Willingness to Pay for Firm Reputation: Paying for Risk Rating in the Annuity Market

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Abstract

In this paper we test the existence of a reputation premium in the context of the annuity market in Chile. This market provides an exceptionally good setting to measure consumers' willingness to pay: retirees choose between a set of offers that vary only in the quote and the risk rating –a measure of the firm's solvency– within each class of product. We find that willingness to pay for the reputation linked to the firm's risk rating is statistically and economically significant. We also find a strong relationship between willingness to pay and intermediation choice, and we explore four potential sources of correlation between them.

1 Introduction

There is a large theoretical literature that studies reputation as a mechanism to solve the adverse selection problem under incomplete information, and as a disciplinary device that may restore incentives for high effort under moral hazard (see Bar-Isaac and Tadelis (2008) and Mailath and Samuelson (2006) for excellent reviews of this literature). In this setting, reputation is the consumers' belief about the provider's type and/or his equilibrium behavior: a "good reputation" is related to a high probability that the firm provides a "good service". On the demand side, the cited mechanism requires that consumers are willing to pay a higher price to better reputed firms, which also requires that their willingness to pay for the product increases as its quality improves. Unfortunately there is scarce empirical evidence of a positive relationship between reputations and prices (see for instance Lei (2011), Jolivet et al. (2013), and Saeedi (2014)). There are three main problems for testing this relationship: reputation is well defined in theory but hard to measure in practice, products are complex and differ in many attributes unobserved by the econometrician, and only transaction prices are usually available instead of the set of prices faced by the consumer.

We provide evidence for a positive relationship between reputation and prices on the demand side. For this, we estimate the consumers' willingness to pay for firm's reputation using data from the annuity market in Chile. This is a very good environment to study this problem, because we can solve the three problems described above.

From the consumers' point of view –the retiree–, immediate annuities are homogeneous products that differ only in a few observable attributes. A mandatory system for retirement products centralizes all communication between the two sides of the market and makes annuity offers particularly simple to compare. Retirees request quotes for annuities with different attributes and receive a set of offers that only differ in the payouts, the attributes, and the risk rating of each insurer. Participating insurers are restricted to give quotes only to the annuities requested, and consequently differentiation in attributes is not a decision of the firms; the only possible quality differentiation is related to the firms' solvency. Retirees are explicitly told that the risk rating measures insurers' default risk, and so we consider rating as a public signal of solvency; in this sense, estimating willingness to pay for a better risk rating is regarded the same as estimating willingness to pay for a higher reputation of solvency.

We observe all quotes received by all retirees from all participating insurers from 2004 to 2013. We define two quotes as belonging to the same class of product if they share the same attributes. A significant fraction of retirees chose an insurer that did not offer the highest payout within the same class. This behavior is more frequent for retirees that chose insurers with better risk rating, both in the cross section and over time. Indeed, 70% of retirees chose an offer from an insurer rated AA or better; of these, only 45% accepted the offer with the highest quote in its class. In contrast, of the other 30% of retirees who chose an offer from an insurer rated worse than AA, 76% accepted the offer with the highest payout in its class. This evidence suggests that retirees may be willing to pay for providers with less default risk.

As can be expected, this pattern summarizes a great amount of individual heterogeneity. In particular, retirees may have different *a priori* preferences for the different insurers; for example, firms with better rating may advertise more and increase the preference for them. Even though economic models have standard methods for dealing with these issues, another interesting feature of the market analyzed is that we can use the choice of an intermediary as additional information to improve on those methods.

Retirees can enter the mandatory system with the intermediation of a sales agent –hired by an insurance company– or an independent adviser –hired by the retiree herself. The use of a sales agent may indicate, at least partially, that the retiree has a larger preference for that insurer; we can include this decision as a first stage in the model, to characterize in more detail the dispersion of preferences.

Indeed, the fraction of retirees that chose an offer from an insurer rated AA or better rises to 83% among those using a sales agent and declines to 53% among those using an independent adviser. Moreover, the fraction of retirees that accepted the offer with the highest payout in its class declines to only 28% among those using a sales agent and is 76% among those using an independent adviser. In other words, retirees intermediated by sales agents tend to choose better-rated companies that offer lower payouts, while those intermediated by independent advisers tend to choose worse-rated companies that offer higher payouts. This correlation may indicate that financial advice has heterogeneous impact on the clients' risk-taking behavior (see Inderst and Ottaviani (2012) for a theoretical analysis and a good review of related empirical work), but it may also indicate that those retirees with a larger preference for better-rated companies tend to use sales agents from those insurers as intermediaries and choose their offers even if they offer lower payouts.

Taking advantage of the detailed individual-level data available, we estimate a two-stage discrete choice model where individuals sequentially choose intermediaries and annuities. Using a mixed logit specification to allow for flexible patterns of substitution, we estimate the willingness to pay for reputation of solvency. We find that willingness to pay for reputation in the anniuty market in Chile is statistically and economically significant: retirees are willing to reduce their lifetime payments up to 2% on average in order to contract an annuity to a better rated company, depending on the retiree's characteristics and intermediation. Willingness to pay is significantly lower among retirees entering the system with an independent adviser than with a sales agent.

The model admits four potential sources of correlation between willingness to pay and intermediaries. The first source is selection on observables: some observable characteristics of firms may affect both the choice of intermediary and of annuity quotes. Indeed, the results show that retirees are more likely to choose sales agents from larger companies and from better-rated companies. In other words, the firm's rating affects both the value of the intermediary and of the annuity quote for that firm.

The second source is selection on unobservables: unmeasured brand characteristics have an aggregate effect that is fixed over time (so a standard firm-level fixed effect is included to capture the potential correlation between these brand characteristics and the payouts) and an idiosyncratic effect that may be correlated with the value of the intermediary, so retirees with a larger preference for an insurer may choose both a sales agent and a quote from that company. Our results suggest that the correlation between unobservable tastes for the firms in the first stage and unobservable tastes for offers placed by those firms in the second stage is not significant, and zero for many firms.

The third source of correlation is general biased advice: independent advisers and sales agents may focus their advice on different attributes of each annuity, affecting differently the valuation of these attributes. The results suggest that this source of correlation is important: the mean valuation of the payouts is larger for retirees whose intermediaries are independent advisers, but also the willingness to pay for better rating is lower, than for retirees whose intermediaries are sales agents.

Finally, the fourth source of correlation is firm-specific biased advice: sales agents only receive commission if retirees choose a quote from their firm, so they may emphasize the value of their firm's quotes over other firms, and sales agents from better-rated companies may over-emphasize the importance of firms' solvency in order to convince their clients to accept their offer. Our results show that this effect is also important: among retirees entering the system with a sales agent, the mean valuation of the payment is lower for the offers placed by their agent's firm; this translates into an even higher willingness to pay for a better rating for this specific firm.

The welfare implications of the four sources of correlation may be very different. The evaluation of the welfare impact of biased advise is non-trivial, however, as there is a trade-off between payments and default risk: retirees being advised to select quotes with higher payments may be subject to a higher default risk. Besides performing administrative tasks, intermediaries provide advice that influences their clients' belief updating about the firms' types; conflict of interest may certainly introduce bias in the advice of sales agents, but also provides an effective channel for better-rated, solvent firms, to attract customers.

The magnitude of the estimated willingness to pay for risk rating implies that better-rated firms can earn significant higher profits than lower-rated firms, supporting the hypothesis of the theoretical literature on the existence of a reputation premium that may provide incentives in a market for an experience good. Our preliminary analysis suggests that better-rated firms charge higher prices: the fraction of quotes that are the highest in its class is lower for firms with better risk ratings than for firms with worse ratings. However, the study of the supply side of the market is left for future work.

Beyond testing the existence of a reputation premium, our results may also be important for the particular analysis of the annuity market. Consider for instance the welfare effect of insurance portfolio transfers: pensioners that buy annuities from better-rated firms are willing to accept lower payments in exchange of a lower default probability; this benefit is lost, however, when their policies are unilaterally transferred by the insurance company to other worse-rated firms. In light of our results, this loss may be equivalent to a loss of 1% to 2% of pensioners' lifetime income, which is not trivial.

The paper is organized as follows: Sections 2 and 3 introduce the main characteristics of the annuity market in Chile and the data available respectively. Section 4 develops a two-stage model where individuals sequentially choose intermediaries and annuities. Section 5 describes the estimations, while Section 5.1 shows the resulting willingness to pay for risk rating. Section 6 concludes.

Related literature

By now there is a long theoretical literature that analyzes the role of the reputation premium in providing incentives for good performance and fostering the operation of the market under information asymmetries. Klein and Leffler (1981) studies how repeat-purchase may suffice to keep incentives aligned, as long as the market punishment for bad behavior –i.e., losing customers or charging lower prices– is high enough to compensate for the differential cost of high quality. This analysis was first introduced in a symmetric information environment, and incomplete information was introduced later in the literature as a key ingredient (as in Mailath and Samuelson, 2001). Although most of the literature focuses on the monopoly case, competition among firms under reputation concerns has also been theoretically studied, as in Tadelis (1999), Tadelis (2002) and Rob and Fishman (2005).

There is also a growing empirical literature that focuses on reputation markets. The evidence reported by McDevitt (2011) and Cabral and Hortacsu (2010) supports the hypothesis that reputation is a state variable that affects firm's behavior at the individual level and determines industry dynamics under reputation concerns, as exit is more likely subsequent to poor consumer reviews or complaints in some industries. The eBay feedback system has been studied in some detail (see Bajari and Hortaçsu (2004) for a good review of the literature), as it provides a good setting to analyze the evolution of consumers' and sellers' decisions across time. Lei (2011), Jolivet et al. (2013) and Saeedi (2014) are some of the recent papers that try to measure the reputation premium.

2 The annuity market in Chile

The Chilean pension system is composed of two phases (see Larrain and Morales (2010)). On the accumulation phase, dependent workers are mandated to save monthly; savings are collected and managed by private pension fund administrator companies (*"Administradoras de Fondos de Pensión"*, AFP). On the payout phase, retirees receive their savings back gradually over time by choosing among four different retirement options: annuities, programmed withdrawals, or two different combinations of them. A detailed description of the retirement options is provided in Appendix A.1. Even though most countries show low annuitization rates, the market in Chile is fairly large: in our data 38.6% of retirees chose immediate annuities. The interface between the accumulation and payout phases is the mandatory SCOMP system (*"Sistema de Consultas y Ofertas de Montos de Pensión"*). This section explains the operation of this system and highlights the features that are important when looking at the data.

