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The Network Composition of Aggregate Unemployment*

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Abstract

We develop a theory of unemployment in which workers search for jobs through a network of firms, the labor flow network (LFN). The lack of an edge between two companies indicates the impossibility of labor flows between them due to high frictions. In equilibrium, firms' hiring behavior correlates through the network, modulating labor flows and generating aggregate unemployment. This theory provides new microfoundations for the aggregate matching function, the Beveridge curve, wage dispersion, and the employer-size premium. Using employer-employee matched records, we study the effect of the LFN topology through a new concept: 'firm-specific unemployment'.

Keywords: *Aggregate unemployment, labor flow networks, job search.*

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1 Introduction

Aggregate unemployment is a fundamental economic problem resulting form several distinct social mechanisms. These include people becoming separated from their jobs and searching for new positions; firms opening vacancies and searching for new workers using diverse strategies; and recruiters finding job seekers through the labor market. Due to the complexity involved in accounting for these and other mechanisms, the composition of aggregate unemployment has been studied under the umbrella of labor market frictions. A simplified way to account for these frictions has been to assume that companies and job seekers meet at random in the job market. Failure to coordinate these encounters can then be attributed to frictions.

The seminal work of Hall (1979), Pissarides (1979), and Bowden (1980) paved the way for the application of random matching models in order to integrate frictions into models of equilibrium unemployment. A reduced way to capture these matching processes is through the aggregate matching function (AMF). In its most typical form, the AMF takes two quantities as inputs: total unemployment and total number of vacancies; and returns the total number of successful matches. If the AMF produces unsuccessful matches, even when there are more vacancies than unemployed, it means that the labor market has frictions. Although somehow elegant, this reduced representation of the matching process sacrifices important structural information about frictions by aggregating and homogenizing the matching process. When we operate on aggregate quantities such as total unemployment and number of vacancies, it is not possible to understand the role that specific workers or firms play in the composition of aggregate unemployment. In other words, this approach assumes well-mixed matching mechanisms that are of limited relevance when there is large heterogeneity present in labor markets.

There have been several contributions that provide micro-foundations of the AMF and account for different types of heterogeneity. Unfortunately, each of these models focus on a specific type of friction (e.g., geographical distance, social networks, skills mismatch, etc.), which makes operationally challenging to account for all of them in a parsimonious way. Combining labor market frictions in an integrated framework is something desirable from both positive (to understand labor dynamics) and normative (for policy purposes) points of view. Moreover, today's availability of detailed labor micro-data makes it possible to account for the empirical patterns of firm-to-firm labor flows arising from the labor market frictions. For these reasons, a theoretical framework that takes the structure of frictions into consideration could be extremely valuable to construct a new understanding of aggregate unemployment.

In this paper we develop a theoretical framework of job search on networks that is empirically motivated by previous work on firm-to-firm labor flows (Guerrero and Axtell, 2013; Guerrero and López, 2015). Here, we capture the structure of labor market frictions through a network of firms. In this network, the presence or absence of an edge represents a categorical relation between two firm, resulting from the frictions that determine the amount of labor mobility between them. More specifically, the absence of an edge means that labor flows between two unconnected firms are highly unlikely due to high frictions, while the opposite is expected for connected firms. Together, firms and edges form the *labor flow network* (LFN) of the economy. The LFN constrains labor mobility, so we assume that an unemployed agent can only apply for jobs in those firms that are connected to his or her last employer. This could be due to a social relationship between former co-workers; a professional relationship between people in similar jobs at different firms; an industry-specific relationship between competitors; geographical proximity between firms; and so on. Therefore, instead of modeling job search as an aggregate random matching process, we model it as random walks on graphs. As we will show in this paper, this approach allows to infer the distribution of unemployment across the economy at the level of the firm; it provides new insights on the effect of the structure of labor market frictions on aggregate unemployment; it increases our understanding of equilibrium outcomes when firm behavior correlates through LFNs; and it provides a new method to estimate firms' hiring behavior without the need for data on vacancies. We show that this framework is consistent with empirical data from employer-employee matched records of two countries, and that the structure of the LFN may be accountable for most of the aggregate unemployment and its temporal variation.

1.1 Arbitrary Aggregations

The idea of limiting job search to groups of firms is not new or uncommon. For example, mismatch models posit that coordination failures between firms and workers are due to frictions that prevent job seekers from freely moving between submarkets. Conventionally, mobility between submarkets is studied by grouping firms into different categories and analyzing the labor flows that take place between such groups. Since the early contribution of Lucas and Prescott (1974), multisector matching models have offered a variety of ways to think about frictions between submarkets. An example is the model of Shimer (2007), where inter-submarket flows are modeled as a process where workers and jobs are randomly reassigned to any submarket every period. This reassignment originates from an exogenous random process under which it is equally likely to move between any two submarkets. Once workers and jobs have been reallocated, matching takes place in each submarket through local AMFs. In contrast, Sahin et al. (2014) assume that, provided with information on vacancies, shocks, and efficiencies, workers periodically choose a submarket to move into. Once labor is reallocated, match creation and destruction take place in each submarket. An alternative approach proposed by Herz and van Rens (2011) assumes that workers can search for vacancies in any submarket and firms can search for workers in the same way. There are costs associated to searching in each submarket. Therefore, matching depends on the optimal decisions of workers and firms about where to search. Other models combine some of these elements in the tradition of Lucas and Prescott (Alvarez and Shimer, 2011; Carrillo-Tudela and Visschers, 2013; Lkhagvasuren, 2009; Kambourov and Manovskii, 2009). On the other hand, a related strand of research studies submarkets as spatially delimited units (generally cities) (Glaeser and Gottlieb, 2009; Moretti, 2011; Manning and Petrongolo, 2011). These models focus on the effect of local shocks when the economy is in spatial equilibrium, which is useful when we know the spatial location of interest. However, as units of aggregation, spatial partitions are quite arbitrary.

Whether it is for the whole economy or for submarkets, there are a number of problems that arise from viewing matching in aggregate terms, and here we mention a few. First, when an AMF is responsible of pairing up workers and vacancies, it is assumed that all matches are equally likely. This neglects the importance that specific firms have in reallocating labor within a submarket. Second, defining a submarket is an arbitrary choice that might be well suited for a specific problem, but not necessarily for a broader context. According to the literature in community detection, (Girvan and Newman, 2002) aggregations should be well defined in terms of minimizing inter-submarket flows and maximizing intra-submarket flows in order to be empirically relevant. Conventional aggregations are not built with this criteria, as it has been pointed out by Jackman and Roper (1987) in their classical paper on structural unemployment:

> ... "there seems no particular reason why unemployed workers should regard themselves as specific to a particular industry, and in practice the unemployed do move between industries reasonably easily." (Jackman and Roper, 1987, pg. 19)

Third, aggregation assumes that any worker from one submarket is equally likely to transition to another submarket. Furthermore, it ignores the fact that only a few firms are responsible for inter-submarket transitions. These firms are crucial to overall labor mobility since they are diffusion outlets or bottlenecks in the process of labor reallocation. Fourth, aggregation 'smooths' the search landscape, enabling firm-to-firm flows that are highly unlikely in the short run. In fact, Guerrero and López (2015) have shown that the hypothesis of an AMF is rejected as an explanation of empirical firm-to-firm flows, even at the level of submarkets. Using community detection methods for network data, independent studies by Guerrero and Axtell (2013) and Schmutte (2014) show that conventional classifications such as industries and geographical regions poorly capture the clusters of labor that are detected in employer-employee matched micro-data. For these reasons, a framework that does not rely on arbitrary aggregations to define submarkets would represent a significant methodological improvement. Petrongolo and Pissarides (2001) suggests the use of graph theory as a potential tool to overcome arbitrary aggregations. We take this approach in order to depart from the established notions of submarkets and instead look at labor dynamics as random walks on a graph.

1.2 A Network Approach

Our theory is inspired in a simple and intuitive mechanism of job search. When a person looks for a job in search of a vacancy, he or she approaches a group of firms that are 'accessible' in the short run. Such group is determined by the frictions of the labor market and we assume that it is specific to the firm where this person was last employed. We represent the correspondence between firms and their respective groups of accessible firms through a LFN. In this network, firms are represented by nodes. An edge between firms i and j means that frictions are such that j will be accessible to employees of i and vice versa. Therefore, edges have a categorical nature that represents the possibility (or impossibility in their absence) of labor flows between firms. Firm i's edges determine its first neighbors, which are equivalent to the group of accessible firms to someone employed in i. We refer to these firms as i's neighbor firms. As a person progresses through his or her career, he or she traverses the economy by taking jobs at the neighbor firms of past employers. This gradual navigation process is fundamentally different from previous approaches because the identity of the firm (i.e., its position in the LFN) matters in order to determine the employment prospects of the job seeker. There is a number reasons why this is important. To mention a few, it allows to study the composition of aggregate unemployment at the firm level; it sheds light on the effect of localized shocks and targeted policies; and it exploits the granularity and inter-firm structure captured in employer-employee matched records. By analyzing the steady-state equilibrium, we obtain analytical solutions that inform us about local unemployment, local flows, firm sizes, and firm hiring behavior. In addition, this framework provides new micro foundations of the AMF that are consistent with important stylized facts of labor markets such as the Beveridge curve and the employer-size premium.

Network theory has been extensively used to study labor markets in the context of information transmission through social networks. The pioneering work of Granovetter (1973) showed the importance that infrequently-used personal contacts have in acquiring non-redundant information about vacancies. Although Granovetter's hypothesis has been recently challenged by studies that use comprehensive social media micro-data (Gee et al., 2014,?), the importance of social networks in diffusing job information is not in question. Other empirical studies about social networks in labor markets look at migration (Munshi, 2003), urban and rural unemployment (Wahba and Zenou, 2005), investment in personal contacts (Galeotti and Merlino, 2014), and local earnings (Schmutte, 2010) among other topics. On the theoretical side, there is a substantial number of models concerning social networks in labor markets, pioneered by Boorman (1975) and Montgomery (1991b). Some studies have focused on labor outcomes as a result of the structure of social networks (Calvó-Armengol and Jackson, 2004; Calvó-Armengol and Zenou, 2005; Calvó-Armengol and Jackson, 2007; Schmutte, 2010). Other works analyze inequality and segregation effects in the job market (Calvó-Armengol and Jackson, 2004; Tassier and Menczer, 2008). For a review of these and other models, we refer to the literature survey provided by Ioannides and Loury (2004).

Despite the wide application of network methods to study labor markets, most of this work was only focused on the role of social networks in communicating information about vacancies. These studies have important applications in long-term policies such as affirmative action laws, but are not so useful for short-term policies such as contingency plans in the presence of shocks. Furthermore, the role of the firm in these models becomes trivial if not absent, which is problematic for policies that aim at incentivizing firms. In fact, little has been done to study labor mobility on networks. To the best of our knowledge, there are only a few studies that analyze labor flows through networks. For example, Guerrero and Axtell (2013) study firm-to-firm labor flows using employee-employer matched records from

Finland and Mexico. They characterize the topology of these labor flow networks and find that network connectivity is highly correlated with employment growth at the firm level. Using US micro-data, Schmutte (2014) constructs job-to-job networks in order to identify four job clusters. Mobility between these clusters is highly frictional and dependent on the business cycle. Both studies find that any clusters identified through community detection methods have little correspondence to standard categorizations such as industrial classification, geographical regions, or occupational groups. The LFN framework provides a new way to analyze labor dynamics, while contributing to the use of methods from network science in economics.

