Structuration processes in complex dynamic systems - an overview and reassessment

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Abstract

Many questions addressing the emergence and dynamics of economic networks are still unresolved, especially regarding dynamics on and of networks. Previous research shows that processes at the micro-level affect socio-economic systems at aggregated levels. These insights facilitated the development of models taking the network structure explicitly into account. However, what is still missing is a systemic network theory that considers the full complexity of socio-economic systems. We argue that sociological, economic and institutional theories are complementary in many respects and have the potential to fill this gap by providing the theoretical ground for an eclectic network theory. In this paper, we address key concepts that are concerned with structuration processes in socio-economic networks, review and reassess the literature in this field and discuss approaches to explain pattern formation processes at higher aggregation levels. We propose to take advantage of the complementarities of the above outlined yet unconnected research programs.

(149 words)

Key words: complex systems, innovation networks, structuration processes, network dynamics, evolutionary economics

1 Why study structuration processes and the dynamics of networks?

Socio-economic systems are complex. A range of their characteristics depend on their structural configuration, the change of which is a multi-level phenomenon:

On the one hand, events at the micro level, i.e. vertex entries and exits as well as tie formation and termination, have structural consequence at higher aggregation levels of the system. On the other hand, actors embedded in these systems are affected by systemic characteristics and by their positioning in a system. Because economic systems are inherently dynamic, both the systemic configurations and the actors' relative positioning in these systems change continuously. Network researchers have done an impressing job in explaining and formalizing link formation and other dynamic mechanisms at the micro-level. Their models have been successfully applied in epidemics, molecular biology, computer science, ecology, sociology, among other fields. These advances also led to an increasing interest in the study of network-related phenomena in economics including financial ownership networks, communication and information networks, trading and supply chain networks, and innovation networks.

However, what is still missing is a systemic network theory that considers the full complexity of socio-economic systems as a whole. We argue that sociological, economic and institutional theories are complementary in many respects and have the potential to fill this gap by providing the theoretical ground for an eclectic network theory in the future. This paper provides a first step into this direction by reviewing the most influential theoretical and empirical network dynamic and structuration models that have been proposed by now.

We focus on socio-economic networks where the vertices represent various types of social and economic actors, i.e. persons, firms, governmental entities etc. These actors are typically interconnected in multiple ways. The further specification of the networks depend on the type of network ties we look at: In innovation networks, our prime illustrative example throughout this article, an edge between two firms represents a joint R&D activity between them. Therefore, innovation networks were usually conceptualized as follows [Cantner and Graf, 2011, Brenner et al., 2011, Kudic, 2015]: (I) they consist of well-defined sets of independent economic actors, (II) these actors are directly or indirectly interconnected and the linkages allow for a unilateral, bilateral or multilateral exchange of ideas, information knowledge and expertise, (III) the network is embedded in a broader socio-economic environment, and (IV) it has a strategic dimension in the sense that the actors involved cooperate to recombine and generate new knowledge enclosed in goods or services to meet market demands and customer needs. This definition applies foremostly to innovation networks but it can also be used as a natural starting point for other instances of networks in institutionalist theory mentioned above.

Three particular aspects have been at the heart of the debate in interdisciplinary innovation network research: firstly, the relatedness between network structure, network positioning and performance outcomes. Secondly, dynamic processes on existing networks, e.g. games, learning, knowledge exchange and diffusion processes. And thirdly, the evolution of the network itself due to the formation and destruction of edges and vertices. While the first two aspects have been studied extensively, many questions about the evolution of networks, and the underlying determinants and mechanism causing structural network change at higher aggregation levels, are still unresolved. The reasons range from conceptual and theoretical issues to data bottlenecks to methodological limitations. This insight provides the vantage point of this article, which aims at contributing to the existing body of literature in at least two ways:

On the one hand, we provide a comprehensive overview of the most important theoretical ideas and methodological concepts addressing the structuration and dynamics of networks. In doing so, we refer to the history of the study of network-related phenomena in economics and sociology before reviewing the contemporary literature.

On the other hand, advances in various scientific disciplines provided us with highly sophisticated tools and methods for exploring and understanding networks from various angles. We review the methodologically oriented network literature in four closely interrelated thematic fields: (i) clustering, (ii) scaling in large scale networks, (iii) small-word characteristics, and (iv) core-periphery patterns. We discuss economic examples and outline in what way the resulting insights can help further institutionalist theory and our understanding of socio-economic systems.

The remainder of the article is structured as follows: Section 2 discusses historical and theoretical concepts in the interdisciplinary field of network research. Section 3 presents an overview of findings on the structuration and dynamics of networks in economics. We conclude in Section 4 with some remarks on limitations of contemporary network research and fruitful avenues for future inquiry.