2.1 Information available through SCOMP

SCOMP is a mandatory electronic system for retirement products where all the information of the pension process is centralized and shared to the relevant parties: retirees request quotes, information about the retiree and her beneficiaries is transferred to pension providers, providers give quotes, and retirees choose their preferred quote. The system maximizes the information available to retirees while restricting the information available to insurance companies. A summary of the information available to each participant is the following:

i. Information about the retiree: The information transferred to providers (insurers and AFPs) includes only basic demographic characteristics of the retiree and her beneficiaries (age, gender, dissability status), her retirement status (early or normal retirement), the size and composition of her pension savings fund, and the attributes of the annuities she requested. Other information is unobserved by providers: in particular, the identity of the retiree and her beneficiaries, socioeconomic information as area of residence, and also the type and identity of the intermediary she is using.

ii. Information about the payout quotes: Annuities are single premium products, irreversible, with fixed payouts in real terms (payouts are quoted in UF, an indexed unit of account that is the standard currency for long term contracts in Chile), and are deposited monthly in a bank account. Payout quotes are privately offered to each retiree and are personalized –they depend on the information transferred to providers described in *i.* They are not disclosed at any moment to providers or the market, neither during or after the retirement process. It is not straightforward to infer individual prices from the quotes: the implicit price that the insurer charges depends on his prediction of the retiree's life expectancy, the performance of the economy, and his return on investments, and on the provisions mandated by the regulatory agency.

iii. Information about the providers: Each retiree is given a certificate that includes the quotes, name and risk rating of each participating company. The certificate explicitly indicates that the risk rating is a measure of the default probability of the insurer. If the insurance company goes bankrupt the state guarantees only part of the payment up to a limit; however, neither the level of the guaranty, its operation, or limit is informed in the certificate.[1]

iv. Information about the intermediary's fee: Offers are net of intermediary's fee and the retiree is told that payouts may increase if that fee is reduced. A maximum fee is set for both sales agents and financial advisers. We observe all the offers received by each retiree, both initial offers and subsequent counteroffers; counteroffers must include higher payouts, either by reductions of the intermediary' fee or by improvements of the insurer's offer.

A more detailed description of the retirement process is provided in Appendix A.2.

2.2 Attributes of annuities

When buying an annuity, the retiree transfers the property of the savings fund to the insurance company in exchange for a lifetime monthly payment fixed in real terms; standard annuities start paying immediately. Annuities may differ in only two attributes: guaranteed period and lump sum withdrawal.

The guaranteed period extends the number of months in which the company pays. The law requires that if the retiree dies the legal beneficiaries will continue receiving a percentage of the monthly payment. Under an annuity with guaranteed period, if the retiree dies before the end of the selected period, the whole payment will be paid to the beneficiaries until it ends, after which

[1]The state's guaranty corresponds to the minimum pension in addition to 75% of the difference between the monthly payment and this minimum pension –if positive– up to 45UF. The minimum pension is determined by law and depends on the age of the retiree. Additionally, the regulatory agency may authorize the transfer of active contracts and available funds to a different insurance company that will continue making payments for a given period of time; after that period, surviving policy holders will receive the state's guaranty. The most recent experience of bankruptcy in Chile was in January 2004. The regulatory agency paid pensions until February 2008, when policies were transferred through a bidding process to another insurance company that committed to continue paying 100% of the annuities until May 2018. The financial strength of the company not only affects the probability that the company will go bankrupt, but also the output of this bidding process, as it influences the decision made by the bidding companies.

payments will continue as legally required. This feature acts as a form of inheritance, particularly for non-legal beneficiaries.[2]

On the other hand, if the size of the pension savings fund is sufficiently high, the retiree may also request a lump sum withdrawal (*"Excedente de Libre Disposición"*) that is removed from the savings fund. The maximum withdrawal allowed is such that enough funds are left to buy an annuity with a monthly payment of 70% of the average wage from the previous 10 years, and at least 150% of the minimal pension –same requisites than for early retirement. If elegible, the retiree can request quotes for annuities with no withdrawal, the maximum, or some amount in between. If the retiree requests zero or other fixed amount of withdrawal the companies may compete offering different monthly payments; if the retiree requests the maximum withdrawal the companies must fix the monthly payment to the amount specified above and may compete offering different withdrawals.

A request for quotes may include combinations of up to three lengths of guaranteed periods and three lump sum specifications (if eligible). Let us define the class of an annuity offer as the subset of quotes received by a retiree that share the same attributes. Two offers are in the same class if:

i. [Zero or fixed withdrawal] the length of the guaranteed period and the size of the withdrawal requested are the same, or

ii. [Maximum withdrawal] the length of the guaranteed period is the same and the withdrawal requested is the maximum allowed.

In case (*i*) all offers in the same class share the same withdrawal but differ in terms of the monthly payments. In this case *the highest payout in its class* is the offer with the highest monthly payment in this subset. In contrast, in case (*ii*) all offers in the same class share the same monthly payment (the minimum annuity with withdrawal) but differ in terms of the withdrawal. In this case the highest payout in its class is the offer with the highest withdrawal in this subset.

The certificate sent to the retiree provides a different table of quotes for each class requested. Each quote in each table shows the name of the company, the payout (the monthly payment and withdrawal), the risk rating, and the discount rate if the annuity includes a guaranteed period. The offers are ordered by payout, from larger to smaller, and from best to worst risk rating in case of a tie. This system makes annuity offers particularly simple to compare for the retiree.

2.3 Intermediaries

To start the retirement process, the retiree must enter a request for quotes into the system through an intermediary. The system considers four different intermediation options. The first one is to go directly to an AFP, either the one that manages the retiree's savings account or a different one. This intermediary is the most independent but also provides the least financial advice: the person at the office will enter all the required information into the system but will usually provide very little and standard information about the system and will not advice the retiree on what product is best for her.

The following two intermediation options involve a life insurer: the second one is to go directly to a life insurance company, and the third one is a sales agent. Both options will naturally be biased towards the company and certain pension products. And finally, the last option is to hire an independent adviser authorized by the regulator.

[2] For annuities with guaranteed period, the company must also offer a discount rate. If the retiree dies before the end of the selected period, the beneficiaries may choose to withdraw the remaining funds, paying this discount rate –most frequently 7.5%.

The last two intermediaries involve paying an intermediation fee, which is subtracted from the savings fund. Intermediation fees are regulated: the maximum is 2.5% of the fund until 2008, and 2% from that date on. Because companies ignore the type of the intermediary, all quotes are entered in the system net of the maximum intermediation fee. The adviser is paid the same fee if the retiree chooses any annuity product, but the sales agent is only paid if the retiree chooses an annuity from his company; both can choose to reduce their fees.[3]

2.4 Risk rating

In the 2004-2013 period 21 insurance companies gave quotes through the system, after taking into account some mergers and name changes. To participate, all must obtain ratings from at least two independent agencies and have ratings of at least BBB-; retirees receive the worst of these two ratings. Table 1 summarizes the history of firms' risk ratings, that fluctuate from AA+ to BBB-.

Despite the size and richness of the data set, it has two important limitations: only one company obtains ratings BBB or BBB- in the period, creating a gap with the other companies. Hence, the effect of obtaining a rating worse than A- cannot be separately identified from that firm's fixed effect. Additionally, variation within companies over time is infrequent, and in all but one case it is between two adjacent ratings.

3 The Data

The data set contains all the requested and received quotes for all retirement processes in the period between 2004 and 2013. The quotes consist of the name and risk rating of each participating company, the payouts (i.e., the monthly payment and withdrawal, if any), and the attributes that define the class as described in section 2.2.

Tables 2 and 3 show the distribution of pension products and intermediary types respectively. Our sample consists of 85,105 retirees with almost four million quotes. These correspond to the retirees that chose an immediate annuity, a 38.58% of all the persons that retire through the system –both standard and early retirement. Regarding intermediation, 20.2% of the sample go directly to an AFP. The rest of the sample is split between those that use an independent advisor (39.6%) and an insurance company (40.2%). Of those that use an insurance company, most of them use a sales agent, and a few go directly to the company. This subgroup (3.3% of the sample) is supposed to not pay any fees, but in the data almost all of them end up paying some fee at the end of the process; thus we assume that the insurance companies redirect them to one of their agents, and we will treat them as the same category in estimation.

Figure 1 shows the relationship between the percentage of accepted quotes with the highest payout in its class and the companies' rating and size –measured as the number of accepted quotes. Each bubble represents an insurer at a given rating grade; those with two different ratings in the period are represented with two connected bubbles. The bubble's size represents the number of accepted quotes from this company at the corresponding rating grade. The horizontal axis shows the risk rating, while the vertical axis shows the ratio of accepted quotes with the highest payout in its class to total accepted quotes from each company at a given rating grade. For those companies rates AA+ a significant fraction of the accepted quotes are not the highest in its class, but for the worst rated company almost all the accepted quotes are the highest.

[3]These fees are for annuity products; in case of choosing a programmed withdrawal the adviser is only paid at most a fee of 1.2% and the sales agent does not receive a fee. This raises another issue with the market for programmed withdrawal that is not explored in this paper and is left for future work.

