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Abstract

We demonstrate how a prospective buyer's optimal home-size purchase can be determined by means of a stochastic-dominance (SD) analysis of historical data. Using monthly property yields in Hong Kong over a 15-year period to illustrate our procedure, and as a case in point to demonstrate the efficacy of the SD approach, we show that regardless of whether the buyer eschews risk, embraces risk, or is indifferent to it, in any adjacent pairing of five well-defined housing classes, the smaller class provides the optimal purchase, and thus that the smallest class affords the buyer the optimal purchase over all classes in this important housing market – at least where *rental* yields are of primary concern. In addition, we find that risk averters focusing on total yield would prefer to invest in the smallest and second- smallest classes than in the largest class.

Keywords: urban studies; stochastic dominance; probability; regional studies

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1. Introduction

The decision to purchase a particular home may involve a variety of considerations that run the gamut from a choice of an affordable lifestyle at one end of the spectrum to a "flippable" investment opportunity at the other end, to some combination of lifestyle and investment considerations in between. We would venture to suggest that few prospective home buyers can afford to overlook the possibility that they might be impelled to sell the home in the future. The focus of the present paper is on the determination of the optimal home-size purchase, *ceteris paribus*, and the stochastic-dominance (SD) approach to the resolution of the problem.

As a case in point we consider the housing market in Hong Kong, which Qiao and Wong (2015) also examined. They use SD test on housing prices in Hong Kong housing market to conclude that smaller houses seems are better investment choice. However, that result can only apply on risk averser as they mainly found higher-ordered SD relationship. Moreover, prices and rent are not separable in considering housing investment, especially for long-term investors like pension funds, REITs or even an individual owner, a combine view of both prices and rent is need to draw further conclusion. Thus, we propose the use of housing yield to tackle this problem.

Within this setting we apply the SD approach to the analysis of uncertain prospects, to compare the performance of five classes of housing. The distinctive contributions of this paper are: (a) to show that in Hong Kong, regardless of whether a prospective home buyer eschews risk, embraces risk, or is indifferent to it, in any adjacent pairing of the classes, the smaller class provides the optimal purchase, and thus that the smallest class affords the buyer the optimal expected wealth from rental yield over all classes, and for buyers focusing on total yield, it is preferred over the largest class; (b) to demonstrate the efficacy of the SD rule in the analysis of *any* real-estate market; and (c) to provide what we believe to be the first demonstration of first-order SD over an extended period of time.

The paper is organized as follows. The next section briefly introduce the housing market in Hong Kong, review the literature and motivate the analysis to follow. After that, we describe the data and the SD approach, following which our empirical results are presented. The final section offers our conclusions.

2. Literature review

2.1 Hong Kong housing market

Hong Kong, which on July 1, 1997, became a Special Administrative Region (SAR) of the People's Republic of China, to be governed under a policy of "one country, two systems." As such, the territory's economy in general and its housing market in particular, is subject to the vicissitudes of both the mainland economy and the other Asian economies. Yet it remains one of the most important, vibrant, and popular housing markets in the world, Association of Foreign Investors in Real Estate listed Hong Kong as the top-10 cities in which to invest in housing in 2012.[1] Hong Kong is also as one of the world's 10 most important over the past 10 years and top 5 in world's largest financial center and the "global City" in 2013 by Knight Frank.[2]

Hong Kong is also famous in high populated densely, with only 7 million people occupying less than 300 km square of built-up areas. Hong Kong's residential density is also one of the world's densest, with near 7000 people living in each km square, higher than Tokyo and London. As a result, Hong Kong is one of the most expensive housing market in the world in terms of prices and rents. Moreover, Hong Kong housing market is also is one of the most volatile housing market (Xiao and Liu, 2010). In 2014, there were 1.14 million private housing units in Hong Kong, and the value of registered agreements for the sale of private housing units reached 257 billion Hong Kong dollars. Besides, the

[1] 2012 Foreign Investment Survey, Association of Foreign Investors in Real Estate, http://afire.org/annual-foreign-investment-survey

[2] The Wealth Report, Knight Frank Research. http://www.thewealthreport.net

vacancy rate of private housing units reached a 17-years low of 3.8 in 2014, which peaked at 6.8% after Asian Financial Crisis in 1997.[3]

Moreover, previous studies mostly focus on the well-developed housing market such as US and Euro Region, but emerging markets were relatively less focused. Thus our study on Hong Kong housing market offers a different view on existing literature in the era of Asia.

2.2 Housing investment

Purchasing a house plays an important role in both consumption and investment decisions (Henderson and Ioannides, 1987). Economists have mainly focused on the consumption aspects of this process, while academics such as Dusansky and Koç (2007), Hiebert and Sydow (2011), and Paciorek and Sinai (2012) have shown that one should not ignore the investment aspects of the purchasing process, since most buyers consider housing to be a "lifetime" investment.

Some authors would advocate buying larger houses for investment portfolios. For example, Ziering and McIntosh (2000) find that housing size is an important factor in determining the risk and return of housing, and conclude that the largest class of housing provides investors with the highest return and also the greatest volatility. Flavin and Nakagawa (2002) and Flavin and Yamishita (2008), however, suggest that investing in larger housing does not necessarily reduce risk. By way of explanation, Krainer (2001), McMillen (2008), and Ihlanfeldt and Mayock (2012) point out that a spectrum of prices, rather than a single price, exist in the housing market. Still further, Zimmer (2012) suggests that huge housing portfolios are a source of instability in the market.

While the "size effect" in housing portfolios has been adequately explored, only a few studies have attempted to link the size of housing directly to housing investment. For example, Kallberg *et al*. (1996) show that since smaller-property values have lower correlations with stocks and bonds, they offer particularly impactful diversification

[3] Census and Statistics Department- The Government of the Hong Kong Special Administrative Region, http://www.censtatd.gov.hk

benefits for investment portfolios with high-return aspirations. By contrast, Graff and Young (1996), Linand Liu (2008), and Zimmer (2012) suggest that the link between housing size and housing performance needs to be further investigated, with a particular need for the accurate measurement of the return and risk for different house sizes.

Turnbull *et al*. (2006) directly broach the optimal house-size question, looking at the sales prices of houses in a Louisiana parish over an almost six-year period. They draw their inferences from the parameter estimates of a two-stage least-squares linear regression that includes, among its independent variables, living area. Their results imply that the answer to the question, even for that parish, which they acknowledge might not apply elsewhere, is "it depends." That is, for example, "smaller houses in a neighborhood of larger houses sell at a premium relative to small houses in small-house neighborhoods" (Turnbull*et al*.,2006, p. 453).

There are different approaches to analyzing housing returns and risk. Early studies such as Topel and Rosen (1988) and Archer *et al.* (1996) suggest that housing returns can be expressed in terms of a set of macroeconomic variables. Harter-Dreiman (2004) studies the housing market by forming a link between housing prices and economic cycles. Cannon *et al.* (2006) explain housing returns by volatility, price level and stock-market risk, whereas Ghent and Owyang (2010) investigate supply and demand to explain movements in the housing market. Qiao and Wong (2015) further examined housing prices by adopting SD test on different housing size in Hong Kong but did not find any superior housing size for all investors.