2 Theoretical Roots of Contemporary Network Research in Economics and Related Disciplines

2.1 The Origins of Network Research

Structuration and the inherent dynamics of networks was already discussed in early writings in sociology, economics and other fields, even though the term "network" is often not explicitly used. This section reviews these early approaches in order to identify the origins of contemporary network research and its constituting theoretical concepts and analytical tools.

2.1.1 A Glance over the Fence – Roots of Network Research in Other Disciplines

In the early 18$^{\text{th}}$ century mathematicians became interested in what later became known as graph theoretical problems. In its most basic sense, graph theory is concerned with abstract mathematical structures which can be fully described by limited number of lines (also called ties or edges) between a well-defined set of objects (also called nodes or vertices). The first graph theoretical problem – known as the seven bridges of Königsberg - was solved by Leonard Euler in 1736. Since then, graph theory has developed greatly with many crucial problems solved and important methods developed only in the 20$^{\text{th}}$ century.

Game theory, also emerging in the 20th century, is another mathematical pillar of modern network theory. It is concerned with strategic interactions of rational players and develops models that allow predicting – under very restrictive idealized conditions – the behavior of the sequentially interacting players in the game. The development of game theory entailed the possibility of models of games on networks.[1] Another closely related strand of game-theoretical literature focuses on the formation of network [Jackson, 2003, Goyal, 2007]. The concepts and models from these two fields turned out to provide a rich theoretical basis for numerical agent-based simulation modeling which is another powerful tool to study networks, particularly dynamics of formation and development of networks of higher complexity.

2.1.2 Networks in Economic Theory

From today's point of view, it may seem obvious that economic systems are always built on a social layer of many interacting entities (agents, firms, and others) and that the microstructure of these systems matters in various ways for their nature as a whole. But this has not always been the case: much of traditional economic theory derives from concepts such as mechanical equilibria in a price-utility system which effectively eliminate any importance of the micro-level

[1] An excellent overview and synthesis of such models is given in Jackson and Zenou [Jackson and Zenou, 2015].

structure or the representative agent which justifies the treatment of micro- and macro-layer as identical and homogeneous.

Network theory entered economics – well after the fixation of the Walrasian-Marshallian microeconomic standard – via the study of nonlinear interactions of aggregate concepts such as industries in supply relationships. These interdisciplinary approaches, e.g. [Goodwin, 1947, Simon, 1953] were soon joined by other traditions of literature that investigated the role of social networks in economics and of different network structures at the micro-layer of economic interactions between firms and agents of all kind (e.g. [Bowles and Gintis, 1975] following advances in sociology, see section 2.1.3). This enabled agent-based models which, based on earlier Schumpeterian and institutionalist groundwork, were very successful in describing industry dynamics realistically (see, e.g. [Nelson and Winter, 1982, Arthur, 1989]).

However, the fact that earlier theories did not take network theory into account does not mean that the respective models are not subject to properties resulting from their implicitly assumed underlying network. General equilibrium theory, for example, presumes in effect either star networks (with a hypothetical auctionator in the center position) or complete graphs - which result from the assumption of perfect homogenity in a hypothetical global market without transaction costs or any institutional obstacles. Naturally, not all economists were content with this theory. Institutionalists criticized the lack of representation of human social reality. Veblen [Veblen, 1898] ridiculed the concept of human nature employed in (in this case Carl Menger's) equilibrium theory as "hedonistic man"; Polanyi [Polanyi, 1944] argued that the economy was not only embedded in social relations but that the project of equilibrium theory was to disentangle this - something he argued to be absurd and impossible. Of course, the tools to apply this to formal models were not available at the time both in terms of theoretical concepts and in terms of computation power.

2.1.3 Sociological contributions to network research

The common ground of social network theorizing is the notion that individuals are embedded in social structures. The explanation of causes and consequences of various types of interrelations among individuals is one of the key topics in social science.

Simmel [Simmel, 1922] already emphasized the fact that the nature of ties among individuals affects their behaviors in multiple ways. In the mid of the 20$^{\text{th}}$ century, sociologists started to employ graph theoretical concepts to operationalize social structures. One of the pioneers in this research area was Barnes [Barnes, 1954] who helped coin the term "social network". The concept attracted a great deal of attention and constituted the starting point for new research in the field. Several important advances in the theory of social networks date back to this period. For instance, Milgram's [Milgram, 1967] letter-passing experiment showed that people in the United States are separated by, on avarage, only six degrees of separation. That is, the average shortest path between any two individuals[2] is no longer than six interaction steps. The findings and implications triggered countless research efforts on "small-world characteristics" in subsequent years (for an overview, see: [Uzzi et al., 2007], see also section 3).