Table 1: Number of offers and rating grade by insurance company, SCOMP 2004-2013

Brand	Number of offers	Risk rating of the Insurance company (years bidding quotes within each rating category)						
id	period 2004-2013	AA+	AA	AA-	A+	A	BBB	BBB-
1	342,454	2005-2013		2004-2005				
2	29,933		2004-2005					
3	46,109		2010-2013					
4	215,645	2008-2009	2004-2008					
			2009-2013					
5	289,745		2008-2013	2004-2008	2004			
6	36,508	2004-2006						
7	15,392			2007-2008	2005-2007			
8	50,345				2007-2008	2007		
9	16,935					2005-2008		
10	304,087			2004-2013				
11	339,612			2004-2013				
12	324,586		2004-2013					
13	288,122		2007-2011	2004-2007				
			2012-2013	2011-2012				
14	104,653			2009-2013				
15	237,939		2004-2013					
16	191,142			2007-2013	2005-2007			
17	36,122	2008-2011						
18	59,466			2011-2013				
19	327,254	2012-2013	2004-2012					
20	345,353	2005-2013	2004-2005					
21	216,781						2013	2008-2013

Table 2: Distribution of pension products, SCOMP 2004-2013

	Number of requests		Number of offers
Total in the System	444,795	100.00%	18,796,012
Standard retirement	194,164	43.65%	8,737,513
Early retirement	125,605	28.24%	4,868,675
Disability pensions	55,158	12.40%	2,179,165
Pensions for beneficiaries	69,868	15.71%	3,010,659
Total retirement	319,769	100.00%	13,606,188
Did not finish process	98,897	30.93%	4,830,782
Programmed withdrawal	84,776	26.51%	2,246,273
Non-immediate annuity	50,885	15.91%	2,319,640
Immediate annuity	85,211	26.65%	4,209,493
Our sample	85,105		3,972,104

Table 3: Distribution of intermediary types, SCOMP 2004-2013

Intermediary type	Frequency	Percentage
AFP	17,190	20.20%
Insurance company	2,816	3.31%
Sales agent	31,395	36.89%
Independent adviser	33,704	39.60%

The figure shows a negative correlation between these two variables. As a firm's rating improves, the fraction of its accepted quotes that have the highest payout in its class decreases in most cases. Additionally, accepted quotes are less frequently the highest payout in its class among better-rated companies. As the retiree is fully aware that she is not choosing the quote with the highest payout in its class, we regard this negative relationship as indicative that retirees are willing to sacrifice payouts in order to obtain a pension from a better-rated company –and companies are aware of that, as the probability that the offers made are the highest in its class is lower for companies with better risk ratings.

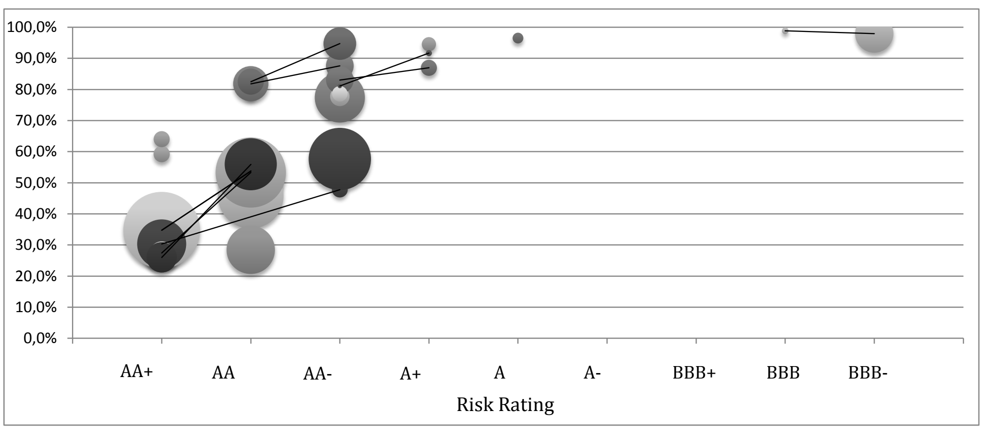
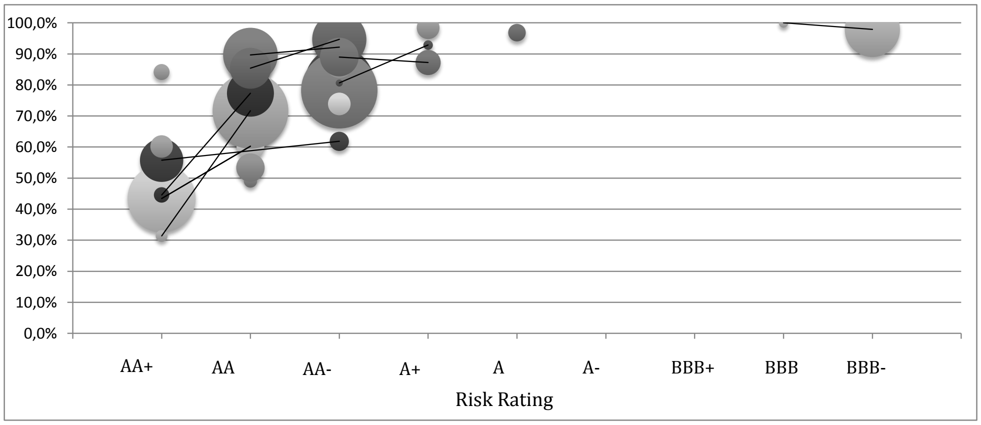


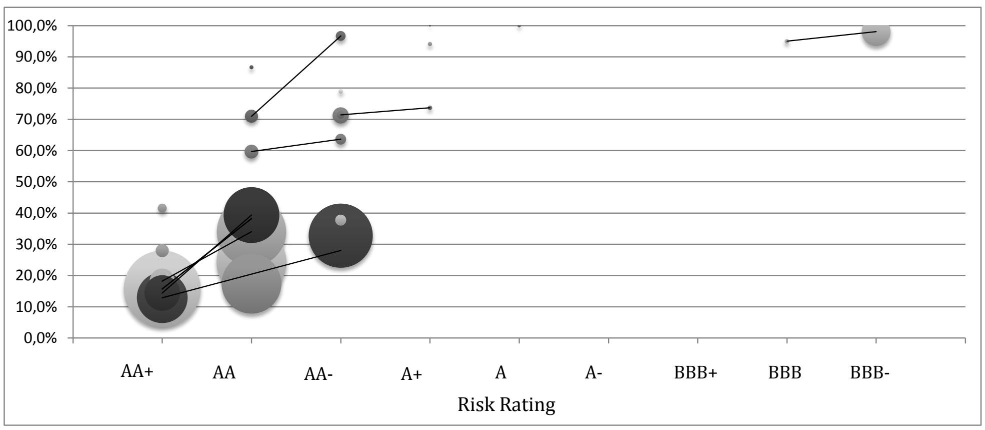
Figure 1: Ratio of accepted quotes with the highest payout to total accepted quotes.

Conditioning on the intermediary strongly affects the results. Figure 2(a) shows the percentage of accepted quotes with the highest payout in its class and the companies' rating and size (exactly as in Figure 1), but only for those retirees who use an independent adviser. The advantage of better-rated firms tends to disappear and there is a negative but weaker relation between the rating of the company and the ratio of highest to accepted quotes, both within and across firms. In contrast, Figure 2(b) shows the same ratio but only for those retirees who use a sales agent. The fraction of accepted quotes that have the highest payout in its class is strongly reduced for all ratings. The advantage of better rated firms increases and there is a stronger relation between the rating of the company and the ratio of highest to accepted quotes, both within and across firms.

This evidence suggest that risk rating and financial intermediation play an important role on the market for annuities in Chile. Because the default probability is the main source of uncertainty for retirees with respect to the quality of the product they are choosing, we can think of the risk rating as a measure of reputation of the firm. In most industries it is difficult to test the empirical relevance of reputation and to measure the reputation premium, as reputation is an unobservable attribute for the econometrician. In our case, however, risk rating is explicitly included as a measure



(a) Retirees using independent advisers.



(b) Retirees using sales agents.

Figure 2: Ratio of highest to accepted quotes by intermediary type.

of the default probability of the insurer, hence making this variable observable both for the retirees and for us.

4 The Model

The model considers a retiree who must choose an annuity once and for all. Retirees are indexed by $i \in \{1, ..., I\}$. The decision process is modelled in two stages: the individual chooses an intermediary of type $k \in \{1, .., K\}$ to enter the system in a first stage, and then chooses a quote $j \in \{1, ..., J\}$ from a firm $f \in \{1, ..., F\}$ among all the quotes received in the second stage. The number of intermediary types is $K = F + 2$. When the individual chooses a sales agent from firm f both indexes take on the same value $k = f \leq F$; independent advisers are indexed by $k = F + 1$, and entering directly through an AFP is denoted by $k = F + 2$.

To estimate heterogeneous willingness to pay for reputation of solvency, we use a mixed logit model. The model admits four potential sources of correlation between this willingness to pay and intermediaries. We can use the choice of intermediary as additional information to capture part of the heterogeneity in preferences for the different insurers or attributes.

The first source is selection on observables: some observable characteristics of firms may affect both the choice of intermediary in the fist stage and the choice of annuity quotes in the second stage. The second source is selection on unobservables: idiosyncratic unmeasured brand characteristics may be correlated with the value of the intermediary, so retirees with a larger preference for an insurer may choose both a sales agent and a quote from that company. The third source of correlation is general biased advice: independent advisers and sales agents may focus their advice on different attributes of each annuity, affecting differently the valuation of these attributes. Finally the fourth source is firm-specific biased advice: sales agents may emphasize the value of their firm's quotes over other firms, and sales agents from better-rated companies may over-emphasize the importance of firms' solvency in order to convince their clients to accept their offer.

To solve for the potential correlation between the aggregate effect of unmeasured brand characteristics and the quotes, we use a firm-level fixed effect, a feature that has become standard in the literature. Because we have individual and time variation in the payouts quoted, we do not require the usual second stage in the estimation to retrieve aggregate parameters (see, for instance, Berry et al. (1995) and Nevo (2001)). Individual prices charged by each firm are not easily obtained, for the reasons outlined above. Because of the centralization of the system, we can assume that firms are only aware of this aggregate effect of their brands, but are not able to predict the idiosyncratic effect when facing a new request for quotes, hence the firm-level fixed effect should be sufficient to solve the endogeneity of quotes.