Alternatively, there are suggestions that the return to housing investment should be viewed in terms of yields. For example, Campbell and Shiller (1988) and Clayton (1996) provide strong support for the theory that the yield in the housing market is similar to the dividend-price ratio in the financial market. Leamer (2002) states that housing prices should reflect the present value of future rent, and further that investors should use the same calculations when purchasing a house as they do when purchasing a stock. Dreman

(1982) and Gallin (2008) show the importance of using yields in housing investment, because yields can be a good indicator of housing investment under the concept that rent is a prime element of the value of housing. Nevertheless, Addae-Dapaah *et al*. (2010) find that the contrary strategy also works in real-estate investment, implying that high-yield housing could precede low-yield housing. Moreover, Ayuso and Restoy (2006) suggest that housing yields possess the merit of circumventing the specification of the user cost and/or of the market price of housing services.

To conclude, using yield could finesse the problems arising from dealing with both prices and rent. In what follows we therefore treat housing investment as a type of annuity and consider yields in the analysis.

2.3 Stochastic dominance

SD theory, which originated with Hadar and Russell (1969) and Hanoch and Levy (1969), is one of the most powerful instruments with which to compare investment prospects under uncertainty, and the Hong Kong real-estate market, particularly during our 15-yearsample period that extends from 1999 through 2013, is an exemplar of such. The period is an especially propitious one for study, since it marks the beginning of the territory's recovery from the almost two-year Asian financial crisis that began in July 1997, and spans the global financial crisis and economic meltdown that began in mid-September 2008. The instrument's power derives from the fact that different SD relationships correspond to different sets of risk preferences.

Early theoretical studies such as Hanoch and Levy (1969) linked SD theory to the selection rule for risk avoiders under various constraints on their risk-preference functions. SD theory has also been extended to other types of investors. As will be seen, our results satisfy first-order SD for risk avoiders, and the latter implies first-order SD for risk takers as well.

Early studies such as Levy and Sarnat (1970), Porter (1973) and Joy and Porter (1974) applied the SD rule empirically but did not discuss the testing procedure. Some significant SD tests have recently been developed. For example, Barrett and Donald (2003) exploit a Kolmogorov-Smirnov-type test and Linton *et al*. (2005) further relax the i.i.d. assumption. In addition, Anderson (1996) and Davidson and Duclos (2000) develop SD tests that examine the underlying distributions at a finite number of grid points. Armed with these powerful tests, the SD approach becomes widely applicable. Most critically, and as detailed in the Technical Appendix, first-order stochastic dominance (FSD) of one investment prospect over another implies that regardless of one's risk preferences, the dominant prospect is the preferred investment. We apply the Davidson and Duclos (2000) DD test to make this determination.

3. The data

We employ monthly property-market rental yields in private domestic units of five different housing classes (saleable area) from January 1999 to December 2013. The data are obtained from the Rating and Valuation Department of the Hong Kong SAR. The monthly rental yields for each class are calculated by dividing the average rent within that class by the average sale price for houses in that class, in that month. These average rents and sales prices are government estimates that are based on that month's transactions. Thus the data provide a broad indication of market yields and trends.

Total yield is defined as monthly rental yields plus the monthly price return. Private domestic units are defined as independent dwellings with separate cooking facilities and bathroom (and/or lavatory). They are sub-divided into five classes by reference to floor area: Class A - saleable area less than 40 m^2; Class B - saleable area of 40 m^2 to 69.9 m^2; Class C - saleable area of 70 m^2 to 99.9 m^2; Class D - saleable area of 100 m^2 to 159.9 m^2; and Class E - saleable area of 160 m^2 or above.

The data comprise private second-hand sales, and exclude public-sector development housing, and primary sales. The descriptive statistics for rental yields are shown in Table 1. The yield data exclude fees and taxes, as Hong Kong is a low-tax region, one in which property tax is charged on a proportional basis at an effective rate of around 12% on rental income. This helps to make Hong Kong an ideal case to study, since the results are not impacted by high or non-proportional transition costs.

Indeed, the dataset has a number of advantages. First, it provides official monthly data for both rents and prices for all five classes. Second, each yield is calculated based on all the rental and sale transactions in each month. Third, since Hong Kong is a city, the data reduce the problems that arise from differences between different areas, thus ameliorating the problems of age, location, and other housing attributes.

4. The empirical results: Rental yield

4.1 The visual evidence

Figure 1 provides the time-series plots of the rental yields for all five classes over the entire period of study, revealing that the yield of Class A rests at the top, with that of Class B following, and so on down the line until we get to the yield of Class E at the bottom, thus suggesting that over the entire period the smallest house provides the highest yield and the largest house the smallest yield.

Figure 2 shows the density plots of rental yields from all five classes. The plots provoke an initial impression that FSD could exist among all yields of different classes, because some probability density functions (PDFs) are on the right-hand side of some others. The plots show that some PDFs have higher yields. For example, Class E ranges from 0.18% to 0.45%, while Class A ranges from 0.26% to 0.54%.

As an initial indicator of whether there might be any dominance relationships, we also plot the cumulative distributions of the rental yields from the five different classes, in Figure 3. The figure shows a provocative phenomenon: namely, the cumulative distribution function (CDF) of Class A is at the bottom, while that of Class B is on top of Class A, and so on until the top line is the CDF of Class E. The class on the bottom is the most preferred because, at any point, the value of its CDF is the smallest, implying that it has the least probability of gaining less (or, equivalently, the least probability of losing more) up to this point, and thus, it has the highest probability of gaining more. In this sense, Figure 3 shows that Class A dominates Class B, which dominates Class C, Class D, and then Class E.

4.2 The Davidson and Duclos test

The dominance relationship exhibited in Figure 3 only gives us some ideas as to the dominance relationship among different classes, but it does not properly *test* the relationship itself. To formally conduct the test, we employ the Davidson and Duclos (2000) DD test statistic to make pairwise comparisons of the five classes. We compute the values of the statistic over a grid of 100 comprising the 10 major, and 10 minor, partitions on the monthly rental yields. Table 2 records the percentage of significant DD statistics, based on the simulated critical value suggested by Bai *et al*. (2011).The DD statistics, T_1,T_2, and T_3, are negative in the entire range of the yield distribution, most of the range is significantly negative, and no portion of T_j is positive.

For an improved perspective of the DD test comparison, we plot the DD test and the corresponding CDFs for each pair of yields from different classes, although only the plots of Classes A and E are displayed in Figure 4, since the plots of all other pairs of distributions are similar to those displayed in the figure. In tandem, the plot of the DD test and the corresponding CDFs implies that Class A dominates Class E in first, second, and third order, as detailed in A.1 below, since T_1,T_2, and T_3 are significantly negative for 94%, 97%, and 99% of the distribution, respectively. Similarly, Class B dominates Class E in first, second, and third order, as T_1, T_2, and T_3 are significantly negative for 89%, 99%, and 99% of the distribution, respectively. The results from Table 3 show FSD among all yields. That is, Class A dominates Classes B, C, D, and E; Class B dominates Classes C, D, and E; Class C dominates Classes D and E; and Class D dominates Class E, in the sense of FSD. We summarize the SD results in Table 3.