The social capital and embeddedness literature [Laumann et al., 1978] emphasizes that economic actions and outcomes are influenced by the context in which they occur. A controversial discussion in social capital literature arose on whether and to what extent weak ties [Granovetter, 1973, Levin and Cross, 2004] or strong ties [Uzzi, 1996, Krackhardt, 1992] affect the actors' behavior and outcomes in social and economic networks.[3] Other seminal contributions [Bourdieu, 1986] paved the way for what we refer to as "closure" theory [Coleman, 1988]. The concept is based on the notion that a network actor's positioning in a "cohesive" network structure, densely interconnected and interdependent agents at least at the local level, goes along with several advantages. Cohesion is typically assumed to facilitate "the build-up of reputation, trust, social norms, and social control, for example by coalition building to constrain actions, which facilitates collaboration" [Nooteboom, 2008, p.619].

By contrast, the structural hole theory [Burt, 1992] put forward an efficiency argument and assumes that a network position to be beneficial when it allows the actor to bridge the gap between two unconnected (or at least less connected) subgroups of the network. Integrative approaches between these two theoretical concepts emerged recently [Burt, 2005, Rowley et al., 2000].

2.2 What are inter-firm networks and why do they exist?

In the late 20th century a controversial debate among sociologists and economists arose on the very nature of hybrid organizational forms. Why do individual economic entities develop decentralized cooperative practices and do not limit their innovative activities to the hierarchies within the firm or exchange their results on the market (if there were such a thing as an ideal free market)?

The common ground of traditional economic explanations of hybrid organizational forms is the use of transaction cost arguments. In this context, economists [Ouchi, 1980, Jarillo, 1988, Williamson, 1991] have argued that hybrids are an organizational form positioned interme-

[2]The implicit network model in this context is that vertices stand for individuals and edges indicate direct acquaintance or interaction.

[3]Granovetter's [Granovetter, 1973] concept of weak and strong ties was designed to capture the overlap between connected agent's direct neighborhood with the strength of the link higher the larger the overlap while weak ties tend to be links to more distant parts of the network.

diately between markets and hierarchies. According to Williamson [Williamson, 1991], the key distinguishing feature of hybrids compared to other forms of governance is a flexible contracting mechanism that facilitates continuity and efficient adaptation [Nee, 1992, p.2]. He conducted a discrete structural analysis in order to compare the three supposed generic forms of economic organization – markets, hybrids and hierarchies – in terms of governance cost efficiency with regard to the level of uncertainty, frequency of disturbance, and asset specificity. One of the key findings from this comparative-static analysis is that transactions characterized by an intermediate level of asset specificity are most efficiently processed by hybrid organizational forms, i.e. innovation networks [Williamson, 1991, p.284].

Many institutionalists, however, reject the argument since boundedly rational agents acting under true uncertainty are neither able nor willing to perform these transaction cost calculations [Hodgson, 1993]. An evolutionary argument could be applied (i.e., that they do not optimize their organizational type but evolutionary selection works in its favor and eliminates other types), but this argument fails to recognize that there is a huge number of environmental and institutional factors beyond transaction costs that would interfere with such a selection mechanism.

Sociologists proposed an alternative explanation for the existence of hybrid organizational forms. They argued that hybrids have to be seen as unique organizational structures and thus should be considered an organizational form in their own right [Powell, 1987, Podolny and Page, 1998]. According to this line of argument, the transaction cost perspective fails to see and explain the enormous variety of forms that cooperative arrangements can take. Powell [Powell, 1987, p.77-82] draws up four factors that explain the emergence, existence and proliferation of hybrid organizational forms: (I) hybrid organizational forms allow greater flexibility and adaptability to rapidly changing environments (II) hybrids allow large organizations, which are usually considered to be structurally inert and thus resistant to change, to overcome, at least to some extent, these limitations; (III) hybrids provide fast and flexible access to information and knowledge located outside the firm's boundaries; (IV) hybrids have to be understood as a variant or application of the "generalized reciprocity concept" (i.e. individual units do not exist in isolation but rather in relation to other units, cf. Podolny and Page [Podolny and Page, 1998]) that creates legitimacy, reputation and mutual trust, and thus generates an efficient and reliable environment for exchange and transfer of information.

The preceding discussion provides very different perspectives on the same phenomenon - i.e. the explanation of the very nature of hybrid organizational forms such as economic networks. The transaction cost logic certainly captures not the multiplicity and complexity of economic network observable in real life. In other words the explanation of the existence of economic networks cannot be reduced to a transaction cost optimization problem. This is in line with institutional and evolutionary arguments on the role of uncertainty and bounded rationality in market processes. The sociological view on networks also represents a contradiction to the transaction cost perspective. The explanation of the very nature of networks is based on a more comprehensive understanding. It incorporates several important aspects which are inherently entailed in institutional and evolutionary lines of argument.

3 Structuration Processes and Network Dynamics

Traditional modeling in economics generally relied on random graphs and most often on complete networks until fairly recently. Before the great advances of network theory in the 1990s, networks were usually not considered an important feature of a model in economics. This section shows why that well-established practice in mainstream economics is far too limited.

