t \tag{4}$$

where $Nb_j^t(c_i)$ is consumer i's network of opinions about firm j. Though we assume no buyer's remorse, we allow for some conflicts in the system before the purchase time, in the sense that the $|B|_i^t$ constraints provided by equations (4) will not be satisfied. The behavioral literature points out that consumers face an "uncertainty reduction" to the choice of the product they will purchase as the purchase time comes closer. The agents change and adapt their behavior over time, reflecting both their own learning and changes in their environment. When they think of a purchase, initially they are more likely to change opinions and examine different options. They have some form of opinion at the beginning, but as they gain experience and acquire more information, their opinion changes and stabilizes over time (see Hoeffler and Ariely, 1999; West et al., 1996). This "uncertainty reduction" is captured by dynamics that slow down until the purchase time when they become completely frozen. These dynamics are implemented with the simulated annealing algorithm.

2.3 Evolution dynamics during consumers' purchasing process

To model how consumers decide during the time period t we consider the following equation (5) that counts the number of the constraints in equations (4) that are violated (Halu et al., 2013):

$$H = \sum_{j=1}^{|B|_i^t} \left\{ \sum_{i=1}^{|C|_t} \left\{ b_i^t(f_j) - \left[1 - \prod_{\lambda \in Nb_j^t(c_i)} (1 - b_\lambda^t(f_j)) \right] \prod_{\substack{k=1 \\ k \neq j}}^{|B|_i^t} (1 - b_i^t(f_k)) \right\}^2 \right\} \tag{5}$$

Initially, active nodes in each network j are distributed according to consumers' consideration sets [equation (3)], and we allow the consumers to change opinion at any time before they make their final choice. These dynamics are implemented using the simulated annealing algorithm, which works as follows. Starting from a relatively high initial temperature,[6] i.e. a large number of potential conflicts [a large-number of non-binding constraints given by equations (4), hence a high H given by equation (5)] due to the stochastic way initial opinions are distributed, we use a Monte Carlo dynamics which will reach an equilibrium following the equation (5). As the time for the final choice approaches, the effective temperature decreases and the consumers tend to have less and less conflicts with their network peers until they reach to zero conflicts at equilibrium, when $H = 0$. These dynamics are implemented using the simulated annealing algorithm.

[6] Since we start with a sufficiently high temperature, the dynamics are not affected by the initial conditions of the system.

At the end of the simulated annealing calculation we will have the configuration of the model for the period t, which is depicted in Figure 1. Hence, we will have "regions" in the average connectivity of consumers' opinion networks space where the active nodes of some networks will percolate the economy, while the nodes of the remaining layers will be concentrated in disconnected clusters indicating high market share inequality (since the active nodes in each network represent the respective firm's customers). Nevertheless, "regions" where the average connectivity of the opinion networks will be comparable manifest a market that sustains low market share inequality.[7]

