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20 October 2015

October 20, 2015 Comments Welcome

*JEP* Symposium on Overconfidence

# Overconfident Investors, Predictable Returns, and Excessive Trading

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# 1 Introduction

The last several decades have seen a shift away from a fully rational paradigm of financial markets towards one in which investor behavior is influenced by psychological biases. One of the main factors contributing to this evolution is a body of evidence showing how psychological bias affects the behavior of economic actors. Another main factor is an accumulation of evidence that is hard to reconcile with fully rational models of security market trading volumes and returns. In particular, asset markets exhibit trading volumes that are high, while individuals and asset managers trade aggressively, even when such trading results in high risk and low net returns. Moreover, asset prices display patterns of predictability that are difficult to reconcile with rational expectations based theories of price formation.

In this paper, we discuss the role of overconfidence as an explanation for these patterns. Overconfidence means having mistaken valuations and believing in them too strongly. It might seem that actors in liquid financial markets should not be very susceptible to overconfidence, because return outcomes are measurable, providing extensive feedback. However, overconfidence has been documented among experts and professionals, including those in the finance profession. For example, overconfidence is observed among corporate financial officers (Ben-David, Graham and Harvey 2013) and among professional traders and investment bankers (Glaser, Langer and Weber 2013). People tend to be overoptimistic about their life prospects (Weinstein 1980), and this optimism directly affects their financial decisions (Puri and Robinson 2007).

We do not mean to suggest that overconfidence is the only phenomenon worth considering in behavioral finance, nor that it should serve as an all-purpose explanation for all financial anomalies. But overconfidence seems likely to be a key factor in financial decision making. Overconfidence is a widespread psychological phenomenon (as discussed by Malmendier and Taylor in their overview for this symposium), and is associated with a cluster of related effects. For example, it includes *overplacement*—overestimation of one's rank in a population on some positive dimension—and *overprecision*—overestimation of the accuracy of one's beliefs. An example is overestimation of one's ability to predict the stock market's future returns. A cognitive process that helps support overconfident beliefs is *self-attribution bias*, in which people give credit their own talents and abilities for past successes, while blaming their failures on bad luck.

To evaluate the importance of overconfidence for financial markets, we proceed as follows. We start by reviewing two of the primary financial market anomalies at odds with rational agent asset pricing theories: the arguments that trading volumes are excessive and the evidence that security returns are in some ways predictable. We then sketch a sequence of models of investor trading and security prices that include various aspects of overconfidence, with increasing complexity, and discuss the empirical implications of each of these models. We hope that this presentation will clarify which aspects of the model are important in delivering specific empirical implications. Finally, we offer some conclusions about how overconfidence contributes to our understanding of financial markets.

# 2 Evidence on Trading Patterns and Return Predictability

The notion of market efficiency, as explained in Fama (1970), is based on the idea that when investors in frictionless asset markets compete with one another, securities will be correctly priced to fully reflect all publicly available information. More generally, rationality on the part of investors has some strong implications.

With surprisingly mild theoretical assumptions, one can show that rational individuals should not agree to disagree. Intuitively, if we start with the same prior beliefs, yet now we disagree, this suggests that at least one party has information that the other party should be taking more fully into account (Aumann 1976). In a similar spirit, rational investors should not place bets with each other; the fact that another investor is willing to take the opposite side of my trade should suggest to me that this investor knows something I do not know (Grossman 1976, Milgrom and Stokey 1982, Tirole 1982). For this reason, leading rational frictionless models of asset pricing—at least in their most simple versions—imply that after a single round of trading everyone should hold the market portfolio. Investors should not bet against each other, each expecting to beat his counterparties. However, we clearly observe high volumes of trade in financial markets.

Moreover, in an efficient market, a trading strategy based on existing information cannot be used to earn abnormal profits. If such trading strategies do exist there is a *return anomaly*: such opportunities suggest either that rational agents are not fully exploiting available profit opportunities, or that risk aversion or market frictions constrain their ability to do so. However, it is now a well-accepted empirical finding—even by those who adhere to a rational-actors explanation—that asset markets do display strong patterns of return predictability. This finding poses a challenge to the hypothesis that investors are rational, because it suggests that investors are making mistakes: they are throwing away money buying overpriced securities that subsequently do poorly, and are missing out on buying underpriced securities that subsequently do well. An alternative explanation for return predictability is that it results from some kind of risk premia—risky assets predictably return more than less risky ones. This explanation then raises the question of whether plausible levels of risk aversion are high enough to explain the size of the predictability, a question we address below.

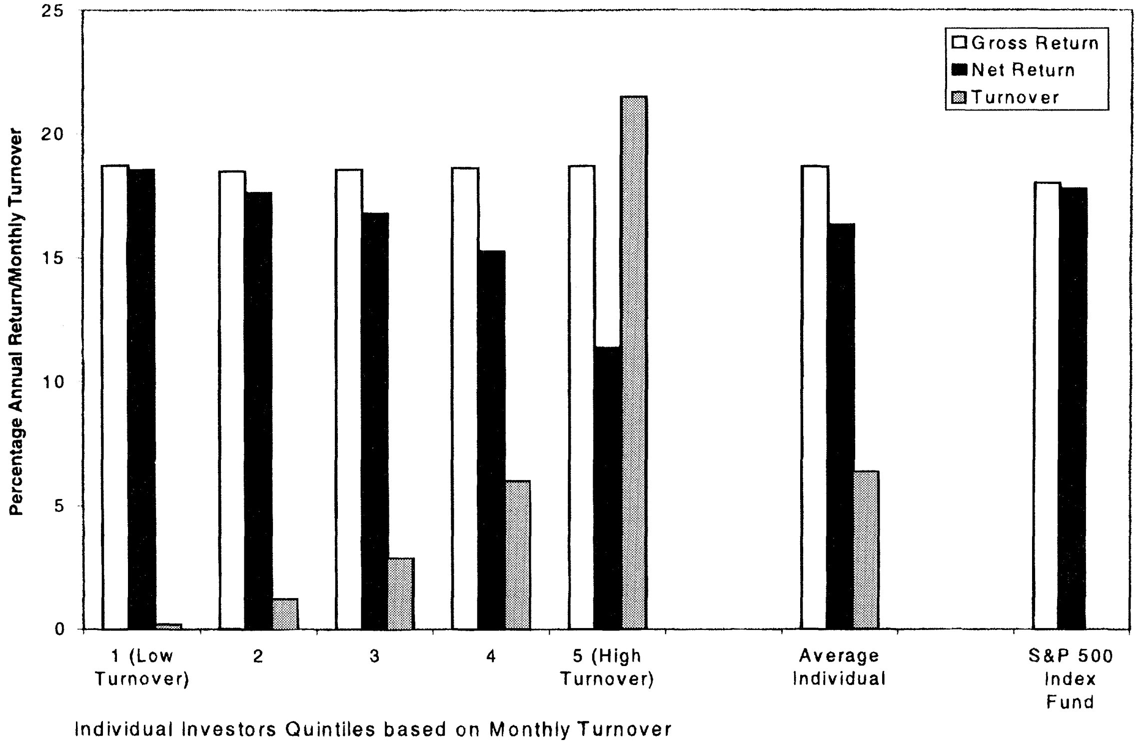
In this section, we will explore the evidence on high trading volumes and predictable returns in greater depth and discuss how overconfidence-based explanations provide some insight into these patterns

## 2.1 Disagreement, Speculative Trade, and Trading Volume

A financial trade requires that two parties agree to disagree, in the sense that at a given price one party believes it is a good idea to sell the asset while the other party believes it is a good idea to buy it. Of course, there are possible reasons for informed agents to trade other than disagreement, such as liquidity motives (such as sending a child to college), or to rebalance to achieve a more diversified portfolio (for example, after a shock to one's labor income or human capital). Speculative trade can arise in rational models if investors in securities markets are periodically required to sell or buy securities as a result of liquidity shocks. Several models starting with Grossman and Stiglitz (1976) have shown that if there are random shocks to security supply (designed to capture the idea that there are investors whose need to cash out of their positions is unpredictable to others) this can add enough noise to make room for some speculative trading.