These results thus imply that investors can increase their expected utility (in the von Neumann-Morgenstern sense) as well as their expected wealth, by shifting their investment from Class E to Class D, from Class D to Class C, from Class C to Class B, and from Class B to Class A.

5. The empirical results: Total yield

Some investors may not be interested solely in getting high rental yields. Rather, their interest could be focused on the *total* yield of their properties. The latter is defined as the rental yield plus the price return or the price-appreciation rate. Thus, in this section we investigate the performance of the monthly total yields for the five different classes of private domestic units, to complement the findings for rental yields alone.

To help analyze the performance of the monthly total yields, and comparable to Figures 1-3 on rental yields, Figures 5-7 show, respectively, the time-series plots of total yields, the density plot of total yields, and the cumulative distribution plot of total yields, from all classes over the period of study.

Focusing first on Figure 5, the time-series plots of total yields for the five housing classes, unlike the pattern depicted in Figure 1 for rental yields alone, we do not detect any apparent dominance from any particular class, where total yields are concerned. But the cumulative distributions plots of all the total yields, shown in Figure 6, reveals the dominance of the smaller houses over the largest house. Inasmuch as in tandem the density and cumulative distribution plots of total yields do not immediately presage the dominance of any one house class, we conjecture that FSD is not present in regard to total yields, and confirm this conjecture from the DD results, which are displayed in Table 5.

Specifically, none of the FSD tests is significant for any pairs other than classes B and E. For those two classes, only one percent are significantly greater than zero, while another one percent are significantly less than zero. Nonetheless, there might well be higher-order SD among the different classes. Thus, we move on to examine the higher-order DD tests, the details of which are presented in A.1.

The second-order DD (SSD) test shows that only Class A SSD-dominates Class E, in that 30 percent of the SSD DD tests are significantly less than zero, and Class B marginally SSD-dominates Class E, in that four percent of the SSD tests are significantly less than zero. Further, the third-order test (TSD) shows that Class A TSD-dominates Classes C and E, and marginally TSD-dominates Class D, in that only one percent of the TSD DD tests are significantly less than zero, while Class B TSD-dominates Class E. Since a hierarchical relationship exists in SD (Levy, 1992), Table 6 only reports the lowest dominance order of SD. Some such as Fong *et al*. (2005) and Qiao *et al.* (2012), however, evince a disdain for "almost-SD" results (Leshno and Levy, 2002; Guo *et al*. 2014) and do not consider that there is an SD relationship between the asses if only a small number, say five percent, of the SD tests are significant. We refer to such as a marginal SD relationship. Based on this standard, as shown in Table 6 we determine that Class B marginally SSD-dominates Class E and strongly TSD-dominates Class E

6. Three additional inferences

Although they come as something of asides, we would be remiss if we failed to mention three additional inferences that may be drawn from our results.

6.1 Arbitrage opportunities

Jarrow (1986) shows that under certain regularity conditions investment Y dominates investment Z in FSD if and only if there is an arbitrage opportunity between Y and Z, and that non-satiated investors can increase their wealth by shifting investments from Y to Z. Arbitrage opportunities, however, may not exist even in the presence of FSD (Wong *et al.*, 2008). Nonetheless, investors can increase their expected wealth and expected utility by shifting their holdings from the dominated asset to the dominant one. Thus, FSD is a necessary but not sufficient condition for the existence of an arbitrage opportunity.

We have established that the smallest class FSD-dominates the other classes in regard to rental yields. But does this imply that there is an arbitrage opportunity? The answer is in the negative, because only the rental yield of the smallest class, and not the total yield, FSD-dominates those of the other classes. The total yield of the smallest class only SSD-or TSD-dominates those of the other classes, not FSD. Thus, if investors shift their housing investments from the other classes of housing to the smallest housing class, they could only increase their rental yield, not the total yield. In this sense, there is no arbitrage opportunity in the Hong Kong housing market.

6.2 Market efficiency

If one is able to earn an abnormal return, the market is considered inefficient. Market efficiency can be examined by the SD rule as follows. When non-satiated investors can increase their expected wealth by switching their choice of housing purchase, this implies an inefficient market (Falk and Levy, 1989).Thus, market efficiency can be rejected if FSD exists. Having established that the smallest class FSD-dominates the other classes in rental yields, does this finding imply that the housing market in Hong Kong is not efficient. Once again our answer is in the negative, and again because only the rental yield of the smallest class FSD-dominates those of the other classes, but not the total yield. The total yield of the smallest class only SSD-dominates or TSD-dominates those of the other classes. There is no first-order domination. Thus, if investors shift their housing investments from the other classes of housing to the smallest housing class, they could only increase their rental yield, not the total yield. In this sense, the market-efficiency hypothesis cannot be rejected for the Hong Kong housing market.

6.3 *Housing consumption*

Our focus has been primarily on the investment aspect of housing. Housing, however, serves as both consumption and investment (Henderson and Ioannides, 1987). In general, the consumption value of housing can be separated into two parts: use value, or the utility derived from living in the home, and ownership value or the utility derived from ownership of the asset that the home represents. There are two ways to enjoy the use value: one is buying a house, and the other is renting a house. Under this condition, we extend our study to housing consumption.

By renting a house and financing it with the rent collected from the small leased house, one can capture the use value of one's preferred house, with the minimum cost. In this sense, our findings suggest that buying a small house is still the best way to fulfill one's accommodation needs and at the same time maximize one's rental return – at least in Hong Kong.

As to ownership value, since some individuals may prefer to own big houses not only for their market value but also for status and other reasons, it is nearly impossible to measure ownership value and compare it for different houses and different individuals.

7. Conclusions

Should one purchase a larger home or a smaller home? We answer that question by treating housing as a long-term investment that provides a type of annuity. This is important for long-term investors like pension funds, REITs or even an individual owner to obtain higher return. We then take advantage of the stochastic-dominance rule to compare the rental yields of different housing classes using data from the Hong Kong housing market to demonstrate the efficacy of the proffered procedure. While the results are in and of themselves of interest, insofar as the Hong Kong housing market is a major component of world investment portfolios, more critically we provide a template for applying the stochastic-dominance approach to other real-estate markets where the results will not necessarily mirror those of Hong Kong.

When we apply the rule in Hong Kong, however, we find that the smallest housing class demonstrates first-order dominance over all other classes in term of rental yields, which implies that by investing in the smallest class of housing, regardless of the shape of one's risk-preference function and attitude towards risk, investors can maximize their expected utility and expected wealth from rentals.

When total yields are taken into consideration, however, we do not find any first-order dominance among all the five classes. In this sense, investors shifting their investment from one class of house to another class of house in Hong Kong cannot obtain a higher total yield.