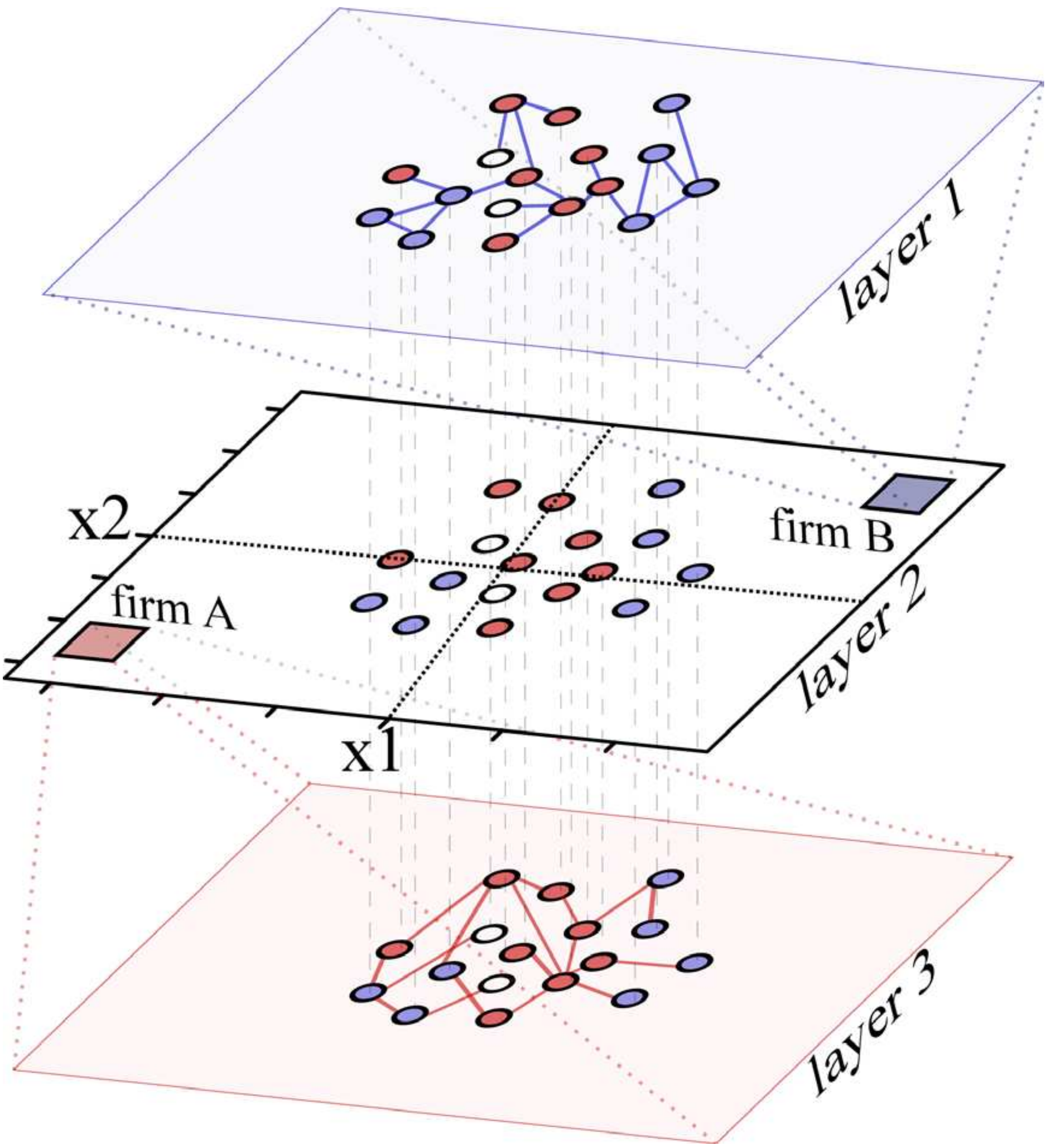
2.4 Firms' 'behavior'

Firms' managers are assumed to use an adaptive decision rule to set product characteristics on a multidimensional product-characteristics space at any given period t. We assume an adaptive rule that models a manager who constantly modifies product characteristics in the search for more customers, and the manager cares only about the firm's market share. If the manager's decision for the characteristics of the product at time t was rewarded by an increase in market share, then the firm makes a unit move at time $t+1$ in the same direction as the move at t. If not, the manager reverses direction and makes a unit move on a heading randomly selected within the half-space being faced now. In other words, the firm's manager relentlessly forages in the product-characteristics space, always searching for more customers and never being satisfied, changing strategic planning in the same direction as long as this is rewarded with more sales, but casting around for a new strategic planning when the previous one was punished with falling or static sales.

Now that we have structurally defined the evolution process of the complex system as a whole, we can proceed to simulations.

