A comment on "Pareto improving taxes"

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In an article appeared in the Journal of Mathematical Economics, J. Geanakoplos and H. Polemarchakis, [Geanakoplos J. and Polemarchakis H.M.: "Pareto improving taxes", *Journal of Mathematical Economics* **44** (2008), 682–696], prove on page 685 the following theorem:

"**Theorem.** *For almost all economies with separable externalities and $L > I$, every competitive equilibrium is constrained Pareto suboptimal, that is, for each competitive equilibrium, there exists an anonymous tax package t and a competitive t-equilibrium allocation which Pareto dominates it.*"

It is the purpose of this comment to show that restrictions must be applied on the limiting cases for the theorem to hold. Proposition 1.3, below, gives a counter-positive result and the ensuing Corollary shows that the Theorem in [Geanakoplos & Polemarchakis 2008][p. 685] does not hold for $I = 2$ and subsequently the example given in Section 6, page 693, of [Geanakoplos & Polemarchakis (2008)] appears to be incorrect.

We keep the notation as in [Geanakoplos & Polemarchakis (2008)]. First, a lemma

Lemma 1.1 *In a pure exchange economy with separable externalities where each commodity is traded (bought or sold) in equal amounts the revenue generated by an anonymous tax package can be compensated by a respective adjustment of prices.*

Proof. We shall prove the lemma in a commodity-wise manner, i.e. we claim that the tax revenue raised by the trade of a commodity that is purchased and sold in equal amounts can be completely absorbed by a readjustment of the price of the commodity. This is trivially true for a commodity that is not traded at all.

Let $\mathcal{B}_j$ the set of $\mu_j := \#(\mathcal{B}_j)$ consumers buying commodity j in equal amounts, say q_j^b, and $\mathcal{S}_j$ the set of $\nu_j := \#(\mathcal{S}_j)$ consumers selling commodity j in equal amounts,

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say q_j^s. A consumer either sells q_j^s units of commodity j or buys q_j^b units of commodity j, i.e. $\mu_j + \nu_j = I$. If a is a vector of $\mathbf{R}^I$, let us denote by $\hat{a}_j$ the vector of $\mathbf{R}^I$ defined by

$$(\hat{a}_j)_i := \begin{cases} a_i\,, & i \neq j \\ 0\,, & i = j \end{cases} .$$

The per capita share of total tax revenue due to the trade of commodity j is

$$\tau_j = \frac{1}{I} \sum_{b \in \mathcal{B}_j} t_j (x_j^s - e_j^s) = \frac{1}{I} \mu_j t_j q_j^b\,.$$

We may, therefore, write the total per capita tax revenue as composed of two parts one part due to the trade of commodity j, τ_j, and another part raised through the trade of other than j commodities,

$$\tau = \frac{1}{I} \mu_j t_j q_j^b + \frac{1}{I} \sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+\,. \tag{1-1}$$

We look at the budget constraint of all consumers distinguishing between the two mutually exclusive groups of buyers and sellers of commodity j. Let i_{b_j} and i_{s_j} denote a typical buyer and a typical seller of commodity j, respectively.

The income constraint of buyer i_{b_j} is

$$(p + t) \cdot (x^{i_{b_j}} - e^{i_{b_j}})_+ - p \cdot (x^{i_{b_j}} - e^{i_{b_j}})_- \leq \tau$$

$$(p_j + t_j)(x_j^{i_{b_j}} - e_j^{i_{b_j}}) + (\hat{p}_j + \hat{t}_j) \cdot (\hat{x}_j^{i_{b_j}} - \hat{e}_j^{i_{b_j}})_+ - p \cdot (x^{i_{b_j}} - e^{i_{b_j}})_- \leq \tau\,. \tag{1-2}$$

Taking into account 1-1 and the fact that $x_j^{i_{b_j}} - e_j^{i_{b_j}} = q_j^b$, inequality 1-2 becomes

$$\begin{aligned} (p_j + (1 - \frac{\mu_j}{I}) t_j)(x_j^{i_{b_j}} - e_j^{i_{b_j}}) + (\hat{p}_j + \hat{t}_j) \cdot (\hat{x}_j^{i_{b_j}} - \hat{e}_j^{i_{b_j}})_+ \\ - p \cdot (x^{i_{b_j}} - e^{i_{b_j}})_- \leq \frac{1}{I} \sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+\,. \end{aligned} \tag{1-3}$$

The corresponding constraint of the random seller i_{s_j} of commodity j is

$$\begin{aligned} (p + t)(x^{i_{s_j}} - e^{i_{s_j}})_+ - p_j(x_j^{i_{s_j}} - e_j^{i_{s_j}})_- - \hat{p}_j \cdot (\hat{x}_j^{i_{s_j}} - \hat{e}_j^{i_{s_j}})_- \\ \leq \frac{1}{I} \mu_j t_j q_j^b + \frac{1}{I} \sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+\,. \end{aligned} \tag{1-4}$$

However, $\mu_j q_j^b = \nu_j q_j^s$ by assumption and 1-4 becomes

$$\begin{aligned} (p + t)(x^{i_{s_j}} - e^{i_{s_j}})_+ - p_j(x_j^{i_{s_j}} - e_j^{i_{s_j}})_- - \hat{p}_j \cdot (\hat{x}_j^{i_{s_j}} - \hat{e}_j^{i_{s_j}})_- \\ \leq \frac{1}{I} \nu_j t_j q_j^s + \frac{1}{I} \sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+ \end{aligned}$$

which together with the fact that $(x_j^{i_{s_j}} - e_j^{i_{s_j}})_- = q_j^s$ yields

$$(p+t)(x^{i_{s_j}} - e^{i_{s_j}})_+ - (p_j + \frac{\nu_j}{I})(x_j^{i_{s_j}} - e_j^{i_{s_j}})_- - \hat{p}_j \cdot (\hat{x}_j^{i_{s_j}} - \hat{e}_j^{i_{s_j}})_- \leq \frac{1}{I}\sum_{i=1}^{I} \hat{t}_j \cdot (\hat{x}_j^i - \hat{e}_j^i)_+ . \tag{1-5}$$

completing the proof since $\frac{\nu_j}{I} = 1 - \frac{\mu_j}{I}$.

Remark 1.2 *A converse to the statement of Lemma 1.1 may be proved provided that the optimum is attained on the boundary.*

Proposition 1.3 *In a pure exchange economy with full trade and separable externalities if all commodities are sold and bought in equal amounts, no anonymous tax package can Pareto improve a competitive equilibrium.*

Proof. According to Lemma 1.1, the extra income generated by the redistribution of taxes collected by application of any tax package can be completely absorbed by a corresponding change in the price each commodity is traded leading to no better reallocation of the initial resources.

Corollary 1.4 *In a pure exchange economy of two consumers and L commodities, with separable externalities, no anonymous tax package can Pareto improve a competitive equilibrium.*

References

[Geanakoplos & Polemarchakis (2008)] Geanakoplos J. and Polemarchakis H.M.: "Pareto improving taxes", *Journal of Mathematical Economics* **44** (2008), 682–696.