4.1 Utilities of the first and second stages

In the first stage, the value for individual i of choosing intermediary k is given by:

$$\tilde{U}_{ik} = W_i\alpha_k + W_{ik}\alpha + \tilde{\varepsilon}^1_{ik} + \tilde{\varepsilon}^2_{ik}, \tag{1}$$

where W_i is a vector of individual characteristics, W_{ik} is a vector of individual-specific attributes of intermediary k, and the error term contains two parts: a normally distributed error $\tilde{\varepsilon}^1_{ik}$ and an *iid* Gumbel error $\tilde{\varepsilon}^2_{ik}$. The choice of individual i in the first stage defines a set of indicator variables as follows:

$$\tilde{Y}_{ik} = \begin{cases} 1 & \text{if } i \text{ chooses intermediary } k\text{, and} \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Conditional on the first stage choice, the value of choosing quote j from firm f in the second stage is given by:

$$U_{ijf} = X_{ijf}\beta_i + \xi_f + \varepsilon^1_{if} + \varepsilon^2_{ijf}, \tag{3}$$

where X_{ijf} is a vector of (individual-specific) attributes of the quote and firm, ξ_f is a firm fixed-effect, and the error term again contains two parts: a normally distributed error ε^1_{if} and an *iid* Gumbel error ε^2_{ijf}. The fixed effect ξ_f only varies by firm but not over time, and is included –as standard– to control for the endogeneity of payouts: firm f is aware only of the "average" value of the firm for consumers and this may be correlated to the implicit price charged by the firm. We assume that the firm is not aware of the idiosyncratic value of the firm for a specific retiree.[4]

To complete the mixed logit specification, the valuation of the attributes of the quote β_i is a normally distributed variable that depends on individual characteristics:

$$\beta_i \sim N(\lambda Z_i', \Sigma_\beta) \tag{4}$$

[4]When making initial offers, the firm is not aware of the intermediation choice, and there is no communication between the sales agent and the company. The average difference between the initial offer and subsequent counteroffers is almost the same for retirees that enter the system through sales agents and through independent advisers.

where Z_i' is a vector of individual's i characteristics and Σ_β is a diagonal matrix ($\Sigma_\beta = diag(\sigma_\beta^2)$).

The choice of individual i in the second stage defines a set of indicator variables as follows:

$$Y_{ijf} = \begin{cases} 1 & \text{if } i \text{ chooses quote } j \text{ from firm } f\text{, and} \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

4.1.1 Variables and normalizations included in the model

Attributes of quotes and firms. The vector X_{ijf} corresponds to the individual-specific attributes of the quotes and firms that affect the choice in the second stage, and can be decomposed in three parts: X^1_{ijf} is a vector of size 5 that captures the valuation of the discounted present value of the future payments; X^2_{ijf} is a vector of size 7 that include attributes of the offer and the insurer; finally, X^3_{ijf} is a vector of size 5 that captures the effect of entering to the system with a sales agent from the same insurance company f that is giving the quote. The three parts $X_{ijf} = [X^1_{ijf}, X^2_{ijf}, X^3_{ijf}]$ are described as follows.

1. X^1_{ijf} includes the variables that enter into the discounted present value of the future payments of the annuity. Suppose a von Neumann-Morgenstern utility function with Bernoulli function $u_i(y) = \ln(y)$; then the present value at time t V_{it} of these payments is given by

$$V_{it}(P_{ijf}) = \ln(P_{ijf}) + \rho_i \pi_{it} \pi_{ft} V_{it+1}(P_{ijf}) + \pi_{it}(1 - \pi_{ft}) G_{ijf} + (1 - \pi_{it}) B_{ijf} \tag{6}$$

where π_{it} is the probability that the retiree survives to the next month, π_{ft} is the probability that the company continues making payments the next month, ρ_i is the monthly discount factor and P_{ijf} is the monthly payment. In turn, G_{ijf} is the payment made by the state guarantee if the company defaults as described in Section 2.1, and B_{ijf} is the payments left for legal beneficiaries after death as described in Section 2.2, both in present value terms and utility units.

Consider for instance the special case where the last two terms in Equation 6 that include G_{ijf} and B_{ijf} are set to zero–or that G_{ijf} and B_{ijf} are linear functions of P_{ijf}.[5] Under the strong assumption that π_{it} and π_{jt} are constant over time, we obtain:

$$V_i(P_{ijf}) = \ln(P_{ijf}) + \rho_i \pi_i \pi_f V_i(P_{ijf}) \tag{7}$$

and

$$V_i(P_{ijf}) = \frac{1}{1 - \rho_i \pi_i \pi_f} \ln(P_{ijf}) \tag{8}$$

Under these assumptions, the present value of the payments of the annuity depends on the retiree's discount factor and probability of survival, and the probability that the company continues making payments, that depends on his risk rating.

Let $r \in \{AA+, AA, AA-, A+, A \text{ or lower}\}$ denote the risk rating. We model the relation between r and π_f in a flexible way: We define the set of risk-rating dummies $\{R_{ifr}\}_r$ as $R_{ifr} = 1$ if firm f has risk rating r when i receives the quote; then

$$X^1_{ijf} \equiv \{R_{ifr} \ln(P_{ijf})\}_r.$$

This is, the present value of the payments of the annuity is captured using the log-monthly payment and interactions between the log-monthly payment and the risk rating dummies. We leave as base alternative $r = A$ or lower.

[5] The values of G_{ijf} and B_{ijf} are not included in the offers and are difficult to estimate for the retiree.

2. X^2_{ijf} includes the observable attributes of the quote and firm characteristics that may affect the value: the number of months of the guaranteed period, a dummy if the quote includes lump sum withdrawal, the amount of the withdrawal, the size of firm f measured as the amount of technical reserves, and the rating dummies R_{fr} (only for $r \in \{AA+, AA, AA-\}$ as other categories cannot be separately identified from ξ_f).

3. X^3_{ijf} includes a dummy that takes on a value of 1 if $\tilde{Y}_{if} = 1$, this is, if the retiree uses a sales agent from the same firm that is giving the quote, and interactions of this dummy with the variables in X^1_{ijf}. The logic for this set of variables is explained next.

Variables affecting mean valuation of quote attributes. The vector Z_i corresponds to the individual characteristics that affect the mean valuation of the quote's attributes $E(\beta_i)$. The vector also includes three parts: $Z_i = [\iota, Z^1_i, Z^2_i]$, where ι is a constant and:

1. Z^1_i of size 12 includes the demographic and financial information available: a dummy if the retiree is female, age, a dummy if married, number of children, a dummy if she lives in the metropolitan area, a set of two dummies that capture different AFPs the retiree may be in, size of her savings fund, a set of dummies describing different composition of stocks and options of her fund, and a dummy if the retiree had already obtained a programmed withdrawal and is changing her pension product.

2. Z^2_i is a vector of indicator variables for the type of intermediary chosen in the first stage,

$$Z^2_i = \left(\tilde{Y}^*_{iF}, \tilde{Y}_{i,F+1} \right) \tag{9}$$

   where $\tilde{Y}^*_{iF} \equiv \sum_{k \in \{1,\ldots,F\}} \tilde{Y}_{ik}$ is a dummy that takes on a value of 1 if individual i chose any sales agent in the first stage, while $\tilde{Y}_{i,F+1} = 1$ if she chose an independent adviser.

To focus on willingness to pay for reputation and avoid a curse of dimensionality, we restrict $Z_i = \iota$ to the variables in X^2_{ijf} and X^3_{ijf}. This is, the mean valuation of observable attributes is constant and we only allow observable variation in the mean valuation for the present value of the annuity's payments; this variation represents differences in discount factors and survival probabilities.[6]

These variables imply that the present value of the annuity's payments varies with individual characteristics but also with intermediation. The effect of Z^2_i captures general (as opposed to firm-specific) biased advice: independent advisers and sales agents may focus their advice on the payments or on the risk of each annuity, affecting differently the valuation of these attributes, but the effect is the same for all firms making quotes with similar attributes. For this reason we include the vector X^3_{ijf} above, that captures firm-specific biased advice: sales agents may increase the value of their firm's quotes over other firms when $\tilde{Y}_{if} = 1$, but also sales agents from better-rated companies may over-emphasize the importance of firms' solvency in order to convince their clients to accept their offer, and this is included as interactions of this dummy with the variables in X^1_{ijf}. Ideally the dummy if $\tilde{Y}_{if} = 1$ would be included in Z_i, but β_i must only vary by i, not f. Hence we include the dummy in X_{ijf} with interactions, so it indirectly changes the valuation of the present value of the annuity's payments. Because the intention is to vary β_i, we additionally assume $\beta_i = \beta$ for the vector X^3_{ijf} ($V(\beta_i) = 0$).

[6] We also restrict $Z_i = \iota$ for $R_{fr} \ln(P_{ijf})$ with r=A or lower, to avoid multicolinearity.

Variables affecting choice of intermediation. The choice in the first stage is affected by W_i, a vector of size 14 with individual characteristics, and W_{ik}, a vector of size 7 with attributes of the alternatives:

1. W_i corresponds to the individual characteristics that affect the value of an intermediary. The vector also includes three parts $W_i = [\iota, Z_i^1, W_i^2]$, where ι is a constant, Z_i^1 are the same demographic and financial variables included in the valuation of attributes in the second stage, and W_i^2 is a vector of individual variables used as exclusion restrictions: if the retiree lives in a city with AFP offices available, and if the retiree's spouse is above the retiring age, to capture the effect of different experience with the pension process. We include the standard normalization that $\alpha_k = 0$ for $K = F + 2$ (entering directly with an AFP).

2. W_{ik} corresponds to individual and alternative-specific characteristics that affect the value of an intermediary. The first three variables are included in X_{ijf}^2: rating dummies R_{fr} for $r \in \{AA+, AA\}$ and firm size measured as the amount of technical reserves. We also include the number of sales agents of firm f available in the retiree's area as an exclusion variable–all this variables take value zero for $k > F$. The number of independent advisers available in the area is also included for $k = F + 1$.