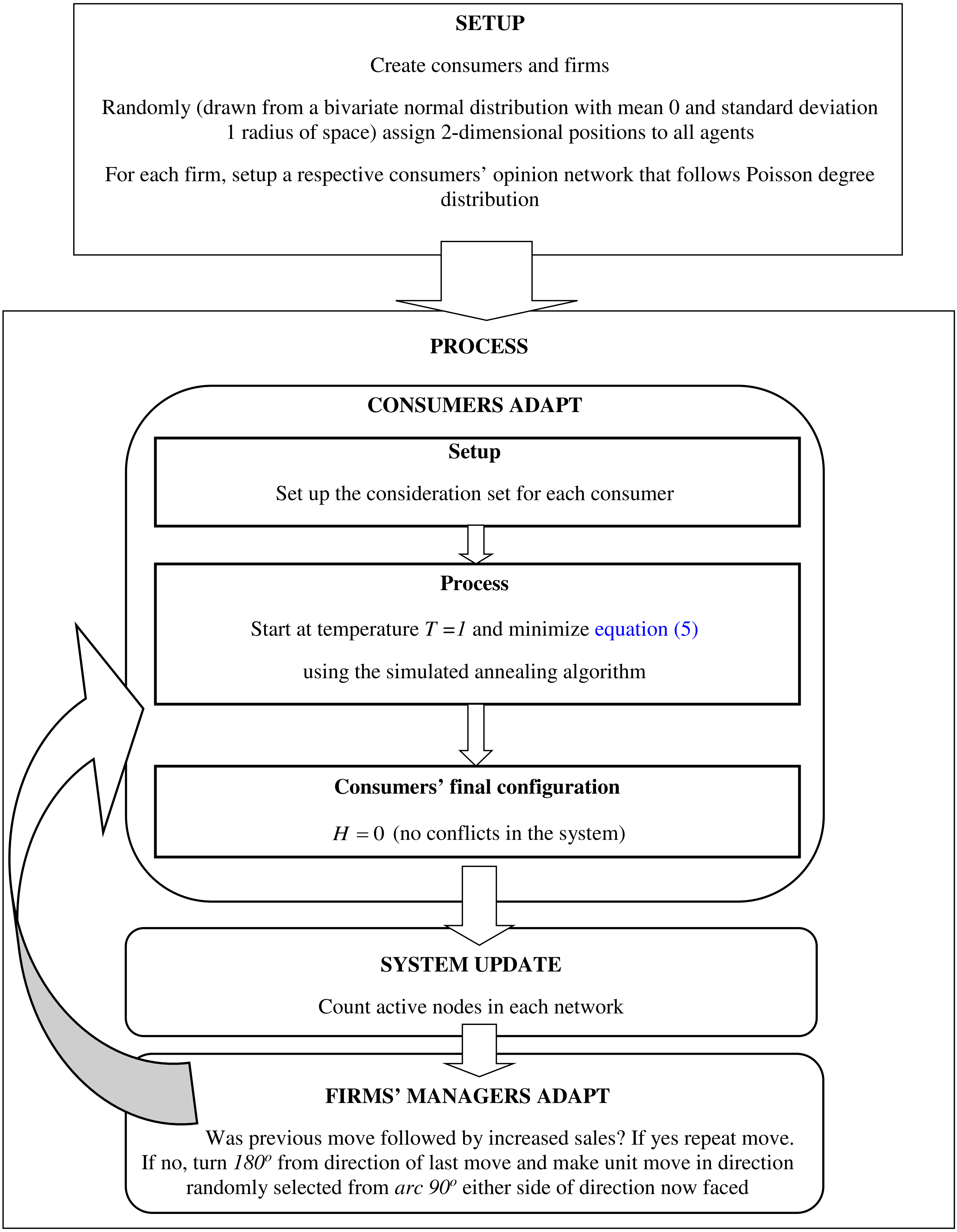
[7] These results arise intuitively -and are confirmed by our model- from the percolation theory on networks literature (see Newman, 2010) and the recent results concerning the role of densely connected social networks on the adoption of a behavior (see Centola, 2010).

Figure 1: Illustration of the system's final configuration



Notes: (Colored figure) *Layer 1* represents consumers' opinion network for firm *B*; *Layer 3* represents consumers' opinion network for firm *A*; *Layer 2* is the (2-dimensional) product-characteristics plane; Each consumer is represented in all layers but can be active either in the firm *B*'s underlying opinion-network (blue node) or in the firm *A*'s underlying opinion-network (red node) or inactive in both networks (white node).

Figure 2: Overview of the evolution of the system



3. Simulations

For clarity in what follows we consider two firms, hence two layers in the consumers' opinion network, in the simulation experiments. Though, our code can be straightforwardly generalized to consider multiple firms/networks. In our setting, consumers are assumed to be intrinsically interested in product characteristics and to have ideal points in a product-characteristics space (again, in order to aid visualization, the simulated version of the model is implemented in two dimensions, i.e. in the Euclidean plane, but the model can be implemented in any number of dimensions). Firms compete with each other by offering products with varied characteristics to consumers. Figure 2 summarizes the model and its evolution process, which was programmed in R.