Our work complements five strands of literature. First it adds to the family of search and matching models in labor economics by introducing the method or random walks on graphs as a new tool to analyze labor mobility and aggregate unemployment. It also pushed the boundaries on how employer-employee matched micro-datasets are used today. Second, it contributes to the field of networks in labor markets by expanding the application of network methods beyond the scope of personal contacts. Social networks are difficult to observe at a large-scales[1]. Since LFNs partially capture labor flows induced by personal contacts (people who worked together may recommend each other in the future), they serve as an additional source of information to study the effect of social networks in the labor market. Third, it complements the literature on micro-foundations of the AMF (Butters, 1977; Hall, 1979; Pissarides, 1979; Montgomery, 1991a; Lang, 1991; Blanchard and Diamond, 1994; Coles, 1994; Coles and Smith, 1998; Stevens, 2007; Naidu, 2007). Because frictions are captured in the form of a network, there is no need to assume an aggregate matching process. Fourth, it strengthens the growing literature of inter-firm networks (Saito et al., 2007; Konno, 2009; Atalay et al., 2011; Acemoglu et al., 2012; di Giovanni et al., 2014). By avoiding aggregation into arbitrary submarkets, the network approach allows to study firm and labor dynamics jointly. Fifth, it contributes to the study of local labor markets by providing a new way of defining localities at the level of the firm, which facilitates the study local shocks and their

[1]Although online social networks provide a rich source of information, they are highly susceptible to biases and multiple factors that incentivize individuals to opt out of this form of communication.

propagation.

This paper is organized in the following way. Section 2 presents the model in two parts. First, we solve the problem of random walks on graphs in order to show that there is a unique steady-state. Second, we introduce a model in which firms maximize their expected steady-state profits by setting the frequency at which they hire new workers. At this point, the model assumes exogenous wages, which allows to obtain parsimonious predictions that can be empirically tested with standard econometrics. In section 3, we use employer-employee matched micro-data to test the model's predictions. We find that our results are significant and robust across 20 annual cross-sections of data. In section 4 we endogenize wages and find that, in equilibrium, firms' hiring behavior correlates through the LFN. This is our main result because the composition of aggregate unemployment depends on the structure of the LFN. We fit the model to the empirical data and find that the LFN topology may be responsible for more than half of the aggregate unemployment and temporal variation. In section 5 we discuss the results, their policy implications, and potential of this framework for future research.

2 Model with an Exogenous Wage

2.1 Setup

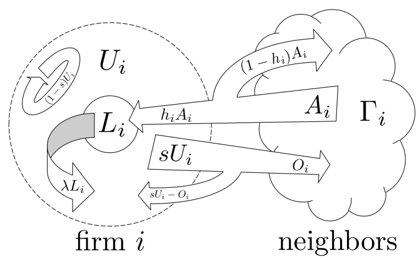
Consider an economy with N firms and H workers. Let G denote a connected, unweighted, and undirected graph that represents the LFN of the economy. G is exogenous and fixed, and nodes represent firms. We assume that G has a single component. However, the results are generalizable for networks with multiple components. The edges in G have no weights because they represent a categorical aspect of the labor market: whether we should expect labor flows between two firms or not. The network is undirected because the edges capture some 'affinity' between firms such that frictions are low in both directions. This is a firm-centric model in the sense that it emphasizes the role of firms and how their hiring behaviour modulates the flows on the LFN.

A specific firm i has k_i edges in G, also known as the degree of i. The set Γ_i contains all firms $j \neq i$ such that i and j are directly connected through an edge, i.e. Γ_i is the set of i's neighbor firms. Every period, firm i may receive an external shock in the form of an investment; this happens with with probability v. This investment enables i to open vacancies, in which case we say that the firm is *open* to receive job applications. With probability $1-v$, firm i is not shocked, so it does not take any applications, and we say that it is *closed*.

We can think of job applications as people dropping their CV in the firm's mailbox every period, regardless if it has vacancies or not. The firm accepts CVs only if when it is open. Since this repeats over time, firms gain knowledge on the average number of CVs received every time that they are open. We denote this quantity as the number A_i of applications received. Opening each vacancy is costly, so it is in the best interest of the firm to use the information that is has on A_i in order to avoid opening vacancies that would remain unfilled (we assume that vacancies expire at the end of every period). For this reason, we assume that firms open no more vacancies than A_i. Consequently, the firm has to pick a fraction of all the applicants if they are more than the number of vacancies. For analytical simplicity, we work with this fraction $h_i \in [0,1]$, which we call the *hiring policy*. Firms do not discriminate between applicants, so h_i is the probability of becoming employed for every worker that applies to firm i.

Workers are homogeneous and can be in one of two states: *employed* or *unemployed*. Regardless of his or her state, each worker is always associated with a firm. Therefore, jobless workers are associated to their last employers. Each worker employed by firm i faces the possibility of becoming unemployed with probability λ. If unemployed, the worker decides to search for a job with probability s or to remain unemployed with the complement. If he or she chooses to search, the worker looks at the set $\gamma_i \subseteq \Gamma_i$ of i's neighbor firms that received investments. Hence, we say that γ_i is the set of open neighbors of i and it may

Figure 1: Firm Dynamics



The diagram illustrates the flows that take place at the level of the firm O_i denotes the total number of unemployed agents associated to firm i who find a job, i.e. i's total outflows.

change every period. If $|\gamma_i| = 0$, the job seeker remains unemployed for the rest of the period. Otherwise, he or she selects a firm $j \in \gamma_i$ at random with uniform probability and submits a job application. For simplicity, we assume that each job seeker can submit at most one application per period. It is possible to return to i when the last job was held at j for which i is an element of Γ_j. This means that we do not allow for direct recall. This omission does not change the qualitative character of the results, but simplifies their intuition[2]. Finally, if the job application is successful, the job seeker becomes employed at j, updating its firm association. Otherwise, it remains unemployed for the rest of the period.

Figure 1 summarizes the model in terms of the inflows and outflows O_i of firm i. These ingredients constitute a stochastic process that can be clearly summarized in the pseudocode

[2]Direct recall can be easily integrated to address recall unemployment (Fujita and Moscarini, 2013).

of algorithm 1.

```
for period t do
    for each firm i in G do
        receive investment shock with probability v;
    end
    for each worker do
        get associated firm i;
        if employed then
            become unemployed with probability λ;
        end
        else
            become active seeker with probability s;
            if active then
                randomly select an open firm j ∈ γ_i;
                submit a job application to j;
            end
        end
    end
    for each open firm i in G do
        hire h_i A_i new workers from the pool of applicants;
    end
end
```

Algorithm 1: Timing

The reader may be concerned about the possibility that a job seeker may occasionally search among firms that are not connected to his or her last employer. If the probability of such event is low, the model preserves the roughly the same characteristics because the LFN induces a dominant effect on job search. When this probability is large, the model becomes an 'urn-balls' model, so the structure of the network is irrelevant. What should be the empirically relevant magnitude of such probability? Previous work shows that the idea of searching on a network is empirically compelling since firm-to-firm labor flows tend to be significantly persistent through time (López et al., 2015). In fact, unrestricted random matching between firms and workers is formally rejected when looking at employer-employee matched records (Guerrero and López, 2015). These results suggests that, in a more general model, the probability of searching 'outside' of the network has to be calibrated with a low value. Such a model can be easily constructed, but its solutions do not have explicit form. In contrast, focusing exclusively on job search 'on' the network yields explicit solutions, which is convenient for building economic intuition.

2.2 Dynamics

The process described in algorithm 1 is a random walk on a graph with waiting times determined by the investment shocks v, the separation rate λ, the search intensity s, and the set of hiring policies $\{h_i\}_{i=1}^{N}$. In order to characterize the dynamics of the economy, we concentrate on the evolution of the probability $p_i(t)$ that a worker is employed at firm i in period t, and the probability $q_i(t)$ that a worker is unemployed in period t and associated to firm i. For this purpose, let us first construct the dynamic equations of both probabilities and then concentrate on the steady-state solution.

In period t, the probability that a worker is employed at firm i depends on the probability $(1-\lambda)p_i(t-1)$ that he or she was employed at the same firm in the previous period and did not become separated. In case that the worker was unemployed during $t-1$, then $p_i(t)$ also depends on: the probability $q_i(t-1)$ that the worker was associated to a neighbor firm j; on the probability $\Pr(\gamma_j^{(i)})$ of having a particular configuration $\gamma_j^{(i)}$ of open and closed neighbors of j such that i is open; and on the probability $1/|\gamma_i^{(i)}|$ that the worker picks i from all of j's open neighbors. Altogether, summing over all possible neighbors and all possible configurations of open neighbors, and conditioning to the search intensity and hiring policy, the probability that a worker is employed by firm i in period t is

$$p_i(t) = (1-\lambda)p_i(t-1) + sh_i \sum_{j\in\Gamma_i} q_j(t-1) \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) \frac{1}{\left|\gamma_j^{(i)}\right|}, \tag{1}$$

where $\{\gamma_j^{(i)}\}$ denotes the set of all possible configurations of open and closed neighbors of j where i is open.

The probability that a worker is unemployed during t while associated to firm i depends on the probability $\lambda p_i(t-1)$ of becoming separated from i in the previous period. On the other hand, if the worker was already unemployed, the probability of remaining in such state depends on: the probability $1-s$ of not searching in that period; the probability $\Pr(\gamma_i = \emptyset)$ that no neighbor firm of i is open; and the probability $1-h_j$ of not being hired by the chosen open neighbor j. Accounting for all possible non-empty sets γ_i of open neighbors, the probability of being unemployed in t and associated to firm i is given by

$$q_i(t) = \lambda p_i(t-1) + q_i(t-1)\left[s\sum_{\gamma_i\neq\emptyset}\Pr(\gamma_i)\frac{1}{|\gamma_i|}\sum_{j\in\gamma_i}(1-h_j) + s\Pr(\gamma_i=\emptyset) + (1-s)\right]. \tag{2}$$

In the steady-state, $p_i(t) = p_i(t-1) = p_i$ and $q_i(t) = q_i(t-1) = q_i$ for every firm i. We concentrate on the steady-state solution in order construct a model that allows to study how firm behavior modulates labor flows and affects aggregate unemployment. The following results follow from solving eqs. (1) and (2).

2.3 Firm Size and Number of Applications

In order to construct a firm-centric model, we are study the steady-state average firm size and the average number of job applications received. Abusing notation, we denote these averages as L_i and A_i respectively. The next propositions follow from solving eqs. (1) and (2).