4.1.2 Correlation between willingness to pay for reputation and intermediaries

The model admits four potential sources of relation between the first and second stages. Selection on observables is captured in α: retirees may be more likely to choose a sales agent from a better-rated or a larger company. Selection on unobservables is captured through correlation between $\tilde{\varepsilon}_{ik}^1$ and ε_{if}^1 when $k = f$: retirees with a larger preference for an insurer may have larger values and be more likely to choose both a sales agent and a quote from that company. In particular, we assume the following correlation structure:

$$
\begin{aligned}
Cov(\tilde{\varepsilon}_{ik}^1, \tilde{\varepsilon}_{ik'}^1) &= 0 && \forall k \neq k' \\
Cov(\varepsilon_{if}^1, \varepsilon_{if'}^1) &= 0 && \forall f \neq f' \\
Cov(\tilde{\varepsilon}_{ik}^1, \varepsilon_{if}^1) &= 0 && \forall k \neq f \\
Cov(\tilde{\varepsilon}_{ik}^1, \varepsilon_{if}^1) &\neq 0 && \text{if } k = f
\end{aligned}
$$

this structure implies that these shocks can be considered as pairs of bivariate normal disturbances:

$$
\begin{pmatrix} \tilde{\varepsilon}_{if}^1 \\ \varepsilon_{if}^1 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{1f}^2 & \sigma_{12f} \\ \sigma_{12f} & \sigma_{2f}^2 \end{pmatrix} \right) \tag{10}
$$

or

$$
\begin{pmatrix} \tilde{\varepsilon}_{i}^1 \\ \varepsilon_{i}^1 \end{pmatrix} \sim N\left(\begin{pmatrix} 0_K \\ 0_F \end{pmatrix}, \begin{pmatrix} \Sigma_1 & \Sigma_{12}' \\ \Sigma_{12} & \Sigma_2 \end{pmatrix} \right) \tag{11}
$$

where

$$
\begin{aligned}
\Sigma_1 &= diag(\sigma_{11}^2, \ldots, \sigma_{1f}^2, \ldots, \sigma_{1F}^2, \sigma_{1,F+1}^2, \sigma_{1,F+2}^2) \\
\Sigma_2 &= diag(\sigma_{21}^2, \ldots, \sigma_{2f}^2, \ldots, \sigma_{2F}^2) \\
\Sigma_{12} &= \begin{pmatrix} \sigma_{121} & \ldots & 0 & \ldots & 0 & 0 & 0 \\ & \ddots & & & & \vdots & \\ 0 & \ldots & \sigma_{12f} & \ldots & 0 & 0 & 0 \\ & & & \ddots & & \vdots & \\ 0 & \ldots & 0 & \ldots & \sigma_{12F} & 0 & 0 \end{pmatrix}
\end{aligned}
$$

The third source of relation between the first and second stages is general biased advice, which is captured by the λ parameters that accompany Z_i^2: independent advisers and sales agents may focus their advice on the payments or on the risk of each annuity, affecting differently the valuation of these attributes, but the effect is the same for all firms making quotes with similar attributes. Finally the fourth source is firm-specific biased advice, which is captured by the β that accompany X_{ijf}^3: sales agents may emphasize the value of their firm's quotes over other firms, and sales agents from better-rated companies may over-emphasize the importance of firms' solvency in order to convince their clients to accept their offer.

4.2 Derivation of Choice Probabilities

In the first stage, retiree i chooses the intermediary that maximizes her first stage utility:

$$\tilde{Y}_{ik} = 1 \Leftrightarrow \tilde{U}_{ik} \geq \max_{k' \neq k} \tilde{U}_{ik'}$$

Because the shocks $\tilde{\varepsilon}_{ik}^1$ and $\tilde{\varepsilon}_{ik}^2$ are independent, and also the shocks $\tilde{\varepsilon}_{ik}^1$ and $\tilde{\varepsilon}_{ik'}^1$ are independent for $k \neq k'$, this implies:

$$Pr(\tilde{Y}_{ik} = 1 | W_i, W_{ik}, \tilde{\varepsilon}_i^1) = \frac{\exp(W_i \alpha_k + W_{ik} \alpha + \tilde{\varepsilon}_{ik}^1)}{\sum_{k'=1}^{K} \exp(W_i \alpha_{k'} + W_{ik'} \alpha + \tilde{\varepsilon}_{ik'}^1)} \tag{12}$$

with the normalization that $\alpha_k = 0$ for $k = F + 2$ (entering directly).

In the second stage, conditional on the intermediary k the retiree i chooses the quote j from insurance company f that maximizes her second stage utility:

$$Y_{ijf} = 1 \Leftrightarrow U_{ijf} \geq \max_{j'f' \neq jf} U_{ij'f'}$$

Because the shocks ε_{if}^1 and ε_{ijf}^2 are independent, and also the shocks ε_{if}^1 and $\varepsilon_{if'}^1$ are independent for $f \neq f'$, this implies that:

$$\Pr(Y_{ijf} = 1 | X_{ijf}, \beta_i, \varepsilon_i^1) = \frac{\exp(X_{ijf} \beta_i + \xi_f + \varepsilon_{if}^1)}{\sum_{j'f'} \exp(X_{ij'f'} \beta_i + \xi_{f'} + \varepsilon_{if'}^1)} \tag{13}$$

Integrating over the dispersion in individual preferences, we obtain:

$$\Pr(Y_{ijf} = 1 | X_{ijf}, Z_i, \tilde{Y}_i^*, \varepsilon_i^1) = \int \frac{\exp(X_{ijf} \beta_i + \xi_f + \varepsilon_{if}^1)}{\sum_{j'f'} \exp(X_{ij'f'} \beta_i + \xi_{f'} + \varepsilon_{if'}^1)} \phi(\beta_i | \lambda, \Sigma_\beta) d\beta_i \tag{14}$$

Since $\tilde{Y}_i^*$ and ε_{if}^1 for $f \in \{1, ..., F\}$ are correlated, the second stage cannot be estimated alone. We use the conditional expectation of $\varepsilon_i^1 | \tilde{\varepsilon}_i^1$ to write:

$$\Pr(Y_{ijf} = 1 | X_{ijf}, Z_i, \tilde{Y}_i, \tilde{\varepsilon}_i^1) = \int \int \frac{\exp(X_{ijf} \beta_i + \xi_f + \varepsilon_{if}^1)}{\sum_{j'f'} \exp(X_{ij'f'} \beta_i + \xi_{f'} + \varepsilon_{if'}^1)} \phi(\beta_i | \lambda, \Sigma_\beta) \phi(\varepsilon_i^1 | \tilde{\varepsilon}_i^1) d\beta_i d\varepsilon_i^1 \tag{15}$$

and the joint likelihood for retiree i is given by

$$\begin{aligned} &\Pr(\tilde{Y}_{ik} = 1, Y_{ijf} = 1 | W_i, W_{ik}, X_{ijf}) \\ &= \int \int \int \frac{\exp(W_i \alpha_k + W_{ik} \alpha + \tilde{\varepsilon}_{ik}^1)}{\sum_{k'=1}^{K} \exp(W_i \alpha_{k'} + W_{ik'} \alpha + \tilde{\varepsilon}_{ik'}^1)} \frac{\exp(X_{ijf} \beta_i + \xi_f + \varepsilon_{if}^1)}{\sum_{j'f'} \exp(X_{ij'f'} \beta_i + \xi_{f'} + \varepsilon_{if'}^1)} \phi(\beta_i | \lambda, \Sigma_\beta) \phi(\varepsilon_i^1 | \tilde{\varepsilon}_i^1) \phi(\tilde{\varepsilon}_i^1) d\beta_i d\varepsilon_i^1 d\tilde{\varepsilon}_i^1 \end{aligned} \tag{16}$$

We maximize the joint log-likelihood, i.e., $L = \sum_i \Pr(\tilde{Y}_{ik} = 1, Y_{ijf} = 1 | W_i, W_{ik}, X_{ijf})$ via Simulated Maximum Likelihood. The estimation algorithm is described in Appendix A.3.

Table 4: First stage: attributes of intermediaries

Attributes of alternative k W_{ik}	Coefficient α
AA+	1.154
	(0.94)
AA	1.806*
	(0.94)
Firm size	0.998**
	(0.14)
Availability of intermediary	0.045**
	(0.01)

Note: Standard errors in parenthesis
Parameters are statistically significant:
(*) at the 10% level, (**) at the 5% level

5 Results

The model requires estimating 289 different parameters –including parameters for the first stage, for the second stage, and for the correlation among both stages. The likelihood function for mixed logit models usually is not globally concave and has local maxima (see Train (2003)); we considered a grid of different starting values for the parameters and 50 Halton draws for the simulation process. Due to computing limitations, we used a random subsample of 1,428 retirees with 84,656 quotes. Tables 4, 5 and 7 show the results for a subset of the estimated parameters.

Table 4 presents the parameters for intermediary attributes in the first stage, α. Firm size and availability of intermediaries are the most important determinants of intermediation choice: sales agents from larger firms and more accessible alternatives are more likely to be chosen. The parameters for socioeconomic characteristics (α_k, not shown in the table) are mostly non-significant and vary across alternatives. There is no clear pattern of individual characteristics in the probability of entering with any of the sales agents; this evidence supports our hypothesis that firms are not able to predict the specific intermediary used by a retiree nor the idiosyncratic value for the different firms that she may have. We are only able to say that the probability of entering the system directly using an AFP (the only option that does not require paying fees) is larger for females, for individuals with larger pension saving funds, for those from an AFP that manages larger pension funds on average, and for those who live in the metropolitan area. The age of the spouse do not seem to play an important role in the decision.

Tables 5 and 7 present some of the parameters of the second stage. Table 5 shows some parameters related to the mean valuation of monthly payments, β_i, and risk rating; in particular, it shows the intercept for the mean valuation λ_0, the effect of the first stage choice on the mean valuation λ_2 and the standard deviation of the mean valuation σ_β. As expected, all retirees value positively the monthly payment –its level increases the probability of choosing an offer– but the valuation is larger when the firm's risk rating is $AA+$ or when the retiree entered the system directly with an AFP. There is substantial individual heterogeneity on the mean valuation of monthly payments: the variation due to observed characteristics is significant (λ_1, not shown in the table) and also due to unobserved factors: the standard deviation is statistically different from zero.

To see more clearly how the mean valuation changes with risk rating and the intermediary, Table 6 computes the mean valuation $E(\beta_i)$ for a 65 year-old male with other characteristics set to their

Table 5: Second stage: monthly payment and risk rating

Variables X^1_{ijf}	Coefficient	
log of monthly payment (ln P)	λ_0	σ_{β_i}
lnP	839.8**	74.69**
	(86.67)	(5.87)
interaction with risk rating (base category: A *or lower)*		
lnP*AA+	6.33*	0.8**
	(3.34)	(0.18)
lnP*AA	2.93	0.09
	(3.06)	(0.09)
lnP*AA-	2.94	0.11
	(3.03)	(0.17)
lnP*A+	-1.84	0.07
	(1.3)	(0.4)
	intermediation (λ_2)	
log of monthly payment (ln P)	*sales agent*	*indep adviser*
lnP	-35.4**	-18.7**
	(8.2)	(7.53)
interaction with risk rating (base category: A+ *or lower)*		
lnP*AA+	-2.43**	-4.19**
	(0.89)	(0.73)
lnP*AA	-1.45	-2.72**
	(0.91)	(0.83)
lnP*AA-	-1.5	-2.36**
	(0.93)	(0.84)

Note: Standard errors in parenthesis

Parameters are statistically significant: (*) at the 10% level, (**) at the 5% level

sample mean. The rows show how the valuation changes for different ratings, and the columns show how the valuation changes for different intermediaries. We can see again that the valuation is larger when the firm's risk rating is $AA+$ or when the retiree entered the system directly with an AFP. Now it is easier to see that the valuation increases with better rating for all intermediaries, that the valuation is less for retirees that entered with an advisor, lower for retirees that entered with a sales agent, and even lower if the quote is from the same firm that the sales agent used to enter.