Section 3.1 offers an overview about the most fundamental stylized facts of real world networks. The particular models and methods that are used to identify and explain these regularities are summarized in an online appendix to this article. Section 3.2 discusses dynamics on and of networks in more detail.

3.1 Some Stylized Facts of Economic Networks

We focus on empirical results regarding the degree of clustering in economic networks, their diameter and their degree distribution mainly for two reasons: Firstly, these characteristics received the most attention in explorative studies and empirical results on their regularities are abundant. Secondly they represent the most distinctive properties of economic networks for which empirical results are available.

3.1.1 Clustering

A cluster is a subset of vertices are characterized by an above average degree of interconnectedness. The precise interpretation of clustering depends on the definition of the edges: usually, an edge exists between two vertices if the corresponding firms hold up close research collaboration (no matter how close they are geographically). Clusters are therefore interpreted in a *functional* way. Functional clusters are a common phenomenon, e.g. if one considers co-operation networks among firms [Storper and Harrison, 1991]. Here, clusters are of particular theoretical interest as they tend to ”outgrow” the market: close relations among the different players may be initiated for economic reasons, but after some time reciprocal relationships yield a higher level of trust. This may be one reason for the stability of industrial clusters.

But there are many other instances where clusters were identified in economic networks, e.g. countries in the world trade networks form clusters of dense trade relationships with each other [Fagiolo et al., 2010], or banks in financial markets form clusters of mutual lending and borrowing [Fricke and Lux, 2014].

In all these cases, the identification of clusters represented a vantage point for more concise models of the phenomenon under study in which the emergence of the cluster, and its economic importance could be considered.

3.1.2 Small-World Property

A straightforward way to account for clustering is the usage of regular grid networks that have been widely employed in economic modeling (see, e.g., Schelling [Schelling, 1971]). This, however, does not sufficiently represent the structure of most social and economic networks. It has been shown that real world networks are clustered and characterized by short average path lengths.

Networks that combine these two properties, strong clustering and small diameter, are referred to as *small world networks*.[4] It has been an important *empirical* contribution to show that small-world networks are common in the economy and to motivate the *theoretical* question of how these networks emerged and what their consequences are. Are there common mechanisms causing the network to be so similar in so different areas? Studies providing potential answers to this questions are summarized in the online appendix to this article.

Empirical exercises addressing the structure of knowledge transfer networks among firms [Cowan and Jonard, 2004], firm ownership [Kogut and Walker, 2001], relations among the boards of directors of the biggest US firms [Davis et al., 2003], the collaboration among research institutes [Cowan and Jonard, 2004], and firm's research collaborations [Phelps and Schilling, 2005].

3.1.3 Heavy Tails and Scaling

Another important statistical property of social and economic networks is their typical degree distribution. The degree of a vertex is the number of links it has to other vertices in the network. A degree of four means the vertex is connected to four other vertices. Information on how degrees are distributed among the different vertices may hint at important underlying mechanisms operating in the system under study.

[4]The name stems from Milgram's [Milgram, 1967] famous study according to which every person on the planet knows any other person with on average only six intermediate steps.

As in the case of clustering and diameter, the distribution of degrees in real world networks shows a surprising regularity: the degree distribution is highly asymmetric with the number of neighbors (the degree) being inversely proportional to the relative frequency of vertices with this number of neighbors. The resulting distribution is called scale-free, so called as the shape of the distribution remains unchanged no matter which part of the distribution is considered or whether the network is scaled to some level of aggregation.

Research into network structures of inter-firm networks and other networks in economic systems found that these networks also were scale-free (e.g., [Kim et al., 2002, Souma et al., 2003, Foster, 2005]). The high probability of tail events in such distributions is particularly relevant since averaging over large numbers of observations may not work since the central limit theorem may not be applicable. Risk management relying on such averaging operations would consequently fail. This is particularly problematic in corporate ownership and corporate lending networks which have indeed been found to be heavy tailed [Iori et al., 2008, Battiston et al., 2007].[5]

It should be noted that some doubt has been cast in recent years on whether all distributions claimed to be scale-free actually belong into that category. It is difficult to statistically differentiate between scale-free (power law) distributions and other candidates including log-normal, exponential with cutoff and less regular distributions [Clauset et al., 2009, Heinrich, 2014]. The property of heavy tails, however, remains unaffected and some or all of the implications may also be preserved for these alternative distributions. Schweizer et al. [Schweitzer et al., 2009] give an overview over more recent empirical findings and the more recent discussion of heavy tails in economic networks.

3.1.4 Core-Periphery Structure

One huge advantage of using the formal apparatus of network theory is to be very precise about the concrete network structure of the system under study. This precise language helps to identify regularities that would not be apparent if the network had not been represented as a graph. The core-periphery structure represents a relevant example:

In its most basic sense, the core-periphery concept is based on the notion of "(...) a dense, cohesive core and a sparse, unconnected periphery" [Borgatti and Everett, 2000a].

