A comment on “Pareto improving taxes”

John Leventides and Nickolas Michelacakis

National & Kapodistrian University of Athens, Department of Economics, University of Piraeus, Economics Department

29 January 2016

Online at https://mpra.ub.uni-muenchen.de/69081/
MPRA Paper No. 69081, posted 31 January 2016 08:38 UTC
A comment on “Pareto improving taxes”

J. Leventides* & N.J. Michelacakis**

National & Kapodistrian University of Athens, Department of Economics,
1 Sofokeous Str., 105 59, Greece

University of Piraeus, Economics Department, 80 Karaoli & Dimitriou Strs., 18534, Greece


“Theorem. For almost all economies with separable externalities and I > 1, every competitive equilibrium is constrained Pareto suboptimal, that is, for each competitive equilibrium, there exists an anonymous tax package t and a competitive t-equilibrium allocation which Pareto dominates it.”

It is the purpose of this comment to show that restrictions must be applied on the limiting cases for the theorem to hold. Proposition 1.3, below, gives a counter-positive result and the ensuing Corollary shows that the Theorem in [Geanakoplos & Polemarchakis 2008][p. 685] does not hold for I = 2 and subsequently the example given in Section 6, page 693, of [Geanakoplos & Polemarchakis (2008)] appears to be incorrect.

We keep the notation as in [Geanakoplos & Polemarchakis (2008)]. First, a lemma

Lemma 1.1 In a pure exchange economy with separable externalities where each commodity is traded (bought or sold) in equal amounts the revenue generated by an anonymous tax package can be compensated by a respective adjustment of prices.

Proof. We shall prove the lemma in a commodity-wise manner, i.e. we claim that the tax revenue raised by the trade of a commodity that is purchased and sold in equal amounts can be completely absorbed by a readjustment of the price of the commodity. This is trivially true for a commodity that is not traded at all.

Let \( B_j \) the set of \( \mu_j := \#(B_j) \) consumers buying commodity \( j \) in equal amounts, say \( q_j^\uparrow \), and \( S_j \) the set of \( \nu_j := \#(S_j) \) consumers selling commodity \( j \) in equal amounts,

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*e-mail address: ylevent@econ.uoa.gr  corresponding author

**e-mail address: njm@uni.gr (tel.: +30 210414 2289)
say \( q_j^s \). A consumer either sells \( q_j^s \) units of commodity \( j \) or buys \( q_j^b \) units of commodity \( j \), i.e. \( \mu_j + \nu_j = I \). If \( a \) is a vector of \( \mathbb{R}^l \), let us denote by \( \hat{a}_j \) the vector of \( \mathbb{R}^l \) defined by

\[
(\hat{a}_j)_i := \begin{cases} 
  a_i, & i \neq j \\
  0, & i = j 
\end{cases}.
\]

The per capita share of total tax revenue due to the trade of commodity \( j \) is

\[
\tau_j = \frac{1}{T} \sum_{i \in B_j} t_j (x_j^s - e_j^s) = \frac{1}{T} \mu_j q_j^b.
\]

We may, therefore, write the total per capita tax revenue as composed of two parts
one part due to the trade of commodity \( j \), \( \tau_j \), and another part raised through the trade of other than \( j \) commodities,

\[
(1-1) \quad \tau = \frac{1}{T} \mu_j t_j q_j^b + \frac{1}{T} \sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+.
\]

We look at the budget constraint of all consumers distinguishing between the two mutually exclusive groups of buyers and sellers of commodity \( j \). Let \( i_{bj} \) and \( i_{sj} \) denote a typical buyer and a typical seller of commodity \( j \), respectively.

The income constraint of buyer \( i_{bj} \) is

\[
(1-2) \quad (p + t)(x_{ij}^{i_{bj}} - e_{ij}^{i_{bj}})_+ - p \cdot (x_{ij}^{i_{bj}} - e_{ij}^{i_{bj}})_- \leq \tau.
\]

Taking into account 1-1 and the fact that \( x_{ij}^{i_{bj}} - e_{ij}^{i_{bj}} = q_j^b \), inequality 1-2 becomes

\[
(1-3) \quad (p_j + (1 - \frac{\mu_j}{T})t_j)(x_{ij}^{i_{bj}} - e_{ij}^{i_{bj}}) + (\hat{p}_j + \hat{t}_j) \cdot (\hat{x}_j^{i_{bj}} - \hat{e}_j^{i_{bj}})_+ \leq \frac{1}{T} \sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+.
\]

The corresponding constraint of the random seller \( i_{sj} \) of commodity \( j \) is

\[
(1-4) \quad (p + t)(x_{ij}^{i_{sj}} - e_{ij}^{i_{sj}})_+ - p_j (x_{ij}^{i_{sj}} - e_{ij}^{i_{sj}})_- - \hat{p}_j \cdot (\hat{x}_j^{i_{sj}} - \hat{e}_j^{i_{sj}})_- \leq \frac{1}{T} \mu_j t_j q_j^b + \frac{1}{T} \sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+.
\]

However, \( \mu_j q_j^b = \nu_j q_j^s \) by assumption and 1-4 becomes

\[
(1-4) \quad (p + t)(x_{ij}^{i_{sj}} - e_{ij}^{i_{sj}})_+ - p_j (x_{ij}^{i_{sj}} - e_{ij}^{i_{sj}})_- - \hat{p}_j \cdot (\hat{x}_j^{i_{sj}} - \hat{e}_j^{i_{sj}})_- \leq \frac{1}{T} \nu_j t_j q_j^s + \frac{1}{T} \sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+.
\]
which together with the fact that \((x_{j}^{i} - e_{j}^{i})_{-} = q_{j}^{i}\) yields

\[
(1-5) \quad (p + t)(x_{j}^{i} - e_{j}^{i}) + (p_{j} + \frac{\nu_{j}}{I})(x_{j}^{i} - e_{j}^{i})_{-} - \sum_{i=1}^{I} \tilde{t}_{j} \cdot (\tilde{x}_{j}^{i} - \tilde{e}_{j}^{i})_{+} \leq \frac{I}{I} \sum_{i=1}^{I} \tilde{t}_{j} \cdot (\tilde{x}_{j}^{i} - \tilde{e}_{j}^{i})_{+}.
\]

completing the proof since \(\frac{\nu}{I} = 1 - \frac{n}{I}.

**Remark 1.2** A converse to the statement of Lemma 1.1 may be proved provided that the optimum is attained on the boundary.

**Proposition 1.3** In a pure exchange economy with full trade and separable externalities if all commodities are sold and bought in equal amounts, no anonymous tax package can Pareto improve a competitive equilibrium.

**Proof.** According to Lemma 1.1, the extra income generated by the redistribution of taxes collected by application of any tax package can be completely absorbed by a corresponding change in the price each commodity is traded leading to no better reallocation of the initial resources.

**Corollary 1.4** In a pure exchange economy of two consumers and \(L\) commodities, with separable externalities, no anonymous tax package can Pareto improve a competitive equilibrium.

**References**