Table 7 shows the parameters of other attributes of the quote and firm characteristics. As expected, the valuation of guaranteed periods and lump sum withdrawal is positive and statistically significant. Contrary to the first stage, firm size does not have a significant effect on the choice of the second stage. Interestingly, when the quote is from the same firm that the sales agent used to enter, there is a large value of that quote that increases the probability of choosing it, but the valuation of the monthly payment is reduced, independently of the rating.

Firm fixed effects (not shown) are statistically different at the 10% level from the base firm for 6 of 20 firms. The estimated correlation between $\tilde{\varepsilon}^1_{if}$ and ε^1_{if} (i.e., between the value of choosing a sales agent from firm f in the first stage and the value of the offer received from the same firm f in the second stage) is not statistically different from zero, and is almost zero (less than 0.001) in 5 of 8 cases.

Table 6: Mean valuation $E(\beta_i)$ for a Male, 65 years old

	Intermediary			
Rating	AFP	Independent adviser	Sales agent from any firm	Sales agent from same firm
AA+	249.01	226.12	211.17	209.14
AA	247.03	225.61	210.17	207.23
AA-	246.86	225.8	209.95	205.75
A+	243.3	224.6	207.89	207.89
A or lower	245.14	226.44	209.74	207.18

Table 7: Second stage: attributes of offers and firms

Variables X_{ijf}^2	Coefficient	
other attributes	λ_0	σ_{β_i}
Months of guaranteed period	0.09**	0.05**
	(0.01)	(0.01)
Has lump sum withdrawal	33.31**	38.57**
	(3.04)	(3.44)
Amount withdrawal	128.71**	206.88**
	(9.91)	(16.9)
Firm size	0.37	0
	(0.4)	(0.24)
AA+	-7.79**	0
	(3.28)	(0.65)
AA	-6.76**	0.08
	(3.16)	(0.32)
AA-	-7.29**	0.56
	(3.17)	(0.51)
Variables X_{ijf}^3	Coefficient	
offer from same firm	β	
Same firm	19.9**	
	(1.61)	
interaction with offer attributes		
Same firm*lnP	-2.56**	
	(1.13)	
Same firm*lnP*AA+	0.53	
	(0.97)	
Same firm*lnP*AA	-0.38	
	(1.07)	
Same firm*lnP*AA-	-1.64	
	(1.11)	

Note: Standard errors in parenthesis
Parameters are statistically significant:
(*) at the 10% level, (**) at the 5% level

5.1 Willingness to pay for risk rating

We use the estimated structural parameters to obtain an estimation of the willingness to pay for better risk rating. We define this willingness to pay as the change in the monthly payment required to keep the level of U_{ijf} constant when the risk rating of firm f improves from r_0 to r_1. Note that this definition keeps constant both the choice and the ranking between different alternatives. If product j from firm f is chosen in the second stage under r_0, then improving the rating to r_1 also affects the value of the other products offered by the firm, while the value of the products offered by other firms is not affected. Hence, the firm could change the payment for all its products in the same proportion and keep constant the ranking among different offers (and also keep constant the probability of choosing the same offer).

We compute the willingness to pay of a given retiree with mean valuation $\mathrm{E}(\beta_i)$. Table 8 shows the willigness to pay of a retiree with different individual characteristics (all other characteristics are set to their sample mean). Most of the heterogeneity in willingness to pay comes from differences in intermediation rather than differences in individual characteristics. Retirees that enter with an adviser have the lowest willingness to pay, and retirees that enter directly with an AFP or with a sales agent and receive a quote from that firm have the largest.

The model allows to disentangle the role of the different sources of correlation between intermediaries and willingness to pay for reputation: selection on observables, selection on unobservables, general biased advice, and firm-specific biased advice.

The effect of general biased advice is captured by comparing the willingness to pay for different intermediation types; i.e., this effect is captured by the differences among the three first columns in Table 8, and proved to be economically significant. When we look at the willingness to pay for improving the rating from AA to AA+, this effect is significant: retirees entering directly with an AFP are willing to pay more for improving the rating than those entering with independent advisers or with a sales agent. When we focus on the willingness to pay for improving the rating from AA- to AA, in contrast, differences are not significant.

The effect of firm-specific biased advise is captured by the differences in willingness to pay among retirees who entered with a sales agent from any firm, and those who entered with a sales agent from the same firm that makes the quote; i.e., this effect is captured by the differences between the third and forth columns in Table 8. The willingness to pay increases substantially when the quote is from the same firm as the sales agent, for any rating.

The effect of selection on unobservables can be captured by comparing the results when we estimate the model only with the second stage (conditional on the selection of intermediaries). Table 9 shows the willingness to pay estimated with and without the first stage (in the first and second panel respectively). The results are similar, which is consistent with the results for the correlation of shocks between both stages: the larger differences are in the fourth column, i.e., for retirees that entered with a sales agent and receive offers from the same firm.

5.2 Unconditional willingness to pay

The willingness to pay defined in the previous section is conditional on the choice of the first stage, and assumes this choice remains constant after an improvement in the firm's rating. The results in Table 4 suggest, however, this is not the case: the probability that a sales agent from firm f is chosen increases when the firm's rating improves (especially when improving from AA- to AA). As the (conditional) willingness to pay is higher for retirees that enter the system with a sales agent from the same firm, this suggests that the results shown in Table 8 may be a lower bound for the unconditional willingness to pay for risk rating. The difference between conditional an unconditional

Table 8: Willingness to pay for risk rating, evaluated at $E(\beta_i)$

	Intermediary			
	AFP	Independent adviser	Sales agent from any firm	Sales agent from same firm
Risk rating: from AA to AA+				
Man, age 65	1.5%**	0.09%**	0.67%**	1.71%**
	(0.0052)	(0.0004)	(0.003)	(0.0138)
+ age 60	1.33%**	0.15%**	0.62%**	1.48%**
	(0.0042)	(0.0005)	(0.0024)	(0.0102)
+ married	1.71%**	0.7%**	1.14%**	1.89%**
	(0.0045)	(0.0018)	(0.0037)	(0.0114)
+ married, one kid	1.7%**	0.73%**	1.16%**	1.87%**
	(0.0041)	(0.0019)	(0.0034)	(0.0098)
+ savings fund * 0.5	1.68%**	0.31%**	0.89%**	1.91%**
	(0.0061)	(0.0012)	(0.0041)	(0.0159)
+ savings fund * 2	1.13%**	-0.37%**	0.2%**	1.27%**
	(0.004)	(0.0016)	(0.0009)	(0.0101)
Woman, age 65	1.47%**	0.48%**	0.89%**	1.61%**
	(0.0038)	(0.0014)	(0.0029)	(0.0085)
+ age 60	1.35%**	0.47%**	0.83%**	1.45%**
	(0.0031)	(0.0012)	(0.0023)	(0.0069)
Risk rating: from AA- to AA				
Man, age 65	0.38%**	0.03%**	0.5%**	1.97%**
	(0.001)	(0.0001)	(0.0018)	(0.011)
+ age 60	0.31%**	0.02%**	0.4%**	1.6%**
	(0.0007)	(0.00004)	(0.0012)	(0.0073)
+ married	0.31%**	0.05%**	0.39%**	1.44%**
	(0.0006)	(0.0001)	(0.001)	(0.0057)
+ married, one kid	0.44%**	0.2%**	0.53%**	1.54%**
	(0.0008)	(0.0004)	(0.0013)	(0.0057)
+ savings fund * 0.5	0.37%**	0.02%**	0.49%**	1.93%**
	(0.001)	(0.0001)	(0.0018)	(0.0108)
+ savings fund * 2	0.4%**	0.05%**	0.53%**	2.05%**
	(0.0012)	(0.0001)	(0.0021)	(0.0117)
Woman, age 65	0.46%**	0.22%**	0.55%**	1.56%**
	(0.0009)	(0.0004)	(0.0014)	(0.0055)
+ age 60	0.4%**	0.18%**	0.47%**	1.35%**
	(0.0007)	(0.0003)	(0.001)	(0.0041)

Note: Standard errors in parenthesis, using the Delta Method.

Parameters are statistically significant: (*) at the 10% level, (**) at the 5% level

Table 9: Willingness to pay for risk rating evaluated at $E(\beta_i)$, with and without the first stage

	Intermediary			
	AFP	Independent adviser	Sales agent from any firm	Sales agent from the same firm
Two stages				
Risk rating: from AA to AA+				
Man	1.5%	0.1%	0.7%	1.7%
Woman	1.3%	0.5%	0.8%	1.5%
Risk rating: from AA- to AA				
Man	0.4%	0%	0.5%	2%
Woman	0.4%	0.2%	0.5%	1.4%
Only second stage				
Risk rating: from AA to AA+				
Man	1.4%	0.2%	0.7%	2.4%
Woman	1.3%	0.6%	0.9%	1.8%
Risk rating: from AA- to AA				
Man	0.4%	0%	0.4%	2.1%
Woman	0.4%	0.2%	0.4%	1.4%

willingness to pay captures the role of selection on observables coming from differences on ratings (i.e., from differences on observable attributes of the firms).

Estimating the unconditional willingness to pay requires a different definition: we must focus on keeping constant the probability of choosing the same quote rather than just U_{ijf}, as the level of utility for all quotes changes when the intermediary is different. Indeed, Table 6 shows a much smaller valuation of the monthly payment when entering with a sales agent than other intermediary, which reduces the value of all quotes received. But this change in value does not need to be compensated to maintain the probability of choosing the same quote. Moreover, estimating unconditional willingness requires simulation, as the choice of the first stage under r_1 depends on the values of $\tilde{\varepsilon}^1_{ik}$ and $\tilde{\varepsilon}^2_{ik}$. The results of this section are pending.