This means that in a core periphery network basically two types of vertices exist: one group of vertices that are very closely connected (the "core"), and another group of vertices (the "periphery") that are sparely connected and typically spread across several small and unconnected components.

[5] The identification of scaling laws is not limited to economic networks but was motivated by findings in several other fields (stock price returns, firm sizes, city sizes, etc., see e.g. Newman, [Newman, 2003]).

Hence, the core of the network occupies a dominant position in contrast to the subordinated network periphery and the identification of core-periphery structures in real world networks may help identifying important differences in power among different actors in a network. Rank and her colleagues [Rank et al., 2006], for instance, have argued that actors in the core of a network have a favorable position for negotiating with peripheral actors in bargaining networks.

Concrete empirical strategies to identify core-periphery patterns were proposed by [Borgatti and Everett, 2000b] and [Holme, 2005] (the latter based on the well known k-core concept of [Doreian and Woodard, 1994]. And indeed, building on the seminal contributions of Craig and von Peter [Craig and Von Peter, 2010], economists found, for example, that the lending behavior of banks can be much more adequately described by core periphery networks than by scale-free networks [Fricke and Lux, 2014].

This empirical finding does not only have descriptive value, it also helps identifying the generative mechanisms that bring about such networks of lending and to study potential outcomes of policy measures in this context. In banking networks, core and periphery elements play different roles in the financial system that should be taken into account for a reasonable regulation to be implemented.

But core-periphery networks can also be found very different contexts, such as the German laser industry sector[Kudic et al., 2015], and supply chains in general [Bair, 2008] and international trade networks [Fagiolo et al., 2010]. In international trade, some countries form a center of the trade network and other countries are connected only to a few of these central vertices, thus being strongly dependent on them. The identified pattern is very stable over time, including time spans of increasing globalization [Fagiolo et al., 2010]. It should be considered e.g. in the discussion about the socio-economic consequences of globalization, a prominent topic in the evolutionary-institutional community: the marginalization of certain countries in such a trade network both polarizes wealth and capabilities of different countries and does not contribute to overall efficiency [Fagiolo et al., 2010].

3.2 Dynamics on Networks

The topology of networks is fundamentally important in economic contexts because the structure of a firm's (or an agent's) environment determines to a large part the risks it has to confront as well as its strategic options and its potential to use them. It is therefore crucial to understand why there are so many stable regularities of the social networks we considered so far.

To answer this question one should pay particular attention on what happens on the networks: the development of such networks tends to depend on both the decisions of the firm or agent representing the vertices and their success in surviving immediate threats both alone and in conjunction with their local environment. Less successful parts of the network will be more volatile or may fail completely and can (in cases in which persistent regularities emerge) lead to an evolution-like self-organization of the network as a whole. It is obvious that structure and function of properties of the network are interlocked in this case and will co-evolve. This structure-function relationship is one of the research frontiers of modern network science.

One of the most natural approaches to model strategic interaction on networks is to study games played on graphs. For simple networks, analytical results on how the structure of networks affects the outcome of games played are available (e.g. [Kets et al., 2011]) but the effect of complex, empirical networks must usually be studied via simulations. Pacheco et al. [Pacheco et al., 2009] for example study how network structure affects the performance of different strategies in the Prisoners Dilemma. This aligns well with the institutionalist literature on the topic of economic trust and recognized interdependence, and many complementarities are to be exploited.

Various studies analyzed the emergence of an commodity-exchange equilibrium on networks [Albin and Foley, 1992, Gintis, 2007, Axtell, 2005] and the emergence of social classes [Axtell et al., 2001] in bargaining games on networks among other things.

Further, it was found that models from epidemics are well-suited to model information diffusion and technology adoption (adapted from models of the diffusion of epidemics). A distinguishing factor for such models is that diffusion speed depends crucially on the current prevalence of the property in question and the size of the reachable population as well as the properties of the network. Further, the case of technology diffusion likely involves network effects which are quite different from global network effects [Arthur, 1989] which are monopolizing. Local network effects in small-world networks and scale-free networks may, depending on the parameters of the network, allow for either monopolization or for the persistence of niches of minority technologies [Uchida and Shirayama, 2008, Pegoretti et al., 2009].

4 Open question and fruitful avenues for further research

Network theory made it possible to apply a realistic representation of the micro-layer of economic interaction to formal models. This is something that indirectly follows from the vehement institutionalist criticism of static equilibrium models [Veblen, 1898, Polanyi, 1944] and that was with the progessive development of network theory gradually applied to economics (see [Schelling, 1971, Bowles and Gintis, 1975, Nelson and Winter, 1982, Kauffman, 1993] and many more).