But such motives for trade are relatively limited, and do not seem to explain the magnitudes of trade, or the willingness of investors to incur the large transaction costs that they sometimes need to pay to make such trades. The total volume of trade in financial markets is vast. Over the period 1980-2014, the annualized average turnover for the 500 largest US stocks has averaged 223 percent, or just over $100 billion per day. Over the year 2014, the total dollar trade in these top 500 stocks was $29.5 trillion (Collin-Dufresne and Daniel 2014)—nearly double the US GDP. Trade in foreign exchange is even larger. Froot and Thaler (1990) report that, as of 1989, average trading in the foreign exchange market was about $430 billion per day as compared to daily US GDP of $22 billion and daily trades in goods and services of $11 billion.

Figure 1: **Monthly Turnover and Annual Return of Individual Investors**




*Source:* Barber and Odean (2000), Figure 1. The white bar (black bar) represents the gross (net) annualized geometric mean return for February 1991 through January 1997 for individual investor quintiles based on monthly turnover (grey bar). The net return on the S&P 500 Fund is that earned by the Vanguard Index 500.

Theories about rational traders reacting to liquidity shocks don't seem sufficient to explain the magnitudes of financial trades that we observe, or the patterns in trading volume. Rather, several findings point to overconfidence as a likely explanation. Everyday experience suggests that there is considerable disagreement across individuals in the economy, with each individual believing that he or she is correct. In overconfidence-based models, investors who are overconfident form judgments about the value of a security that put too much weight on their own views, and insufficient weight on the views of other investors (as reflected in the security's price). As a result, overconfident investors expect high profits from trading on their opinions..

The excessive trading of individual investors can be called *the active investing puzzle.* Individual investors trade individual stocks actively, and on average lose money by doing so. The more actively investors trade, the more they typically lose (Odean 1999). In particular, Barber and Odean (2000) find that in a sample of trades of 78,000 clients of a large discount brokerage firm from 1991-1996, some households trade much more than others. The turnover and gross- and net-returns to the clients in different turnover quintiles are summarized in

Figure 1, reproduced from their paper. The gray bars give the average monthly turnover of the accounts in each quintile. Strikingly, the average monthly turnover in the fifth quintile is over 20 percent per month. The white bars give gross returns (that is, without accounting for the costs associated with trading) and show that, across quintiles, there is little variation in average gross returns. However, the black bars show that the net returns are quite different. The high-turnover investors pay large fees, given their high volume of trade, which drives down their net returns. The net returns of all quintiles except the lowest are lower than the net return from investing in a Standard & Poor's 500 index fund.

Tests that aggregate across individual investors also find that the stocks that individual investors buy tend to subsequently underperform. Investor losses can be astonishingly large; in the aggregate, the annual losses of Taiwanese individual investors amount to 2.2 percent of Taiwan's gross domestic product and 2.8 percent of total personal income (Barber, Odean and Zhu 2009). In experimental markets as well, some investors overestimate the precision of their signals, are more subject to the winner's curse, and have inferior trading performance (Biais et al. 2005). Greater ease of trading gives investors free rein to harm themselves by more aggressive trading, as occurred with the rise of online trading (Barber and Odean 2002, Choi, Laibson and Metrick 2002). A similar point applies to individuals who invest in active mutual funds instead of index funds for better net-of-fees performance. Indeed, the existence of actively managed mutual fund that charge high fees without providing correspondingly high gross performance provides evidence that a number of individual investors are overconfident about their ability to select the high-performing active fund managers (French 2008, Malkiel 2013)

A range of evidence from a wide variety of sources suggests that overconfidence provides a natural explanation for the active investing puzzle, because it causes investors to trade more aggressively even in the face of transactions costs or adverse expected payoffs (Odean 1998). In one of the rare studies of investor trading that measures overconfidence directly, Grinblatt and Keloharju (2009) associate the trading behavior of Finnish investors with the results of a psychometric test given to all Finnish males at age 19 or 20. The study finds that overconfident investors (as well as investors who are prone to sensation seeking) trade more often. In a different study consistent with overconfidence as an explanation for the active investing puzzle, Kelley and Tetlock (2013) construct a structural model of market trading which includes informed rational investors as well as uninformed investors who trade either for hedging reasons, or to make an (overconfident) bet on perceived information. They estimate this model using a dataset on trades, prices and information releases for US traded firms, and conclude that, without overconfidence-based trading, volumes would be smaller by a factor of 100. Finally, motivated by psychological evidence that men are more overconfident than women in decision domains traditionally perceived as masculine, such as financial matters, Barber and Odean (2001) compare the trading behavior and performance of men and women. Consistent with higher confidence, the average turnover for accounts opened by men is about 1.5 times higher than accounts opened by women, and as a result men pay 0.94 percent per year in higher transaction costs. The gross (benchmark-adjusted) returns of the men in the sample are lower, though this difference is not statistically significant. As a result, the net-of-fees returns of men are far lower.

Other aspects of investor trading behavior are also consistent with overconfidence and the psychological processes that accompany it. Individual investors tend to trade more after they experience high stock returns. For example, early adopters of online trading tended to make the switch after unusually good personal performance, and subsequently traded more actively (Barber and Odean 2002, Choi, Laibson and Metrick 2002). This connection may help to explain why stock market trading volume increase after high returns, as has been documented in a large number of countries (Griffin, Nardari and Stulz 2007). For example, annualized turnover in US common stocks was at levels of over 100 percent late in the bull market of the 1920s, fell through the 1930s and 1940s, and then rose dramatically from the 1990s up through the financial crisis of 2007-2008 (Collin-Dufresne and Daniel 2014). Statman, Thorley and Vorkink (2006) find that US market turnover is positively correlated with lagged monthly market returns, and that turnover of individual securities is positively associated with lagged market turnover (after controlling for past values of turnover and returns in each security).

How can these patterns of overconfidence and high turnover persist over time, despite the high risks and costs they impose upon investors? Overconfidence in general is supported by bias in self-attribution, as modeled by Daniel, Hirshleifer and Subrahmanyam (1998) and Gervais and Odean (2001); investors who have experienced high returns attribute this to their high skill, and become more overconfident, while investors who experience low returns attribute it to bad luck, rather than experiencing an offsetting fall in their overconfidence level.

Overconfidence is likely to be especially important when security markets are less liquid, and when short-selling is difficult or costly. When short-selling is constrained, pessimists about a stock find it harder to trade on their views than optimists. If some of the optimists do not adequately take into account that pessimists are sidelined by short-sale constraints, the optimists will overvalue the stock, resulting in equilibrium overpricing. Thus, when overconfidence is combined with short sales constraints, we expect the security to become overpriced (Miller 1977).

Motivated by this hypothesis, Diether, Malloy and Scherbina (2002) document that firms for which the analysts disagree more—measured by the dispersion in analysts' forecasts of the firm's future earnings—on average earn lower returns. This finding is usually interpreted as evidence that investor disagreement matters; overconfidence provides a natural explanation for *why* disagreement exists and matters. Because volatility creates greater scope for disagreement, this approach also suggests overpricing of more volatile stocks. Consistent with this insight, Ang *et al.* (2006, 2009) and Baker, Bradley and Wurgler (2011) show that high idiosyncratic-volatility stocks earn lower subsequent returns than low volatility stocks. This hypothesis is also consistent with the finding that stocks and other assets with high systematic risk (i.e. high market beta) typically earn too low a return premium relative to the risk-return tradeoff implied by equilibrium models such as the Capital Asset Pricing Model (Black, Jensen and Scholes 1972, Frazzini and Pedersen 2014),

During the high-tech boom at the turn of the millennium, episodes of strong disagreement in which, remarkably, the market value of a parent firm was sometimes substantially less than the value of its holdings in one of its publicly-traded divisions (Lamont and Thaler 2003). Such patterns reflected the fact that an optimistic set of investors were excited about the prospect of a glamorous division, and the relatively pessimistic investors who were setting the price of the parent firm found it too costly or troublesome to short-sell the glamorous division to bring its price in line with that of the parent. Also consistent with overvaluation induced by investor disagreement, stocks with tighter short-sale constraints have stronger return predictability (Nagel 2005). Such asymmetry between the long and the short side of return anomalies is especially strong during optimistic periods, when overvaluation is most severe (Stambaugh, Yu and Yuan 2012).

Overconfident disagreement, combined with short sale constraints, can also cause dynamic patterns of increasing overpricing. Building on Harrison and Kreps (1978), Scheinkman and Xiong (2003) present a model in which overconfidence generates disagreement among agents regarding asset fundamentals. Owing to short sale constraints, investors buy stocks that they know to be overvalued in the hope of selling at even higher prices to more optimistic buyers. This magnifies the pricing effects of disagreement. Such bubbles should be more severe in markets with lower available supply of shares ("float") (Hong, Scheinkman and Xiong 2006), as seems to have occurred during a bubble in Chinese warrants (Xiong and

Yu 2011).