Our findings and inferences fill in the missing part of Qiao and Wong (2015), which they examined housing prices in Hong Kong, that they did not consider rental yields for different classes of housing, nor did they obtain any first-order SD results. Moreover, we herein employ a more recently-developed SD test, exposed in Bai et al. (2011, 2015), which simulations reveal that our test is more powerful, and that our critical value is closer to the true critical value. In addition, our test overcomes the problem of dependence among the chosen grid points.

Finally, “Should one purchase a larger home or a smaller home?” Our answer is clear in Hong Kong housing market. For long-term investors, smaller home; for others, it depends.

A. Technical appendix

A.1 Stochastic-dominance theory

Let F and G be the cumulative distribution functions (CDFs), and f and g be the corresponding probability density functions (PDFs) of two investments, Y and Z, respectively, with common support of $[a, b]$, where $a < b$, and respective means of μ_y and μ_z.

Define

$H_0 = h$ and $H_j(x) = \int_a^x H_{j-1}(t)\,dt$ for $h = f, g$, $H = F, G$, and $j = 1, 2, 3$. (1)

The most frequently-used SD rule is compatible with three broadly-defined risk-avoiders' risk-preference functions: notably, first-order, second-order, and third-order SD, denoted FSD, SSD, and TSD, respectively. The SD rules are: Y dominates Z by FSD, denoted $Y \succ_1 Z$, if and only if $F_1(x) \le G_1(x)$; Y dominates Z by SSD, denoted $Y \succ_2 Z$, if and only if $F_2(x) \le G_2(x)$; and finally Y dominates Z by TSD, denoted $Y \succ_3 Z$, if and only if $F_3(x) \le G_3(x)$ for all possible returns x, $\mu_y \ge \mu_z$, and a strict inequality for a non-empty interval of x.

Investigating the SD relationship among different investments is equivalent to examining the choice of investments by expected-utility maximization. SD implies that holding the dominant asset always gives a higher expected utility than holding the dominated assets. Under FSD, investors will exhibit non-satiation (more is preferred to less); under SSD, investors will have the additional characteristic of risk aversion; and under TSD they also have decreasing absolute risk aversion. Since a hierarchical relationship exists in SD (Levy 1992), which means FSD implies SSD and TSD, only the lowest dominance order of SD is reported.

A.2 The Davidson and Duclos test

Let $\{(y_i, z_i)\}$ be pairs of observations drawn from the random variables Y and Z, with distribution functions $F(x)$ and $G(x)$, respectively, for market yields of private domestic units from two different classes. The integrals $F_j(x)$ and $G_j(x)$ for F and G are defined in (1) for j = 1, 2, 3. For a grid of pre-selected points x_1, x_2... x_k, the j^{th}- order DD test statistic, $T_j(x)$ (j = 1, 2 and 3), is:

$$T_j(x) = \frac{\hat{F}_j(x) - \hat{G}_j(x)}{\sqrt{\hat{V}_j(x)}}, \quad (2)$$

where $\hat{V}_j(x) = \hat{V}_{F,j}(x) + \hat{V}_{G,j}(x) - 2\hat{V}_{FG,j}(x)$,

$$\hat{H}_j(x) = \frac{1}{N(j-1)!}\sum_{i=1}^{N}(x-h_i)_+^{j-1},$$

$$\hat{V}_{H,j}(x) = \frac{1}{N}\left[\frac{1}{N\big((j-1)!\big)^2}\sum_{i=1}^{N}(x-h_i)_+^{2(j-1)} - \hat{H}_j(x)^2\right],\ H = F, G, h = f, g,$$

$$\hat{V}_{FG,j}(x) = \frac{1}{N}\left[\frac{1}{N\big((j-1)!\big)^2}\sum_{i=1}^{N}(x-f_i)_+^{j-1}(x-g_i)_+^{j-1} - \hat{F}_j(x)\hat{G}_j(x)\right].$$

It is not possible to test the null hypothesis for the full support of distributions, empirically. Thus, Bishop *et al*. (1992) propose testing the null hypothesis for a pre-designated finite number of values of x. The following four hypotheses are tested:

$H_0: F_j(x_i) = G_j(x_i)$, for all $x_i, i = 1,2,\dots,k$;

$H_A: F_j(x_i) \neq G_j(x_i)$, for some x_i;

$H_{A1}: F_j(x_i) \leq G_j(x_i)$, for all x_i, $F_j(x_i) < G_j(x_i)$ for some x_i;

$H_{A2}: F_j(x_i) \geq G_j(x_i)$, for all x_i, $F_j(x_i) > G_j(x_i)$ for some x_i. (3)

To conduct the DD test, the $T_j(x)$ at each grid point is computed and the null hypothesis, H_0, is rejected if $T_j(x)$ is significant at any grid point. Accepting either H_0 or H_A implies that there is no SD relationship between F and G. If H_{A1} (H_{A2}) of order one is accepted, F (G) dominates G (F) in FSD. If H_{A1} or H_{A2} is accepted for orders two or three, SD exists in the second or third order. In this situation, switching from the dominated asset to the dominant one will only increase risk-avoiders' expected utilities but not their expected wealth (Falk and Levy, 1989).

The DD test compares the distributions at a finite number of grid points. In order to make the comparisons comprehensive, we make 10 major partitions with 10 minor partitions within any two consecutive major partitions in each comparison. Bai *et al*.

(2011, 2015) improve the DD test by deriving the limiting process of the DD statistic $T_j(x)$, and hence, the DD test can be performed by $\max_x |T_j(x)|$ to take care of the dependency of the partitions. Their procedure is employed in carrying out the test.

References

Addae-Dapaah K, Webb JR, Ho KH and Tan YF (2010). Industrial real estate investment: Does the contrarian strategy work? *Journal of Real Estate Finance and Economics* **41**(2):193-227.

Anderson G (1996). Nonparametric tests of stochastic dominance in income distributions. *Econometrica* **64**(5): 1183-1193.

Archer WR, Gatzlaff DH and Ling DC (1996). Measuring the importance of location in house price appreciation. *Journal of UrbanEconomics* **40**(3):334-353.

Ayuso J and Restoy F (2006). House prices and rents: An equilibrium asset pricing approach. *Journal of Empirical Finance* **13**(3): 371-388.

Bai ZD, Li H, Liu HX and Wong WK (2011). Test statistics for prospect and Markowitz stochastic dominances with applications. *Econometrics Journal* **14**(2): 278-303.

Bai ZD, Li H, McAleer M and Wong WK (2015). Stochastic dominance statistics for risk averters and risk seekers: An analysis of stock preferences for USA and China. *Quantitative Finance*, forthcoming.

Barrett G and Donald S (2003). Consistent tests for stochastic dominance. *Econometrica*, **71**(1): 71-104.

Bishop JA, Formly JP and Thistle PD (1992). Convergence of the south and non-South income distributions. 1969-1979. *American Economic Review* **82**(1): 262-272.

Campbell JY and Shiller RJ (1988). Stock prices, earnings, and expected dividends. *Journal of Finance* **43**(3): 661-676.

Cannon S, Miller NG and Pandher GS (2006). Risk and return in the U.S. housing market: A cross-sectional asset-pricing approach. *Real Estate Economics* **34**(4): 519-552.