Proposition 1. *The process specified in algorithm 1 has a unique steady-state where probabilities p_i and q_i are time-invariant for every firm i.*

Existence follows from a standard result in random walks on graphs (Bollobás, 1998) (see appendix). Uniqueness comes from condition

$$1 = \sum_{i=1}^{N} p_i + \sum_{i=1}^{N} q_i,$$

which indicates that all probabilities should add up to one, implying that every worker is either employed or unemployed, and associated to only one firm. This result implies that a unique steady-state is always reached regardless of how the hiring policies in $\{h_i\}_{i=1}^{N}$ are assigned to each firm in the LFN. López et al. (2015) provide more general results for heterogeneous separation rates and heterogeneous investment shocks. However, this version is more suitable for economic modeling because it yields explicit solutions with intuitive economic meaning.

Proposition 2. *The steady-state average size of a firm i that follows eqs.* (1) *and* (2) *is*

$$L_i = \frac{\varphi}{\lambda} h_i \bar{h}_{\Gamma_i} k_i, \tag{3}$$

where $\bar{h}_{\Gamma_i}$ is the average hiring policy of i's neighbor firms and φ is a normalizing constant.

For now, let us defer the explanation of φ for a few paragraphs. Equation (3) suggests that, *ceteris paribus*, the size of a firm increases with its degree. As expected, firms can increase their own sizes through larger hiring policies. Equation (3) captures an externality: a firm's hiring policy affects the size of its neighbor firms. This result follows from an intuitive mechanism. If firm i hires more people from its pool of applicants, it increases its own size. In consequence, more people will become separated from i through the exogenous separation process governed by λ (which also reduces the size of the firm). More unemployed individuals associated to i translates into a larger pool of job seekers that will potentially apply for a job at i's neighbor j. Therefore, if everything else is constant, A_j increases, contributing to j's growth. This mechanism becomes evident in the following result.

Proposition 3. *The steady-state average number of applications received by a firm i that follows eqs.* (1) *and* (2) *is*

$$A_i = \lambda \varphi \bar{h}_{\Gamma_i} k_i. \tag{4}$$

The proof follows from the fact that, in the steady-state, the number of separated employees λL_i must equal the number of newly hired ones $h_i A_i$ in order for L_i to remain constant through time (see appendix).

2.4 Hiring Policy and Profits

Once with firm size and number of applications, we propose a simple profit-maximization model inspired in (Barron et al., 1987). This model captures the interdependence between economic behaviors of connected firms. This model is simple in the sense that it contains the most basic ingredients to account for how firms adapt to the model parameters, but not for more sophisticated behavior such as discriminating between job candidates or investing in human and physical capital. These and other factors can be incorporated in more complicated versions. However, introducing more parameters and mechanisms defeats the purpose of gaining a clean intuition about how hiring behavior modulates labor flows and determines the composition of aggregate unemployment. For this reason, this model is ideal for the task.

The goal of firm i is to maximize its expected steady-state profit Π_i. All firms produce with labor as their only input and have linear technologies such that productivity y is additive. When a firm engages in production, it pays the market wage $w \in (0, 1)$ only to those workers who are not separated in the corresponding period. Firm size L_i is important in the maximization process because it determines the size of the output. Another variable that firms take into account is the average number of applications A_i received in the steady-state. Firm i expects to hire $h_i A_i$ new workers, so the hiring policy h_i serves as an instrument to compensate for the separated workers. We capture the cost of opening vacancies through a cost associated to the hiring policy of the firm. Let us assume that larger firms incur in marginally higher costs because they invest more in recruiting, screening, and other related administrative processes. The overall hiring cost is normalized by a parameter $c \in (0, 1)$, so the cost incurred by firm i is $cL_i h_i$.

Since hiring only takes place when the firm receives an investment, only a fraction of these costs are incurred when a firm is closed. Let $\kappa \in [0, 1]$ denote such fraction. These sunk costs can be interpreted as setup expenses for screening future applications. The firm's problem is to maximize profits by setting an optimal hiring policy h_i^*. Therefore, the firm solves the problem

$$\max_{h_i} \Pi_i = (1 - \lambda)(y - w)L_i + v(y - w)h_i A_i - vcL_i h_i - (1 - v)\kappa c L_i h_i, \tag{5}$$

We assume that firms understand how hiring policies affect A_i and L_i, and that they take their neighbors' hiring policies as given. This is formalized by substituting eqs. (3) and (4) in eq. (5), which provides convexity to the profit function in order to obtain the optimal hiring policy

$$h^* = \frac{\psi}{2\phi}(y - w), \tag{6}$$

where $\psi = (1 - \lambda + v\lambda)$ and $\phi = c(v + \kappa - v\kappa)$. We have removed sub-index i because the optimal hiring policy is independent of k_i. This result is quite intuitive in a neoclassical sense, since higher wages are compensated with lower hiring policies. It also suggests that, with a unique exogenous wage, all firms set the same optimal hiring policy. This means that we can rewrite some of these results exclusively as functions of k_i. More specifically, we rewrite the firm size as

$$L_i = \varphi h^{*2} k_i, \tag{7}$$

and the profit as

$$\Pi_i^* = \frac{\varphi \psi^3}{8\lambda\phi^2}(y-w)^3 k_i, \tag{8}$$

which later will be useful for empirical testing.

2.5 Firm-Specific Unemployment and Aggregation

Solving eqs. (1) and (2) yields the average number of unemployed individuals associated to firm i in the steady-state. This is a bottom-up construction of unemployment that takes into account how it is distributed across firms, so we term it *firm-specific unemployment.* This new measure provides information about the employment prospects of a firms' ex-employees and the tools to identify pools of local unemployment. Firm-specific unemployment is obtained from the following result.

Proposition 4. *The steady-state average unemployment associated to a firm i that follows eqs.* (1) *and* (2) *is*

$$U_i = \frac{\varphi h_i k_i}{s[1-(1-v)^{k_i}]}. \tag{9}$$

The normalizing constant φ captures the population conservation condition $H = \sum_i L_i + \sum_i U_i$, so it takes the form

$$\varphi = \frac{H}{\sum_{i \in G} h_i \bar{h}_{\Gamma_i} k_i \left[\frac{1}{\lambda} + \frac{1}{s\bar{h}_{\Gamma_i}[1-(1-v)^{k_i}]}\right]}. \tag{10}$$

Equation (9) becomes more intuitive when multiplying by $\frac{\lambda \bar{h}_{\Gamma_i}}{\lambda \bar{h}_{\Gamma_i}}$, in which case we obtain

$$U_i = \frac{\lambda L_i}{s\bar{h}_{\Gamma_i}[1-(1-v)^{k_i}]}. \tag{11}$$

Note that $s\bar{h}_{\Gamma_i}[1-(1-v)^{k_i}]$ is the transition probability from unemployment to employment for a worker associated to firm i. The reciprocal of this probability is the average duration $\bar{t}_i^u$ of an unemployment spell for an individual whose last job was in i. Therefore, we can rewrite eq. (9) as

$$U_i = \lambda \bar{t}_i^u L_i, \tag{12}$$

which will become useful for empirical testing. In general, firm-specific unemployment is an interesting measure because it not only provides a highly granular unit of the composition of aggregate unemployment, but also yields information about how good will be the employment prospects of someone working at a particular company.

Due to the independence between degree and hiring policy implied by eq. (6), aggregation of unemployment is straightforward given that the firm-specific unemployment rate is defined as

$$u_i = \frac{U_i}{U_i + L_i} = \frac{\lambda}{\lambda + sh^*[1-(1-v)^{k_i}]}, \tag{13}$$

which is non-increasing and convex in k_i. Note that for a LFN where all firms have the same degree, eq. (13) is equivalent to Beveridge curve obtained in 'urn-balls' models.

Let the LFNs of two economies be represented by graphs G and G', with corresponding degree distributions P and P', and aggregate unemployment rates $u = \sum_{k=1}^{k_{\max}} u_k P(k)$ and $u' = \sum_{k=1}^{k_{\max}} u_k P'(k)$. Then, the next results follow from network stochastic dominance (Jackson and Rogers, 2007a,b; López-Pintado, 2008).

Proposition 5. *If P strictly first-order stochastically dominates P', then $u < u'$.*

Proposition 5 is quite intuitive since the average firm connectivity of G is higher than in G'. An LFN with higher connectivity reflects an economy with lower labor market frictions. Under these conditions, job seekers have better chances of finding open firms and new job opportunities.

Proposition 6. *If P' is a strict mean-preserving spread of P, then $u < u'$.*

Proofs of propositions 5 and 6 follow from direct differentiation of eq. (13), which shows that u is non-increasing and convex in k_i. Proposition 6 means that more degree heterogeneity translates into higher unemployment. Heterogeneity in a LFN reflects the 'roughness' of the search landscape. It is analogous to heterogeneity in search and matching models. However, there is the fundamental difference: agents traverse the economy by gradually navigating the LFN, instead of being randomly allocated to any firm. As we will learn ahead, this subtle difference in the reallocation process induces significant effects in aggregate unemployment when hiring policies are heterogeneous. We will show that the LFN not only has an ordinal effect on aggregate unemployment, but also a significant impact on its level and temporal variation.

At this point, it is important to summarize what we have achieved so far. We introduced a model of job search as random walks on a graph, influenced by the optimal hiring policies of firms. In doing so, we characterized the dynamics of the model and constructed the equations that describe them. We showed that the model has a unique steady-state, which yields economically intuitive expressions for L_i and A_i. The firm size captures an externality through which the hiring behavior of a firm affects the size of its neighbors. Assuming a unique exogenous market wage, we solved the profit-maximization problem of the firm, suggesting independence between hiring policies and degree. This allowed us to rewrite some of the results in a form in which k_i is identifiable, which we will exploit ahead in order to perform empirical tests. Finally, we obtained an expression for U_i, which is a new granular measure of how a firm contributes to unemployment. Using network stochastic dominance, we learnt that degree heterogeneity in the LFN induces higher aggregate unemployment rates. In the next section, we will test some of our theoretical results using empirical data.

3 Empirical Support

In this section, we test the model's predictions when wages are considered exogenous and homogeneous. For this purpose, we use employer-employee matched micro-data from two countries and we introduce a method to reconstruct LFNs. Given the simple form of our results, these tests should not be interpreted as an attempt to provide definite empirical measures. Instead, we use them as a way to show that our theory is empirically sound.

3.1 Data

We use different datasets of employer-employee matched records. The first is the Finnish Longitudinal Employer-Employee Data (FLEED), which consists of an annual panel of employer-employee matched records of the universe of firms and employees in Finland. The panel was constructed by Statistics Finland from social security registries by recording the association between each worker and each firm (enterprise codes, not establishments), at the end of each calendar year. If a worker is not employed, it is not part of the corresponding cross-section. The result is a panel of 20 years that tracks every firm and every employed individual at the end of each year (approximately 2×10^5 firms and 2×10^6 workers).