Although overconfidence causes problems in markets, it brings some benefits, as well. Overconfidence can induce investors to investigate more, and/or to trade more aggressively based on their signals. This sometimes results in greater incorporation of information into price (Hirshleifer, Subrahmanyam and Titman 1994, Kyle and Wang 1997, Odean 1998, Hirshleifer and Luo 2001). Furthermore, overconfidence encourages investors to participate in asset classes, such as the stock market or international investing, that they might otherwise neglect (owing to concerns such as fear of the unfamiliar). Empirically, a greater feeling of competence about investing is associated with more active trading and with greater willingness to invest in foreign stock markets (Graham, Harvey and Huang 2009).

## 2.2 Return Predictability

Here, we lay out the documented patterns in return predictability that are at odds with the efficient markets hypothesis and potentially attributable to overconfidence. We first concentrate primarily on the nature and direction of the patterns, as opposed to their magnitudes. Of course, it is possible that the abnormal returns generated by "anomaly portfolios" based on patterns of predictable returns are not anomalous at all. A strategy may earn high returns relative to some benchmark by virtue of exposure to some systematic risk factor that the benchmark does not capture. (A factor in the asset pricing literature refers to a statistical source of common variation in security returns—usually the return on a portfolio. For example, the returns of individual stocks can be explained in part by realizations of the stock market as a whole, as is verified by regressing stock returns on the market portfolio.) In the next subsection, we will argue that the large premia earned by a combination of these anomaly-based strategies is too large to be explained plausibly in this way. We consider evidence on return predictability of three types: (1) predictability based on the market price of the firm, scaled by measures of fundamental value; (2) predictability based on a recent history of past returns (momentum and reversal); and (3) predictability based on underreaction to, or neglect of, public information about fundamentals.

One of the earliest anomalies uncovered in academic research was the *size anomaly* (Banz 1981, Keim 1983)—the phenomenon that "small" firms, defined in terms of low-market-capitalization, earn higher returns than large firms. Even stronger predictability is obtained when scaling the firm's market capitalization by a measure of the firm's fundamental value. Fama and French (1992) find that the *book-to-market* ratio—that is, the book-value of equity, scaled by the firm's market capitalization—predicts returns. In particular, so-called "value firms" with high book-to-price ratio firms substantially outperform "growth firms" with low book-to-price ratios. Many other fundamental-to-price measures, including earnings-to-price, sales-to-price, and cash-flow-to-price ratios, also positively forecast future returns (Lakonishok, Shleifer and Vishny 1994).

A pattern of long-term price reversal (DeBondt and Thaler 1985) can also be understood as related to the fundamental-to-price ratio. Intuitively, a stock that is mispriced now probably did not share the same mispricing years ago. Daniel and Titman (2006) add an additional dimension to this point; if past long-term returns are decomposed into a component associated with public-information and an orthogonal component, a long-term reversal of prices is only observed for the orthogonal component. The component of the past return associated with public information does not reverse.

*post-earnings announcement drift* or *earnings momentum* is the phenomenon in which firms that announce high earnings relative to forecasts, or whose price jumps up on an announcement date, tend to earn high returns over the subsequent 3-6 months (Bernard and Thomas 1989, Bernard and Thomas 1990). *Price momentum* is the tendency for returns over the past 3-12 months to continue in the same direction in the future in many asset classes. A related but distinct phenomenon is The overconfidence explanation for momentum involves a pattern of continuing overreaction and slow correction.

More specifically, price momentum in the US stock market has several key features. First, it is predominantly associated with lagged price changes that can be attributed to public information releases. In contrast, price changes that cannot be associated with news tend to exhibit reversal rather than continuation (Chan 2003, Tetlock 2011). Second, in the long run momentum tends to reverse (Griffin, Ji and Martin 2003, Jegadeesh and Titman 2011). Third, momentum effects are weak for value stocks, but strong for growth stocks (Daniel and Titman 1999). Fourth, momentum strategies generate especially strong returns in calm periods when the past return on the market is high (Cooper and Hameed 2004, Daniel and Moskowitz 2015), but exhibit strong negative skewness and earn lower returns in turbulent (high volatility) bear markets (Daniel and Moskowitz 2015, Daniel, Jagannathan and Kim 2015).

Asness, Moskowitz and Pedersen (2013), among many others, document strong value and momentum anomalies in non-US data, and in other asset classes including currencies, commodity futures and government bonds. Moskowitz (2015) shows that the same momentum and value/reversal patterns observed in other asset classes are also present in sports betting venues. Sports betting markets are a useful test-bed for overconfidence-based theories because the outcomes of these contests are unlikely to be interdependent with other economic outcomes that may affect the marginal utility of individuals. Moskowitz argues that the presence of value and momentum effects in sports betting markets is consistent with delayed overreaction theories of asset pricing. Consistent with the model in Daniel, Hirshleifer and Subrahmanyam (2001), he finds that higher ambiguity predicts stronger momentum and value returns, consistent with what is observed in financial markets. .

Many items reported in financial statements can be useful in forecasting the future earnings, but investors do not appear to make full use of such information. One prominent example is "operating accruals," which are the accounting adjustments made to a firm's cash flows to obtain earnings, a standard measure of profitability. Such adjustments may include sales transactions whose payments have not yet arrived or expense transactions for which actual payments have not yet been made. Sloan (1996) shows that market prices don't fully reflect the extent to which earnings arise from cash flows or accruals.

A common pattern in event studies is continuation of the event-date return, so that events that are on average good news experience high subsequent returns, and the opposite for bad news events (see the summary in Hirshleifer (2001)). For example, the issuance of new securities tends to convey bad news about future cash flows, while the repurchase of existing securities tends to convey good news. Consistent with return continuation, repurchases tend to be followed over a long period by high returns (Ikenberry, Lakonishok and Vermaelen 1995), and equity and debt issues in many countries by negative abnormal returns (Loughran and Ritter 1995, Spiess and Affleck-Graves 1995, Henderson, Jegadeesh and Weisbach 2006). Daniel and Titman (2006) and Pontiff and Woodgate (2008) develop more comprehensive measures of share issuance over a given time period, and show that lagged measure of issuance strongly forecast returns. At the aggregate level as well, the share of equity issues in total new equity and debt issues has been a negative predictor of US stock market returns (Baker and Wurgler 2000).

## 2.3 Return Predictability—Magnitudes

The return patterns documented in the preceding section might reflect certain kinds of rational risk premia, rather than mistakes or biases on the part of investors. Here, we summarize evidence on the risk and rewards of strategies based upon these effects to see if this explanation is plausible. A portfolio which simultaneously exploits several the patterns of return predictability documented in the preceding section generates an exceptionally high reward-to-risk ratio. Using insights from Hansen and Jagannathan (1997), accommodating these premia within any frictionless rational expectations model would require extreme (and we will argue, unrealistic) variation in investor marginal utility across states of the world.

We start with a set of seven "zero-investment" portfolios designed to capture the return predictability patterns described in the preceding section.

First, the "Small Minus Big" portfolio, or SMB, proposed by Fama and French (1993), captures the difference in average returns between small and large market-capitalization firms. This portfolio, at the beginning of each month, takes a long position in \$1 worth of small-market-capitalization stocks, financed by taking a short position in \$1 worth of large-market capitalization stocks. Historically, investors should have been able capture the returns of this *zero-investment* or *\$1-long/\$1-short* portfolio with minimal transaction costs (despite the need to sell short). This portfolio has been used in numerous academic studies, and yearly, monthly and daily returns from 1926 on are available on Kenneth French's website.

Second, the "High minus Low" or HML portfolio is formed to exploit the persistently higher returns of stocks with high book-to-market ratios and those with low book-to-market ratios. The portfolio involves buying value stocks—stocks with ratios of book-value of equity to market-value of equity in the top 30 percent of all stocks on the New York Stock Exchange—and shorting growth stocks, with book-to-market ratios in the bottom 30 percent.

Third, the "Up minus Down" or UMD portfolio is a price momentum portfolio (Carhart (1997), Fama and French (1993)). It is formed by buying stocks that rose in price in the previous time period (often 12 months) and taking a short position in stocks that declined in price in the previous time period. Thus, it is based on momentum in stock prices.