3.1 Firm System Dynamics

Initiation of the model randomly distributes a discrete set of firms' locations and consumers' ideal points across the product-characteristics plane.[8] Consumers are initially present and active in the opinion-networks/firms which lie within their consideration sets. As discussed above (see section 2.3), we allow the consumers to change opinion at any time before they make their final choice, and we model their dynamics using the simulated annealing aiming to reduce to zero the number of conflicts with their network peers (the number of violated mathematical constraints given by equation (4) for all consumers) when reaching equilibrium. More precisely, the algorithm starts with an initial temperature $T = 1$ and at every time step we select a node at random from either one of the two networks with equal probability and we change it from active to inactive or vice versa. After this change, the equation (5) is recalculated, and if the difference with respect to its previous value, ΔH, is negative i.e., the number of conflicts in the system is reduced, the change is accepted. If $\Delta H > 0$, the change can still be accepted but with a small probability given by $e^{-\Delta H/kT}$. This random selection process is repeated $2N$ time steps, in order to update the whole system on average once and then we advance the system time by one Monte Carlo step. The whole process is repeated by slowly reducing the temperature until we reach

[8] We assign 2-dimensional positions to all agents, drawn from a bivariate random normal distribution with mean zero and standard deviation one radius of space.

equilibrium at $H = 0$, where there are no more conflicts in the network. The final configuration of the "uncertainty reduction" process just described is depicted in Figure 1.

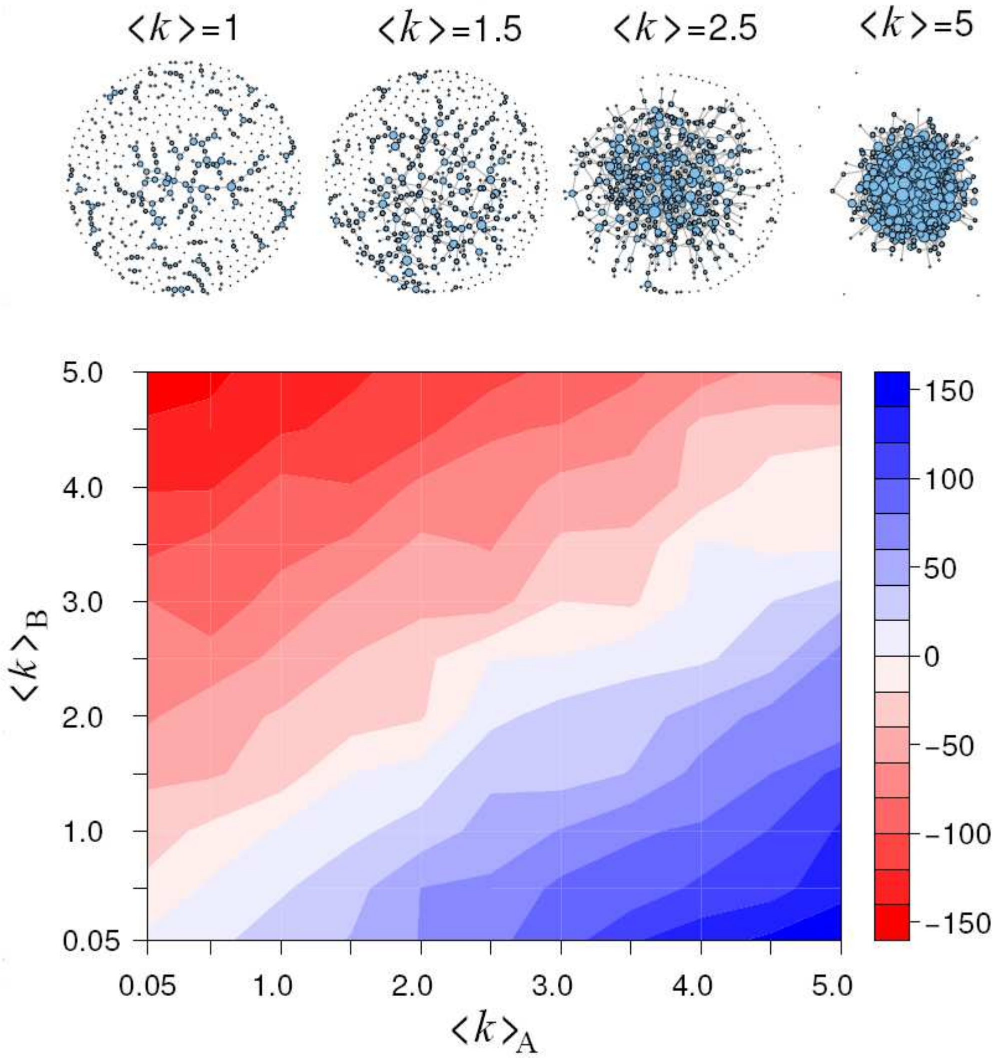
We assume that consumers can share opinions with others randomly, therefore the consumers' opinion networks follow Poisson degree distributions, i.e. they fall in the class of Erdos-Renyi random networks (in the top panel of Figure 3 we illustrate how these networks look like for different average degrees). Using average degrees above the percolation threshold, we visualize the evolution of the giant component against the average degree of the network (see Newman, 2010). In the bottom panel of Figure 3 the final configuration of the two-firm model for different average-degrees of consumers' opinion networks is shown. In particular we contour-plot the difference between the total number of customers of firm A (total number of nodes active in network A) and the total number of customers of firm B (total number of nodes active in network B) as a function of the average connectivities of the two networks, $\langle k \rangle_A$ and $\langle k \rangle_B$ respectively.[9]