Only more recent advances from the field of complex networks allowed the systematic investigation of the various kinds of networks in found in economics systems. It is now understood that social networks found in reality exhibit surprising statistical regularities: the small-world property, clustering, scale-free degree distributions or the core-periphery structure are very common structures found in very different contexts.

However, there are still gaping holes in our understanding of networks and many promising paths of research are not yet exhaustively explored. The reasons are twofold. First, for many characteristics of real world networks we are still not able to mathematically describe their generating mechanisms, or their properties. Particularly at a more fine grained level of description, and particularly for more complex networks, only descriptive, but no *mechanismic* models are available yet.

The term *mechanismic* comes from [Bunge, 2011] who uses it to distinguish mechanismic from instrumental or hermeneutic explanations. A *mechansmic* explanation proposes a particular *mechanism* that has led to the creation of the network. It provides a much deeper insight into the system under investigation than the alternative modes of explanation because it not only illustrates *what* is the state of the system under investigation, but also *why* the system is in this particular state.[6] Developing more mechanismic models for social networks is a key area for further research.

Second, the available data is limited. There is some data on corporate ownership networks, trade, and supply networks. Many of these are, however, from particular contexts or particular isolated sources which does not allow continuous and comprehensive observation of the network's development. Innovation networks can be inferred from patent and funding co-applications. But this is an indirect observation; it does not allow to make any observations on the number and network theoretic characteristics of unsuccessful R&D cooperations nor does it yield conclusive evidence on the lengths of cooperations and the possible role of the network in this. In fact, models often assume a homogenous cooperation duration of 3 years, a potentially unrealistic assumption. Tie termination processes are as important as tie formations. Hence, we need to spend much more time and resourcen developing models and gathering comprehensive data on tie termination processes [Schilling and Phelps, 2007]. Data on social and professional networks is scarce for reasons of privacy and the difficulty

[6] See [Gräbner and Kapeller, 2015] for a more detailed epistemological and ontological discussion of the concepts of *social mechanisms* and *mechanismic explanations* and their usefulness for economic research.

to gather such data. Thus, a potentially large number of networks relevant for economics remains completely unstudied.

Despite these open issues, we argue that the progress of network theory and its application to economics is good news for social and institutionalist economics. Scholars in these traditions have maintained the importance of studying direct interactions between humans in a realistic, non-aggregated, non-idealized way [Elsner, 2014]. The infeasibility of representing this in formal models has complicated progress in these areas. This, however, is changing: with network theory and agent-based simulations, the study of social interactions, institutions, and many other issues, can also be extended to and beyond what has already been accomplished for network models in innovation economics, financial economics (where structural instability is thoroughly investigated since the recent economic crisis), and information economics [Gräbner, 2015].

We therefore close this article by emphasizing that the two camps of network scientists and socio-economists have much to offer to each other: network theorists developed a huge set of tools to model networks and to identify their empirical regularities. Socio-economists produced a tremendous amount of knowledge about socio-economic mechanisms that may help to develop the mechanismic explanations of the structure of the socio-economic systems we see. Only with the knowledge about these mechanisms we will be able to proceed significantly in our understanding of the complex socio-economic systems we are interested in.

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Clustering			
Reference	*Content*	*Methods*	*Main findings*
Schelling, T. C. (1971). Dynamic models of segregation. *Journal of Mathematical Sociology*, 1:143–186.	- Segregation as result from the interplay and dynamics of interactive individual choices - Distribution of members in neighborhoods defined by reference to their own location - Second model deals with compartmented space - Deals also with 'neighborhood tipping'	- One simulation model and one analytic model	- No simple correspondence of individual incentive to collective results - Exaggerated separation and patterning result from the dynamics of movement - Inferences about individual motives cannot be drawn from aggregate patterns
Krugman, P. (1997). How the economy organizes itself in space: A survey of the new economic geography. In Arthur, W. B., Durlauf, S., and Lane, D., editors, *The Economy as an Evolving Complex System II*, pages 339-362.Westview Press, Reading, MA.	- Geography of the economy / spatial economy as self-organizing system characterized by interaction of individual decisions - Evolution of spatial economy and emergence of spatial structure	- Illustration and description of models and major lines of research - Discussion of implication for future work	
Arthur, W. B. (1990). 'Silicon Valley' locational clusters: When do increasing returns imply monopoly? *Mathematical Social Sciences*, 19(3):235-251.	- Increasing returns and dominant markets in the context of industrial location - Model of industry location: when do economies of agglomeration lead to a single dominant location monopolizing the industry?	- Modelling industrial location	i) No upper bound to locational increasing returns due to agglomeration, leads to a clustered dominant location ii) An upper bound can produce a monopoly by certain sequences of firm entry, or can lead to a sharing of industry
Elsner, W. (2014). Social economics and evolutionary institutionalism today. Theoretical components and heterodox convergence in a socio-economic perspective. *Forum for Social Economics*.	- Theoretical and methodological discussion of the constitution of social economics - Elements of evolutionary (Veblenian) institutional economics are considered - Focus on "heterodox" convergence - Explanation real-world forms of market, hierarchies and spatial clusters	- Discussion of theoretical concepts and methodology (complex modeling, game theory, computer simulations)	
Karrer, B. and Newman, M. E. J. (2011). Stochastic blockmodels and community structure in networks. *Phys. Rev. E*, 83:016107.	- Stochastic blockmodels as a tool for detecting community structure in networks - Variation in vertex degree and generalization of blockmodels which leads to an improved objective function for community detection in complex networks	- Generalized blockmodels - Heuristic algorithm using an objective function or its non-degree-corrected counterpart	- Degree-corrected version outperforms the uncorrected one in real-world and synthetic networks