Fourth, the "ISsUance" or ISU portfolio buys a value-weighted portfolio of firms that over the preceding three years repurchased stock, and shorts a portfolio of stocks that issued new equity, based on the Daniel and Titman (2006) measure.

Fifth, the "ACcRual" or ACR portfolio goes long a portfolio of firms which had the lowest the ratio of accruals to earnings over the past year, and goes short on the firms which had the highest accruals.

Sixth, the "Betting-Against-Beta" or BAB portfolio is constructed following the description in Frazzini and Pedersen (2014). The long side of the portfolio is a leveraged portfolio of low-beta stocks. The portfolio takes a short position in high-beta stocks.

Finally, the "Idiosyncratic-VOLatility" or IVOL portfolio each month takes a long position in the set of firms that had the lowest idiosyncratic volatility of daily returns over

Table 1: **Anomaly-Based Strategy Sharpe Ratios**

This table presents the realized *ex-post* optimal strategy Sharpe-ratios from 1963:07-2014:05 for a set of long-short portfolios based on a set of anomalies taken from the finance literature: Mkt-Rf, SMB, HML are the three Fama and French (1993) portfolios; UMD is the Carhart (1997) price momentum portfolio; "ISsUance" (ISU) and "ACcRual" (ACR) are long-short portfolios based on the Daniel and Titman (2006) cumulative issuance and Sloan (1996) accruals measures, respectively; BAB is the Frazzini and Pedersen (2014) Betting-Against-Beta portfolio; and finally IVOL is the Ang et al. (2006) idiosyncratic-volatility portfolio.

| Portfolio Weights (%) | | | | | | | | Sharpe |
|---|---|---|---|---|---|---|---|---|
| Mkt-Rf | SMB | HML | UMD | ISU | ACR | BAB | IVOL | Ratio |
| 100.0 | – | – | – | – | – | – | – | 0.39 |
| 34.9 | 18.7 | 46.4 | – | – | – | – | – | 0.76 |
| 25.8 | 10.5 | 33.0 | 30.7 | – | – | – | – | 1.07 |
| 8.0 | 4.5 | 33.9 | 17.7 | 26.8 | 9.1 | – | – | 1.37 |
| 7.7 | 12.4 | 13.8 | 4.5 | 18.0 | 10.2 | 9.5 | 24.0 | 1.78 |

the preceding one month, and shorts the highest idiosyncratic volatility stocks, measured following the procedure specified in Ang et al. (2006).

In working with these portfolios, remember that the "Sharpe Ratio" of a portfolio is the ratio of its reward to its risk. More specifically, we define it here to be the ratio of the annualized excess return on the portfolio to the annualized return standard deviation of the portfolio. To summarize how an investor might exploit these anomalies, it is useful to examine the Sharpe ratios achieved by combining the anomaly portfolios into super-portfolios.

Table 1 presents Sharpe ratios for portfolios consisting of the US market portfolio—specifically the Center for Research in Security Prices value-weighted index return—along with various mixtures of the seven candidate anomaly portfolios.[1] Each row of Table 1 represents a different combination of the set of anomaly portfolios designed to achieve a high Sharpe ratio. The first eight columns show the weights on each of the anomaly portfolios, and the number in the ninth (and last) column gives the annualized Sharpe ratio of the overall portfolio that combines them. The component portfolios are normalized so that each of has the same volatility over the 1963:07-2014:05 sample period. Thus, the weights given in the table are proportional to the volatility of that component.

The first row of the table shows that during this sample period, a portfolio that was

[1] Mkt-Rf is the notation used by Fama and French (1993) for the excess return of the CRSP value weighted index, relative to the 1-month US treasury-bill return in the same month.

100 percent invested in the market index (Mkt-RF) experienced an annualized Sharpe ratio of 0.39. Specifically, the annualized return, net of the one-month Treasury-bill rate, was 6 percent, and the annualized volatility was 15.5 percent. The second row shows how much an investor could have improved on the market Sharpe Ratio by also investing in the size-based SMB and value-based HML portfolios. The optimal combination of these three portfolios results in a Sharpe ratio of 0.76, a vast improvement relative to the market portfolio on its own. The next few lines of the table show that the ability to invest in the momentum factor brings the Sharpe ratio up to 1.07, and the ability to invest in the issuance and accrual portfolios brings it up further to 1.37. Finally, if the investor had been free to invest in any of these eight portfolios, and knew beforehand the distribution of returns over this period (not the returns themselves but only the distribution), that investor could have earned a Sharpe ratio of 1.78, more than *four times* higher than that of the market.

The numbers presented in this table are the Sharpe ratios for the optimal portfolios, calculated as if investors knew up front the realized distribution of returns. But our main conclusions still apply if investor do not have full foreknowledge of distributions. For example, an equal-weighted combination of the eight portfolios (weights which do not require assumptions about future performance of an of the portfolios) earns an annualized Sharpe-ratio of 1.54. Similarly, Asness, Moskowitz and Pedersen (2013) document that a 50/50 combination of only the value and momentum portfolios, but diversified across different regions and asset classes, produces an annualized Sharpe ratio of 1.59.

Any asset-pricing model—whether rational or behavioral—needs to explain why investors are apparently passing up these very high-return, low volatility investments. In a rational expectations setting, asset premia arise only when the asset's returns are risky, meaning that returns are high when the investor is relatively rich (and marginal utility of wealth is low) and are low when the investor is poor (and marginal utility is high). To explain the such a large Sharpe ratio, marginal utility must be quite variable. The Hansen and Jagannathan (1991) bound shows that, to explain the existence of a portfolio with a Sharpe-ratio of 1.8 requires that the annualized standard deviation in marginal utility growth be almost as large or larger—that is, greater than approximately 170 percent. Both casual observation and macroeconomic data suggest that marginal utility growth does not vary nearly this much. For example, the annualized volatility of aggregate US consumption growth is 100 times smaller (1.8 percent). Also, the macroeconomics profession is still with the *equity premium puzzle*—the finding that the Sharpe ratio of the equity market portfolio, which is about 0.4 (annualized), is so high relative to the low volatility of consumption growth (Hansen and

Singleton 1983, Mehra and Prescott 1985, Weil 1989), which seems to imply that the return to the US equity market portfolio is much higher than can be justified by its riskiness. The far higher Sharpe ratio associated with these anomaly portfolios is even harder to reconcile with a rational investor model.

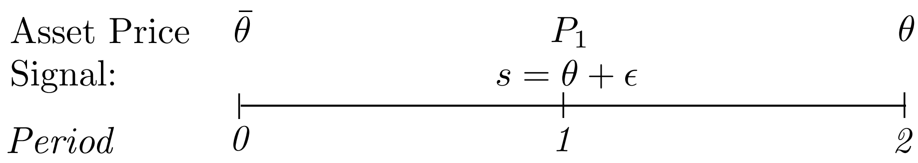
Perhaps an answer to these puzzles can be found in trading frictions that make it costly for rational investors to trade to exploit perceived profit opportunities. However, the magnitude of such frictions, as captured by bid-ask spreads, is too small to explain why investors would forego the combination of return and risk described here. For moderate-sized trades in large firms, such as those used to construct the zero-investment portfolios described here, such spreads are relatively small. Alternatively, maybe these results arise from data mining, and if one looked at different time periods, or a limited set of these portfolios, or weighted the portfolios differently, then the pricing anomalies would disappear. One can tinker with different time periods, or different portfolios, or different weights. But the opportunities presented by these anomaly portfolios appear robust.

What other theories can explain the patterns in Table 1? Could it be that the decision processes or beliefs of investors are biased in ways that induce the seven pricing anomalies listed earlier? Overconfidence-based models suggest that the answer is "yes." In these models, investors continue to optimize, but do so based on incorrect beliefs about the state probabilities. Under this explanation, investors think that the state probabilities are such that the expected returns of the anomaly portfolios are not abnormally high, despite the evidence in Table 1. This explanation need not presume that all investors are overconfident. There could still be rational investors who correctly perceive the high available Sharpe ratios, but if such investors are relatively small in number, and capital constrained, their trading to exploit the profit opportunities will not fully eliminate them. How might overconfidence generate the anomalies that underlie Table 1, so that overconfident investors do not believe that these portfolios outperform? In the next section, we lay out overconfidence models that can potentially explain these patterns.