FLEED can be merged with other datasets that provide information about companies. For this, we employ Statistics Finland's Business Register, an annual panel providing number of employees and profits per firm. The Business Register is constructed from administrative data from the Tax Administration, and from direct inquiries from Statistics Finland to business with more than 20 employees. FLEED and the Business Register provide data on labor flows, firm sizes, and profits from different sources. Unfortunately, their temporal aggregation prevents us from measuring firm-specific unemployment because it is not possible to observe whether a person underwent an unemployment spell between jobs. For this purpose, we employ an additional dataset.

We use a dataset from Mexico consisting of employer-employee matched records with daily resolution. The data was obtained by sampling raw social security records from the Mexican Social Security Institute. Approximately 4×10^5 individuals who were active between 1989 and 2008 were randomly selected and their entire employment history was extracted (hence, covering dates prior to 1989). This procedure generates a dataset with nearly 2×10^5 firms. The records contain information about the exact date in which a person became hired/separated by/from a firm. Therefore, it is possible to identify unemployment spells, duration of each spell, and associations between job seekers and their last employer.

Although the datasets show firm-to-firm flows, they do not contain explicit information about the LFN. In order to construct k_i from the data, we identify those firm-to-firm flows that most likely took place in the network, as opposed to those that result from random hops generated from a more aggregate process (e.g., through an AMF). For this purpose, we identify *significant edges*. If the labor flows between a pair of firms have a higher volume than what we would expect under an AMF, that is an indicator of a significant edge between firms.

3.2 Significant Edges

The idea of significant edges begins with the null hypothesis of the AMF being responsible for every firm-to-firm labor flow observed in data. Consider the total number of matches M that take place in a given period with U unemployed and V vacancies. Suppose that an AMF $M = f(U, V)$ is responsible for these matches. This assumption implies that these M matches are created with homogeneous probability. A second, less obvious assumption, is that any distribution of vacancies across firms is acceptable. To explain this point, consider the number of vacancies V_i in firm i. Under the AMF, the number of matches is $M \leq \sum_i^N V_i = V$. In an AMF, any sequence $\{V_1, V_2, ..., V_N\}$ such that $\sum_i^N V_i = V$ is permissible. The same applies to any sequence $\{M_1, M_2, ..., M_N\}$ such that $\sum_i^n M_i = M$, where M_i denotes the number of matched individuals whose last employer was firm i. Employer-employee matched records provide information about these sequences.

We use the information contained in FLEED in order to infer the statistical significance of labor flows between pairs of firms. Here we describe the spirit of the test, but all the details are provided in the appendix. Using empirical data, we construct a weighted directed graph G' where an edge $e_{i,j}$ indicates that there was at least one flow from i to j. The total number of flows from i to j are captured in the weight of $e_{i,j}$. The test consists of comparing the observed weight of $e_{i,j}$ against its expected weight under the hypothesis that labor reallocation takes place through an AMF. When its weight is larger than the expectation under the null, we say that $e_{i,j}$ is a significant edge. Obviously, the null hypothesis can be further refined using alternative models. However, Guerrero and López (2015) have shown that the number of significant edges tends to be high, even when considering local AMFs.

Figure 2(a) shows that more than 80% of the edges are significant in 20 different cross-sections of FLEED. Moreover, most of the labor flows in the economy take place on significant edges. Figure 2(b) illustrates the topology revealed by these labor flows, and highlights the small number of edges that are not significant. These non-significant edges tend to be concentrated among the largest firms; they are so large that flows between them can be expected under a homogeneous matching process. In order to reconstruct a LFN we generate an undirected unweighted graph with edges corresponding to the significant edges found G'. The resulting network provides the degree k_i of each firm. We perform this procedure for every annual cross-section in FLEED in order to test our results.

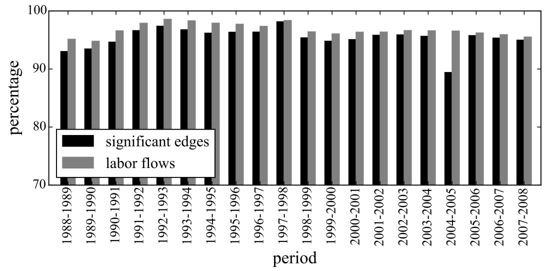
3.3 Empirical Testing

We test the prediction concerning degree and firm size, expressed in eq. (7) by estimating the model

$$L_i = \beta_L k_i + \epsilon_i, \tag{14}$$

Figure 2: Significant Edges a the Finnish LFN

(a) Significant Edges



(b) Labor Flow Network

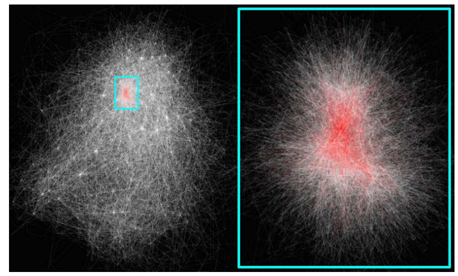


Figure 2(a) shows the percentage of edges that can be considered significant. Each bar corresponds to one cross-section from FLEED. Figure 2(a) provides a graphical representation of the network obtained from significant edges. The image shows the 2,000 most connected firms in the period 2007-2008. Significant edges are colored in white.

where ϵ_i is an error term and $\beta_L = \varphi h^{*2}$. If the model is empirically consistent, then the null hypothesis of $\beta_L = 0$ should be rejected. In a similar spirit, we test the predicted positive relationship between degree and profits suggested in eq. (8) by estimating the model

$$\Pi_i = \beta_\Pi k_i + \epsilon_i, \tag{15}$$

where $\beta_\Pi = \frac{\varphi\psi^3}{8\lambda\phi^2}(y-w)^3$.

We estimate both models for each annual cross-sections in FLEED. Table 1 shows that the data validates the model's prediction eq. (7) in all cross-sections, confirming a positive relationship between degree and firm size. Equation (8) is valid in most of the cross-sections. These tests provide support of the empirical soundness of the model, allowing us to proceed with further tests and theoretical developments.

We proceed to test the theoretical prediction connecting degree and firm-specific unemployment, as expressed in eq. (9). For this, we estimate the model

Year	Firm Size			Firm Profit		
	β_L	N	R^2	β_Π	N	R^2
1988	7.135*** (4.963e-01)	19,251	0.466	9.031e+04** (4.963e-01)	19,251	0.060
1989	8.200*** (6.766e-01)	19,184	0.549	1.352e+05*** (6.766e-01)	19,184	0.136
1990	11.595*** (1.294e+00)	16,164	0.398	2.291e+05*** (1.294e+00)	16,164	0.088
1991	15.706*** (1.409e+00)	12,752	0.437	2.975e+05*** (1.409e+00)	12,752	0.059
1992	17.776*** (1.620e+00)	12,306	0.416	2.250e+05 (1.620e+00)	12,306	0.019
1993	15.763*** (1.746e+00)	13,750	0.248	2.662e+05* (1.746e+00)	13,750	0.030
1994	15.156*** (2.320e+00)	16,074	0.461	3.074e+05*** (2.320e+00)	16,074	0.049
1995	15.308*** (2.278e+00)	16,675	0.432	2.561e+05*** (2.278e+00)	16,675	0.049
1996	14.193*** (1.879e+00)	20,188	0.541	3.710e+05*** (1.879e+00)	20,188	0.097
1997	11.478*** (1.436e+00)	20,220	0.627	2.405e+05* (1.436e+00)	20,220	0.065
1998	10.803*** (1.902e+00)	25,485	0.570	4.247e+05* (1.902e+00)	25,485	0.069
1999	8.509*** (1.147e+00)	27,340	0.380	7.583e+05* (1.147e+00)	27,340	0.076
2000	9.074*** (1.577e+00)	27,575	0.419	2.506e+05** (1.577e+00)	27,575	0.024
2001	10.879*** (2.048e+00)	26,882	0.500	7.449e+04 (2.048e+00)	26,882	0.007
2002	10.229*** (1.669e+00)	26,546	0.515	5.255e+05 (1.669e+00)	26,546	0.042
2003	10.021*** (1.661e+00)	27,350	0.518	3.511e+05 (1.661e+00)	27,350	0.036
2004	8.855*** (1.751e+00)	29,719	0.506	2.245e+05 (1.751e+00)	29,719	0.021
2005	7.991*** (1.180e+00)	34,089	0.596	2.172e+05 (1.180e+00)	34,089	0.032
2006	7.221*** (9.297e-01)	36,813	0.575	3.474e+05* (9.297e-01)	36,813	0.021

Table 1: Empirical test of theoretical predictions eqs. (7) and (8). The corresponding estimated models are eqs. (14) and (15). Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

$$U_i = \beta_\lambda x_i + \epsilon_i, \tag{16}$$

where $\beta_\lambda = \lambda$ is the estimated separation rate and $x_i = \bar{t}_i^u L_i$.

Using data from Mexico, we measure the number of associated employed and unemployed individuals in each firm during a single day of every year. By measuring for a single day, we guarantee that the unemployed individuals are different from the employed ones. For each firm, we choose the day when they have the maximum amount of both employed and unemployed, in other words the day of the year that maximizes $U_i L_i$ for each firm i. We compute $\bar{t}_i^u$ by averaging the duration (in number of days) of unemployment spells (shorter than 24 months) associated to firm i. If the sample size of unemployment spells per firm is high, the total number of firms in the sample becomes too low. On the other hand, the data becomes highly noisy (many firms with $U_i L_i \leq 1$) if the sample size of unemployment spells per firm is too low. Therefore, we select firms with at least 80 associated unemployment spells in order to maximize both the number of unemployment spells per firm and the number of firms in the sample.

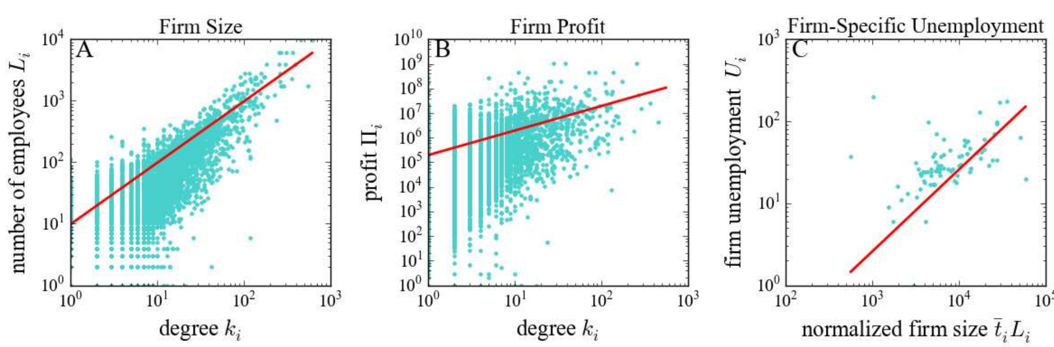
Table 2 shows that the theoretical prediction in eq. (12) is empirically consistent. Moreover, all the estimated separation rates fall in the interval $(0, 1)$, which is reassuring if we think of the model as a new way to estimate the separation rate.