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Small World Property			
Reference	*Content*	*Methods*	*Main findings*
Milgram, S. (1967). The small world problem. *Psychology today*, 2(1):60-67.	- Concept and structure of small world phenomenon - Examination of the average path length for social networks (six degrees of separation)	- Experiments with randomly selected citizens of the United States	- Social networks are small world networks in which in average everyone is six or fewer steps away
Watts, D. J. and Strogatz, S. H. (1998). Collective dynamics of 'small-world' networks. *Nature*, 393(6684):440-442	- Characteristics and dynamics of small world networks and connection topology - Neural network of the worm *Caenorhabditis elegans*, the power grid of the western United States, and the collaboration graph of film actors are shown to be small-world networks	- Exploration of simple models of networks with rewiring by introduction of increasing amount of disorder - Generating algorithm for small-world networks	- Small world networks can be highly clustered, like regular lattices, and have small path lengths, like random graphs - Enhanced signal propagation speed, computational power and synchronizability - Infectious diseases spread more easily in small-world networks than in regular lattices
Baum, J. A. C., Calabrese, T., and Silverman, B. S. (2000). Don't go it alone: alliance network composition and startups' performance in Canadian biotechnology. *Strategic Management Journal*, 21(3):267-294.	- Combination of theory and research on alliance networks - Impact of variation in startups' alliance network composition on early performance	- Analysis of Canadian biotech startups' performance with panel data	- Alliance network configuration at the time of founding affect early performance - Enhancement of performance by established alliances, access to information alliances with rivals - Explanation of how and why firm age and size affect firm performance
Uzzi, B. and Spiro, J. (2005). Collaboration and creativity: The small world problem. *American Journal of Sociology*, 111(2):447-504.	- Dynamics of small world networks - Impact of small world properties on performance	- Analysis of the small world network of the creative artists who made Broadway musicals from 1945 to 1989	- Varying "small world" properties of the systemic-level network affects creativity in terms of financial and artistic performance
Fleming, L., Charles King, I., and Juda, A. I. (2007). Small worlds and regional innovation. *Organization Science*, 18(6):938-954.	- Small-world networks in regional context - Investigation of the effects of collaboration networks on innovation	- Development and exploitation of a novel database on patent coauthorship using statistical models	- Existence of regional small-world structures enhance innovative productivity within geographic regions - Shorter path lengths and larger connected components correlate with innovation
Schilling, M. A. and Phelps, C. C. 2007). Interfirm collaboration networks: The impact of large-scale network structure on firm innovation. *Management Science*, 53(7):1113–1126.	- Influence of alliance networks structure on potential knowledge creation / innovation	- Longitudinal study of the patent performance of 1,106 firms in 11 industry-level alliance networks	- Interfirm collaboration networks with short path lengths and which are highly clustered have greater innovative output