Clayton J (1996). Rational expectations, market fundamentals and housing price volatility. *Real Estate Economics* **24**(4): 441-470.

Davidson R and Duclos JY (2000). Statistical inference for stochastic dominance and or the measurement of poverty and inequality. *Econometrica* **68**(6): 1435-1464.

Dreman D. (1982). The New Contrarian Investment Strategy. Random House: New York.

Dusansky R and Koç C (2007). The capital gains effect in the demand for housing. *Journal of Urban Economics* **61**(2): 287-298.

Falk H and Levy H (1989). Market reaction to quarterly earnings' announcements: A

stochastic-dominance based test of market efficiency. *Management Science* **35**(4): 425-446.
Flavin M and Nakagawa S (2008). A model of housing in the presence of adjustment costs: A structural interpretation of habit persistence. *American Economic Review* **98**(1): 474-495.
Flavin M and Yamashita T (2002). Owner-occupied housing and the composition of the household portfolio over the life cycle. *American Economic Review* **92**(1): 345-362.
Fong WM, Wong, WK and Lean HH (2005). Stochastic dominance and the rationality of the momentum effect across markets. *Journal of Financial Markets* **8**: 89-109.
Gallin J. (2008).The long-run relationship between house prices and rents. *Real Estate Economics* **36**(4): 635-658.
Ghent AC and Owyang MT (2010). Is housing the business cycle? Evidence from US cities. *Journal of Urban Economics* **67**(3): 336-351.
Graff RA and Young MS (1996). Real estate correlations: Real-world limitations on relationships inferred from NCREIF data. *Journal of Real Estate Finance and Economics* **13**(2): 121-142.
Guo X, Post T, Wong WK and Zhu LX (2014). Moment conditions for almost stochastic dominance. *Economics Letters* **124**(2): 163–167.
Hadar J and Russell WR (1969). Rules for ordering uncertain prospects. *American Economic Review* **59**(1): 25-34.
Hanoch G and Levy H (1969). The efficiency analysis of choices involving risk. *Review of Economic Studies* **36**(3): 335-346.
Harter-Dreiman M (2004). Drawing inferences about housing supply elasticity from house price responses to income shocks. *Journal of Urban Economics* **55**(2): 316-337.
Henderson JV and Ioannides Y (1987). Owner occupancy: Consumption vs. investment demand. *Journal of Urban Economics* **21**(2):228-241.
Hiebert P and Sydow M (2011). What drives returns to Euro area housing? Evidence from a dynamic dividend–discount model. *Journal of Urban Economics* **70**(2): 88-98.
Ihlanfeldt K and Mayock T (2012). Information, search, and house prices: revisited. *Journal of Real Estate Finance and Economics* **44**(1-2): 90-115.
Jarrow R (1986).The relationship between arbitrage and first order stochastic dominance. *Journal of Finance* **41**(4): 915-921.
Joy OM and Porter RB (1974). Stochastic dominance and mutual fund performance. *Journal of Financial and Quantitative Analysis* **9**(1): 24-31.
Kallberg JG, Liu CH and Greig DW (1996). The role of real estate in the portfolio allocation process. *Real Estate Economics* **24**(3): 359-359.
Krainer J (2001). A theory of liquidity in residential real estate markets. *Journal of Urban Economics* **49**(1): 32-53.
Leamer EE (2002). Bubble trouble? Your home has a P/E ratio too. *UCLA Anderson*

Forecast June.
Leshno M and Levy H (2002). Preferred by "all"" and preferred by "most" decision makers: Almost stochastic dominance. *Management Science* **48**(8): 1074-1085.
Levy H (1992). Stochastic dominance and expected utility: Survey and analysis. *Management Science* **38**(4): 555–593.
Levy M and Levy H (2002). Prospect Theory: Much ado about nothing? *Management Science* **48**(10): 1334-1349.
Levy H and Sarnat M (1970). Alternative efficiency criteria: An empirical analysis. *Journal of Finance* **25**(5): 1153-1158.
Lin Z and Liu Y (2008). Real estate returns and risk with heterogeneous investors. *Real Estate Economics* **36**(4): 753-776.
Linton O, Maasoumi E and Whang Y-J (2005). Consistent testing for stochastic dominance under general sampling schemes. *Review of Economic Studies* **72**(3): 735-765.
McMillen DP (2008). Changes in the distribution of house prices over time: structural characteristics, neighborhood, or coefficients? *Journal of Urban Economics* **64**(3): 573-589.
Paciorek A and Sinai T (2012). Does home owning smooth the variability of future housing consumption? *Journal of Urban Economics* **71**(2): 244-257.
Porter RB (1973). An empirical comparison of stochastic dominance and mean-variance portfolio choice criteria. *Journal of Financial and Quantitative Analysis* **8**(4): 587-608.
Qiao Z, Clark E and Wong, WK (2012). Investors' preference towards risk: evidence from the Taiwan stock and stock index futures markets. *Accounting & Finance* DOI: 10.1111/j.1467-629X.2012.00508.x.
Qiao Z and Wong, WK (2015). Which is a better investment choice in the Hong Kong residential property market: a big or small property? Applied Economics 47(16), 1670-1685.
Topel R and Rosen S (1988). Housing investment in the United States. *Journal of Political Economy* **96**(4): 718-740.
Tsang,CK, Gao, JJ, Li, XL and Wong, WK (2012). Big house or small house, which one should we buy? Evidence from Hong Kong, Social Science Research Network Working Paper Series 2182403.
Turnbull GK, Dornbrow JD and Simans CF (2008). Big house, little house: Relative size and value. *Real Estate Economics* **34**(3): 439-456.
Wong WK, Phoon KF and Lean HH (2008). Stochastic dominance analysis of Asian hedge funds. *Pacific-Basin Finance Journal* **16**(3): 204-223.
Xiao, Q and Liu, Y (2010). The residential market of Hong Kong: rational or irrational? *Applied Economics* **42**: 923–33.
Ziering B and McIntosh W (2000). Property size and risk: Why bigger is not always better. *Journal of Property Management* **5**(2): 62-70.

Zimmer DM (2012). The role of copulas in the housing crisis. *The Review of Economics and Statistics* **94**(2): 607-620.

Table 1 Descriptive statistics of rental yield

	Class				
	A	B	C	D	E
Mean (%)	**0.412%**	0.346%	0.326%	0.309%	**0.280%**
Std Dev (%)	**0.083%**	**0.063%**	0.074%	0.079%	0.080%
Maximum (%)	0.540%	0.460%	0.470%	0.460%	0.450%
Minimum (%)	0.260%	0.230%	0.220%	0.200%	0.180%
Skewness	-0.196	-0.026	0.259	0.347	0.452
Kurtosis	-1.038	-1.014	-1.137	-1.145	-1.129
Jarque–Bera	9.237***	7.733***	11.716***	13.452***	15.692***

Note: Classes A and Bare the "most outstanding classes" in which Class A has the highest monthly mean yield (0.412%); Class B has the smallest standard deviation (0.063%); and Class E has the smallest monthly mean yield (0.280%);Class A has the largest standard deviation (0.083%). Results in bold are extreme values. Readers may refer to Section 3 for information on the different classes.

*, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table 2 Results of the Davidson-Duclos (DD) test of rental yield for risk averters

Sample	FSD		SSD		TSD	
	$\%T_1>0$	$\%T_1<0$	$\%T_2>0$	$\%T_2<0$	$\%T_3>0$	$\%T_3<0$
Class A – Class B	0	93	0	96	0	96
Class A – Class C	0	94	0	96	0	94
Class A – Class D	0	94	0	97	0	97
Class A – Class E	0	94	0	97	0	99
Class B – Class C	0	50	0	95	0	93
Class B – Class D	0	65	0	96	0	96
Class B – Class E	0	89	0	99	0	99
Class C – Class D	0	33	0	96	0	96
Class C – Class E	0	90	0	96	0	99
Class D – Class E	0	74	0	99	0	99

Note: The DD test statistics are computed over a grid of 100 on monthly yields. The table reports the percentage of DD statistics that are significantly negative or positive at the 5% significance level, based on the simulated critical value recommended by Bai *et al.* (2011). $T_j$is the Davidson and Duclos (DD) statistic for risk-averters with *j=1,2,*and *3* defined in equation (2)with F being the first series and G being the second series stated in the first column.

Table 3 Pairwise comparison between rental yields by the Davidson-Duclos (DD) tests

Class	A	B	C	D	E
A		FSD	FSD	FSD	FSD
B	NS		FSD	FSD	FSD
C	NS	NS		FSD	FSD
D	NS	NS	NS		FSD
E	NS	NS	NS	NS	

Notes: The results in this table are read based on row versus column. For example, the cell in row A and column B tells us that Class A stochastically dominates Class B at first-order SD, while the cell in row B and column A means that Class B does not stochastically dominate Class A.
*NS: no stochastic dominance, FSD: first-order stochastic dominance.

Table 4 Descriptive statistics of total yield

	Class				
	A	B	C	D	E
Mean (%)	**0.952%**	0.815%	**0.813%**	0.837%	0.843%
Std Dev (%)	**2.294%**	2.299%	2.620%	2.637%	**3.063%**
Maximum (%)	7.730%	7.220%	9.520%	10.760%	9.600%
Minimum (%)	-7.030%	-7.860%	-8.780%	-9.560%	-9.230%
Skewness	-0.205	-0.309	0.047	-0.028	-0.129
Kurtosis	0.717	1.158	2.073	2.504	0.965
Jarque–Bera	5.116*	12.922***	32.300***	47.055***	7.482**

Note: Classes A is the "most outstanding class" in which Class A has the highest monthly mean yield (0.952%) and the smallest standard deviation (2.294%); and Class Chas the smallest monthly mean yield (0.813%); Class E has the largest standard deviation (3.063%). Results in bold are extreme values. Readers may refer to Section 3 for information on the different classes.
*, **, *** denote significance at the 10%, 5%, and 1% levels, respectively.

Table 5 Results of the Davidson-Duclos (DD) test of total yield for risk averters

Sample	FSD		SSD		TSD	
	$\%T_1$>0	$\%T_1$<0	$\%T_2$>0	$\%T_2$<0	$\%T_3$>0	$\%T_3$<0
Class A – Class B	0	0	0	0	0	0
Class A – Class C	0	0	0	0	0	16
Class A – Class D	0	0	0	0	0	1
Class A – Class E	0	0	0	30	0	48
Class B – Class C	0	0	0	0	0	0
Class B – Class D	0	0	0	0	0	0
Class B – Class E	1	1	0	4	0	40
Class C – Class D	0	0	0	0	0	0
Class C – Class E	0	0	0	0	0	0
Class D – Class E	0	0	0	0	0	0

Note: The DD test statistics are computed over a grid of 100 on monthly yields. The table reports the percentage of DD statistics that are significantly negative or positive at the 5% significance level, based on the simulated critical value recommended by Bai *et al.* (2011). T_j is the Davidson and Duclos (DD) statistic for risk-averters with *j=1,2,* and *3* defined in equation (2) with F being the first series and G being the second series stated in the first column.

Table 6 Pairwise comparison between total yields by the Davidson-Duclos (DD) tests

Class	A	B	C	D	E
A		NS	TSD	TSD#	SSD
B	NS		NS	NS	SSD
C	NS	NS		NS	NS
D	NS	NS	NS		NS
E	NS	NS	NS	NS	

Notes: The results in this table are read based on row versus column. For example, the cell in row A and column B tells us that Class A stochastically dominates Class B at first-order SD, while the cell in row B and column A means that Class B does not stochastically dominate Class A.
*NS: no stochastic dominance, FSD, SSD, TSD: first-order, second-order, third-order stochastic dominance.
# indicates it is marginally SD.