Figure 3 provides a graphical illustration of the three theoretical predictions that are tested using empirical data. It is clear that the predicted relationships are not only statistically significant but positive. Each panel corresponds to an annual cross-section of the datasets: panel A corresponds to eq. (7), panel B to eq. (8), and panel C to eq. (12). With this qualitative corroboration of the empirical soundness of the model, we proceed to extend the model in order to analyze equilibrium unemployment.

Year	β_λ	N	R^2
1999	0.001* (4.203e-04)	83	0.457
2000	0.001*** (3.726e-04)	89	0.502
2001	0.002*** (3.153e-04)	92	0.506
2002	0.002*** (3.448e-04)	93	0.581
2003	0.002*** (3.250e-04)	93	0.729
2004	0.002*** (3.894e-04)	84	0.731
2005	0.003*** (4.936e-04)	86	0.610
2006	0.003*** (6.643e-04)	80	0.507
2007	0.003*** (4.880e-04)	76	0.695
2008	0.003*** (5.379e-04)	72	0.702
2009	0.001** (3.868e-04)	69	0.426
2010	0.001*** (2.242e-04)	63	0.670

Table 2: Empirical test of theoretical prediction eq. (12). The corresponding estimated model is eq. (16). Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

Figure 3: Model Prediction vs. Empirical Data



The figures were constructed using the cross-section corresponding to 2006 of each dataset. The scattered dots correspond to empirical observations and the solid lines to the fitted models eqs. (14) to (16).

4 Equilibrium and Aggregate Unemployment

So far we have developed a model where there is a unique exogenous market wage. This allows us to gain new insights about the effect of the LFN on firm dynamics and unemployment. One of the main findings is an externality through which firms affect their neighbors' sizes through their hiring policies. However, it is important to study how this externality interacts with the LFN topology when h_i^* is heterogeneous. For this purpose, we endogenize wages and study the equilibrium of the economy. We define equilibrium as a sextuple

$$\Theta = \left(\{h_i^*\}_{i=1}^N, \{w_i^*\}_{i=1}^N, \{L_i^*\}_{i=1}^N, \{A_i^*\}_{i=1}^N, \{U_i^*\}_{i=1}^N, \varphi^*\right)$$

of optimal hiring policies, wages, firm sizes, applications, firm-specific unemployment, and a condition of population conservation.

Obtaining the equilibrium of the economy boils down to solving for the set of optimal hiring policies $\{h_i^*\}_{i=1}^N$. For this, we first introduce a wage generating mechanism and then solve for the equilibrium wage. This enables us to obtain $\{h_i^*\}_{i=1}^N$, and all subsequent quantities of interest. Next, we characterize $\{h_i^*\}_{i=1}^N$ and present theoretical results for three stylized networks. Finally, we calibrate the model to the Finnish empirical LFNs to show that their topology can have a dramatic impact on aggregate unemployment and its temporal variation.

4.1 Hiring Policies

In order to endogenize wages, we have chosen an aggregate supply approach. We make this choice for simplicity and analytical convenience, but other types of labor supply could be incorporated if needed. Let us assume that firms demand labor at different moments, so the aggregate supply responds to each one independently with a wage w_i. The inverse labor supply has the form

$$w_i = \frac{y\ell_i}{b + \ell_i}, \tag{17}$$

where ℓ_i is the individual demand of firm i and $b > 0$ is a parameter that affects the price elasticity. The wage is asymptotic to the productivity because we assume that workers are aware of the firms' incentives for not paying more than productivity y.

On the other hand, the labor demand of firm i is equivalent to the number of new hires. We assume that firms are wage takers, so their profit-maximization problem remains unchanged. Therefore the labor demand of firm i takes the form

$$\ell_i = h_i^* A_i. \tag{18}$$

Substituting eq. (18) in eq. (17) and using eq. (3) yields the equilibrium wage

$$w_i^* = \frac{y\varphi h_i^* \bar{h}_{\Gamma_i}^* k_i}{b + \varphi h_i^* \bar{h}_{\Gamma_i}^* k_i} = \frac{y\lambda L_i}{b + \lambda L_i}, \tag{19}$$

which explicitly shows that larger firms pay higher wages. In other words this result captures the the well-known employer size premium (Brown and Medoff, 1989; Brown et al., 1990). It also suggests that firms with a higher degree pay higher salaries when compared to other firms with the same h_i and $\bar{h}_{\Gamma_i}$.

Substituting eq. (19) in eq. (6) yields i's equilibrium hiring policy

$$h_i^* = \min\left(1, \frac{\phi b - \sqrt{\phi^2 b^2 + \phi\psi\varphi 2by\bar{h}_{\Gamma_i}^* k_i}}{-2\phi\varphi\bar{h}_{\Gamma_i}^* k_i}\right), \tag{20}$$

where the firm sets either a fraction in $(0, 1)$, or a corner solution where it hires all applicants.

Note that eq. (20) is continuous and maps $[0,1]$ into itself because $h_i \in [0,1]$ and $\bar{h}_{\Gamma_i} \in [0,1]$. Therefore, by Kakutani's fixed point theorem we know that a unique set $\{h_i^*\}_{i=1}^N$ exists, so does equilibrium Θ.

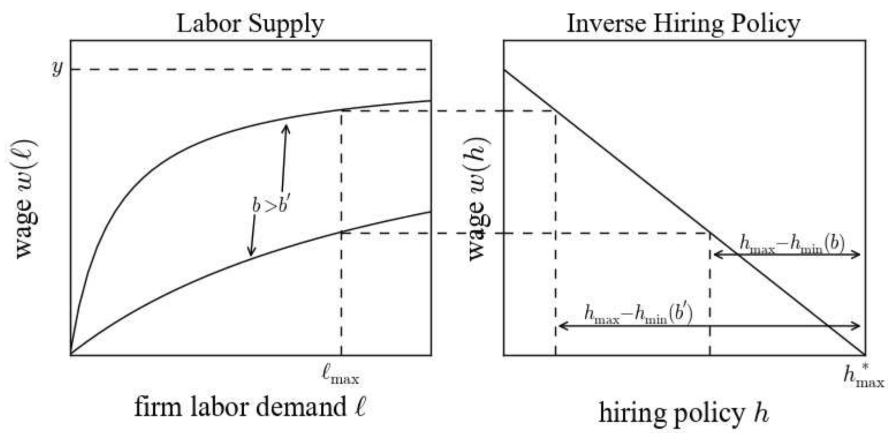
Equation (20) captures captures the interaction between the hiring behavior of firm i (expressed through h_i) and the hiring behavior of its neighbors. This interaction correlates hiring policies across the LFN. This has important implications on how we understand labor reallocation. For example, if a worker leaves a firm with a low hiring policy, he or she will most likely have immediate access to firms with slightly different hiring policies. In contrast, a standard random matching process allows job seekers to jump between firms with strikingly different hiring policies. This difference has a profound effect on our understanding of local shocks and unemployment traps due to the congestion effects generated by the navigation process on the LFN. In order to elaborate on this point, we present further theoretical predictions and their economic intuition.

4.2 Theoretical Implications

Let us build some intuition about the relationship between the equilibrium aggregate unemployment and network topology. Figure 4 illustrates the the effect that the supply elasticity has on wage dispersion and equilibrium hiring policies. Consider the firm with the largest labor demand $\ell_{\max}$, which determines the maximum wage in the economy. The latter is higher in an economy with a more inelastic labor supply, considering everything else constant. A higher wage implies a lower hiring policy for this firm, increasing the dispersion between the maximum hiring policy $h_{\max}$ and the lowest one $h_{\min}$. Firms with different degrees set different hiring policies (assuming that $\bar{h}_{\Gamma_i}$ does not cancel the effect of k_i). In a heterogeneous network, diversity of hiring policies plays a central role in determining the level of unemployment because because it correlates degrees with hiring policies in a negative way.

We know by eq. (3) that higher a k_i induces a larger firm size. Then, the negative

Figure 4: Wage Dispersion and Hiring Policies



The left panel shows two aggregate labor supplies with different elasticities obtained from eq. (17). It also presents the corresponding wages that the firm with the largest demand $\ell_{\max}$ would have to pay when confronting each supply. The right panel maps these wages through eq. (6), into the hiring policies that would be set by the firm with the largest demand.

correlation between k_i and h_i means that a larger proportion of workers (those in the largest firms) are searching for jobs in firms with lower hiring policies (their neighbors). Following this logic, we can expect that a LFN with a degree distribution that is a mean-preserving spread of another one induces higher a level of unemployment. In this example, we have introduced wage dispersion through the supply elasticity. However, the model is flexible enough to allow firm heterogeneity in parameters such as the separation rate λ, the productivity y, the hiring cost c, the sunk cost κ, and the search intensity s. This is an important strength of the model because it facilitates more realistic calibrations that consider the cross-sectional variation of firms; an important feature to understand other things such as the effect and propagation of local shocks.

In order to demonstrate the previous intuition, we solve the model for three stylized networks that are representative of real-world topologies: (i) a regular graph with a delta degree distribution, (ii) an Erdős-Rényi graph with a binomial degree distribution, and (iii) a scale-free network with a Pareto degree distribution. Solving the model for the regular network is straightforward since all firms have the same $k_i = k$. Therefore, substituting $\bar{h}^*_{\Gamma_i}$ by h^* in eq. (20) together with eq. (10) yields

$$h^* = \frac{bN(y\psi\theta - 2\lambda\phi) + \sqrt{b^2N^2(2\lambda\phi + y\psi\theta)^2 + 8byNH\lambda^2\phi\psi\theta}}{4\phi\theta(bN + H\lambda)}, \tag{21}$$

where $\theta = [1-(1-v)^k]$. For the case of the networks with heterogeneous degrees, we solved eq. (20) numerically.

Panel A in fig. 5 shows the Beveridge curves generated by the model. Here, we portray the Beveridge curve as the relationship between the unemployment rate and the average hiring policy. The curves are generated by solving the equilibria of different levels of the hiring cost c in the interval $[0.1, 0.9]$. Two notable features stand out in this diagram. First, curve from the scale-free network is significantly distant from the other two. Second, the three curves collapse when $\bar{h}^* = 1$. This is quite intuitive when we consider the sampling process that workers undergo in the LFN. If all firms set hiring policies near 1, the likelihood of getting a job depends mostly on the investment shocks, which happen uniformly across firms. In this situation, a job seeker at a firm with few edges has almost the same chance of finding a job as a worker at a firm with many connections. This also relates to the dispersion of $\{h^*_i\}_{i=1}^N$ because when firms hire all applicants there is no diversity of hiring policies, which nullifies the effect of the LFN topology.

Panel B in fig. 5 shows the employer-size premium across the three networks. It is clear that the network with largest degree heterogeneity also has the largest wage dispersion. The topology of the network does not shift the $L - w$ curve so we cannot expect significant changes in the average wage due to network structure. Panel C demonstrates the interaction between firms' hiring behavior and their neighbors'. As suggested in eq. (20), there is a negative relationship between h^*_i and $\bar{h}^*_{\Gamma_i}$. These correlations are clustered by levels of h^*_i and their dispersion is larger in the scale-free network.