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Heavy Tails and Scaling			
Reference	*Content*	*Methods*	*Main findings*
Price, D. D. S. (1976). A general theory of bibliometric and other cumulative advantage processes. *Journal of the American Society for Information Science*, 27(5):292-306.	- Cumulative Advantage Distribution is proposed which models the situation in which success breeds success	- Modelling Distributions	- Stochastic law is governed by the Beta Function containing only one free parameter which approximated diverse social science phenomena
Barabási, A.-L. and Albert, R. (1999). Emergence of scaling in random networks. *Science*, 286(5439):509- 512.	- Emergence of networks - Property of large, complex networks: vertex connectivities following a scale-free power-law distribution	- Modelling scale-free distributions	- Networks expand continuously by the addition of new vertices - New vertices attach preferentially to sites that are already well connected - Development of large networks is governed by robust self-organizing phenomena
Podolny, J. M. (1994). Market uncertainty and the social character of economic exchange. *Administrative Science Quarterly*, 39(3):458-483.	- Organizations overcome the problems of market uncertainty in selecting exchange partners	- Study of investment banking relationships in the investment grade and non-investment-grade debt markets from 1981 to 1987	- The greater the market uncertainty the more organizations engage in exchanges relations they already have transacted in the past - The greater uncertainty, the more that organizations engage in transactions with those of similar status
Stuart, T. E., Hoang, H., and Hybels, R. C. (1999). Interorganizational endorsements and the performance of entrepreneurial ventures. *Administrative Science Quarterly*, 44(2):315-349.	- Examination of the ecological consequences of initial public offerings (IPOs) and acquisitions - Focus on how the spatial distribution influence the location-specific founding rates of new companies	- Count models of biotechnology firm	- IPOs of organizations located contiguous to or within an MSA (metropolitan statistical area) accelerate the founding rate within that MSA - Acquisitions of biotech firms situated near to or within an MSA accelerate the founding rate within the MSA when acquirer enters from outside of the biotech industry - Enforceability of post-employment non-compete covenants moderate these effects
Vázquez, A. (2003). Growing network with local rules: Preferential attachment, clustering hierarchy, and degree correlations. *Phys. Rev. E*, 67:056104.	- Linear preferential attachment hypothesis as explanation for the existence of networks with power-law degree distributions	- Analytical and numerical results of different local rules	- Effective linear preferential attachment is the natural outcome of growing network models based on local rules - Local models offer an explanation to other properties like the clustering hierarchy and degree correlations
Ghoshal, G., Chi, L., and Barabási, A.-L. (2013). Uncovering the role of elementary processes in network evolution. *Scientific Reports*, 3(2920).	- Identification of elementary mechanism and their role on network evolution - Focus on formation and deletion of connections	- Formulating and solving a model with minimal processes of network evolution	- Contribution to growth by formation of connections between existing pair of vertices, while others capture deletion - Dependence of the removal of a node with its corresponding edges, or the removal of an edge between a pair of vertices

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Core-Periphery Patterns			
Reference	*Content*	*Methods*	*Main findings*
Karrer, B. and Newman, M. E. J. (2011). Stochastic blockmodels and community structure in networks. *Phys. Rev. E*, 83:016107.	- Stochastic blockmodels as a tool for detecting community structure in networks - Variation in vertex degree and generalization of blockmodels	- Generalized blockmodels - Heuristic algorithm for community detection using an objective function or its non-degree-corrected counterpart	- Improved objective function for community detection - Degree-corrected version outperforms the uncorrected one in the real-world and synthetic networks
Hidalgo, C. A., Klinger, B., Barabási, A.-L., and Hausmann, R. (2007). The product space conditions the development of nations. *Science*, 17(5837):482-487.	- Characteristics and adaption of new products - Network of relatedness between products or “product space” and national technology and income level	- Empirical methods	- More-sophisticated products are located in a densely connected core - Less-sophisticated products occupy a less-connected periphery
Holme, P. (2005). Core-periphery organization of complex networks. *Phys. Rev. E,* 72:046111	- Measurement of the core-periphery dichotomy for a number of real-world and model networks - Focus on statistical properties of the core and of the *n* neighbors of the core vertices for increasing *n*	- A coefficient for the measurement of the core-periphery dichotomy is proposed	- Geographically embedded transportation networks have a strong core-periphery structure - Almost all networks have many edges within n neighborhoods at a certain distance from the core
Doreian, P. and Woodard, K. L. (1994). Defining and locating cores and boundaries of social networks. *Social Networks* 16:276-293.	- General procedure for locating the boundary of a network and for discerning the boundaries within a network - First: Expanding (snowball) selection procedure - Second: specification of two critical parameters: the value of *k* for a *k*-core and the threshold, *w*, for the quantitative magnitude of network ties	- Single sector and multi-sector social service inter-agency networks are used	- Method for locating cores and boundaries generates a sequence of nested cores as *k* and *w* are systematically changed
Borgatti, S. P. and Everett, M. G. (1999). Models of core/periphery structures. *Social Networks* 21:375-395.	- Concept and formalization of a core/periphery structure - algorithm for the detection of a dense, cohesive core and a sparse, unconnected periphery	- Statistical tests for testing a priori hypotheses	- Different models are presented for different kinds of graphs (directed and indirected, valued and nonvalued)
Silva, M. R. D., Ma, H., and Zeng, A. P. (2008). Centrality, network capacity, and modularity as parameters to analyze the core-periphery structure in metabolic networks. *Proceedings of the IEEE*, *96*(8), 1411-1420.	- Deals with genome-scale metabolic networks of organism and their core-periphery modular organization - Focus on hierarchical and modular structure of metabolic networks	Development of method with genome-scale metabolic networks of five representative organisms, which include Aeropyrum pernix, Bacillus subtilis, Escherichia coli, Saccharomyces cerevisiae, and Homo sapiens	- Proposes a parameter: the core coefficient which quantitatively evaluate the core-periphery structure of a metabolic network and which is based on the concepts of closeness centrality of metabolites and network capacity