6 Concluding remarks

The Chilean annuity market provides an exceptional opportunity to measure consumers' willingness to pay for reputation. Within each class of retirement product requested, retirees receive quotes that vary only in two dimensions: the payout (monthly payment and lump sum withdrawl, if any) and the firm risk rating. By comparing the selected quote with the set of alternatives available for each retiree, we can estimate willingness to pay for risk rating. As the firm rating is presented as a measure of its probability of default, willingness to pay for a better rating is regarded the same as willingness to pay for a higher reputation of solvency.

While some retirees choose the offer with the highest payout in its class, a significant fraction of them choose an offer with a lower payout from a better-rated company. The prevalence of this stylized fact, however, depends on the intermediation mode chose by the retiree. We estimate a two-stage model where individuals sequentially choose intermediaries and a pension product among all the quotes received in the second stage. We find that willingness to pay for risk rating is statistically and economically significant: retirees are willing to reduce their lifetime payments around 1% or 2% in order to contract an annuity to a better rated company.

The preliminary analysis shows that retirees intermediated by sales agents tend to choose better-rated companies that offer lower payouts, while those intermediated by independent advisers tend to choose worse-rated companies that offer higher payouts. This analysis is consistent with our results: willingness to pay for a better risk rating is higher among retirees intermediated by sales agents than among those who are intermediated by independent advisers. Heterogeneity in willingness to pay may reflect ex-ante consumers' heterogeneity, buy it is also the consequence of biased advice. The welfare impact of this biased advise is non-trivial, however: retirees being advised to select quotes with higher payments may be subject to a higher default risk.

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Appendix

A.1 Basic retirement options

To access any funds saved in the pension system during the accumulation phase, the retiree must retire through SCOMP and choose one of the options available to her. The retiree has four basic retirement options: annuity, programmed withdrawal, combined annuity with programmed withdrawal, and temporary allowance with deferred annuity. As their names indicate, the last two are combinations of the first (commonly known as combined annuities), and are less common as retirement options.

The annuity is the most natural example of retirement option and the usual economic prescription for risk averse individuals, but similarly as other countries it is not the most common option, which is still a puzzle to the literature. In Chile the annuitization rate is larger than in other countries. According to the regulatory agency, 55.7% of retirees choose an annuity or a combined annuity; in our sample data, 38.6% of retirees choose a standard annuity and 23% choose a combined annuity. Under the standard annuity, the retiree transfers the property of the savings fund to the insurance company in exchange for a fixed monthly payment up to the death of the retiree. This transfer is irreversible. The law in Chile requires that after the death of the retiree, the payments continue to the legal beneficiaries: 60% of the monthly payment while the spouse is alive, 40% of the monthly payment while there are children under 25 years old, and 40% of the monthly payment while children with disability are alive; this requirements are naturally considered by the insurance company when giving quotes. In any case, it is usually understood that the upside of buying an annuity is the insurance against the longevity risk –outliving your own assets– and the insurance against inflation –due to the real currency used for these transactions–while the downside is the loss of inheritance, specially if the retiree dies early.

The second retirement option available is a programmed withdrawal. According to the regulatory agency, 44.3% of retirees choose a programmed withdrawal; in our sample data it is a 38.4%. This option allows the withdrawals of predetermined amounts of funds on a regular monthly basis, so even under this option the retiree cannot obtain all the savings fund at once. The retiree maintains the property of her savings fund but it continues under the administration of an AFP (the same one that managed the savings fund in the accumulation phase or a different one). The AFP invests the funds, pays the retiree a sum each month, and charges an administration fee. The sum paid each month varies with a schedule, and it is updated each year according to the size of the available funds, the rate of return on investments, the retiree's conditional life expectancy, and the rules set by the regulatory agency. This updating rule has the result that the schedule of payments is decreasing over time at an increasing rate. The retiree may die before the savings fund runs out –in which case the rest is used as inheritance– or the savings may ran out while the retiree is still alive –in which case she may qualify for the minimal pension provided by the state.

Because of these characteristics, it is usually understood that programmed withdrawal may be a better option for retirees that are less risk averse, that have other savings outside the pension system, that have large fund sizes that are less likely to run out, and that may want to continue working after retirement: the system allows to adjust the sum paid each month to the minimal pension provided by the state, decreasing the progressive tax burden and leaving more savings for the future. The law also forces retirees with low savings to choose a programmed withdrawal: if the funds available to the retiree are not enough to buy an annuity higher than the minimal pension provided by the state, then she must choose a programmed withdrawal and receive the minimal pension as payment until the savings fund runs out, and receive the minimal pension from the state from that point on. Therefore, programmed withdrawal is a more popular option in the two extremes of the socioeconomic spectrum. Retirees with programmed withdrawal may come back to the SCOMP system to buy an annuity with the remaining funds; these individuals are included in our data and represent aproximately 25% of the retirees.

The last two options available –combined annuities– are combinations of annuities and programmed withdrawal. In the combined annuity with programmed withdrawal, the retiree splits her savings fund in two parts as she wishes, and with each part buys an annuity and a programmed withdrawal respectively. In the temporary allowance with delayed annuity, the retiree again splits her savings fund in two parts: one is used to buy an annuity that will start paying in a future date, and the other one is used meanwhile as a programmed withdrawal.

A.2 The retirement process

Workers can legally retire once they reach 65 years old for males and 60 years old for females. Workers in the private sector can keep their jobs after retiring, but this legal process allows them to access their pension savings funds. Workers may also opt for an early retirement if their savings fund is large enough to meet certain conditions.[7] Around 40% of retirees correspond to early retirement.

The retirement process starts when the soon-to-be retiree approaches an office of her pension fund administrator (AFP) and fills the Retirement Application and Declaration of Beneficiaries. The AFP has 10 days to issue the Certificate of Balance, which contains the information about the savings fund – specific accounts, amounts, and composition of stocks and options– and the personal and demographical information of the retiree and the legal beneficiaries –spouse, children, and parents. This certificate is sent to the retiree and entered into SCOMP, and expires after 35 days. According to the regulatory agency (Superintendencia de Valores y Seguros SVS), for our sample period (August 2004 to April 2013) the system has issued 492,986 Certificates of Balance.

With this certificate, the retiree can request quotes of different pension products. As noted above, there are four basic retirement options: annuities, programmed withdrawals, or two different combinations of them. The retiree decides the options to request and the atributes for each option. This request is entered into SCOMP by the intermediary. Of the total of Certificates of Balance issued, 333,560 are used to enter a request for quotes. The retiree can enter up to three requests for each Certificate of Balance issued –in average the retirees enter 1.34 requests per Certificate of Balance, which adds to 446,421 requests. In our sample we have information for 424,623 requests.

The SCOMP system assigns a unique numeric code to the Certificate of Balance and the corresponding request for quotes, and sends the (otherwise anonymous) financial and demographical information to all the authorized life insurance companies – the personal information of the retiree and the type of intermediary are omitted and thus unknown to the companies. There are 21 authorized companies in the sample period. If a company wants to give a quote to the retiree for some or all the pension products requested, it enters the offer(s) into SCOMP before a due date set by the system. The payouts of the quotes are entered in UF (Unidades de Fomento), a currency that is daily corrected for inflation and is the standard unit of account for long term contracts in Chile; therefore the offers are set in real terms. Within the next 4 working days after the request for quotes was entered, the system issues a Certificate of Offers to the retiree. This Certificate of Offers is a letter sent to the retiree's home address, containing all the quotes arranged in a previously specified fashion, and is valid for 12 working days.

With the Certificate of Offers, the retiree may do one of the following things: choose any of the quotes in the Certificate of Offers, ask for an outside offer, ask for an auction, make a new request to the system, or cancel the application for retirement and start the process again in the future.

[7] A worker is elegible for early retirement if her savings fund is large enough to buy an annuity at least 70% of her average wage from the previous 10 years, and at least 150% of the minimal pension provided by the state.

According to the regulatory agency, 128,506 retirees choose an offer included in the Certificate of Offers. The outside offer occurs when the retiree goes directly to an insurance company with a valid quote and negotiates individually; the outside offer must also be entered into SCOMP and must be from the same class and a higher payout than the offer in the Certificate of Offers. This option is very popular; according to the regulatory agency, 114,903 retirees choose an outside offer. In our sample –only retirees that choose an immediate annuity– 87.3% of the retirees that choose annuities ask for an outside offer but to a few companies –these represent 3.74% of the total offers– and 84% choose an outside offer.The auction occurs when the retiree goes to her AFP and selects up to three insurance companies with a valid quote from the same class to run a first-price auction; if at least two companies decide to participate then the retiree is automatically assigned to the company that bids the highest monthly payment. Auctions are very rare: in 10 years of operation the system has only run 89 auctions. Finally, the application can be cancelled actively by the retiree (for example, if she wants to enter the system with a different intermediary), but also when the Certificate of Balance or the Certificate of Offers expire before a final decision is entered into the system. This also happens very frequently: of the 446,421 requests made to the system, 202,923 (45%) of the requests are cancelled; similarly, in our data 30.93% of the requests are cancelled. The retiree can only access her savings funds if she retire, hence everyone that cancelled a request should come back to the system in the future.

To accept one of the quotes in the Certificate of Offers or one received afterwards, the retiree must sign a Certificate of Acceptance with the chosen insurance company or with her AFP, which has to enter the information into SCOMP. If the retiree is choosing a quote that does not have the higher payout in its class, the system will issue a form with a declaration that must be signed by the retiree acknowledging the gap. The retirement process finishes when the AFP that manages the retiree's savings account transfers the savings fund to the selected provider. After the retirement process finishes, SCOMP can only inform each company on the relative position in terms of payout that its quotes had in each respective class. It is not allowed to disclose information on the selected provider, the relative position of other companies, or the payouts quoted.

The SCOMP system can also be used by people that already retired under programmed withdrawal but would like to switch to a different option (25% of our sample data), by people that qualify for disability pension (12.5% according to the regulatory agency, 13% in our sample data), or by the legal beneficiaries of a deceased worker that inherit the savings funds (11.23% according to the regulatory agency, 1.7% in our sample data). These three cases are subject to the same process described above, but the law specifies some differences for each case.