# 3 Overconfidence-Based Models of Asset Price Formation

In the standard frictionless rational expectations framework, investors process information perfectly. Thus, asset prices are always equal to rationally discounted expected cashflows,

Figure 2: **Model 1: Basic One-Signal Model—Timeline**




This figure illustrates the timeline for the basic three-date, one-signal model discussed in Section 3.1. At time 0, the investor knows only the prior distribution from which the time 2 final payoff is drawn. At time 1 the investor observes a noisy signal $s$. At $t = 2$ the payoff $\theta$ is revealed.

where discount rates are equal to rational expectations of returns. Investors earn returns that are, on average, exactly what they expect.

As discussed in the previous section, so-called zero investment portfolios constructed to reveal anomalies have produced high Sharpe ratios—high average excess returns with low volatility—and which have low correlation with macroeconomic shocks that might plausibly represent risk. Thus some researchers have turned to behavioral models in an attempt to explain these patterns. The behavioral models rely on either non-standard preferences, or biased beliefs.

In models with non-standard preferences, investors correctly expect that high excess returns are achievable with these anomaly-based portfolios. In these models investors choose not to invest more in these portfolios because they find certain kinds of risk extraordinarily painful to bear. In contrast, biased belief models posit that investors make mistakes in the way that they form expectations about asset payoffs. Overconfidence-based models fall into this category.

We now provide a sequence of models that illustrate some insights of the overconfidence-based approach. The first model is a bare-bones setting which captures the fact that an overconfident investor overreacts to a signal that is perceived as private, resulting in overreaction and correction, consistent with evidence of long-run return reversals. We then present models that show how refinements to this basic model, grounded in the psychological evidence on overconfidence, can plausibly generate other anomalies described above.

## 3.1 Model 1: One Signal

Consider a static overconfidence model, which involves a three-date, one-signal example. Figure 2 provides a timeline. For the moment, assume that the overconfident representative investor in the model is risk neutral. There are three dates, $t \in \{0, 1, 2\}$, and two securities:

a riskfree asset with a riskfree rate of zero, and a security which will pay an uncertain liquidating dividend $\theta$ at time 2. The prior distribution for $\theta$ is known. At $t = 1$, the investor receives a private signal of the form $s = \theta + \epsilon$. In a representative agent model, signals cannot of course be truly private, but the model can be viewed as one where there is also a very small mass of investors who do not receive the signal. Also, labeling this signal as "private" captures this idea that this agent believes that she has used her skill to process information and generate new and unique insights about the payoff $\theta$. The psychology literature suggests that agents will be more overconfident about these "private" signals than they will be about "public" information such as earnings announcement that are more easily translated into estimates of future firm value. The date 1 price in this setting is a weighted average of the prior expectation and the signal, with relative weights proportional to the investor's perceived precisions of the prior and of the signal.

We are interested in whether the asset return is forecastable. If the investor is rational (not overconfident) then the price $P_1$ is equal to the rational expectation of the payoff $E[\theta]$, and in this case the price change from date 1 to date 2 is unforecastable—that is, it is not correlated with any price change from time 0 to time 1, nor with the signal received in time 1. The information the signal provides for fundamental value is correctly impounded into price at date 1, and the market is efficient.

However, if an investor overestimates the precision of the signal, that the investor will overreact. Thus, a positive signal will cause the date 1 price to be too high, resulting in too high a price change between dates 0 and 1. On average the price then falls back between dates 1 and 2, which is a pattern of return reversal. In contrast, if the investor underestimates the precision of the signal, the date 1 price will underreact, and the subsequent price change will on average be positive again for a second period, which would be a case of return momentum.

What might cause the investor's estimate of the signal precision to differ from the true precision? One possible answer is investor overconfidence. In the simplest version of the Daniel, Hirshleifer and Subrahmanyam (1998) model, the representative investor observes only a private signal and is overconfident about that signal, resulting in price reversal. Alternatively, in Eyster, Rabin and Vayanos (2013), investors make a different error; they fail to infer fully the private signals received by other investors from the price. Effectively, the representative investor underestimates the information implicit in price—namely , the precision of the aggregate private signal. In consequence, the investor underreacts to this signal, which implies that in equilibrium price underreacts to new information. Their model implies price momentum, but because this is a result of pure underreaction to information,

Figure 3: **Model 2: Separate Public and Private Signals – Timeline**

| | | | | |
|---|---|---|---|---|
| Asset Price | $\bar{\theta}$ | $P_1$ | $P_2$ | $\theta$ |
| Signal: | | $s_V = \theta + \epsilon_V$ | $s_B = \theta + \epsilon_B$ | |
| *Period* | *0* | *1* | *2* | *3* |

This figure presents the timeline for the four-date, two-signal model discussed in Section 3.2. Now, the investor observes distinct pri$V$ate and pu$B$lic signals $s_V$ and $s_B$ at $t = 1$ and $t = 2$, respectively. At $t = 3$ the asset payoff $\theta$ is revealed.

there is no reversal.

Neither pure underreaction nor pure overreaction, as reflected in Model 1, fully captures the return predictability evidence discussed earlier, in which there is momentum at shorter horizons and reversal at longer horizons. In addition, Model 1 does not allow for public information signals prior to the terminal date, and therefore does not allow consideration of whether returns can be predicted based on public information such as the news of a new equity issue by the firm. To capture these patterns, we need to move to a richer model.

## 3.2 Model 2: Public and Private Signals

In Model 2, we introduce separate public and private signals. This model is the "static-overconfidence" model of Daniel, Hirshleifer and Subrahmanyam (1998). The timeline for Model 2 is given in Figure 3. There are now four dates, and two signals: $s_V$ is a priVate signal, and $s_B$ is puBlic. The investor is overconfident, and therefore overestimates the precision of the private signal at time 1. However, the investor correctly estimates the precision of the public signal and the prior.

This approach delivers several additional features. First, as in Model 1, the market overreacts to the private signal, therefore the price change from time 1 to time 3 is in the opposite direction of the price change from time 0 to time 1. In addition, the market underreacts to the public signal: that is, price changes are autocorrelated, $cov(R_{2,3}, R_{1,2}) > 0$. Given this positive return autocorrelation, it is tempting to jump to the conclusion that following the public release of good news at date 2, the share price will continue rising between dates 2 and 3, but his turns out to be incorrect. Intuitively, consider the rationally updated expectation of the fundamental $\theta$ conditional on the public signal. We want to see if, on average, the date 2 price differs from this expectation. If so, the public signal can be used to predict the subsequent return. For example, assuming that the precisions of the prior and public signal are equal, the prior is 0, and the public signal is 100, then the rational updated expectation of the payoff will be 50. On average, the unbiased private signal will 50 (the expected fundamental plus mean zero noise). So even though the private signal is overweighted relative to the public signal in market price, there is no mispricing *on average*, conditional on the public signal. On average the private signal has zero effect on the expectation, which is a weighted average of 50. So on average there is no conditional mispricing.[2]

To explain the evidence that share prices underreact to corporate announcements documented earlier, a further refinement is needed. Suppose that a good- or bad-news public signal is an event chosen by the firm or some other party in opposition to the private signal. For example, perhaps the firm announces a new equity issue—a bad news event—when the firm is overvalued (that is, overconfident investors received a positive private signal). There is evidence that firms that issue equity are indeed overvalued (Loughran and Ritter 1995, Dong, Hirshleifer and Teoh 2012). In a similar way, evidence suggests that firms engage in repurchase—a good news event—in response to undervaluation (Ikenberry, Lakonishok and Vermaelen 1995). We call such public signals "selective." To the extent that public signals are selectively undertaken in opposition to preexisting mispricing, such signals will show return continuation, wherein the long-run return after the event is on average of the same sign as the initial market reaction to the event. This implication is consistent with the strong performance of the ISU (issuance) portfolio described earlier.

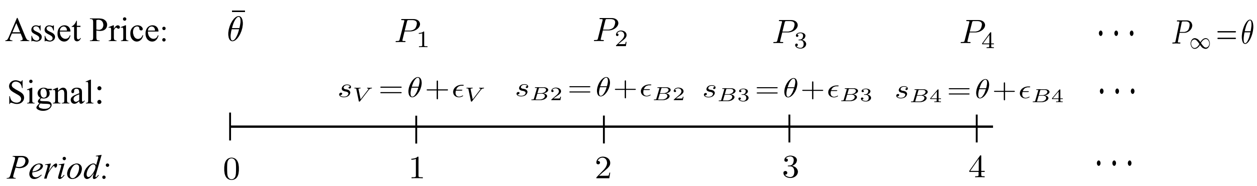
However, Model 2 still does not deliver the key empirical predictions that there will be both medium-term price momentum (Jegadeesh and Titman 1993) and long-term reversal (DeBondt and Thaler 1985). To deliver these implications we need to consider the psychology of how overconfidence changes over time as people receive feedback from their environments.