The simulation results visualized in Figure 3 disclose the following three cases that appear in our model:

- *Case 1*: In the regions where the mean degree of both networks is smaller than one, there are no giant components in the networks. This means that the networks are fragmented and the opinion of a consumer is not affected by peer opinions. In this case each firm possesses only a marginal market share and noticeable market share inequality is unlikely.
- *Case 2*: In the regions where the mean degree of either one of the two networks is smaller than one and for the other network bigger than one, the giant component in the latter network emerges and market share inequality occurs.
- *Case 3*: In the regions where the mean degree of both networks is greater than one, giant components emerge in both networks. In these regions we have the pluralism solution of the consumers' choices and hence, no noticeable market share inequality is apparent.

[9] For all the experiments we pin down consideration set's radius, at $\varepsilon = 0.7$. We have also simulated the model with different values of ε obtaining similar results.

Figure 3: The difference between the total number of firm A's customers and the total number of firm B's customers as a function of the two underlying opinion networks' average connectivities, $\langle k \rangle_A$ and $\langle k \rangle_B$.



Notes: (Colored figure) *Top panel:* Examples of Erdos Renyi networks with 500 nodes and different average degrees $\langle k \rangle$. Note that while the first graph is at the percolation threshold and the largest cluster has already emerged, the graph is still very sparse. *Bottom panel*: The contour plot for the excess number of customers of firm A over firm B, *sA*-*sB*, for a system of 500 customers that evolved for 1000 steps. The underlying social networks are Erdos-Renyi random graphs with average degrees $\langle k \rangle$ as shown in the axis of the figure. The results are averages of 100 realizations. Consumers' consideration sets' radius is pinned down at $\varepsilon = 0.7$.

It is clear from Figure 3 that the larger the difference in average degrees of the two networks, the larger the advantage of the firm with the more opinion-based connected consumers. Overall, Figure 3 and the above cases imply that *the firm with the most connected customers gets the lion's share of the market*.

Once consumers have purchased products, firms' managers adapt their locations to reflect the pattern of consumers' preferences. Firms' managers are assumed to use an "unconstrained" adaptive rule that constantly modifies product characteristics in the search for more customers. The rule searches for customers using a "win-stay, lose-shift" algorithm (Nowak and Sigmund,1993; Bendor et al., 2003; Laver, 2005): if the previous move increased market share the manager makes another unit move in the same direction. If the previous move did not increase market share, the manager makes a unit random move in the opposite direction chosen randomly within the half space toward which it now faces.[10] Managers use no information whatsoever about the global geography of the product-characteristics plane. They have no knowledge of the ideal point of any consumer but applying recursively the limited feedback from their local environment they pick up effective clues about the best direction in which to move.

Once firms' managers have adapted firms' positions, consumers readapt and once more are active in the opinion-networks/firms which lie inside their consideration sets. Then they are again affected by peers' opinions till their final decision on which product to purchase, then firms readapt to the new configuration of consumers' preferences and the process iterates continuously.