As shown in panel D of fig. 5, firms with more edges tend to set lower hiring policies.

The mechanism is simple: with more neighbors, A_i grows and so does i's demand for labor. More demand implies a higher wage to be paid by the firm, which shifts its profit curve to the left. In order to compensate for higher salaries, the firm needs to re-adjust h_i^* to a lower level. Finally, as predicted by eqs. (3) and (9), firms with higher connectivity tend to be larger and have more associated unemployed agents. In addition, the network with a Pareto degree distribution also exhibits a larger firm size dispersion, which is consistent with real-world economies.

4.3 Empirical Implications

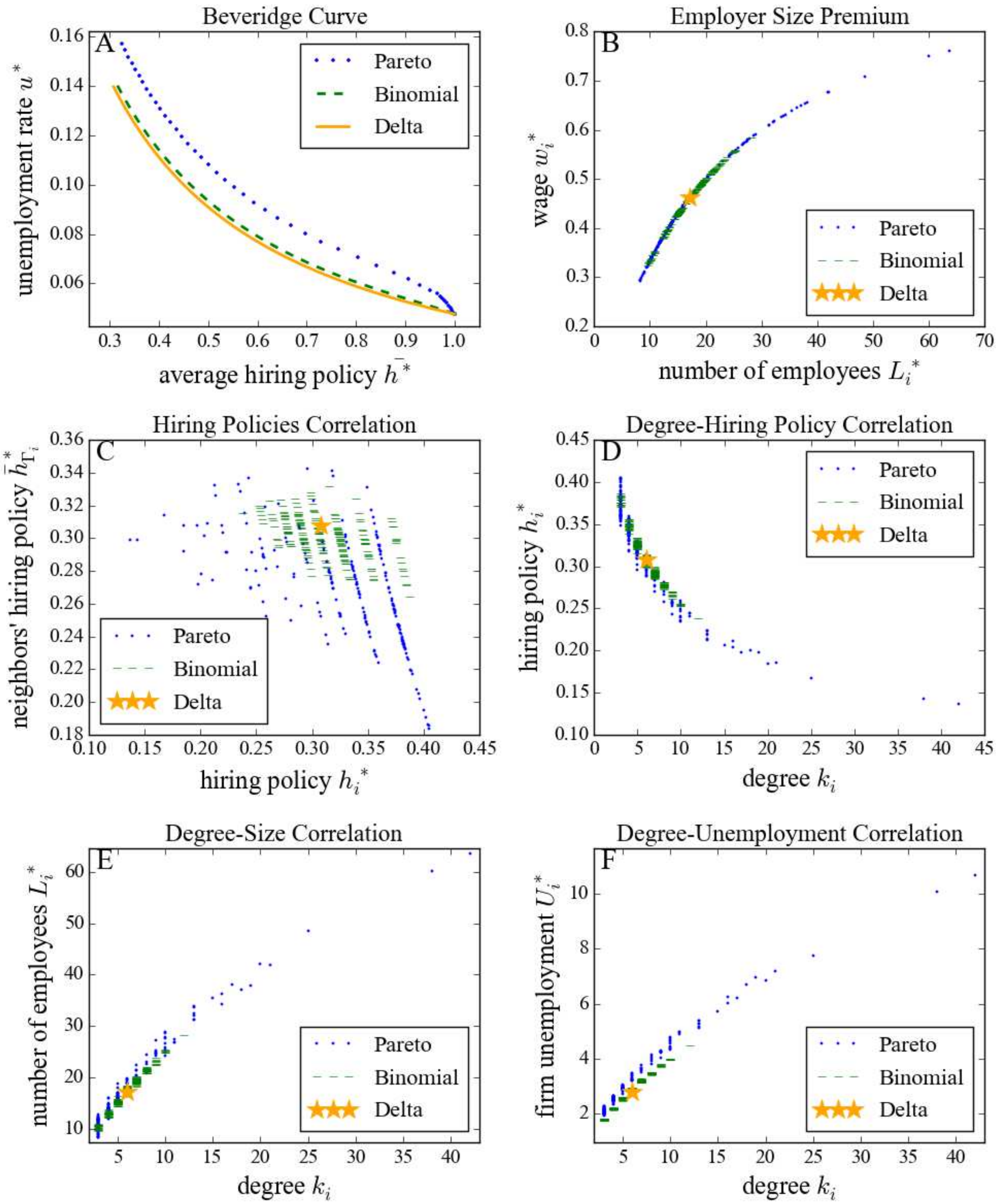
We have shown that the LFN topology has important theoretical implications in the composition of aggregate unemployment. We would like to conclude by analyzing real-world LFNs and learning something about the empirical implications of their topologies. For this purpose, we calibrate the model to match the observed aggregate unemployment rates of Finland throughout 20 years, while controlling for its LFNs and separation rates. In order to estimate λ, we make use of our last theoretical result

Proposition 7. *The steady-state average number of unemployed who become employed after being associated to a firm i that follows eqs.* (1) *and* (2) *is*

$$O_i = \varphi h_i \bar{h}_{\Gamma_i} k_i. \tag{22}$$

The proof follows from the fact that, in the steady-state, $O_i = \lambda L_i$ (see appendix). The intuition is simple: we can consider firm-specific unemployment as a pool of people that is constant through time. The inflows into U_i are λL_i while the outflows are O_i. In order for U_i to be constant, the inflows and the outflows must be equal.

Figure 5: Equilibrium Outcomes on Different Network Topologies



Equilibrium solutions for an example calibration: $\{N = 200,\ H = 4000,\ \lambda = .05,\ y = 1,\ v = .8,\ c = .1,\ \kappa = .5,\ s = 1,\ b = 1\ \}$, and different network topologies with the same average degree of 6. The solution for the network with a Dirac delta degree distribution was obtained through eq. (21), while the ones for the binomial and Pareto degree distributions were obtained numerically. Panel a shows the solutions for different levels of c. The rest of the panels show the cross-sectional variation of the solution for representative networks.

Taking advantage of eq. (22), we use the steady-state condition $O_i = \lambda L_i$ in order to estimate the model

$$O_i = \beta_\lambda L_i + \epsilon_i, \tag{23}$$

where $\beta_\lambda = \lambda$. We calibrate the model to a daily frequency, so the estimated separation rate becomes $\hat{\beta}^d_\lambda = 1 - (1 - \hat{\beta}_\lambda)^{\frac{1}{365}}$ (see appendix).

We use the annual unemployment rates in Finland from Eurostat for each period covered in FLEED. It should be noted that eq. (20) provides a new way of estimating the hiring behavior of firms. This is an important contribution because the method takes advantage of the labor inflows and outflows of each firm and does not depend on data about vacancies.

An important consideration in the calibration process is avoiding trivial solutions. In other words, we should carefully choose a set of parameters that yields an equilibrium where firms set heterogeneous hiring policies (as opposed to all firms setting the corner solution). This is important because empirical LFNs have degree distributions with a wide spread (Guerrero and Axtell, 2013). If all firms (or most) set corner solutions, the equilibrium would not be consistent with wage dispersion and the skewed firm size distributions observed in real data. Therefore, parameters c, κ, and b play a crucial role. As previously discussed, b allows wage dispersion, so an inelastic labor supply is desirable in order to generate heterogeneous hiring policies. Parameter c determines the overall level of w_i, hence of h_i^*. Finally, κ limits the maximum w_i by making the firm more sensitive to the investment shocks, even when it is closed. We normalize $y = 1$, assume full search intensity ($s = 1$), and allow v to be a degree of freedom to calibrate the model and match the observed level of aggregate unemployment.

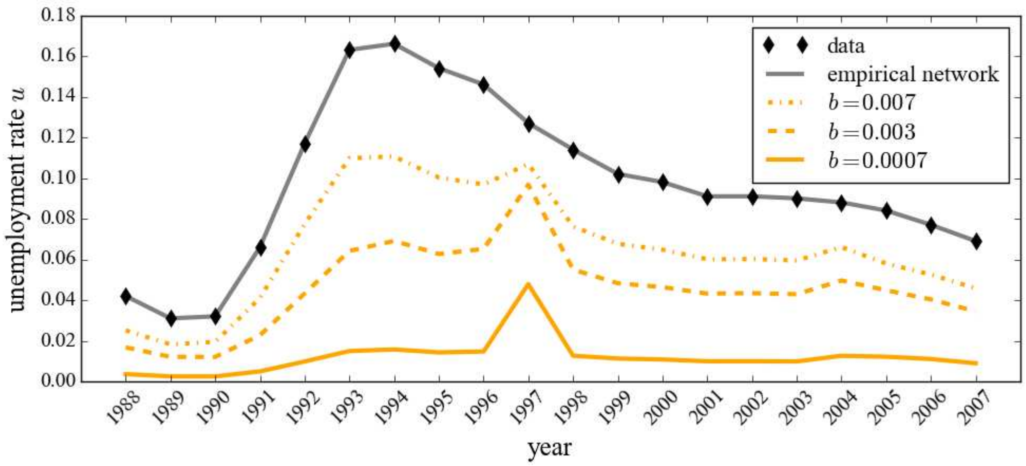
Once calibrated, we use the model to compute a counter-factual. This counter-factual consists of evaluating the model under a different network structure, while keeping everything else constant. More specifically, we estimate what would be the aggregate unemployment rate in Finland if the frictions of the labor market would have a homogeneous structure. In other words, we compute aggregate unemployment when $k_i = k$, which is given by eq. (13), where h^* corresponds to the solution of the homogeneous case in eq. (21). We perform this exercise for different supply elasticities in order to gain some insights about the minimum and maximum effects of the network topology.

Figure 6 shows the difference in aggregate unemployment between the fitted model and the counter-factual. We present results for three levels of supply elasticity[3]. As discussed previously, a more inelastic labor supply generates more wage dispersion, which contributes to a larger difference in unemployment between the real LFN and the regular network. We interpret this difference as the contribution of the network structure to aggregate unemployment. Under a very elastic labor supply, the contribution is marginal. However, if the supply is highly elastic, the contribution of the network topology can account for more than 90% of the unemployment rate. Given that real economies exhibit wage dispersion, the LFN is likely to have a significant effect on aggregate unemployment.

Finally, the LFN topology not only affects the level of aggregate unemployment, but also its variation through time. In this exercise, it is evident that more degree heterogeneity also increases the magnitude of annual variations of the unemployment rate. This is an important result considering that the origins of unemployment volatility is a highly debated topic (Mortensen and Nagypál, 2007; Pissarides, 2009; Shimer, 2010; Obstbaum, 2011). If structural changes or shocks take place (e.g., changes in λ or v), the labor reallocation process is smoother on a regular structure than on a heterogeneous one. This is quite intuitive when thinking in terms of job search as a gradual navigation on the LFN. A shock or a structural change generates heterogeneous adjustments of hiring policies when the network is not regular (and assuming wage dispersion). If the LFN has firms that concentrate many connections, labor reallocation becomes susceptible to the congestion effects that these companies generate by re-adjusting their hiring policies. In a regular LFN the reallocation process is smoother because the shock or structural change generates the same re-adjustment across

[3]The bump in the counter-factual of 1997 is caused by an anomaly in the data. Due to changes in data administration, 1997 registers a substantial increase in N (see table 1). Most of these firms have $k_i = 1$, so the average degree drops nearly 50% with respect to 1996.