## 3.3 Model 3: Dynamic Overconfidence

Confidence changes over time as people receive feedback about their judgements and decisions. When people learn that their recent forecasts were accurate, they tend to revise their confidence upward, and when they learn that they were wrong they tend to revise it downward. However, this process is not symmetric, owing to self-attribution bias, which

[2]Thus, knowing the public signal does not allow one to forecast the future return from time 2 to time 3. For the interested reader, a formal proof of this assertion is given as Appendix B.

Figure 4: **Model 3: Dynamic Overconfidence Model – Timeline**




This figure illustrates the dynamic overconfidence model timeline. At time 1, the informed investors receive a private signal $s_I$. At each subsequent time (2, 3, ...), the investor receives additional public signals, with uncorrelated noise terms.

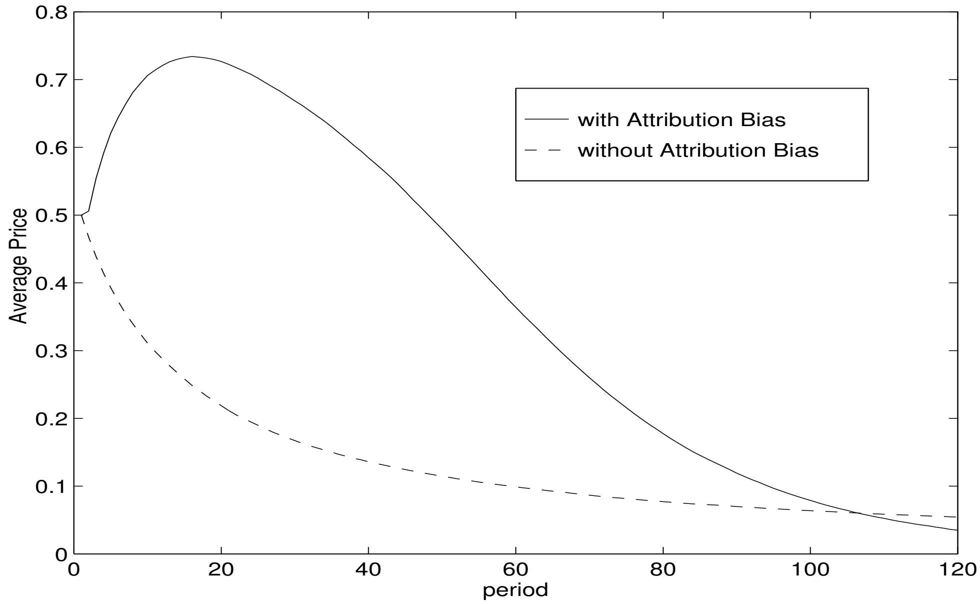
is the tendency of people to treat successes as mainly a reflection of their own skills and failures as mainly a matter of bad luck—the "heads I win, tails it's chance" fallacy (Langer and Roth 1975). Self-attribution bias explains how overconfidence can persist over time.

Incorporating the dynamics of overconfidence into our price formation model allows us to derive more realistic predictions for patterns of return continuation and reversal. To do so, we need to give investors opportunities to update their estimate of their private signal precision. Thus we adopt the structure illustrated in Model 3 (Figure 4), with the change that there are now an unlimited number of public signals arriving at times 2, 3, 4, ....

Consistent with findings from the psychology literature, we specify that the investor's estimate of private signal precision shifts through time as a function of whether the investor's private signal proves to be consistent with subsequently arriving public signals. This specification for confidence updating is admittedly ad hoc, but is roughly consistent with the psychology literature. In particular, investors update their estimates of their signal accuracy based on their historical forecast success, but in a biased way.

Think of the "cumulative public signal" as the average of all previous public signals. The investor's perceived precision evolves over time based on public signal arrival. The updating rule is that when the arrival of the next public signal pushes the cumulative public signal (and market price) in the direction of the investor's private signal, then the investor becomes more confident in her private signal. So the investor's estimated signal precision increases by a factor of $1 + \overline{k}$. In contrast, if the new public signal pushes the price away from the investor's valuation, the investor loses confidence, and the investor's estimate of $\tau_V$ falls by a factor of $(1 - \underline{k})$. Biased self-attribution is captured by the assumption that $\overline{k} > \underline{k}$: the investor's estimated precision increases more with a good outcome than it decreases with a bad outcome.

Figure 5: **Response to a unit private signal—static and dynamic overconfidence models.**




This figure illustrates the impulse response to a private signal $s_V = 1$ at time 1 when the true security value is $\theta = 0$. In this simulation the prior and private-signal precisions are equal. The dashed line illustrates the impulse response in the static overconfidence setting. The solid line is the impulse response in the dynamic overconfidence setting. (from Daniel, Hirshleifer and Subrahmanyam (1998)).

Figure 5 illustrates the impulse response to to a private signal of $s_V = 1$ at time 1, when the prior of $\bar{\theta} = 0$, the true security value is $\theta = 0$, and the prior and private signal precisions are equal. The dashed line illustrates the price path with static overconfidence (as presented earlier in Model 2). Here, because of the equal precisions, the price at time 1 is 0.5—the average of $\bar{\theta}$ and $s_V$. However, starting at $t = 2$, with the arrival of the first public signal, the price on average starts to decline, as the average public signal is equal to $\theta = 0$, and converges to the true security value of $\theta = 0$.

The solid line in Figure 5 illustrates the average price path with dynamic overconfidence as in Model 3). As in the static overconfidence setting, the price initially moves to $P_1 = 0.5$. However now, on observing the sequence of (noisy) public signals, the investor's estimate of her private signal precision increases, resulting in continuing overreaction to this original signal—in this example up to about 15 periods. Eventually, as more public signals arrive, the cumulative public signal becomes more precise and the mispricing necessarily converges to zero. The result is a hump-shaped impulse response function. If instead we began with a private signal that was negative, there would be a trough-shaped impulse response function—the reflection across the $x$-axis of the solid line in Figure 5.

This shape implies momentum at short lags and reversal at long lags. To build some intuition on this point, consider the hump-shape (the long side). The upward slope in the overreaction phase indicates that positive returns tend to be followed by positive returns. The downward slope in the correction phase indicates there negative returns tend to be followed by negative returns. Similar reasoning applies on the short side. In contrast, with a long lag, a positive return on the left side of the hump tends to be followed by a negative return in the correction phase. In sum, a model with self-attribution and dynamic shifts in confidence implies positive short-lag autocorrelations and negative long-lag autocorrelations and is therefore consistent with evidence of momentum and long-run reversal discussed earlier. It is also consistent with the strong performance of the UMD (Up Minus Down) momentum-based portfolio described earlier.

## 3.4 Models with Both Rational and Overconfident Investors

In the models so far, prices are set by overconfident investors. How would these conclusions change were we to introduce a mass of rational investors into these models? These investors would act as arbitrageurs, pushing prices toward fundamental values.

Daniel, Hirshleifer and Subrahmanyam (2001) explore such a setting as an extension of the three-date static-overconfidence model explored earlier. In this approach, the market has a continuum of risk-averse investors, who start identical to each other. There are $N$ securities, and the joint distribution of their fundamental payoffs is common knowledge. At time 1, investors receive different private signals. Some receive signals about what we call "factor realizations"—common influences that affect the returns of all securities—while other receive signals about what we call "residual payoff components"—the pieces of security payoffs that are not explained by common factors.

Investors are overconfident about the signal they receive: they believe that the precision of that signal is higher than it is actually is. However, the investors who do not receive a signal instead infer the signal as it manifests itself through prices, assess precision correctly, and act as arbitrageurs. Owing to risk aversion, these arbitrageurs eliminate only *some of* the mispricing.

This setting yields a number of implications for the relationships between risk and return. First, just as in the Model 2 setting, size and fundamental/price ratios are predictors of future security returns. Size is a negative predictor, because a firm that is large in market value will on average be large in part because it is overvalued. This ability of size to predict returns can help to explain the performance of the SMB (Small Minus Big) portfolio described earlier. For a similar reason, fundamental/price ratios (such as earnings-to-price or book-value-to-price) are positive return predictors. Indeed, scaling of price by a fundamental measure can improve return predictability, because a firm can have high price for fundamental reasons, not just because of mispricing. These effects can explain the performance of the HML (High Minus Low) book-to-market-based portfolio described earlier.