Figure 1 Time-series plot of rental yields from all classes

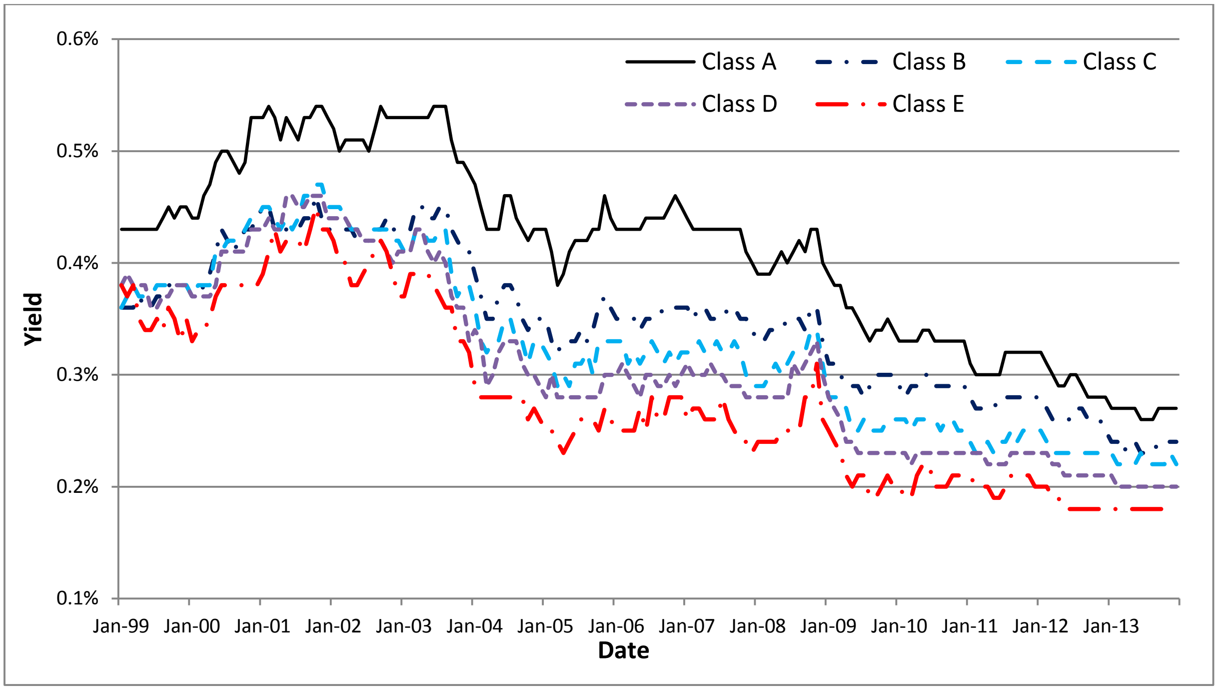


Figure 2 Density plot of rental yields from all classes

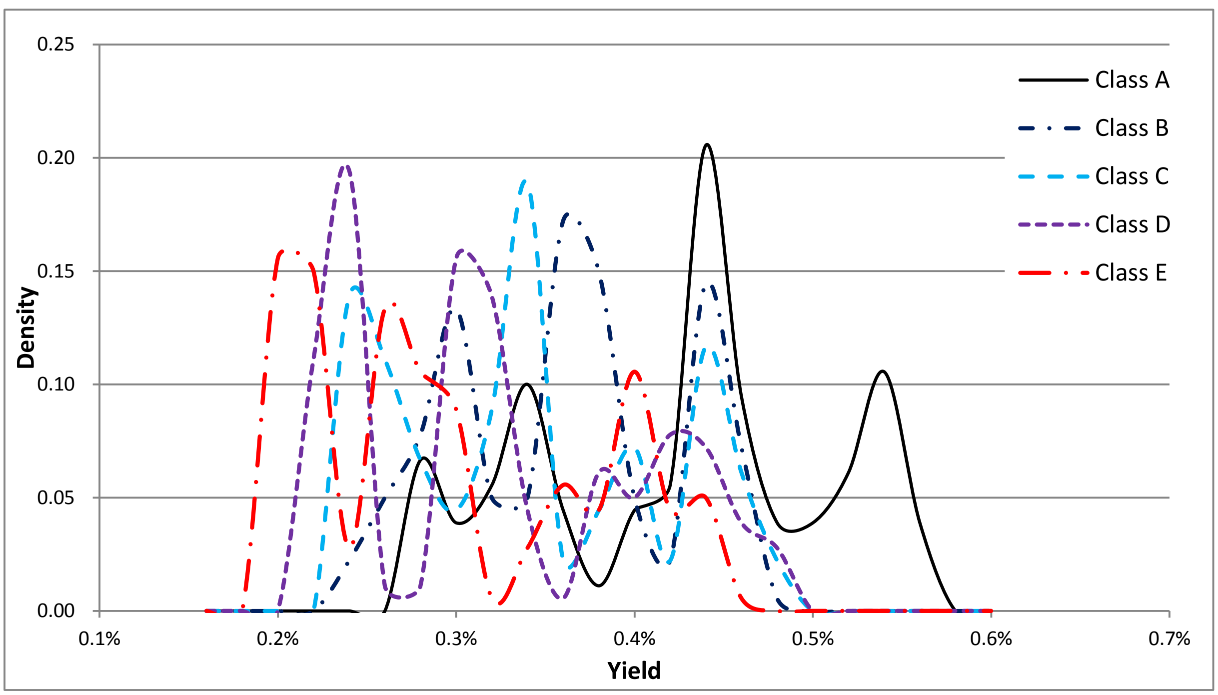


Figure 3 Cumulative distribution plot of rental yields from all classes

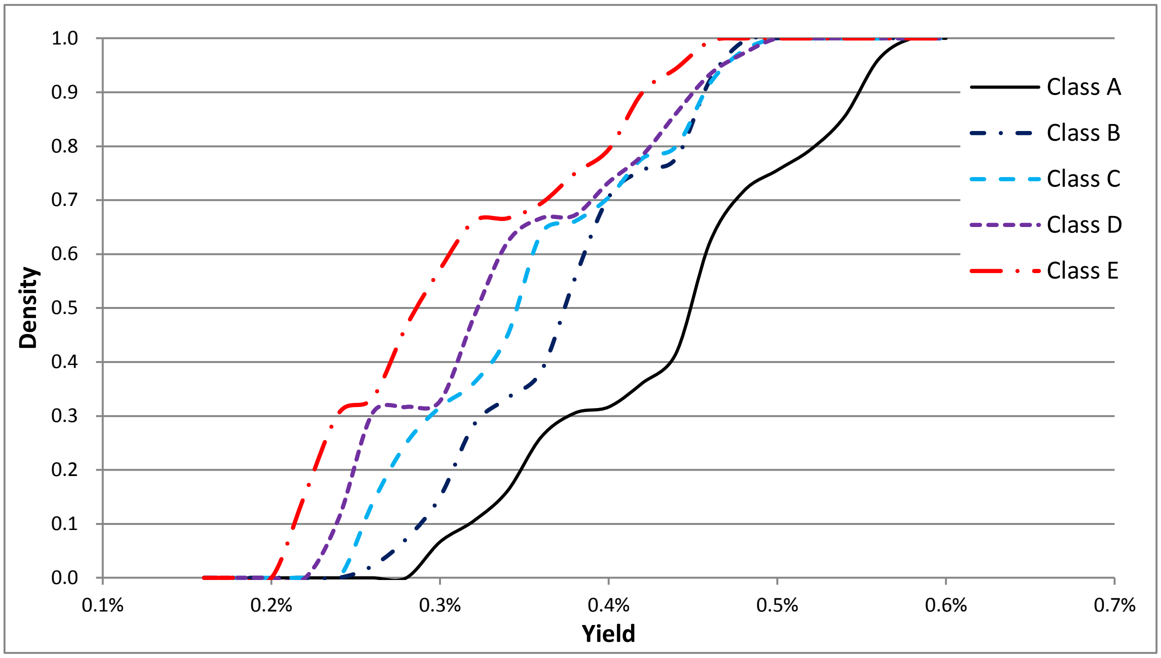
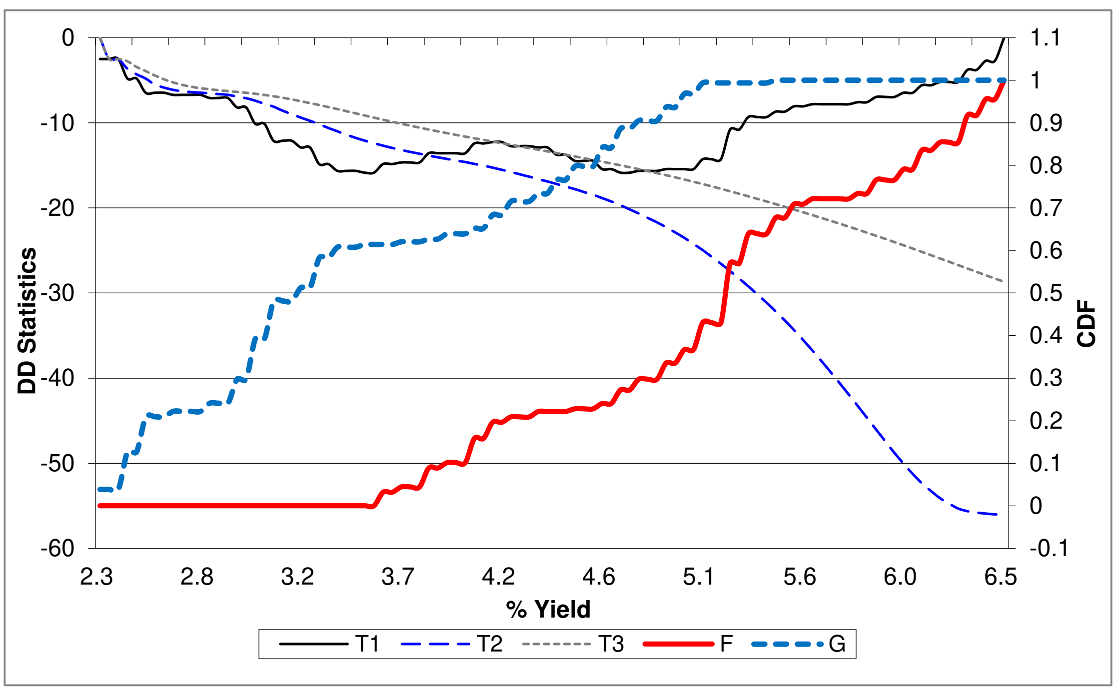