Figure 6: Equilibrium Unemployment and Counterfactuals



The diamonds correspond to the observed annual aggregate unemployment rate. The grey line was obtained by calibrating the model to match the observed unemployment rates of each year using parameter values: $y = 1$, $s = 1$, $c = .1$, $\kappa = .5$, and $H = 2,000,000$ (the size of the Finnish labor force). N is the number of firms in the data, λ was estimated form the data, and v varies between years due to the fitting procedure.

all firms, which happen to have the same number of employees and associated unemployed. Therefore, the LFN offers a new perspective to study unemployment volatility and points towards the need to understand the propagation of shocks and structural changes through the gradual reallocation of labor that takes place on the network, something that we leave for future work.

5 Discussion and Conclusion

We developed a framework to study aggregate unemployment from new network-theoretic micro-foundations of job search as a gradual navigation process on a LFN. By employing the method of random walks on graphs, we solved the model for the steady-state and equilibrium. The framework allows to study the composition of aggregate unemployment with a resolution at the level of each firm. It also shows that an externality emerges between neighbor firms: 'my growth affects your growth'. We found that when labor is reallocated through networks with degree heterogeneity, hiring policies correlate through neighbor firms. Depending on the elasticity of the labor supply, the model generates wage dispersion and the topology of the LFN contributes in a significant way to the level of aggregate unemployment. This means that the way in which labor market frictions are structured (the network topology) plays a central role in the process of labor reallocation because this structure determines the pathways that labor uses to navigate through different firms. Through their hiring behavior, firms modulate the flows of labor, generating pockets of local unemployment and congestion effects. This framework provides a rich and elegant, description of decentralized labor markets with the possibility of preserving important information that is lost through arbitrary aggregations.

Our theory is empirically supported by comprehensive micro-data on employer-employee matched records. It suggests that the role of firm connectivity is key to link individual firm dynamics to aggregate unemployment. Moreover, we found that, in the case of Finland, the structure of the LFN may account for most of the aggregate unemployment rate and its temporal variation. The framework also provides a new way to estimate separation rates and hiring policies. In addition, our results suggest that the collection of new information such as firm-specific unemployment could be useful to complement our knowledge about aggregate unemployment and the role of labor policy. For example, it would shed light on the origins of unemployment volatility and mismatch unemployment.

On the theoretical side, the LFN framework can be employed to consider firm-specific phenomena such as recall unemployment. In addition, this framework is particularly well suited to study the propagation of local shocks and structural changes, a major issue in labor policy discussions. Its localized nature allows it to be implemented through other methods such as computer simulation and agent-computing models (Freeman, 1998; Geanakoplos et al., 2012) in order to study the impact and timing effects of specific policies. This facilitates the study of a much richer set of problems that are difficult to address from an aggregate perspective. For example, we could use employer-employee matched records to calibrate an agent-computing model with the real LFN and then simulate local shocks to groups of firms. The computational model would allow us to obtain information about how labor would flow out of the affected parts of the economy, and gradually find its way to firms with better employment prospects. Characterizing this gradual navigation process would be extremely helpful in designing policies that aim not only to alleviate unemployment, but to smooth transitional phases of the economy.

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Appendix (For Online Publication)

Proof of proposition 1

Let $p_i(t)$ and $q_i(t)$ be the probabilities of being employed and unemployed at firm i in period t respectively. Both quantities are dynamically described by

$$p_i(t) = (1-\lambda)p_i(t-1) + sh_i \sum_{j\in\Gamma_i} q_j(t-1) \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) \frac{1}{\left|\gamma_j^{(i)}\right|}, \tag{24}$$

and

$$q_i(t) = \lambda p_i(t-1) + q_i(t-1)\left[s\sum_{\gamma\neq\emptyset} \Pr(\gamma_i)\frac{1}{|\gamma_i|}\sum_{j\in\gamma_i}(1-h_j) + s\Pr(\gamma_i=\emptyset) + (1-s)\right]. \tag{25}$$

There, where $\gamma_j^{(i)}$ indicates a configuration of open and closed neighbors of j, such that i is open. The symbol $\{\gamma_j^{(i)}\}$ denotes the set of all possible configurations of open and closed neighbors of j where i is open. The set γ_i contains all open neighbors of i, and we denote $\emptyset$ the set of neighbors of i when all of them are closed.

In the steady-state, $p_i(t) = p_i(t-t) = p_i$ and $q_i(t) = q_i(t-t) = q_i$. Note that $\sum_{\gamma\neq\emptyset}\Pr(\gamma_i) + \Pr(\gamma_i=\emptyset) = 1$, so the system defined by eqs. (24) and (25) becomes

$$0 = -\lambda p_i + sh_i \sum_{i\in\Gamma_i} q_j \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) \frac{1}{\left|\gamma_j^{(i)}\right|} \tag{26}$$

$$0 = \lambda p_i - q_i s \sum_{\gamma\neq\emptyset} \Pr(\gamma_i)\bar{h}_{\Gamma_i}. \tag{27}$$

Let us write q_i in terms of p_i as

$$q_i = \frac{\lambda}{s\sum_{\gamma\neq\emptyset}\Pr(\gamma_i)\bar{h}_{\Gamma_i}} p_i, \tag{28}$$

and then substitute p_i with eq. (40) to obtain

$$q_i = \sum_{i\in\Gamma_i} \frac{q_j h_i \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) / \left|\gamma_j^{(i)}\right|}{\sum_{\gamma\neq\emptyset}\Pr(\gamma_i)\bar{h}_{\Gamma_i}}, \tag{29}$$

To understand this further, we write the previous equation in matrix form making use of the adjacency matrix of the graph, $\mathbf{A}$, for which $\mathbf{A}_{ij} = \mathbf{A}_{ji} = 1$ if i and j have an edge connecting them, and zero otherwise. This produces the expression

$$\sum_{j=1}^{N} \left[\mathbf{A}_{ij} \frac{h_i \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) / \left|\gamma_j^{(i)}\right|}{\sum_{\gamma \neq \emptyset} \Pr(\gamma_i) \bar{h}_{\Gamma_i}} - \delta[i,j] \right] \lambda p_j = 0 \tag{30}$$

for all i. This represents a homogeneous system of linear equations, which always has the trivial null solution, and has non-trivial solutions if and only if the matrix contained inside brackets is singular which, among other things, implies that the matrix does not have full rank. To show that our model has non-trivial solutions indeed, we define the matrix $\mathbf{\Lambda}$, with element $\mathbf{\Lambda}_{ij}$ corresponding to the expression inside brackets

$$\mathbf{\Lambda}_{ij} := \mathbf{A}_{ij} \frac{h_i \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) / \left|\gamma_j^{(i)}\right|}{\sum_{\gamma \neq \emptyset} \Pr(\gamma_i) \bar{h}_{\Gamma_i}} - \delta[i,j]. \tag{31}$$

This matrix does not possess full rank as can be explicitly seen from the fact that all columns add to zero. To show this, we first sum $\mathbf{\Lambda}_{ij}$ over i

$$\sum_{i=1}^{N} \mathbf{\Lambda}_{ij} = -1 + \sum_{i=1}^{N} \mathbf{A}_{ij} \frac{h_i \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) / \left|\gamma_j^{(i)}\right|}{\sum_{\gamma \neq \emptyset} \Pr(\gamma_i) \bar{h}_{\Gamma_i}} \tag{32}$$

where -1 comes from $-\sum_i \delta[i,j]$. We can now show that the numerator and denominator of the second term are indeed equal. To see this in detail, we organize the elements of $\{\gamma_j^{(i)}\}$ by cardinality $|\gamma_j^{(i)}|$, and rewrite the numerator as

$$\sum_{i=1}^{N} \mathbf{A}_{ij} h_i \sum_{\{\gamma_j^{(i)}\}} \Pr(\gamma_j^{(i)}) / |\gamma_j^{(i)}| = \sum_{c=1}^{|\Gamma_j|} \frac{1}{c} \sum_i \mathbf{A}_{ij} h_i \sum_{|\gamma_j^{(i)}| = c} \Pr(\gamma_j^{(i)}), \tag{33}$$

where the last sum is over all elements of $\{\gamma_j^{(i)}\}$ with equal size c. Now, the sum over i guarantees that each neighbor of j belonging to a particular $\gamma_j^{(i)}$ is summed, along with the corresponding h_r, where $r \in \gamma_j^{(i)}$. Therefore, the sum over i can be rewritten as

$$\sum_i \mathbf{A}_{ij} h_i \sum_{|\gamma_j^{(i)}| = c} \Pr(\gamma_j^{(i)}) = \sum_{|\gamma_j| = c} \left(\sum_{r \in \gamma_j} h_r \right) \Pr(\gamma_j) \tag{34}$$

and inserting this into the sum over c leads to

$$\sum_{c=1}^{|\Gamma_j|} \frac{1}{c} \sum_{|\gamma_j| = c} \left(\sum_{r \in \gamma_j} h_r \right) \Pr(\gamma_j) = \sum_{\gamma_j \neq \emptyset} \frac{\sum_{r \in \gamma_j} h_r}{|\gamma_j|} \Pr(\gamma_j) = \sum_{\gamma_j \neq \emptyset} \langle h \rangle_{\gamma_j} \Pr(\gamma_j) \tag{35}$$

Therefore,

$$\sum_{i=1}^{N} \mathbf{A}_{ij} h_i \sum_{\{\gamma_j^{(i)}\}} \Pr(\gamma_j^{(i)})/|\gamma_j^{(i)}| = \sum_{\gamma_j \neq \emptyset} \langle h \rangle_{\gamma_j} \Pr(\gamma_j) \tag{36}$$

which means that for all j, (eq. (32)) is identically zero, guaranteeing that the system has non-trivial solutions.