A second key implication is that the amount of mispricing will be constrained by the return factor structure, meaning the set of random variables ("factors") that affect the returns of different stocks, and the sensitivities of returns to the different factors. The factor structure affects how risky it is to arbitrage mispricing. When all investors are overconfident, relatively extreme mispricing is feasible. However, when there are arbitrageurs with rational perceptions, high Sharpe ratios become an attractive opportunity to exploit. Such exploitation acts as a constraint on possible mispricing.[3] In particular, in the limit as the number of securities in the market becomes arbitrarily large, it is possible to form portfolios that hedge away factor risk and exploit any mispricing of residual payoff components. Such portfolios are virtually riskfree. This implies that, owing to arbitrage activity, there will be almost no security-specific mispricing (with the possible exception of a small number of securities).

In contrast, to arbitrage the mispricing of a factor (such as the excess return on the market portfolio, or the return on the HML portfolio, both discussed at Table 1), an investor must bear substantial factor risk—the risk that the factor portfolio return could turn out high or low. This implies that in equilibrium, the factor portfolio can remain substantially mispriced. This contrast between almost perfect arbitrage of idiosyncratic mispricing, but not of factor mispricing comes in part from the assumption that markets are perfectly liquid. For illiquid stocks, arbitrage is more costly, so all stocks can have some idiosyncratic mispricing.

In this setting, regressing across stocks on $\beta$ (the classic risk measure of the Capital Asset Pricing Model) as well as the fundamental-to-price ratio generally helps disentangle risk premium versus mispricing effects. However, if overconfidence about signals is extreme and the fundamental is measured perfectly, even though $\beta$ is priced, it has no *incremental* power to predict future returns. Intuitively, the fundamental-to-price ratio captures both standard risk effects and mispricing effects—both drive market price down relative to expected future cash flows. In the limiting case in which the firm-specific signal the overconfident investors receive is pure noise, and the fundamental proxy is perfect (the best rational forecast of future

[3] More precisely, the flow of wealth from irrational to rational investors becomes arbitrarily large, which clearly is not sustainable.

cash flows), $\beta$ does not provide any additional useful information to predict returns. The fundamental/price ratios will eliminate $\beta$ in a multiple regression when forecasting the cross-section of future returns. This implication is consistent with empirical studies mentioned earlier in which book-to-market eliminates $\beta$ in predicting returns.

Finally, this model displays excessive disagreement, because overconfident investors insist on relying too heavily on the signals they possess, and then will trade against rational arbitrageurs who do not possess those signals and do not overweight the signals' precision. An excessively large volume of trade will result. In this way, overconfidence helps to explain the remarkably high volumes of trade in liquid securities.

### 3.5 Summing Up: Linking the Models to the Trading Strategies

We have already discussed how the models in this section can explain the strong performance of the first four trading strategies summarized in Table 1. We close this section by discussing whether overconfidence can help explain the performance of the remaining three trading strategies: ACR (long on low-accrual firms, short on high accrual firms), BAB (long on low-beta stocks, short on high-beta stocks), and IVOL (long on stocks with low idiosyncratic volatility, short on stocks with high idiosyncratic volatility).

We begin with the strong performance of the BAB and IVOL portfolios, which reflects, respectively, the underperformance of stocks with high systematic and idiosyncratic risk. As noted earlier, in a model such as that of Miller (1977) in which there is both investor disagreement about firm value and short-sales constraints, irrational optimists dominate price setting. This implies that when investors disagree more about a firm's future prospects, that firm will be more overpriced and thus will earn lower returns on average. Overconfidence provides a natural explanation for the irrational tendency for investors to be too insistent in disagreeing, and for optimists to fail to fully adjust for the fact that there are pessimists who have been sidelined by short-sale constraints. High risk firms have greater scope for overconfidence and disagreement, so we expect this source of overpricing to be greatest for high risk firms. In these ways, overconfidence provides a natural explanation for the idiosyncratic volatility and betting against beta effects.

ACR (the accrual anomaly) is usually understood as arising from limited investor attention. The earnings for a firm are the sum of its cash flow and accrual components. The cash flow component of earnings is a much more favorable indicator than the accrual component of high future profits (Sloan 1996). Investors who do not delve into earnings to evaluate these components separately will tend to overvalue firms with high accruals and undervalue firms with low accruals.

In our view, overconfidence is an important part of understanding return anomalies that are usually attributed solely to limited investor attention. Limited attention has a much bigger effect on price if the investors are overconfident and so fail to recognize that the information they are neglecting is important. A similar point is made by Kahneman (2011), who discusses the tendency of people to be overconfident about fast heuristic judgements (which he calls "System 1").

# 4 Cursedness: A Related Approach to Asset Pricing

We make no attempt at a systematic review of behavioral approaches to investment here, but one alternative, cursedness, is notable for its potential overlap with the overconfidence approach.[4] Indeed, Eyster, Rabin and Vayanos (2013) point out that cursedness can potentially explain several financial economic phenomena that are often understood in terms of overconfidence.

In cursedness, a game theoretic equilibrium concept developed in Eyster and Rabin (2005), individuals underweight the information implicit in the actions of others. An example is provided by the winner's curse—the phenomenon that those who win a sealed-bid auction often have submitted too high a bid, in which the very fact of winning is an indication that others do not value the object as highly. A sophisticated bidder will make a subtle inference: if I win, others have information that is more adverse than mine. Someone who understands the winner's curse will then tend to bid more conservatively to adjust for the danger of overbidding, or at times choose not bid at all and thus receive a safe outcome of zero.

An overconfident individual who overweights his own signal will, accordingly, also underweight the information implicit in the actions of others, so the overconfidence and cursedness approaches yield overlapping implications. However, the cursedness approach does have some distinct implications. The behavior of an overconfident individual is too aggressive even when others have no signals; in contrast, cursedness only arises when others have signals that the

[4]Other behavioral approaches include representativeness and conservatism (Edwards (1968), Kahneman and Tversky (1972), Barberis, Shleifer and Vishny (1998)); realization utility (Barberis and Xiong (2012)); mental accounting and prospect theory (Kahneman and Tversky (1979), Thaler (1985), Barberis and Huang (2001), Grinblatt and Han (2005)); limited attention (Kahneman (1973), Hirshleifer and Teoh (2003), Peng and Xiong (2006)); and anchoring (Tversky and Kahneman (1974), George and Hwang (2004)).

cursed individual might fail to take into account. These distinctions matter for a key argument of Eyster, Rabin and Vayanos (2013) in favor of cursedness over overconfidence as an explanation for overly aggressive trading. According to this argument, an overconfident investor should still trade little, because the investor should recognize that one personal signal is minor relative to the aggregated signals of millions of other investors (some of whom might be highly expert). In contrast, a cursed investor ignores those other signals, and hence trades too readily.

However, an investor could be overconfident about the uniqueness of a personal signal, not just its quality. Consider a setting where a security's payoff will be $\theta = \theta_1 + \theta_2$, that the investor believes he has a unique signal about $\theta_1$, but that millions of others are observing signals about $\theta_2$. Even with only a moderate level of overconfidence about signal precision, such an investor may trade quite aggressively, despite being fully aware that there are many other informed players in the market.

Furthermore, we believe that cursedness does not go far to explain the phenomenon of aggressive trading. Many financial economists now believe that the great bulk of individual investors—those who are not insiders, financial professionals, or remarkable amateurs—have little or no useful private information that would allow them to trade profitably in individual stocks. But a poorly informed investor who is only cursed, not overconfident, understands perfectly well that the expected profitability of making a trade is quite small, and moreover, is costly, owing to brokerage fees, time costs, and risk. These frictions or a modest degree risk aversion should easily deter aggressive trading by investors who are cursed but understand that they are ill-informed.

Finally, the empirical evidence summarized earlier in this paper documents short-term return momentum and long-term return reversal in numerous markets. The model of cursedness in Eyster, Rabin and Vayanos (2013) explains momentum as a pure underreaction phenomenon. As such, it explains momentum but not long-run reversal. The overconfidence approach, in contrast, explains momentum and reversal jointly as parts of a phenomenon of continuing overreaction and sluggish correction (Daniel, Hirshleifer and Subrahmanyam 1998).