Figure4 SD test statistics and the distribution functions of rental yields from classes A and E



Note: T_j is the test statistic defined in (2) for $j = 1;$ 2, and 3 with F = yield from *Class A* and G = yield from *Class E*.

Figure 5 Time-series plot of total yields from all classes

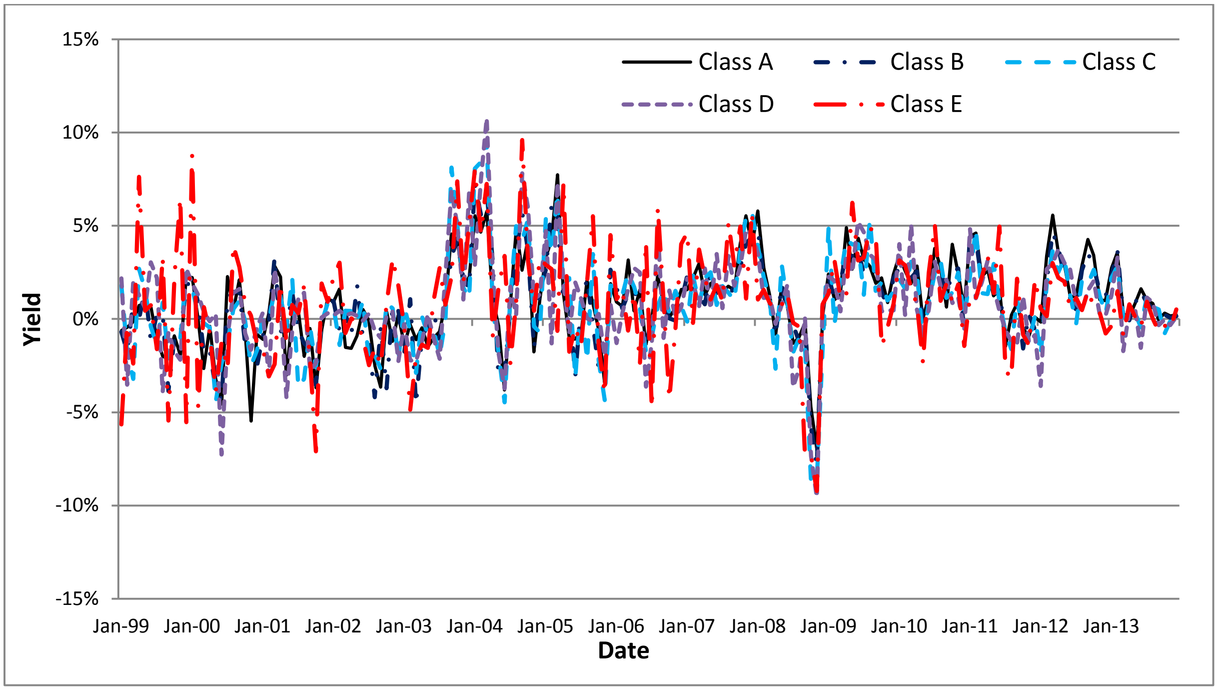


Figure 6 Density plot of total yields from all classes

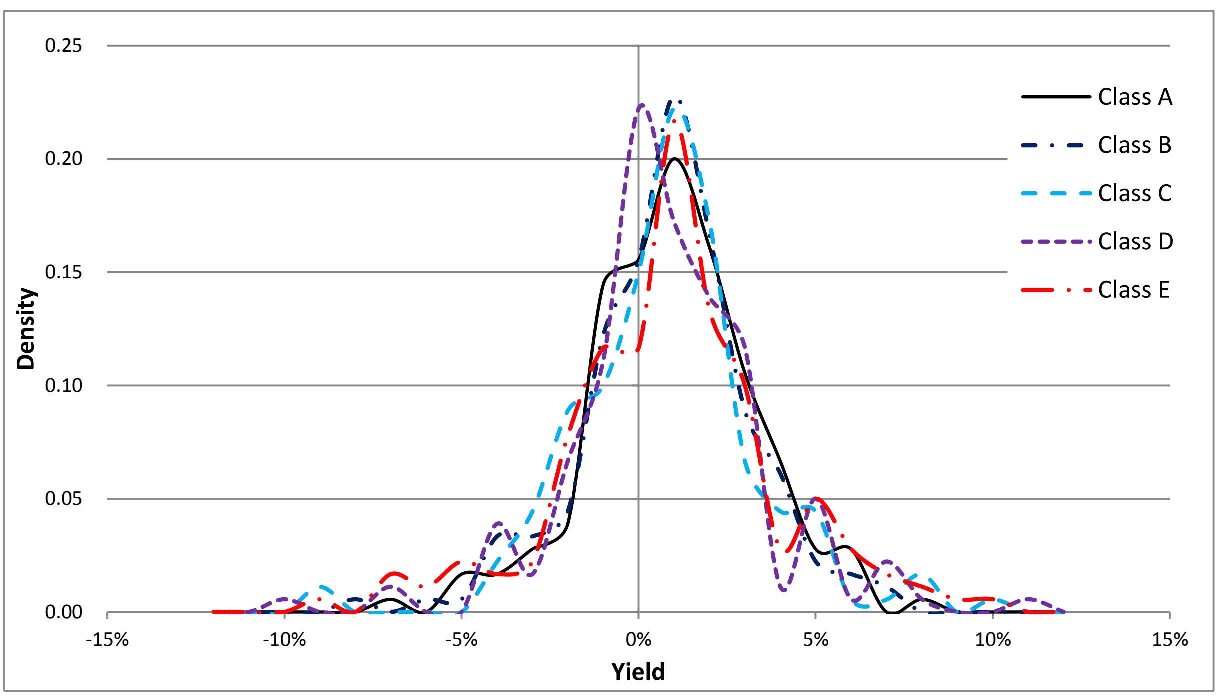


Figure 7 Cumulative distribution plot of total yield from all classes