Since the matrix for a connected graph has rank $N-1$, its kernel is one-dimensional, and thus, to choose a unique solution that belongs to the kernel of $\mathbf{\Lambda}$ we need a single additional condition. In our case, this condition corresponds to

$$\sum_{i=1}^{N} (p_i + q_i) = 1, \tag{37}$$

which guarantees that each individual is either employed or unemployed and associated to only one firm each period. *Q.E.D.*

Proof of proposition 2

Let us consider eqs. (26) and (27) and note that the probability $\Pr(\gamma_i)$ of obtaining a specific configuration γ_i of open and closed neighbors follows the binomial $v^{|\gamma_i|}(1-v)^{k_i-|\gamma_i|}$. Then, we obtain that

$$\sum_{\{\gamma_j^{(i)}\}} \Pr(\gamma_j^{(i)})/|\gamma_j^{(i)}| \to \sum_{|\gamma_j^{(i)}|=1}^{k_j} \binom{k_j-1}{|\gamma_j^{(i)}|-1} \frac{v^{|\gamma_j^{(i)}|}(1-v)^{k_j-|\gamma_j^{(i)}|}}{|\gamma_j^{(i)}|} = \frac{1-(1-v)^{k_j}}{k_j}. \tag{38}$$

For the sum $\sum_{\gamma_j\neq\emptyset} \bar{h}_{\Gamma_i}\Pr(\gamma_j)$, we note that each hiring policy h_i for $i \in \Gamma_j$ appears $\binom{k_j-1}{|\gamma_j|-1}$ times among all the terms where there are $|\gamma_j|$ open neighbors to j. We can then write

$$\sum_{\gamma_j\neq\emptyset} \bar{h}_{\Gamma_i}\Pr(\gamma_j) \to \sum_{|\gamma_j|=1}^{k_j} \binom{k_j-1}{|\gamma_j|-1} \frac{\sum_{i\in\Gamma_j} h_i}{|\gamma_j|} v^{|\gamma_j|}(1-v)^{k_j-|\gamma_j|} = \bar{h}_{\Gamma_i}(1-(1-v)^{k_j}), \tag{39}$$

where $\bar{h}_{\Gamma_i} := \sum_{i\in\Gamma_j} h_i/k_j$, i.e., the average hiring policy of the full neighbor set of j. Therefore, eqs. (26) and (27) simplify into

$$0 = -\lambda p_i + s h_i \sum_{i\in\Gamma_i} q_j \frac{1-(1-v)^{k_j}}{k_j} \tag{40}$$

$$0 = \lambda p_i - q_i s \bar{h}_{\Gamma_i}[1-(1-v)^{k_i}]. \tag{41}$$

It is easy to see by inspection that the solution to the system is

$$p_i = \frac{\chi h_i \bar{h}_{\Gamma_i} k_i}{\lambda} \tag{42}$$

$$q_i = \frac{\chi h_i k_i}{s[1-(1-v)^{k_i}]} \tag{43}$$

$$\chi = \frac{1}{\sum_{i\in G} h_i \bar{h}_{\Gamma_i} k_i \left[\frac{1}{\lambda} + \frac{1}{s\bar{h}_{\Gamma_i}[1-(1-v)^{k_i}]}\right]}. \tag{44}$$

Given that the workers' actions are independent from each other, the evolution of the firm size follows the binomial

$$\Pr(L_i) = \binom{H}{L_i} p_i^{L_i}(1-p_i)^{H_i - L_i}, \tag{45}$$

so the steady-state average firm size L_i (abusing notation) is

$$L_i = Hp_i = \frac{\varphi h_i \bar{h}_{\Gamma_i} k_i}{\lambda}, \tag{46}$$

where $\varphi = H\chi$. *Q.E.D.*

Proof of proposition 3

Consider the probability $a_i(t)$ that a worker submits a job application to firm i in period t. This depends on: the worker becoming an active searcher; on the probability $q_j(t-1)$ of being unemployed in a neighbor $j \in \Gamma_i$ during the previous period; on the probability $\Pr(\gamma_j^{(i)})$ of j having a configuration $\gamma_j^{(i)}$ of open of closed neighbors in which i is open; and on the probability of choosing i over all other alternative neighbors of j. Accounting for all possible events and configurations of neighbors, this probability is written as

$$a_i(t) = s \sum_{j \in \Gamma_i} q_j(t-1) \sum_{\{\gamma_j^{(i)}\}} \Pr\left(\gamma_j^{(i)}\right) \frac{1}{\left|\gamma_j^{(i)}\right|}. \tag{47}$$

In the steady-state $a_i(t) = a_i(t-1) = a_i$ and $q_i(t) = q_i(t-1) = q_i$, and by replacing eqs. (38) and (43) we obtain

$$a_i = \chi \bar{h}_{\Gamma_i} k_i. \tag{48}$$

Since the workers' behaviors are independent from each other, the number of job applications received by firm i in any period follows the binomial

$$\Pr(A_i) = \binom{H}{A_i} a_i^{A_i} (1-a_i)^{H_i - A_i}, \tag{49}$$

so the steady-state average number of applications A_i (abusing notation) is

$$A_i = H a_i = \varphi \bar{h}_{\Gamma_i} k_i, \tag{50}$$

where $\varphi = H\chi$. A_i fulfills the steady-state balance condition $\lambda L_i = h_i A_i$. *Q.E.D.*

Proof of proposition 4

Let us consider the steady-state solution for the probability q_i of being unemployed and associated to firm i, as written in eq. (43). Given that the workers' actions are independent from each other, the evolution of the firm-specific unemployment follows the binomial

$$\Pr(U_i) = \binom{H}{U_i} q_i^{U_i}(1-q_i)^{H_i - U_i}, \tag{51}$$

so the steady-state average firm-specific unemployment U_i (abusing notation) is

$$U_i = Hq_i = \frac{\varphi h_i k_i}{s[1-(1-v)^{k_i}]}, \tag{52}$$

where $\varphi = H\chi$. *Q.E.D.*

Proof of proposition 7

Consider the probability $o_i(t)$ that a worker associated to firm i finds a job at a different firm in period t. This event depends on: the probability $q_i(t-1)$ that the worker was unemployed and associated to firm $i \in \Gamma_j$ during the previous period, on the worker's search intensity, on the probability $\Pr(\gamma_i)$ of i having a configuration γ_i of open of closed neighbors; and on the probability of choosing one particular firm over all other alternatives available in Γ_i. Altogether, these factors constitute probability

$$o_i(t) = q_i(t-1)s \sum_{\gamma_i \neq \emptyset} \Pr(\gamma_i) \frac{1}{|\gamma_i|}. \tag{53}$$

In the steady-state $o_i(t) = o_i(t-1) = o_i$ and $q_i(t) = q_i(t-1) = q_i$, and by replacing eqs. (39) and (43) we obtain

$$o_i = \chi h_i \bar{h}_{\Gamma_i} k_i. \tag{54}$$

Since the workers' behaviors are independent from each other, the number of i's outflows in any period follows the binomial

$$\Pr(O_i) = \binom{H}{O_i} o_i^{O_i} (1 - o_i)^{H_i - O_i}, \tag{55}$$

so the steady-state average outflows O_i (abusing notation) is

$$O_i = H o_i = \varphi h_i \bar{h}_{\Gamma_i} k_i, \tag{56}$$

where $\varphi = H\chi$. O_i fulfills the steady-state balance condition $O_i = \lambda L_i$. *Q.E.D.*

Test for significant edges

In order to introduce the test, consider 4 workers who become separated from firm i and eventually find jobs at different firms. Workers 1 and 2 find jobs in firm j, while workers 3 and 4 become employed by l and m respectively. These flows can be represented as the elements $a_{ij} = 2$, $a_{il} = 1$, and $a_{im} = 1$ of the adjacency matrix of a directed multigraph G^m, defined as an ordered pair with a set of nodes and a multiset of edges. The element $a_{ij} = 2$ means that an edge from i to j appears two times in G^m. We generalize this example to the case where firm i has $k_i^{in} = V_i$ incoming matched workers and $k_i^{out} = M_i$ outgoing matched job seekers. Then, we can think of the AMF as a generating mechanism of G^m, for any degree sequences $\{k_1^{in}, k_2^{in}, ..., k_N^{in}\}$ and $\{k_1^{out}, k_2^{out}, ..., k_N^{out}\}$ such that $\sum_i^N k_i^{in} = \sum_i^N k_i^{out} = M$. By randomly matching the job seekers and vacancies of each firm, the AMF generates a random graph G^m. This process takes into account the fact that some firms receive more workers than others. In fact, this network formation process corresponds to a well established framework called the *directed configuration model* (Holland and Leinhardt, 1981; Molloy and Reed, 1995; Bollobás, 1998).

The directed configuration model hypothesizes that edge formation is the result of random matching between nodes. It begins with nodes that possess severed incoming and outgoing edges, also known as *stubs*. Outgoing stubs can only be paired with incoming stubs. The procedure consist of randomly taking one outgoing stub and pairing it to an incoming one. This sequential process generates a topology for G^m. Under reasonable assumptions, it is possible to approximate the probability distribution of the presence an edge as a binomial distribution.

Let $\hat{a}_{ij}$ denote the number of edges from i to j generated by the directed configuration model. Suppose that G^m has a degree distribution with finite mean. A result by Wilson et al. (2013) shows that, for $N \to \infty$, $\hat{a}_{ij}$ is a random variable such that

$$\hat{a}_{ij} \sim \text{Binomial}\left(k_i^{out}, k_j^{in}/n\right), \tag{57}$$

where n is the total number of edges in G^m. The first parameter corresponds to the number of outgoing stubs of i (the number of trials), and the second corresponds to the probability that an outgoing stub will be matched with an incoming stub from j.

Equation (57) allows to statistically test the significance of individual edges in G^m. The test consists of comparing a_{ij} against the expectation $\hat{a}_{ij}$ and it takes the from of the p-value of a Binomial test

$$p(i \to j) = \Pr(\hat{a}_{ij} \geq a_{ij}). \tag{58}$$

We compute eq. (58) for each edge of every network. If the p-value is less than 0.05, we consider that edge significant and take it as an indicator that there is a significant edge underlying those labor flows. Recall that an edge has a categorical nature in the sense that it indicates whether we would expect (under the null) labor flows between the firms that it connects or not.

Estimation of Separation Rates for Finland

Year	β_λ	N	R^2
1988	0.188*** (2.900e-02)	34,279	0.407
1989	0.102*** (2.224e-02)	32,771	0.301
1990	0.105*** (2.442e-02)	25,260	0.246
1991	0.049*** (9.445e-03)	19,143	0.252
1992	0.028*** (4.575e-03)	16,810	0.141
1993	0.188 (1.270e-01)	17,667	0.174
1994	0.150* (7.513e-02)	20,756	0.279
1995	0.068*** (1.759e-02)	21,012	0.151
1996	0.059*** (6.019e-03)	24,076	0.382
1997	0.065*** (8.652e-03)	51,493	0.509
1998	0.088*** (1.590e-02)	31,322	0.281
1999	0.208* (8.141e-02)	33,648	0.409
2000	0.154** (4.993e-02)	34,008	0.342
2001	0.088*** (1.667e-02)	33,331	0.323
2002	0.066*** (1.103e-02)	33,031	0.376
2003	0.070*** (1.041e-02)	33,842	0.367
2004	0.592*** (1.528e-01)	35,924	0.609
2005	0.074*** (1.029e-02)	41,978	0.415
2006	0.127*** (3.286e-02)	44,403	0.524
2007	0.086*** (1.171e-02)	42,767	0.470

Table 3: Estimation of annual separation rates for Finland via eq. (23). Robust standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.

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