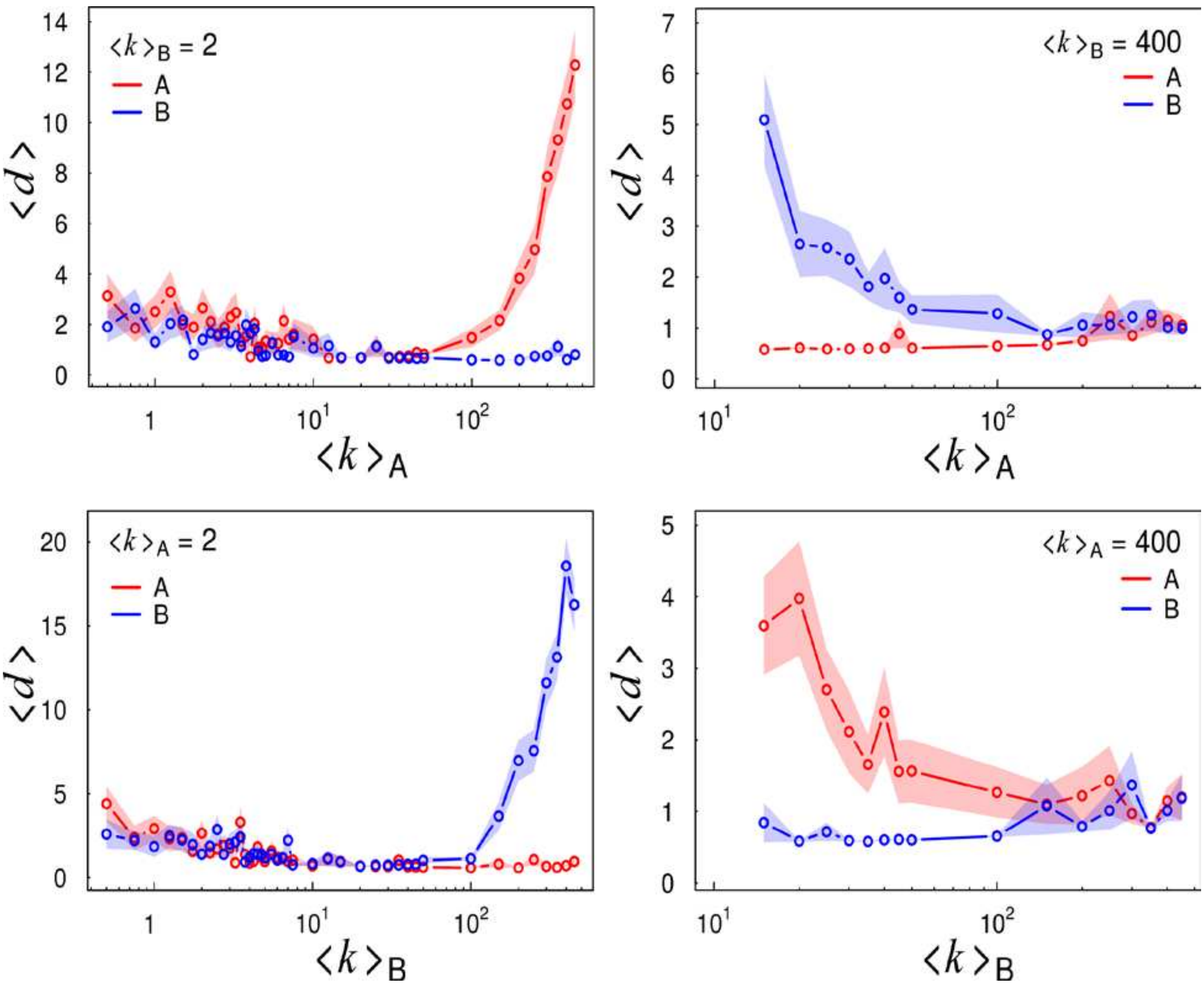
3.2 Location choice

In the previous section we analyzed consumers' behavior when their opinions are affected by the opinions of their network peers and the effect of this process in firms' market share. Next, we consider firms' location choice on the product-characteristics plane with origin (0,0) assuming that consumer ideal points' distribution is normal on both product-characteristics' dimensions.[11] We check whether certain product differentiation patterns emerge.

[10] The manager makes a unit move in a random direction on the first iteration.

[11] Bivariate random normal distribution with mean zero and standard deviation one radius of space.

Figure 4: The average distance $\langle d \rangle$ of the two firms (A, B) from the origin of the product-characteristics plane (0,0), as a function of the logarithms of the underlying opinion-networks' average degrees, $\langle k \rangle_A$ and $\langle k \rangle_B$.



Notes: (Colored figure) The system consists of 500 consumers whose ideal points/tastes for product-characteristics are distributed following the bivariate $N(0,1)$. The distance $\langle d \rangle$ is calculated at the end of a 1000 step trajectory, and each point is averaged from 100 realizations. The shaded area around the curves represents the standard error in the calculation of the mean value. Consumers' consideration sets' radius is pinned down at $\varepsilon = 0.7$.