A.3 Estimation Algorithm

We estimate the model described outlined in Section 4 by using Simulated Maximum Likelihood, starting from Kenneth Train's Matlab code "mxlmsl".[8] We augment this code to incorporate the first stage and to account for the correlation between the first and second stage. Taking D draws, the likelihood function corresponds to:

$$\begin{aligned}\ln L &= \sum_{i=1}^{N} \ln L_i \\ &\sum_{i=1}^{N} \ln \left[\frac{1}{D} \sum_{d=1}^{D} \Pr(\tilde{Y}_i^d = k) \Pr(Y_i^d = jf | \tilde{Y}_i^d = k) \right]\end{aligned}$$

[8]Ww thank Professor Train for his code available online at http://eml.berkeley.edu/ train/software.html

To incorporate the individual heterogeneity in preferences in the second stage through $E(\beta_i)$, and the correlation between the first and second stage through the joint distribution of $(\tilde{\varepsilon}_i^1, \varepsilon_i^1)$, we use two simple "tricks":

1. The distribution of β_i implies that

$$\beta_i = \lambda_0 + \lambda_1 Z_i^{1\prime} + \lambda_2 Z_i^{2\prime} + \Lambda_\beta u_{\beta i}$$

where Λ_β is the Cholesky decomposition of Σ_β and $u_{\beta i}$ is a vector of independent $N(0,1)$ shocks. So far we assume Σ_β is diagonal, therefore Λ_β is also diagonal and contains the standard deviation of each parameter β_i, σ_β. Then, for each draw d, in the second stage the value of a quote j from firm f is given by

$$U_{ijf}^d = X_{ijf}\lambda_0 + X_{ijf}\lambda_1 Z_i^{1\prime} + X_{ijf}\lambda_2 Z_i^{2\prime} + \xi_f + X_{ijf}\sigma_\beta u_{\beta i}^d + \varepsilon_{if}^{1d} + \varepsilon_{ijf}^2 \tag{17}$$

Therefore our model for the second stage can be easily adapted to Train's algorithm by including the interactions between X_{ijf} and $[Z_i^1, Z_i^2]$ with fixed parameters, and adding a random parameter (with positive standard deviation) for X_{ijf}. We also include the normalizations discussed in Section 4.1.1.

2. According to equation (10), the shocks for the first and second stage can be considered as pairs of bivariate normal disturbances:

$$\begin{pmatrix} \tilde{\varepsilon}_{if}^1 \\ \varepsilon_{if}^1 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{1f}^2 & \sigma_{12f} \\ \sigma_{12f} & \sigma_{2f}^2 \end{pmatrix} \right)$$

By the Cholesky decomposition, this implies that there exists numbers (l_{1f}, l_{2f}, l_{3f}) such that:

$$\begin{pmatrix} l_{1f} & 0 \\ l_{2f} & l_{3f} \end{pmatrix} \begin{pmatrix} l_{1f} & l_{2f} \\ 0 & l_{3f} \end{pmatrix} = \begin{pmatrix} \sigma_{1f}^2 & \sigma_{12f} \\ \sigma_{12f} & \sigma_{2f}^2 \end{pmatrix} \tag{18}$$

and therefore

$$\begin{pmatrix} \tilde{\varepsilon}_{if}^1 \\ \varepsilon_{if}^1 \end{pmatrix} = \begin{pmatrix} l_{1f} & 0 \\ l_{2f} & l_{3f} \end{pmatrix} \begin{pmatrix} \tilde{u}_{1if} \\ u_{1if} \end{pmatrix} = \begin{pmatrix} l_{1f}\tilde{u}_{1if} \\ l_{2f}\tilde{u}_{1if} + l_{3f}u_{1if} \end{pmatrix} \tag{19}$$

where again, $\tilde{u}_{1if}$ and u_{1if} correspond to pairs of independent $N(0,1)$ shocks, with:

$$\begin{aligned} l_{1f} &= \sigma_{1f} \\ l_{2f} &= \rho_{12f}\sigma_{2f} = \frac{\sigma_{12f}}{\sigma_{1f}} \\ l_{3f} &= \sigma_{2f}\sqrt{1-\rho_{12f}^2} \end{aligned}$$

where ρ_{12f} is the correlation between the two shocks ($\rho_{12f} = \frac{\sigma_{12f}}{\sigma_{1f}\sigma_{2f}}$).

With this "trick", for each draw d, in the first stage the value of an intermediary k is

$$\tilde{U}_{ik}^d = W_i\alpha_k + W_{ik}\alpha + \sigma_{1k}\tilde{u}_{1ik}^d + \tilde{\varepsilon}_{ik}^2 \tag{20}$$

and in the second stage the value of an offer jf is:

$$\begin{aligned} U_{ijf}^d =& X_{ijf}\lambda_0 + X_{ijf}\lambda_1 Z_i^{1\prime} + X_{ijf}\lambda_2 Z_i^{2\prime} + \xi_f \\ &+ X_{ijf}\sigma_\beta u_{\beta i}^d + \rho_{12f}\sigma_{2f}\tilde{u}_{1if}^d + \sigma_{2f}\sqrt{1-\rho_{12f}^2}\,u_{1if}^d + \varepsilon_{ijf}^2 \end{aligned}$$

This "trick" of including the same draw $\tilde{u}^d_{1if}$ from the value of the first stage into the value of the second stage when $k = f$ –with a different parameter– is the same as using the conditional distribution $\phi(\varepsilon^1_i|\tilde{\varepsilon}^1_i)$ in the second stage.

Note also that the term $\sigma_{2f}\sqrt{1-\rho^2_{12f}}u^d_{1if}$ is equivalent to adding in the code a random parameter to the firm fixed effects ξ_f, with the normalization of $E(\xi_{f'}) = 0$ for a given f'.

With these two tricks, in each simulation a draw d corresponds to a vector of independent $N(0,1)$ shocks $u^d_i = [u^d_{\beta i}, \tilde{u}^d_{1i}, u^d_{1i}]$, where the vector $u^d_{\beta i}$ is used to estimate the heterogeneity in the second stage valuation parameters β_i, the vector $\tilde{u}^d_{1i}$ is used to estimate the heterogeneity in the value of the first stage, and the vector u^d_{1i} is used to estimate the heterogeneity in the value of the second stage.

These two tricks imply that the parameters estimated by the code correspond to the following parameters in our model:

1. $\hat{F}$: fixed parameters of the second stage, correspond to:
   (a) $\hat{\lambda}_1$: effect of the interactions between X_{ijf} and Z^1_i, or the effect of individual characteristics on the mean valuation of the present value of monthly payments.
   (b) $\hat{\lambda}_2$: effect the interactions between X_{ijf} and $Z^2_i = \tilde{Y}^*_i$, or the effect of general biased advice from the intermediary.
2. $\hat{B}$: mean of random coefficients of the second stage, correspond to:
   (a) $\hat{\lambda}_0$: "intercept" of the mean valuation of quotes attributes.
   (b) $\hat{\xi_f}$: to control for endogeneity of prices we include a dummy for each firm f - the mean will correspond to the firm fixed effect. We require the normalization that $E(\xi_{f'}) = 0$ for a given f'.
3. $\hat{W}$: standard deviations of random coefficients of the second stage, correspond to:
   (a) $\hat{\sigma_\beta}$: SD of the valuations for the quotes attributes.
   (b) $\hat{\sigma}_{2f}\sqrt{1-\hat{\rho}^2_{12f}}$: the SD of the firm dummies estimate the conditional variance of the distribution $\phi(\varepsilon^1_i|\tilde{\varepsilon}^1_i)$, to control for selection on unobservables.
4. $\hat{F0}$: fixed parameters of the first stage, correspond to $\hat{\alpha}_k$, the parameters for W_i.
5. $\hat{F0}_{ij}$: fixed parameters of the first stage, correspond to $\hat{\alpha}$, the parameters for W_{ik} to control for selection on observables.
6. $\hat{W0}$: SD of random coefficients of the first stage, correspond directly to $\hat{\sigma}_{1k}$, the standard deviation of $\tilde{\varepsilon}^1_i$. The estimation of $\hat{W0}$ was included simply as the parameter that goes with the draws $\tilde{u}^d_{1ik}$.
7. $\hat{WS}$: the draws for the first stage $\tilde{u}^d_{1ik}$ go directly into the second stage when $k = f$, with parameter $\hat{WS} = \hat{\rho}_{12f}\hat{\sigma}_{2f}$ that estimate the conditional mean of the distribution $\phi(\varepsilon^1_i|\tilde{\varepsilon}^1_i)$, to control for selection on unobservables.

With the estimated parameters for the Cholesky decomposition, it is possible to obtain the bivariate normal distribution that accounts for the correlation between the first and second stage:

$$\hat{\sigma}_{2f} = \sqrt{\hat{WS}_f^2 + \hat{W}_f^2} = \sqrt{l_{2f}^2 + l_{3f}^2} = \sqrt{\hat{\rho}_{12f}^2\hat{\sigma}_{2f}^2 + \hat{\sigma}_{2f}^2(1-\hat{\rho}_{12f}^2)}$$

$$\hat{\rho}_{12f} = \frac{\hat{WS}_f}{\sqrt{\hat{WS}_f^2 + \hat{W}_f^2}} = \frac{l_{2f}}{\hat{\sigma}_{2f}}$$

To obtain the SD of the estimated parameters it is possible to use the Delta Method:

$$g(WS_f, W_f) = \begin{pmatrix} g_1(WS_f, W_f) \\ g_2(WS_f, W_f) \end{pmatrix} = \begin{pmatrix} \sqrt{WS_f^2 + W_f^2} \\ \frac{WS_f}{\sqrt{WS_f^2+W_f^2}} \end{pmatrix}$$

Then, if $\Sigma_{WS,W,f}$ is the covariance matrix of the estimates WS_f, W_f, then the covariance matrix for the parameters of the bivariate normal distribution is given by

$$\nabla g' \Sigma_{WS,W,f} \nabla g$$

with

$$\begin{aligned} \nabla g &= \begin{pmatrix} \frac{\partial g_1}{\partial WS_f} & \frac{\partial g_2}{\partial WS_f} \\ \frac{\partial g_1}{\partial W_f} & \frac{\partial g_2}{\partial W_f} \end{pmatrix} \\ &= \begin{pmatrix} \frac{WS_f}{\sqrt{WS_f^2+W_f^2}} & \frac{W_f^2}{\sqrt{WS_f^2+W_f^2}^3} \\ \frac{W_f}{\sqrt{WS_f^2+W_f^2}} & \frac{-WS_f W_f}{\sqrt{WS_f^2+W_f^2}^3} \end{pmatrix} \end{aligned}$$