In summary, we believe that cursedness offers a rich approach for understanding economic phenomena. We do not, however, see cursedness, at least taken in isolation, as offering an explanation for the key patterns presented here—excessive trading, short-term momentum, long-term reversal—that have motivated the use of overconfidence in models of securities markets.

# 5 Conclusion

This essay has two main themes: 1) There are anomalies in financial markets—unprofitable active trading, and patterns of return predictability—that are puzzling from the perspective of traditional purely rational models; and 2) models of overconfidence, and of the dynamic psychological processes that underlie overconfidence, can plausibly explain why these patterns exist and persist.

For those readers who are uncomfortable with an explanation for anomalies based on imperfect rationality, we would point out that the empirical patterns of unprofitable active trading and of return predictability are more-or-less agreed upon both by the leading fans of the efficient markets hypothesis and those with a more behavioral bent. For example, the data underlying the three- and five-factor models of Fama and French (1993, 2015) suggest that portfolios can be built that provide high returns can be achieved with relatively low volatility. The main disagreement is not over the empirical facts described in this paper, but about what components should be added to an asset pricing model to describe them.

We believe that overconfidence offers a useful component, both because of how it explains the agreed-upon facts emphasized here, and also because overconfidence promises to help integrate other elements of behavioral finance theory. For example, some authors have emphasized the importance of investor disagreement in understanding financial markets (Hong and Stein 2007). Overconfidence provides a natural explanation for why investors who process the same public information end up disagreeing so much. Limited investor attention has also recently been offered as an explanation for various empirical patterns in trading and prices. Overconfidence explains why investors who neglect important information would nevertheless trade aggressively, so that such neglect can influence price. In these ways and others, overconfidence offers a microfoundation for other important building blocks of behavioral finance models.

# Appendices

## A An Alternative Formulation of Overconfidence

As an extension of the overconfidence model in Section 3.2, in this section we explore an alternative formulation of overconfidence first proposed in Scheinkman and Xiong (2003, SX).[5] In contrast with the specification of the model presented in the main text, in which the overconfident investor perceives that the investor's private signal has lower variance than it actually does, in the SX specification the overconfident investor thinks that the investor is observing a signal that is highly correlated with innovations in firm value, when in reality the signal is only loosely correlated with firm value innovations.[6]

This formulation has the advantage of generating momentum effects without biased-self-attribution. However, this formulation embeds multiple assumptions about the agent's information processing. In standard modeling of overconfidence,[7] as presented in Section 3, the agent receives signal which are unbiased, but imprecise, and the overconfident agent overestimates the signal precision. In this alternative formulation, the signal the agent receives is biased toward the prior, or to put it differently, it is more strongly aligned with old information than the individual realizes, and it is this bias that generates the momentum effect. While *overprecision* is well documented in the psychology literature, we are not aware of psychological evidence for the idea that people underestimate the degree to which their signals are aligned with with old information (after taking into account any overprecision).

To illustrate this alternative formulation of overconfidence, we construct a simple model like that in Section 3.2, but with a structure that captures the SX structure. As in the model in Section 3.2, true asset value $\theta$ is drawn from a common knowledge prior distribution:

$$\theta = \bar{\theta} + \epsilon_0, \tag{1}$$

where $\epsilon_0 \sim \mathcal{N}(0, 1/\tau_0)$ The timeline for the model is as follows: at date 0, the agent knows only the prior distribution, and the price $P_0 = \bar{\theta}$. At time 1 the agent observes distinct hard

[5]This specification of is also used in Alti and Tetlock (2013) and Kelley and Tetlock (2013).

[6]See pp. 1189-1190, Scheinkman and Xiong (2003)

[7]By 'standard' we mean the signal as truth plus noise, as used in the models of Kyle and Wang (1997), Odean (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Daniel, Hirshleifer and Subrahmanyam (2001), Hirshleifer and Luo (2001), and others.

($h$) and soft ($s$) signals of the form:[8]

$$
\begin{aligned}
\sigma_h &= \theta + \epsilon_h \\
\sigma_s &= \bar{\theta} + \eta\epsilon_0 + \sqrt{1-\eta^2}\ \epsilon_s.
\end{aligned}
\tag{2}
$$

At time 2, $\theta$ is revealed and $P_2 = \theta$.

$\epsilon_h$ and $\epsilon_s$ are mean zero, normally distributed with precisions $\tau_s$, and $\tau_h$.[9] However, SX model the investor's overconfidence as leading her to believe that the private/soft signal is:

$$\sigma_s = \bar{\theta} + \eta_C\epsilon_0 + \sqrt{1-\eta_C^2}\ \epsilon_s, \tag{3}$$

where $\eta_C > \eta$.

To see how this is distinct from the standard overconfidence setting, note that equations (2) and (3) can be rewritten as:

$$
\begin{aligned}
\sigma_s &= \eta\theta + (1-\eta)\bar{\theta} + \sqrt{1-\eta^2}\ \epsilon_s \\
\sigma_s &= \eta_C\theta + (1-\eta_C)\bar{\theta} + \sqrt{1-\eta_C^2}\ \epsilon_s
\end{aligned}
$$

In the setting in Section 3.2, overconfident investors underestimate the variance of $\epsilon_s$. In contrast, in this setting an "overconfident" investor (with $\eta_C > \eta$) not only underestimates the signal variance, but also overestimates the extent to which $\sigma_s$ is pushed away from the prior $\bar{\theta}$ and towards the true value of $\theta$. As a result, when the agent's overconfidence is of this form, $P_1$ will be pushed away from $\theta$ and towards $\bar{\theta}$.

To better illustrate the importance of this assumption, consider an extreme setting where $\eta = 0$, implying $\sigma_s = \bar{\theta} + \epsilon_s$—so that the informed investor's signal is equal to the mean of the prior distribution plus pure noise. However, assume that the investor is severely overconfident, meaning that $\eta_C = 1$, implying that the investor believes that her signal is unbiased, and infinite precision — *i.e.*, $\sigma_s = \theta$. Thus, $P_1 = \bar{\theta}$, while the rational expected time 2 price, conditional on the hard signal $\sigma_h$, is:

$$\mathbb{E}^R[P_2|\sigma_h] = \mathbb{E}^R[\theta|\sigma_h] = \left(\frac{\tau_0}{\tau_0+\tau_h}\right)\bar{\theta} + \left(\frac{\tau_h}{\tau_0+\tau_h}\right)\sigma_h.$$

[8] Alti and Tetlock (2013) label these signals as hard and soft, rather than as public and private.

[9] In the Section 3.2 specification, the public and private signals were revealed at times 1 and 2 respectively. Here, they arrive simultaneously at time 1.

Thus,

$$\mathbb{E}^R[r_{12}|\sigma_h] = \left(\frac{\tau_h}{\tau_0 + \tau_h}\right)(\sigma_h - \bar{\theta}).$$

Thus, in a setting where the investor both underestimates the noise variance in the private signal, and underestimates the extent to which the signal is shrunk towards the prior, there will be underreaction to public information signals, and a form of public-signal linked momentum will result. However, if all signals are unbiased—i.e., the true value $\theta$ plus noise—then additional model structure is necessary to generate the observed underreaction to public information, and price momentum.

How consistent is are the psychological evidence on overconfidence with these two possible specifications? The SX specification assumes a combination of overestimating signal precision (as in the standard overconfidence approach) and a distinct second bias of believing (above and beyond the effects of any misperception of signal precision) that the realized signal is closer to the true value than it really is. We view the psychological underpinning of overconfidence (that people think they are good at generating high quality signals) and the psychological evidence of overprecision as more supportive of the first bias than the second.

# B A proof that in the Model 2 setting, $\mathbb{E}[R_{2,3}|s_B] = 0$

From the equation:

$$P_2 = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \hat{\tau}_V s_V + \tau_B s_B\right),$$

$$\mathbb{E}[P_2|s_B] = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B + \hat{\tau}_V \mathbb{E}[s_V|s_B]\right). \tag{4}$$

However,

$$\mathbb{E}[s_V|s_B] = \mathbb{E}[\theta|s_B] = \frac{1}{\tau_0 + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B\right).$$

Substituting this into equation (4) yields:

$$\mathbb{E}[P_2|s_B] = \frac{1}{\tau_0 + \hat{\tau}_V + \tau_B}\left(\tau_0\bar{\theta} + \tau_B s_B\right)\left(1 + \frac{\hat{\tau}_V}{\tau_0 + \tau_B}\right).$$

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