Figure 4 summarizes the results of a simulation involving 100 independent 1000-cycle runs of the system with two firms (A and B) and 500 consumers. To avoid any dependence from the initial configuration we discard output from the first 150 cycles of each run, before the 1000 cycles were recorded. Figure 4 shows the average distance $\langle d \rangle$ of the two firms from the origin of the product-characteristics plane (0,0) calculated at the end of a 1000 step trajectory, and each point is averaged for the 100 realizations. The shaded area around the curves represents the standard error in the calculation of the mean value. For each panel we fix the average degree of the underlying opinion-network of one firm to either a very large or a very low value, and we measure $\langle d \rangle$ as a function of the average degree of the other opinion-network/firm.

From the left panels we see that, despite some fluctuations, the firms are moving closer to the center of the plane when both their underlying opinion-networks have relatively small average degrees, but when the average degree of only one of the two networks increases enough ($\langle k \rangle > 100$), the corresponding firm moves away from the center while the other firm still moves towards the center of the plane. The right panels represent the reverse behavior; if one firm has an underlying opinion-network with large average degree and the other one with low, we find again that the one with the high degree roams anywhere in the product-characteristics plane, on average further away from the center than its competitor. On the other hand, the firm with the small average degree network stays close to the center in order to keep its consumers. However, by keeping fixed the large average degree and allowing the small average degree to increase, we observe that the firm which corresponds to the fixed degree systematically moves toward the center of the product-characteristics plane as the average degree of the network of its competitor increases, and when the average degrees are comparable, the two firms move at similar distances from the center.

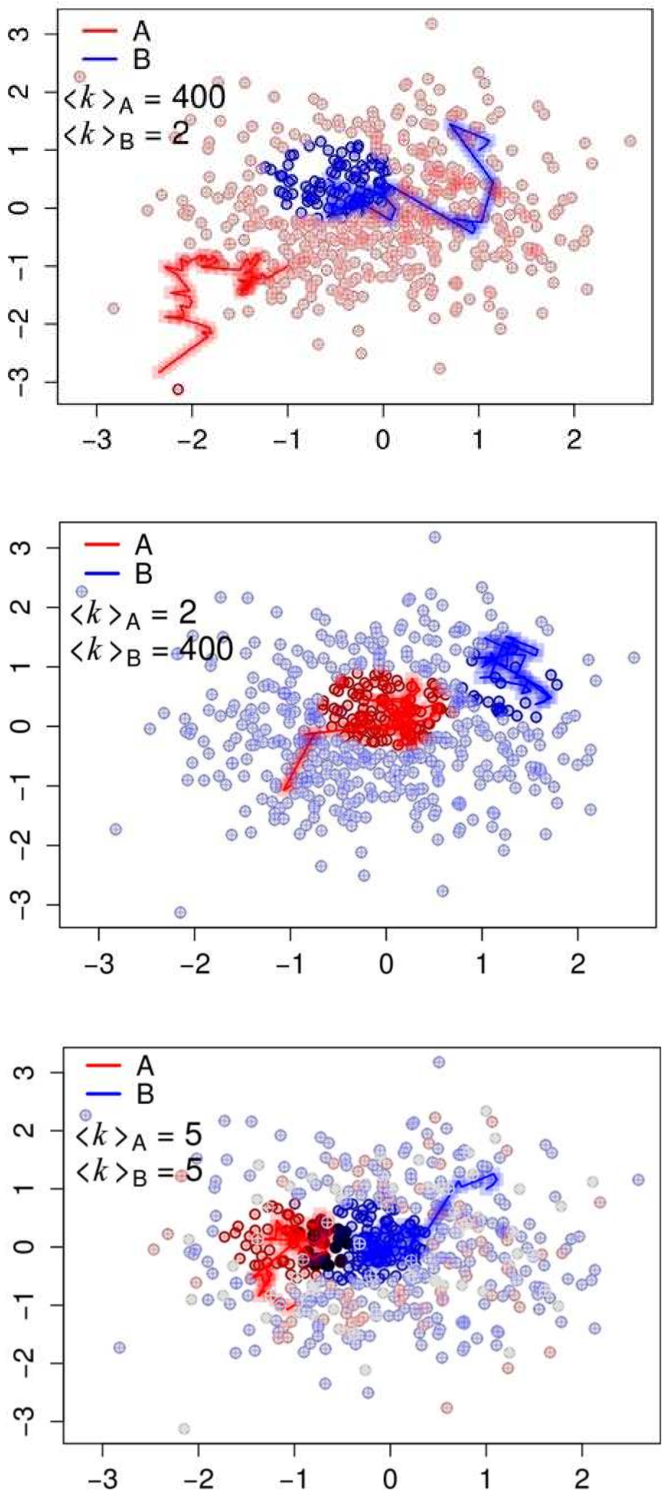
Summarizing, Figure 4 discloses the following scenarios for the different phases of the model:

- *Scenario I:* $\langle k \rangle_A \gg \langle k \rangle_B$. In this case, firm B moves toward the center of the product-characteristics plane searching for customers while the firm A with better connected consumers can afford to roam anywhere in the product characteristics plane.

- *Scenario II:* $\langle k \rangle_A \approx \langle k \rangle_B$. In this case, no firm has a significant advantage over its competitor due to the presence of the opinion-network. Therefore, very much along the lines predicted by the traditional spatial competition model, both firms *systematically move toward the center of the product-characteristics plane*.
- *Scenario III:* $\langle k \rangle_A \ll \langle k \rangle_B$. In this case, it is firm B that can afford to roam anywhere in the product-characteristics plane when the firm A with the poorly connected underlying opinion-network is hunting customers moving towards the highest density of consumers.

Constructively, the different phases of the model show that for the case of strong inequality in the density of firms' underlying opinion-networks, the firm with the highly connected network is roaming anywhere in the product-characteristics plane away from its origin, despite the fact that this is the location of the highest density of consumers. The opposite is true, with the firms approaching the center more often, when their underlying networks are weak. The behavior behind this pattern is easily understood if the system is watched in motion (see supplementary videos). Firms' locational moves have no effect in attracting consumers since the result of the maneuver is overshadowed by the effect of the network (the existence of links-connections between the consumers). On the other hand, the absence of opinion- networks reproduces the pattern observed in many simulations of two-party systems where the agents search for votes in the location of the highest vote densities. However, when there is no network-inequality apparent, firms go up against in attracting every single potential client, both moving toward the highest density location of consumers. In this case, the two firms go to the center of the plane (0,0) and attack each other's customer bases repelling one another from the dead center, along the lines predicted by the traditional spatial competition model.

Figure 5: Trajectory of the system with two firms (A, B) for different pairs of average degrees, $\langle k \rangle_A$ and $\langle k \rangle_B$ for the respective underlying opinion-networks.



For clarity, Figure 5 shows the trajectory of the simulated system with two firms (A and B), each artificially started at the edges of the product-characteristics plane, $(-1,-1)$ and $(1,1)$, with 500 uniformly randomly scattered consumers, for different -representative for the three phases of the model- pairs of average degrees for the underlying networks: $\langle k \rangle_A = 400, \langle k \rangle_B = 2$; $\langle k \rangle_A = 2, \langle k \rangle_B = 400$; $\langle k \rangle_A = \langle k \rangle_B = 2$. The first panel shows firm B's manager systematically moving toward the location of the highest consumer densities, while firm A's manager enjoys the support of her customers' opinion-network and relentlessly zigzags anywhere in the product-characteristics plane in a highly atypical way. The opposite pattern is apparent in the middle panel, while the bottom panel depicts the traditional spatial model, where the two firms go to the center of the plane and attack each others' customer bases head to head.

4. Conclusions

In conclusion, we have put forth a product-characteristics competition spatial model based on complex systems science. Moving away from the forward-thinking strategic game theoretic models, we have argued that consumers' boundedly rational behavior within their opinion exchange networks can result in concentrated power nodes in the network structure of the market. We have pointed out that the key feature to get the higher share of the overall product-purchases is the connectivity of the consumers' opinion networks corresponding to different firms. Regarding firms' location, we use agent-based modeling to study multi-characteristics competition in the evolving market. We are interested in location competition among multiple firms in a multidimensional product-characteristics environment in which consumers and firms care more than one product characteristic. Though we miss the analytical tractability of the relevant theoretical models, our simulated results demonstrate the feasibility of agent-based techniques to describe and explore firms' locational choices, as well as the capability of the proposed model to further advance the analysis in alternative ways.

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